Competing coherent and dissipative dynamics close to quantum criticality

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Outlook

- Finite-size scaling in the quantum realm
- Dynamic finite-size scaling
  - → for the ***unitary quantum dynamics***
  - → for ***open quantum systems***

Work in collaboration with *Davide Nigro, Andrea Pelissetto, Ettore Vicari*

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Finite-size scaling (FSS)

FSS describes the behavior of a system ***around a given critical point***, when the correlation length ξ becomes comparable with the size *L*

→ $\xi, L \to \infty, \;\; \xi \sim L, \quad \xi/L \text{ fixed}$

→ universal features ruled by critical exponents

→ asymptotic FSS predictions affected by sizable scaling corrections

C. Domb and J. L. Lebowitz, Eds. (Academic Press, New York) "*Phase Transitions and Critical Phenomena*", Vol. **6** (1976), **8** (1983), **14** (1991)

S. Sachdev, "*Quantum Phase Transitions*" (Cambridge Univ. Press 1999)

S.L. Sondhi, S.M. Girvin, J.P. Carini, and D. Shahar, *Rev. Mod. Phys.* ***69****, 315 (1997)*

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Relies on the ***renormalization group*** (RG) theory of critical phenomena

First developed in **classical systems** (thermal fluctuations)

Later extended to the **quantum** realm (quantum fluctuations)

C. Domb and J. L. Lebowitz, Eds. (Academic Press, New York)
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FSS: standard quantum scenario

$$H(\lambda) = H_u + \lambda P$$

H_u: critical Hamiltonian

λP: perturbation

CQT @ λ = 0
continuous quantum transition

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FSS behavior of a generic observable O :

$$O(L, \lambda, T) \approx L^{-y_o} \mathcal{O}(\lambda L^{y_\lambda}, T L^{y_T})$$

system size (L)

control parameter (λ)

temperature (T)

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y_O : RG dimension of the observable O

y_λ : RG dimension of the control parameter

y_T : RG dimension of the temperature $(y_T = z)$

$\mathcal{O}$: ***scaling function*** associated to O

$\kappa \equiv \lambda L^{y_\lambda}$ scaling variables

$\tau \equiv T L^z$

z: dynamic critical exponent

$$\Delta(L) \sim L^{-z}$$

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Dynamic finite-size scaling (DFSS)

Quantum quench framework:

$$\lambda = \begin{cases} \lambda_i & \text{for } t < 0 \\ \lambda_f & \text{for } t > 0 \end{cases}$$

$$H(\lambda) = H_u + \lambda P$$

$$|\Psi(0)\rangle \equiv |0_{\lambda_i}\rangle \longrightarrow |\Psi(t)\rangle = e^{-iH(\lambda_f)t}|\Psi(0)\rangle$$

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a further scaling variable associated to the ***time*** has to be introduced:

$$\theta \equiv t L^{-z} \sim t\,\Delta(L)$$

A. Pelissetto, DR, E. Vicari, *PRB **97**, 094414 (2018)*; *PRE **97**, 052148 (2018)*

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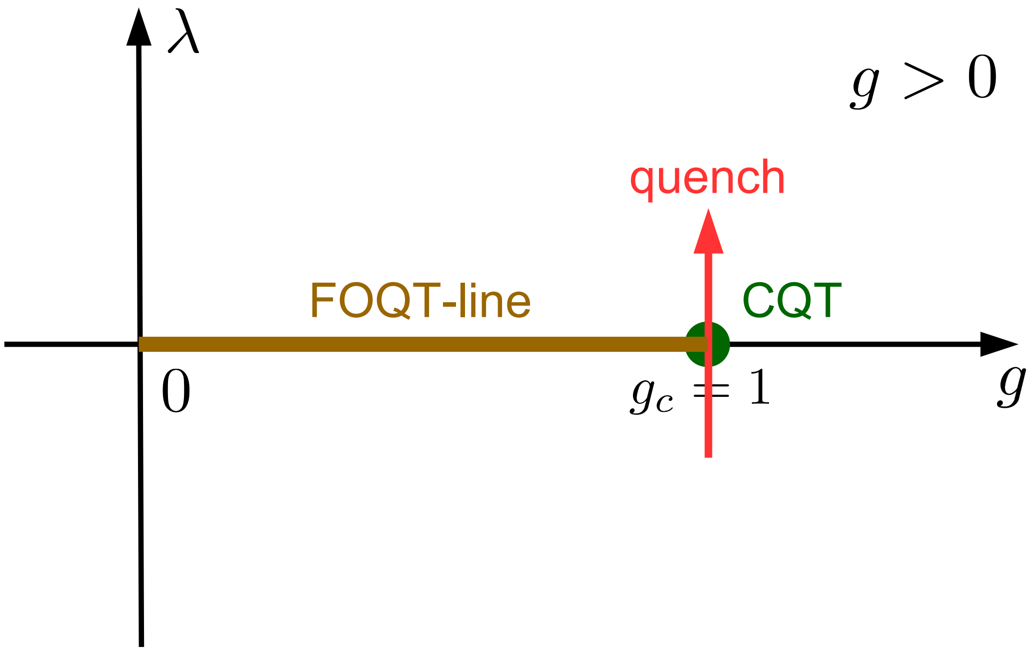
Example: quantum Ising chain

