Skyrmions and rho mesons, a numerically demanding relationship for nuclei

Carlos Naya Rodríguez

Instituto Nazionale di Fisica Nucleare
INFN - Sezione di Lecce

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1. Introduction
2. A BPS Skyrme model of vector mesons
3. Numerical results

[based on work in collaboration with P. Sutcliffe: Phys. Rev. Lett. **121**, 232002 (2018)]

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1 Introduction

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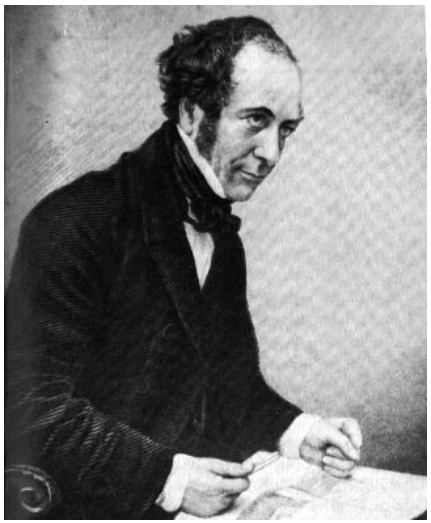
Solitons: First seen by J. Scott Russell in the Edinburgh canal as a solitary wave (1834).



- Localized solutions.
- Conserve their shape.
- Propagation at constant velocity.
- After collisions with other solitons they emerge unchanged.

Topological Solitons

- Exact solutions to classical non-linear field equations.
- Collective excitations of the relevant degrees of freedom.
- Existence and stability due to the global topological structure of the base and field space.
- Topological charge conservation (number of particles).

Recreation: 12 July 1995 during a conference on non-linear waves at Heriot-Watt University.

Soliton wave receives crowd of admirers

London. **Scientists at Heriot-Watt University in Edinburgh, Scotland's capital city, last month took to the water (right) to honour a Victorian civil engineer for a discovery he made 161 years ago.**

John Scott Russell, better known for his successes in ship hull design and the first experimental demonstration of the Doppler effect, is also the first person to have correctly identified a soliton wave, while watching a boat on Edinburgh's Union Canal.

Last month, scientists attending a conference at Heriot-Watt University on nonlinear waves in physics and biology witnessed a reconstruction of that first sighting. The occasion was part of a ceremony to name a new aqueduct after Russell.

A soliton is a nonlinear wave that is able to propagate without spreading out, breaking up or losing strength over distance. It is now the basis of an established method for transporting data through fibre optic cables.

Russell witnessed such a wave while watching a boat being drawn along the Union Canal by a pair of horses. When the boat stopped, he noticed that water around the vessel surged ahead in the form of a single wave, whose height and speed remained virtually unchanged. Intrigued, Russell pursued the wave on horseback for more than a mile before returning home to reconstruct the event in an experimental tank in his garden.

Until the 1960s, soliton waves were considered a curiosity. But Russell believed he had discovered an important phenomenon, describing the sighting as the happiest day of his life. □

Extracted from: Nature v. 376, 3 Aug 1995, pg 373.

- **Tony Hilton Royle Skyrme (1922-1987)**

EFT: The Skyrme model (as one of the most popular and succesful)

- Baryons as "vortices" in a mesonic fluid $\Rightarrow$ topological solitons, i.e., emergent, *non-perturbative* objects

$$\mathscr{L} = \mathscr{L}_0 + \underbrace{\underbrace{\lambda_2\mathscr{L}_2 + \lambda_4\mathscr{L}_4}_{\text{massless } \mathscr{L}_{\text{Skyrme}}} + \lambda_6\mathscr{L}_6}_{\text{massive } \mathscr{L}_{\text{Skyrme}}}$$

$$\mathscr{L}_2 = -\mathrm{Tr}(L_\mu L^\mu), \quad \mathscr{L}_4 = \mathrm{Tr}([L_\mu, L_\nu]^2), \quad \mathscr{L}_6 = -\mathcal{B}_\mu \mathcal{B}^\mu$$

$$\mathcal{B}^\mu = \frac{1}{24^2}\mathrm{Tr}(\epsilon^{\mu\nu\rho\sigma} L_\nu L_\rho L_\sigma), \quad L_\mu = U^\dagger \partial_\mu U, \quad U \in \mathrm{SU}(2) \cong \mathbb{S}^3$$

- Baryon number $\equiv$ topological charge: $B = \int d^3x \mathcal{B}^0$
- Success in the 80's with the study of the nucleon:
  - **Massless:** G. S. Adkins, C. R. Nappi, and E. Witten, Nucl. Phys. B **228**, 552 (1983).
  - **Massive:** G. Adkins and C. Nappi, Nucl. Phys. B **233**, 109 (1984).

