Lattice Calculations for the anomalous magnetic moment of the Muon

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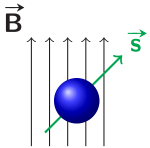
Magnetic moment of leptons $(\mathbf{e}, \mu, \tau)$

- magnetic moment $\vec{\mu}$ of the lepton ℓ due to its spin $\vec{s}$ and electric charge $\mathbf{e}$

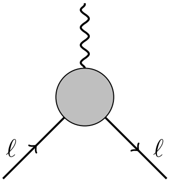
$$\vec{\mu} = g \frac{e}{2m_\ell} \vec{s}$$

torque $\quad \vec{\tau} = \vec{\mu} \times \vec{B}$



- **g**-factor: without quantum fluctuations for a lepton one finds $\mathbf{g = 2}$
- deviation from the value "2" due to quantum loops $\to$ anomalous magnetic moment of lepton ℓ

$$a_\ell = \frac{g_\ell - 2}{2}$$



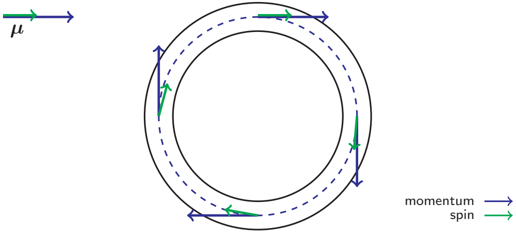
$$\langle \ell(p') | j^\gamma_\mu | \ell(p) \rangle = (-ie)\bar{u}(p') \left[\gamma_\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\ell} F_2(q^2) \right] u(p)$$

- $\mathbf{F_1(0) = 1}$ (electric charge) $\qquad \mathbf{F_2(0) = a_\ell}$ (anomalous magnetic moment)

a_μ: Experiment vs. Theory

- measured and calculated very precisely $\longrightarrow$ test of the Standard Model
- experiment: polarized muons in a magnetic field [Bennet et al., Phys.Rev. **D73**, 072003 (2006)]

$$a_\mu = 11659209.1(5.4)(3.3) \times 10^{-10}$$



$$\omega_a = a_\mu \frac{eB}{m_\mu}$$

- new experiments at Fermilab and JPARC $\rightarrow$ reduce error by $\approx$ **4**
  $\rightarrow$ experiment at Fermilab is running
  $\rightarrow$ first results expected soon

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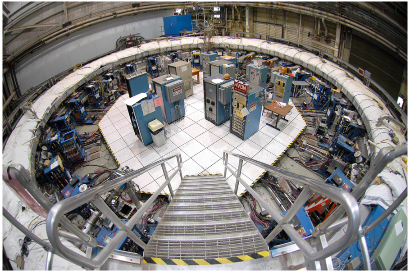
[http://muon-g-2.fnal.gov/bigmove/gallery.shtml]



[Credit: Brookhaven National Laboratory]

[Credit: Fermilab]

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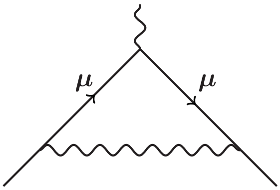

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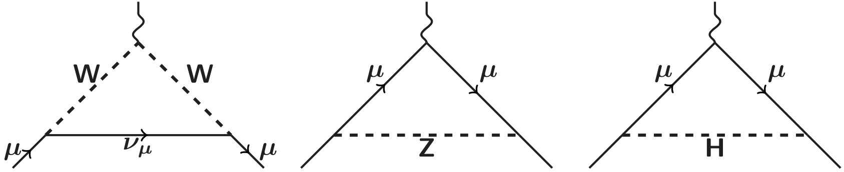

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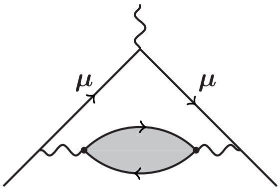

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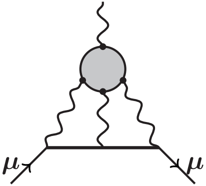

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- Comparison of theory and experiment: **3.8σ** deviation

$$\Delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 27.9(6.3)^{\text{Exp}}(3.6)^{\text{SM}} \times 10^{-10}$$

required precision to match upcoming experiments $\Delta a_\mu^{\text{hvp}} \lesssim 0.2\%$ $\Delta a_\mu^{\text{lbl}} \lesssim 10\%$

