Interevent time distributions of avalanche dynamics

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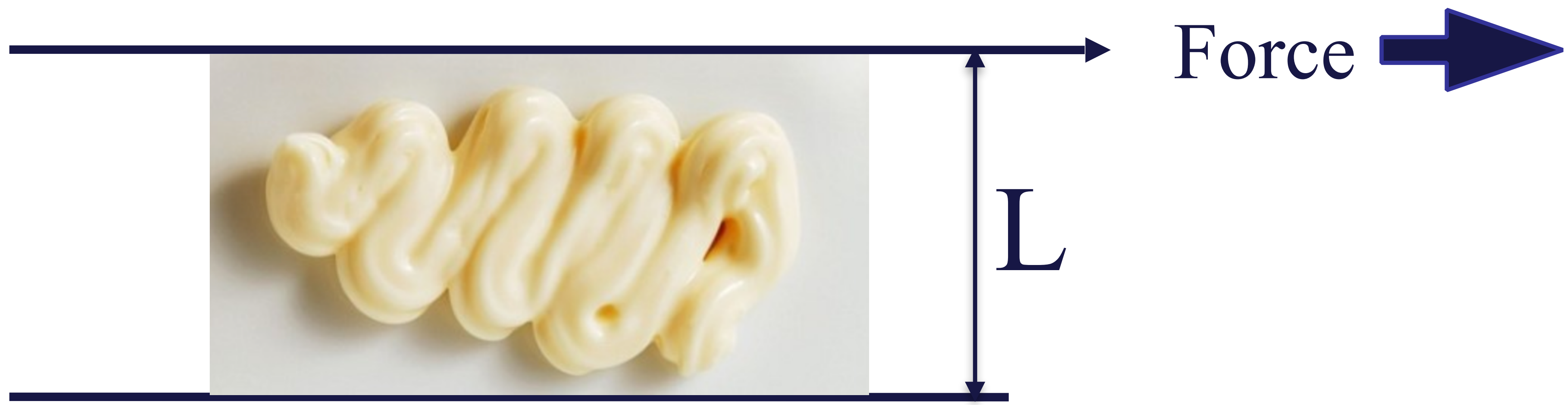
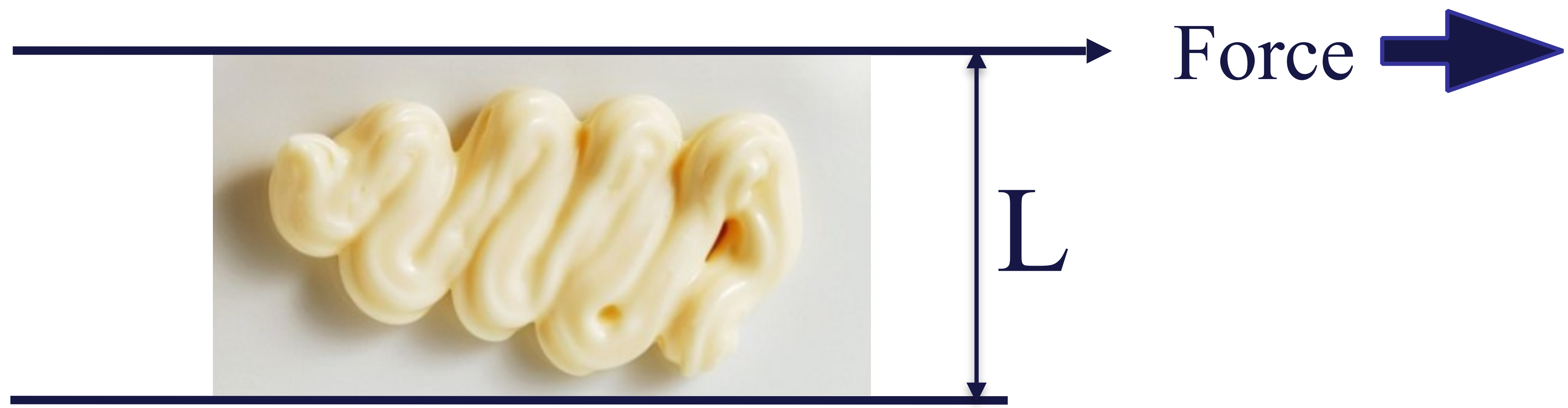
Solid / Liquid transition

The basic statement: liquids flow and solids do not



Force >0 ⇒ { the system flows → fluid; the system does not flow → solid

Maionese is an example of “soft glass”:
two liquids with peculiar surface forces between them.



