Non-perturbative generation of elementary fermion mass: a numerical study

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- The existence of a non-perturbative fermion mass generation mechanism was conjectured in R. Frezzotti, G. Rossi Phys. Rev. D92 (2015)
- We test this conjecture in the "simplest" appropriate d = 4 "toy model": may be the first non-perturbative simulation of gauge + scalars + fermions S. Capitani *et al.*, Phys. Rev.Lett. 123 (2019)

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}{\rm Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big] + \frac{m_\phi^2}{2}{\rm Tr}\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big({\rm Tr}\big[\Phi^\dagger\Phi\big]\big)^2$$
$$+\overline{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \overline{Q}_R\gamma_\mu\mathcal{D}_\mu Q_R + \eta\left(\overline{Q}_L\Phi Q_R + h.c.\right)$$
$$+\frac{b^2}{2}\rho\big(\overline{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + h.c.\big), \qquad \Phi \equiv \varphi_0 1\!\!1 + i\tau_i\varphi_i$$

- "Wilson-like" $\propto \rho$ (naively irrelevant)
- UV cutoff $\sim b^{-1}$

$$\mathcal{L}_{\rm toy}(Q,A,\Phi) = \frac{1}{4}F^a_{\mu\nu}F^a_{\mu\nu} + \frac{1}{2}{\rm Tr}\big[\partial_\mu\Phi^\dagger\partial_\mu\Phi\big] + \frac{m_\phi^2}{2}{\rm Tr}\big[\Phi^\dagger\Phi\big] + \frac{\lambda_0}{4}\big({\rm Tr}\big[\Phi^\dagger\Phi\big]\big)^2$$
$$+\overline{Q}_L\gamma_\mu\mathcal{D}_\mu Q_L + \overline{Q}_R\gamma_\mu\mathcal{D}_\mu Q_R + \frac{b^2}{2}\rho\big(\overline{Q}_L\overleftarrow{\mathcal{D}}_\mu\Phi\mathcal{D}_\mu Q_R + h.c.\big)$$
$$+\eta\,\big(\overline{Q}_L\Phi Q_R + h.c.\big), \qquad \Phi \equiv [\varphi| - i\tau^2\varphi^*]$$

- Symmetries & power counting $\begin{pmatrix}\text{in suitable}\\ \text{UV-regul.}\end{pmatrix}$ $\Longrightarrow$ Renormalizability
- Invariant under χ (global) SU(2)$_L\times$SU(2)$_R$ transformations

$$\bullet\,\chi_{L,R}: \quad (\Phi\to\Omega_L\Phi)\otimes\tilde{\chi}_L \quad \text{and/or} \quad (\Phi\to\Phi\Omega_R^\dagger)\otimes\tilde{\chi}_R$$
$$\tilde{\chi}_{L,R}: \begin{cases} Q_{L,R}\to\Omega_{L,R}Q_{L,R} \\ \overline{Q}_{L,R}\to\overline{Q}_{L,R}\Omega^\dagger_{L,R}\end{cases} \qquad \Omega_{L,R}\in \text{SU}(2)_{L,R}$$

- χ invariance forbids $\frac{1}{b}\overline{Q}Q$ terms and softens power like U.V. divergences
- Fermionic chiral transformations $\tilde{\chi}$ are not a symmetry if $(\rho,\eta)\neq(0,0)$

- Purely fermionic $\tilde{\chi}_L$ tranformations yield bare Schwinger Dyson Eq.s (SDEs)

$$\partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\, \hat{\mathcal{O}}(0)\rangle = -\eta\, \langle O_{Yuk}^{L,i}(x)\, \hat{\mathcal{O}}(0)\rangle - b^2 \langle O_{Wil}^{L,i}(x)\; \hat{\mathcal{O}}(0)\rangle \qquad x \neq 0$$

$$\tilde{J}_\mu^{L,i} = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$O_{Yuk}^{L,i} = \Big[\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \text{h.c.}\Big] \qquad O_{Wil}^{L,i} = \frac{\rho}{2}\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \text{h.c.}\Big]$$

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$$\tilde{J}^{L,i}_\mu = \overline{Q}_L \gamma_\mu \frac{\tau^i}{2} Q_L - \frac{b^2}{2}\rho\Big(\overline{Q}_L \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \overline{Q}_R \overleftarrow{\mathcal{D}}_\mu \Phi^\dagger \frac{\tau^i}{2} Q_L\Big)$$

$$O^{L,i}_{Yuk} = \Big[\overline{Q}_L \frac{\tau^i}{2}\Phi Q_R - \text{h.c.}\Big] \qquad O^{L,i}_{Wil} = \frac{\rho}{2}\Big[\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \frac{\tau^i}{2}\Phi \mathcal{D}_\mu Q_R - \text{h.c.}\Big]$$

- Mixing of $O^{L,i}_{Wil}$ under renormalization

$$b^2 O^{L,i}_{Wil} = (Z_{\partial\tilde{J}} - 1)\partial_\mu \tilde{J}^{L,i}_\mu - \overline{\eta}(\eta; g_s^2, \rho, \lambda_0)\, O^{L,i}_{Yuk} + \underline{\ldots} + \mathrm{O}(b^2)$$

- Renormalized SDEs at $x \neq 0$ read

$$Z_{\partial\tilde{J}}\partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0)\rangle = -(\eta - \overline{\eta}(\eta))\, \langle O^{L,i}_{Yuk}(x)\, \hat{\mathcal{O}}(0)\rangle + \underline{\ldots} + \mathrm{O}(b^2)$$

where the ellipses ($\underline{\ldots}$) stand for possible NP mixing contributions

