Quantum Simulation of Parton Physics

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1908.10439

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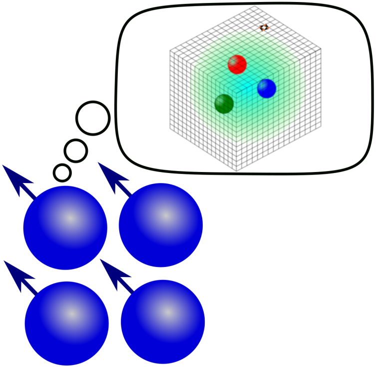
UNIVERSITY OF MARYLAND



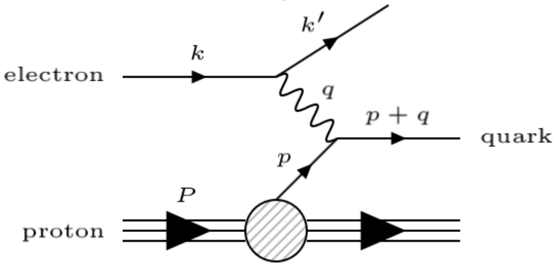
Parton Distribution Function

PDF of parton species q in a hadron with momentum P

$f_q(P,x)$= probability of finding q with momentum fraction p ($p = xP$)

(In longitudinal direction)
With $f_q(x)$, hadron high energy scattering processes' cross section;

$$\sigma_{e^- p \to e^- + X} = \int_0^1 dx \sum_q f_q(P,x)\ \sigma_{e^- q \to e^- + X}$$



On the Euclidean Lattice

PDF:

$$f_q(P,x) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itx(n\cdot P)} \langle P | \bar{\psi}_q(tn^\mu)\gamma^0 W_n(tn^\mu,0)\psi_q(0)|P\rangle$$

Real-time correlators $\rightarrow$ (potential) Sign problem

Many methods available: Quasi PDFs, Pseudo PDFs...

Parton Physics on Euclidean Lattice

Xiangdong Ji[1,2]

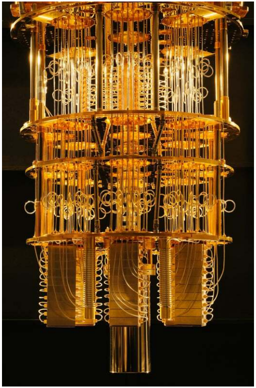
Parton Distribution Functions from Ioffe time pseudo-distributions

Bálint Joó[a], Joseph Karpie[b,a], Kostas Orginos[b,a], Anatoly Radyushkin[c,a], David Richards[a] and Savvas Zafeiropoulos[d]

Quantum Computer is a quantum system evolved in real-time

PDFs are *natural* in a quantum simulation.

Reality



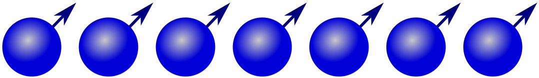
Machine available now is:

- 50 qubits
- circuit depth 20
- noisy..

We'll have to wait for XX years for PDF...

A Quantum Computer - Qubits

Qubits are quantum spins:



So the Hilbert space is 2^N-dimensional for N qubits. A state on qubits is

$$|\phi\rangle = a|0000000\rangle + b|1000000\rangle + c|0100000\rangle + \cdots$$

but once you do measurement, it collapse into one of those basis state

$$|\phi\rangle \to |0101010\rangle$$

A Quantum Computer - Gates

Gates apply to qubits and change the state

- 1 -qubit gates in matrix form...

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}, \; T = \begin{pmatrix} e^{i\pi/8} & 0 \\ 0 & e^{-i\pi/8} \end{pmatrix}$$

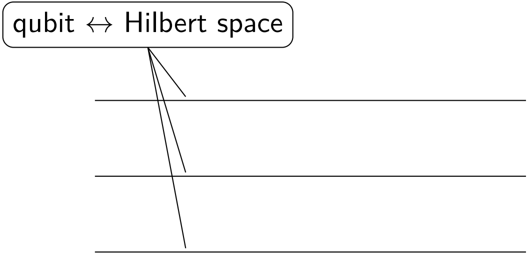
- 2 -qubit gates in matrix form ... example Controlled-not (CNOT)

$$CNOT = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \end{pmatrix}$$

$$|00\rangle \mapsto |00\rangle$$
$$|01\rangle \mapsto |01\rangle$$
$$|10\rangle \mapsto |11\rangle$$
$$|11\rangle \mapsto |10\rangle$$

