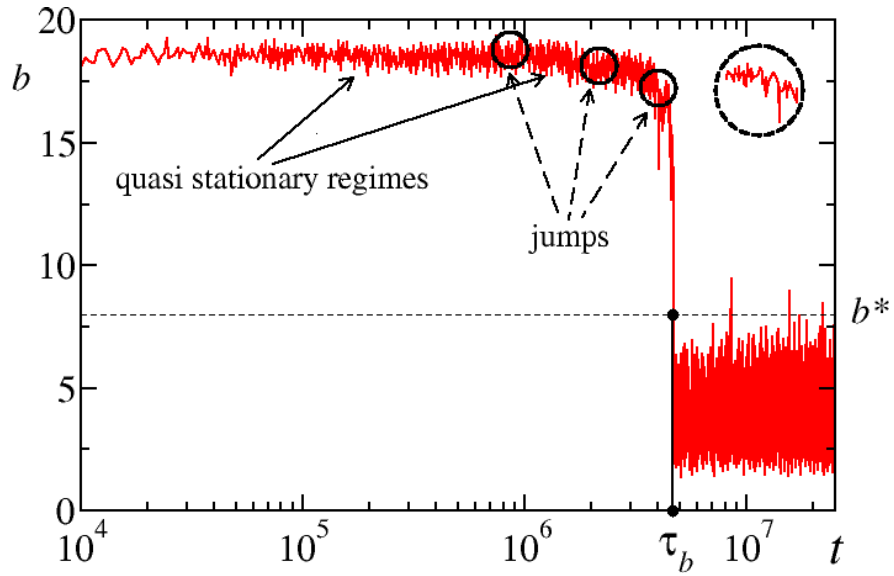
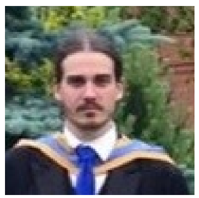
Frozen relaxation dynamics in the presence of breathers

Paolo Politi

Institute for Complex Systems

National Research Council (CNR), Florence (Italy)

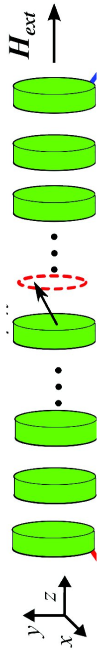


Collaborators:     see at the end

The 1d Discrete NonLinear Schrödinger (DNLS) Equation

$$i\dot{z}_n = -2|z_n|^2 z_n - \gamma(z_{n+1} + z_{n-1})$$

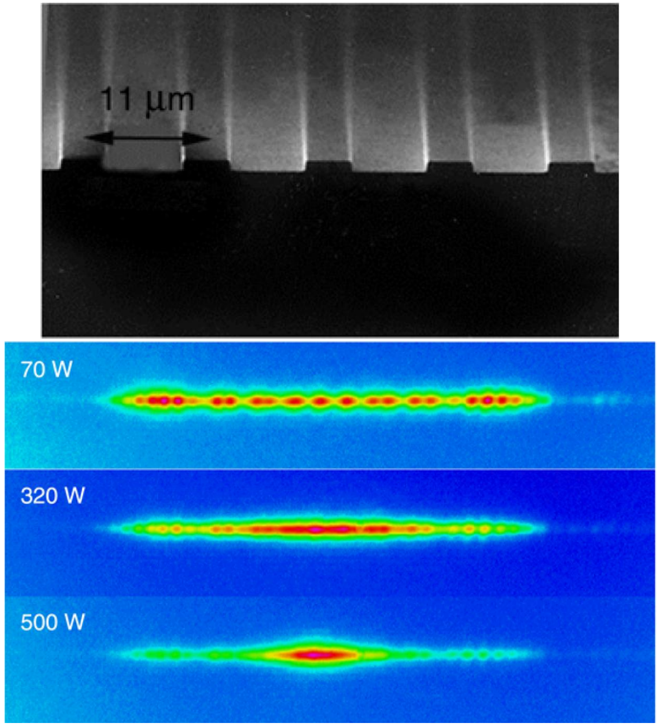
Arrays of nanodiscs



Micromagnetic dynamics

$$z_n = \frac{M_x^n - iM_y^n}{\sqrt{2M_s(M_s + M_z^n)}}$$

Arrays of optical waveguides with Kerr nonlinearity

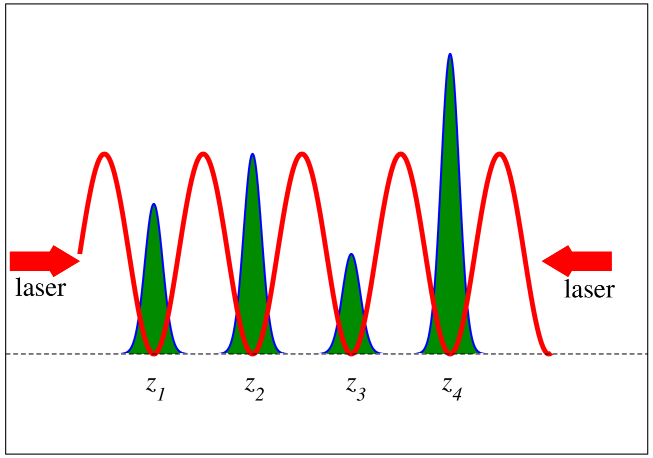


Discrete optical solitons

(H.S. Eisenberg et al 1998)

(F. Lederer et al 2008)

BEC in optical lattices



TB approximation

z_n = wave function in the n$-th$ well

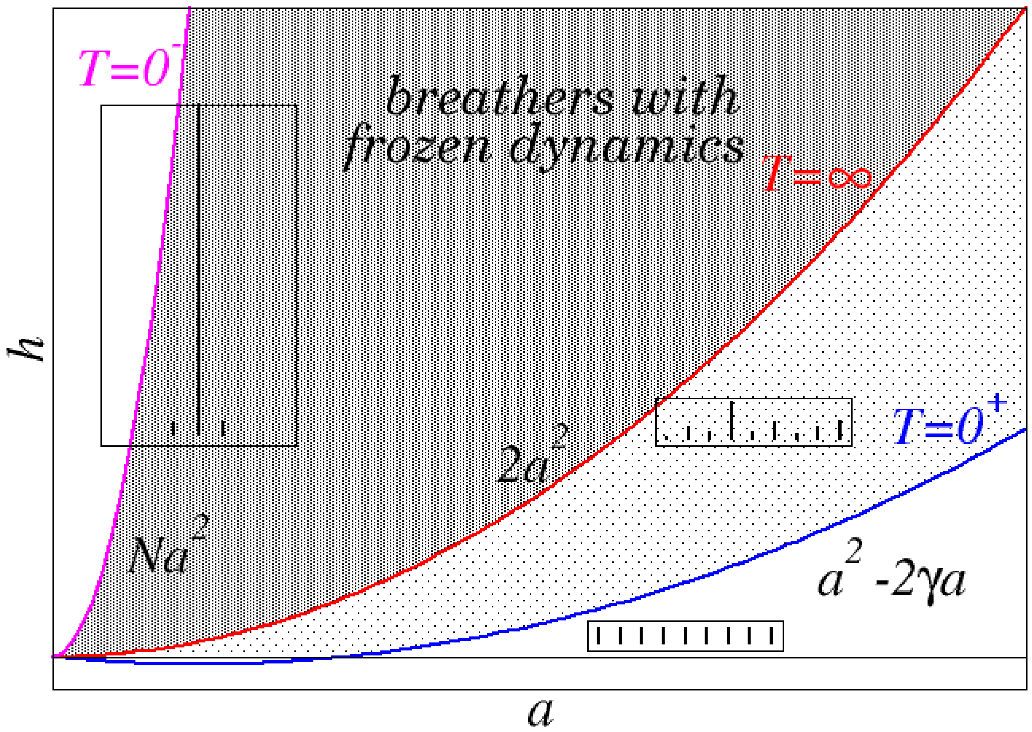
The DNLS equation: $i\dot{z}_n = -2|z_n|^2 z_n - \gamma(z_{n+1} + z_{n-1})$

