Tensor networks: simulating quantum many-body systems and beyond

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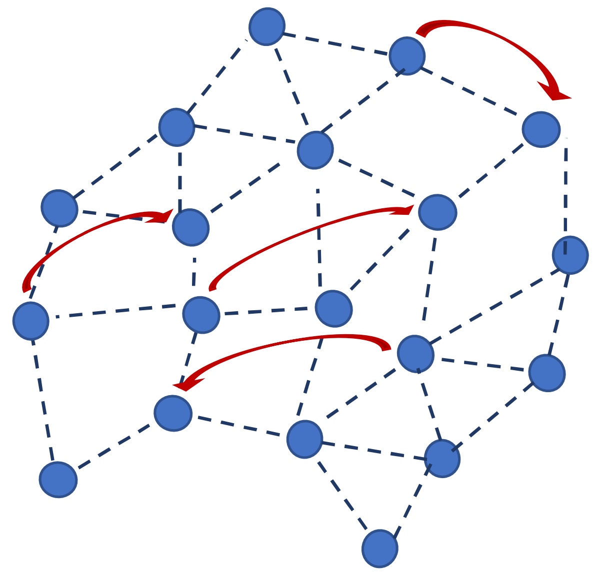
SM&FT, Bari – 11 December 2019

Introduction: quantum many-body systems

Quantum many-body systems

- Quench dynamics
- Ground state properties
- …

Correlations ⟶ Entanglement



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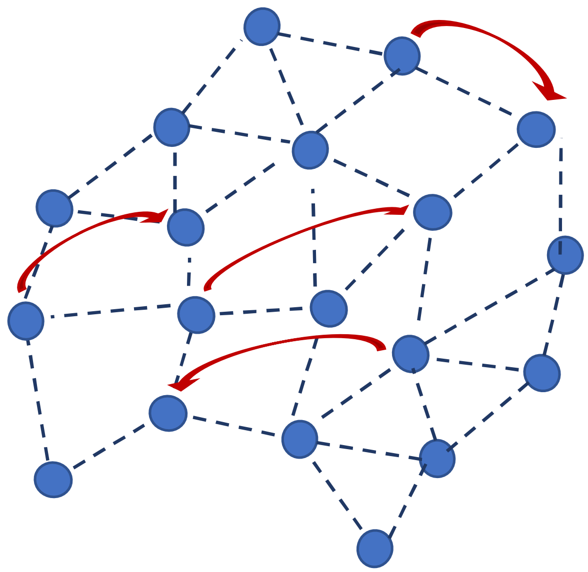
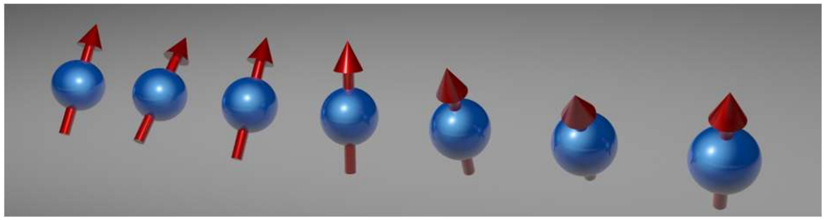
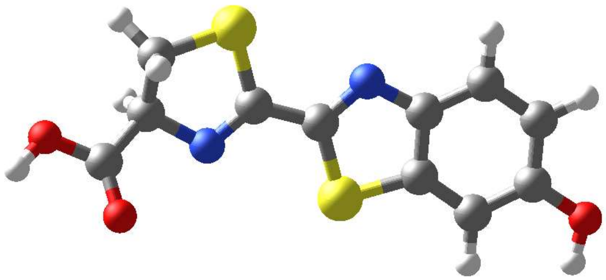


- Condensed matter
- Quantum chemistry
- …



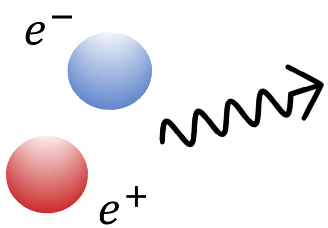
Phys. Rev. Lett. 119, 080501 (2017)

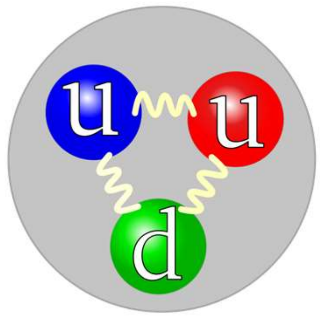


Credits: Maple Quantum Chemistry

Gauge theories

Fundamental interactions of matter





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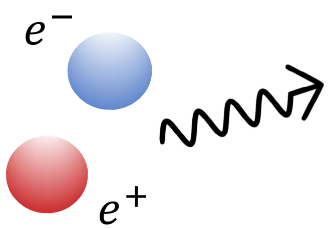
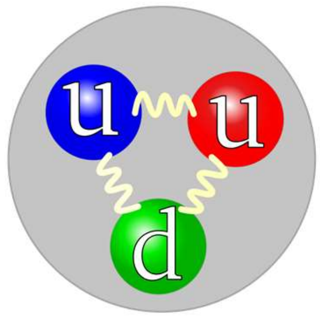
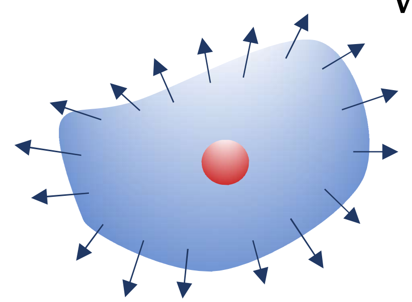




Interactions arise by imposing

local conservation Gauss' laws

$\nabla \vec{E} = \rho$



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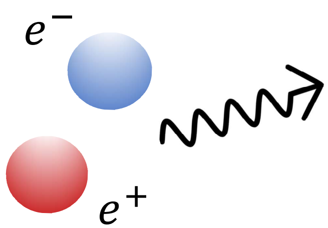

