Calculation of ^{6}Li ground state within the Hyperpsherical Harmonics basis

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Why ^{6}Li?

- First nucleus beyod $A = 5$ mass gap
- Weakly bound $\Rightarrow$ prominent $\alpha + d$ structure
- Good laboratory to study nuclear forces

Chiral interaction (χEFT)

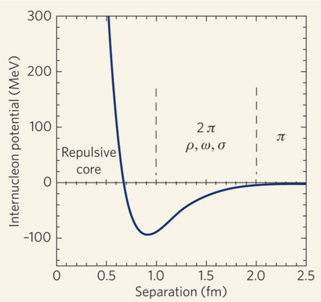
$$\text{QCD} \xrightarrow{\text{chiral symmetry}} \chi\text{EFT}$$

- Low Energy Theory ($\Lambda_\chi \sim 1$ GeV)
  - N, π as d.o.f.
  - high energy d.o.f. integrated out $\rightarrow$ Low Energy Constants
- Perturbative expansion ($\propto (Q/\Lambda_\chi)^\nu$)
- Regularization with a cutoff ($\Lambda = 400 - 600$ MeV)
- Theoretical uncertainties
  - order-by-order expansion
  - Λ-dependence
- It is possible to derive in a self-consistent way the interaction with various probes: electro-weak, dark matter, ...

Nuclear potential

$$H = \sum_i \frac{p_i^2}{2M} + \sum_{i<j} V(i,j) + \sum_{i<j<k} W(i,j,k) + \dots$$

Search for accurate solution of $H\Psi = E\Psi$



The Hyperspherical Harmonics method

- Variational approach
- Expansion on a base $\Rightarrow$ Hyperspherical Harmonics (HH)

$$\Big(\sum_{i=1}^{A-1}\nabla_i\Big)\mathcal{Y}_{[K]}(\Omega_{A-1}) = K(K+3(A-1)-2)\mathcal{Y}_{[K]}(\Omega_{A-1})$$

- The variational wave function

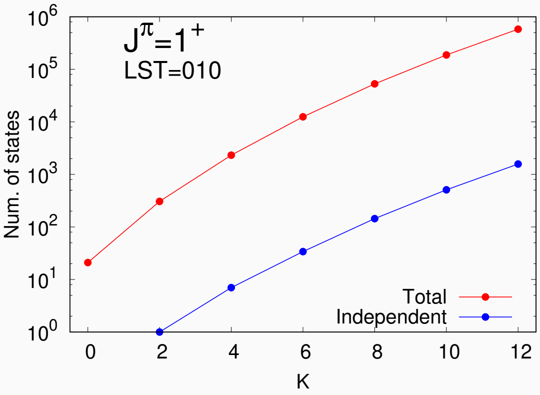
$$\psi_A = \sum_{l,[K]} a_{l,[K]} f_l(\rho)\mathcal{Y}_{[K]}(\Omega_{A-1})\Big[\chi_S \otimes \chi_T\Big],$$

- Fermions $\Rightarrow$Complete anti-symmetrization
- Increase K up to convergence
- Applied for A=3,4 $\Rightarrow$ now A=6 using a new computational approach

[A. Kievsky, *et al.*, J. Phys. G: Nucl. Part. Phys. **35**, 063101(2008).]

The HH basis

- Ground state of ^{6}Li is $J^\pi = 1^+$ (isospin $T = 0$)



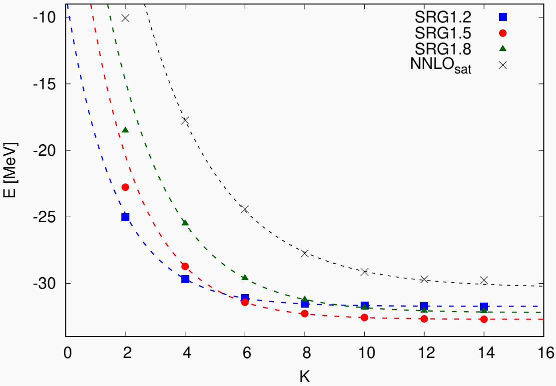
- Up to $K = 14$ we use an equivalent of 1.5×10^8 quantum states
- Number of independent states 10^5

Warning!

