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Real time dynamics, confinement and phase transitions in $\mathbb{Z}_n$ Schwinger-Weyl lattice models for 1+1 QED

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Motivation

QED in (1+1) dimensions: How is that interesting?

- simple example of gauge field (Abelian, 1D) coupled with matter
- comparison with consolidated results [Coleman and Susskind 1975, Coleman 1976, Melnikov and Weinstein 2000, …]
- rich phenomenology, with some interesting analogies to QCD (phase transitions, string breaking)
- accessible real-time dynamics
- optimal experimental testbed

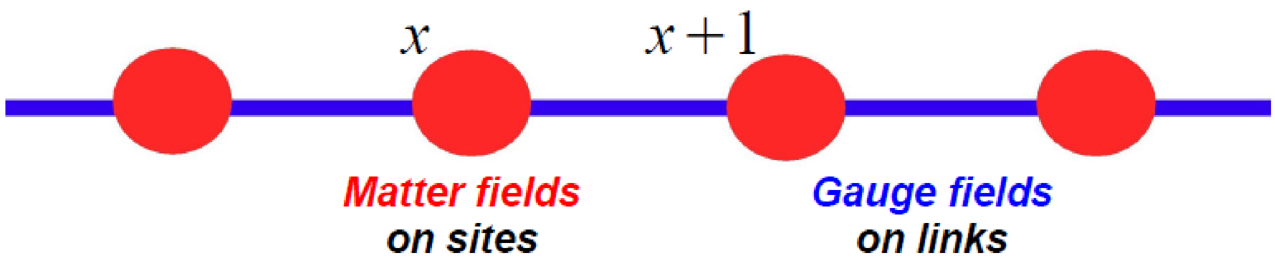
LETTER

doi:10.1038/nature18318

Real-time dynamics of lattice gauge theories with a few-qubit quantum computer

Esteban A. Martinez[1*], Christine A. Muschik[2,3*], Philipp Schindler[1], Daniel Nigg[1], Alexander Erhard[1], Markus Heyl[2,4], Philipp Hauke[2,3], Marcello Dalmonte[2,3], Thomas Monz[1], Peter Zoller[2,3] & Rainer Blatt[1,2]

One-dimensional lattice QED



Matter field (spinless fermion) ψ_x defined on sites

$$\{\psi_x, \psi_y\} = 0 \qquad \{\psi_x, \psi_y^\dagger\} = \delta_{xy}$$

Electric field $E_{x,x+1}$ and vector potential $A_{x,x+1}$ defined on links

$$[E_{x,x+1}, E_{y,y+1}] = 0 \qquad [A_{x,x+1}, A_{y,y+1}] = 0 \qquad [E_{x,x+1}, e^{-iA_{y,y+1}}] = \delta_{xy} e^{-iA_{y,y+1}}$$

Hamiltonian

$$H = \sum_x \left[\underbrace{-\tau\left(\psi_x^\dagger e^{-iA_{x,x+1}} \psi_{x+1} + \mathrm{H.c.}\right)}_{\text{Minimal coupling}} + \underbrace{m(-1)^x \psi_x^\dagger \psi_x}_{\text{Staggered fermion mass}} + \underbrace{\frac{g^2}{2} E_{x,x+1}^2}_{\text{Coulomb energy}} \right]$$

Minimal coupling

Staggered fermion mass

Coulomb energy

One-dimensional lattice QED

$$H = \sum_x \left[-\tau\left(\psi_x^\dagger e^{-iA_{x,x+1}} \psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

CANONICAL QUANTIZATION of the electromagnetic field

The field operators **do not satisfy Gauss' law**

$$G_x = \psi_x^\dagger \psi_x + \frac{(-1)^x - 1}{2} - (E_{x,x+1} - E_{x-1,x}) \neq 0$$