But where are the mesons?

- **Pions:** small perturbations of the Skyrme field

$$U = \exp\left(\frac{2i}{F_\pi}\boldsymbol{\pi}\cdot\boldsymbol{\tau}\right) = 1 + \frac{2i}{F_\pi}\boldsymbol{\pi}\cdot\boldsymbol{\tau} + \frac{1}{2!}\left(\frac{2i}{F_\pi}\right)^2\boldsymbol{\pi}^2 + \dots$$

$$\partial_\mu U = \frac{2i}{F_\pi}\partial_\mu\boldsymbol{\pi}\cdot\boldsymbol{\tau} + \dots$$

Translated into the contributions to the Lagrangian

$$\mathscr{L}_2 = \frac{1}{16}F_\pi^2\mathrm{Tr}(\partial_\mu U\,\partial^\mu U^\dagger) = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} + O(\pi^4)$$

$$\mathscr{L}_0 = \frac{1}{8}m_\pi^2 F_\pi^2(\mathrm{Tr}U - 2) = -\frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + O(\pi^4)$$

$$\mathscr{L}_4 = \frac{1}{32e^2}\mathrm{Tr}\left([(\partial_\mu UU^\dagger, \partial_\nu UU^\dagger)]^2\right) = O(\pi^4)$$

$$\Rightarrow \mathscr{L} = \frac{1}{2}\partial_\mu\boldsymbol{\pi}\cdot\partial^\mu\boldsymbol{\pi} - \frac{1}{2}m_\pi^2\boldsymbol{\pi}^2 + O(\pi^4)$$

Is the Skyrme model so great? Almost! Unphysical high binding energies and lack of clustering structures for light nuclei.

Looking for improvement, different approaches.

- The lightly bound model: Inclusion of a term proportional to $\mathrm{Tr}(1-U)^4$

  M. Gillard, D. Harland, M. Speight, Nucl. Phys. B **895** (2015) 272-287

- BPS models (lower topological energy bound).

  C. Adam, J. Sanchez-Guillen, A. Wereszczynski, Phys. Lett. B **691** (2010) 105.

  P. Sutcliffe, JHEP **1008** (2010) 019.

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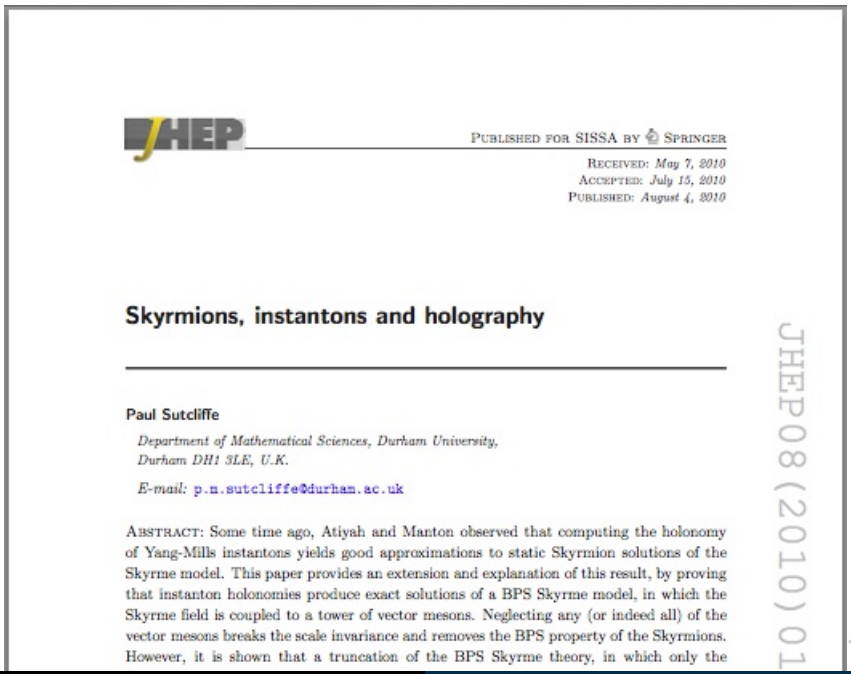

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Skyrmions, instantons and holography

Paul Sutcliffe

Department of Mathematical Sciences, Durham University, Durham DH1 3LE, U.K.

