Condensate in quasi two-dimensional turbulence

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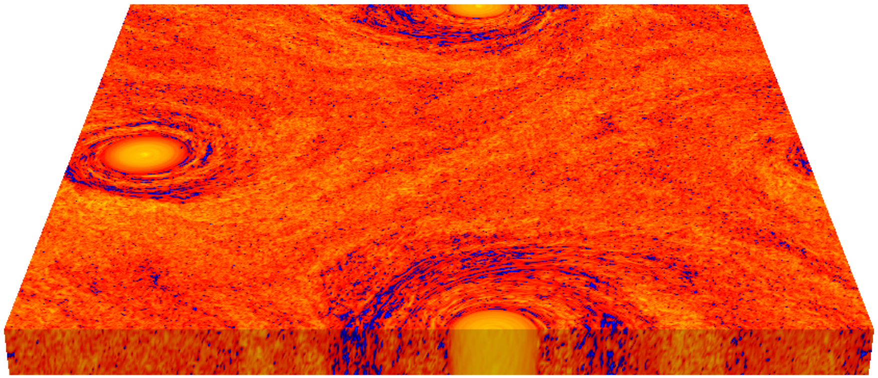
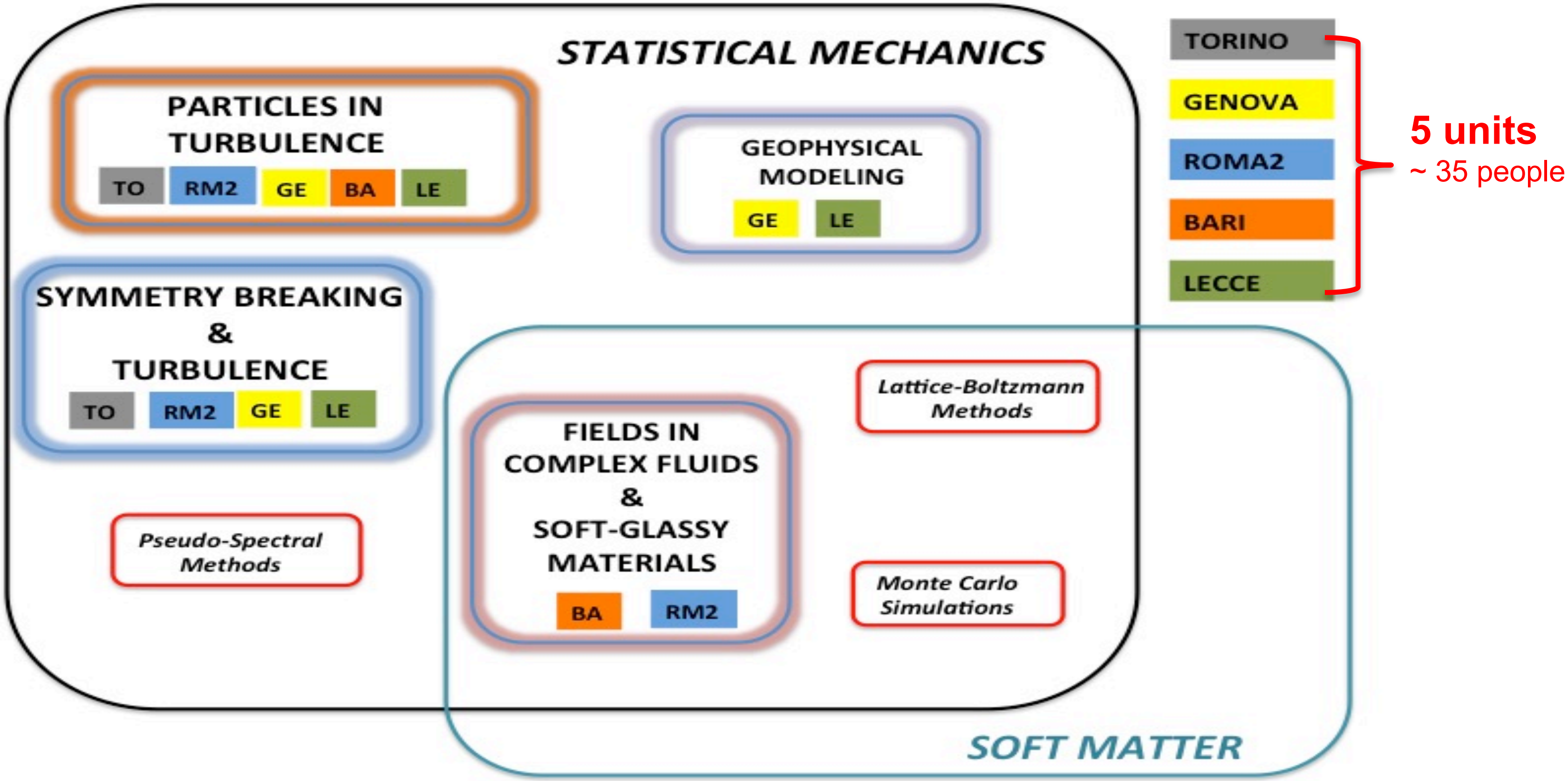
11-13 December 2019, Bari, Italy



FIELDTURB: PARTICLES AND FIELDS IN TURBULENCE AND IN COMPLEX FLOWS



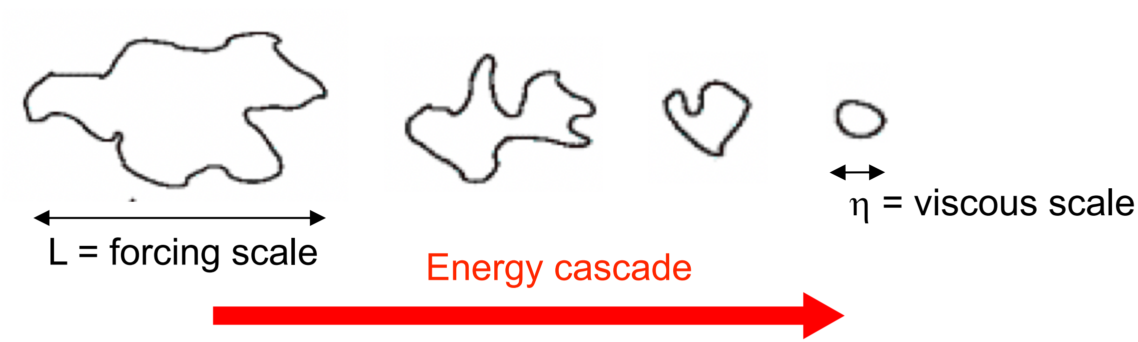
Guido Boffetta - Torino
Andrea Mazzino - Genova
Luca Biferale - Roma 2
Giuseppe Gonnella - Bari
Alessandra Lanotte - Lecce

Turbulence

Turbulence: Classical non-linear field theory, out of equilibrium, non-perturbative, with non-Gaussian, anomalous fluctuations

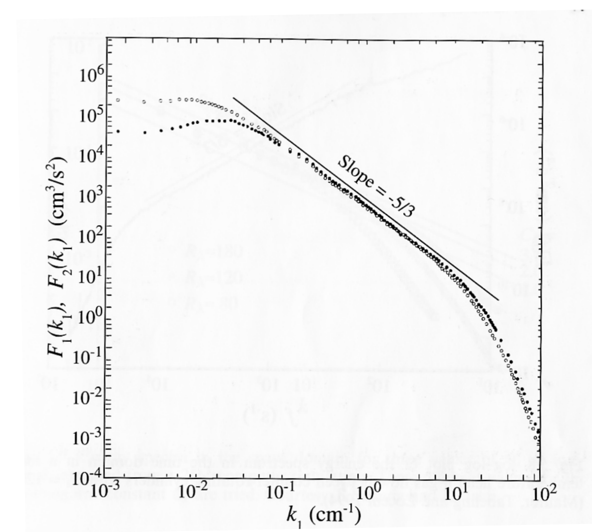
Navier-Stokes equations: $\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}$

Non-equilibrium system: Forcing + dissipation (viscosity)



$$Re = \frac{UL}{\nu} = \left(\frac{L}{\eta}\right)^{4/3}$$

Turbulent cascade of kinetic energy (Richardson, 1920)



Universality of small-scale statistics (for $Re \to \infty$)

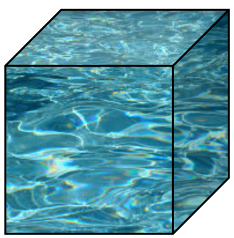
Kolmogorov spectrum $E(k) \simeq \varepsilon^{2/3} k^{-5/3}$

Turbulence & Dimensions

Navier-Stokes Eq. in 2D has 2 inviscid invariants:

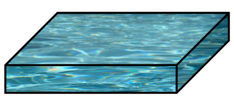
Energy $E = \frac{1}{2}\langle|\boldsymbol{u}|^2\rangle$ Enstrophy $Z = \frac{1}{2}\langle\omega^2\rangle$ $\boldsymbol{\omega} = \nabla \times \boldsymbol{u}$

Reversal of the direction of the turbulent cascade in 2D (Fjørtoft 1953)