Quantum Simulation is simple... in theory

Quantum Computer is a quantum system evolved in real-time



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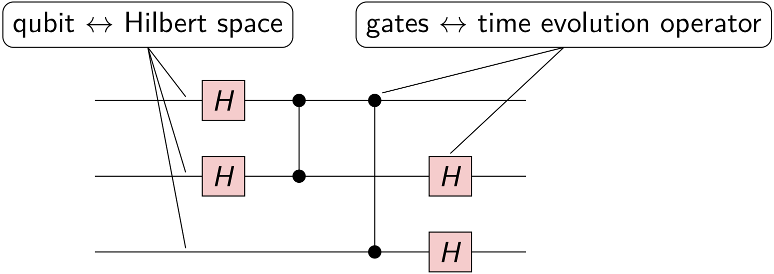

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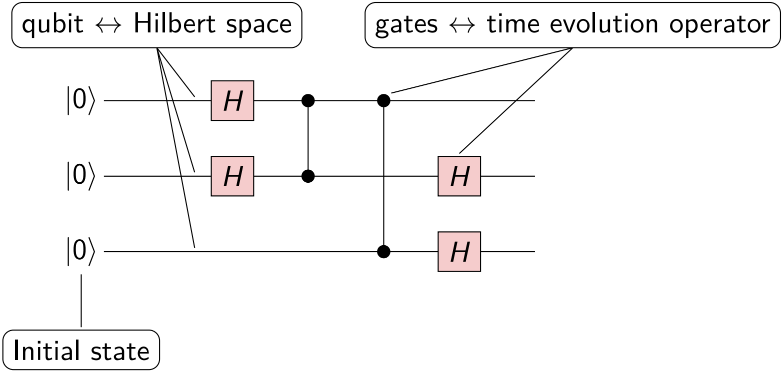

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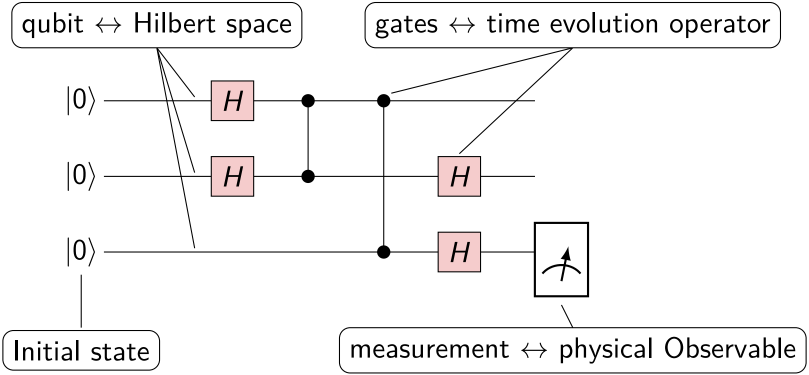
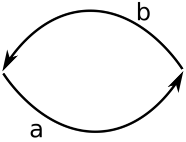


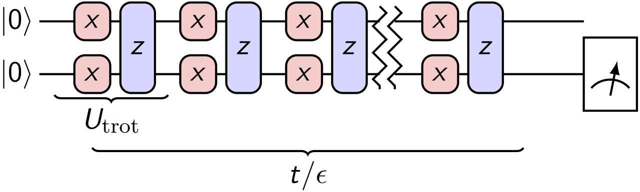
Example - $\mathbb{Z}_2$, One 'Plaquette'

$$H = \sigma_x(a) + \sigma_x(b) + \sigma_z(a)\sigma_z(b)$$



Trotterization:

$$e^{-iHt} = \left[e^{-iH\epsilon}\right]^{t/\epsilon} \approx \Big[\overbrace{e^{-i\epsilon\sigma_x(a)}\,e^{-i\epsilon\sigma_x(b)}\,e^{-i\epsilon\sigma_z(a)\otimes\sigma_z(b)}}^{U_{\mathrm{trot}}}\Big]^{t/\epsilon}$$