E-mail: p.m.sutcliffe@durham.ac.uk

Abstract: Some time ago, Atiyah and Manton observed that computing the holonomy of Yang-Mills instantons yields good approximations to static Skyrmion solutions of the Skyrme model. This paper provides an extension and explanation of this result, by proving that instanton holonomies produce exact solutions of a BPS Skyrme model, in which the Skyrme field is coupled to a tower of vector mesons. Neglecting any (or indeed all) of the vector mesons breaks the scale invariance and removes the BPS property of the Skyrmions. However, it is shown that a truncation of the BPS Skyrme theory, in which only the

JHEP08(2010)01

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The origins:

P. Sutcliffe, JHEP **08** (2010) 019

- Based on Atiyah and Manton's idea: Skyrme field generated from the holonomy of a Yang-Mills instanton in $\mathbb{R}^4$.

$$U(\mathbf{x}) = \pm\mathcal{P}\exp\int_{-\infty}^{\infty} A_4(\mathbf{x}, x_4)dx_4$$

- Yang-Mills energy given by ($I = 1, 2, 3, 4$, and $z = x_4$):

$$E = -\frac{1}{8}\int \mathrm{Tr}(F_{IJ}F_{IJ})d^3x\,dz$$

- Fixing the gauge $A_z = 0$ $\Rightarrow$ gauge field expanded in terms of Hermite functions, ψ_n

$$A_i = -\partial_i U U^{-1}\psi_+(z) + \sum_{n=0}^{\infty} V_i^n(\mathbf{x})\psi_n(z)$$

Resulting theory: Skyrme field coupled to an infinite tower of vector mesons with a BPS bound.

The first in the queue: the ρ meson

Gauge field with one vector meson ($V_i^0 = V_i$)

$$A_i = -\partial_i U\, U^{-1}\psi_+(z) + V_i\psi_0(z) \qquad \left(\psi_0(z) = \frac{1}{\pi^{1/4}}e^{-z^2/2}\right)$$

and field strength

$$F_{zi} = -R_i\frac{\psi_0}{\sqrt{2}\pi^{1/4}} - V_i\frac{\psi_1}{\sqrt{2}},$$

$$F_{ij} = [R_i, R_j]\psi_+(\psi_+ - 1) + (\partial_i V_j - \partial_j V_i)\psi_0 + [V_i, V_j]\psi_0^2 - ([R_i, V_j] - [R_j, V_i])\psi_+\psi_0$$

Different contributions to the energy

$$E = E_{\rm S} + E_{\rm V} + E_{\rm I} = 1.060 \times 2\pi^2$$

Skyrme energy:

$$E_S = \int \left(-\frac{c_1}{2} \mathrm{Tr}(R_i R_i) - \frac{c_2}{16} \mathrm{Tr}([R_i, R_j]^2) \right) d^3x$$

where

$$c_1 = \frac{1}{4\sqrt{\pi}}, \qquad c_2 = \int_{-\infty}^{\infty} 2\psi_+^2 (\psi_+ - 1)^2 dz = 0.198$$

- Energy bounds

$$E_S \geq 12\pi^2 \sqrt{c_1 c_2} |B| = 2.005\pi^2 |B|, \qquad E_{YM} \geq 2\pi^2 |B|$$

- Feeding the family (pion mass)

$$E_S = \int \left(-\frac{c_1}{2} \mathrm{Tr}(R_i R_i) - \frac{c_2}{16} \mathrm{Tr}([R_i, R_j]^2) + \hat{m}^2 \mathrm{Tr}(1 - U) \right) d^3x,$$

where $\hat{m}^2 = m^2 \frac{c_1^2}{c_2}$ and $m = \frac{2m_\pi}{F_\pi e} = 0.526$

Vector meson energy:

$$E_{\rm V} = \int -{\rm Tr}\left\{\frac{1}{8}(\partial_i V_j - \partial_j V_i)^2 + \frac{1}{4}m^2 V_i^2 + c_3(\partial_i V_j - \partial_j V_i)[V_i, V_j] + c_4[V_i, V_j]^2\right\} d^3x,$$