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where the ellipses ($\underline{\ldots}$) stand for possible NP mixing contributions

- At $\eta = \eta_{cr}$ such that

$$\eta_{cr} - \bar{\eta}(\eta_{cr}, g_s, \rho, \lambda_0) = 0$$

the current $\tilde{J}^{L,i}_\mu$ is conserved up to $O(b^2)$ and possible NP mixing contributions ($\underline{\ldots}$)

- At $\eta = \eta_{cr}$ the SDE equation

$$Z_{\partial \tilde{J}} \partial_\mu \langle \tilde{J}^{L,i}_\mu(x)\, \hat{\mathcal{O}}(0) \rangle = O(b^2) + \ldots$$

- At $\eta = \eta_{cr}$ the SDE equation

$$Z_{\partial \tilde{J}} \partial_\mu \langle \tilde{J}^{L,i}_\mu(x) \, \hat{\mathcal{O}}(0) \rangle = O(b^2) + \ldots$$

Wigner phase $\big(\langle \Phi \rangle = 0\big)$

$$Z_{\partial \tilde{J}} \partial_\mu \langle \tilde{J}^{L,i}_\mu(x) \, \hat{\mathcal{O}}(0) \rangle = O(b^2)$$

$$M_{PS} = 0$$

- At $\eta = \eta_{cr}$ the SDE equation

$$Z_{\partial\tilde{J}}\partial_\mu\langle \tilde{J}^{L,i}_\mu(x)\,\hat{\mathcal{O}}(0)\rangle = O(b^2) + \underline{\ldots}$$

Wigner phase $\big(\langle\Phi\rangle = 0\big)$	conjecture Nambu-Goldstone phase $\big(\langle\Phi\rangle = v\big)$
$Z_{\partial\tilde{J}}\partial_\mu\langle \tilde{J}^{L,i}_\mu(x)\,\hat{\mathcal{O}}(0)\rangle = O(b^2)$	$Z_{\partial\tilde{J}}\partial_\mu\langle \tilde{J}^{L,i}_\mu(x)\,\hat{\mathcal{O}}(0)\rangle = O(c_1\Lambda_s) + O(b^2)$
$M_{PS} = 0$	thus $M_{PS} \neq 0$

- At $\eta = \eta_{cr}$ the SDE equation

$$Z_{\partial\tilde{J}}\partial_\mu\langle\tilde{J}^{L,i}_\mu(x)\,\hat{\mathcal{O}}(0)\rangle = O(b^2) + \underline{\ldots}$$

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- In the NG phase, the occurrence of $\propto c_1\Lambda_s$ in the SDEs is equivalent to a NP term in the Low Energy Effective Lagrangian

$$\mathcal{L}^{LEEL}_{4,NG} \supset c_1\Lambda_s[\overline{Q}_L\mathcal{U}Q_R + \text{hc}], \quad \mathcal{U} = \frac{\Phi}{\sqrt{\Phi^\dagger\Phi}} = \frac{(v+\sigma+i\vec{\tau}\vec{\varphi})}{\sqrt{(v+\sigma)^2+\vec{\varphi}\vec{\varphi}}} = \mathbb{1} + i\frac{\vec{\tau}\vec{\varphi}}{v} + \ldots$$

- The term $c_1\Lambda_s[\overline{Q}_L\mathcal{U}Q_R + \text{hc}]$ contains a quark mass

$$c_1\Lambda_s[\overline{Q}_L\mathcal{U}Q_R + \text{hc}] \supset c_1\Lambda_s\overline{Q}_L Q_R \begin{cases} \text{Natural mass} \\ \neq \text{Yukawa term} \\ c_1 = O(\alpha_s^2) \rightarrow \text{Hierarchy} \end{cases}$$

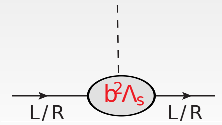
Simulations strategy to check the occurrence of the mechanism:

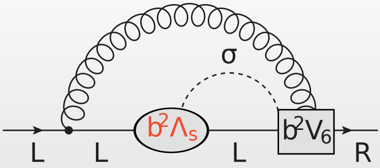
- We regularise the toy model on the lattice
- Renormalized SDEs at $x \neq 0$ read

$$Z_{\partial \tilde{J}} \partial_\mu \langle \tilde{J}_\mu^{L,i}(x)\, \hat{\mathcal{O}}(0) \rangle = -(\eta - \overline{\eta}(\eta))\, \langle O_{Yuk}^{L,i}(x)\, \hat{\mathcal{O}}(0) \rangle + \ldots + \mathrm{O}(b^2)$$

- In Wigner phase ($\langle \phi \rangle = 0$): We look for the critical value $\eta = \eta_{cr}$ of the Yukawa bare parameter
- In Nambu-Goldstone phase ($\langle \phi \rangle = v$): At $\eta = \eta_{cr}$ we measure M_{PS} and the PCAC mass
- Simulations at three values of the lattice spacing

- Quenched approximation sufficient R. Frezzotti, G. Rossi Phys. Rev. D92 (2015) .
- NP effective vertices $\supset b^2\Lambda_s\sigma\overline{Q}\not{D}Q$ with $\Phi = (v+\sigma+i\vec{\tau}\vec{\varphi})$ combined with vertices coming from the Wilson-like term as $b^2\overline{Q}\overleftarrow{D}\Phi DQ \Rightarrow$ NP mass