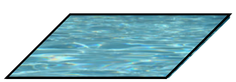


3D: Direct energy cascade:

Kinetic energy is transferred
from large to small eddies
(Kolmogorov ‘41 theory)



Quasi-2D Turbulence



2D: Inverse energy cascade

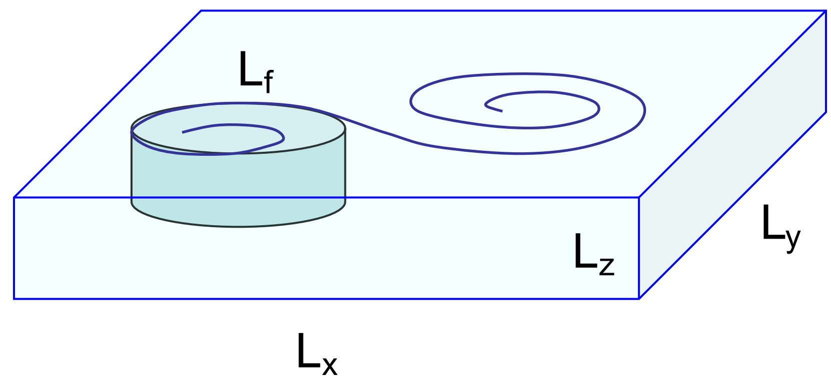
Kinetic energy is transferred
from small to large eddies
(Kraichnan-Batchelor-Leith ‘70 theory)

Split energy cascade in thin fluid layers

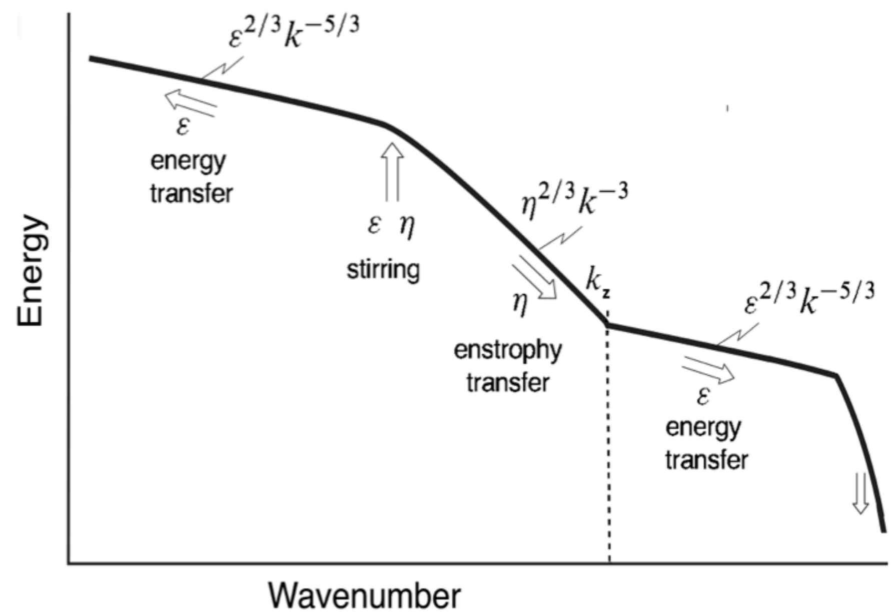
3D Navier-Stokes equations

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

Confined in a thin domain



Smith, Chasnov,Waleffe (1996)
Celani, SM, Vincenzi (2010)
Benavides & Alexakis (2017)
SM & Boffetta (2017)



2D inverse energy cascade
at large scales L > L_f

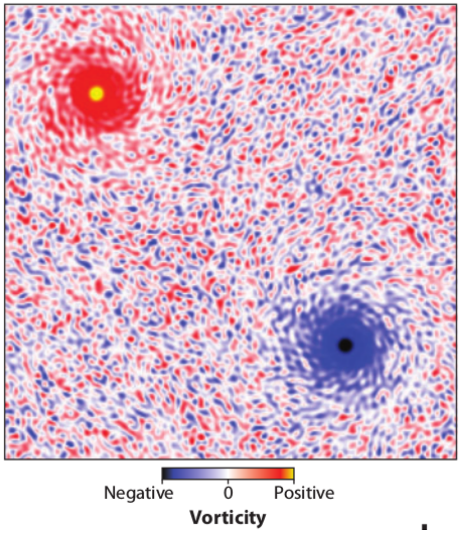
2D direct enstrophy cascade
at intermadiate scales L_z < L < L_f

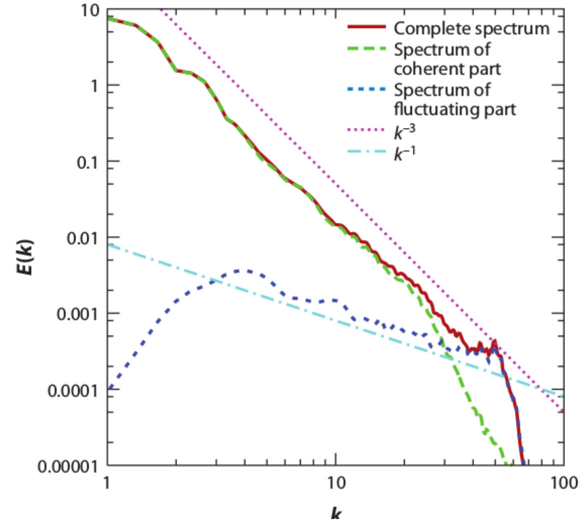
3D direct energy cascade
at small scales L < L_z

Condensates in two dimensional turbulence

Numerics

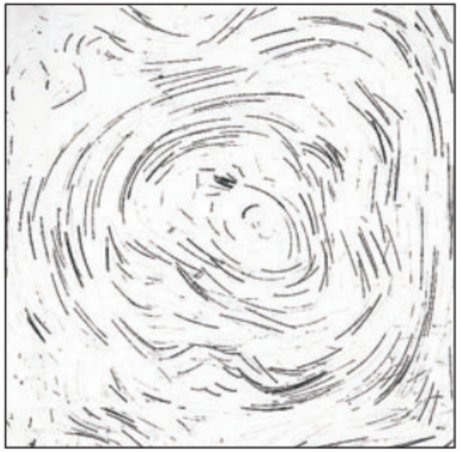
Hossain et al. (1983)
Smith &Yakhot (1993,1994)
Chertkov et.al. (2007)
Bouchet et al. (2009)
Gallet & Young (2013)
Laurie et al. (2014)
Frishman et al. (2017)

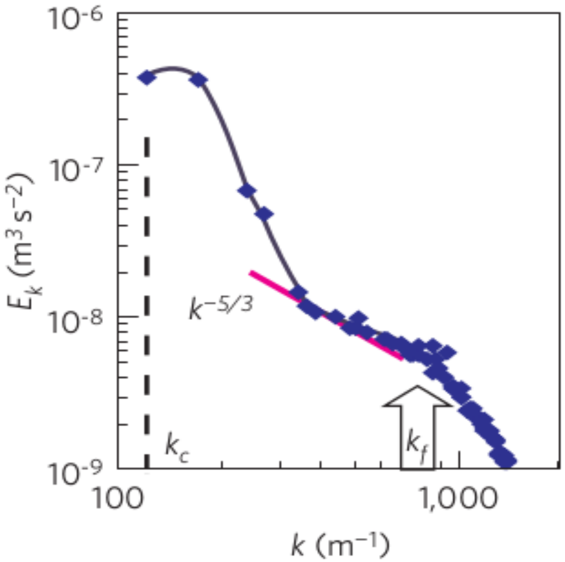




Experiments

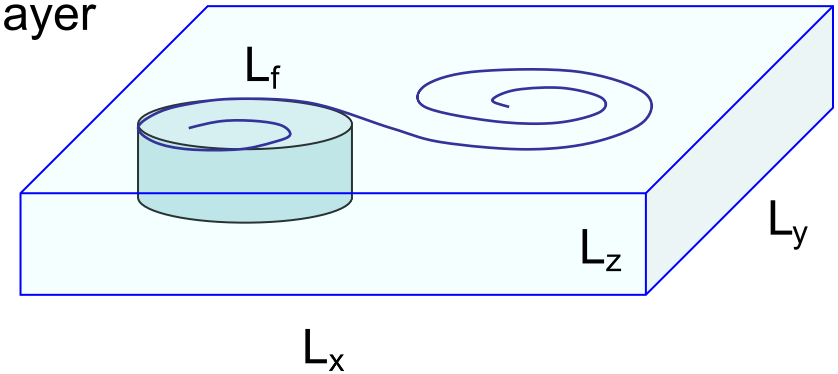
Sommeria (1986)
Xia et al. (2008,2009,2011)





Self-organization of the flow in large-scale structures

Numerical simulations of thin fluid layers



3D Navier-Stokes equations for a thin fluid layer

$$\partial_t \boldsymbol{u} + \boldsymbol{u} \cdot \boldsymbol{\nabla} \boldsymbol{u} = -\boldsymbol{\nabla} p + \nu \nabla^2 \boldsymbol{u} + \boldsymbol{f}$$