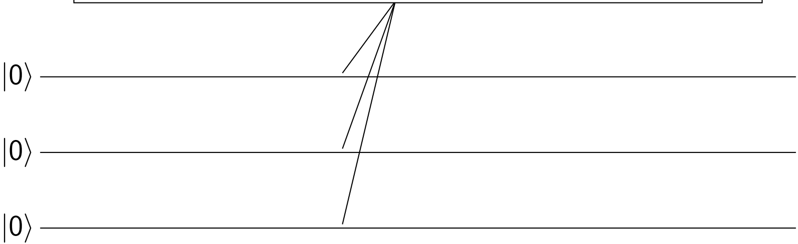
X : $e^{-i\epsilon\sigma_x}$

Z : $e^{-i\epsilon\sigma_z\otimes\sigma_z}$

What is needed for PDFs of Proton

$$f_q(P,x) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itx(n\cdot P)} \langle P|\bar{\psi}_q(tn^\mu)\gamma^0 W_n(tn^\mu,0)\psi_q(0)|P\rangle$$

Map Hilbert space of QCD to qubits (truncate $SU(3)$)



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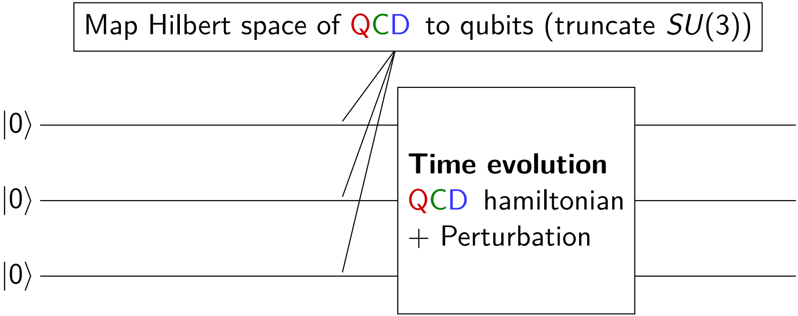

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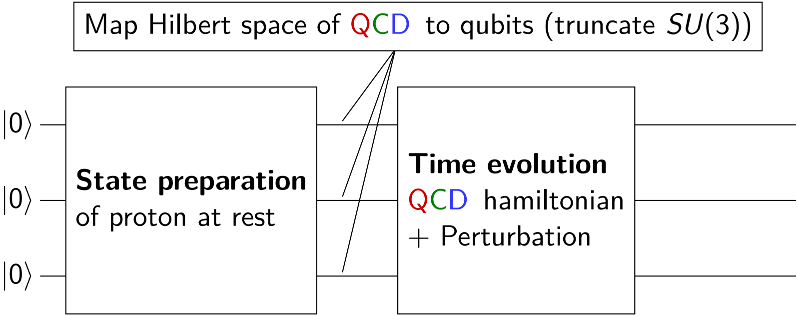

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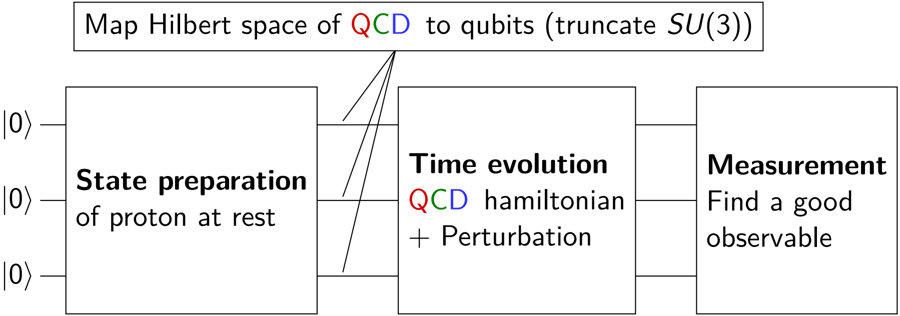