$$H(g,\lambda) = -\sum_{j=1}^{L} \left(\sigma_j^z \sigma_{j+1}^z + g\sigma_j^x + \lambda\sigma_j^z\right)$$

CQT @ g = 1, λ = 0, T = 0

quench of λ close to zero



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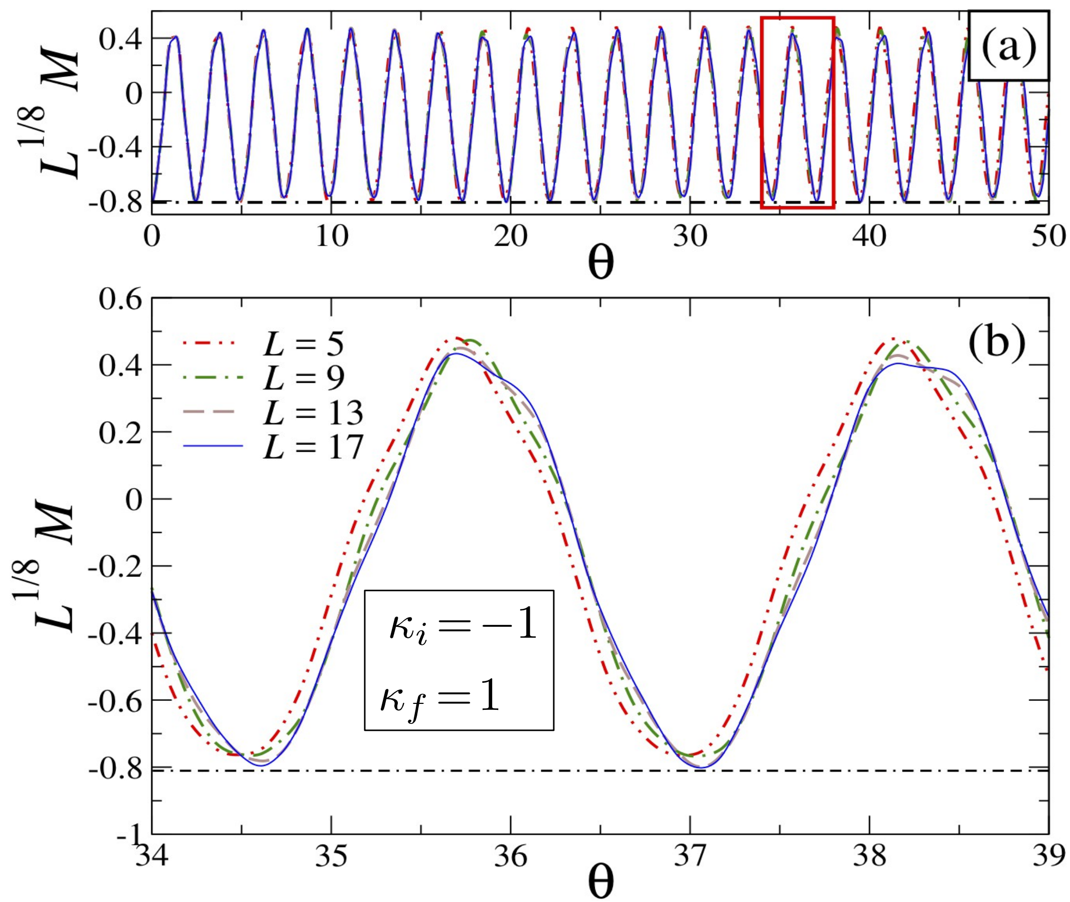
$$M(t;\, L, \lambda_f, \lambda_i) \approx L^{-\beta/\nu} \mathcal{M}(\theta;\, \kappa_f, \kappa_i)$$