is Hamiltonian (and not integrable)

$$\dot{z}_n = \frac{\partial \mathcal{H}}{\partial(-iz_n^*)} \qquad \mathcal{H} = \sum_{n=1}^{N} \left(|z_n|^4 + \gamma(z_n^* z_{n+1} + z_n z_{n+1}^*)\right) \qquad A = \sum_n |z_n|^2$$

$$z_n = \sqrt{a_n} e^{i\varphi_n} \qquad \mathcal{H} = \sum_{n=1}^{N} \left(a_n^2 + 2\gamma\sqrt{a_n a_{n+1}} \cos(\varphi_n - \varphi_{n+1})\right) \qquad A = \sum_n a_n$$

Equilibrium phase diagram



$$h = \mathcal{H}/N \qquad a = A/N$$

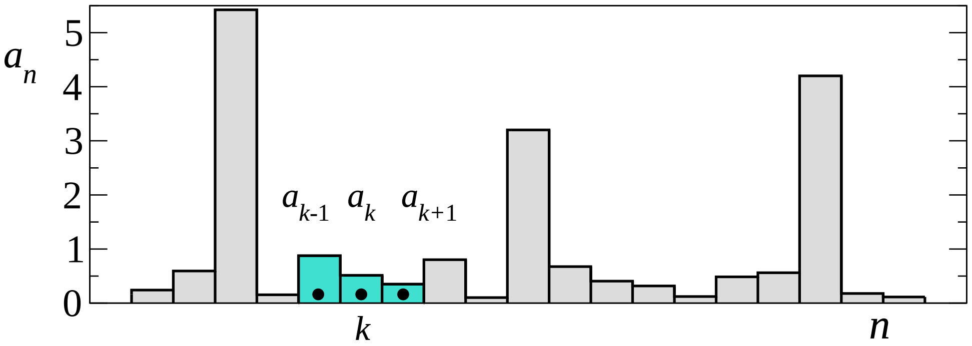
$$h > 2a^2 \qquad (T < 0)$$

Condensation phenomenon

A breather is localized and oscillates with time ($\omega = 2b$)

What is the (sole) effect of having two conservation laws (no coupling) ?

$$\gamma = 0 \quad \Rightarrow \quad i\dot{z}_n = -2|z_n|^2 z_n \quad \Rightarrow \quad \begin{cases} a_n & \equiv \text{constant} \\ \varphi_n(t) & = 2a_n t \end{cases}$$



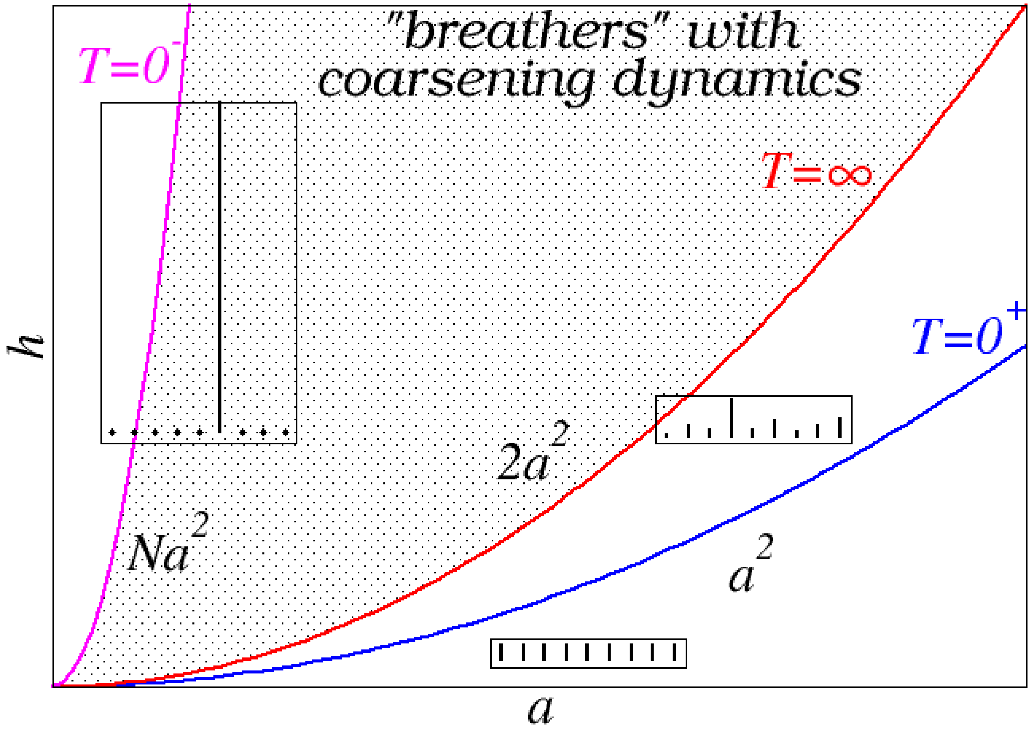
Purely stochastic model

$$A = \sum_n a_n$$

$$\mathcal{H} = \sum_n a_n^2$$

$$a_n \geq 0 \quad \forall n$$

Conservation laws with stochastic dynamics (MMC)