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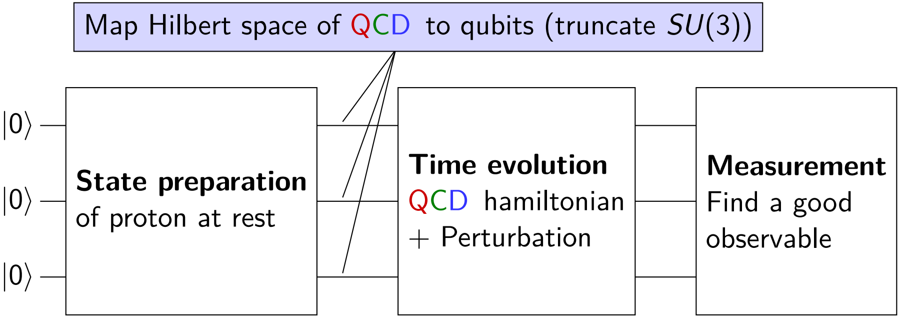


Mapping quarks and gluons to qubits

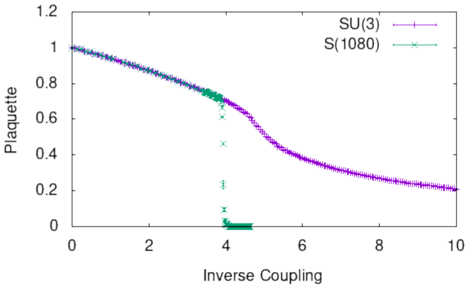
$SU(3)$ has an infinite dimensional Hilbert space
N qubits have 2^N dimensional Hilbert space

$$2^N = \infty \text{ ???}$$

Well, we have only finite number of qubits $\rightarrow$ need to truncate $SU(3)$

Largest finite subgroup of $SU(3)$: $S(1080)$

$S(1080)$ **gauge theory** $\rightarrow$ **11 qubits per gauge link** **(**$1080 < 2^{11}$**)**



"Modified" $S(1080)$ action

Modify the action:

$$S = -\sum_p \left(\frac{\beta_0}{3} \mathrm{Re\,Tr}\, U_p + \beta_1 \mathrm{Re\,Tr}\, U_p^2 \right)$$

Measure two scales, and compare the ratio to $SU(3)$: Wilson flow, center symmetry

$\rightarrow$ **Smallest lattice spacing is $a = 0.08$ fm.**

- At this spacing, $S(1080)$ and $SU(3)$ agree on the low-energy observable $T_c\sqrt{t_0}$
- Beyond this spacing, they disagree.

Do other low-energy quantities agree?

In progress: spectroscopy, further modified actions

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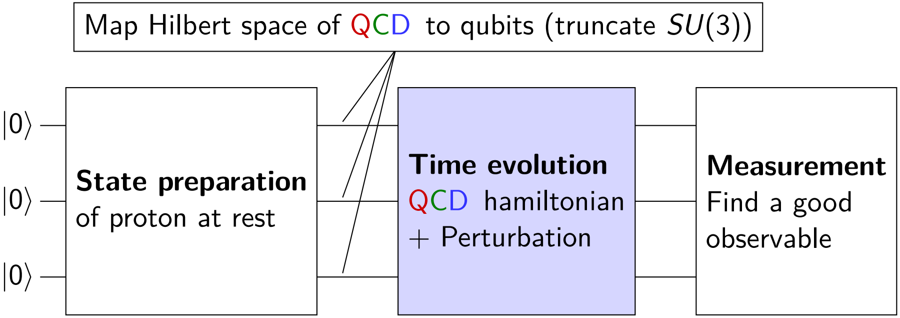
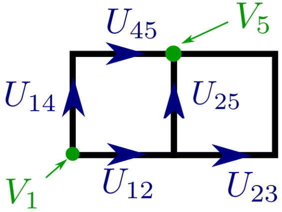
$$f_q(P,x) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itx(n\cdot P)} \langle P|\bar{\psi}_q(tn^\mu)\gamma^0 W_n(tn^\mu,0)\psi_q(0)|P\rangle$$



Hilbert Space of $S(1080)$

$G = S(1080)$ lattice gauge theory $\rightarrow$ each link U_{ij} has Hilbert space $\mathbb{C}G$



$$\mathcal{H} = \mathbb{C}G \otimes \mathbb{C}G \otimes \cdots$$

$$U_{ij} \rightarrow V_j U_{ij} V_i^\dagger$$

Only Gauge-Invariant States Allowed!