- We will use SRG evolved N^3LO500 *NN* interaction [1-2]
  - The Coulomb interaction is included as "bare" (not SRG evolved)
  - SRG evolution parameter $\Lambda = 1.2, 1.5, 1.8$ fm^{-1}
- Explorative study with $NNLO^*_{sat}$ [3]
- No 3-body forces (for now)
- We compute the mean values of "bare" operators

[1] S.K. Bogner, R.J. Furnstahl, and R.J. Perry, PRC **75**, 061001(R) (2007)
[2] D.R. Entem and R. Machleidt, PRC **68**, 041001(R) (2003)
[3] A. Ekström, *et al.*, PRC **91**,051301 (2015)

Convergence



- Exponential behavior [1]

$$E(K) = E(\infty) + Ae^{-bK}$$

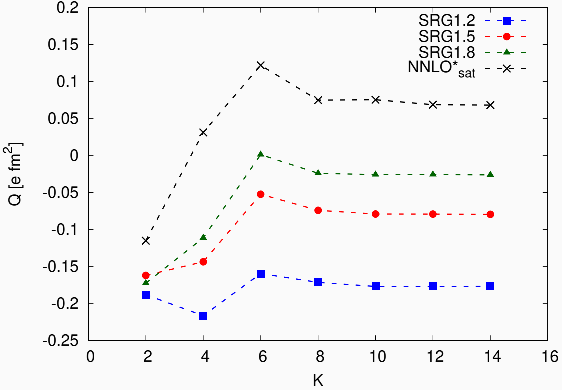
[1] S.K. Bogner *et al.*, NPA **801**, 21 (2008)

Final extrapolation

	B_{full} [MeV]	$B(\infty)$ [MeV]	Exp.
SRG1.2	31.735	31.767(7)	31.99
SRG1.5	32.699	32.789(15)	31.99
SRG1.8	32.093	32.305(25)	31.99
$NNLO^*_{sat}$	29.77	30.71(15)	31.99

- The errors come form the fit
- Error on the extrapolated energy less $< 3\%$
- Difference with the experimental values:
  - No pure 3-body forces
  - No induced 3- and 4-body forces

Electric quadrupole moment



	Q [e fm2]
SRG1.2	-0.177
SRG1.5	-0.080
SRG1.8	-0.026
NNLO$^*_{sat}$	$+0.068$
Ref. [1]	$-0.066(40)$
Exp.	$-0.0806(8)$

- Large cancellations between different K

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Electric quadrupole moment

Matrix elements between different waves

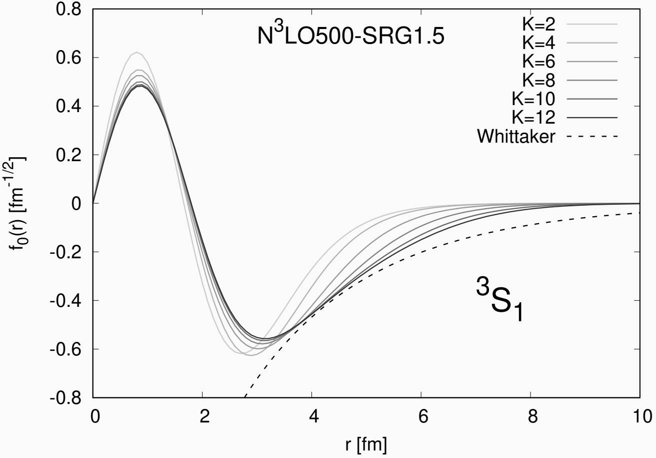
	$S-D$	$D-D$	$P-P$	$P-D$
SRG1.2	-0.173	-0.022	0.009	0.009
SRG1.5	-0.080	-0.021	0.012	0.010
SRG1.8	-0.028	-0.020	0.012	0.010
NNLO$^{*}_{sat}$	0.058	-0.016	0.015	0.011

- Direct connection with the strength of the tensor term in the potential
- Two-body currents contribution could be necessary!!

Cluster form factor $\alpha + d$

Visualize the wave function

$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



Conclusions and future prospective

- extension of HH basis for $A > 4$ (up to now only for "soft" potentials)
- Technically possible to increase the basis
  - Use of not SRG potentials
  - Enlarge the number of particles
- Ground state of ^{6}Li within the HH approach
  - Good convergence for SRG potentials
  - Electromagnetic structure
- The $\alpha + d$ clusterization $\Rightarrow$ first step towards scattering