$\sim \Lambda_S b^0$

- Lattice regularization of $\int d^4x \mathcal{L}_{toy}$
- "Naive" fermions (convenient in the valence) with $\tilde{\chi}$ breaking terms

$$\mathcal{S}_{lat} = b^4 \sum_x \left\{ \mathcal{L}_{kin}^{YM}[U] + \mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) + \sum_g \overline{\Psi}_g D_{lat}[U,\Phi]\Psi_g \right\}$$

$\mathcal{L}_{kin}^{YM}[U]$: SU(3) plaquette action

$$\mathcal{L}_{kin}^{sca}(\Phi) + \mathcal{V}(\Phi) = \frac{1}{2}\,\mathrm{tr}\,[\Phi^\dagger(-\partial_\mu^*\partial_\mu)\Phi] + \frac{m_\phi^2}{2}\,\mathrm{tr}\left[\Phi^\dagger\Phi\right] + \frac{\lambda_0}{4}\left(\mathrm{tr}\left[\Phi^\dagger\Phi\right]\right)^2$$

where $\Phi = \varphi_0 \mathbb{1} + i\varphi_j\tau^j$ and $F(x) \equiv [\varphi_0\mathbb{1} + i\gamma_5\tau^j\varphi_j](x)$

$$(D_{lat}[U,\Phi]\Psi)(x) = \gamma_\mu \widetilde{\nabla}_\mu \Psi(x) + \eta F(x)\Psi(x) - b^2\frac{\rho}{2}F(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) +$$

$$-b^2\rho\frac{1}{4}\left[(\partial_\mu F)(x)U_\mu(x)\widetilde{\nabla}_\mu\Psi(x+\hat{\mu}) + (\partial_\mu^* F)(x)U_\mu^\dagger(x-\hat{\mu})\widetilde{\nabla}_\mu\Psi(x-\hat{\mu})\right]$$

- Wilson-like ($d=6$) term $\propto \rho$ does not remove the doublers, setting $F = v\mathbb{1}$
  $-b^2\rho\frac{\rho}{2}v\overline{\Psi}(x)\widetilde{\nabla}_\mu\widetilde{\nabla}_\mu\Psi(x) \supset \frac{\rho}{8}v\overline{\Psi}(x)\Psi(x)$ no diverging mass of the doublers

Properties of the lattice action in S. Capitani *et al.*, Phys.Rev.Lett. 123 (2019)

- Consider two generations of valence fermions $\overline{\Psi}_\ell D_{latt} \Psi_\ell + \overline{\Psi}_h D_{latt} \Psi_h$ in order to have valence correlators involving no fermionic disconnected diagrams

  e.g. $\langle \overline{\Psi}_\ell(x) \Gamma \tau \Psi_h(x) \overline{\Psi}_h(y) \Gamma \tau \Psi_\ell(y) \rangle \qquad \ell = (u, d) \quad h = (c, s)$

- Wilson-like $(d = 6)$ term $\propto \rho$ does not remove the doublers
- Flavour content: (16 doublers) $\times$ (2 isospin)$\times$ (2 generations)
- Spectrum Doubling Symmetry $\Longrightarrow$ at $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all tastes up to cutoff effects

Gauge and scalar Sector

- Quenched $(U, \Phi) \implies$ exceptional configurations: at large $|\eta|$ and $|\rho|$ enhanced by Φ fluctuations
- Add twisted mass term: $S_{lat}^{toy+tm} = S_{lat} + i\mu b^4 \sum_x \overline{\Psi}\gamma_5\tau_3\Psi$
  control over exceptional confs. at the price of a soft breaking of $\chi_{L,R}$ (and $\widetilde{\chi}_{L,R}$ when restored at η_{cr}), safe $\mu \to 0$ extrapolation

Simulation details and Renormalization

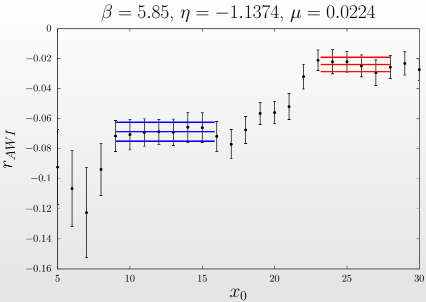
- Quenched lattice study: independent generations and renormalization of U and Φ
- In the fermionic sector renormalised Yukawa coupling vanishes at $\eta = \eta_{cr}$
- Gauge coupling renormalized keeping $r_0 \sim 0.5 fm$
  S. Necco, R. Sommer Nucl. Phys. B622 (2002)
  $\beta = 5.75$ ($b = 0.15$ fm), $\beta = 5.85$ ($b = 0.12$ fm) & $\beta = 5.95$ ($b = 0.10$ fm)
- Scalars parameters m_ϕ^2, λ_0 fixed by the renormalization condition
  $m_\sigma^2 r_0^2 = 1.285$ and $\lambda_R = \frac{m_\sigma^2}{2v_R^2} = 0.4408$
- Wilson-like coupling $\rho \sim 1.96$: free parameter since we are only interested to see whether the mechanism exists (in quenched approximation ρ relevant for the magnitude of the NP mass)
- In Wigner phase we keep fixed $(m_\phi^2 - m_{cr}^2)r_0^2 = 1.22$