scaling variables:

$$\theta = tL^{-z}$$
$$\kappa_f = \lambda_f L^{y_\lambda}$$
$$\kappa_i = \lambda_i L^{y_\lambda}$$

$$y_g = 1/\nu$$
$$y_\lambda = (d + z + 2 - \eta)/2$$

Ising parameters & critical exponents:

$$\nu = 1,\ z = 1,\ \beta = 1/8$$
$$d = 1,\ \eta = 1/4$$

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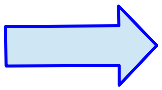
D. Nigro, DR, E. Vicari, *Phys. Rev. A* ***100***, *052108 (2019)*

DR, E. Vicari, *Phys. Rev. B* ***100***, *174303 (2019)*

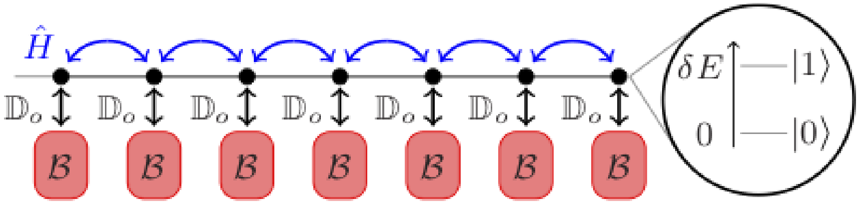
DFSS for open quantum systems

system-bath coupling
weak, local & Markovian



Lindblad master equation

$$\frac{\partial \rho}{\partial t} = -i[H,\rho] + u\sum_o \mathbb{D}_o[\rho]$$



$$\mathbb{D}_o[\rho] = L_o\rho L_o^\dagger - \tfrac{1}{2}(\rho L_o^\dagger L_o + L_o^\dagger L_o \rho)$$

DFSS for open quantum systems

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weak, local & Markovian → Lindblad master equation

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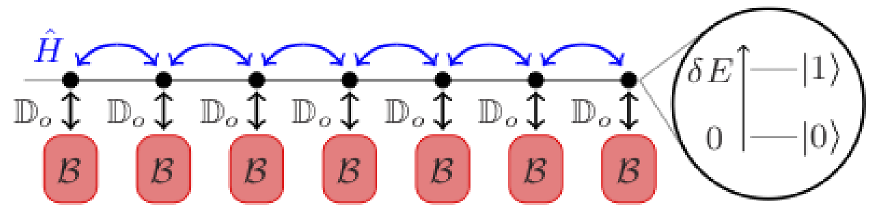

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we need a further scaling variable associated to ***dissipation***:

$$\gamma \equiv u L^\zeta$$

$$\zeta = ??$$

D. Nigro, DR, E. Vicari, *Phys. Rev. A* ***100***, *052108 (2019)*

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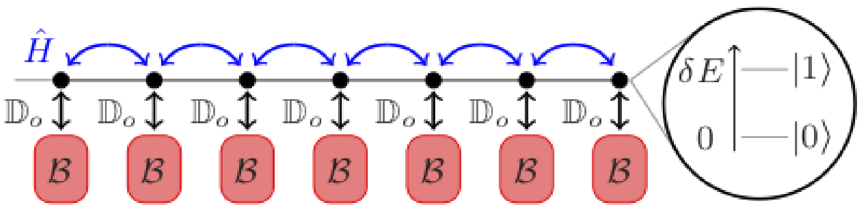

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$$O(t;\, L, \lambda_f, \lambda_i, T, u) \approx L^{-y_o} \mathcal{O}\big(\theta;\, \kappa_f, \kappa_i, \tau, \gamma\big)$$