~~$|U_{12}\rangle$~~ $\qquad \int \mathrm{d}V_1 \mathrm{d}V_2 \left| V_2^\dagger U_{12} V_1 \right\rangle$

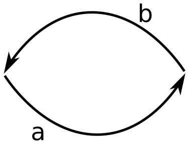
Projection operator to $\mathcal{H}_P$ (Physical subspace):

$$P\,|U_{12}\cdots\rangle = \int (\mathrm{d}V_1 \mathrm{d}V_2 \cdots) \left| V_2^\dagger U_{12} V_1 \cdots \right\rangle$$

We keep the entire Hilbert space on qubits

Gauge invariance - $\mathbb{Z}_2$, One 'Plaquette' again

$$H = \sigma_x(a) + \sigma_x(b) + \sigma_z(a)\sigma_z(b)$$

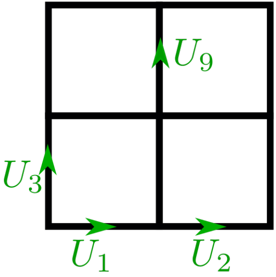


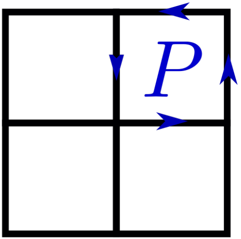
The gauge transformation operator: $\sigma_x(a)\sigma_x(b)$.

$$|00\rangle \leftrightarrow |11\rangle \text{ and } |01\rangle \leftrightarrow |10\rangle$$

Physical states: $|00\rangle + |11\rangle$, $|01\rangle + |10\rangle$
Unphysical states: $|00\rangle - |11\rangle$, $|01\rangle - |10\rangle$

Hamiltonian lattice gauge theory





$$H = \beta_K \sum_L \nabla_L^2 + \beta_P \sum_P \mathrm{Re\,Tr}\, P + \cdots$$

Hamiltonian is gauge-invariant

gauge invariant initial state $\rightarrow$ gauge invariant final state

Real-Time evolution

How do we implement e^{-iHt} with local gates?

$$H = \overbrace{\beta_K \sum_L \nabla_L^2}^{H_K} + \overbrace{\beta_P \sum_P \mathrm{Re}\,\mathrm{Tr}\, P}^{H_V}$$

Kinetic	**Potential**
One link only	Four links
Diagonal in Fourier space	Diagonal (in our basis)

Trotterization:

$$\begin{aligned} e^{-iHt} &= \left[e^{-iH\epsilon}\right]^{t/\epsilon} \\ &\approx \left[\left(e^{-i\epsilon\beta_K \nabla_1^2} e^{-i\epsilon\beta_K \nabla_2^2} \dots\right)\left(e^{-i\epsilon\beta_P \,\mathrm{Re}\,\mathrm{Tr}\, P_1} e^{-i\epsilon\beta_P \,\mathrm{Re}\,\mathrm{Tr}\, P_2} \dots\right)\right]^{t/\epsilon} \end{aligned}$$

with taking ϵ to be small

$S(1080)$ Circuits under construction

- Inversion gate

$$\mathfrak{U}_{-1}\left|g\right\rangle = \left|g^{-1}\right\rangle$$

- Multiplication gate

$$\mathfrak{U}_{\times}\left|g\right\rangle\left|h\right\rangle = \left|g\right\rangle\left|gh\right\rangle$$

- Trace gate

$$\mathfrak{U}_{\mathsf{Tr}}(\theta)\left|g\right\rangle = e^{i\theta\,\mathsf{Re\,Tr}\,g}\left|g\right\rangle$$

- Fourier Transform gate

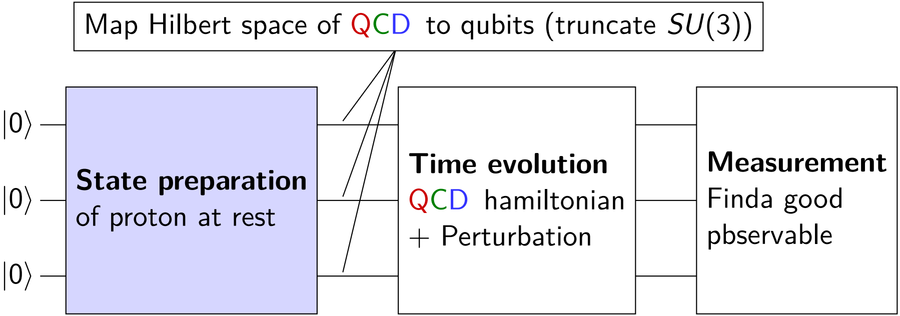
$$\mathfrak{U}_{F}\sum_{g\in G} f(g)\left|g\right\rangle = \sum_{\rho\in\hat{G}} \hat{f}(\rho)_{ij}\left|\rho, i, j\right\rangle$$

- Phase gate for kinetic term $\mathfrak{U}_{\text{phase}}$

Some in progress!