Aspect ratio: $r = L_x / L_z = [32, 64]$

$s = L_z / L_f = [1/4, 1/8]$

Resolution: $N_x = N_y = r N_z = 1024$

Periodic b.c: no boundary layers

Hyperviscosity: $\nu \nabla^2 \boldsymbol{u} \to (-1)^{p+1} \nu_p \nabla^{2p} \boldsymbol{u}$ $\quad p = 8 \;\; \nu_p = \; 10^{-37}$

Forcing: two-dimensional force (2D2C) $\boldsymbol{f}(\boldsymbol{x}) = (f_x(x, y), f_y(x, y), 0)$

forcing wavenumber $K_f = 2\pi / L_f = 8$

random in time: fixed energy input ε and enstrophy input $\beta = K_f^2 \varepsilon$

Time scale: $t_f = \beta^{-1/3}$ Energy scale: $E_f = \varepsilon t_f$

Numerical simulations

Numerical challenges:

Resolve 3D turbulence at small scales << L_z
Resolve 2D dynamics at large scales >> L_z
Resolve the fast dissipative scales and the slow dynamics of the condensate

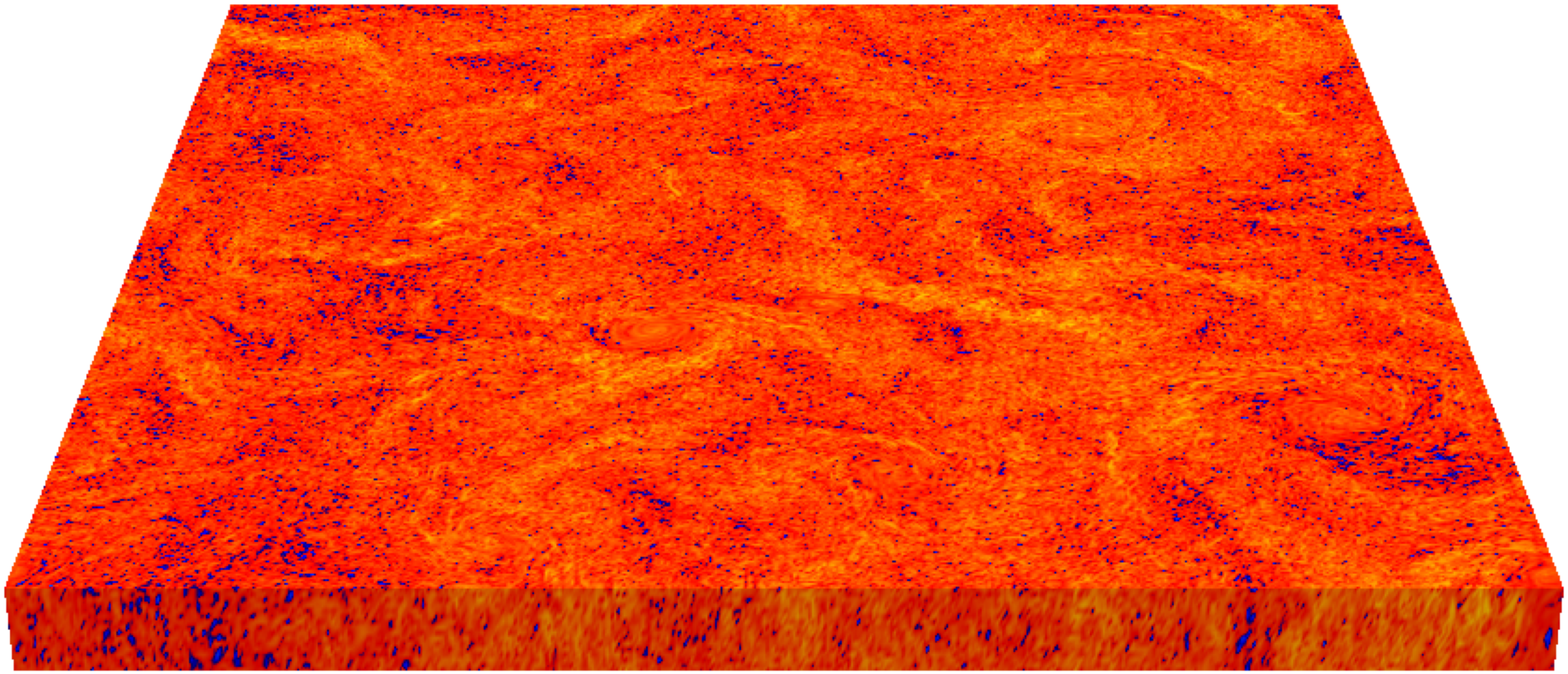
Simulations performed at CINECA on Marconi

CINECA

CINECA HPC Grant: INF18_fldturb

Condensate

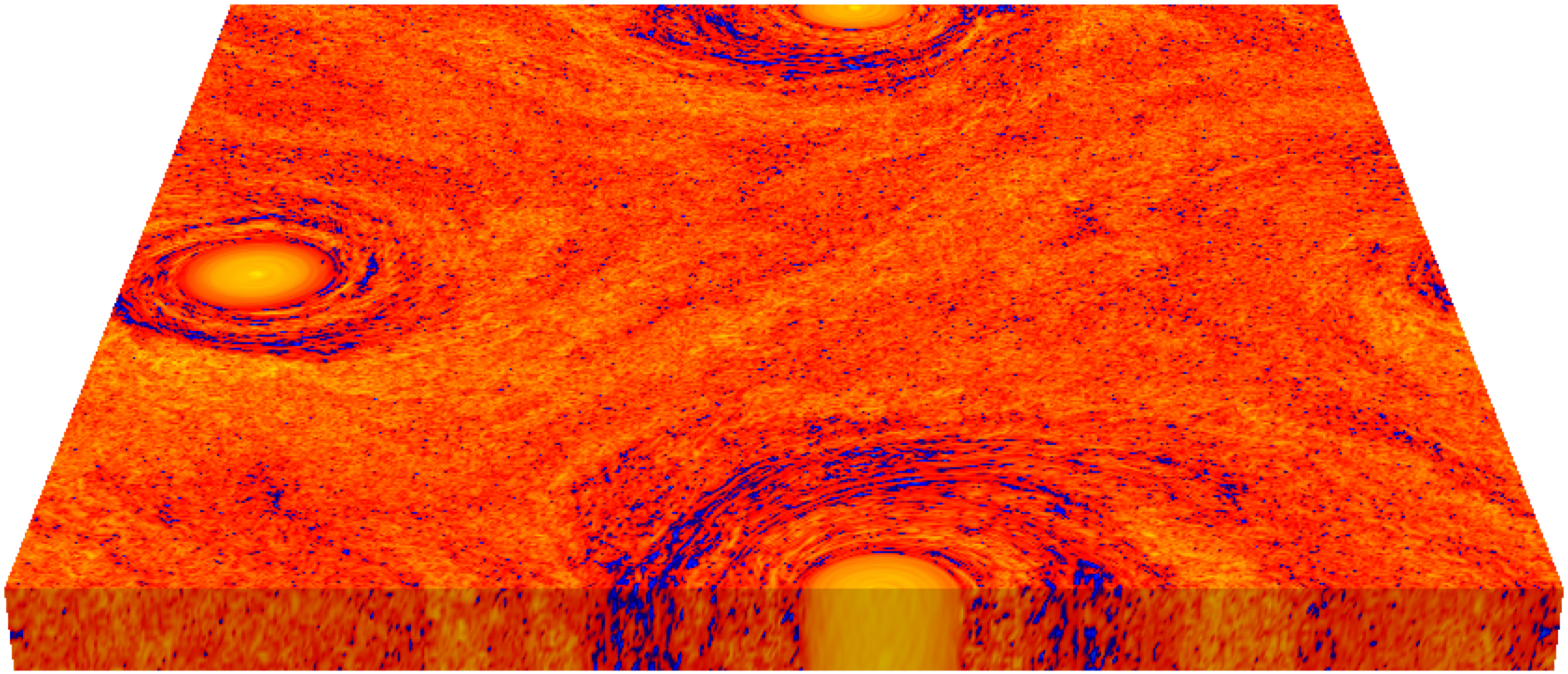
Snapshots of the square vertical vorticity
for the simulation with $L_z/L_f = 1/4$ at times $t = 24\ t_f$



Logarithmic color scale with blue (yellow) representing small (large) values.
The vertical scale has been stretched by a factor 2.

Condensate

Snapshots of the square vertical vorticity
for the simulation with $L_z/L_f = 1/4$ at times $t = 1200\ t_f$



Logarithmic color scale with blue (yellow) representing small (large) values.
The vertical scale has been stretched by a factor 2.