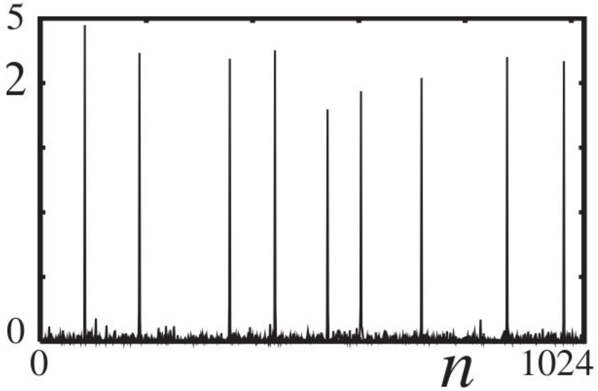


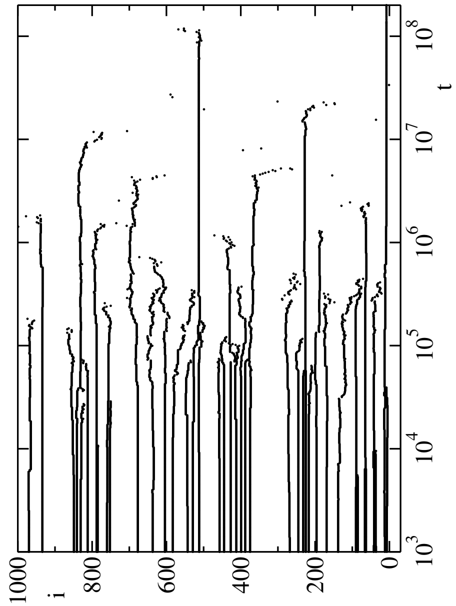
$$h = \mathcal{H}/N \qquad a = A/N$$

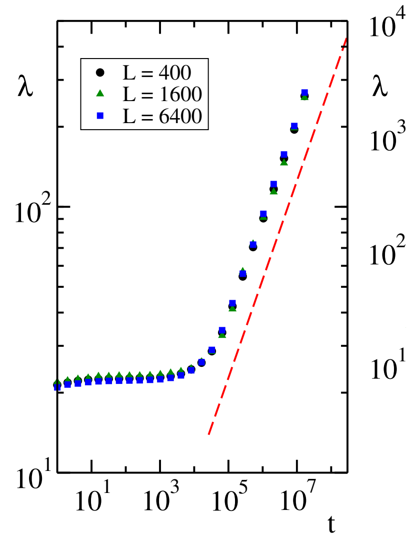
$$h > 2a^2 \qquad (T < 0)$$

Condensation phenomenon

Breather dynamics



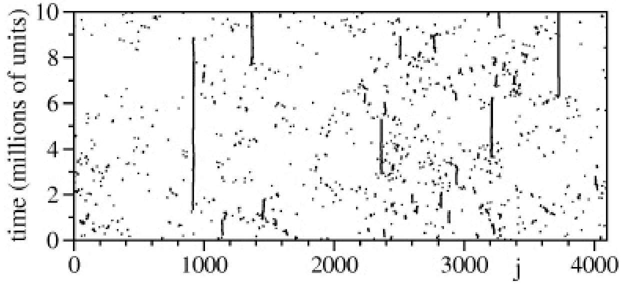




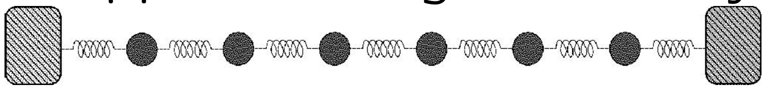
$$\lambda \simeq t^{1/3}$$

When do breathers appear? (In the full DNLS model)

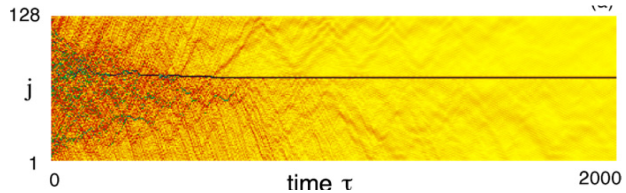
They appear naturally when $T < 0$



They can appear through boundary cooling

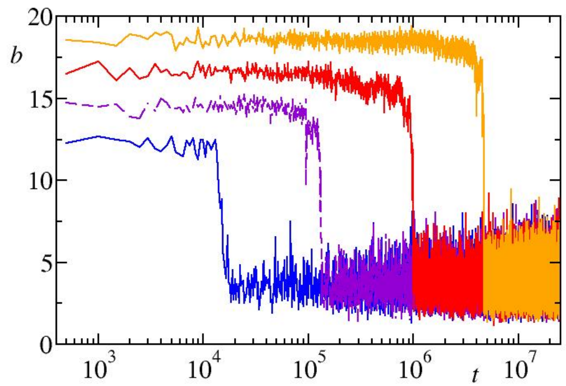


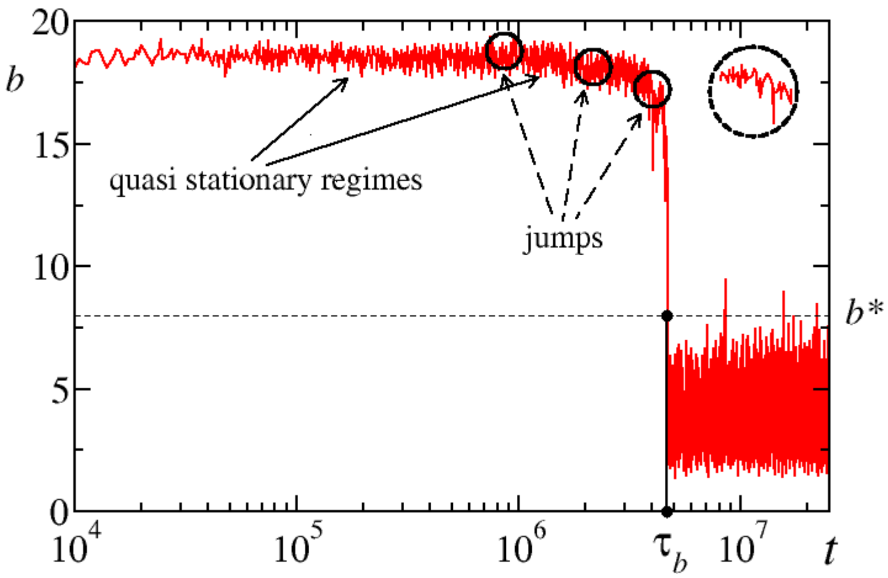
Dissipator Dissipator

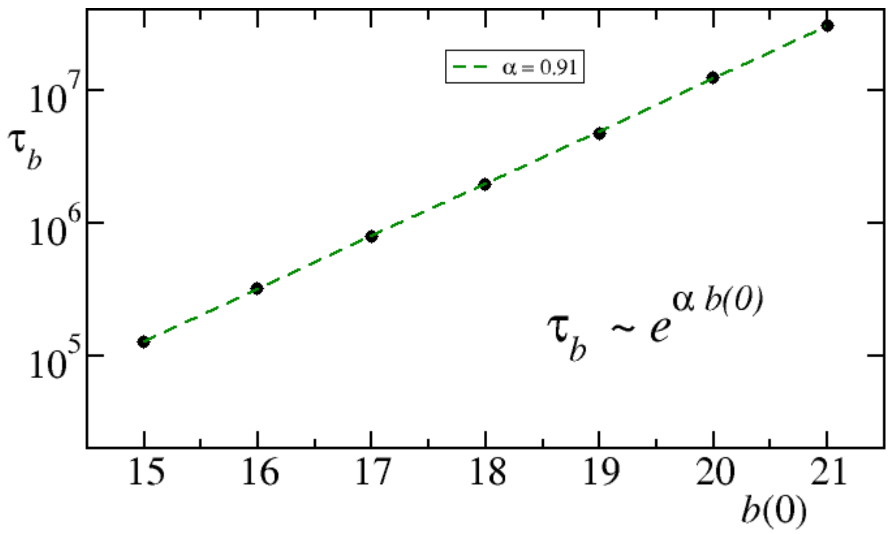


They can be choosen as initial condition at $T > 0$

We study this relaxation process







- Laminar regimes are characterized by an exponentially small diffusion/drift

- Jumps are mainly due to exponentially rare bound states between the breather and a neighbour

