UNIVERSITÀ DEGLI STUDI DI BARI ALDO MORO
DIPARTIMENTO INTERATENEO DI FISICA "M. MERLIN"
INFN Istituto Nazionale di Fisica Nucleare

Spin-waves and multimerization for many-body bound states in the continuum in one-dimensional qubit arrays

Domenico Pomarico

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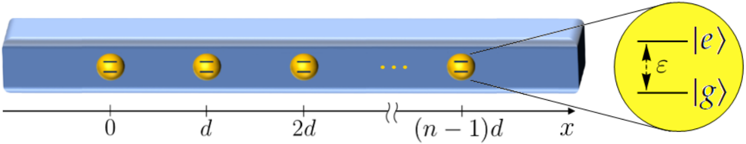
The XVIII Workshop on Statistical Mechanics and Nonperturbative Field Theory

in collaboration with: P. Facchi, D. Lonigro, S. Pascazio, F. V. Pepe (Bari)

Experimental platform: Waveguide QED

- n two-level emitters, spacing d;
- structured 1D photon continuum with $\omega(k) = \sqrt{k^2 + m^2}$.



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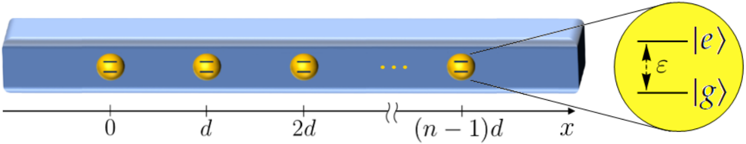
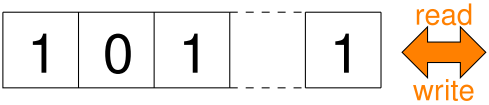


A possible implementation of noise-free memory in processors:

REGISTER



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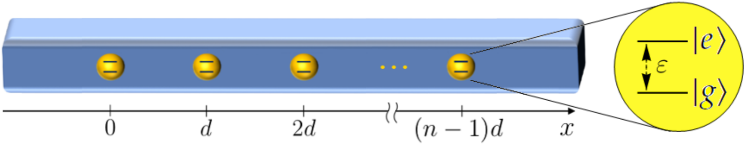

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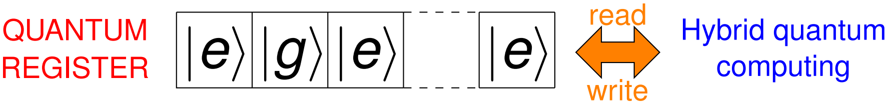

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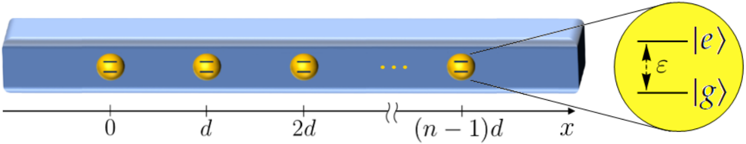
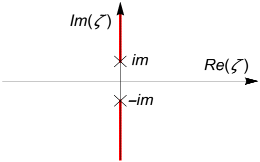


A possible implementation of noise-free memory in processors:

QUANTUM REGISTER $|e\rangle\,|g\rangle\,|e\rangle \cdots |e\rangle$ read ⟷ write Hybrid quantum computing

A generic dispersion relation is conditioned by:

$\omega(k) = \omega(-k)$ with complex extension for $\zeta = k + \mathrm{i}y$ analytic in a strip of the complex plane $\mathbb{R} \times (-m, m)$



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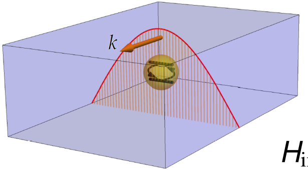

$$H = H_0 + H_{\text{int}}$$

$$H_0 = \varepsilon \sum_{j=1}^{n} |e_j\rangle\langle e_j| + \int \mathrm{d}k\, \omega(k) b^\dagger(k) b(k)$$

$$H_{\text{int}} = \sum_{j=1}^{n} \int \mathrm{d}k \left[F(k)\, \mathrm{e}^{\mathrm{i}(j-1)kd} |e_j\rangle\langle g_j| b(k) + h.c. \right]$$

with $[b(k), b^\dagger(k')] = \delta(k-k')$ and form factor $F(k) = \sqrt{\frac{\gamma}{2\pi\omega(k)}}$. ← coupling constant

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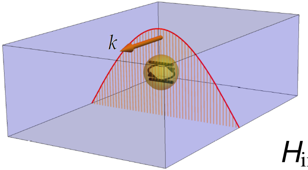