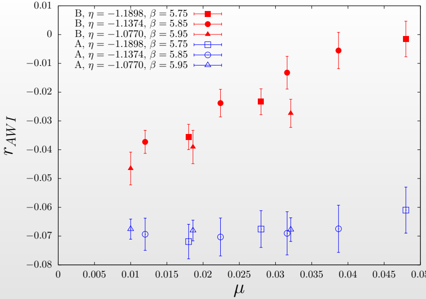
Wigner phase $\langle\phi\rangle = 0$

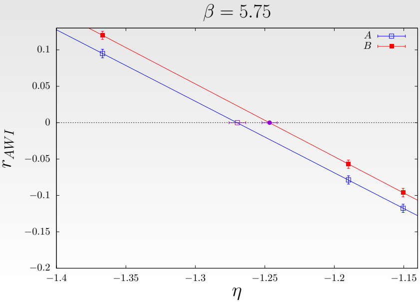
- find η_{cr} where $\tilde{J}_0^{A,i} = \overline{\Psi}\gamma_\mu\gamma_5\frac{\tau^i}{2}\Psi\Big|_{1pt.\,split}$ is conserved

$$r_{AWI}(\eta; g_s^2, \lambda_0, \rho, \mu)|_{\eta_{cr}} = \frac{\sum_{\vec{x}}\sum_{\vec{y}}\langle P^1(0)[\partial_0 \tilde{J}_0^{A1}](x)\phi^0(y)\rangle}{\sum_{\vec{x}}\sum_{\vec{y}}\langle P^1(0)D^{P1}(x)\phi^0(y)\rangle}\Bigg|_{\eta_{cr}}^{\mu\to 0} = 0\,, \quad y_0 = x_0 + \tau$$

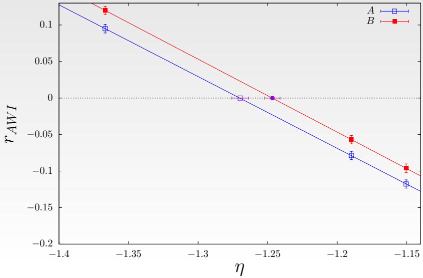
$$D^{P1} = \overline{\Psi}_L\left\{\Phi, \frac{\tau^\pm}{2}\right\}\Psi_R - h.c., \quad \frac{\tau}{fm} \sim 0.6 \;\; \frac{x_0}{fm} \overset{A}{\sim} [0.9, 1.8] \;\; \frac{x_0}{fm} \overset{B}{\sim} [2.7, 3.3]$$



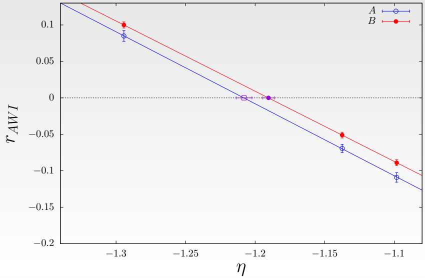



$\beta = 5.75$
A
B
0.1
0.05
0
−0.05
−0.1
−0.15
−0.2
−1.4
−1.35
−1.3
−1.25
−1.2
−1.15
r_{AWI}
η

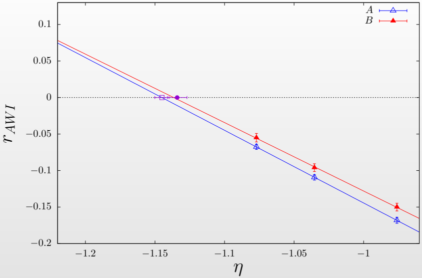
$\beta = 5.75$



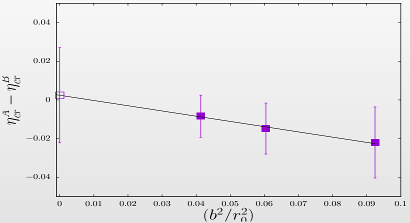
$\beta = 5.85$



$\beta = 5.95$

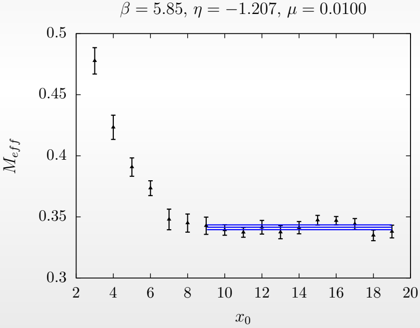


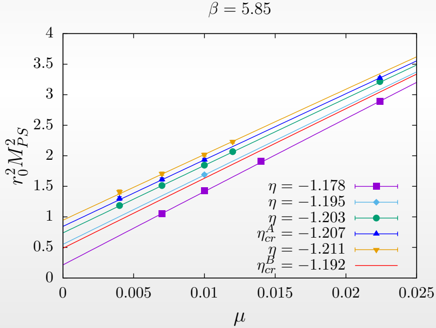
β	5.75	5.85	5.95
η_{cr}^{A}	-1.271(10)	-1.207(8)	-1.145(6)
η_{cr}^{B}	-1.249(8)	-1.192(6)	-1.136(6)



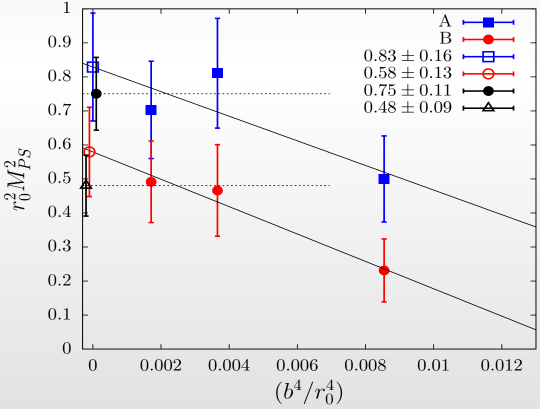
Nambu-Goldston phase $\langle\phi\rangle = v$

- Calculate M_{PS} at $\eta = \eta_{cr}$ and $\mu \to 0$
- Global fit of $M_{PS}^2 = a + b\mu + c\eta + d\mu^2 + e\eta^2 + f\eta\mu$