phenomenological scaling argument

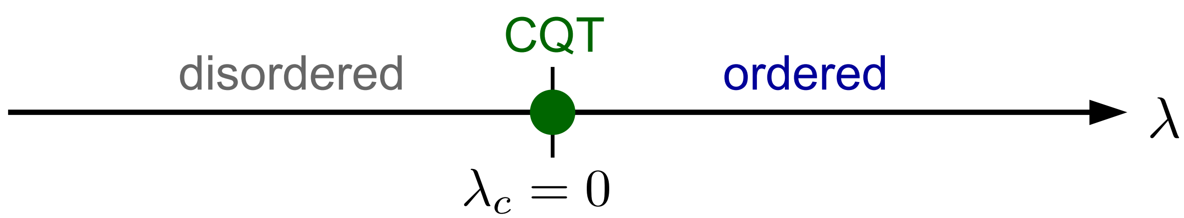
$$\zeta = z$$

D. Nigro, DR, E. Vicari, *Phys. Rev. A* ***100***, *052108 (2019)*

Numerical verification:
Kitaev quantum wire with local dissipative mechanisms

$$H = -\sum_{j=1}^{L} \left[(c_j^\dagger c_{j+1} + c_j^\dagger c_{j+1}^\dagger + h.c.) + (\lambda - 2) c_j^\dagger c_j \right]$$

A. Kitaev, *Phys. Usp.* ***44**, 131 (2001)*



$$\lambda < 4$$

Markovian baths modeled through local jump operators:

$$L_j^{(1)} = c_j^\dagger$$
pumping

$$L_j^{(2)} = c_j$$
losses

$$L_j^{(3)} = c_j^\dagger c_j$$
dephasing

T. Prosen, *NJP* ***10**, 043026 (2008)*
V. Eisler, *J. Stat. Mech. (2011) P06007*
B. Horstmann, J.I. Cirac, G. Giedke, *PRA* ***87**, 012108 (2013)*
M. Keck, S. Montangero, G.E. Santoro, R. Fazio, DR, *NJP* ***19**, 113029 (2017)*

Unitary vs. dissipative dynamics

$$G_{12}(x,t;\, L,\lambda_f,\lambda_i,u) \approx L^{-(y_1+y_2)}\mathcal{G}\Big(X,\theta;\, \kappa_f,\kappa_i,uL^{\zeta}\Big)$$

$$X \equiv x/L$$

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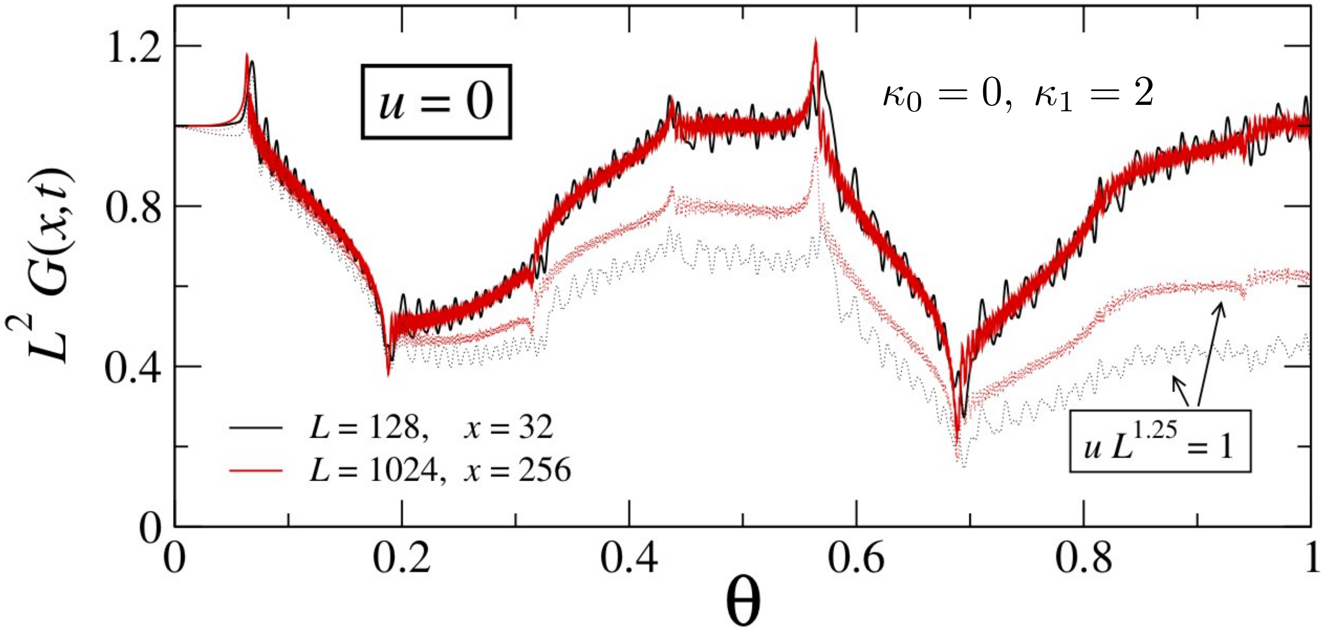
$$G(x,t) = \mathrm{Tr}\big[\rho(t)n_j n_{j+x}\big] - \mathrm{Tr}\big[\rho(t)n_j\big]\mathrm{Tr}\big[\rho(t)n_{j+x}\big] \quad \Rightarrow \quad y_1 = y_2 = 1$$