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	$\Delta a_\mu^{\text{hvp}}$
target	$\lesssim 0.2\%$
current **R**-ratio	$\approx 0.5\%$
current lattice	$\approx 2 - 3\%$

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- light quark contribution
- strange and charm quark contribution
- disconnected contribution
- Isospin Breaking corrections to the HVP
- Summary and Prospects

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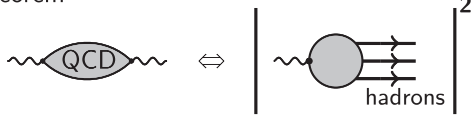
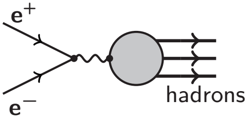
Hadronic Vacuum Polarisation (HVP) from the **R**-ratio

- current best theoretical estimate uses experimental data
- optical theorem

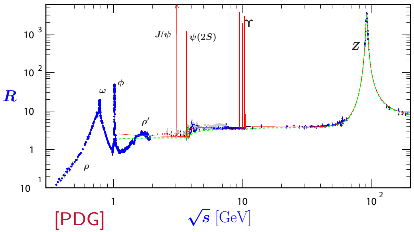


- **R**-ratio $$R(s) = \frac{\sigma(e^+ e^- \to \text{hadrons}, s)}{\sigma(e^+ e^- \to \mu^+\mu^-, s)}$$



$$a_\mu^{hvp} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{m_\pi^2}^{\infty} ds \, \frac{R(s)K(s)}{s^2}$$

- first principles calculation of HVP → lattice QCD



recent results:

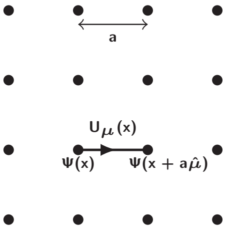
$a_\mu^{hvp} = 689.46(3.25)$ [Jegerlehner 18]

$a_\mu^{hvp} = 693.9(4.0)$ [DHMZ 19]

$a_\mu^{hvp} = 693.37(2.46)$ [KNT 18]

$\approx$ **0.5%** precision

QCD on the lattice



- Wick rotation $(t \to -ix_0)$ to Euclidean space-time
- Discretize space-time by a hypercubic lattice Λ
- Quantize QCD using Euclidean path integrals

$$\langle A \rangle = \frac{1}{Z} \int \mathcal{D}[\Psi, \overline{\Psi}]\mathcal{D}[U]\, e^{-S_E[\Psi,\overline{\Psi},U]}\, A(U, \Psi, \bar{\Psi})$$

$\longrightarrow$ can be split into fermionic and gluonic part

- Calculate gluonic expectation values using Monte Carlo techniques:

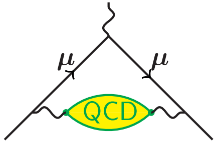
$$\langle\langle A \rangle_F\rangle_G = \int \mathcal{D}[U]\, \langle A \rangle_F\, P(U) \approx \frac{1}{N_{cfg}} \sum_{n=1}^{N_{cfg}} \langle A \rangle_F$$

average over gluonic gauge configurations U distributed according to

$$P(U) = \frac{1}{Z} (\det D)^{N_f}\, e^{-S_G[U]}$$

- extrapolate to the continuum $(a \to 0)$ and infinite volume $(V \to \infty)$

Hadronic Vacuum Polarisation (HVP) from the Lattice



- $\Pi_{\mu\nu}(Q) \equiv \int d^4x \, e^{iQ\cdot x} \left\langle j^\gamma_\mu(x) \, j^\gamma_\nu(0) \right\rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2)\, \Pi(Q^2)$
- electromagnetic current $\quad j^\gamma_\mu = \frac{2}{3}\bar{u}\gamma_\mu u - \frac{1}{3}\bar{d}\gamma_\mu d - \frac{1}{3}\bar{s}\gamma_\mu s + \frac{2}{3}\bar{c}\gamma_\mu c$
- hadronic contribution to the anomalous magnetic moment of the muon
  [T. Blum, Phys.Rev.Lett.91, 052001 (2003)]

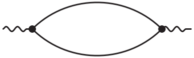
$$a_\mu^{\mathrm{hvp}} = \left(\frac{\alpha}{\pi}\right)^2 \int_0^\infty dQ^2 \, K(Q^2)\, \hat{\Pi}(Q^2) \qquad \text{with} \qquad \hat{\Pi}(Q^2) = 4\pi^2 \left[\Pi(Q^2) - \Pi(0)\right]$$