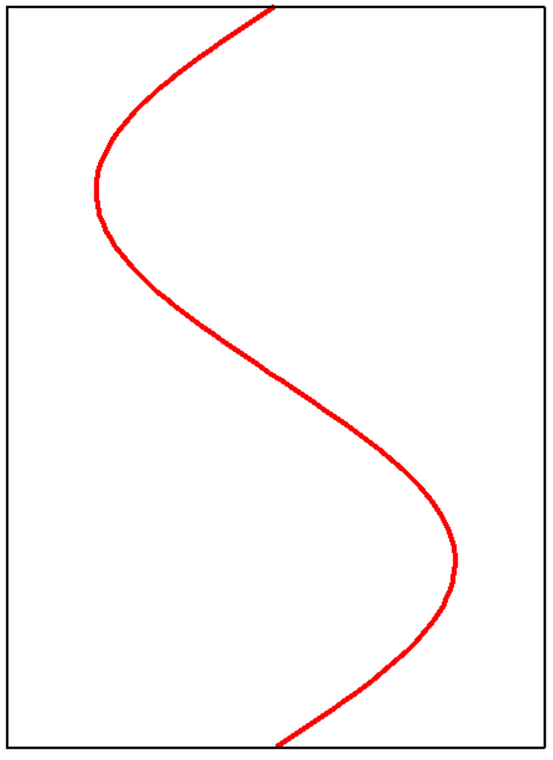
Force $< F_c \Rightarrow$ the system does not flow $\rightarrow$ solid

Force $> F_c \Rightarrow$ the system flows $\rightarrow$ liquid

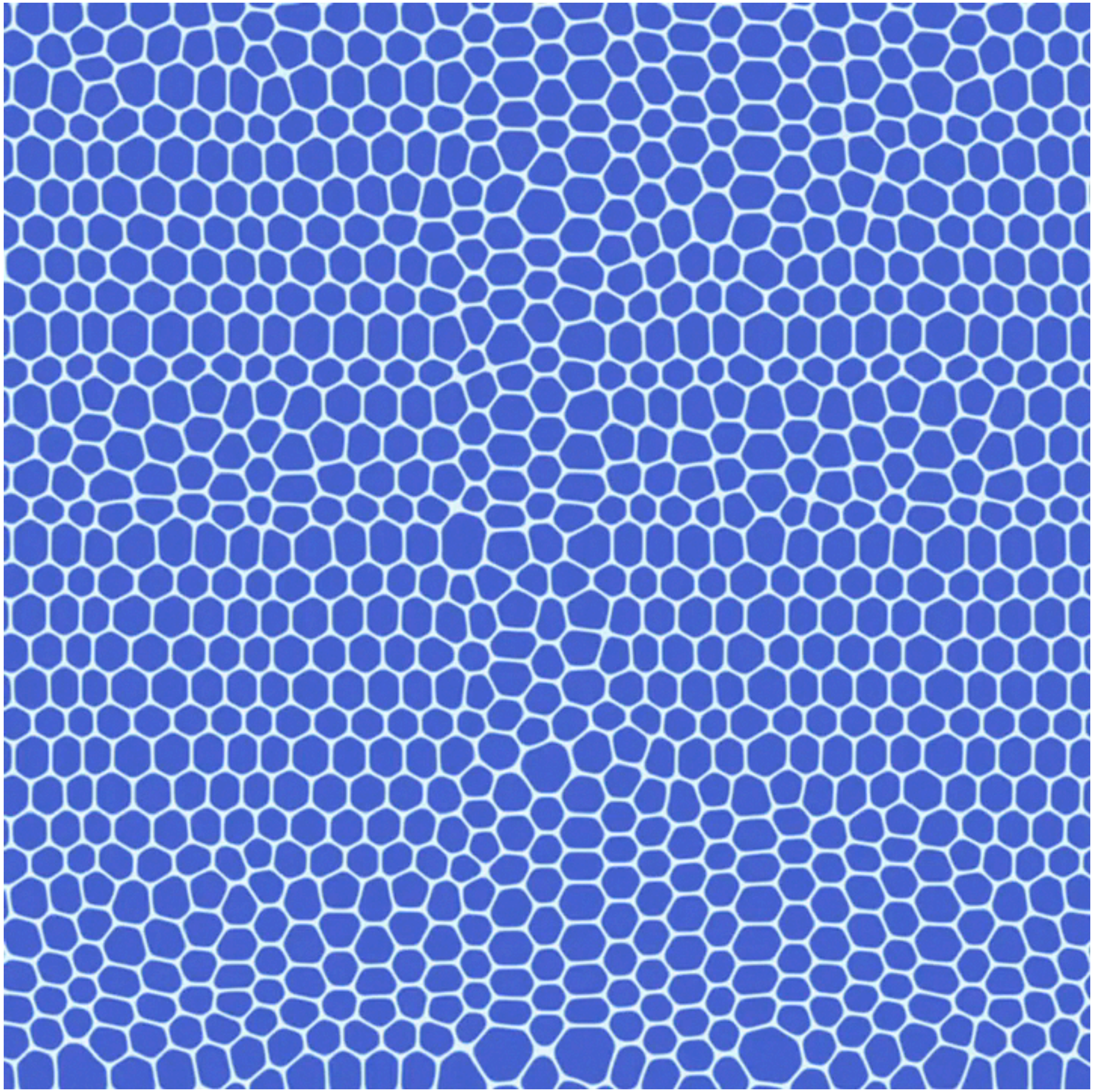
F_c is known as yield stress

P. Sollich, F Lequeux, P Hébraud, ME Cates PRL (2000), L Berthier, G Biroli Rev. Mod. Phys. (2007), A. Cavagna Phys. Rep. (2009), M Megard, G Parisi, MA Virasoro (1987)

