Critical properties of three-dimensional lattice multiflavor scalar chromodynamics

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based on
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Outline

Critical phenomena and symmetries

Landau-Ginzburg-Wilson (LGW) approach: the critical properties of a statistical system depend just on the global symmetry breaking pattern and some "kinematic" parameters (space dimensionality, number of fields components, ...). They are encoded in the ϕ^4 theory sharing the same global symmetries and "kinematic" parameters.

Does something change when local symmetries are also present? What is the role of the gauge degrees of freedom in the effective theory?

Question relevant for several systems of physical interest, like QCD at finite temperature, condensed matter systems

Up to now this problem was investigated only in system with abelian local symmetries: sometimes the gauge degrees of freedom are relevant (antiferromagnetic 3D CP^{N-1} models) sometimes they are not (multiflavour 3D abelian Higgs models).

The lattice model

The basic variables of our model are $N_c \times N_f$ complex matrices $Z_{\mathbf{x}}^{cf}$ and the action before gauging is

$$S_{\rm inv} = -J \sum_{\mathbf{x},\mu} {\rm Re\,Tr}\, Z_{\mathbf{x}}^\dagger Z_{\mathbf{x}+\hat\mu}\,, \quad {\rm Tr}\, Z_{\mathbf{x}}^\dagger Z_{\mathbf{x}} = 1\,,$$

where $\mathbf{x}$ stands for the site of a 3D cubic lattices. This action is $O(2N_cN_f)$ symmetric, as seen by explicitly rewriting it in term of Re and Im parts of $Z_{\mathbf{x}}^{cf}$.

Next we gauge the N_c color degrees of freedom, by coupling them to a $SU(N_c)$ gauge field $U_{\mathbf{x},\hat\mu}$

$$S_g = -\beta N_f \sum_{\mathbf{x},\mu} {\rm Re\,Tr}\left[Z_{\mathbf{x}}^\dagger\, U_{\mathbf{x},\hat\mu}\, Z_{\mathbf{x}+\hat\mu}\right] - \frac{\beta_g}{N_c} \sum_{\mathbf{x},\mu>\nu} {\rm Re\,Tr}\Box_{\mathbf{x},\mu\nu}$$

The lattice model and its symmetries

$$S_g = -\beta N_f \sum_{\mathbf{x},\mu} \mathrm{Re}\,\mathrm{Tr}\left[Z^\dagger_{\mathbf{x}}\, U_{\mathbf{x},\hat\mu}\, Z_{\mathbf{x}+\hat\mu}\right] - \frac{\beta_g}{N_c} \sum_{\mathbf{x},\mu>\nu} \mathrm{Re}\,\mathrm{Tr}\Box_{\mathbf{x},\mu\nu}$$

S_g is invariant under the local transformation SU(N_c)

$$Z_{\mathbf{x}} \to G_{\mathbf{x}} Z_{\mathbf{x}}\ , \quad U_{\mathbf{x},\hat\mu} \to G_{\mathbf{x}} U_{\mathbf{x},\hat\mu} G^\dagger_{\mathbf{x}+\hat\mu}\ ,$$

and under the global transformation U(N_f)

$$Z_{\mathbf{x}} \to Z_{\mathbf{x}} V\ , \quad U_{\mathbf{x},\hat\mu} \to U_{\mathbf{x},\hat\mu}\ .$$

The $N_c = 2$ case is somehow peculiar: since SU(2) is pseudoreal the largest global symmetry is in fact Sp(N_f) (the subgroup of $M \in \mathrm{U}(2N_f)$ such that $MJM^T = J$). Moreover we will need SO(5)=Sp(2)/$\mathbb{Z}_2$.