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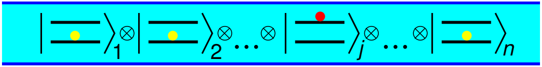
One excitation sector:

$$|\Psi\rangle = \sum_{j=1}^{n} a_j |E_j^{(n)}\rangle \otimes |\mathrm{vac}\rangle + |G^{(n)}\rangle \otimes \int \mathrm{d}k\, \xi(k) b^\dagger(k) |\mathrm{vac}\rangle = \begin{pmatrix} |\mathbf{a}\rangle \\ |\xi\rangle \end{pmatrix} = \begin{pmatrix} a_1 \\ a_2 \\ \vdots \\ a_n \\ |\xi\rangle \end{pmatrix}$$

with $|E_j^{(n)}\rangle =$



Bound states for $n = 3, 4$

Inverse propagator: $G^{-1}(z) = (z-\varepsilon)\mathbb{1} - \boxed{\Sigma(z)} \longleftarrow$ Self-energy matrix

$$G^{-1}(E)|\mathbf{a}\rangle = 0$$

Resonance eigenvalues

$$E_\nu(d) = \sqrt{\frac{\nu^2\pi^2}{d^2} + m^2}$$

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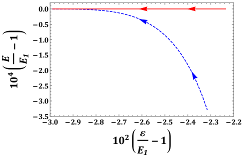
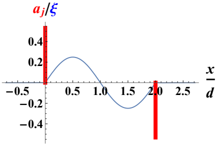
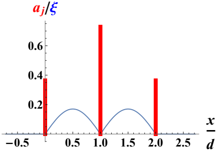


Eigenstates components



$\frac{a_1}{a_3} = -1,\ a_2 = 0$



$\frac{a_1}{a_3} = 1,\ a_2 \approx 2$

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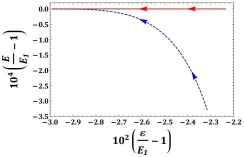

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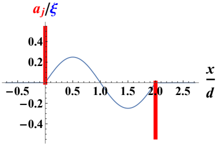

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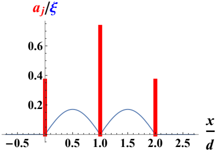

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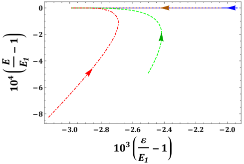

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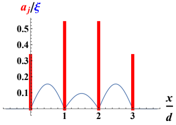
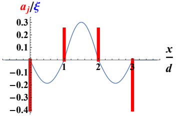
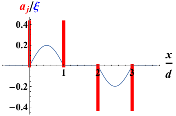


$\frac{a_1}{a_4} = \frac{a_2}{a_3} = 1,\ \frac{a_1}{a_2} \approx \frac{\sqrt{5}-1}{2}$



$\frac{a_1}{a_4} = \frac{a_2}{a_3} = 1,\ \frac{a_1}{a_2} \approx -\frac{\sqrt{5}+1}{2}$



$\frac{a_1}{a_2} = \frac{a_3}{a_4} = -\frac{a_2}{a_3} = 1$

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Graph Laplacian self-energy matrix

$$\mathcal{U}_n \Sigma_n(\varphi) \mathcal{U}_n^\dagger = \Sigma^-_{\lfloor n/2 \rfloor}(\varphi) \oplus \Sigma^+_{\lceil n/2 \rceil}(\varphi)$$

with $\mathcal{U}_n$ given by Toeplitz-Hankel eigenvectors central symmetry

$n = 2h + 1$

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$$\Sigma_h^-(\varphi) = \begin{pmatrix} 2\varphi & 1 & & & \\ 1 & 2\varphi & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2\varphi & 1 \\ & & & 1 & 2\varphi \end{pmatrix}$$

Dirichlet-Dirichlet boundary: $a_0 = a_{h+1} = 0$

$$\lambda_j = -4 \sin^2\left(\frac{j\pi}{2(h+1)}\right)$$

$$a_\ell^{(j)} = \sqrt{\frac{2}{h+1}} \sin\left(\frac{j\pi}{h+1}\ell\right)$$

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$n = 2h$

$$\Sigma^\pm_h(\varphi) = \begin{pmatrix} 2\varphi & 1 & & & \\ 1 & 2\varphi & 1 & & \\ & \ddots & \ddots & \ddots & \\ & & 1 & 2\varphi & 1 \\ & & & 1 & 2\varphi \pm 1 \end{pmatrix}$$

Dirichlet-(anti)Neumann boundary: $a_0 = 0$, $a_h = \pm a_{h+1}$

$$\lambda_j = \mp 4\sin^2\left(\frac{(j-1/2)\pi}{2h+1}\right)$$

$$a^{(j)}_\ell = \sqrt{\frac{2}{h+\frac{1}{2}}} (\pm 1)^{\ell+1} \sin\left(\frac{\left(j-\frac{1}{2}\right)\pi}{h+\frac{1}{2}}\ell\right)$$

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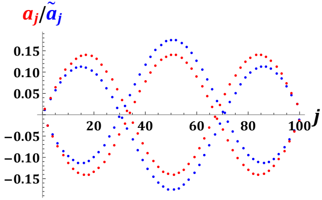
Rank-one modification of the symmetric eigenproblem

$$\widetilde{\Sigma}_{\lceil n/2 \rceil}(b) = b\Sigma_{\lceil n/2 \rceil}(\varphi) - n\,\mathrm{i}\,|\mathbf{u}\rangle\langle\mathbf{u}| \quad \text{with eigenvectors } |\widetilde{\mathbf{a}}^{(j)}\rangle$$

$$\Sigma_{\lceil n/2 \rceil}(\varphi) = \Xi \Lambda \Xi^T \Longrightarrow \widetilde{\Sigma}_{\lceil n/2 \rceil}(b) = \Xi\,(b\Lambda - n\,\mathrm{i}\;|\mathbf{z}\rangle\langle\mathbf{z}|)\,\Xi^T = \widetilde{\Xi}\,\widetilde{\Lambda}\,\widetilde{\Xi}^T$$