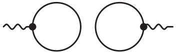
- subtracted HVP from vector correlator [Bernecker and Meyer, Eur.Phys.J. **A47**, 148 (2011)]

$$C(t) = \frac{1}{3}\sum_{k=0}^{2}\sum_{\vec{x}} \langle j^\gamma_k(\vec{x},t) j^\gamma_k(0)\rangle \qquad \hat{\Pi}(Q^2) = 4\pi^2 \int_0^\infty dt\, C(t) \left[\frac{\cos(Qt)-1}{Q^2} + \frac{1}{2}t^2\right] \qquad a_\mu^{\mathrm{hvp}} = \int_0^\infty dt\, f(t) C(t)$$

- flavour decomposition (isospin symmetric QCD)

$$C(t) = \frac{5}{9}C^\ell(t) + \frac{1}{9}C^s(t) + \frac{4}{9}C^c(t) + C^{\mathrm{disc}}(t)$$





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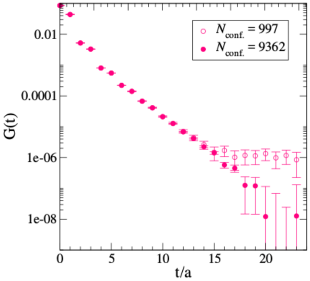
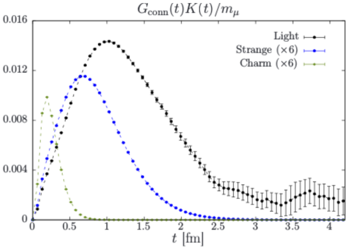
Vector correlator and long distance Signal-to-Noise problem

- examples for light-quark vector correlator at physical point



[C. Davies *et al*, arXiv:1902.04223]



[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510]

- signal deteriorates for large **t**
- need noise reduction techniques to control statistical error on raw data
  - all-mode-averaging (AMA) [T. Blum *et al.*, Phys. Rev. **D88**, 094503 (2013)], [G. Bali *et al.*, Comput.Phys.Commun. 181 (2010) 1570-1583]
  - huge reduction in error when using low-mode-averaging (LMA)
    [T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)], [C. Aubin *et al*, arXiv:1905.09307]
- possible strategy: replace correlator by (multi-) exponential fit for $\mathbf{t} > \mathbf{t_c}$

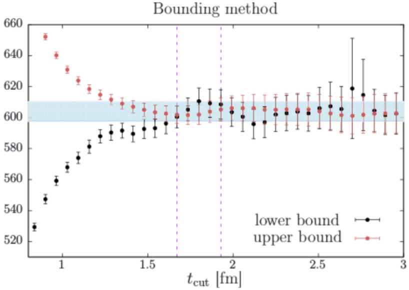
Bounding method

- spectral representation of the vector correlator

$$C(t) = \sum_n \frac{A_n^2}{2E_n} e^{-E_n t} \qquad A_n^2 > 0$$

- bound for the correlator for $t \geq t_c$
[S. Borsanyi *et al.*, Phys. Rev. D 96, 074507 (2017)],
[T. Blum, VG, *et al.*, Phys. Rev. Lett. 121, 022003 (2018)]

$$0 \leq C(t_c)\, e^{-E_{t_c}(t-t_c)} \leq C(t) \leq C(t_c)\, e^{-E_0(t-t_c)}$$



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- E_{t_c}: effective mass of the correlator at t_c
- E_0: finite volume ground state energy, two pions with one unit of momentum
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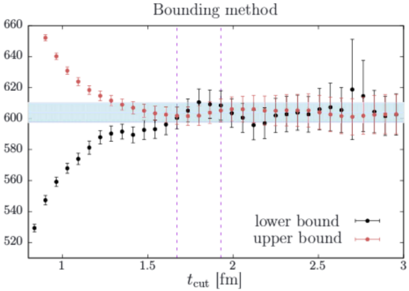

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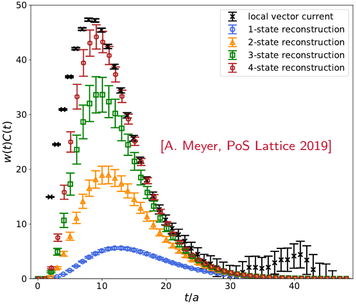