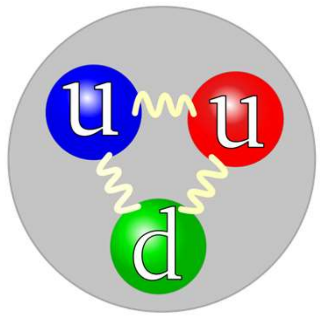

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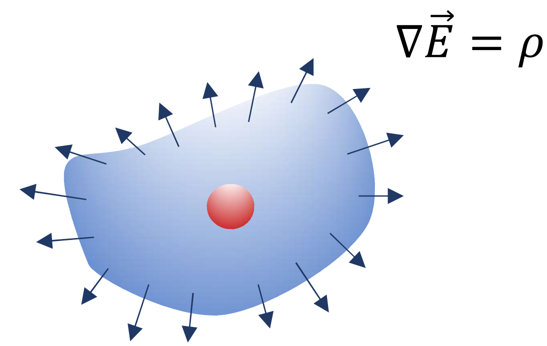
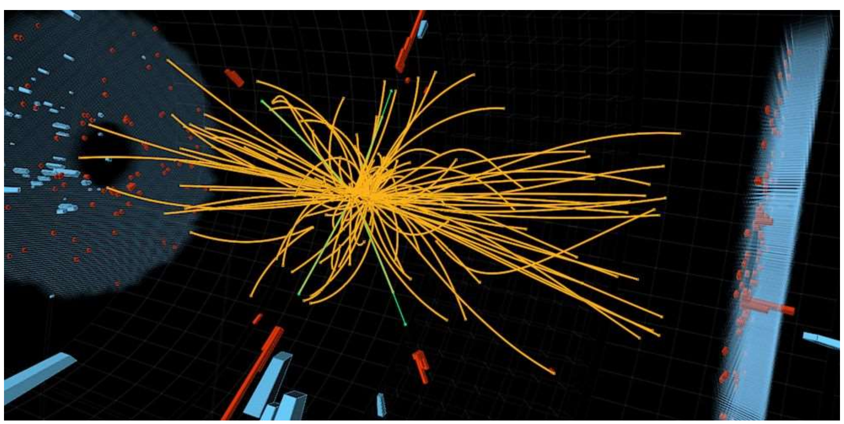




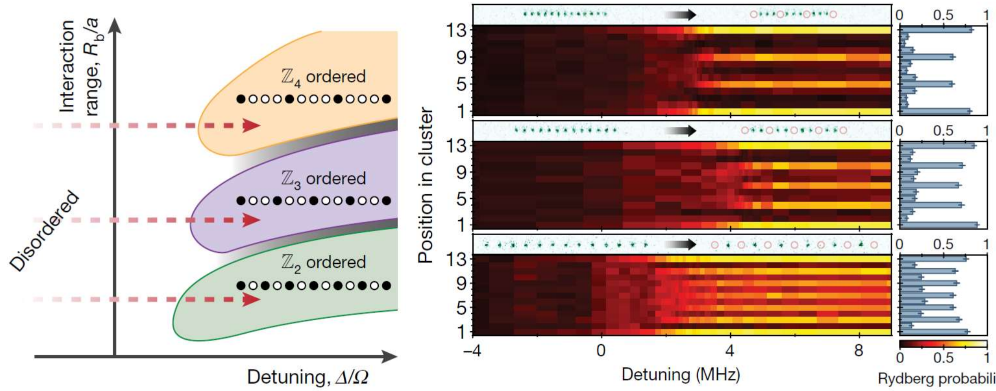
Simulating gauge theories:

- Huge Hilbert space dimesion
- Sign problem

Introduction: quantum computations

Recent technological achievement:

- Rydberg atoms chains

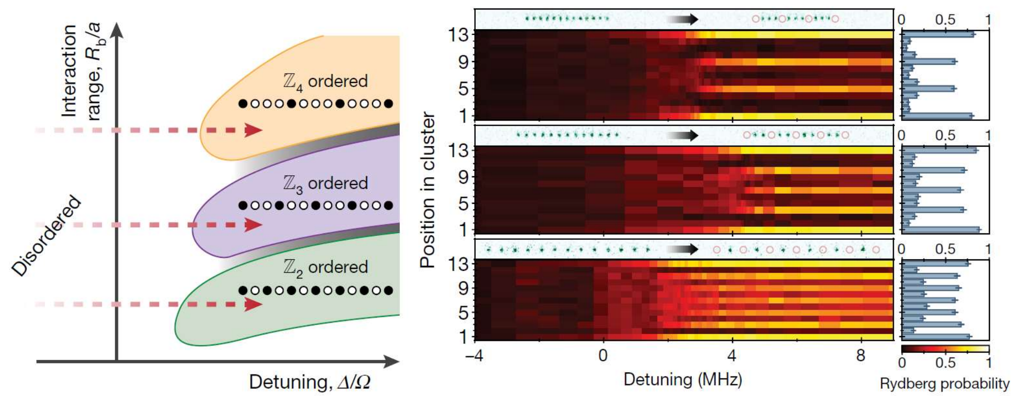


Nature **551**, 579 (2017)

Introduction: quantum computations

Recent technological achievement:

- Rydberg atoms chains
- Trapped ions
- Superconducting-qubits quantum computers



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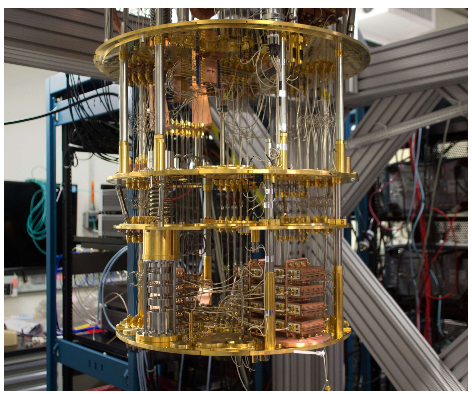


IBM quantum computer

Google quantum computer

Needed:

- Benchmark
- Validation

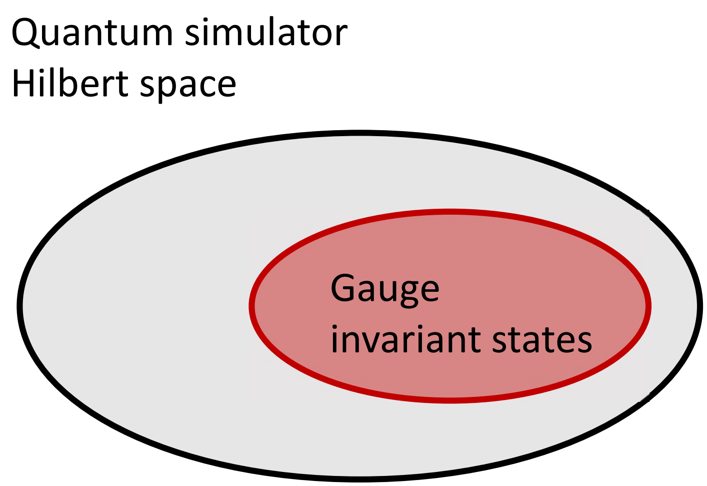
Quantum simulation of Lattice Gauge Theories

Simulating a quantum theory through a controllable quantum system

Quantum simulation of Lattice Gauge Theories

Simulating a quantum theory through a controllable quantum system

- Mapping between the Hilbert spaces
- Mapping between the Hamiltonians



Quantum simulation of Lattice Gauge Theories

Simulating a quantum theory through a controllable quantum system

- Mapping between the Hilbert spaces
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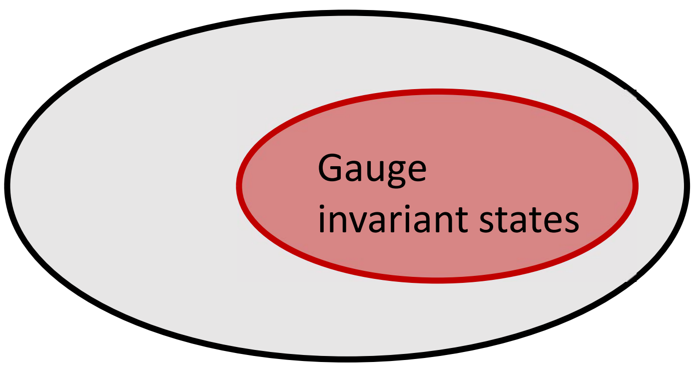
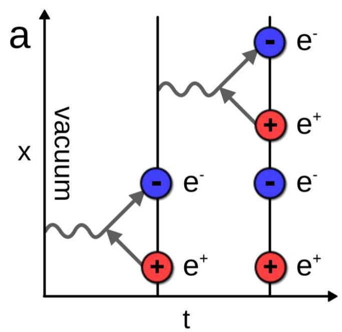
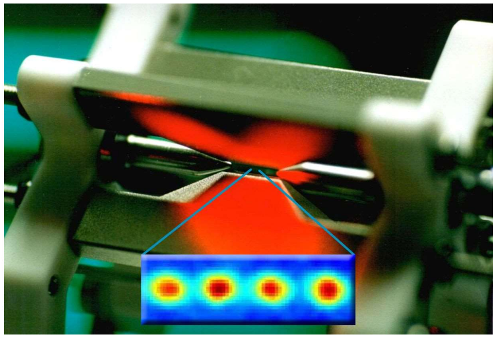
Quantum simulator
Hilbert space



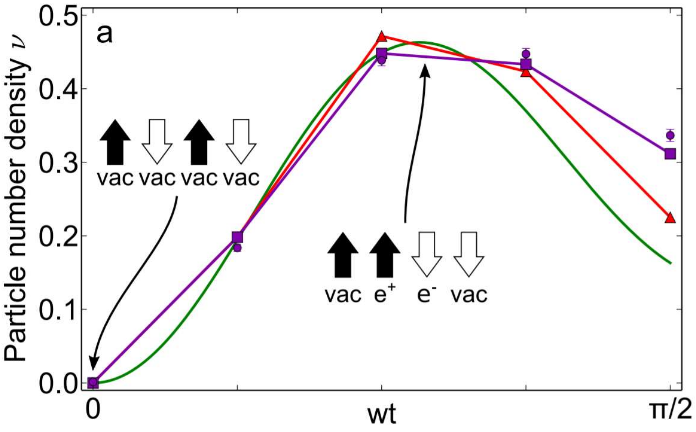
1D Schwinger model:
Fermions coupled to an electric gauge field

Trapped ions:
two relevant internal states implement a qubit





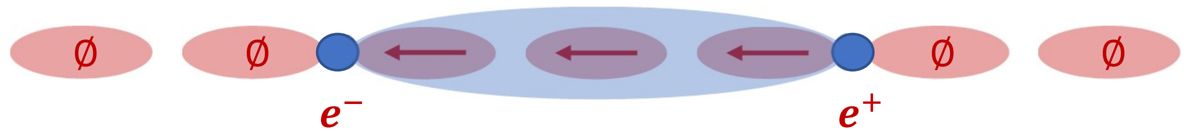
Innsbruck University



Nature **534**, 516–519 (2016)

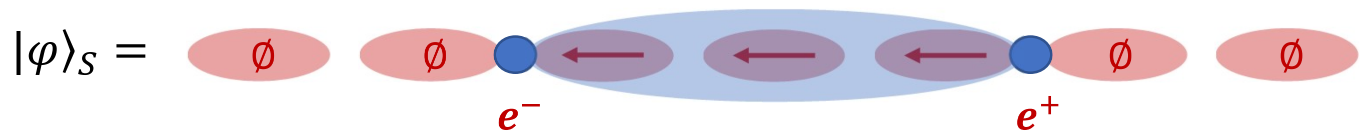
Quantum simulation of Lattice Gauge Theories