low-dissipation regime
$\zeta > z$

⇓

unitary dynamics
asymptotically wins



Unitary vs. dissipative dynamics

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low-dissipation regime

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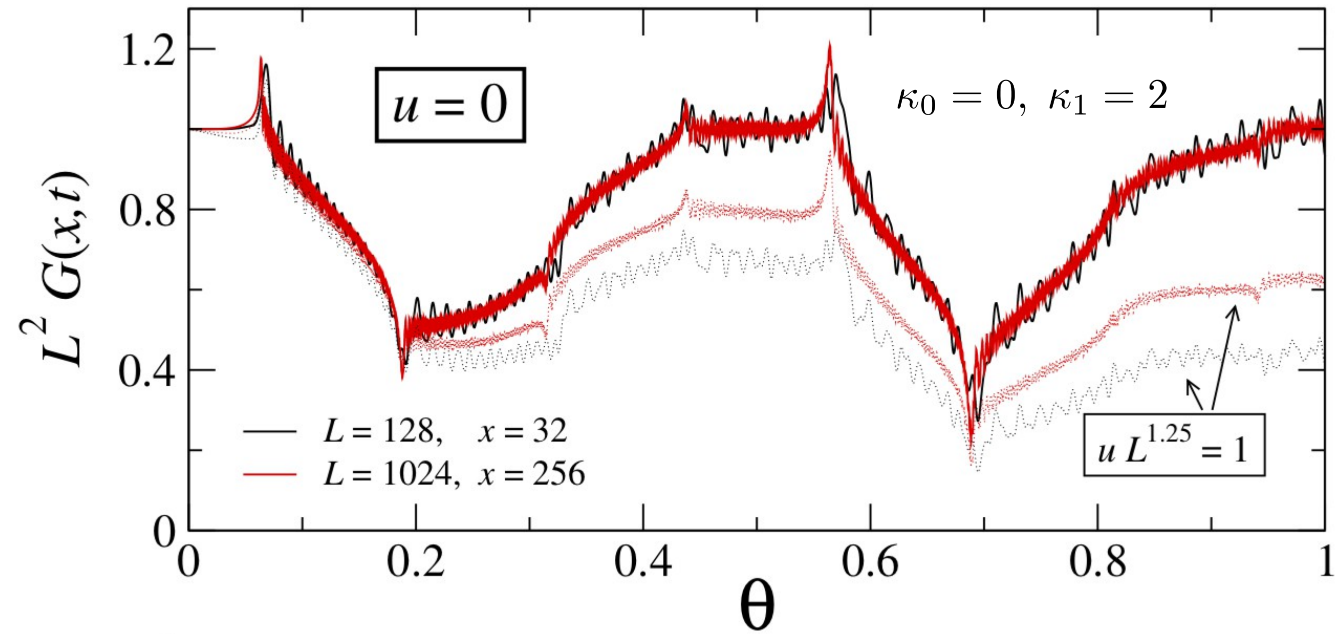
⇓