Continuum EFT 1: gauge degrees of freedom are relevant

The natural continuum EFT is in this case continuum scalar chromodynamics:

$$\mathcal{L} = \frac{1}{4g^2}\mathrm{Tr}\, F_{\mu\nu}^2 + \mathrm{Tr}[(D_\mu Z)^\dagger (D_\mu Z)] + V(\mathrm{Tr} Z^\dagger Z)\,,$$

At one loop, the β functions of the (properly rescaled, $f \propto g^2$, $u \propto \lambda$) couplings are

$$\begin{aligned}
\beta_f(u,f) &= -\varepsilon f - (22N_c - N_f)\, f^2\,,\\
\beta_u(u,f) &= -\varepsilon u + (N_f N_c + 4)\, u^2\\
&\quad - \frac{18\,(N_c^2-1)}{N_c}\, uf + \frac{27(N_c-1)(N_c^2+2N_c-2)}{N_c^2}\, f^2
\end{aligned}$$

and stable FP exist only for very large numbers of flavours (e.g. $N_f > 259 + O(\varepsilon)$ for $N_c = 2$), hence we expect a first order phase transition for all N_c and "reasonable" values of N_f.

Continuum EFT 2: gauge degrees of freedom are irrelevant

We use a LGW approach with the gauge invariant order parameter

$$Q_{\mathbf{x}}^{fg} = \sum_a \bar{Z}_{\mathbf{x}}^{af} Z_{\mathbf{x}}^{ag} - \frac{1}{N_f}\delta^{fg}\,.$$

$Q_{\mathbf{x}}$ is hermitian, traceless and $Q_{\mathbf{x}} \to M^\dagger Q_{\mathbf{x}} M$ under the global symmetry.

Most general 4^{th}-order polynomial consistent with the global symmetry:

$$\mathrm{Tr}(\partial_\mu Q)^2 + r\,\mathrm{Tr}\,Q^2 + w\,\mathrm{tr}\,Q^3 + u\,(\mathrm{Tr}\,Q^2)^2 + v\,\mathrm{Tr}\,Q^4$$

For $N_f > 2$, $\mathrm{Tr}\,Q^3 \neq 0$ and a first order phase transition is expected.

For $N_f = 2$ and $N_c > 3$ we obtain the standard LGW for the 3D O(3) universality class (more explicit if rewritten by using $\varphi_{\mathbf{x}}^k = \bar{Z}_{\mathbf{x}}^{af}\sigma_{fg}^k Z_{\mathbf{x}}^{ag}$)

For $N_f = 2$ and $N_c = 2$ we obtain Sp(2)≈O(5) universality class.
Beyond $\varphi_{\mathbf{x}}$ we also have the complex scalar $\phi_{\mathbf{x}} = \epsilon_{ab}\epsilon_{fg} Z_{\mathbf{x}}^{af} Z_{\mathbf{x}}^{bg}$.

Lattice observables and Finite Size Scaling

On a L^3 lattice with periodic b.c. we can define

$$Q_{\mathbf{x}}^{fg} \equiv \sum_a \bar{Z}_{\mathbf{x}}^{af} Z_{\mathbf{x}}^{ag} - \frac{1}{N_f}\delta^{fg}\,, \quad G(\mathbf{x}-\mathbf{y}) = \langle \mathrm{Tr}\,(Q_{\mathbf{x}} Q_{\mathbf{y}})\rangle,$$

from which we get the susceptibility χ, the correlation lenght ξ and the Binder cumulant U

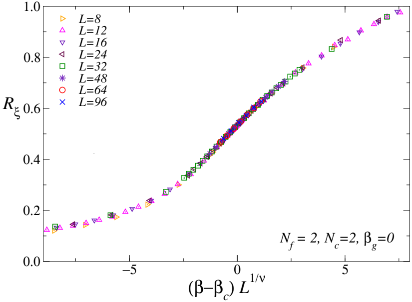
$$\chi = \sum_{\mathbf{x}} G(\mathbf{x})\,, \quad \xi^2 = \frac{1}{4\sin^2(\pi/L)} \frac{\widetilde{G}(\mathbf{0}) - \widetilde{G}(\mathbf{p}_m)}{\widetilde{G}(\mathbf{p}_m)}$$