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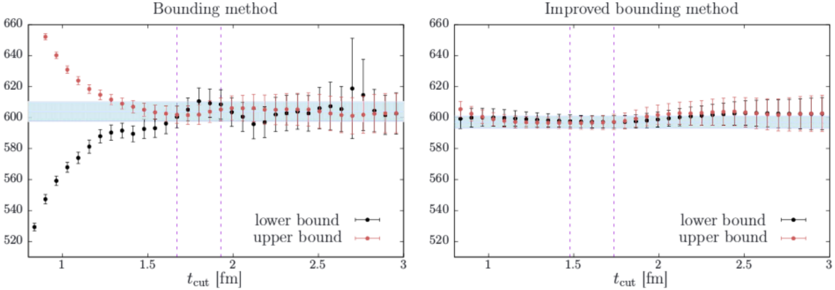

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Light quark contribution systematics

- Finite volume effects
  - dominated by two pion state - important at large t
  - finite volume effects of $\sim \mathcal{O}(20 - 30 \times 10^{-10})$ for typical lattice sizes $\sim \mathcal{O}(5 - 6\text{ fm})$ at physical point, see e.g. [E. Shintani, Y. Kuramashi, arXiv:1902.00885], [C. Lehner @ Lattice 2019], [C. Aubin *et al*, arXiv:1905.09307]

- scale setting
  - a_μ^{hvp} depends on the scale through am_μ in the kernel
  - relative error on quantity used for scale setting amplified by ≈ 1.8 in relative error for a_μ [M. Della Morte, VG, *et al*, JHEP 1710 (2017) 020]

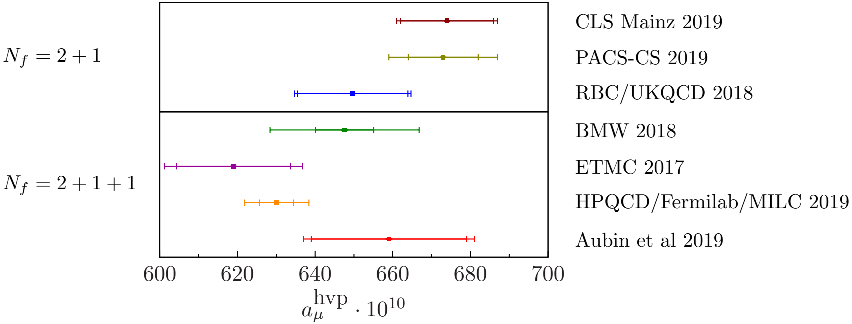
    $\rightarrow$ for 0.2% error on a_μ^{hvp} need $\lesssim 0.1\%$ on lattice spacing

- extrapolation to the physical point
  - chiral extrapolation (if necessary)
  - continuum extrapolation

    $\rightarrow$ work in fully $\mathcal{O}(a)$ improved setup

    $\rightarrow$ ideally at least three lattice spacings

comparison - light quark results



- errors from **1.3% − 3.3%**
- $\approx$ **2σ** discrepancy between smallest and largest results

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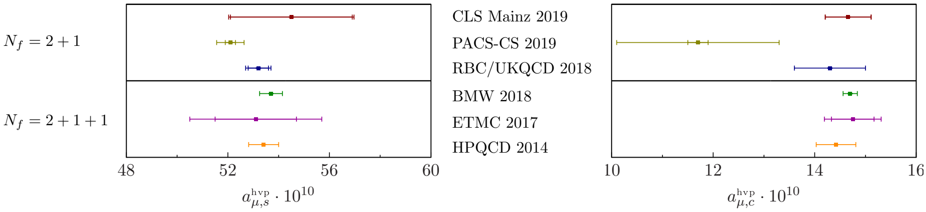
Strange and Charm HVP

- suffers less from long-distance noise-to-signal problem and finite volume effects than light contribution
- charm usually large discretization effects



- errors on total HVP $\lesssim$ **0.4%** $\lesssim$ **0.3%**

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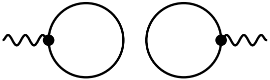
disconnected HVP

- quark-disconnected Wick contraction
- **SU(3)** suppressed
- quark loop



$$\Delta^f_\mu(t) = \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_\mu S^f(x,x)\right]$$