Kolmogorov flow



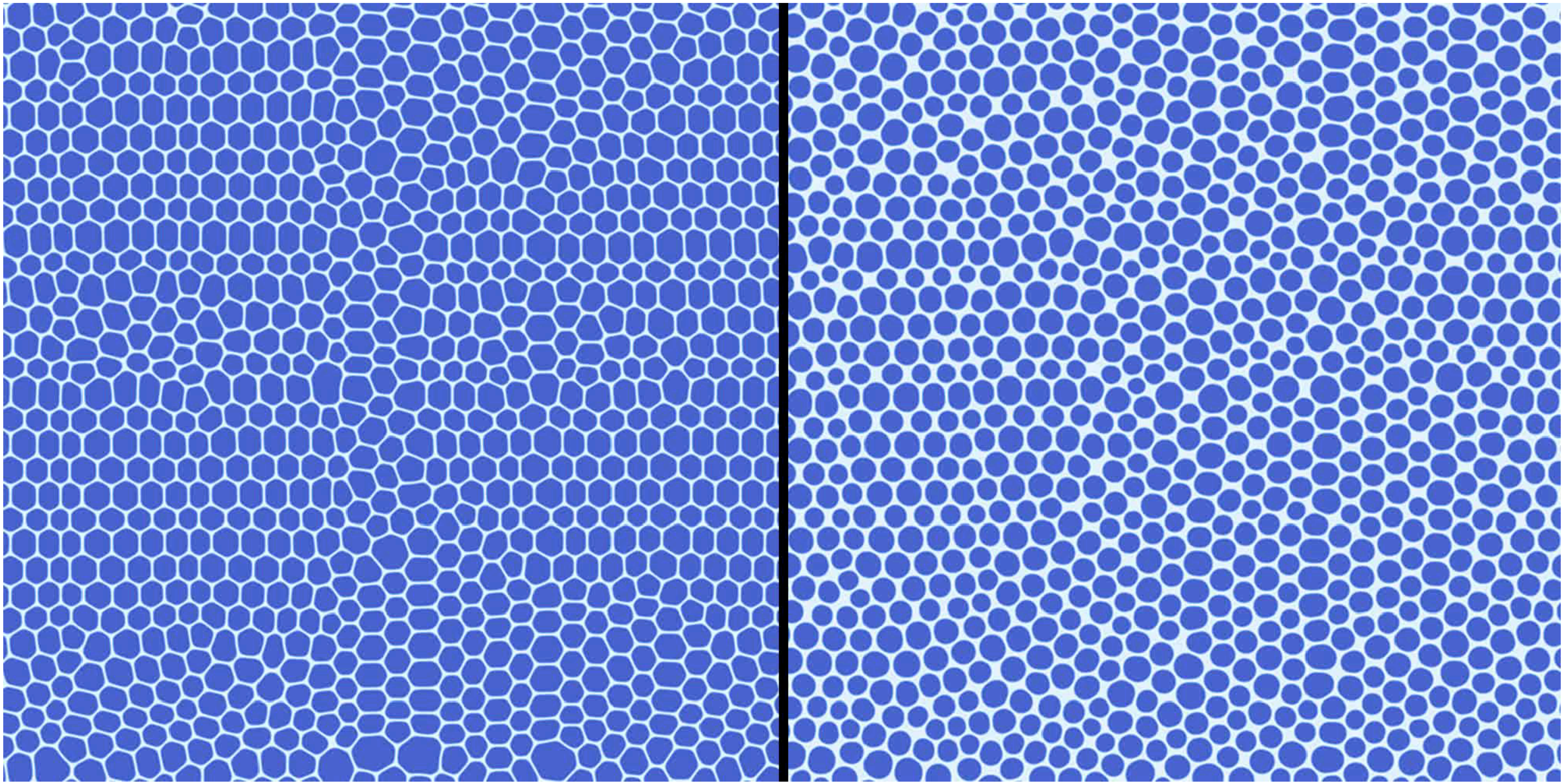
Forcing

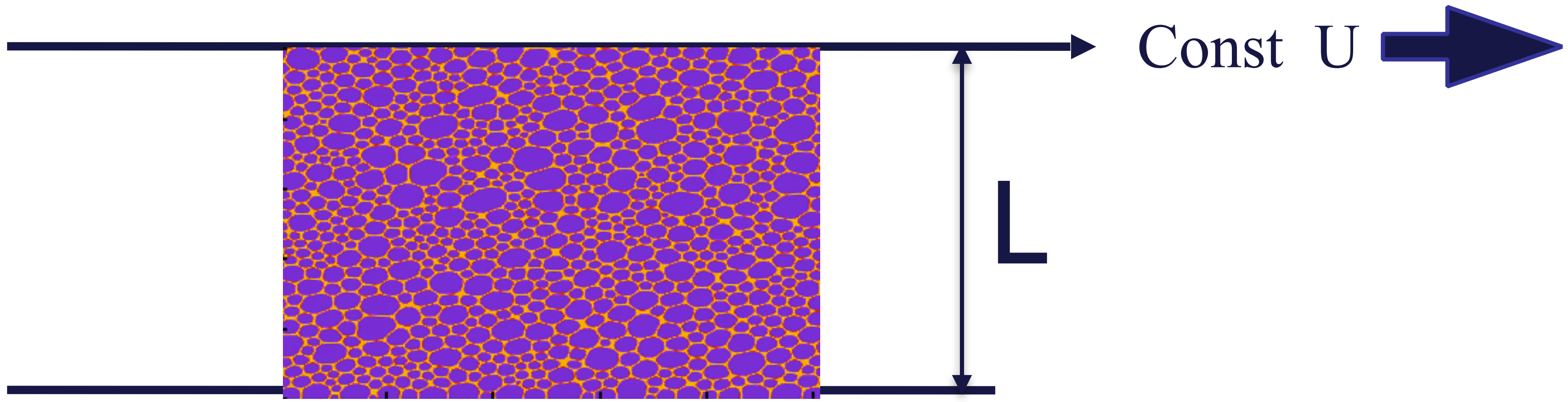


RB, M. Sbragaglia, S. Succi, M. Bernaschi, S. Chibbaro,
J.Chem. Phys. 2009
PRL 2009
RB, M. Sbragaglia, P. Perlekar, M. Bernaschi, S. Succi, F.Toschi
Soft Matter, 2014
Soft Matter, 2015