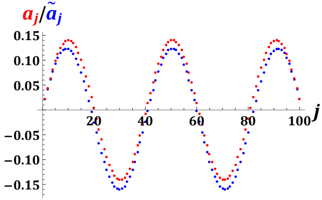
$\nu = 1$

$|\widetilde{\mathbf{a}}^{(1)}\rangle$



$\nu = 2$

$|\widetilde{\mathbf{a}}^{(2)}\rangle$



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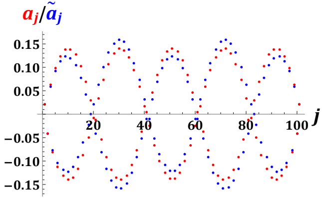

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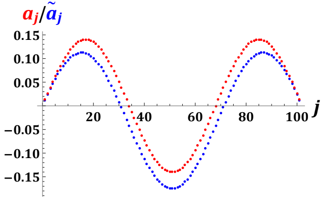

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Determinant factorization: $n = 2h + 1$

$$G^{-1}(E) = -\frac{\mathrm{i}\gamma}{\sqrt{E^2 - m^2}} A_n(E), \quad \det A_h^-(E) = U_h(E)$$

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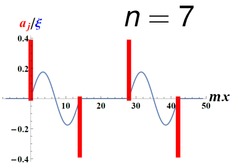
↑ Chebyshev polynomial of the second kind



$$h = 2p + 1 : \det A_h^-(E) = U_h(E) = 2T_{\lceil h/2 \rceil}(E) U_{\lfloor h/2 \rfloor}(E)$$

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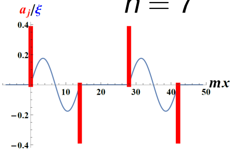
Chebyshev polynomial of the second kind

$n = 7$



$h = 2p + 1$: $\det A_h^-(E) = U_h(E)$

$$= 2\boxed{T_{\lceil h/2 \rceil}(E)} U_{\lfloor h/2 \rfloor}(E)$$

Irriducible factor: Chebyshev polynomial of the first kind

Property: $2T_n U_l = U_{l+n} + U_{l-m}$

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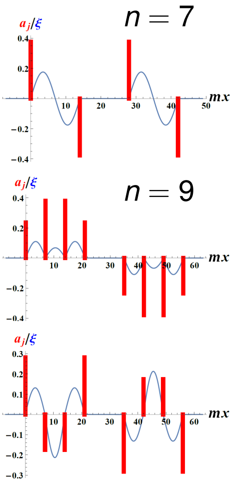
$$= \left(U_{\frac{h}{2}}(E) + U_{\frac{h}{2}-1}(E)\right)\left(U_{\frac{h}{2}}(E) - U_{\frac{h}{2}-1}(E)\right)$$

Property: $U_n U_l = \sum_{k=0}^{l} U_{n-l+2k}$



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Conclusions & Outlook

- Fermi's golden rule: $|\Psi(0)\rangle = \begin{pmatrix} |\mathbf{a}^{(j)}\rangle \\ |\xi\rangle \end{pmatrix} \longrightarrow |\Psi(t)\rangle$?

$$\begin{pmatrix} \Xi \\ \widetilde{\Xi} \end{pmatrix} \begin{pmatrix} G & 0 \\ 0 & \widetilde{G} \end{pmatrix} \begin{pmatrix} \Xi^T & \widetilde{\Xi}^T \end{pmatrix} = \mathrm{diag}\left(\left\{ \frac{1}{E - \varepsilon - \Delta_{\mathbf{a}^{(j)}} + \frac{\mathrm{i}}{2}\Gamma_{\mathbf{a}^{(j)}}} \right\}_j \right)$$

Breit-Wigner expansion: $\Gamma_{\mathbf{a}^{(j)}}(\varepsilon) = 2\pi\kappa_{\mathbf{a}^{(j)}}(\varepsilon)$

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Experimental study of dark states, with the aim of implementig a noise-free memory able to represent a quantum register;

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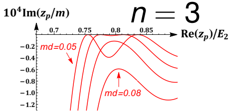
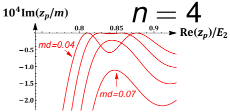
- High-energy eigenstates pair:

$n = 3$



$n = 4$



k nearest-neighbors study for a high qubits number;

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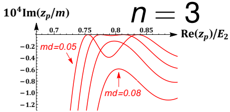

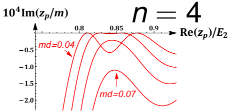

k nearest-neighbors study for a high qubits number;

- Multimode Waveguide QED: implementation of qubit arrays coupled with more than a single mode.