$|\varphi\rangle_S =$

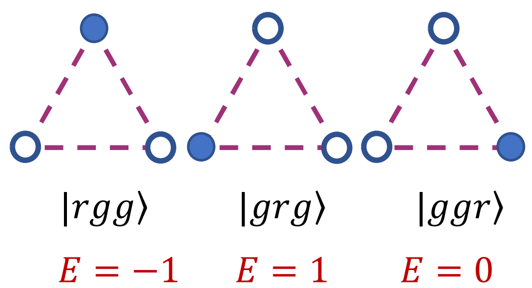


S.N., M. Collura, S. Montangero Arxiv: 1907.12579

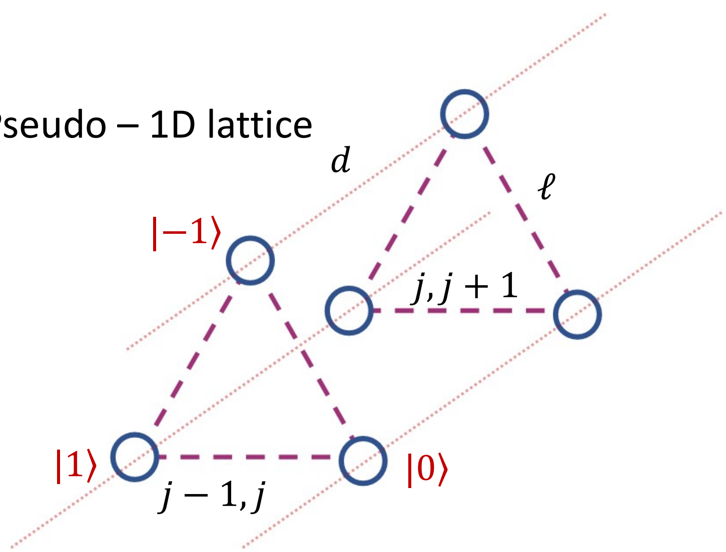
Quantum simulation of Lattice Gauge Theories



Three atoms represent a link



Pseudo – 1D lattice



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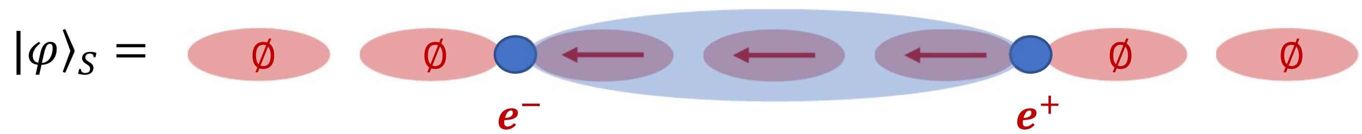

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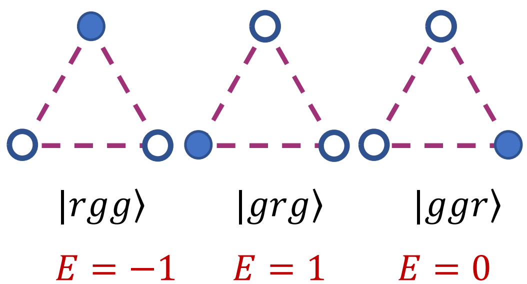

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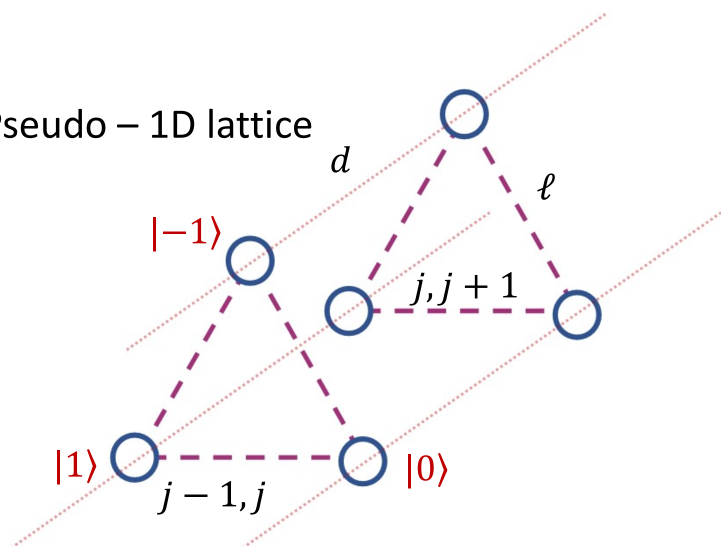
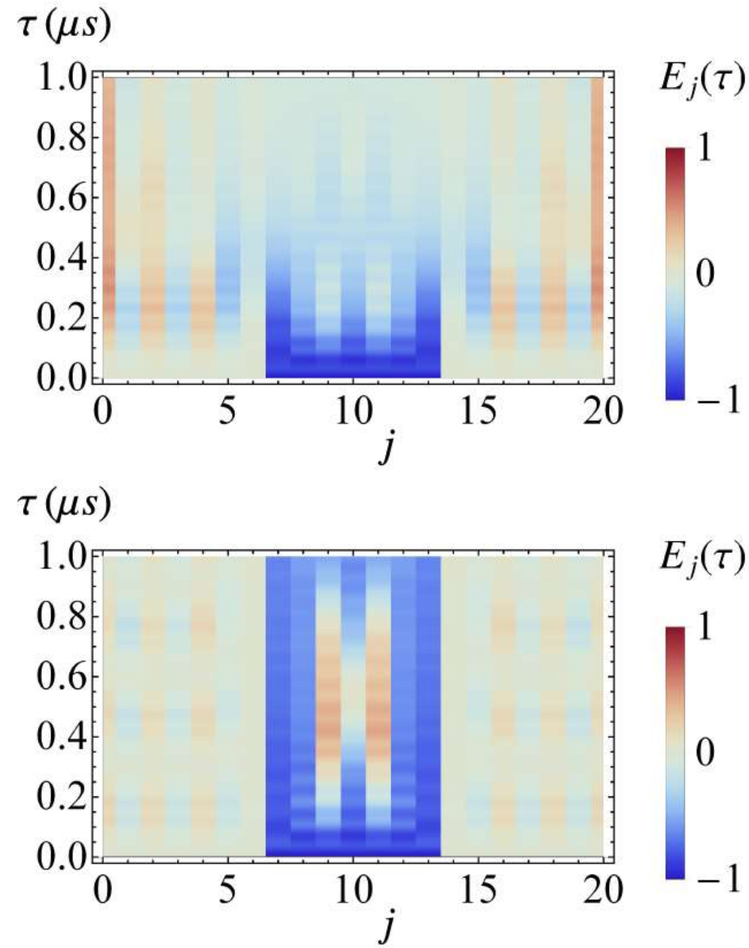