Decomposition of the velocity field

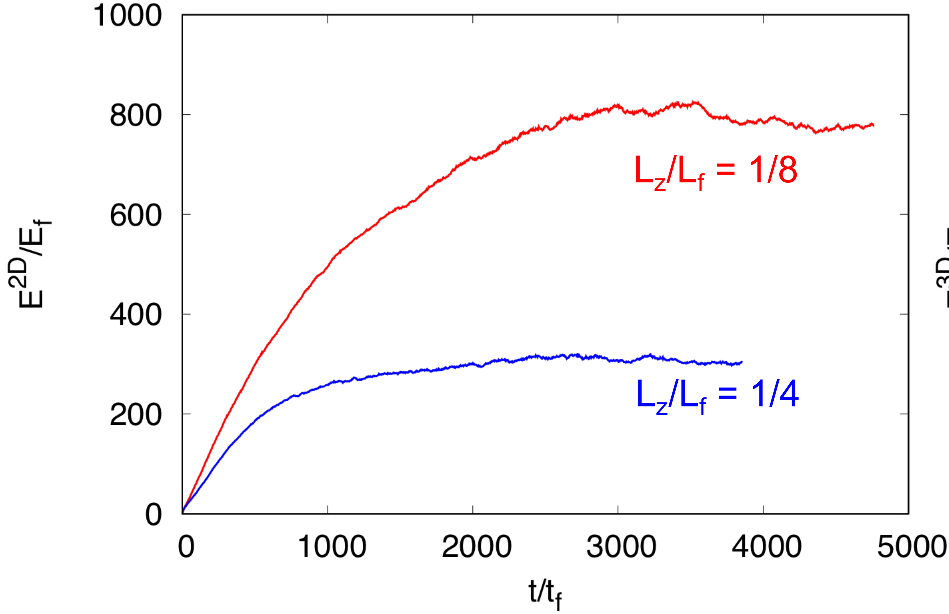
Decomposition of velocity in 2D and 3D modes $\boldsymbol{u} = \boldsymbol{u}^{2D} + \boldsymbol{u}^{3D}$

2D modes: horizontal components u_x u_y averaged in the vertical direction z

$$\boldsymbol{u}^{2D} = (u_x^{2D}(x,y), u_y^{2D}(x,y), 0)$$

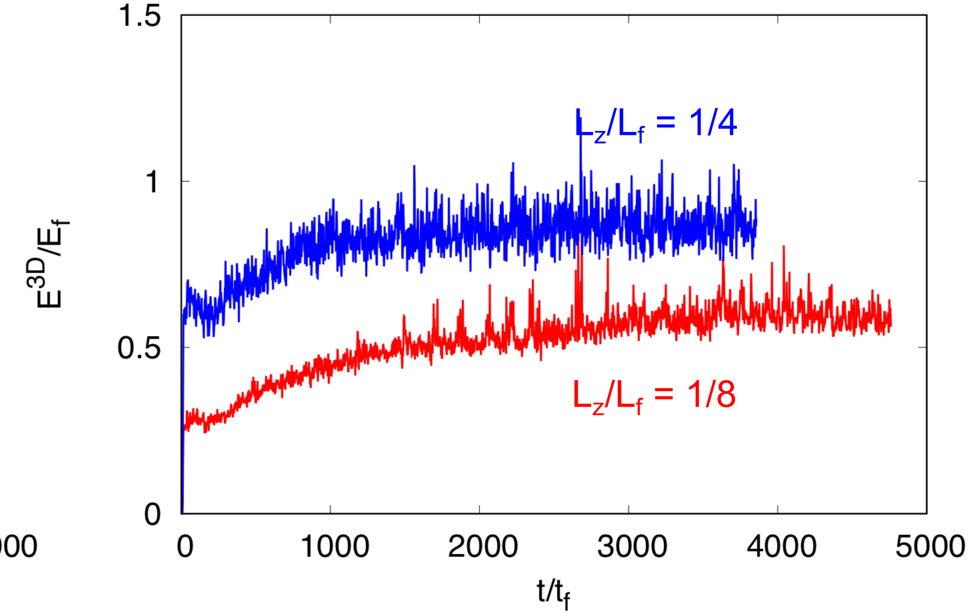
Kinetic energy of 2D modes

$$E^{2D} = (1/2)\langle|\boldsymbol{u}^{2D}|^2\rangle$$



Kinetic energy of 3D modes

$$E^{3D} = (1/2)\langle|\boldsymbol{u}^{3D}|^2\rangle$$

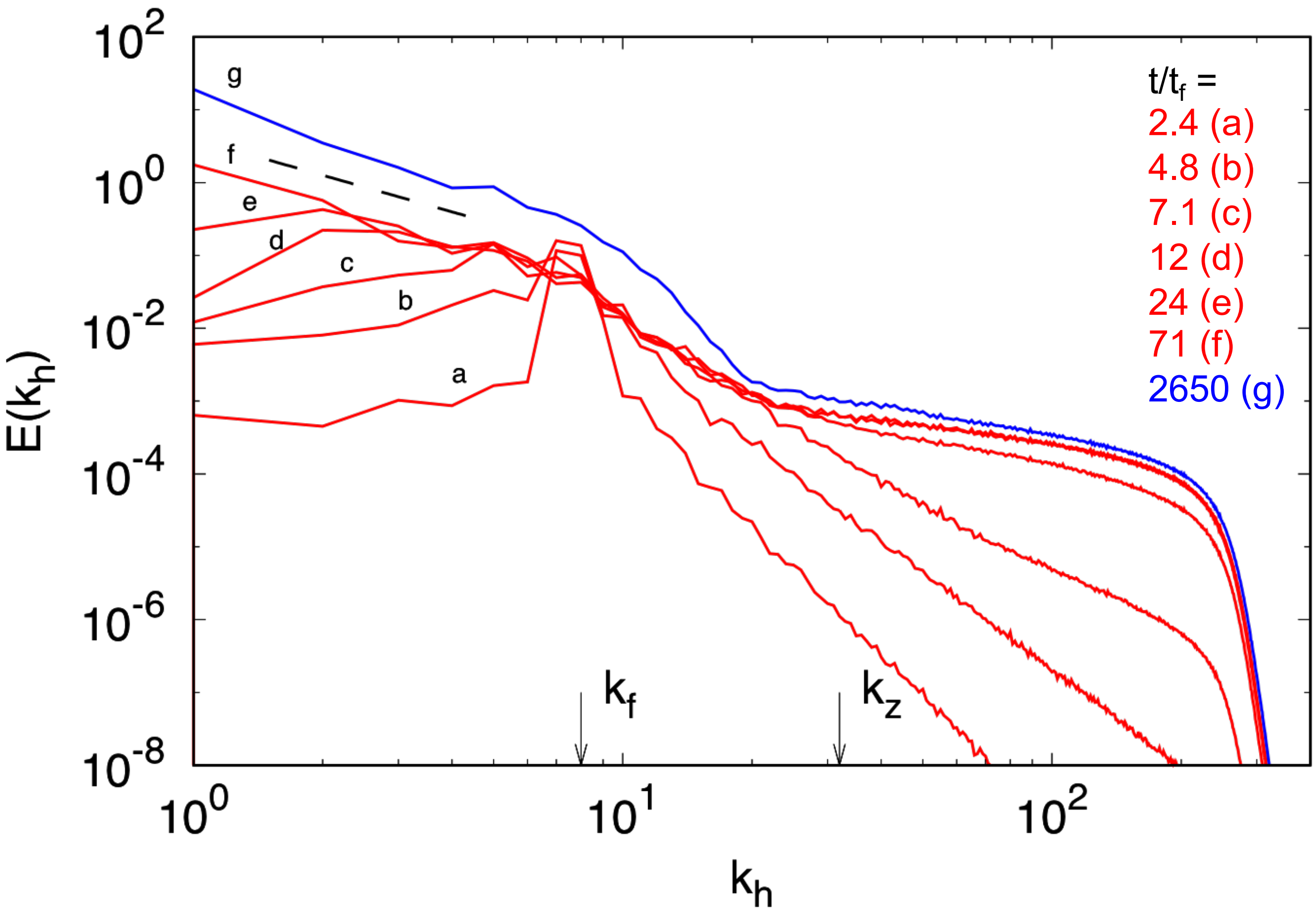


2D Energy spectra

2D spectrum: sum over shells with fixed horizontal wavenumber $k_h = (k_x^2 + k_y^2)^{1/2}$

$$E(k_h) = \frac{1}{2} \sum_{\substack{\boldsymbol{k} \\ k_x^2+k_y^2=k_h^2}} |\hat{\boldsymbol{u}}_{\boldsymbol{k}}|^2$$