unitary dynamics asymptotically wins

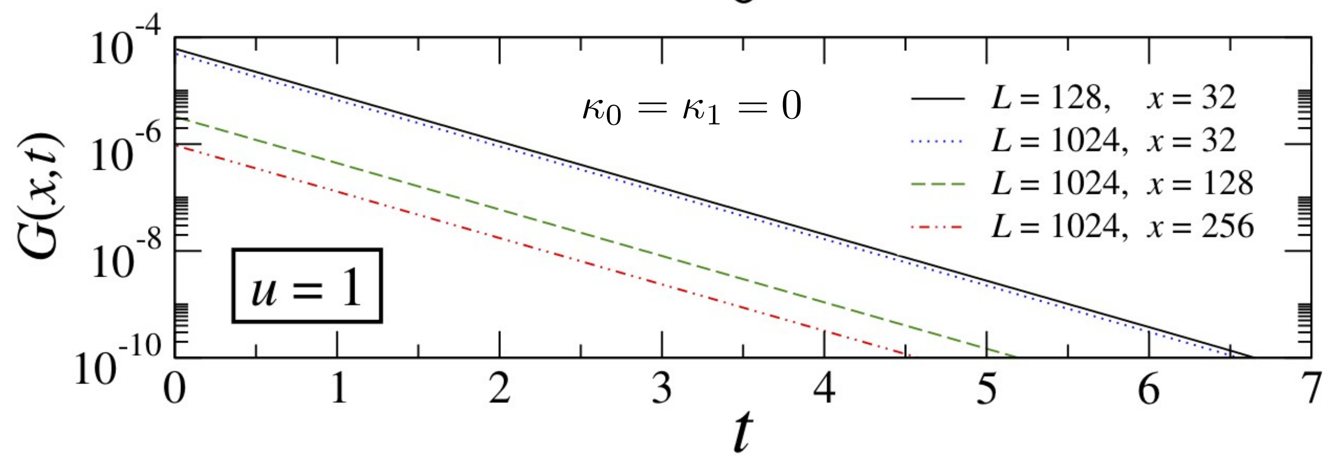


strong-dissipation regime

$\zeta < z$

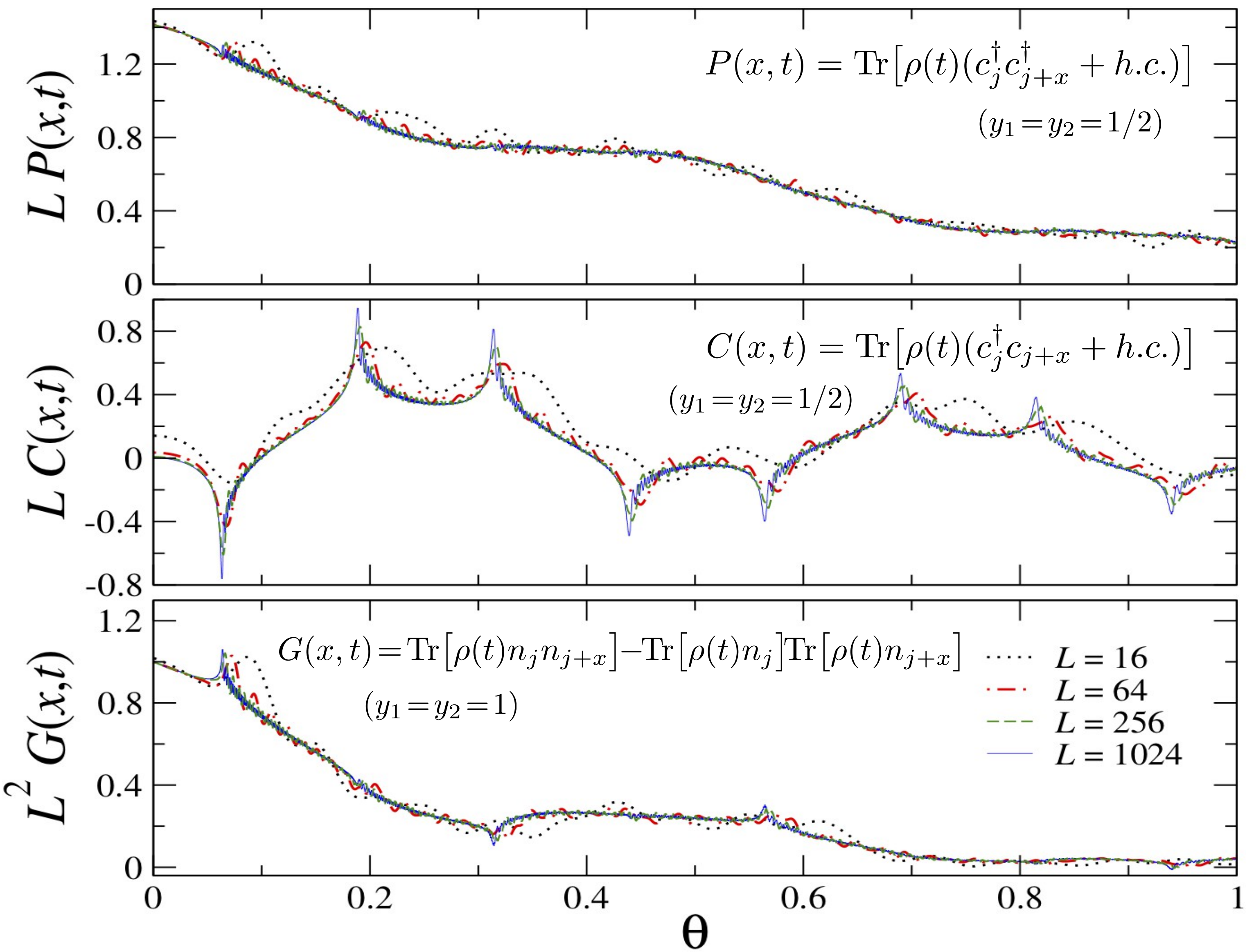
⇓

dissipation effects asymptotically win



Phenomenological DFSS theory

$$G_{12}(x,t;\,L,\lambda_f,\lambda_i,u) \approx L^{-(y_1+y_2)}\mathcal{G}\big(X,\theta;\,\kappa_f,\kappa_i,\gamma\equiv uL^z\big)$$



$\gamma = 1$

$X = 1/4$

$\kappa_i = 0$

$\kappa_f = 2$

Conclusions