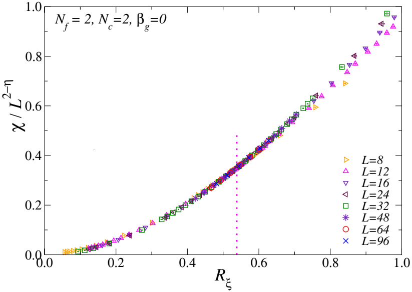
$$U = \frac{\langle \mu_2^2 \rangle}{\langle \mu_2 \rangle^2}, \quad \mu_2 = \frac{1}{V^2}\sum_{\mathbf{x},\mathbf{y}} \mathrm{Tr}\, Q_{\mathbf{x}} Q_{\mathbf{y}}\,.$$

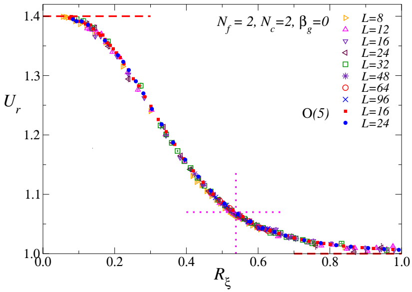
U and $R_\xi = \xi/L$ are RG invariants, hence close to the transition they satisfy $R(\beta, L) = f_R(X) + L^{-\omega} g_R(X) + \cdots$ with $X = (\beta - \beta_c)L^{1/\nu}$ in particular

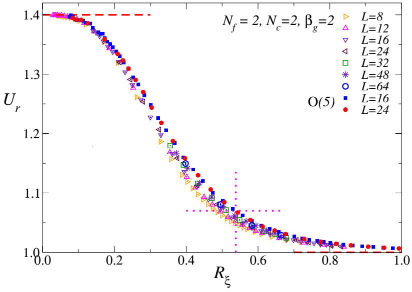
$$U(\beta, L) = F_U(R_\xi) + O(L^{-\omega})\,.$$

The case $N_f = 2$, $N_c = 2$: O(5)

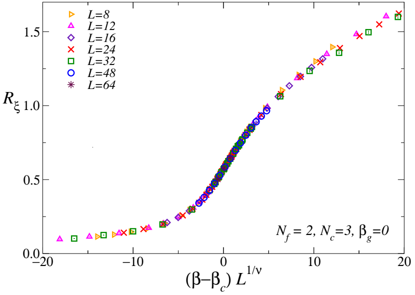


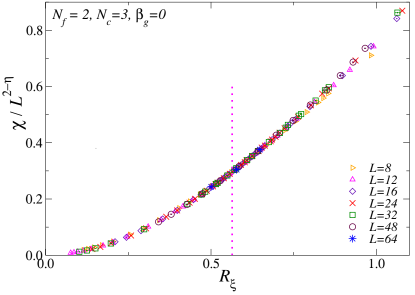


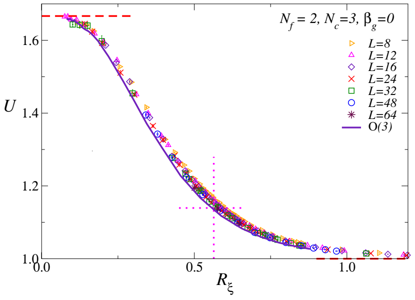


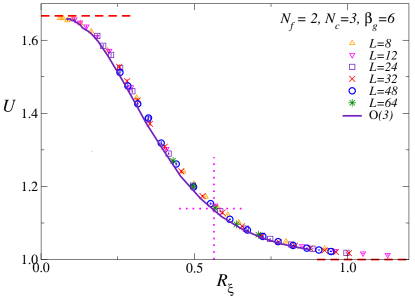


The case $N_f = 2$, $N_c = 3$: O(3)

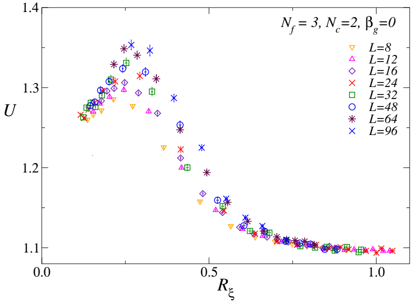


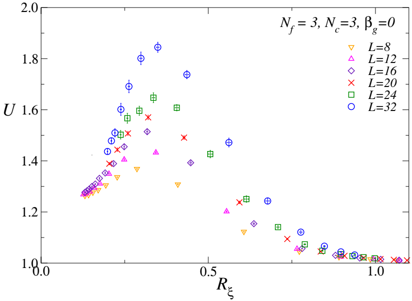






The case $N_f \geq 3$: 1^{st} order





No clear signal of metastabilites, but data of U versus R_ξ do not seem to converge to a scaling curve, as they should for a second order phase transition.