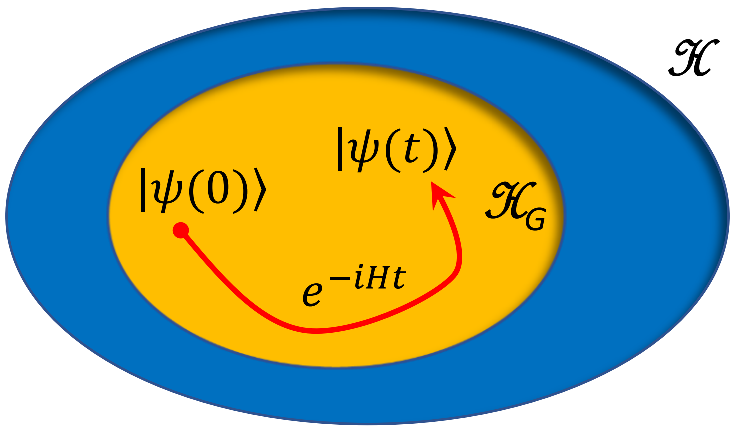
Generator of gauge transformations

Particle-hole picture:
- Occupied even site = particle
- Empty odd site = antiparticle

PHYSICAL SPACE of gauge-invariant states

$$\mathcal{H}_G = \{ |\psi\rangle \in \mathcal{H} \,|\, G_x |\psi\rangle = 0 \text{ for all } x \}$$

$$[G_x, H] = 0$$



Finite link dimensions

In view of the implementation of (1+1) QED in atomic, molecular and optical quantum simulators, the dimension of link Hilbert spaces should become finite.

A proposed solution: **QUANTUM LINK models**

Horn 1981, Orland and Rohrlich 1990, Wiese 2000s

Spin variables attached to each link $\quad S^{\pm} = S^1 \pm iS^2 \quad [S^3, S^{\pm}] = \pm S^{\pm}$

emulates ↓

Identification $\quad S^3 \longrightarrow E \quad S^{\pm} \longrightarrow e^{\mp iA} \quad \left[E, e^{\mp iA}\right] = \pm e^{\mp iA}$

U(1) gauge invariance preserved, same form of the Gauss' law, *but*

- Unitarity of gauge connection is lost (no Hermitian vector potential)

$$(S^+)^\dagger S^+ = S^- \, S^+ \neq \mathbb{I}$$

- Transition amplitudes depend on Clebsch-Gordan coefficients

$$\psi_x^\dagger e^{-iA_{x,x+1}} \psi_{x+1} \longrightarrow \psi_x^\dagger (S^+)_{x,x+1} \psi_{x+1}$$

Finite link dimensions

Our proposal: [Notarnicola, Ercolessi, Facchi, Marmo, Pascazio, Pepe, JPA **48**, 30FT01 (2015)]

Generalization of the Weyl *group* relation

$$e^{-i\eta A}e^{-i\xi E} = e^{i\eta\xi}e^{-i\xi E}e^{-i\eta A} \quad \text{with } \xi, \eta \in \mathbb{R}$$

Correspondence $\quad V \longrightarrow e^{-i\sqrt{\frac{2\pi}{n}}E} \qquad U \longrightarrow e^{-i\sqrt{\frac{2\pi}{n}}A}$

with U and V **unitary operators** on an n-dimensional Hilbert space, such that

$$U^p V^q = e^{\frac{2\pi i}{n}pq} V^q U^p \quad \text{with } p, q \in \mathbb{Z}$$

Unitarity of gauge connection $\quad \psi_x^\dagger e^{-iA_{x,x+1}}\psi_{x+1} \longrightarrow \psi_x^\dagger U_{x,x+1}\psi_{x+1}$

Gauss' law in exponential form: $T_x\,|\psi\rangle = |\psi\rangle$

$$T_x = \exp\left[\frac{2\pi i}{n}\left(\psi_x^\dagger\psi_x + \frac{(-1)^x - 1}{2}\right)\right] V_{x,x+1}V_{x-1,x}^\dagger \Longrightarrow (T_x)^n = \mathbb{I}$$