Rydberg – atom quantum simulation



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Tensor Networks and Matrix Product States

- Efficient method to compress information

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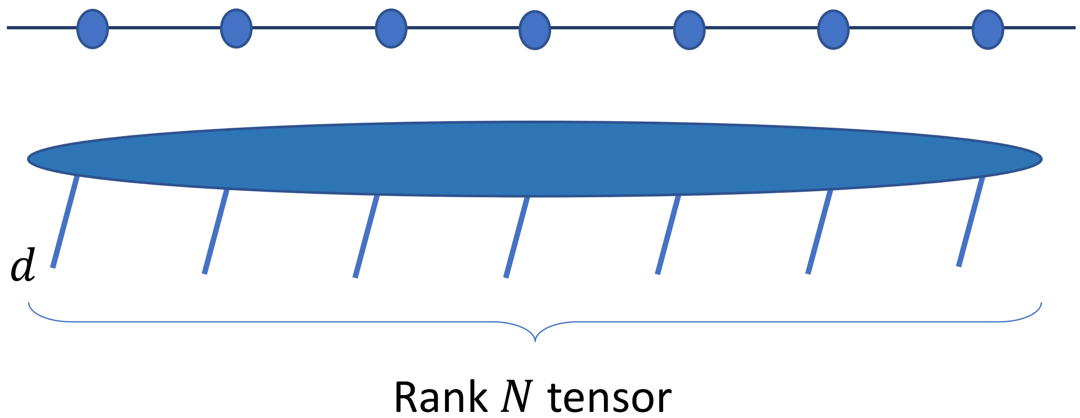
- Lattice model, N sites, local dimension d $\rightarrow \dim \mathcal{H}\ d^N$

$$|\Psi\rangle = \sum_{\alpha_1,\ldots,\alpha_N} c_{\alpha_1,\ldots,\alpha_N} |\alpha_1,\ldots,\alpha_N\rangle$$



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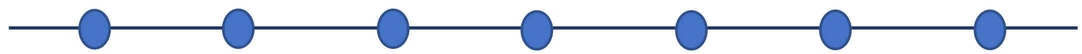

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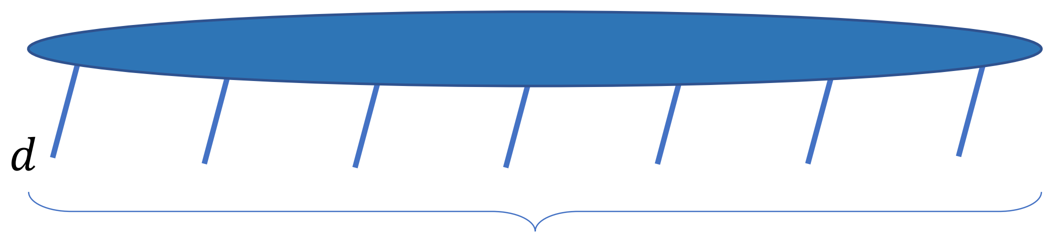
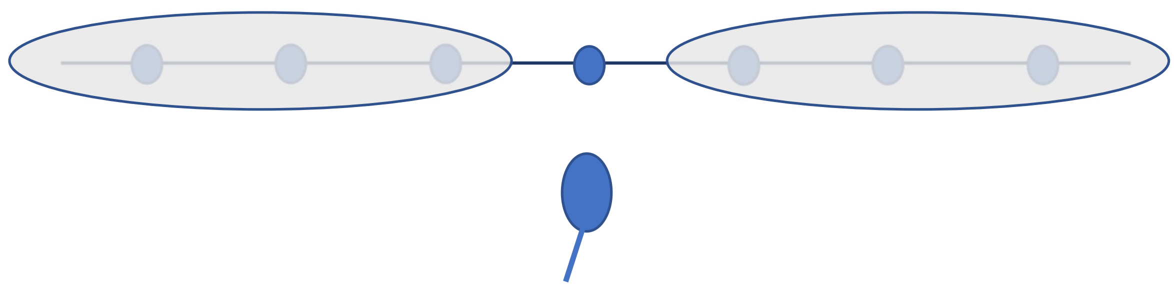


Rank N tensor

$$\rho_{MF} = Tr[|\Psi\rangle\langle\Psi|]$$



Mean field representation

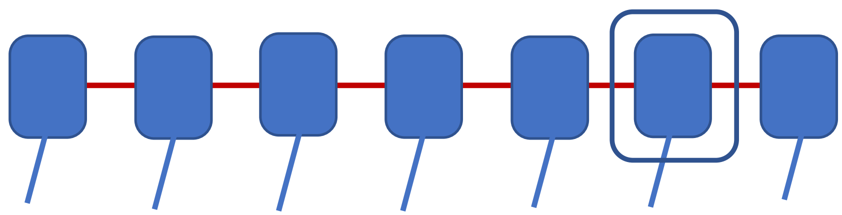
Nd components for describing each site's state

Tensor Networks and Matrix Product States

$$|\Psi\rangle_{MPS} = \sum_{\alpha_{1,\dots,}\alpha_N} T_{\alpha_1}^{d_1}\, T_{\alpha_2}^{d_1 d_2} \dots T_{\alpha_{N-1}}^{d_{N-1} d_N}\, T_{\alpha_N}^{d_N}\, |\alpha_1, \dots, \alpha_N\rangle$$

Bond dimension χ:

- Artificial link
- Allows to keep correlations



Each tensor is a $d \times \chi \times \chi$ matrix

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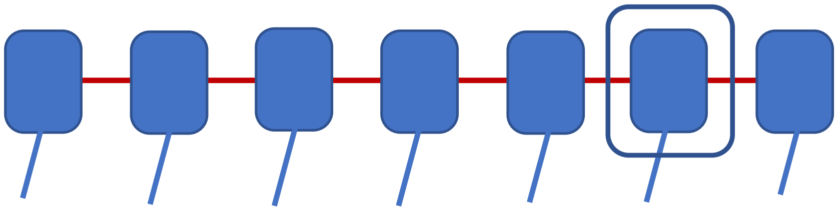