Conclusions

We have numerically investigated a lattice 3D model with SU(N_c) gauge symmetry and U(N_f) global symmetry, and verified that the EFT that correctly describe its critical behaviour is the LGW theory based on a gauge-invariant order parameter.

This supports the Pisarski-Wilczek approach to massless QCD but leaves open questions:

- what happen if we start from an initial symmetry that is not the maximal O($2N_cN_f$)?
- why the LGW approach sometimes fails for abelian gauge field?
- do systems with non-abelian gauge symmetry exist for which the LGW approach is not the correct one?

Thank you for your attention!

Backup with something more

Some abelian cases examples

- 3D ACP^{N-1} model
  $H = J\sum_{\langle \mathbf{x},\mathbf{y}\rangle} |\bar{\mathbf{z}}_\mathbf{x} \cdot \mathbf{z}_\mathbf{y}|^2$ with $\mathbf{z} \in \mathbb{C}^N$, $|\mathbf{z}| = 1$ and $J > 0$
  LGW predicts O(3) for $N = 2$, O(8) for $N = 3$ and 1^{st} order for $N = 4$ (RG flow studied up to five/six loops in different renormalization schemes)
  A second order phase transition is found numerically for $N = 4$
  Pelissetto, Tripodo, Vicari PRD **96**, 034505 (2017)
- 3D CP^{N-1} models and abelian Higgs models
  same as before but with $J < 0$ and
  $H = -\beta N \sum_{\mathbf{x},\mu}(\lambda \bar{\mathbf{z}}_\mathbf{x} \cdot \mathbf{z}_{\mathbf{x}+\hat{\mu}} + \mathrm{c.c.}) - 2\beta_g \sum_{\mathbf{x},\mu>\nu} \mathrm{ReTr}\Box_{\mathbf{x},\mu\nu}$
  LGW predicts O(3) for $N = 2$ and 1^{st} order for $N \geq 3$, in agreement with lattice simulations
  Pelissetto, Vicari PRE **100**, 022122 (2019), PRE **100**, 042134 (2019)

The global symmetry for $N_c = 2$ (I)

$$S_h = \frac{1}{2}\sum_{f,a,b}\left[\bar{Z}^{af}_{\mathbf{x}}\, U^{ab}_{\mathbf{x},\hat{\mu}}\, Z^{bf}_{\mathbf{x}+\hat{\mu}} + Z^{af}_{\mathbf{x}}\, \bar{U}^{ab}_{\mathbf{x},\hat{\mu}}\, \bar{Z}^{bf}_{\mathbf{x}+\hat{\mu}}\right] =$$

$$= \frac{1}{2}\sum_{f,a,b}\left[\bar{Z}^{af}_{\mathbf{x}}\, U^{ab}_{\mathbf{x},\hat{\mu}}\, Z^{bf}_{\mathbf{x}+\hat{\mu}} + \bar{Y}^{af}_{\mathbf{x}}\, U^{ab}_{\mathbf{x},\hat{\mu}}\, Y^{bf}_{\mathbf{x}+\hat{\mu}}\right] ,$$

where

$$Y^{af}_{\mathbf{x}} \equiv i\sigma_2^{ab}\bar{Z}^{bf}_{\mathbf{x}} , \quad \bar{U} = \sigma_2 U \sigma_2 , \quad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} .$$