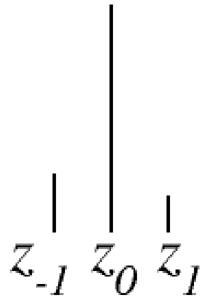
Qualitatively: $b(0) \uparrow \quad \Rightarrow \quad \omega \uparrow \quad \Rightarrow \quad$ breather-backgroud coupling $\downarrow$

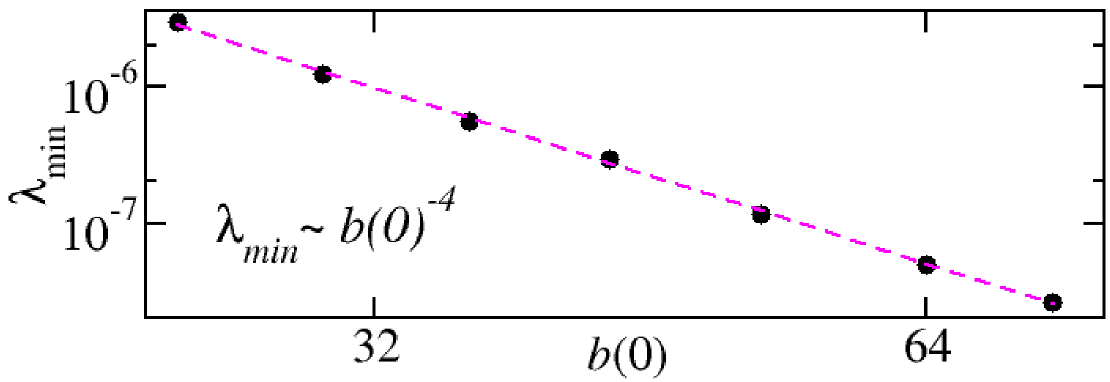
Laminar regime (when $|z_{\pm 1}|$ are small)

Laminar regimes are quasi-stationary with no apparent drift:
what is the diffusion coefficient? $b(t)$ is too noisy so we use PCA analysis.

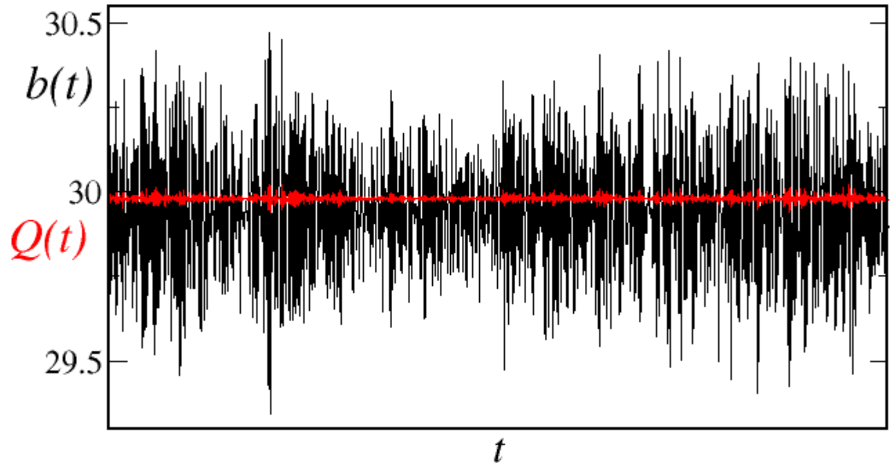
PCA analysis on the triplet:

$$K_{ij} = \langle u_i u_j \rangle - \langle u_i \rangle \langle u_j \rangle$$

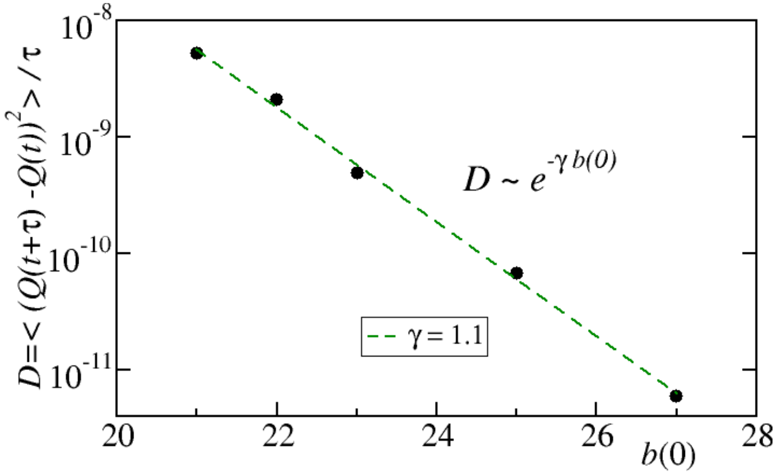


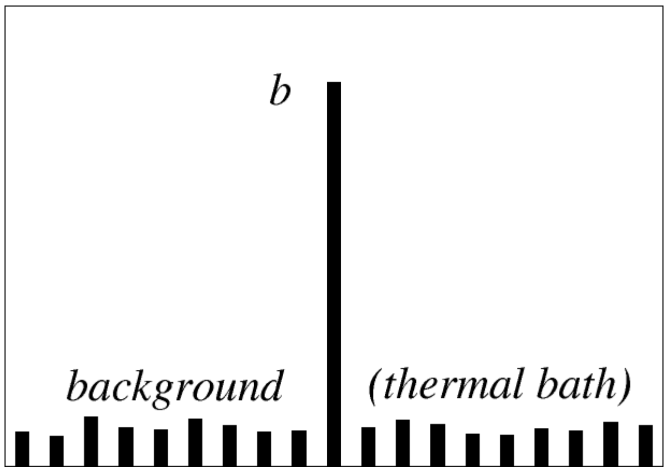


$$Q \simeq E_b^{1/2} \qquad E_b = |z_0|^4 + \frac{1}{2}[z_0^*(z_1 + z_{-1}) + c.c.]$$