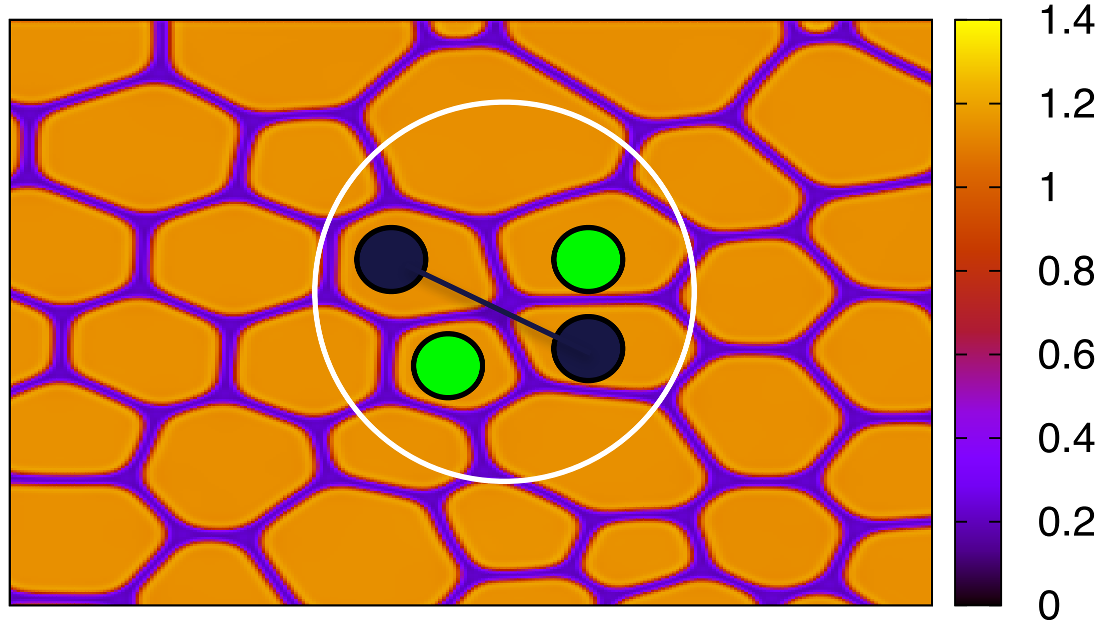
high packing fraction
yield stress

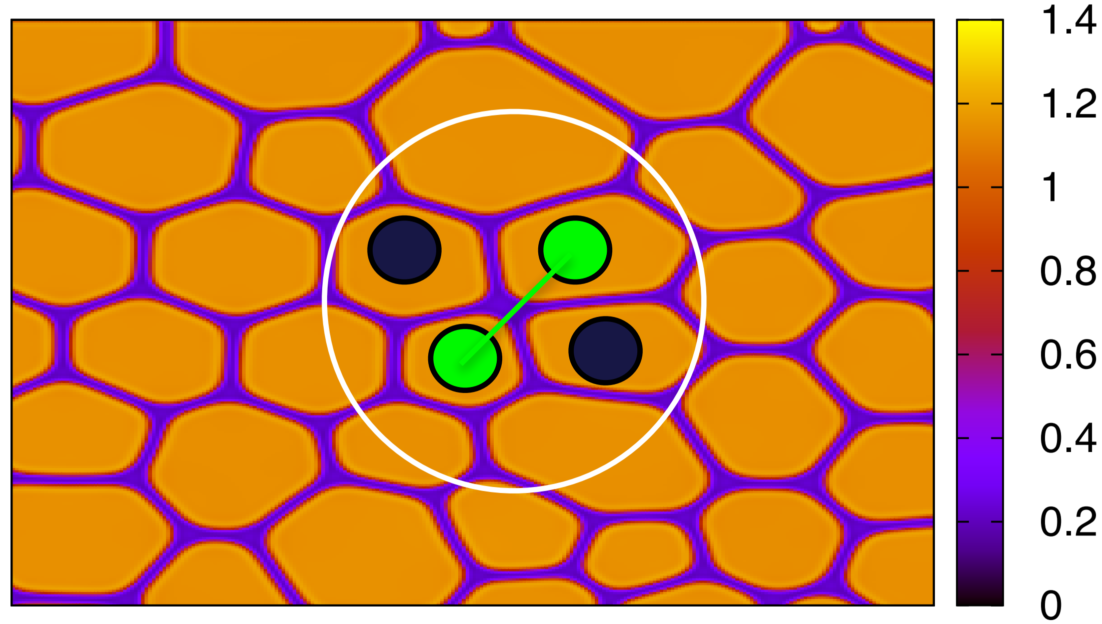
low packing fraction
no yield stress



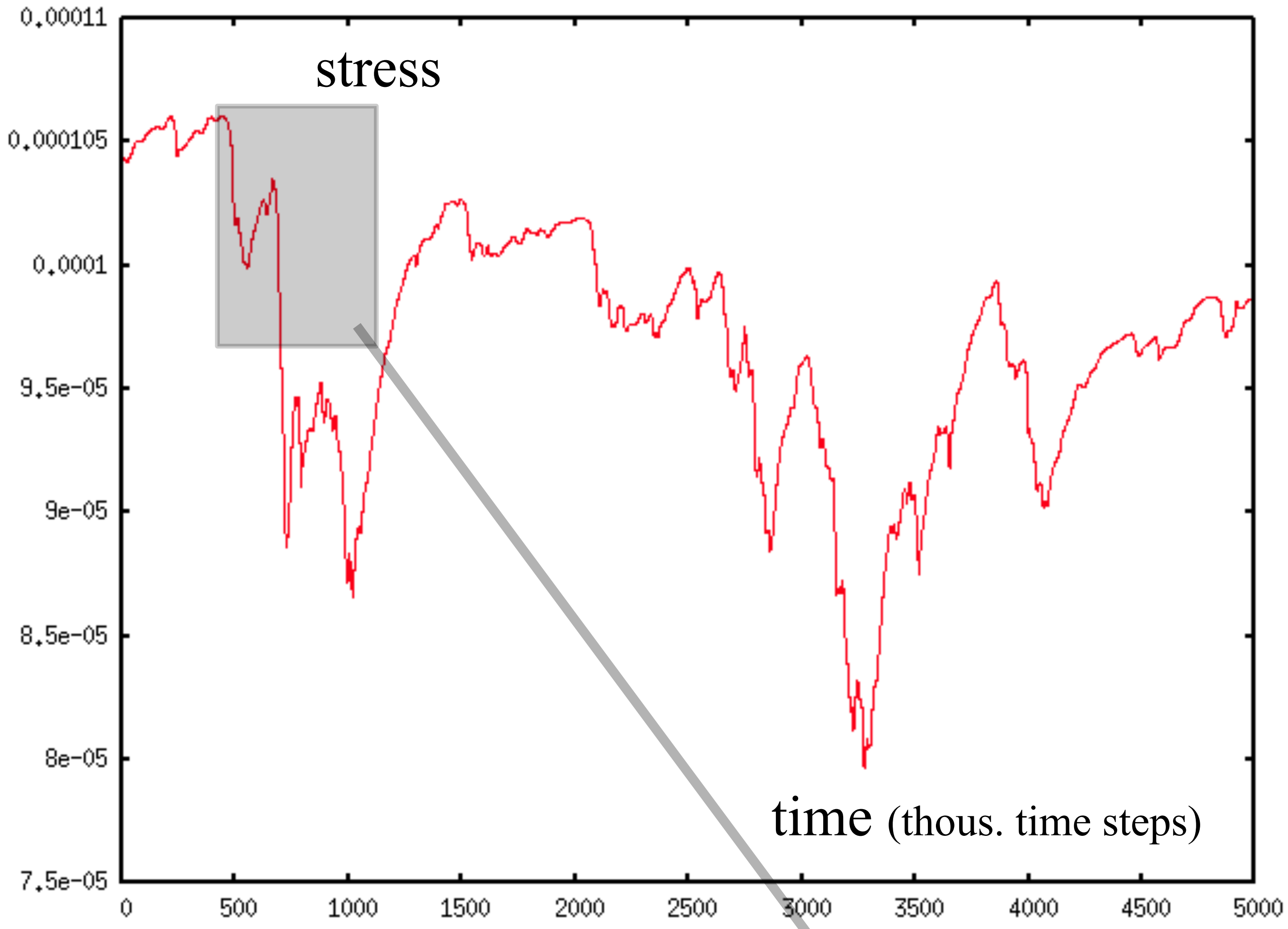