We can now define

$$\Gamma^{a\gamma}_{\mathbf{x}} = \begin{cases} Z^{a\gamma}_{\mathbf{x}} & \text{if} \quad 1 \le \gamma \le N_f \\ Y^{a\,(\gamma - N_f)}_{\mathbf{x}} & \text{if} \quad N_f + 1 \le \gamma \le 2N_f \end{cases} ,$$

then

$$S_h = \frac{1}{2}\sum_{\gamma,a,b}\bar{\Gamma}^{a\gamma}_{\mathbf{x}}\, U^{ab}_{\mathbf{x},\hat{\mu}}\Gamma^{b\gamma}_{\mathbf{x}+\hat{\mu}} = \frac{1}{2}\mathrm{Tr}(\Gamma^{\dagger}_{\mathbf{x}} U_{\mathbf{x},\hat{\mu}}\Gamma) \quad .$$

The global symmetry for $N_c = 2$ (II)

$$\mathrm{Tr}(\Gamma^\dagger_{\mathbf{x}} U_{\mathbf{x},\hat{\mu}} \Gamma)$$

is invariant under the local transformation $\Gamma_{\mathbf{x}} \to G_{\mathbf{x}} \Gamma_{\mathbf{x}}$ with $G_{\mathbf{x}} \in \mathrm{SU}(N_c)$ and it is invariant under the global $\mathrm{U}(2N_f)$ transformation

$$\Gamma_{\mathbf{x}} \to \Gamma_{\mathbf{x}} M \;, \quad M \in \mathrm{U}(2N_f)\,. \tag{1}$$

However Γ variables are not generic, they have the structure

$$\Gamma = \left(Z\,,\ Y = i\sigma_2 \bar{Z}\right) \;,$$

which is equivalent to say that they satisfy the relation

$$i\sigma_2 \bar{\Gamma} = \Gamma J \;, \quad J = \begin{pmatrix} 0 & -I \\ I & 0 \end{pmatrix} \;,$$

where I is the $N_f \times N_f$ identity matrix. The global invariance is thus the subgroup of matrices $M \in \mathrm{U}(2N_f)$ which leave invariant this relation, i.e.

$$M J M^T = J\,.$$

The complete order parameter for $N_c = 2$

Instead of $Q^{fg}_{\mathbf{x}}$ we can define

$$\mathcal{T}^{\alpha\beta}_{\mathbf{x}} = \sum_a \bar{\Gamma}^{a\alpha}_{\mathbf{x}} \Gamma^{a\beta}_{\mathbf{x}} - \frac{\delta^{\alpha\beta}}{2N_f} \sum_{a\gamma} \bar{\Gamma}^{a\gamma}_{\mathbf{x}} \Gamma^{a\gamma}_{\mathbf{x}} \,,$$

but it is not difficult to show that these quantities can be expressed in terms of $Q^{fg}_{\mathbf{x}}$ and of $D^{fg}_{\mathbf{x}} = \sum_{ab} \epsilon^{ab} Z^{af}_{\mathbf{x}} Z^{bg}_{\mathbf{x}}$.

Since $\mathcal{T}^{\alpha\beta}_{\mathbf{x}}$ is not independent of $Q^{fg}_{\mathbf{x}}$ (in particular $\mathcal{T}^{ab}_{\mathbf{x}} = Q^{ab}_{\mathbf{x}}$ for $a, b = 1, \ldots, N_f$), its critical behaviour can be investigated by studying just $Q^{fg}_{\mathbf{x}}$.

We only have to pay attention to the fact that

$$\frac{\langle (\sum_{k=1}^{3} M^k M^k)^2 \rangle}{\langle \sum_{k=1}^{3} M^k M^k \rangle^2} = \frac{21}{25}\, U_{\mathrm{O}(5)} \,.$$

therefore the correct quantity to be studied to achieve matching with the universal Binder parameter of the O(5) vector model is $U_r \equiv \frac{21}{25}\, U$.

Critical exponents and RG-invariant quantities

- 3D O(5) universality class

$$\nu = 0.779(3)\,, \quad \eta = 0.034(1)\,, \quad \omega = 0.79(2)$$
$$R_\xi^* = 0.538(1)\,, \quad U^* = 1.069(1)\,.$$

- 3D O(3) universality class

$$\nu = 0.7117(5)\,, \quad \eta = 0.0378(3)\,, \quad \omega = 0.782(13)\,,$$
$$R_\xi^* = 0.5639(2)\,, \quad U^* = 1.1394(3)\,.$$