- all-to-all propagators, calculate stochastically
- light-strange cancellation [V.G. *et al*, PoS LATTICE2014 (2014) 128]

$$C^{disc}(t) = \frac{1}{9}\left\langle (\Delta^\ell(t) - \Delta^s(t)) \cdot (\Delta^\ell(0) - \Delta^s(0)) \right\rangle$$

- further noise reduction
  - [T. Blum *et al*, Phys. Rev. Lett. 116, 232002 (2016)] low-mode averaging and sparsened noise sources for high modes
  - [A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510] hierarchical probing [A. Stathopoulos *et al*, arXiv:1302.4018]
  - frequency-splitting estimators [L. Giusti *et al*, arXiv:1903.10447]

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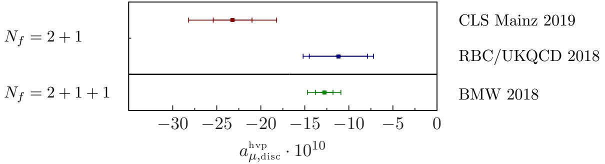
- **SU(3)** suppressed
- quark loop

$$\Delta^f_\mu(t) = \sum_{\vec{x}} \mathrm{Tr}\left[\gamma_\mu S^f(x,x)\right]$$



- errors on total HVP **0.3 – 0.7%**

- all-to-all propagators, calculate stochastically
- light-strange cancellation [V.G. *et al*, PoS LATTICE2014 (2014) 128]

$$C^{\mathrm{disc}}(t) = \frac{1}{9}\left\langle \left(\Delta^\ell(t) - \Delta^s(t)\right)\cdot\left(\Delta^\ell(0) - \Delta^s(0)\right)\right\rangle$$

- further noise reduction
  - [T. Blum *et al*, Phys. Rev. Lett. 116, 232002 (2016)] low-mode averaging and sparsened noise sources for high modes
  - [A. Gérardin *et al*, Phys.Rev. D100 (2019) no.1, 014510] hierarchical probing [A. Stathopoulos *et al*, arXiv:1302.4018]
  - frequency-splitting estimators [L. Giusti *et al*, arXiv:1903.10447]

Outline

Isospin Breaking Corrections

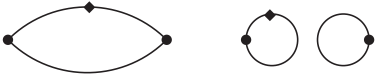
- lattice calculations usually done in the isospin symmetric limit
- two sources of isospin breaking effects
  - different masses for up- and down quark (of $\mathcal{O}((m_d - m_u)/\Lambda_{QCD}))$
  - Quarks have electrical charge (of $\mathcal{O}(\alpha)$)
- lattice calculation aiming at $\lesssim$ **1%** precision requires to include isospin breaking

- separation of strong IB and QED effects requires renormalization scheme
- definition of "physical point" in a "QCD only world" also scheme dependent
  $\rightarrow$ results shown above without QED and isospin breaking for $m_\pi \approx 135$ MeV

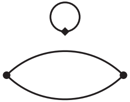
Isospin Breaking Corrections from the Lattice

- strong Isospin Breaking on the Lattice
  - use different up, down quark masses
  - perturbative expansion in $\mathbf{\Delta m = (m_u - m_d)}$ [G.M. de Divitiis *et al*, JHEP 1204 (2012) 124]

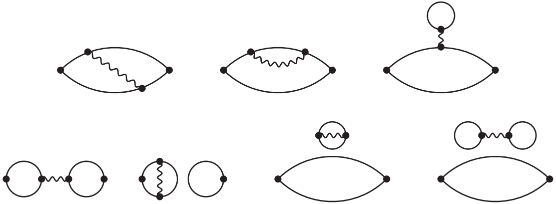
$$\langle O\rangle_{m_u\neq m_d} = \langle O\rangle_{m_u=m_d} + \Delta m \frac{\partial}{\partial m}\langle O\rangle\Big|_{m_u=m_d} + \mathcal{O}\left(\Delta m^2\right)$$



sea quark effects:
quark-disconnected diagrams



- QED: perturbative expansion of the path integral in α [RM123 Collaboration, Phys.Rev. **D87**, 114505 (2013)]



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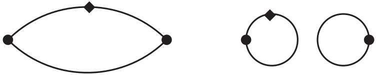

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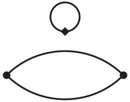