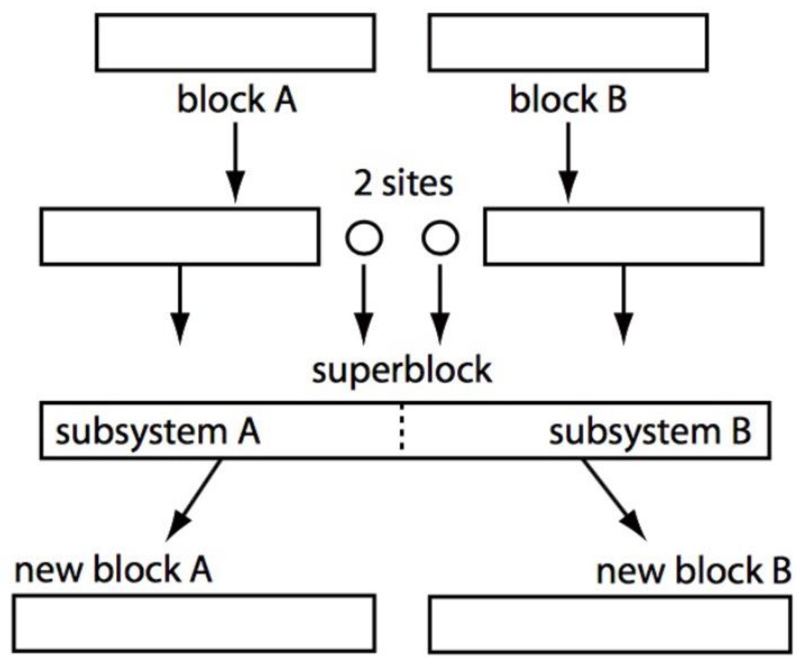
New gauge group: $\mathbb{Z}_n$ transformations $\quad \mathcal{T} = \prod_x (T_x)^{n_x}$

Density Matrix Renormalization Group

Numerical technique based on the truncation of reduced density matrices according to their eigenvalues

[see e.g. Schollwöck, RMP **77**, 259 (2005)]

- Limits the growth of computational time for systems with small entanglement
- Initially developed for GS search, then extended to thermal states and dynamics (tDMRG)
- Automatically incorporates local constraints (Gauss' law)



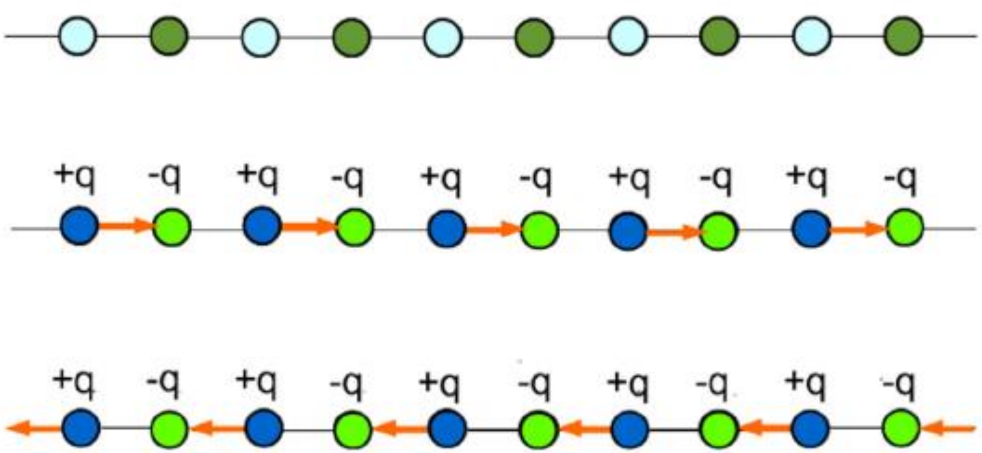
Numerical work by **Giuseppe Magnifico** (Bologna/Padova)

$\mathbb{Z}_n$ model: energy cost of pair creation

$$H = \sum_x \left[-\tau\left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