Interaction energy:

$$\begin{aligned} E_{\rm I} = \int -{\rm Tr}\Big\{ & c_5([R_i, V_j] - [R_j, V_i])^2 - c_6[R_i, R_j](\partial_i V_j - \partial_j V_i) - c_7[R_i, R_j][V_i, V_j] \\ & + \frac{1}{2}c_6[R_i, R_j]([R_i, V_j] - [R_j, V_i]) - \frac{1}{8}([R_i, V_j] - [R_j, V_i])(\partial_i V_j - \partial_j V_i) \\ & - \frac{1}{2}c_3([R_i, V_j] - [R_j, V_i])[V_i, V_j]\Big\} d^3x, \end{aligned}$$

where the constants are

$$m = \frac{1}{\sqrt{2}}, \qquad c_3 = \frac{1}{2\sqrt{6}\pi^{\frac{1}{4}}}, \qquad c_4 = \frac{1}{8}\sqrt{\frac{1}{2\pi}},$$

$$c_5 = 0.038, \qquad c_6 = \frac{\pi^{\frac{1}{4}}}{12\sqrt{2}}, \qquad c_7 = 0.049$$

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Numerical setup

- **How?**
- **Where?**
- **When?**

Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

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Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

  Regular lattice:

$$n = 128, \qquad h = 0.08$$

- **When?**

Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

  Regular lattice:

$$n = 128, \qquad h = 0.08$$

- **When?**

  After a reasonable running time.

Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

  Regular lattice:

$$n = 128, \qquad h = 0.08$$

- **When?**

  After a reasonable running time.

- **Really?**

Numerical setup

- **How?**

  Second order time dynamics with 4th order Runge-Kutta method

- **Where?**

  Regular lattice:

$$n = 128, \qquad h = 0.08$$

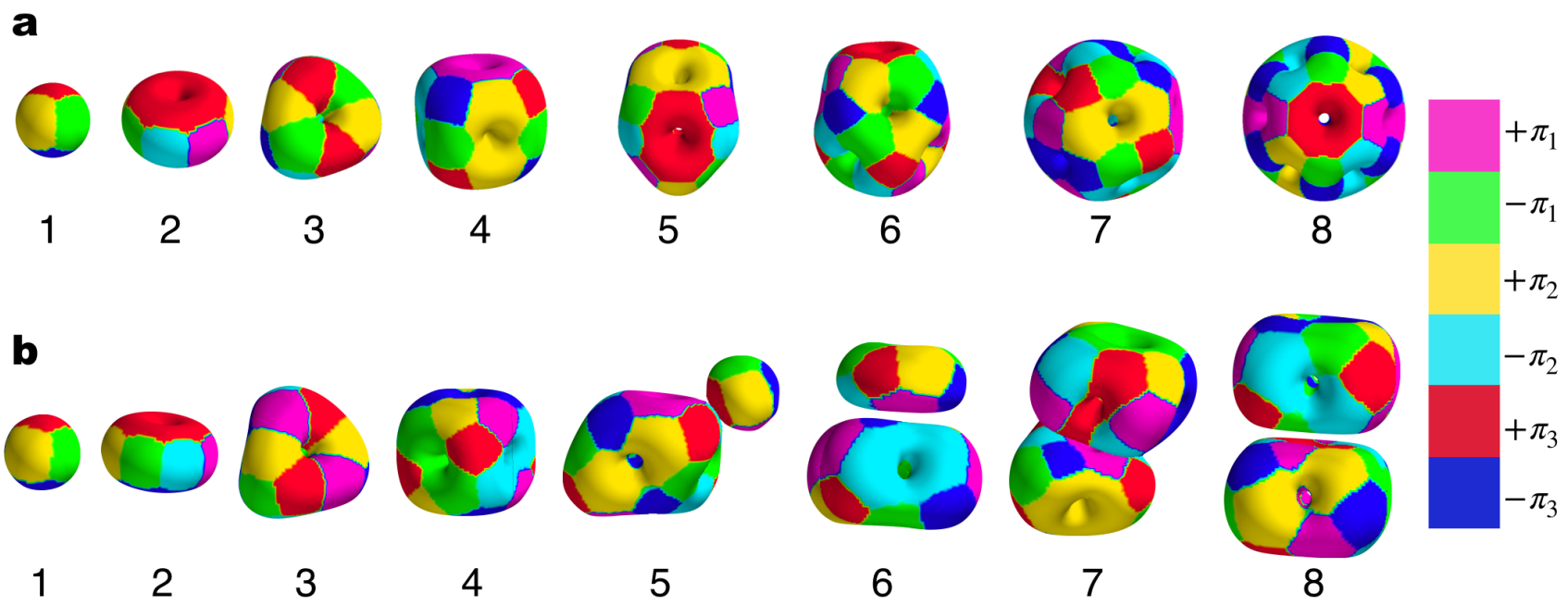
- **When?**

  After a reasonable running time.