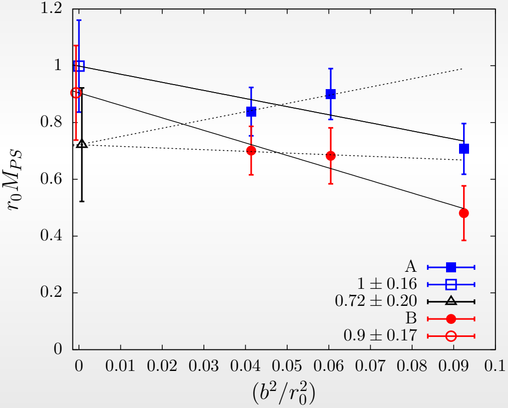


- If we assume the no mechanism hypothesis $Z_{\tilde{J}}\partial_\mu\langle \tilde{J}^{A,i}_\mu(x)\,\hat{\mathcal{O}}(0)\rangle = O(b^2)$ then $M^2_{PS} = O(b^4) \Rightarrow$ à la Symanzik: $\mathcal{L}^{EFF}_4 + b^2\mathcal{L}^{EFF}_6 + \ldots$
- We simulate at $\eta = \eta_{cr}$ and $\mu > 0$ and extrapolate to $\mu \to 0^+$ thus twist angle $\omega = \pi/2$. All terms in $b^2\mathcal{L}^{EFF}_6$ are either parity odd or $\tilde{\chi}$ invariant $\lim_{\mu\to 0^+}\langle PS|\mathcal{L}^{EFF}_6|PS\rangle(\mu) = 0$



No mechanism hypothesis
NOT supported by data

- Check occurrence of the mechanism: $r_0 M_{PS}(\mu \to 0^+)$ vs b^2/r_0^2
  - M_{PS} at η_{cr}^A and η_{cr}^B give consistent continuum limit
  - Dotted lines, combined fit imposing common continuum limit & excluding the coarsest lattice



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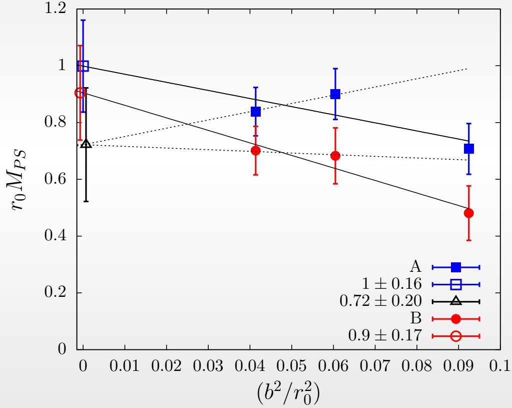

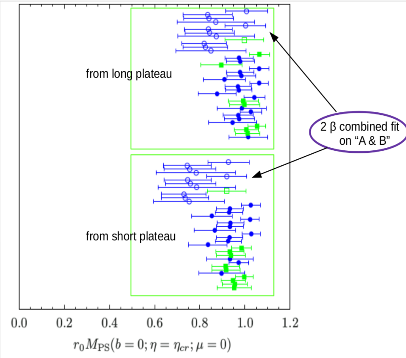

green point have $\chi^2 > 2$

Median over 72 analyses (excluding $\chi^2 > 2$) $r_0 M_{PS} = 0.927 \pm \overset{stat.}{0.094} \pm \overset{sys.}{0.095}$

- WI mass $m_{AWI}(\eta_{cr}) = \left.\frac{\partial_0 \langle \widetilde{J}_0^{A1}(x) P^1(y) \rangle}{\langle P^1(x) P^1(y) \rangle}\right|_{\eta_{cr}}$

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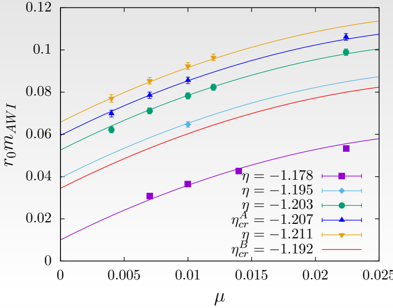
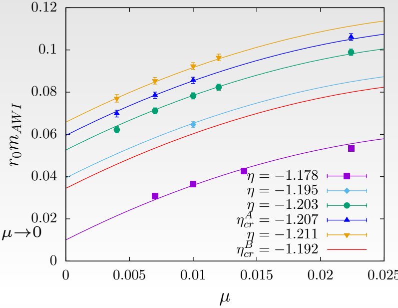
- Global fit $m_{AWI} = a + b\mu + c\eta + d\mu^2$



- WI mass $m_{AWI}(\eta_{cr}) = \frac{\partial_0 \langle \widetilde{J}_0^{A1}(x) P^1(y) \rangle}{\langle P^1(x) P^1(y) \rangle}\Big|_{\eta_{cr}}$
- Global fit $m_{AWI} = a + b\mu + c\eta + d\mu^2$
- Renormalized quantity $r_0^3 m_{AWI} Z_V G_{PS}^W$
- Z_V from $Z_V \langle \widetilde{J}_0^{V2}(x) P^1(y) \rangle\Big|_{\mu\to 0} = 2\mu \langle P^1 P^1 \rangle\Big|_{\mu\to 0}$
- In the Wigner phase $G_{PS}^W = \langle 0|P^1|PS\rangle$