$\tau = 0$ → **Ground state degeneracies**

Odd n



$2m + \frac{\pi g^2}{n} > 0$ Dirac sea

$2m + \frac{\pi g^2}{n} < 0$ Mesons + electric flux

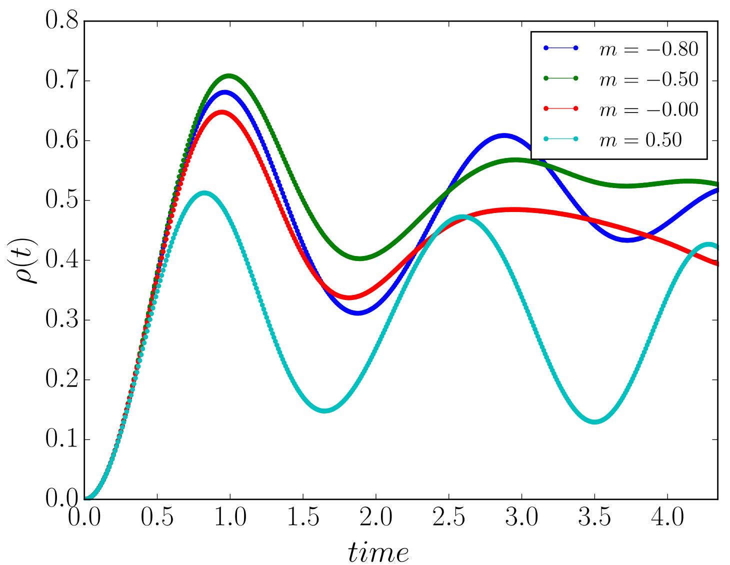
$\mathbb{Z}_n$ model: real-time dynamics

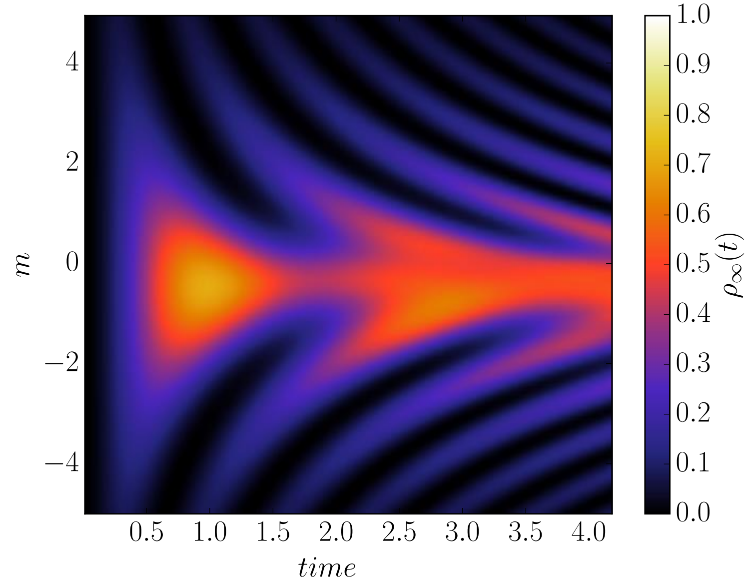
$$H = \sum_x \left[-\tau\left(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

Dirac sea instability by spontaneous pair creation

Mean particle density ($\mathbb{Z}_3$ model):

Time evolution for $N = 40$ … … and asymptotic value





Instability is enhanced when $2m + \pi g^2/n \approx 0$

Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

$\mathbb{Z}_n$ model: real-time dynamics

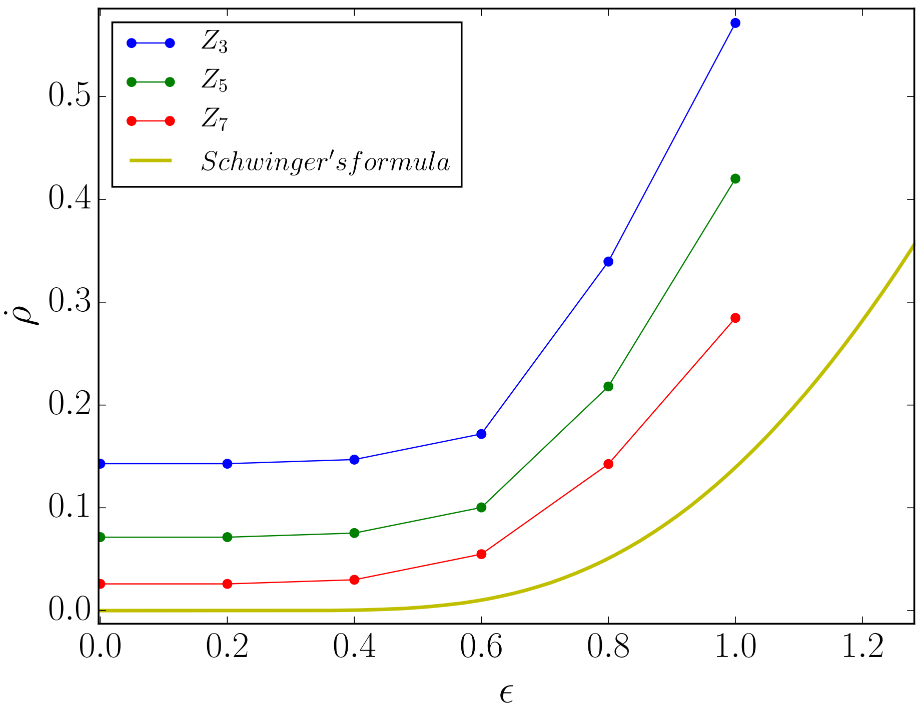
$$H = \sum_x \left[-\tau\left(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