Interpretation of the quasistationary regime

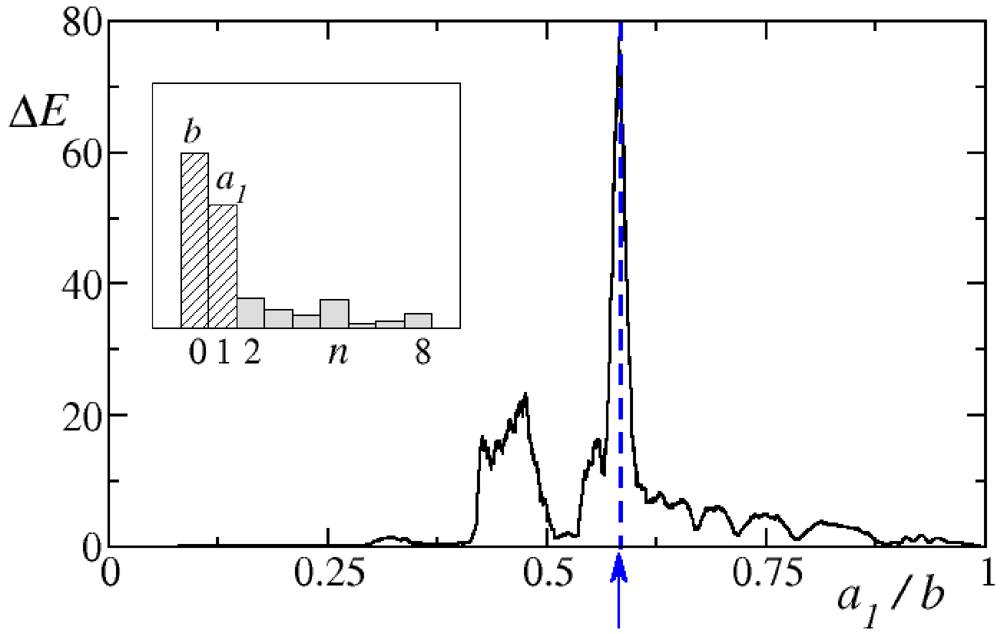


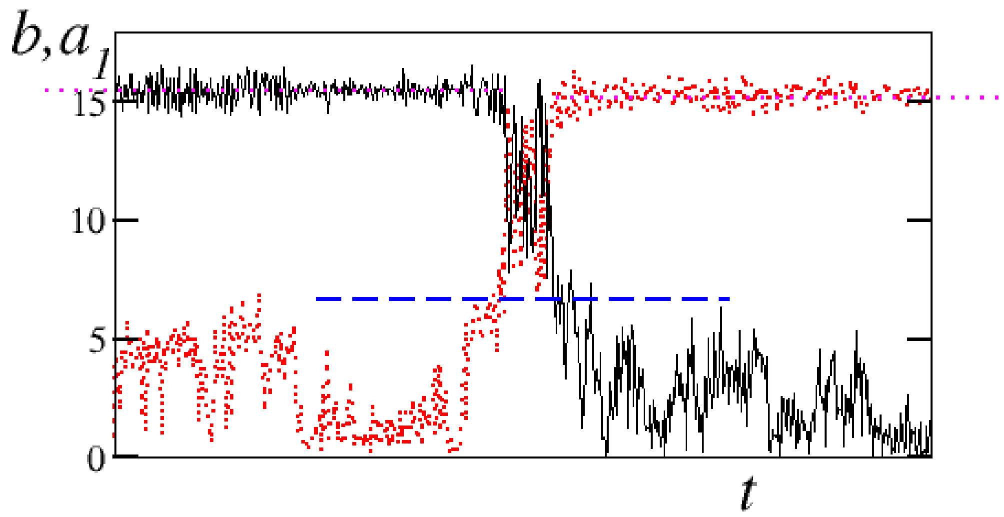


$$D \quad \Rightarrow \quad v \approx D/T \quad \Rightarrow \quad \tau \sim b(0)e^{\gamma b(0)}$$

The hypothesis that $|z_{\pm 1}|$ are small is not always satisfied:

what does it happen when it is not?





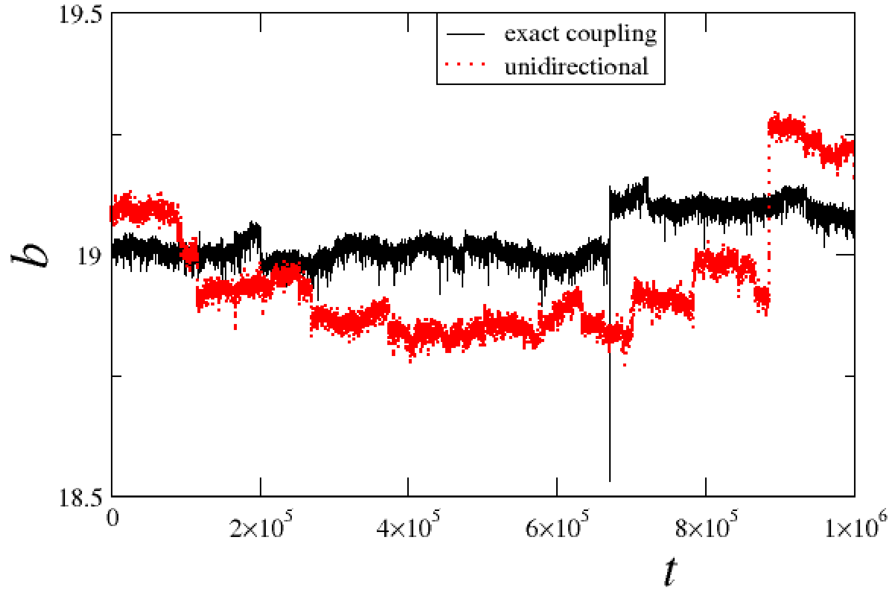
Dimer formation: $\sqrt{a_1} = \sqrt{b} - \sqrt{2}$

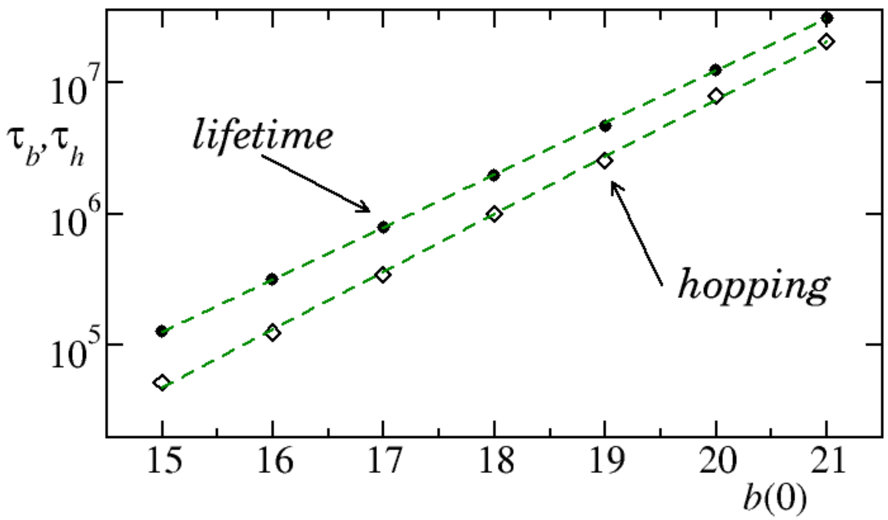
Dimer formation and hopping

$$P(\theta) = \sqrt{4\beta/\pi}\mathrm{e}^{-\beta(\theta^2-\mu\theta+\mu^2/4)}/[1+\mathrm{erf}(\mu\sqrt{\beta}/2)]$$