Pisa group

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Calculations supported by

CINECA

Sparse

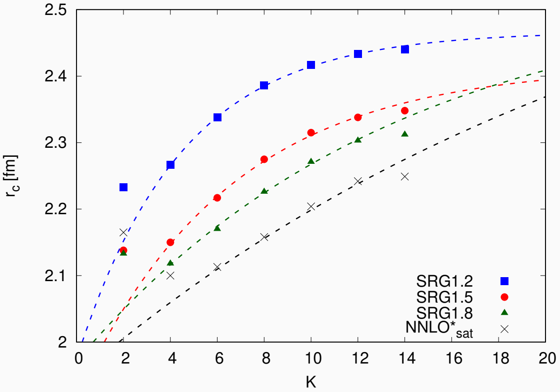
Validation

K	This work (HH)	NSHH [1]
2	-61.142	-61.142
4	-62.015	-62.015
6	-63.377	-63.377
8	-64.437	-64.437
10	-65.354	-65.354
12	-65.884	-65.886

Volkov potential

[1] Nonsymmetrizied HH. M. Gattobigio *et.al.* PRC **71**, 024001 (2005)

Charge radius



	$r_c(\infty)$ [fm]
SRG1.2	2.47(1)
SRG1.5	2.42(3)
SRG1.8	2.54(11)
NNLO^*_{sat}	2.7(3)
Ref. [1]	2.40(6)
Exp.	2.540(28)

- Extrapolation $r_c(K) = r_c(\infty) + Ae^{-bK}$

[1] CDB2k-SRG1.5 C. Forssén, E. Caurier, P. Navrátil, PRC **71**, 021303 (2009)

Magnetic dipole moment

$$^6\text{Li} \simeq \alpha + d \Rightarrow \mu_z(^6\text{Li}) \simeq \mu_z(d)$$

Experiment tells us $\mu_z(^6\text{Li}) < \mu_z(d)$

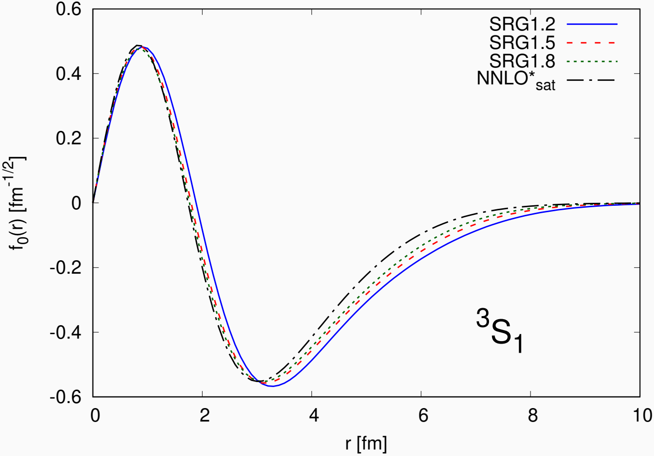
	$\mu_z(d)$	$\mu_z(^6\text{Li})$
SRG1.2	0.872	0.865
SRG1.5	0.868	0.860
SRG1.8	0.865	0.856
NNLO^*_{sat}	0.860	0.850
Exp.	0.857	0.822

- Negative contribution only from the $L = 2$ $S = 1$ component $\Rightarrow$ NOT SUFFICIENT
- We need two body currents contribution!! [1]

[1] R. Schiavilla, *et al.*, PRC **99**, 034005 (2019)

Cluster form factor $\alpha + d$

$$\frac{f_L(r)}{r} = \langle \left[(\Psi_\alpha \otimes \Psi_d)_S \, Y_L(\hat{r}) \right]_J | \Psi_{^6\mathrm{Li}} \rangle$$



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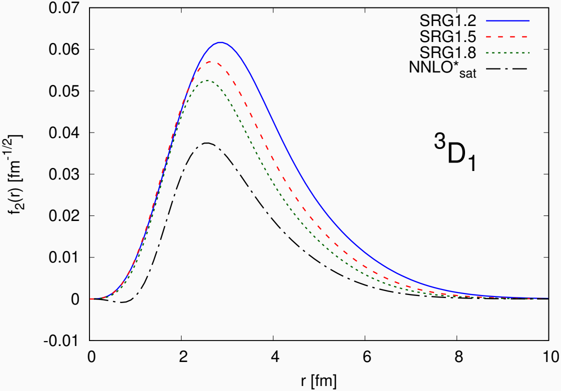

- For the NNLO^*_{sat} a node appears ⇒ strength of the tensor forces[1]

[1] V.I. Kukulin, *et al.* NPA **586**,151 (1995)