Pair production in a constant external field E_0

Schwinger formula of pair production rate

$$\dot{\rho}(t) = \frac{m^2 \epsilon}{2\pi} \exp\left(-\frac{\pi}{\epsilon}\right) \qquad \left[\epsilon = \frac{g E_0}{m^2}\right]$$

approximated with increasing precision as *n* increases



Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

$\mathbb{Z}_n$ model: real-time dynamics

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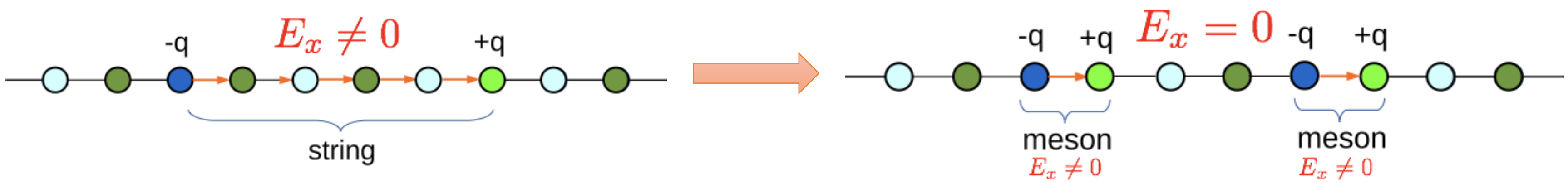
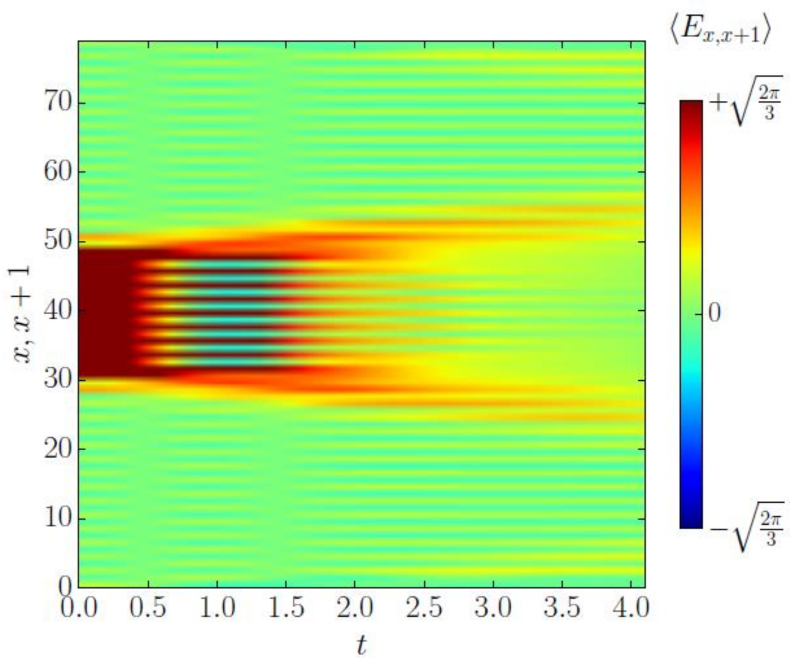
String breaking in different regimes



According to the values of *m* and *g*, the string can:

- break into two propagating mesons

$$m = 0.1, g = 0.1$$

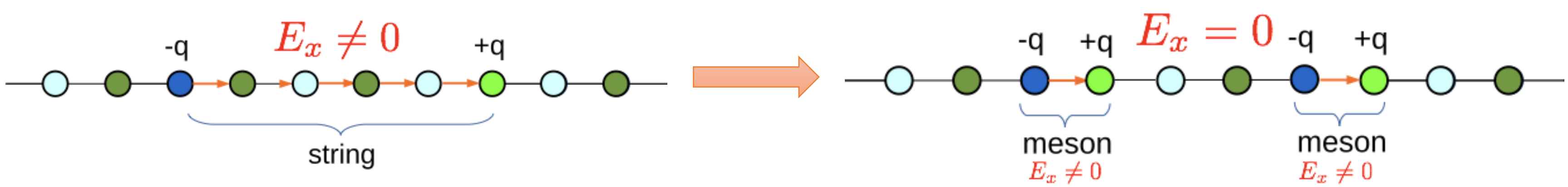


Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

$\mathbb{Z}_n$ model: real-time dynamics

$$H = \sum_x \left[-\tau\left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

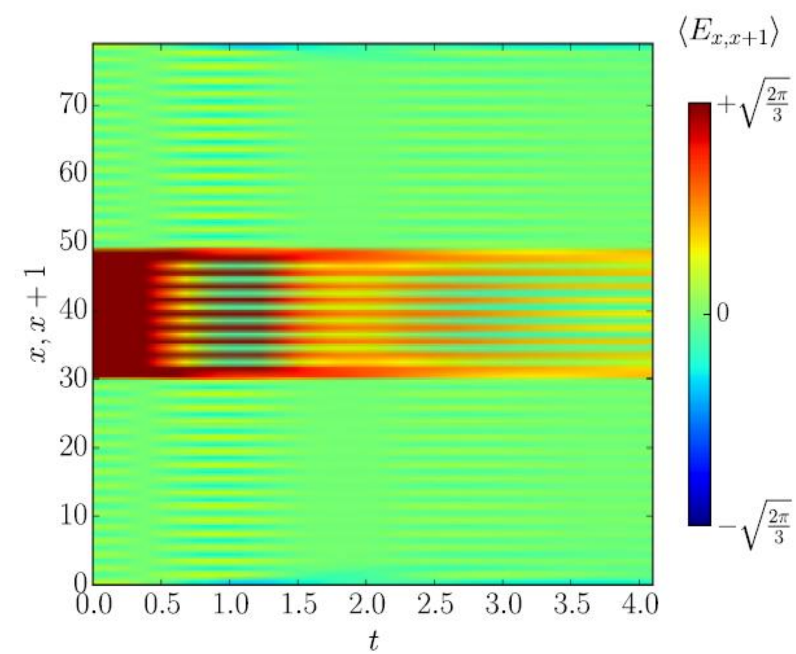
String breaking in different regimes



According to the values of *m* and *g*, the string can:

- show (damped) oscillations without propagation

$$m = 0.3, g = 0.8$$

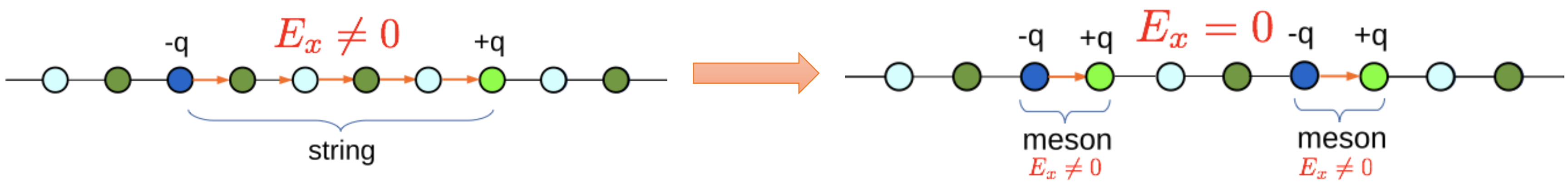


Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

$\mathbb{Z}_n$ model: real-time dynamics

$$H = \sum_x \left[-\tau(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \mathrm{H.c.}) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