Plastic events are irreversible topological changes of the interface

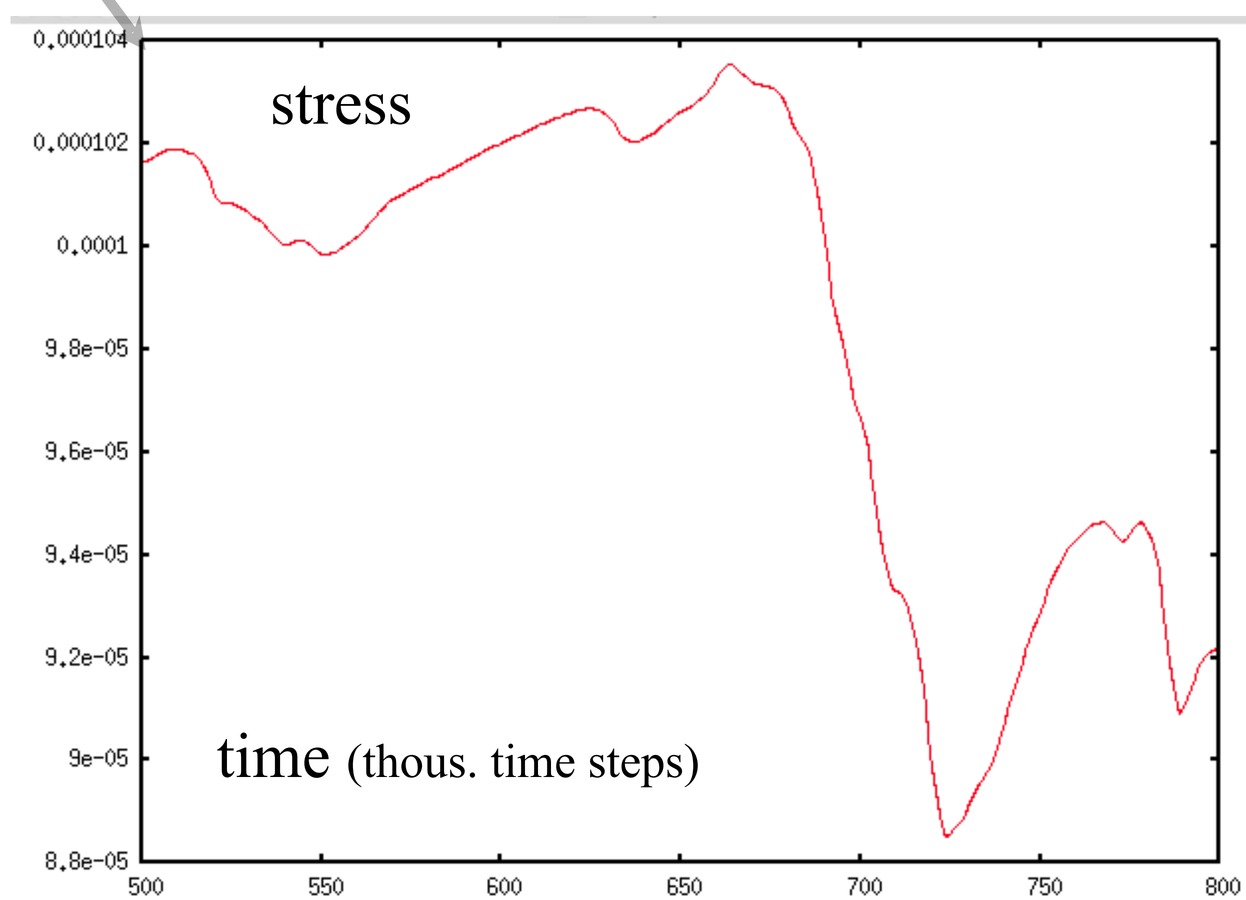


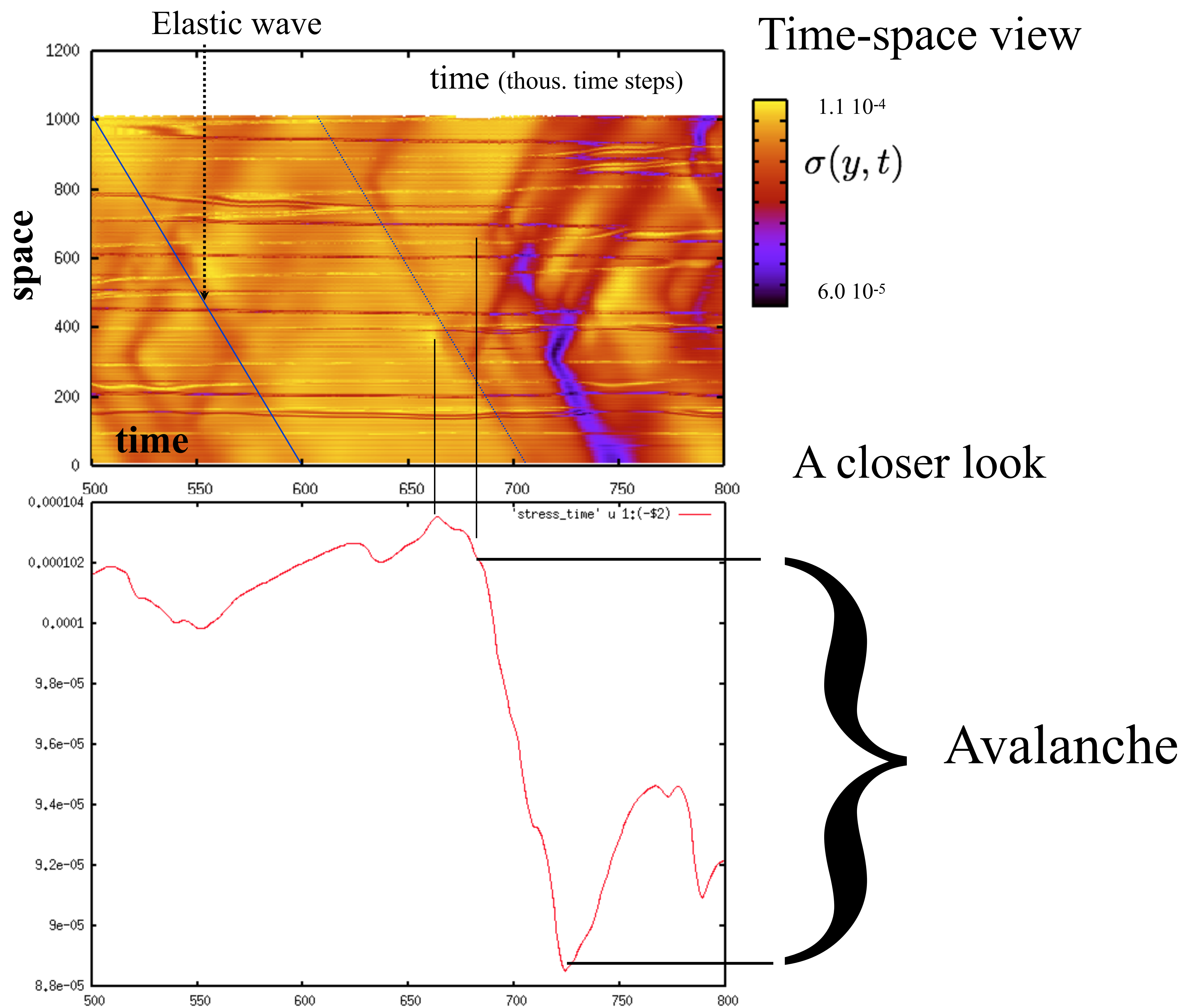


Plastic events are responsible