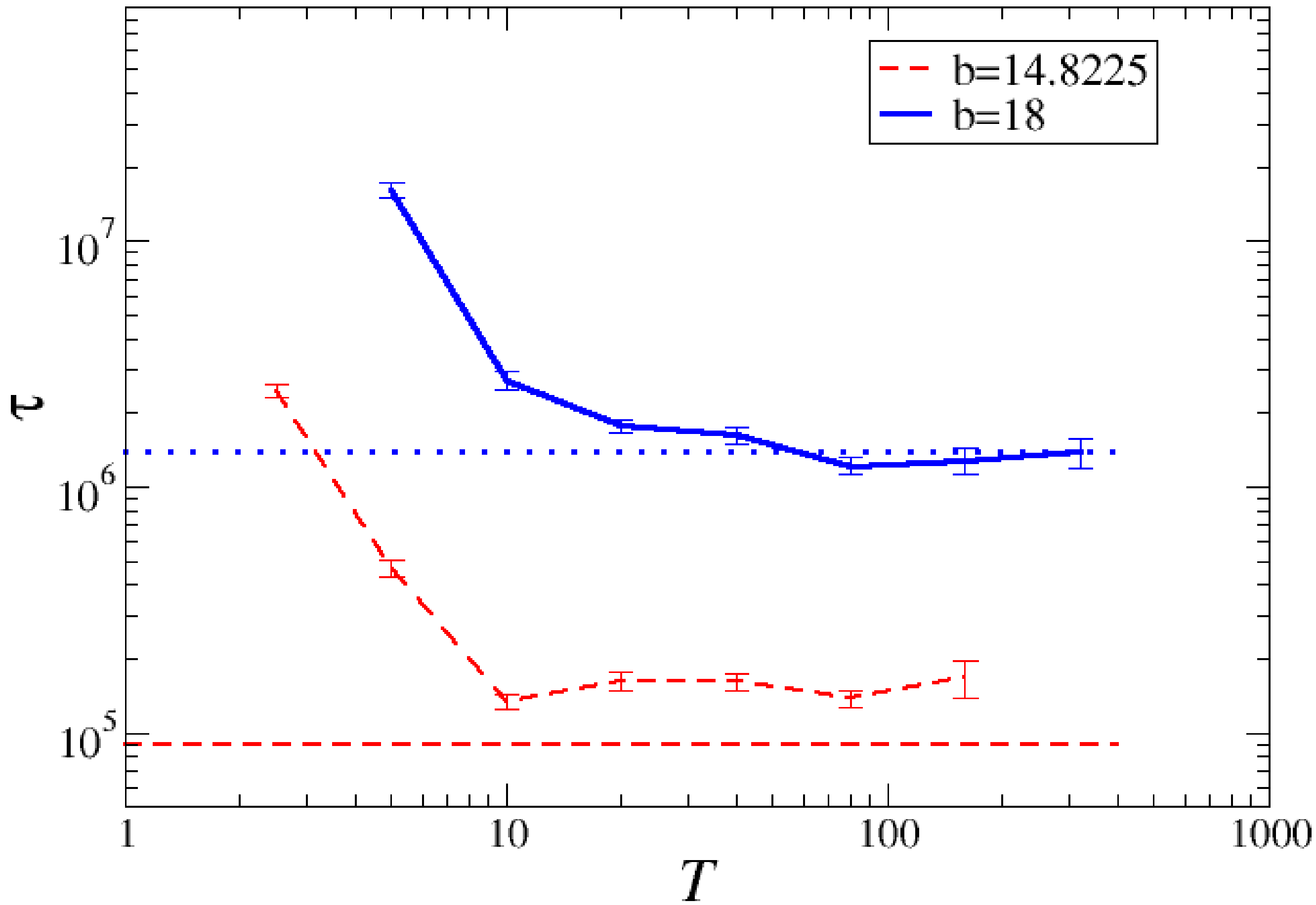
$$\tau_b \approx 1/P(a_1^*) \approx \exp(\beta b^2)$$

(We cannot really discriminate between exponential and superexponential growth)





Relaxation at different temperatures



Conclusions/Discussion

- Two regimes:
  - Laminar regime ($a_{\pm 1} \ll b$) with an exponentially small diffusion/drift
  - Jump regime ($a_{\pm 1} \simeq b$) with an exponentially small rate

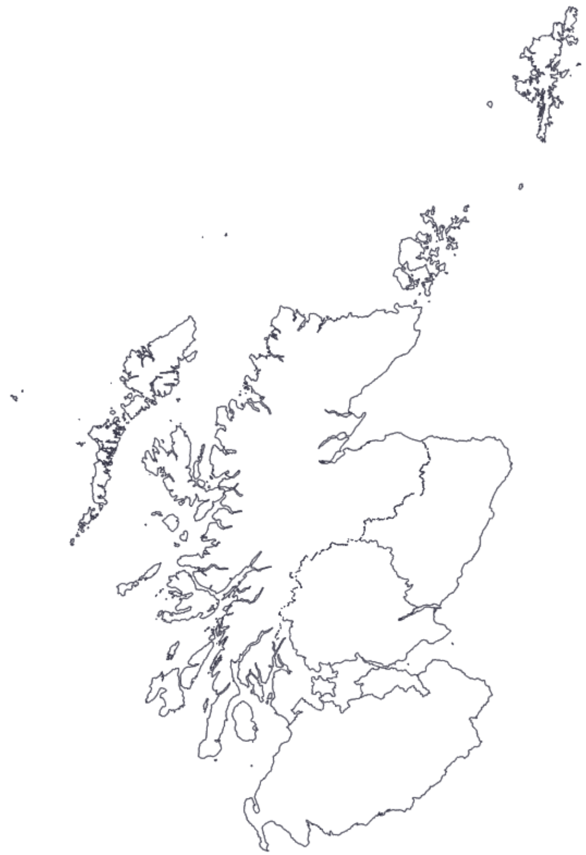
  Are they distinct relaxation channels?

- Why is dynamics frozen?
  We have proposed the existence of an adiabatic invariant,
  but a perturbative approach to determine it does not work

- $T < 0$ regime: ensemble equivalence breaks (Gradenigo et al, 2019)
  Dynamics ?

Ongoing collaboration:

Stefano Iubini (formerly at Uni Padova)



Antonio (Aberdeen)
Gian-Luca Oppo (Glasgow)
Liviu Chirondojan

Thermal bath attached to site 1

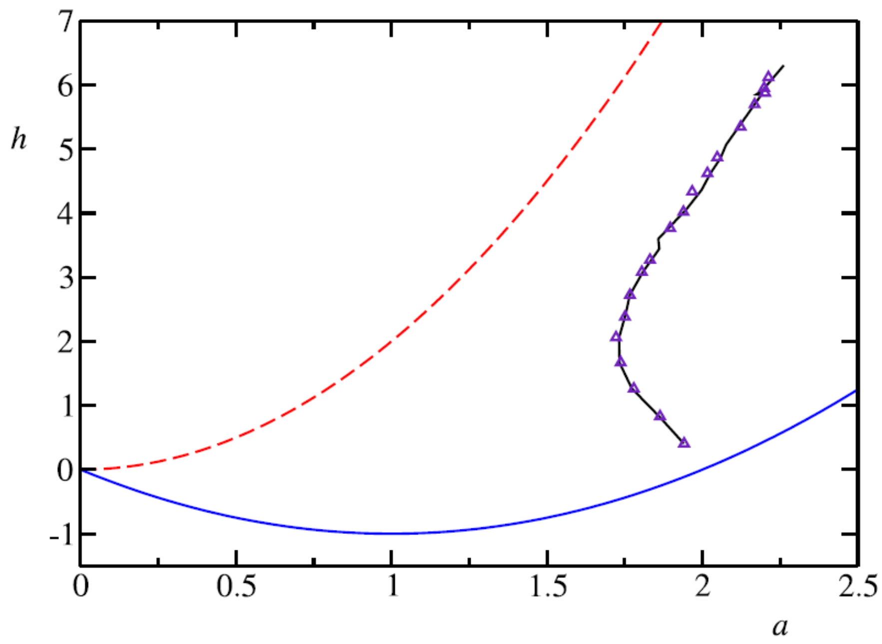
Langevin-type thermal bath (H is not separable)

$$i\dot{z}_1 = (1 + i\Gamma)[\textbf{det. term}] + i\Gamma\mu z_1 + \sqrt{\Gamma T}\eta(t)$$