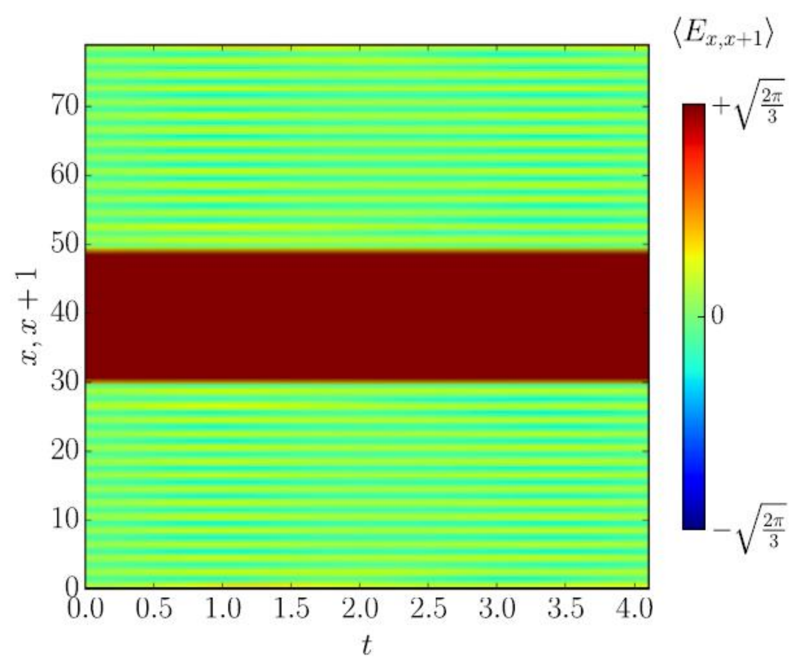
String breaking in different regimes



According to the values of *m* and *g*, the string can:

- remain stable

$$m = 3.0, g = 1.42$$

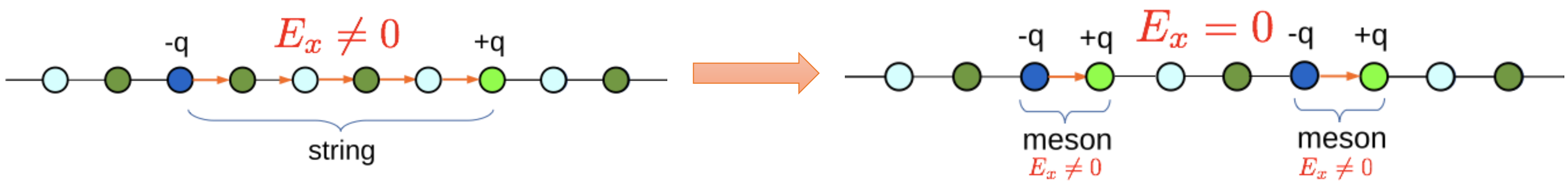


Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

$\mathbb{Z}_n$ model: real-time dynamics

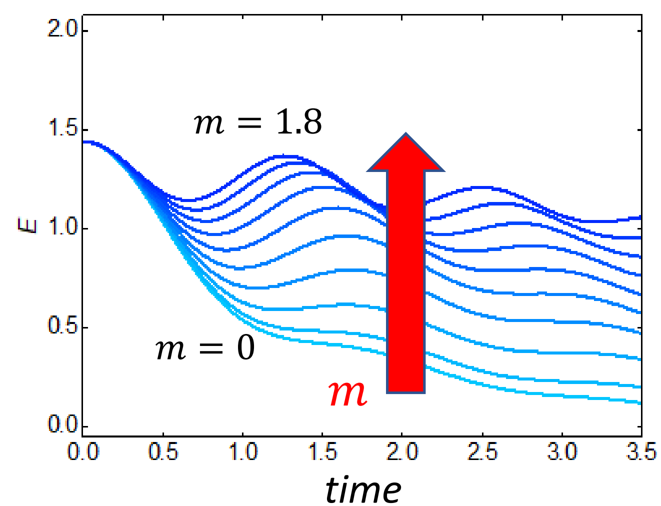
$$H = \sum_x \left[-\tau \left(\psi_x^\dagger U_{x,x+1} \psi_{x+1} + \text{H.c.} \right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