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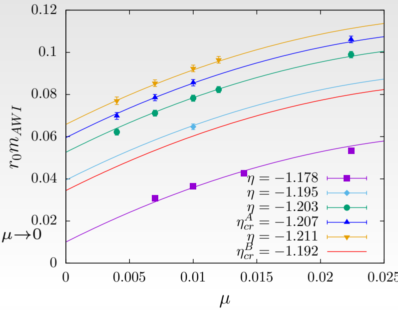
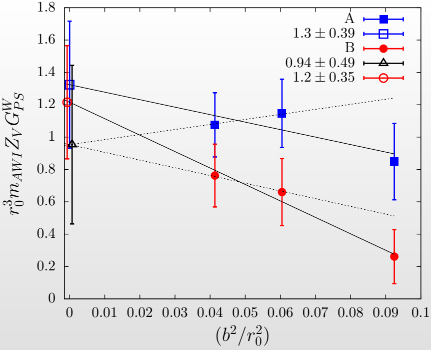




Median over 72 analyses (as for M_{PS} excluding $\chi^2 > 2$)

$$r_0^3 m_{AWI} Z_V G_{PS} = 1.204 \pm \overset{stat.}{0.394} \pm \overset{sys.}{0.193}$$

Conclusions and Remarks

- We have evidence that the mechanism conjectured by R. Frezzotti, G. Rossi Phys. Rev. D92 (2015) is supported by numerical simulation S. Capitani *et al.*, Phys.Rev.Lett. 123 (2019) .
- Other explanations of our data? Possible and searched for: till now none found
- NP mass term in $\tilde{\chi}$ SDEs $\Longleftrightarrow$ NP mass term plus further $\tilde{\chi}$-breaking terms in the LE effective action valid at the scale $\Lambda_S \ll p \ll \Lambda_{UV} \sim b^{-1}$
- $d > 4$ $\tilde{\chi}$-breaking terms control the NP mass
- Fermion mass $\sim \Lambda_S$:
    - expected to be unrelated to v (in the limit $v \gg \Lambda_S$) & vanishing as $v \to 0$
    - natural a‘ la 't Hooft (at least in weak sense:$\tilde{\chi}$ recovery)
- Extension of this mechanism to weak interactions and possible phenomenological under investigation [R. Frezzotti and G. Rossi in preparation]

Thank you for your attention

Backup slides

Independence of η_{cr} from the scalar mass m_ϕ

- $\bar{\eta}$ controls the mixing of the Wilson-like term $b^2\hat{O}_6 = \frac{b^2}{2}\rho\big(\overline{Q}_L \overleftarrow{\mathcal{D}}_\mu \Phi \mathcal{D}_\mu Q_R + h.c.\big)$ with the Yukawa $\hat{O}_{Yuk} = \eta(\overline{Q_L}\Phi Q_R + h.c.)$.

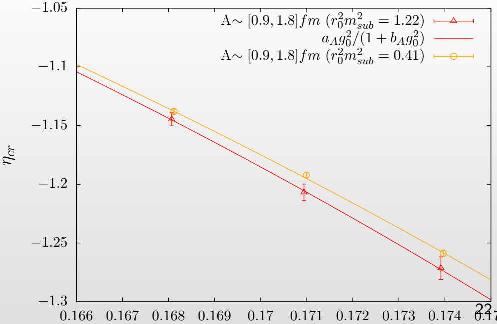
$$\hat{O}_6 = [O_6]_{sub} + \frac{Z_J - 1}{b^2}\tilde{J} + \frac{\bar{\eta}}{b^2}\hat{O}_{Yuk}$$

- The divergence $1/b^2$ in the mixing pattern are independent on the scalar mass m_ϕ^2

$$\frac{1}{p^2+m^2} = \frac{1}{p^2} - \frac{1}{p^2}m^2\frac{1}{p^2} + \frac{1}{p^2}m^2\frac{1}{p^2}m^2\frac{1}{p^2} + \ldots$$

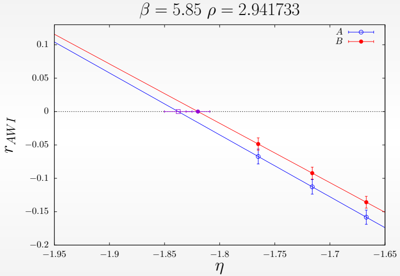
mass insertion lowers the degree of divergence of a diagram

- Numerical test at two values of $r_0^2(m_\phi^2 - m_{cr}^2) = 1.22\,,\ 0.41$

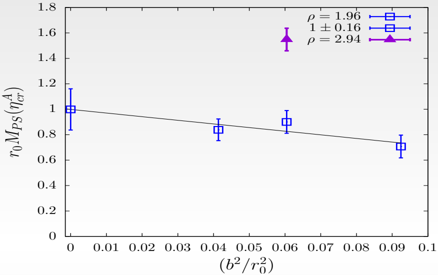


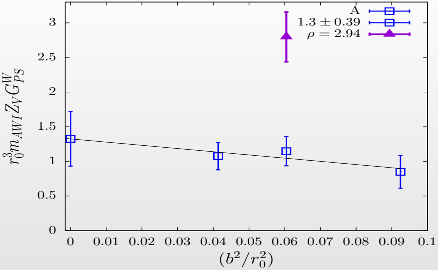
Increasing $\rho : 1.96 \to 2.94$

increase of m_{AWI} (and M_{PS}) with ρ is expected according to R. Frezzotti, G. Rossi ('15) mechanism



$\beta = 5.85$	$\rho = 1.86$	$\rho = 2.94$
η_{cr}^{A}	$-1.207(8)$	$-1.838(13)$
η_{cr}^{B}	$-1.192(6)$	$-1.820(11)$
$r_0 M_{PS}(\eta_{cr}^{A})$	0.90(9)	1.54(9)
$r_0 m_{AWI}(\eta_{cr}^{A})$	1.1(2)	2.8(4)