- **Really?**

  Parallel programming: MPI

Solutions B = 1 - 8: n = 128, h=0.08, m = 0.526

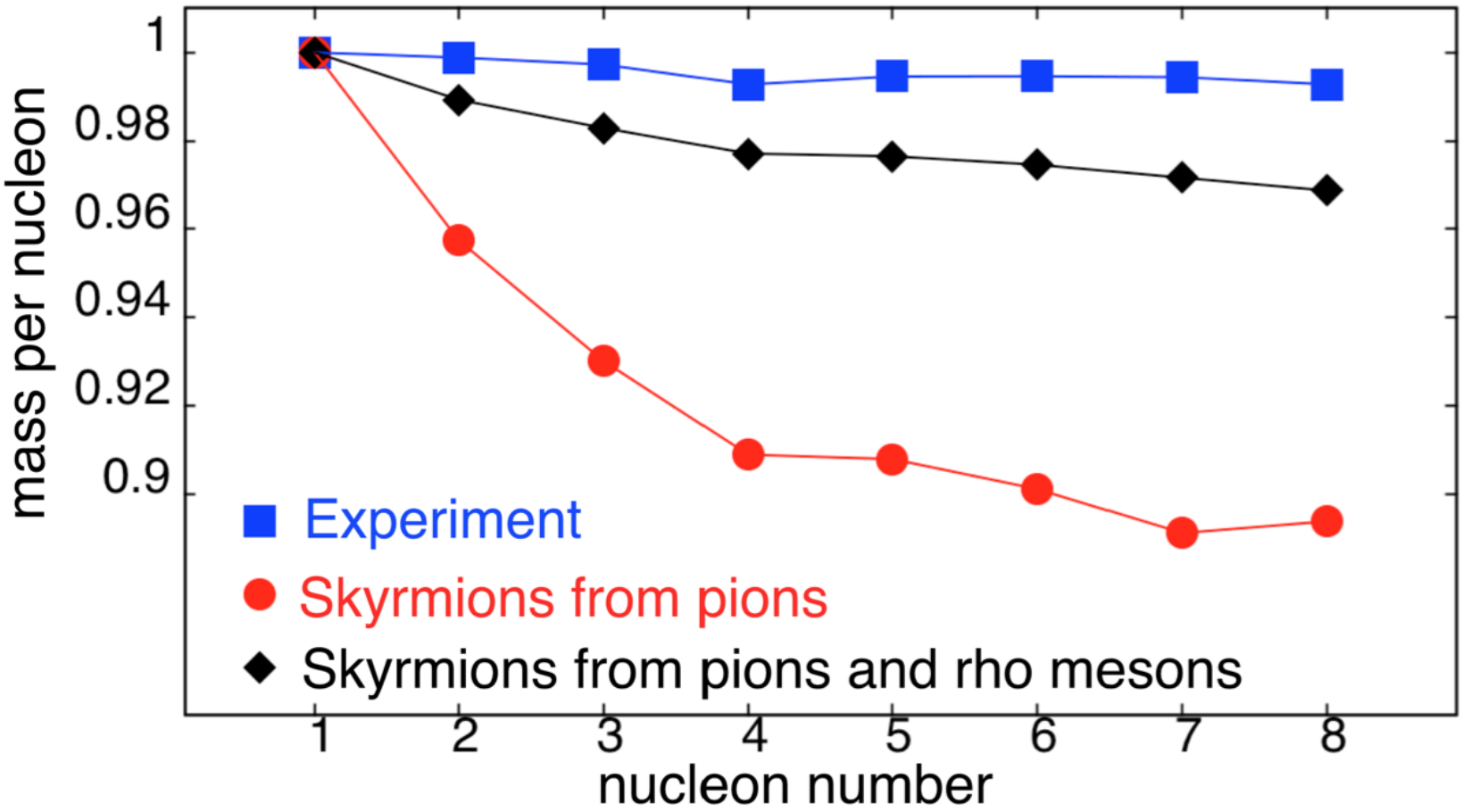


Baryon density isosurfaces: (a) Standard Skyrme model (b) Extended version with massive pions and rho mesons.