Here $L_z/L_f = 1/4$



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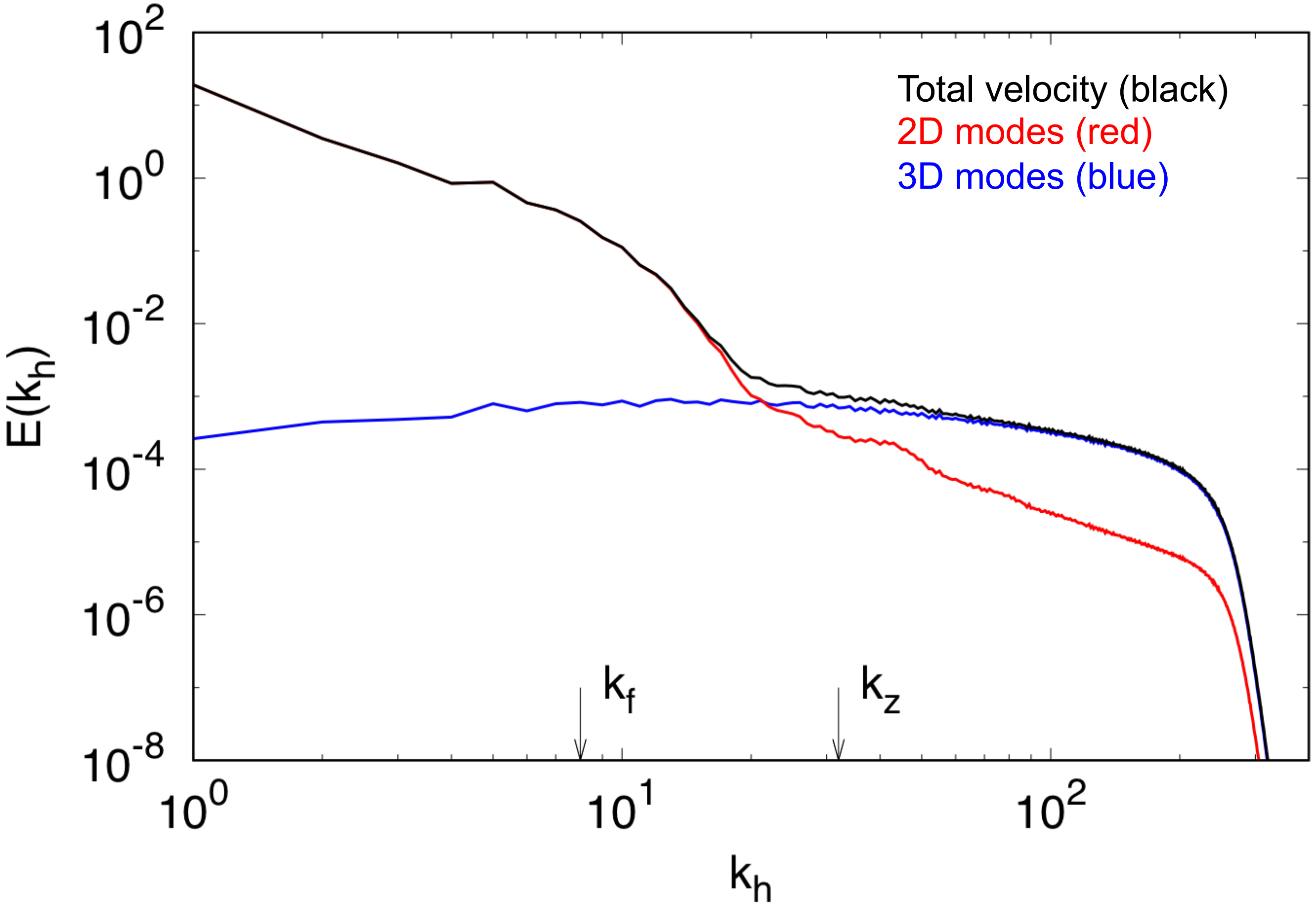


Spectral energy transfer

Partition of the Fourier space
in non-overlapping spherical shells

$$K \leq |k| < K + \Delta K \qquad \Delta K = 4$$

Decomposition of the velocity field
in shell-filtered velocities (Alexakis et al. 2005)

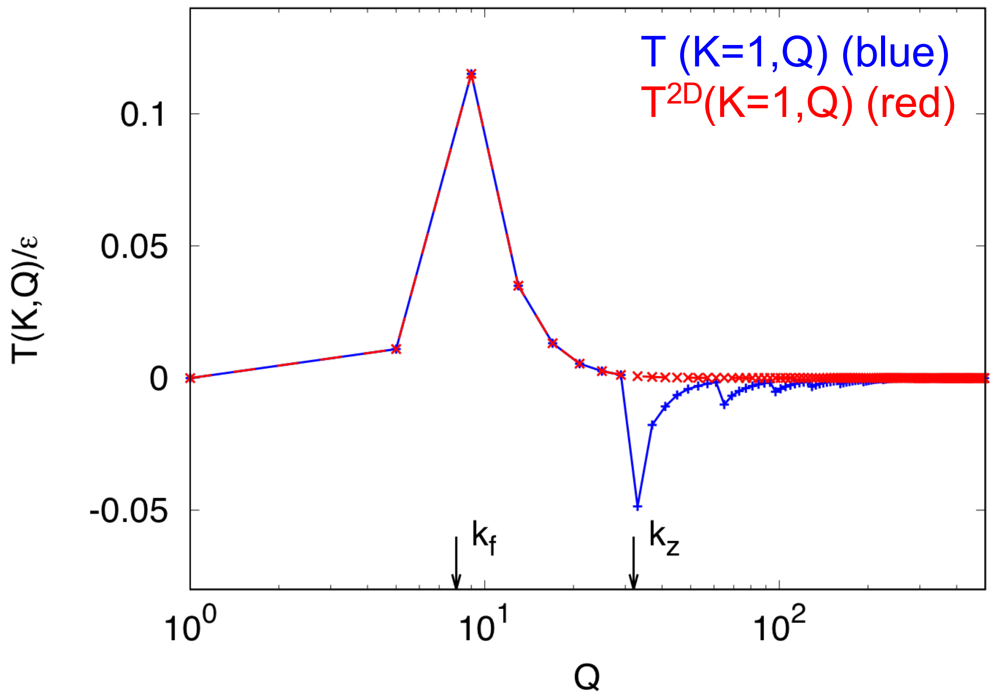
$$\boldsymbol{u} = \sum_K \boldsymbol{u}_K$$

Rate of energy transfer T(K,Q)
from the shell Q to the shell K

$$T(K,Q) = -\sum_P \int d\boldsymbol{x}^3 \boldsymbol{u}_K \cdot (\boldsymbol{u}_P \cdot \boldsymbol{\nabla})\, \boldsymbol{u}_Q$$

Interactions with large-scale
2D modes feed the condensate

Non-local interactions with
3D modes subtract energy
from the condensate



Saturation of the energy of the condensate

2D: balance between the flux of the inverse cascade and the viscous dissipation at the scale of the condensate (Eyink 96)

$$\varepsilon_{inv} \simeq 2\nu E_c / L^2$$

Energy of the condensate $E_c \sim \varepsilon_{inv} L^2 / \nu$

Condensation time scale $t_c \simeq E_c / \varepsilon_{inv} \sim L^2 / \nu$

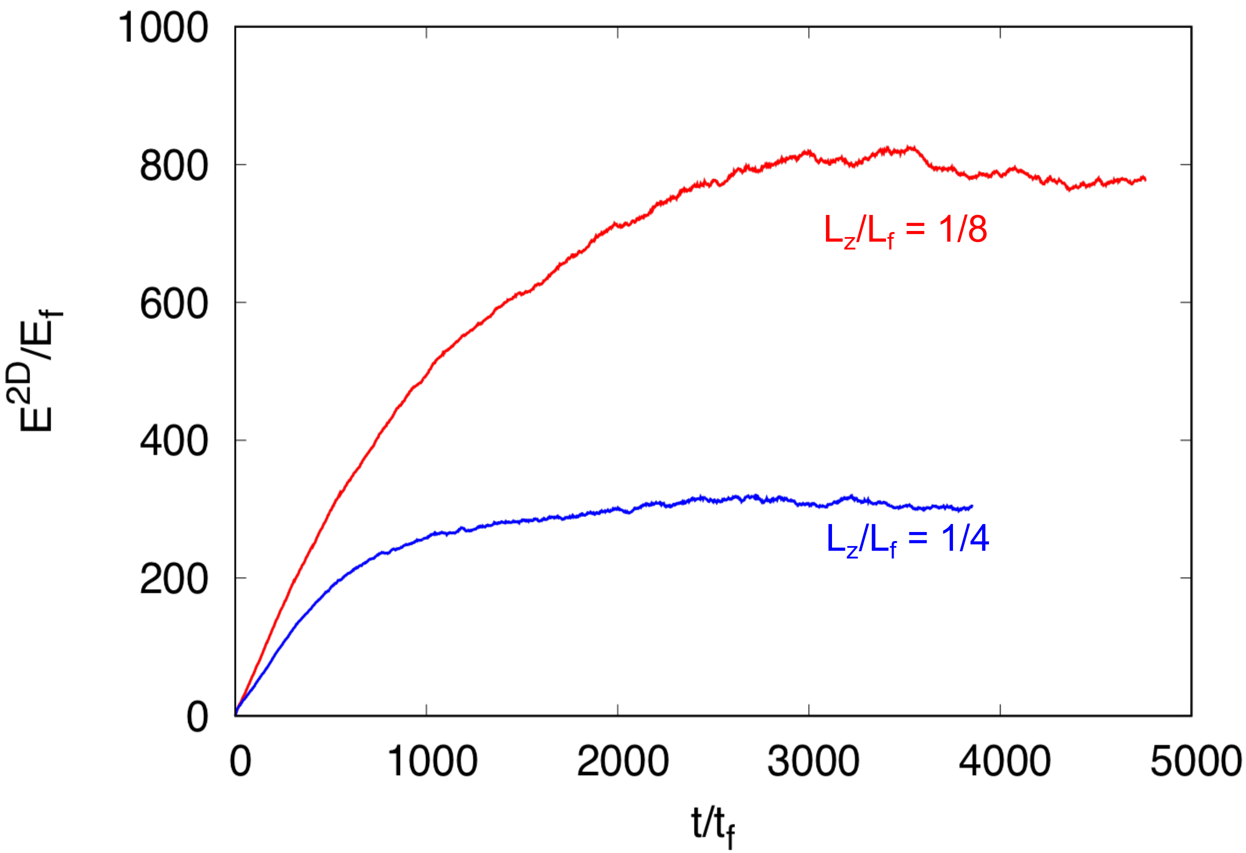
2D - 3D: balance between the flux of the inverse cascade and the eddy viscosity of the small-scale 3D flow