- Using staggered formalism to analyze naive valence fermions
- $\Psi(x)$ contain 4 replicas $B = 1, .., 4$
  $\Psi(x) = \mathcal{A}_x \chi(x), \quad \mathcal{A}_x = \gamma_1^{x_1}\gamma_2^{x_2}\gamma_2^{x_2}\gamma_2^{x_2}, \quad$ Spin diagonalization of S_{latt} in χ^B basis
- $\chi(x)$ contain 4 tastes a=1,...,4
  $q^B_{\alpha,a}(y) = \frac{1}{8}\sum_\xi \overline{U}(2y, 2y+\xi)[\Gamma_\xi]_{\alpha,a}(1 - b\sum_\mu \xi_\mu \widetilde{\nabla}_\mu)\chi^B(2y+\xi),$
  $q^B_{\alpha,a}(y)$ taste basis, $\quad x_\mu = 2y_\mu + \xi_\mu, \quad \xi_\mu = 0, 1$

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- Flavour content: $\underbrace{(4 \text{ replicas:}B) \times (4 \text{ tastes:}a)\times}_{16 \text{ doublers}}$ (2 isospin)$\times$ generations

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- Following Kluberg-Stern et al. ('83),..., Sharpe et al. ('93), Luo ('96) and adding scalars we get the small b expansion of S^{fer}_{lat} on smooth U,Φ configuration

$$S^{fer}_{lat} = \sum_{y,B} \bar{q}^B(y)\left\{\sum_\mu (\gamma_\mu \otimes \mathbb{1})D_\mu + (\eta - \bar{\eta})\mathcal{F}(y)\right\} q^B(y) + O(b^2)$$

$$\mathcal{F}(y) = \varphi_0(2y)(\mathbb{1}\otimes\mathbb{1}) + S^B i\tau^i\varphi_i(2y)(\gamma_5 \otimes t_5), \; S^A = \pm 1, \text{ taste matrices } t_\mu = \gamma^*_\mu$$

- Quark bilinear in Ψ basis that have the classical continuum limit in q^B basis
- Point split vector current

$$\widetilde{J}_\mu^{V^i}(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu \frac{\tau^i}{2} U_\mu^\dagger(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}_\mu^{V^i}(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu \otimes \mathbb{1})\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Point split axial current

$$\widetilde{J}_\mu^{A^i}(x) = \overline{\Psi}(x-\hat{\mu})\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu(x-\hat{\mu})\Psi(x) + \overline{\Psi}(x)\gamma_\mu\gamma_5 \frac{\tau^i}{2} U_\mu^\dagger(x-\hat{\mu})\Psi(x-\hat{\mu})$$

$$\sum_\xi \widetilde{J}_\mu^{A^i}(2y+\xi) = \sum_{B=1}^{4} \overline{q}^B(y)(\gamma_\mu\gamma_5 \otimes t_5)\frac{\tau^i}{2} q^B(y) + O(b^2)$$

- Loop effects do not generated $d \leq 4$ operator besides $F_{\mu\nu}F_{\mu\nu}$, $\partial_\mu \Phi^\dagger \partial_\mu \Phi$ $q^B(\gamma_\mu \otimes 1\!\!1)\tilde{\nabla} q^B$, $\Phi^\dagger \Phi$, $(\Phi^\dagger \Phi)^2$, $\eta \overline{q}^B(y)\mathcal{F}^B(y)q^B(y)$ (all in S_{lat})

- Argument: In S_{latt} there are only $\widetilde{\nabla}_\mu$ acting on fermions $\Longrightarrow$
  Spectrum Doubling Symmetry :
  $\Psi \to e^{-ix\cdot\pi_H} M_H \Psi$ $\overline{\Psi} \to \overline{\Psi} M_H^\dagger e^{+ix\cdot\pi_H}$, $H = \{\mu_1, ..., \mu_h\}$ ordered,
  16 vectors $\pi_H(\pi_{H,\mu} = \pi$ if $\mu \in H)$ with $M_H = (i\gamma_5\gamma_1)...(i\gamma_5\gamma_{\mu_h})$

- It is a symmetry of S_{latt}, thus also of the $\Gamma_{lat}[U, \Phi, \Psi]$. So the latter can only have terms with symmetric covariant derivatives $\widetilde{\nabla}_\mu$ acting on Ψ.
  Close to the continuum limit among the local terms of Γ_{lat} only those with no or one $\widetilde{\nabla}_\mu$ fermionic derivative are relevant

- At $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all tastes up to cutoff effects

Property of the fermions recap

- Wilson-like $(d = 6)$ term $\propto \rho$ does not remove the doublers
- Flavour content: (16 doublers) $\times$ (2 isospin)$\times$ (2 generations)
- Spectrum Doubling Symmetry $\implies$ at $\eta = \eta_{cr}$ $\widetilde{\chi}$ gets simultaneously restored for all tastes up to cutoff effects

No mechanism hypothesis $M^2_{PS}|_{cont} = 0$

- If we assume the no mechanism hypothesis $Z_{\tilde{J}} \partial_\mu \langle \tilde{J}^{A,i}_\mu(x)\, \hat{\mathcal{O}}(0) \rangle = O(b^2)$ then $M^2_{PS} = O(b^4) \Rightarrow$ à la Symanzik: $\mathcal{L}^{EFF}_4 + b^2 \mathcal{L}^{EFF}_6 + \ldots$

$$M^2_{PS}\big|_L = M^2_{PS}\big|_{cont} + b^2 \langle \mathrm{PS} | \mathcal{L}^{NG}_{6,loc} | \mathrm{PS} \rangle + \mathrm{O}(b^4)\,,$$