Monte Carlo thermal bath

$$z_1 = \frac{1}{\sqrt{2}}(p_1 + iq_1), \qquad p_1 \to p_1 + \delta p \qquad q_1 \to q_1 + \delta q$$

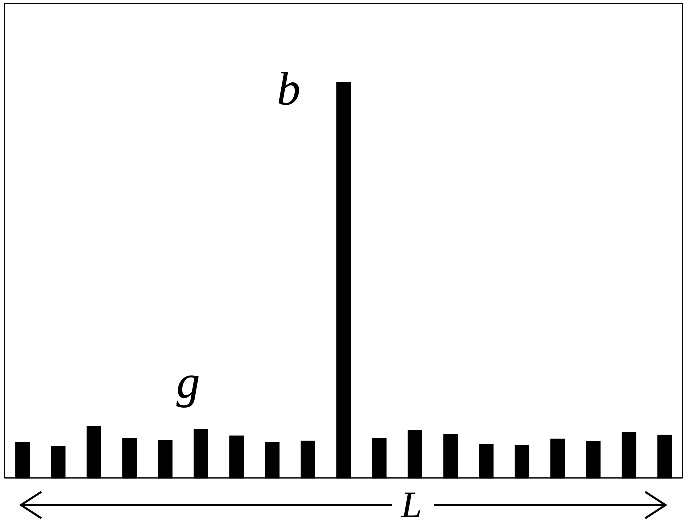
$t \in [t_{min}, t_{max}]$, $\delta p, \delta q \in [-R, R]$, Metropolis algorithm, $e^{-(\Delta H - \mu \Delta A)/T}$



$\mu = 2$

$0.5 \leq T \leq 10$

Why do breathers appear?



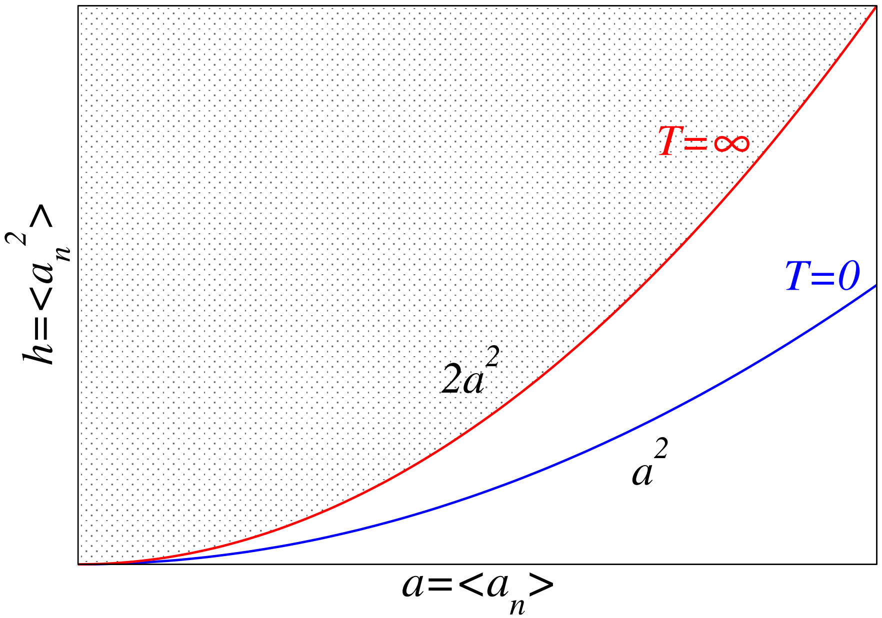
$$a = \langle g \rangle + \frac{b}{L}$$

$$h = \langle g^2 \rangle + \frac{b^2}{L}$$

$$\Longrightarrow$$

$$a \simeq \langle g \rangle$$

$$b \sim \sqrt{L}$$



What is the origin of the coarsening dynamics?

Evolution of a triplet with a breather:

$$\delta(b^2) = 2b\,\delta b \sim g^2 \Longrightarrow \delta b \sim \frac{1}{b}$$

- The typical variation of breather mass depends on its mass
- The typical variation of breather energy is constant

We can interpret dynamics as as exchange of energy between neighbouring breathers, mediated by the background

Why is dynamics frozen? (Simple answer)

Tall breather (mass b)

$\Downarrow$

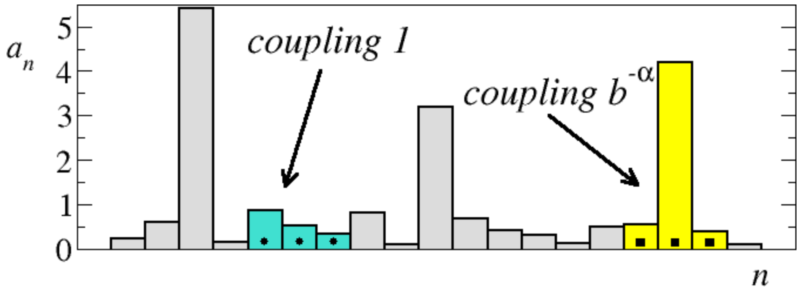
High rotational frequency ($\omega = 2b$)

$\Downarrow$

Weak coupling between sites

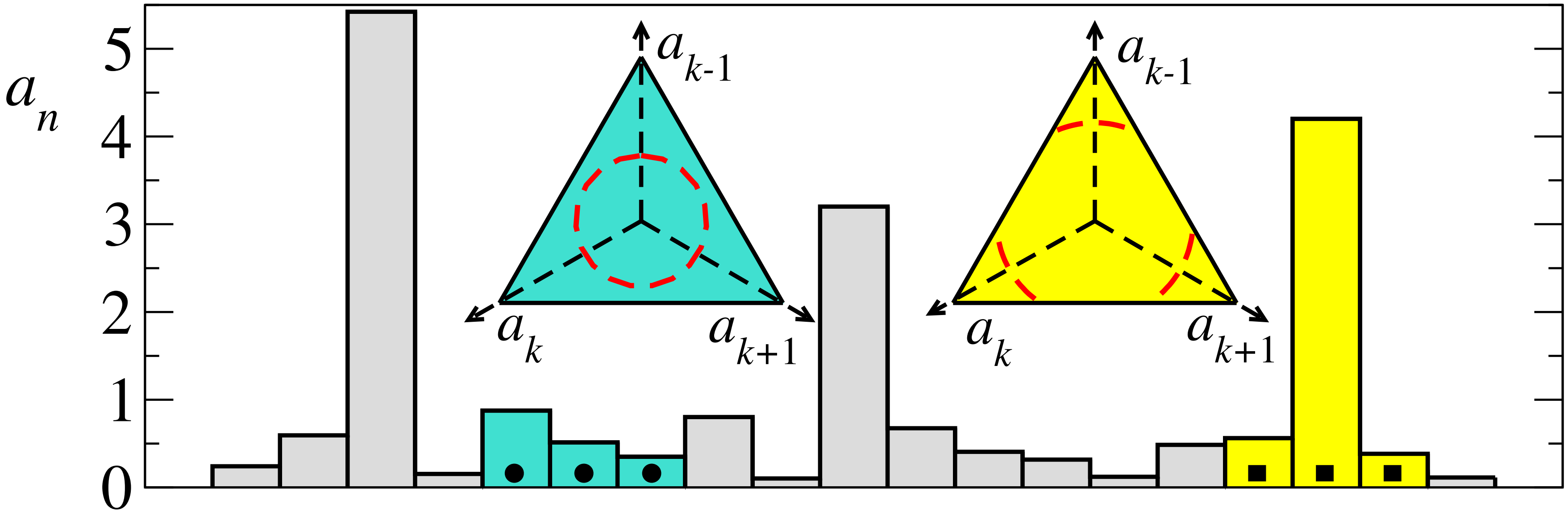
$$\frac{db}{dt} \sim \sqrt{ba}\,\sin(\omega t - \varphi_{\pm 1})$$