$$\varepsilon_{inv} \simeq \nu_{eddy} E_c / L_x^2 \qquad \nu_{eddy} \simeq E^{3D} t_f$$

$$\frac{E_c}{E_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f} \right)^2 \frac{\varepsilon_{inv}}{\varepsilon} \qquad \frac{t_c}{t_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f} \right)^2$$

$$E_f = \varepsilon t_f \qquad t_f = \beta^{-1/3}$$

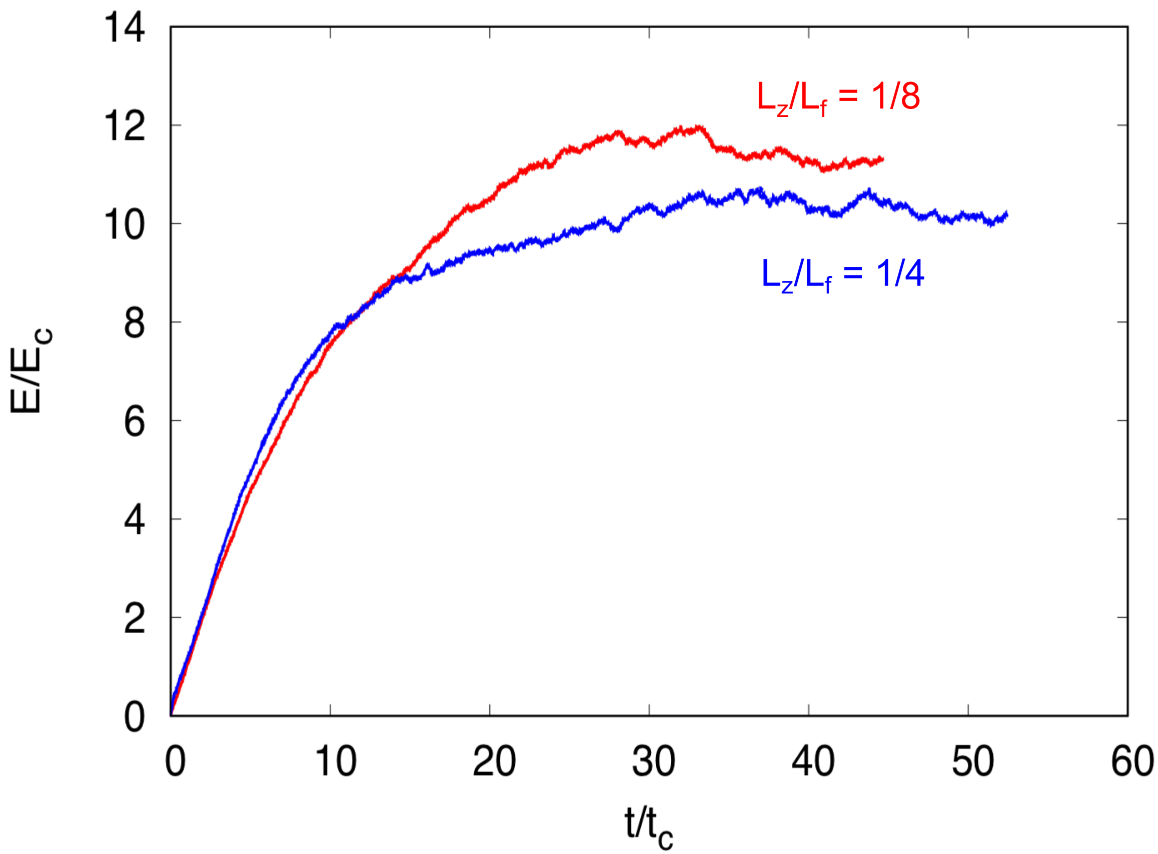
Saturation of the energy of the condensate



$$\frac{E_c}{E_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f}\right)^2 \frac{\varepsilon_{inv}}{\varepsilon} \qquad \frac{t_c}{t_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f}\right)^2$$

$$E_f = \varepsilon t_f \qquad t_f = \beta^{-1/3}$$

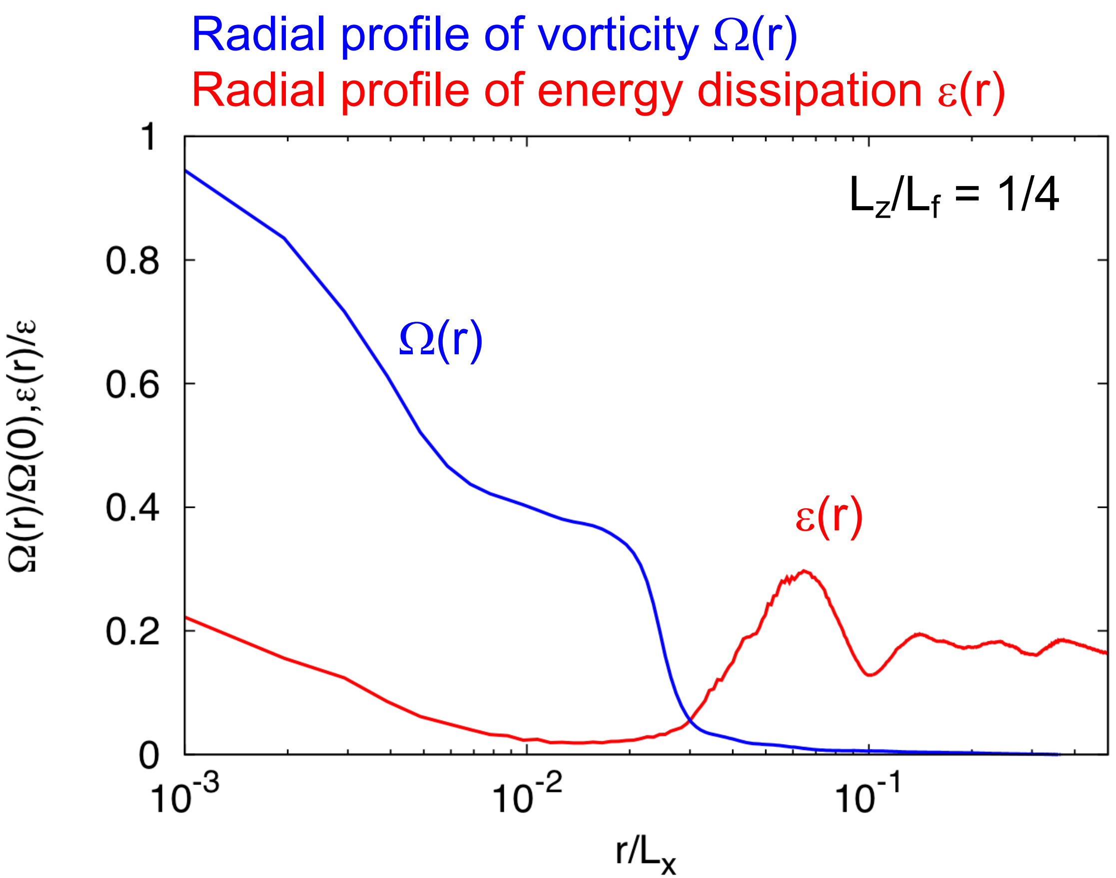
Saturation of the energy of the condensate