C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

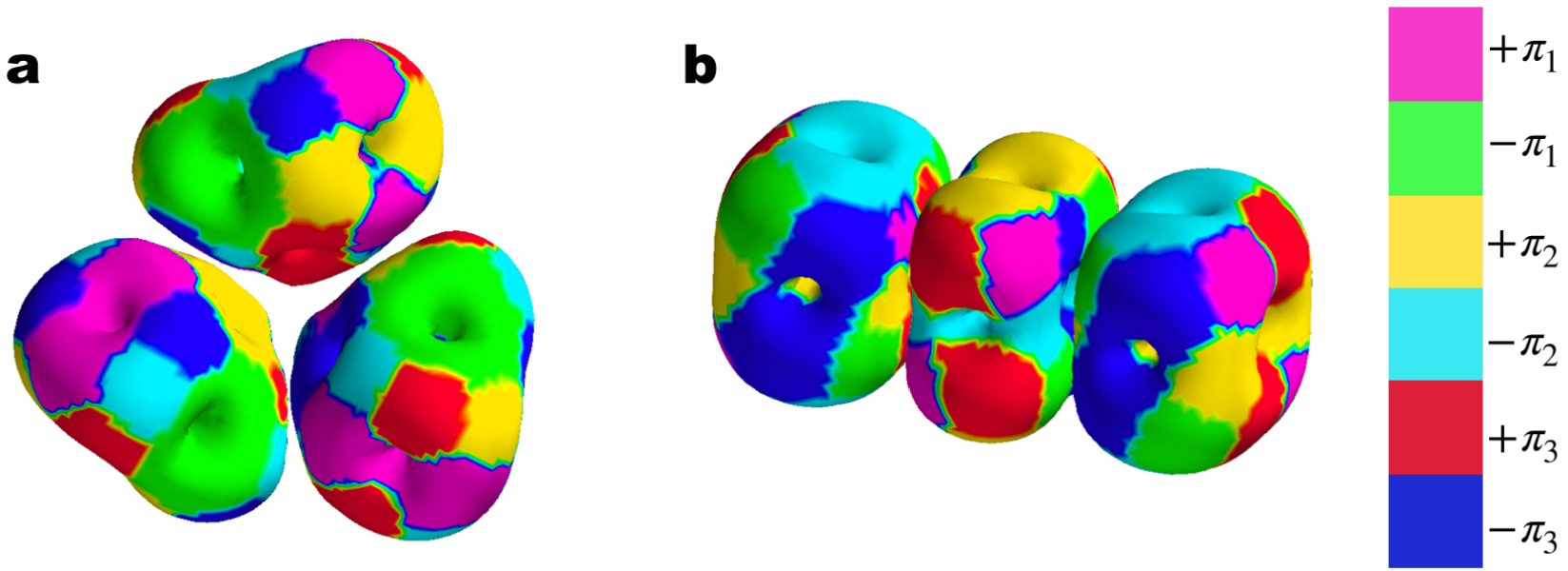
Mass per nucleon normalised by the nucleon (proton) mass:

n = 128, h=0.08, m = 0.526



C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

Solution B = 12: n = 128, h=0.08, m = 0.526



Baryon density isosurfaces: (a) 3α triangular structure (^{12}C ground state) (b) 3α linear chain (Hoyle state).

C. Naya and P. Sutcliffe, Phys. Rev. Lett. **121** 232002 (2018)

Conclusions

- The Skyrme model as an effective field theory in the low energy limit of QCD.
- Baryons as excitations of the mesonic degrees of freedom (pions).
- Further mesons can be included.
- BPS model with the Skyrme field coupled to an infinite tower of vector mesons.
- Even the first vector meson moves the energy closer to the bound in a significative way.
- Dramatic effect of pion mass wit the two main problems solved: lower binding energies and clustering!

obrigado
Dank U
Merci
mahalo
Köszi
спасибо
Grazie
Thank you
mauruuru
Takk
Gracias
Dziękuję
Děkuju
danke
Kiitos