What is needed for PDFs of Proton

$$f_q(P,x) = \int_{-\infty}^{\infty} \frac{dt}{2\pi} e^{-itx(n\cdot P)} \langle P|\bar{\psi}_q(tn^\mu)\gamma^0 W_n(tn^\mu,0)\psi_q(0)|P\rangle$$



Adiabatic State Preparation

Suppose you want the ground state of Hamiltonian H_f

1. Have a time-varying Hamiltonian $H(t)$ with: the ground state of $H(0)$ known and $H_f = H(t_f)$
2. Prepare the ground state of $H(0)$ on quantum computer
3. Time-evolve the state with $H(t)$ until $t = t_f$

Adiabatic theorem guarantees:

When $\dot{H}/\Delta^2 \ll 1$, time-evolution will keep us in the ground state.

where Δ is the gap between ground state and 1st excited state

How much does the method cost?

Time slices in $H(0) \to H(t)$ needed to prepare the ground state $= \Delta^{-2}$

Adiabatic Preparation of Proton

To have a proton as the ground state, restrict to a certin sector of Hilbert space:

- Gauge-invariant states
- Zero total momentum
- Baryon number 1

- Free 3 fermions and gluons
- Small gap $\left(O\left(\frac{1}{L}\right)\right)$

- Hadrons
- Large gap (m_π)

Total circuit size for state preparation; $L^2 \times V, \ (V = L^3)$

What is needed for PDFs of Proton

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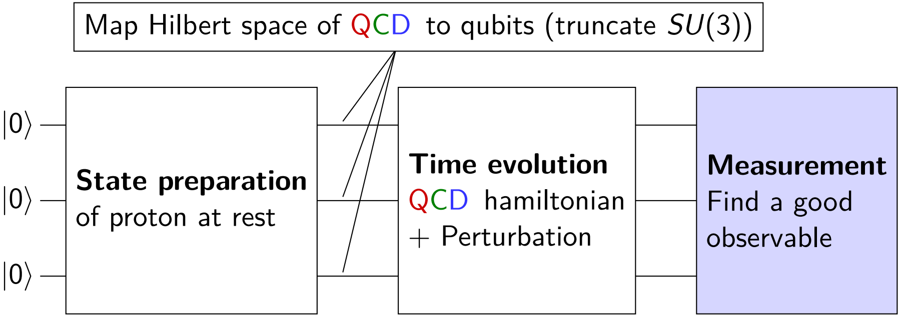


Measurement of Correlators

Real-time correlator

$$\langle J(t)J(0)\rangle = \langle\Psi|\, e^{iHt} J e^{-iHt} J\, |\Psi\rangle$$

This is not a Hermitian operator!
May need to evaluate $\langle\Psi|U(\epsilon_x, \epsilon_0)|\Psi\rangle$, where

$$U(\epsilon_x, \epsilon_0) = e^{iHt} e^{iJ^\mu(\vec{x})\epsilon_x} e^{-iHt} e^{-iJ^\nu(\vec{0})\epsilon_0}$$

and then estimate the derivative

$$\frac{d}{d\epsilon_x}\frac{d}{\epsilon_0}\langle|\Psi U(\epsilon_x, \epsilon_0)|\Psi\rangle = \langle\Psi|J^\mu(x)J^\nu(0)|\Psi\rangle$$

Well, $U(\epsilon_x, \epsilon_0)$ in still not Hermitian...
Though unitary $\rightarrow$ Use ancillary qubit!