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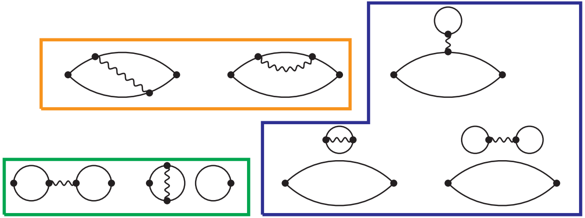

quark-connected

quark-disconnected

sea-quark effects

Results QED corrections

- Summary results lattice calculations for isospin breaking corrections

ETMC 19	$\mathbf{7.1(2.9) \times 10^{-10}}$	connected QED and connected strong IB no sea quark effects
RBC/UKQCD 18	$\mathbf{9.5(10.2) \times 10^{-10}}$	connected and leading disconnected QED and connected strong IB no sea quark effects
HPQCD/FermiLab/MILC 18	$\mathbf{9.5(4.5) \times 10^{-10}}$	dynamical strong IB (including sea-quarks) no QED effects

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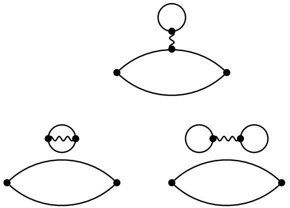
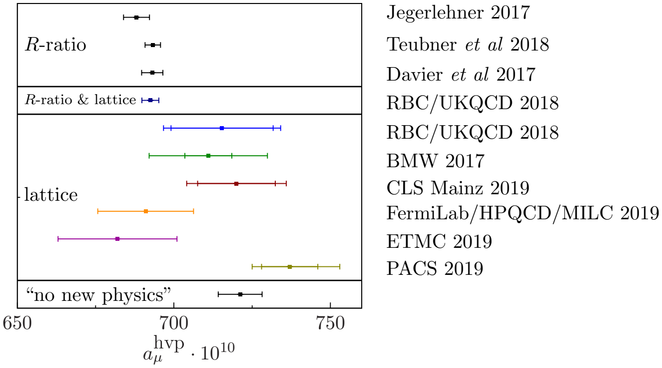
- QED corrections from sea-quark effects
- diagrams at least $\mathbf{1/N_c}$ suppressed
  $\rightarrow$ could be **33%** of connected
  $\rightarrow$ need to be studied for sub-percent precision on total HVP



Outline

Full HVP comparison



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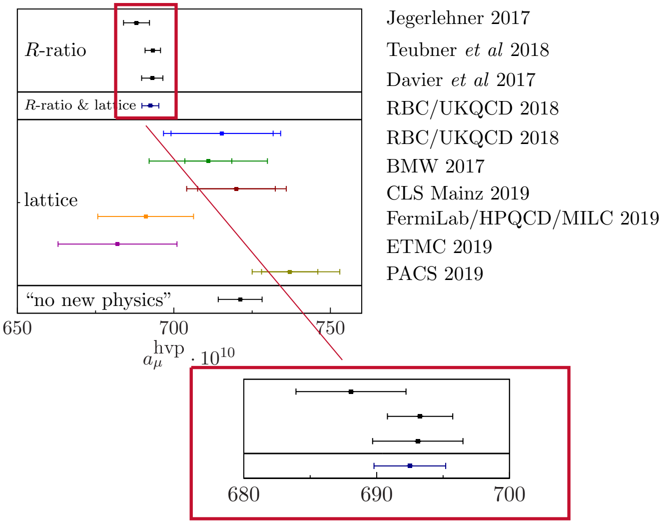

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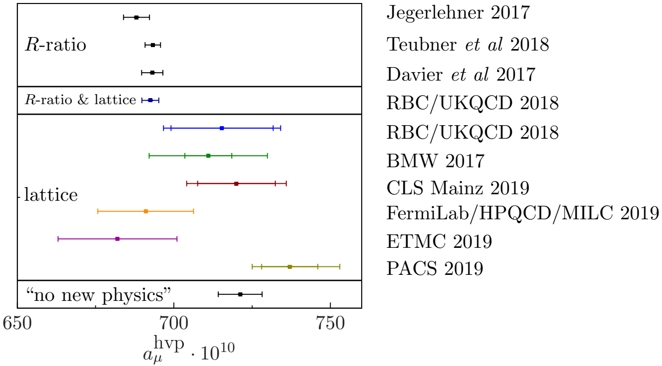
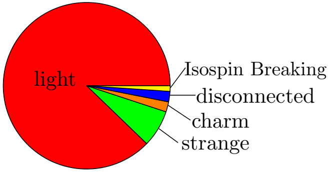