$\mathcal{L}^{NG}_{6\ \mathrm{loc}} \supset O^{\tilde{\chi}-\mathrm{inv}}_{6,glu}\ ;\ [\sum_{\Gamma_A \Gamma_B} c_{AB} (\bar{Q}\Gamma_A Q)(\bar{Q}\Gamma_B Q)]^{\tilde{\chi}-\mathrm{inv}}\ ;\ [D_\lambda \bar{Q}_L \Phi D_\lambda Q_R + h.c.]\,,\ \ldots\,;$

- $\langle \mathrm{PS} | \mathcal{O}^{\tilde{\chi}-inv}_{6,loc} | \mathrm{PS} \rangle = O(M^2_{PS}|_{cont})$ for the $\tilde{\chi}$ invariance
- $\langle \mathrm{PS} | [\sum_{\Gamma_A \Gamma_B} c_{AB} (\bar{Q}\Gamma_A Q)(\bar{Q}\Gamma_B Q)]^{\tilde{\chi}-\mathrm{inv}} | \mathrm{PS} \rangle = O(M^2_{PS}|_{cont})$ for the $\tilde{\chi}$ invariance
- In the No Mechanism hypothesis the twist angle is $\omega = \frac{\pi}{2}$, thus the physical basis is defined as $\Psi = \exp(i\frac{\pi}{4}\gamma_5 \tau^3) Q$.
  The term $[D_\lambda \bar{Q}_L \Phi D_\lambda Q_R + h.c.]$ and similar fermionic bilinears are parity odd

- Simulation parameters of the scalar sector
- Renormalization condition in the NG phase

$$m_\sigma^2 r_0^2 = 1.258\,, \quad \lambda_R = \frac{m_\sigma^2}{2v_R^2} = 0.4408\,, \quad Z_\phi = [m_\phi^2 \langle \varphi^0(x)\varphi^0(0)\rangle - V_4 v^2]^{-1}$$

where $G(0)$ is the two point function at zero momentum

$b(fm)$	β	$r_0^2 M_\sigma^2$	λ_{NP}	$b^2 m_\phi^2$	λ_0
0.152	5.75	1.278(4)	0.437(2)	-0.5941	0.5807
0.123	5.85	1.286(4)	0.441(2)	-0.5805	0.5917
0.102	5.95	1.290(5)	0.444(2)	-0.5756	0.6022

κ: code hopping parameter, s.t. $\kappa^{-1} - 2\kappa\lambda_0 - 8 = b^2 m_0^2$

- In Wigner phase we keep fixed $(m_\phi^2 - m_{cr}^2) r_0^2 = 1.22$

β	r_0/b	$(m_\phi^2 - m_{cr}^2) b^2$	$b^2 m_{cr}^2$	$b^2 m_\phi^2$	λ_0	κ
5.75	3.29	0.1119(12)	-0.5269(12)	-0.4150	0.5807	0.129280
5.85	4.06	0.0742(11)	-0.5357(11)	-0.4615	0.5917	0.130000
5.95	4.91	0.0504(10)	-0.5460(10)	-0.4956	0.6022	0.130521

Wigner phase simulations parameters

β	$a^{-4}(L^3 \times T)$	η	$a\mu$	$N_U \times N_\phi$
5.75	$16^3 \times 32$	-1.1505	0.0180 0.0280 0.0480	60x8
($a \sim 0.152$ fm)		-1.1898	0.0180 0.0280 0.0480	
		-1.3668	0.0180 0.0280 0.0480	
5.85	$16^3 \times 40$	-1.0983	0.0224 0.0316 0.0387	60x8
($a \sim 0.123$ fm)		-1.1375	0.0120 0.0172 0.0224 0.0387 0.0600	
		-1.2944	0.0224 0.0387	
5.95	$20^3 \times 48$	-0.9761	0.0186, 0.0321	60x8
($a \sim 0.102$ fm)		-1.0354	0.0186, 0.0321	
		-1.0771	0.0100 0.0186, 0.0321	

β	5.75	5.85	5.95
η_{cr}^A	-1.271(10)	-1.207(8)	-1.145(6)
η_{cr}^B	-1.249(8)	-1.192(6)	-1.136(6)

Nambu-Goldstone phase simulations parameters

β	$a^{-4}(L^3 \times T)$	η	$a\mu$	$N_U \times N_\phi$
5.75 ($a \sim 0.152$ fm)	$16^3 \times 40$	-1.2714	0.0050 0.0087, 0.0131, 0.0183, 0.0277	60x1
		-1.2656	0.0131	
		-1.2539	0.0183	
		-1.2404	0.0131	
		-1.231	0.0087, 0.0183	
5.85 ($a \sim 0.123$ fm)	$20^3 \times 40$	-1.2105	0.0040, 0.0070, 0.0100, 0.0120	30x2
		-1.2068	0.0040, 0.0070, 0.0100 ,0.0224	
		-1.2028	0.0040, 0.0070, 0.0100, 0.0120, 0.0224	
		-1.1949	0.0100	
		-1.1776	0.0070, 0.0100, 0.0140, 0.0224 , 0.0316	
5.95 ($a \sim 0.102$ fm)	$24^3 \times 48$	-1.1474	0.0066, 0.0077, 0.0116, 0.0145, 0.0185	30x1
		-1.1449	0.0060, 0.0077, 0.0116, 0.0145	
		-1.1215	0.0077	
		-1.1134	0.0077, 0.0108	

blue point $\sim \eta_{cr}^{A}$