Thirring model PDF

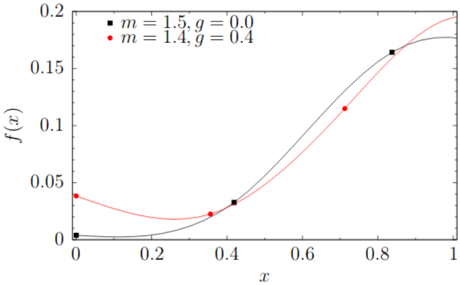
$$H = \int \mathrm{d}x \; \bar{\psi} \left(\partial\!\!\!/ + m \right) \psi + g^2 \left(\bar{\psi}\psi \right)^2$$

Discretization with staggered fermion:

$$H = \sum_r \frac{1}{2}(-1)^r \left(\chi_r^\dagger \chi_{r+1} + \chi_{r+1}^\dagger \chi_r \right) + m(-1)^r \chi_r^\dagger \chi_r - g^2 \chi_r^\dagger \chi_r \chi_{r+1}^\dagger \chi_{r+1}$$

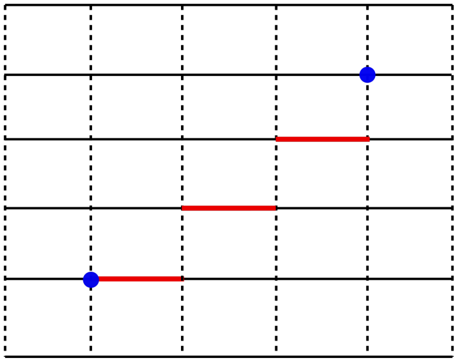
Parton distribution function (No Wilson line!):

$$f(x) = \int \mathrm{d}z \; e^{ixP^+z} \langle P| \, e^{iHz} \bar{\psi}(z) e^{-iHz} \gamma^+ \psi(0) \, |P\rangle$$



With gauge fields, Light cone is hard

Wilson line has to be on "light cone"



$$W(y;0) \approx e^{-iHa} W(y;y-a) e^{-iHa} \cdots e^{-iHa} W(a;0)$$

Lots of finite differencing...

Hadronic tensor

Hadronic tensor is more 'physical',
and PDF can be extracted from it in principle
And, Cross sections may be determined from hadronic tensor.

$$\frac{d^2\sigma}{dx\,dy} = \frac{\alpha^2 y}{Q^4} L_{\mu\nu} W^{\mu\nu}$$

Hadronic tensor:

$$W^{\mu\nu}(q) = \int \mathrm{d}x\; e^{iqx} \left\langle P | e^{iHx^0} J^\mu(\vec{x}) e^{-iHx^0} J^\nu(\vec{0}) | P \right\rangle$$

No Wilson line needed!
J^μ is a *physical* current. (Hermitian so can be added as perturbation)

$$H = H_0 + \epsilon_x(t) J^\mu(\vec{x}) + \epsilon_0(t) J^\nu(\vec{0})$$

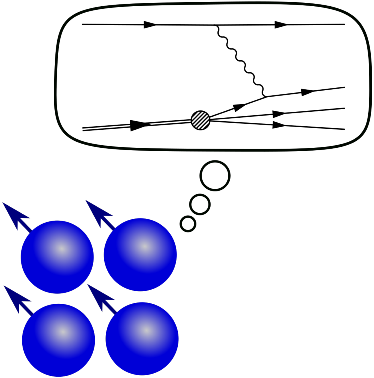
$$\frac{d}{d\epsilon_x} \frac{d}{\epsilon_0} \langle | \Psi U(\epsilon_x, \epsilon_0) | \Psi \rangle = \langle \Psi | J^\mu(x) J^\nu(0) | \Psi \rangle$$

Future

For QCD PDFs:

1. $\sim 10^6$ qubits needed (20^3 lattice)
2. $S(1080)$ exact circuits
3. State preparation details

Other real-time observables?
TMD, viscocity...