contribution to a_μ^{hvp}



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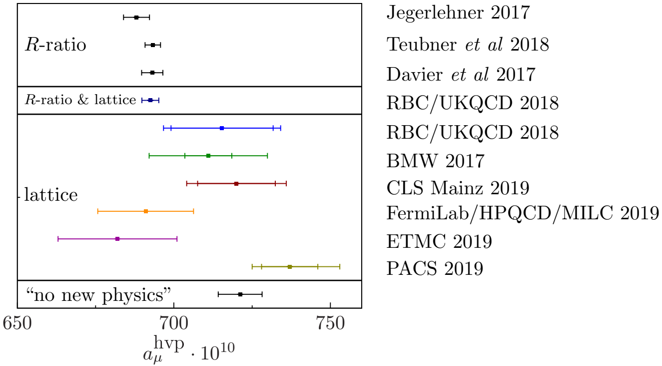
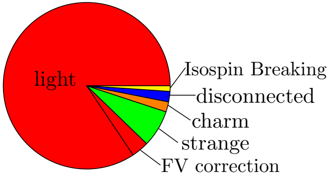


contribution to $\mathbf{a}_{\boldsymbol{\mu}}^{\rm hvp}$



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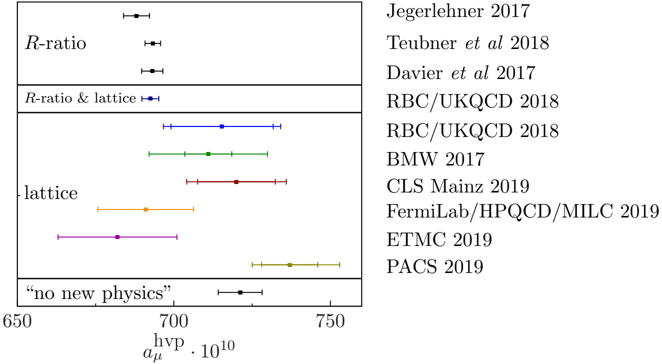

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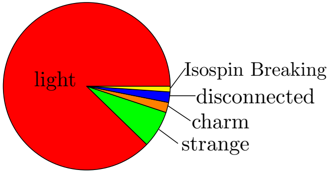
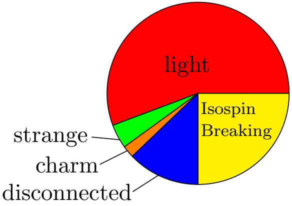


contribution to $\mathbf{\Delta a}_{\boldsymbol{\mu}}^{\rm hvp} \approx \mathbf{2.5\%}$



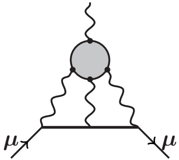
Conclusions and Prospects

- most important issues:
  - noise reduction and control of long-distance tail of the light quark correlator
  - careful estimate of finite volume effects
  - first lattice calculations of isospin breaking and QED corrections
    $\longrightarrow$ study also sea quark effects
  - achieve consensus between lattice results

Outline

Introduction

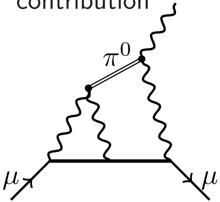
- hadronic light-by-light scattering enters at α^3



- Glasgow consensus [J. Prades, E. de Rafael, A. Vainshtein, Adv.Ser.Direct.High Energy Phys. **20** (2009) 303-317]

π^0, η, η'	$(11.4 \pm 1.3) \times 10^{-10}$
charged π loop	$(-1.9 \pm 1.9) \times 10^{-10}$
axialvector	$(1.5 \pm 1.0) \times 10^{-10}$
scalar	$(-0.7 \pm 0.7) \times 10^{-10}$
charm loops	0.2×10^{-10}
total	$(10.5 \pm 2.6) \times 10^{-10}$