Out-of-equilibrium *dynamics of many-body quantum systems*:

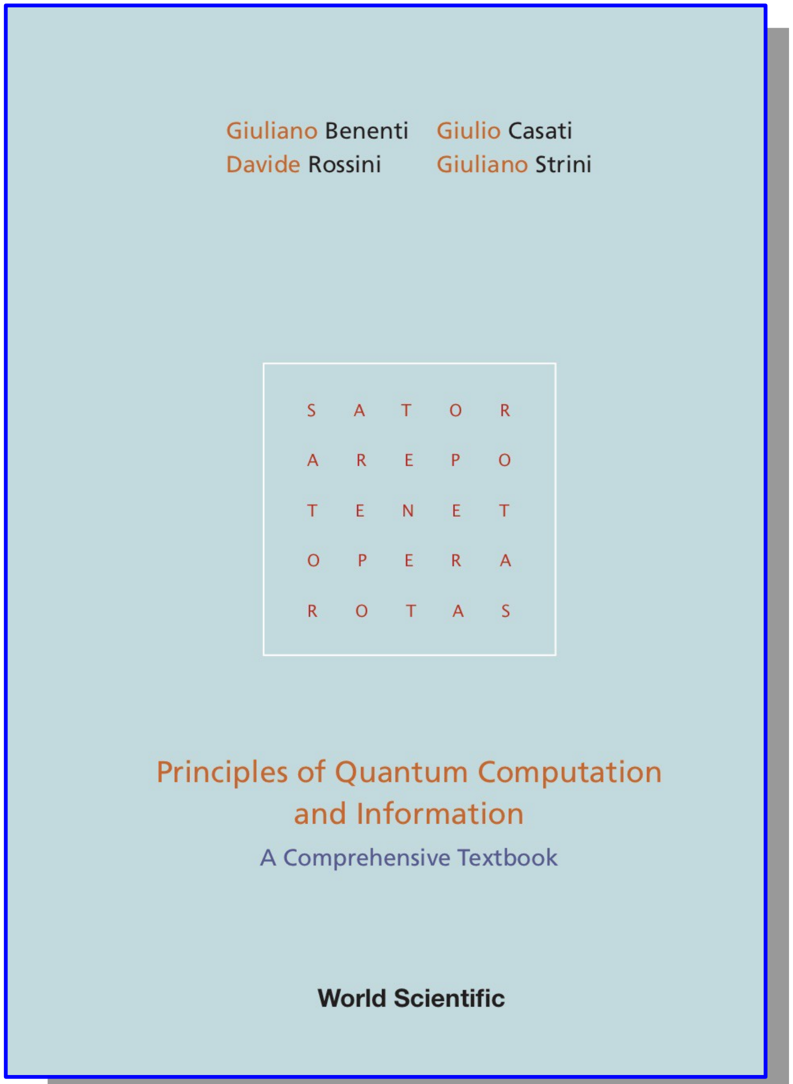
a look at their behavior ***near criticality***

→ general ***dynamic scaling behaviors***
(without and with dissipation)

→ numerical checks on Ising-like quantum spin models

Principles of Quantum Computation and Information:
A Comprehensive Textbook

G. Benenti, G. Casati, D. Rossini, G. Strini



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World Scientific, Singapore, 2019