String breaking in different regimes



Generally, higher absolute values of *m* stabilize the string

Time evolution of mean electric field in the 20 central sites



Magnifico, Dalmonte, Facchi, Pascazio, Pepe, Ercolessi, arXiv:1909:04821

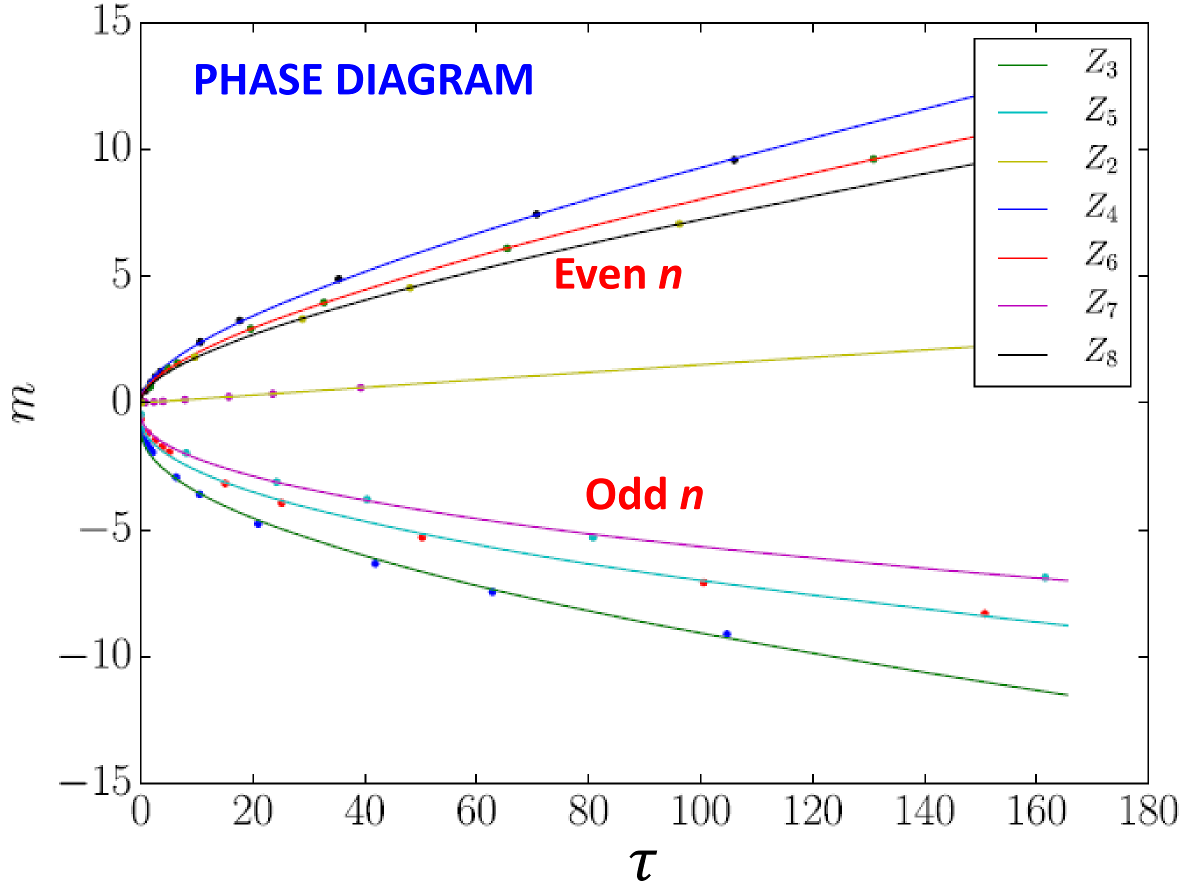
$\mathbb{Z}_n$ model: ground state and phase transitions

$$H = \sum_x \left[-\tau\left(\psi_x^\dagger U_{x,x+1}\psi_{x+1} + \mathrm{H.c.}\right) + m(-1)^x \psi_x^\dagger \psi_x + \frac{g^2}{2} E_{x,x+1}^2 \right]$$

$\tau \neq 0$

Phase transition in the Ising universality class

Prediction of the **critical mass** of the U(1) model by extrapolation to $n \to \infty$



Ercolessi, Magnifico, Facchi, Pascazio, Pepe, PRD **98**, 074503 (2018)

Outlook

- Suitable experimental platform

- Constrain dynamics in the gauge-invariant subspace

  Zohar and Reznik 2011, Banerjee, Dalmonte, Mueller, Rico, Stebler, Wiese, Zoller 2012
  Stanniger, Hauke, Marcos, Hafezi, Diehl, Dalmonte, Zoller 2014
  Notarnicola, Collura, Montangero 2019

- Extend to the non-Abelian case

Thank you
for your attention