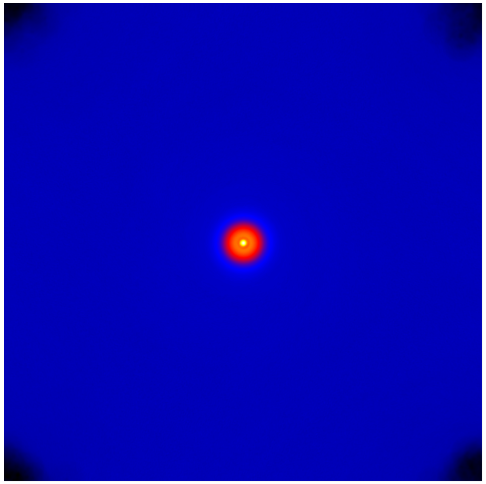
$$\frac{E_c}{E_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f}\right)^2 \frac{\varepsilon_{inv}}{\varepsilon} \qquad \frac{t_c}{t_f} \simeq \frac{E_f}{E^{3D}} \left(\frac{L_x}{L_f}\right)^2$$

$$E_f = \varepsilon t_f \qquad t_f = \beta^{-1/3}$$

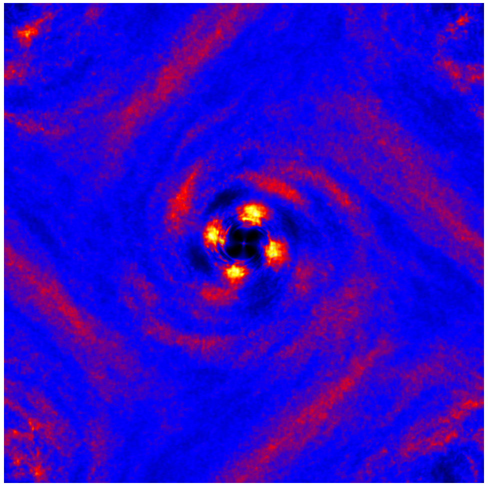
Radial profile of energy dissipation



Maximum energy dissipation around the vortex core:
3D turbulence + strong shear



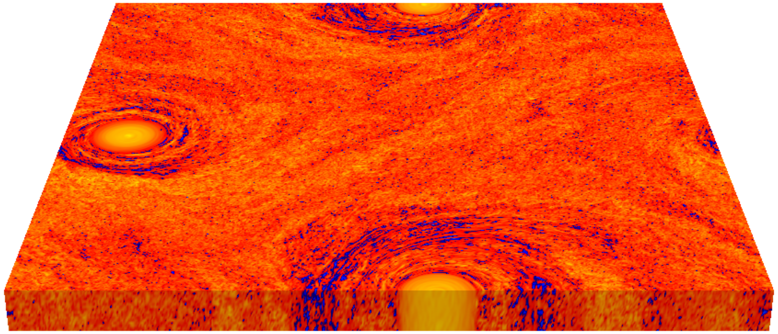
Vorticity



Dissipation

Conclusions

We have investigated the formation of a 2D, statistically steady condensed state in a 3D turbulent flow confined in a thin layer.



The saturation of the kinetic energy of the condensate is due to the balance between an inverse energy cascade of 2D energy and a direct energy transfer from the condensate toward the 3D turbulent flow.

Dimensional prediction for the energy of the condensate based on an eddy viscosity model of the 3D interactions.

SM, G. Boffetta, Phys. Rev. Fluids. 4, 022602(R) (2019)

Thank You!

HPC in Turbulence

Complementary to experiments (lower Re, full statistics)

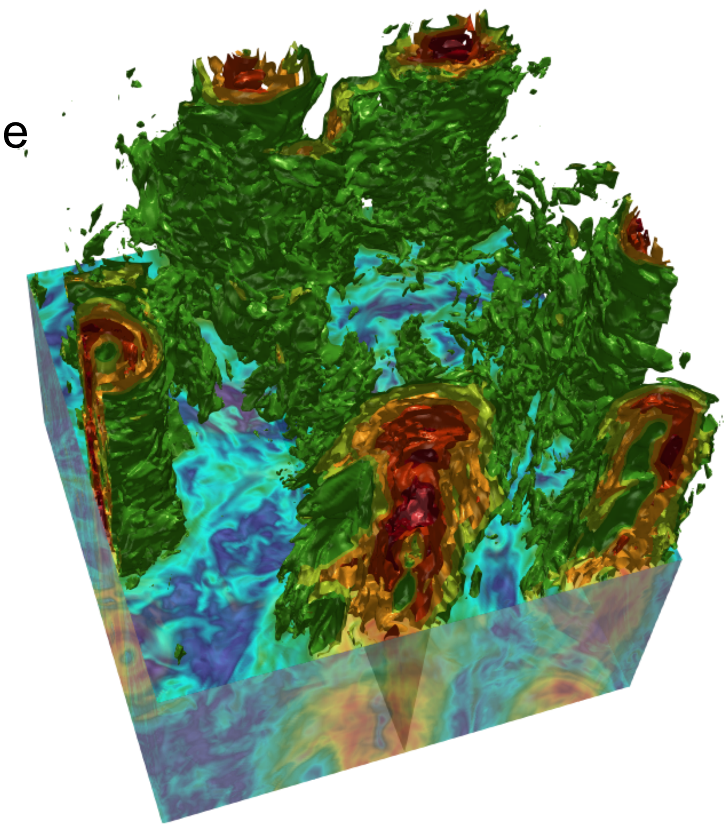
A tool for investigating conditions non accessible to experiments:

- the effect of dimensionality (2D, quasi-2D, nD)
- role of conservation laws (more or less than the physical ones)
- role of boundary conditions (and absence)

Numerical methods:

Pseudo-spectral
Lattice-Boltzmann
Immerse-boundary

MPI-openMP decompositions



Rotating turbulence (courtesy of L. Biferale)

Energy dissipation

Energy balance

$$\frac{dE}{dt} = \varepsilon - \varepsilon_{dis}$$

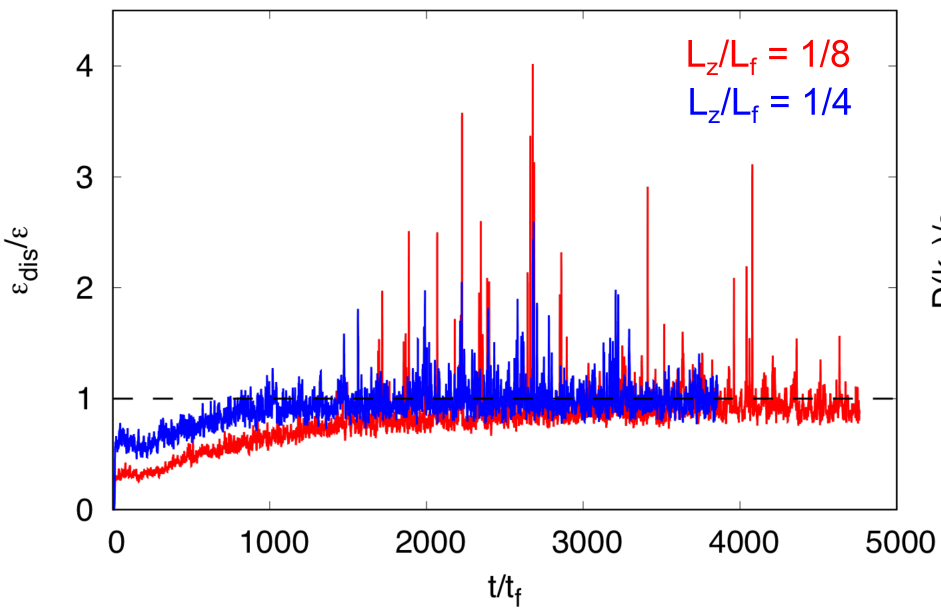
Energy dissipation rate

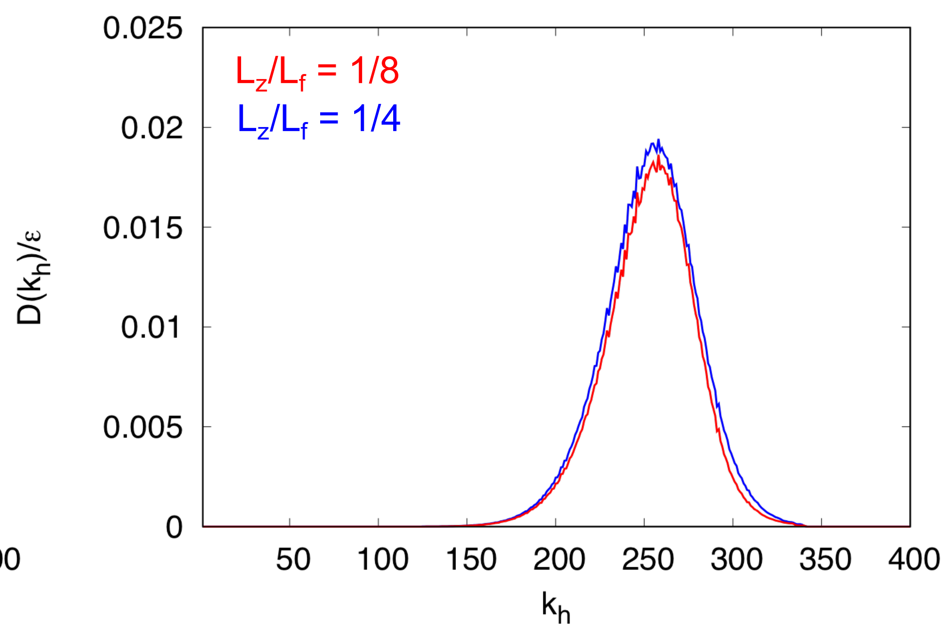
$$\varepsilon_{dis} = \langle 2(-1)^{p-1}\nu_p(\nabla^{2p}\mathbf{u})\cdot\mathbf{u}\rangle$$