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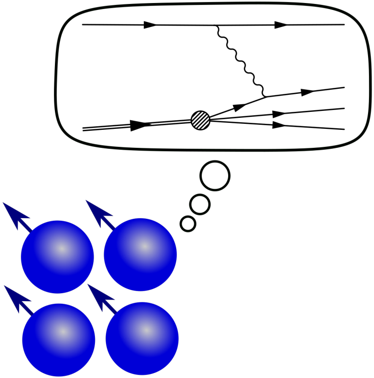


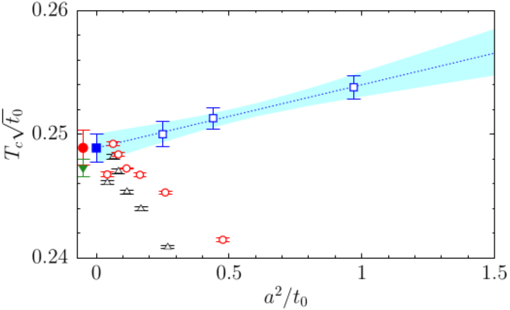
Other real-time observables?
TMD, viscocity...

Thank you!

"Modified" $S(1080)$ action

Measure two scales, and compare the ratio to $SU(3)$: Wilson flow, center symmetry

And test if they agee....



- T_c: center symmetry breaking
- t_0: solution to $0.3 = t_0^2 \langle E \rangle_{t_0}$
- Dimensionless ratio $T_c\sqrt{t_0}$.

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$\rightarrow$ Smallest lattice spacing is $a = 0.08$ fm.

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Do other low-energy quantities agree?

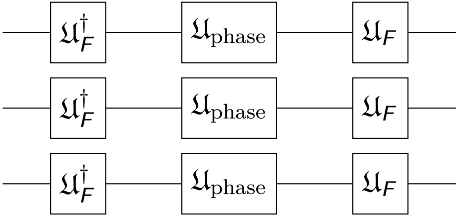
In progress: spectroscopy, further modified actions

Building blocks of Time evolution - Kinetic part

$$H = \overbrace{\beta_K \sum_L \nabla_L^2}^{H_K} + \beta_P \sum_P \mathrm{Re}\,\mathrm{Tr}\, P$$

Diagonal in momentum basis

$\rightarrow$ need Fourier transform gate $\mathfrak{U}_F$ and phase gate $\mathfrak{U}_{\text{phase}}$ (diagonal)



Building blocks of Time evolution Potential part

$$H = \beta_K \sum_L \nabla_L^2 + \overbrace{\beta_P \sum_P \operatorname{Re}\operatorname{Tr} P}^{H_V}$$

We need an operator:

$$\mathcal{U}(\theta)\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle = e^{-i\beta_P \operatorname{Re}\operatorname{Tr}(ABCD)}\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle$$

and thus need inversion $\mathfrak{U}_{-1}$, multiplication $\mathfrak{U}_{\times}$, and trace $\mathfrak{U}_{\mathrm{Tr}}$ gates

$$\begin{aligned}
&|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle \\
\rightarrow\,&|A\rangle\,|B\rangle\,|C\rangle\,|CD\rangle \\
\rightarrow \cdots \rightarrow\,&|A\rangle\,|B\rangle\,|C\rangle\,|ABCD\rangle \\
\rightarrow\,&|A\rangle\,|B\rangle\,|C\rangle\, e^{-i\beta_P \operatorname{Re}\operatorname{Tr}(ABCD)}\,|ABCD\rangle \\
\rightarrow\,& e^{-i\beta_P \operatorname{Re}\operatorname{Tr}(ABCD)}\,\left|A^\dagger\right\rangle\left|B^\dagger\right\rangle\left|C^\dagger\right\rangle|ABCD\rangle \\
\rightarrow \cdots \rightarrow\,& e^{-i\beta_P \operatorname{Re}\operatorname{Tr}(ABCD)}\,|A\rangle\,|B\rangle\,|C\rangle\,|D\rangle
\end{aligned}$$

Measurement of Correlators - Imaginary Part

For a Hermitian operator J,

$$\langle J(t)J(0)\rangle = \langle\Psi| e^{iHt} J e^{-iHt} J |\Psi\rangle$$

This is not a Hermitian operator!

Perturb the Hamiltonian:

$$H'_\epsilon(t) = H + \epsilon\delta(t)\mathcal{O}$$

And now estimate the derivative:

$$\mathrm{Im} \langle\Psi| J(t)J(0) |\Psi\rangle = \frac{1}{2}\frac{\partial}{\partial\epsilon} \langle\Psi| e^{iJ\epsilon} e^{iHt} \mathcal{O} e^{-iHt} e^{-iJ\epsilon} |\Psi\rangle$$