e.g. π^0 contribution



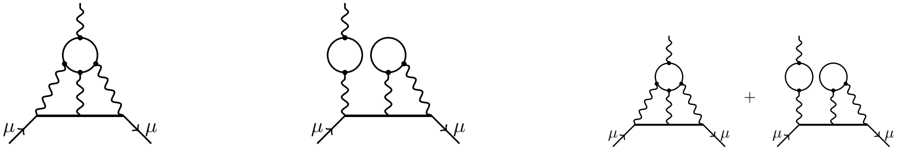
- work in progress using dispersion relations, e.g. [G. Colangelo *et al*, JHEP 1902 (2019) 006], [G. Colangelo *et al*, Phys.Rev.Lett. 118 (2017) no.23, 232001], [G. Colangelo *et al*, JHEP 1704 (2017) 161], [M. Hoferichter *et al*, Phys.Rev.Lett. 121 (2018) no.11, 112002], [V. Pauk *et al*, Phys.Rev. D90 (2014) no.11, 113012], . . .

- lattice calculations: two collaborations working on this: RBC/UKQCD and Mainz

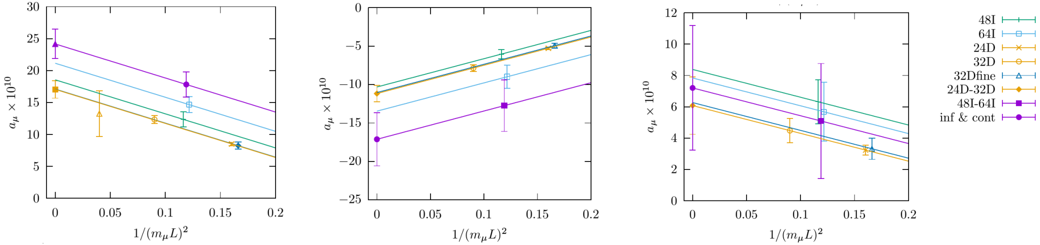
Outline

Results RBC/UKQCD connected + leading disconnected light-by-light

- recent results [T. Blum *et al*, arXiv:1911.08123]



- continuum and infinite volume extrapolation $\mathbf{QED_L}$



$$\mathbf{a_\mu^{lbl} = 7.20(3.98)(1.65) \times 10^{-10}}$$

Outline

Conclusions - light-by-light

- two collaborations working on lattice calculations
  - RBC/UKQCD: result extrapolated to physical point [T. Blum *et al*, arXiv:1911.08123]
  - Mainz: work in progress, see e.g. [N. Asmussen *et al*, PoS Lattice 2019, arXiv:1911.05573]
- important check: consistency with Glasgow Consensus

  $\rightarrow$ would need $\approx \mathbf{3}\times$ larger $\mathbf{a}_\mu^{\mathbf{lbl}}$ than Glasgow Consensus to explain $\mathbf{a}_\mu$ discrepancy

  $\rightarrow$ lattice results suggest this is unlikely

- lattice calculations of the pion transition form factor $\pi^0 \rightarrow \gamma\gamma$
  [A. Gérardin *et al*, Phys.Rev. D94 (2016) no.7, 074507],[A. Gérardin *et al*, Phys.Rev. D100 (2019) no.3, 0345201]

  $\rightarrow$ pion pole contribution to $\mathbf{a}_\mu^{\mathbf{lbl}}$

  $\rightarrow$ constrain long-distance tail to $\mathbf{a}_\mu^{\mathbf{lbl}}$ lattice calculation

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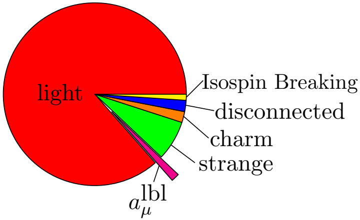

size of light-by-light vs HVP

Outline

Final remarks

- $\mathbf{a}_\mu$ measured and calculated very precisely
  $\rightarrow$ test of the Standard Model
  $\rightarrow$ new experiment running at Fermilab
  $\rightarrow$ largest uncertainty in Standard Model prediction from hadronic contributions
- huge effort in the lattice community to calculate hadronic contributions from first principles
- work in progress on g-2 Theory Whitepaper from the Muon g-2 Theory Initiative,
  several workshops since 2017, last workshop: September 9 - 13, 2019 at INT

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- hadronic vacuum polarisation contribution to $\mathbf{a}_\mu$
  - first lattice calculations of $\mathbf{a}_\mu^{\mathrm{hvp}}$ with $\lesssim \mathbf{1\%}$ precision available within $\mathcal{O}(\text{year})$
  - $\lesssim \mathbf{0.2\%}$ within a few years

Thank you

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