2D dissipation spectrum

$$D(k_h) = \sum_{\substack{\boldsymbol{k} \\ k_x^2+k_y^2=k_h^2}} \nu_p|\boldsymbol{k}|^{2p}|\hat{\boldsymbol{u}}_{\boldsymbol{k}}|^2$$

$$k_h = (k_x^2 + k_y^2)^{1/2}$$





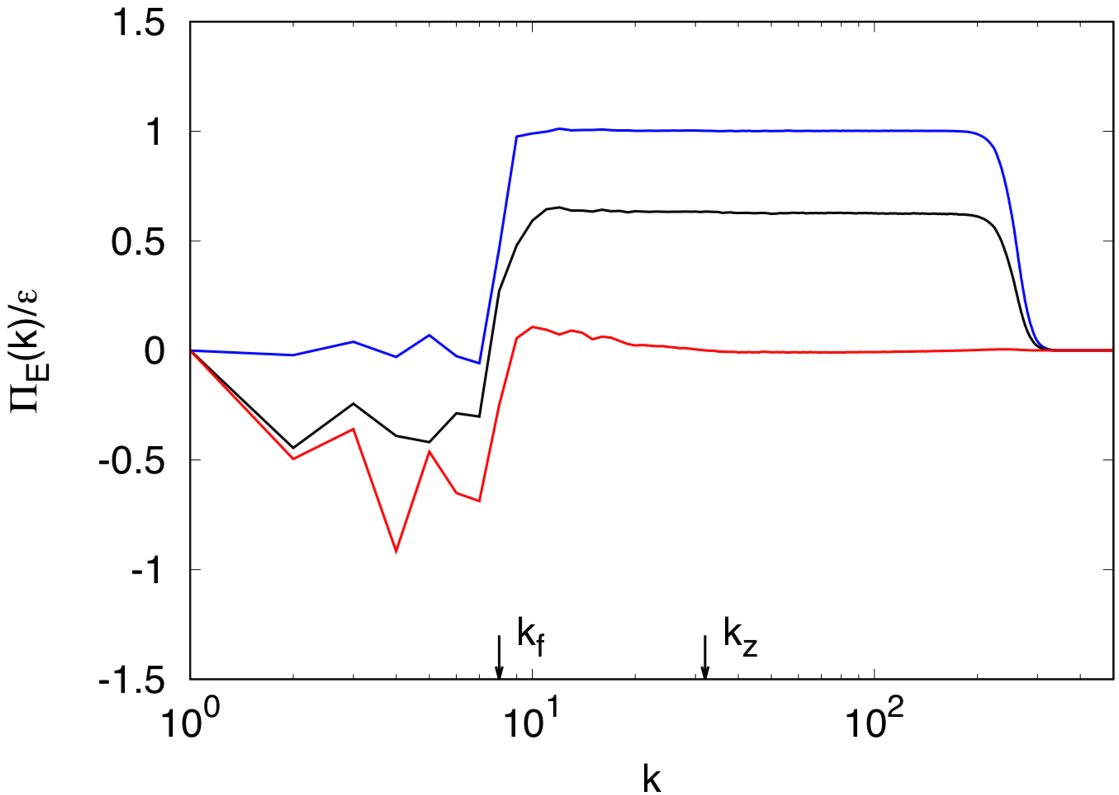
Spectral energy flux

Spectral energy flux (total velocity field)
at time $t = 25\ t_f$ (black)
$t = 2650\ t_f$ (blue)

$$\Pi(k) = -\frac{1}{2} \sum_{\substack{\boldsymbol{k},\boldsymbol{p},\boldsymbol{q} \\ |\boldsymbol{k}|\leq k \\ \boldsymbol{k}+\boldsymbol{p}+\boldsymbol{q}=0}} \hat{\boldsymbol{u}}_{\boldsymbol{k}}^* \cdot (i\boldsymbol{k} \cdot \hat{\boldsymbol{u}}_{\boldsymbol{p}})\, \hat{\boldsymbol{u}}_{\boldsymbol{q}} + c.c.$$

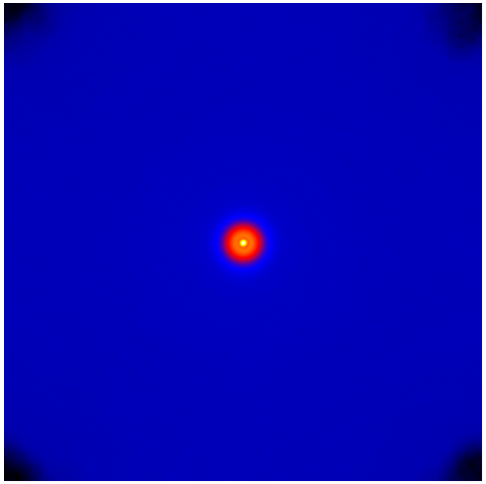
Spectral energy flux (2D modes only)
at time $t = 2650\ t_f$ (red)

Here $L_z/L_f = 1/4$



Radial vorticity profile of the condensate

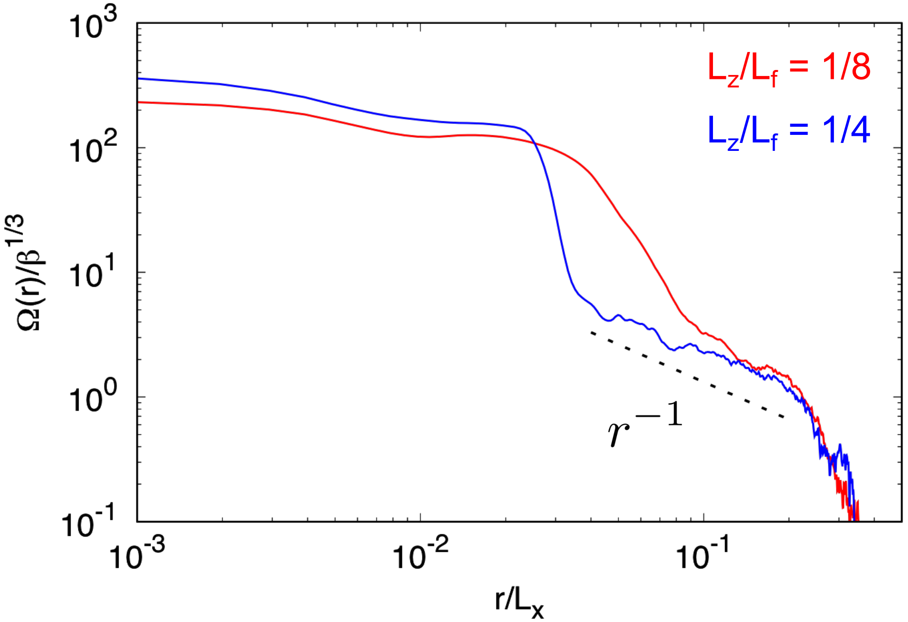
1) Find the center
2) Center the fields
3) Average in time
4) Compute the radial vorticity profile Ω(r)

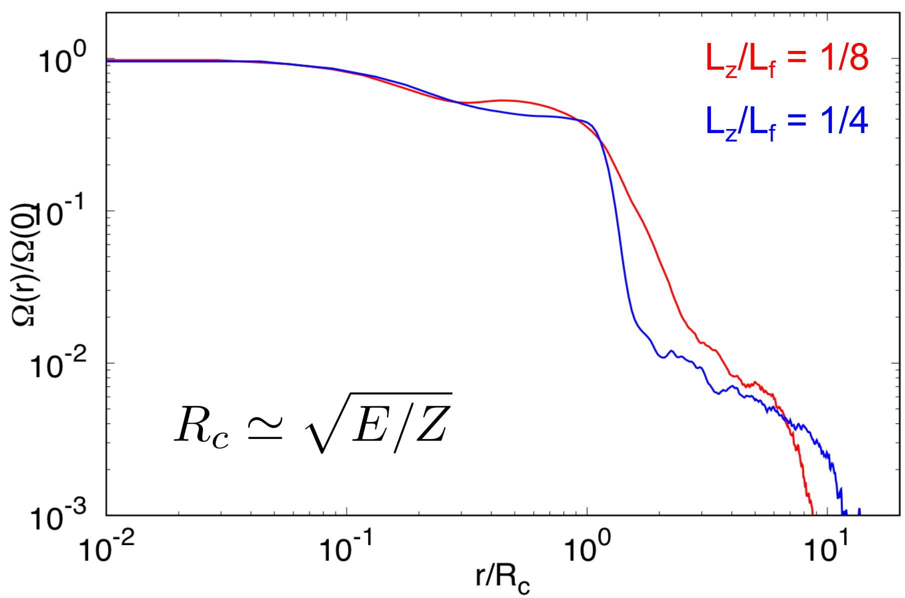


Predictions for the radial vorticity profile

Laurie et al. (2014)
Kolokolov & Lebedev (2015)

$$\Omega(r) \sim r^{-1}$$





Energy dissipation field