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Given an Hamiltonian $\widehat{H}$:

- Real time dynamics (Time Evolving Decimation Blocks - TEBD)
- Computing the ground state of: variational ansatz → $\min \langle\Psi|\widehat{H}|\Psi\rangle_{MPS}$

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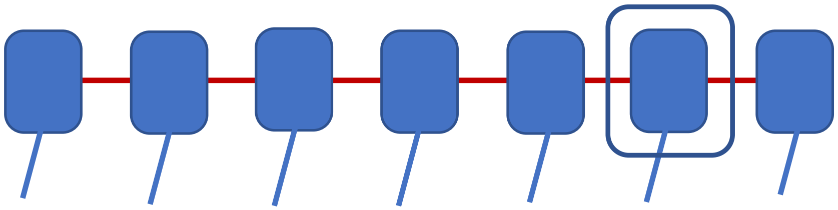

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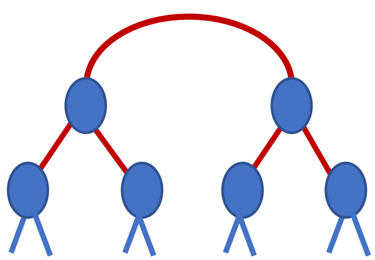
Different geometries available:

Tree Tensor Networks (TTN)



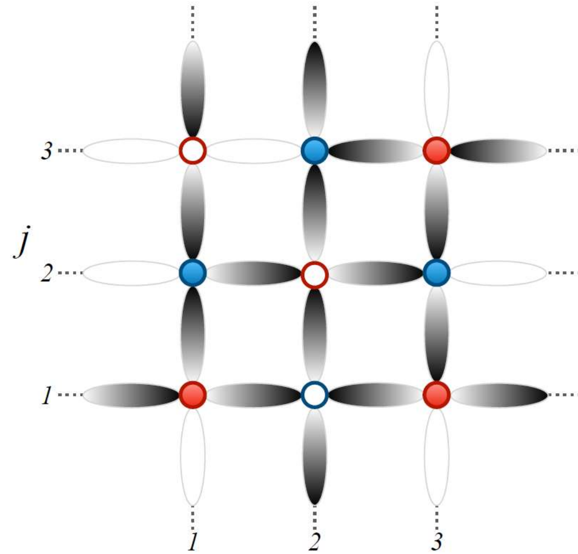
Wave function representation:

No sign problem →

Finite - density simulations are possible

2D Abelian lattice gauge theories

$$\hat{H} = -t\sum_{x,\mu}\left(\hat{\psi}_x^\dagger\,\hat{U}_{x,\mu}\,\hat{\psi}_{x+\mu} + h.c.\right)$$
$$+\,m\sum_x(-1)^x\hat{\psi}_x^\dagger\hat{\psi}_x + \frac{g_e^2}{2}\sum_{x,\mu}\hat{E}_{x,\mu}^2$$
$$-\,\frac{g_m^2}{2}\sum_x\left(\hat{U}_{x,\mu_x}\hat{U}_{x+\mu_x,\mu_y}\hat{U}^\dagger_{x+\mu_y,\mu_x}\hat{U}^\dagger_{x,\mu_y} + h.c.\right)$$

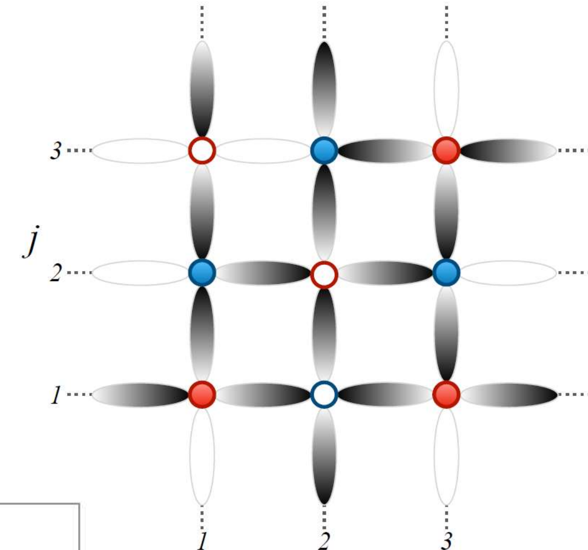


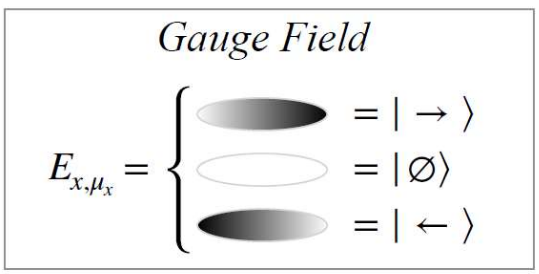
T. Felser, P. Silvi, M. Collura,

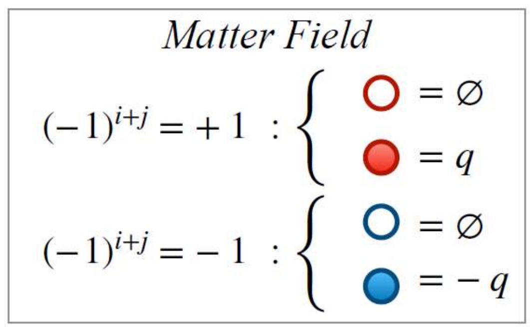
S. Montangero. Arxiv: 1911.09693

2D Abelian lattice gauge theories

$$
\begin{aligned}
\hat{H} = -t \sum_{x,\mu} \left(\hat{\psi}_x^\dagger \, \hat{U}_{x,\mu} \, \hat{\psi}_{x+\mu} + h.c. \right) \\
+ m \sum_{x} (-1)^x \hat{\psi}_x^\dagger \hat{\psi}_x + \frac{g_e^2}{2} \sum_{x,\mu} \hat{E}_{x,\mu}^2 \\
- \frac{g_m^2}{2} \sum_{x} \left(\hat{U}_{x,\mu_x} \hat{U}_{x+\mu_x,\mu_y} \hat{U}^\dagger_{x+\mu_y,\mu_x} \hat{U}^\dagger_{x,\mu_y} + h.c. \right)
\end{aligned}
$$