In the MMC model, power-law coarsening persists



$$\lambda(t) \sim \begin{cases} t^{1/3} & \alpha \le 1 \\ t^{1/(2+\beta)} & \alpha > 1 \end{cases}$$

(simple answer is wrong)



a_n
0
1
2
3
4
5
a_{k-1}
a_k
a_{k+1}
a_{k-1}
a_k
a_{k+1}

The adiabatic invariant

One degree of freedom

$$H(q,p,\lambda), \qquad \lambda = \lambda_T(t)$$

$$I(E,\lambda) = \frac{1}{2\pi}\oint p\,dq$$

$$(\Delta I)_T = \begin{cases} \alpha \exp(-\beta T) & \lambda(t) \in \mathcal{C}^\infty \\ \alpha T^{-(m+1)} & \lambda(t) \in \mathcal{C}^m \end{cases}$$

Klein Gordon lattice

[Carati & Maiocchi (2012)]

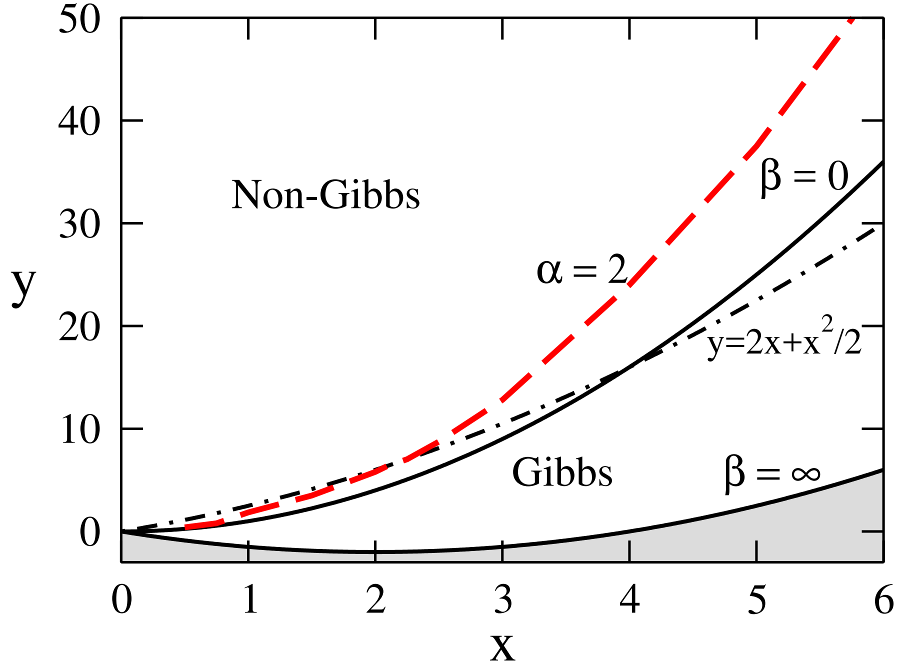
$$H = \sum_i \left[\left(\frac{q_i^2 + p_i^2}{2} \right) + \epsilon \left(q_i q_{i+1} + q_i^4 \right) \right]$$

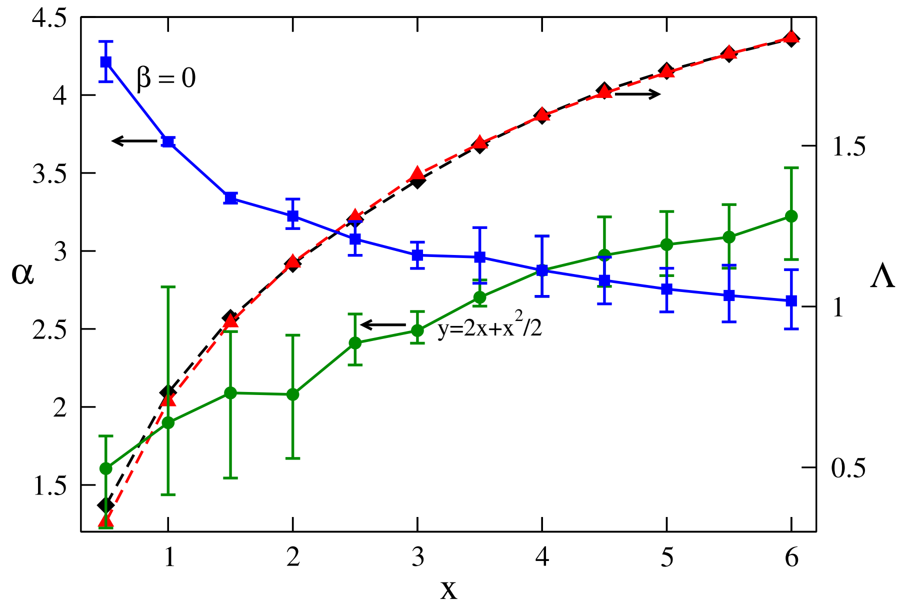
I is obtained recursively

$$\frac{||[I,H]||}{\sigma_I} \leq \exp(-\beta/\epsilon)$$

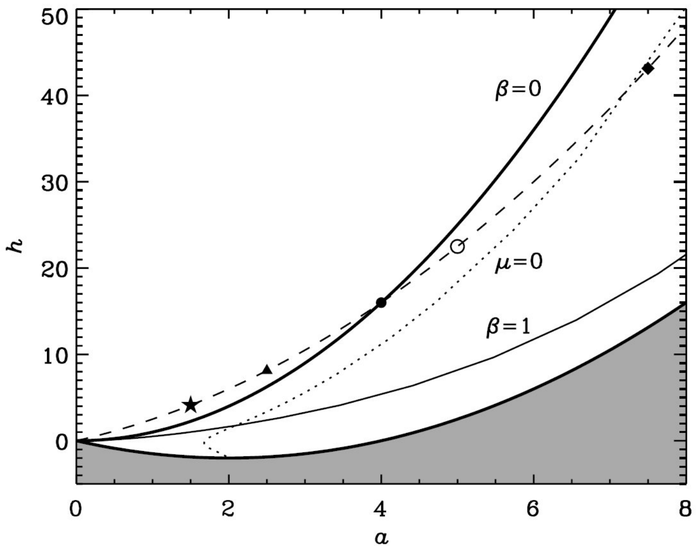
Weakly nonergodic dynamics

[Mithun, Kati, Danieli & Flach (2018)]





K. Ø. Rasmussen et al. (PRL 2000)



Change of curvature of $\ln p(A)$, i.e. change of sign of β