- Truncated electric field
- Staggered matter

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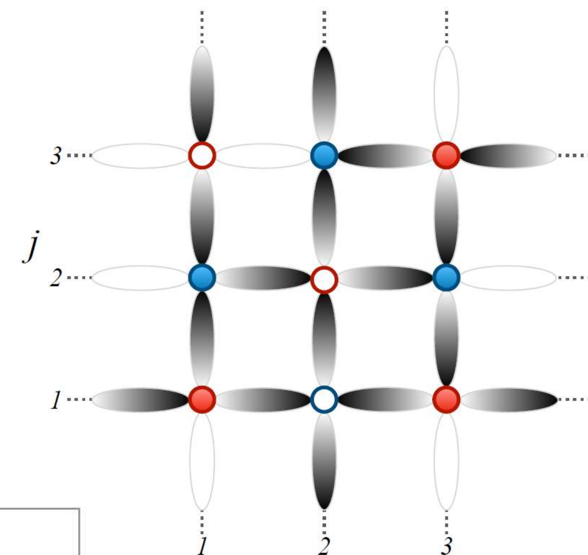

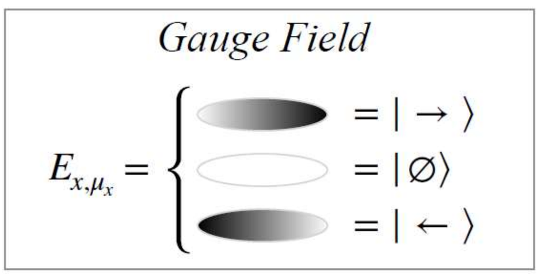

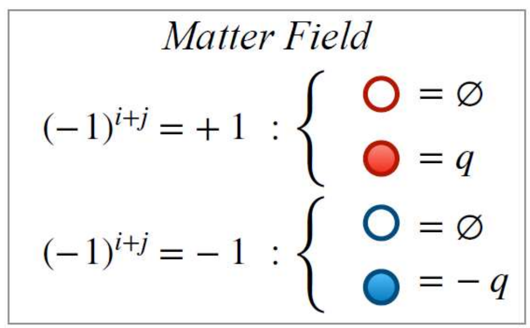

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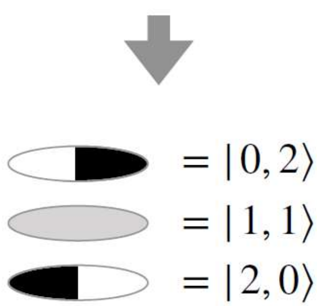
Local gauge + matter state:



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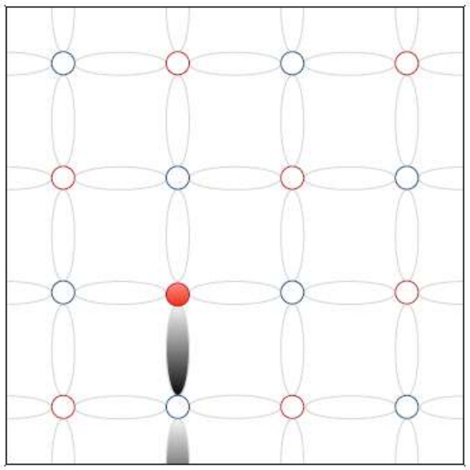
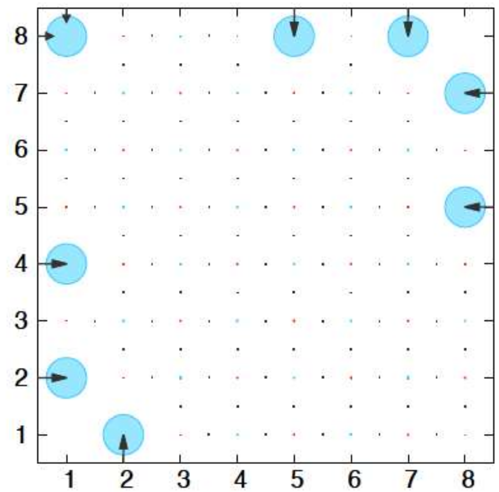
2D Abelian lattice gauge theories

$t = 0$ | $t \neq 0$

Vacuum phase



$\langle \hat{H} \rangle = m + 3g_e^2/4$

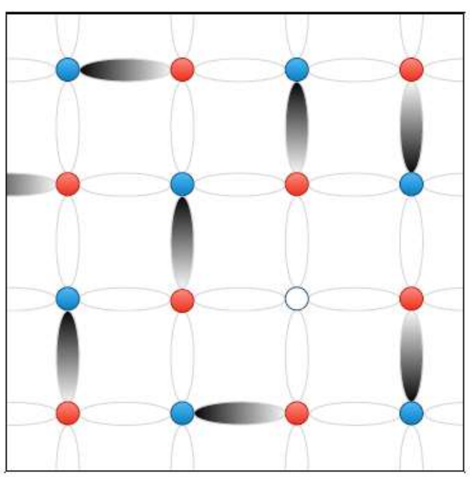


Finite charge density:

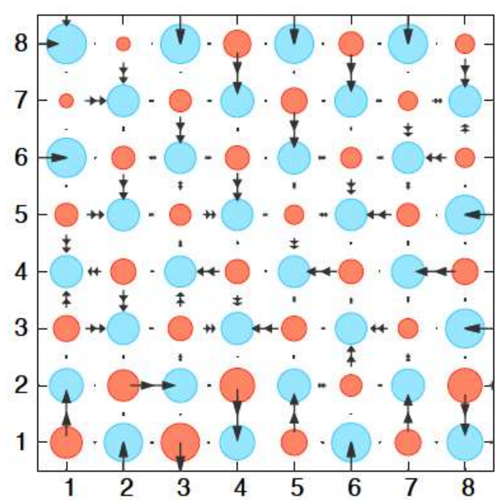
Charges behave as in a conductor

$$m > \frac{g_e^2}{2}$$

Charge-crystal phase



$\langle \hat{H} \rangle = (L^2/2 - 1/2)(2m + g_e^2/2)$

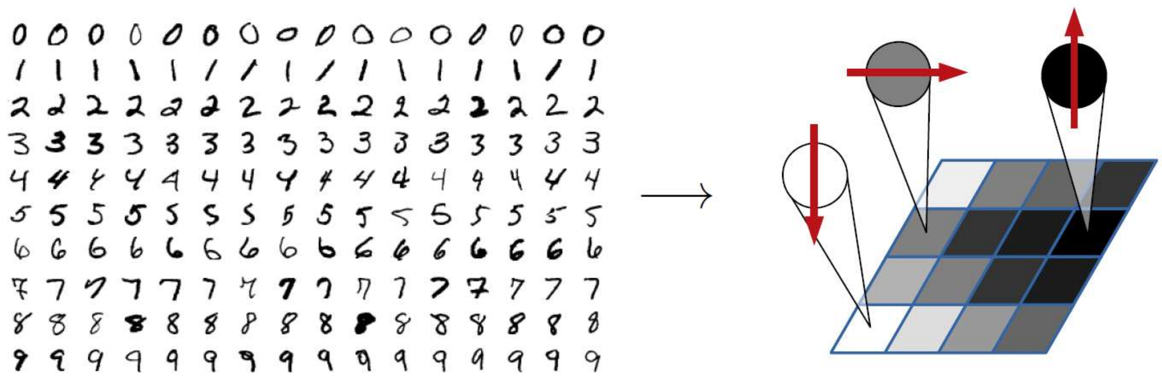


$$m < \frac{g_e^2}{2}$$

MPS applications

MPS applications

Machine learning

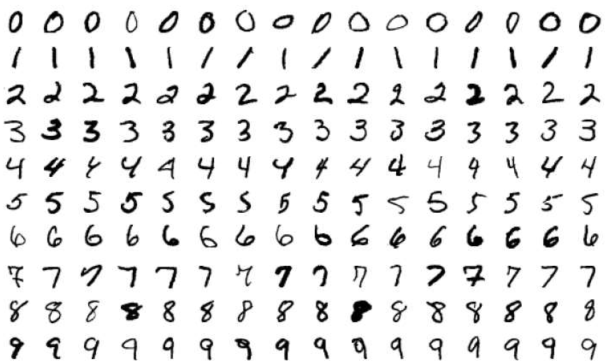


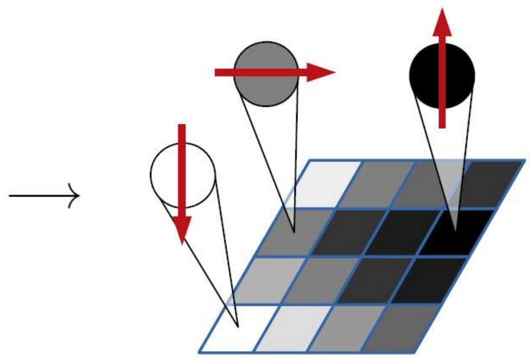
Model	Accuracy
GoogLeNet	93, 7%
AlexNet	89, 9%
XGBoost	89, 8%
MPS	89.0%
2-Layer CNN (Keras)	87.6%

Machine learning: Stoudenmire, Quantum Science and Technology 3, 034003 (2018), arXiv:1801.00315 [stat.ML]

MPS applications

Machine learning





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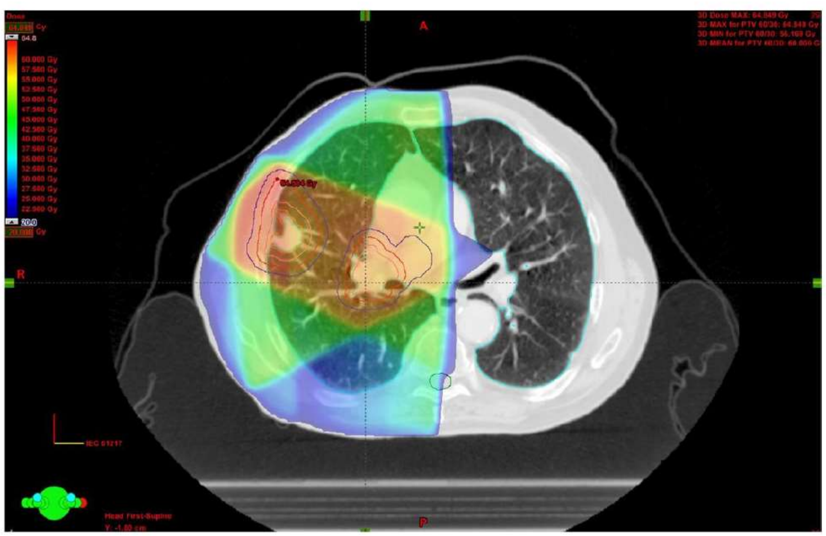
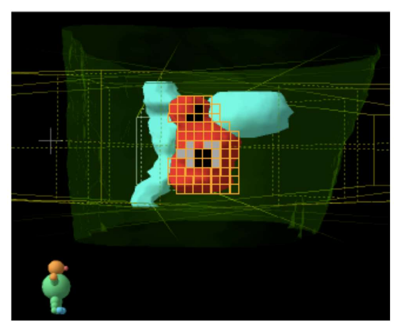


Dosing radiation in cancer therapy



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S. Montangero

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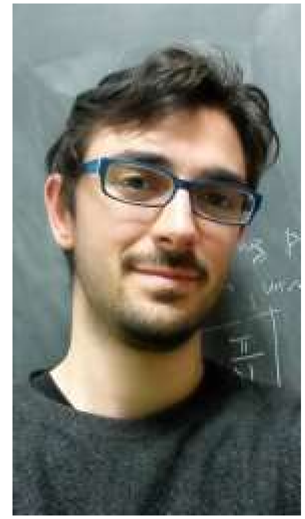
Thank you for your attention



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