of sharp decreases in stress



A closer look





Elastic wave
Time-space view
time (thous. time steps)
space
time
1.1 10-4
σ(y, t)
6.0 10-5
A closer look
'stress_time' u 1:(-$2)
Avalanche

Motivation

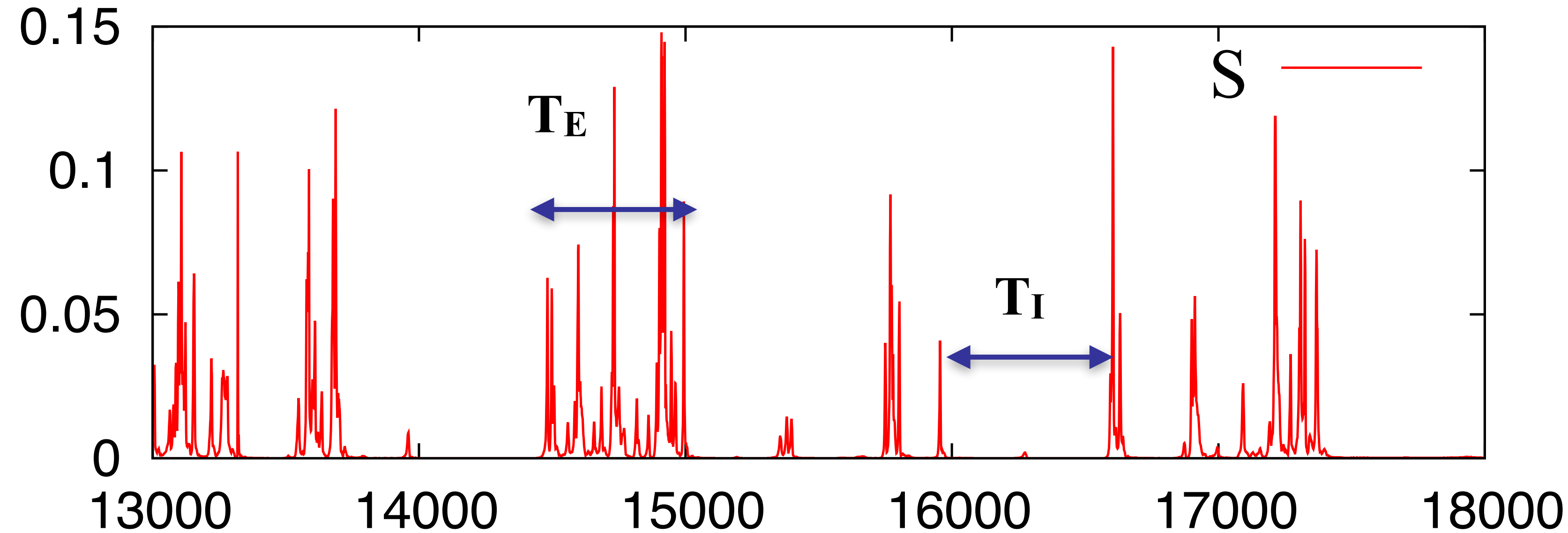
We are interested to study the statistical properties of avalanche dynamics. In principle there are at least three quantities to consider:

- avalanche sizes **S**
- avalanche duration time $\mathbf{t_E}$
- inter event time between two successive avalanche $\mathbf{t_i}$

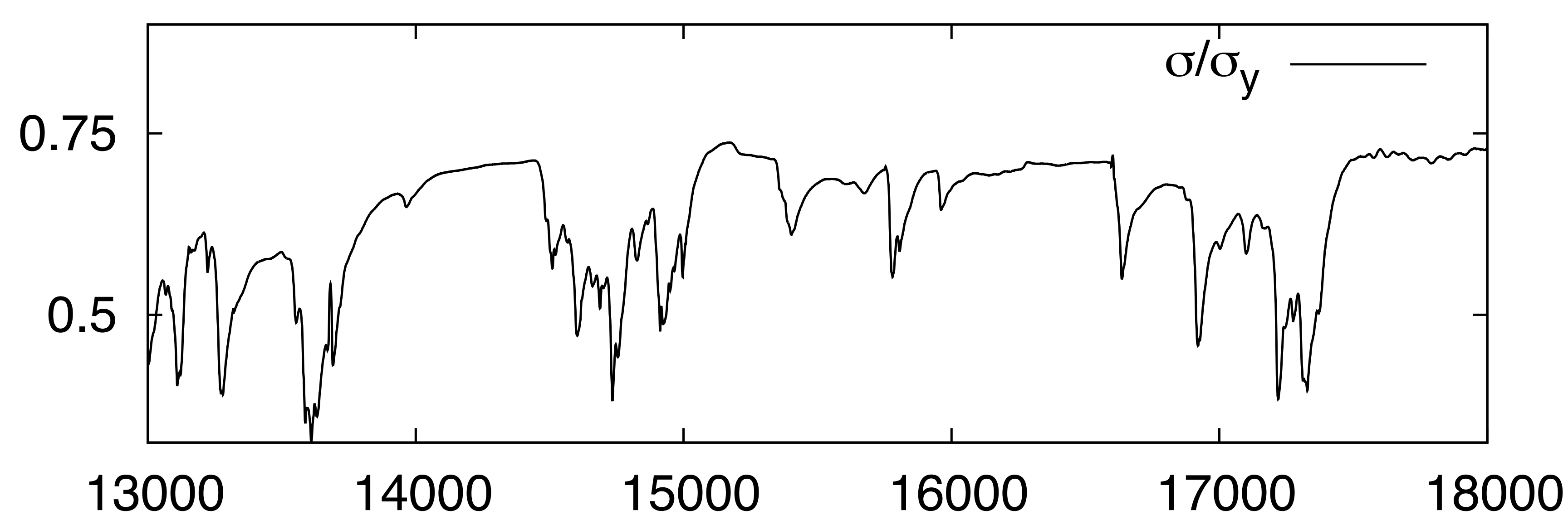
$$S \equiv Energy\, release \sim \int \sigma |\frac{d\sigma}{dt}| dt$$

$$P(S) \sim S^{-\tau}$$

$$P(t_E) \sim t_E^{-\beta}$$



There exist extensive and detailed studies on the probability distribution of **P(S)** and $\mathbf{P(t_E)}$ showing clear scaling behavior:



Motivation

Much less is known on $\mathbf{t_i}$. Why?

Several reasons:

- the statistical properties of inter event time distribution depend critically on how you define an avalanche;
- there exists almost no theory for the inter event time distribution;

Physically, $P(t_i)$ measures the statistical properties of the system relaxation time.

Motivation

Here **we consider scale invariance in the most general form**, namely by the studying the probability distribution of the inter event time distribution occurring for avalanche of size given by some threshold S_* :

$$P(t_i|S_*)$$

<u>Iff $\mathbf{P(t_i|S_*)}$ retains the same functional form upon increasing S_*, then the system shows scale invariance.</u>
(Warning: this does not mean that T_i and S_* are necessarily correlated as it occurs for S and T_E).

Remark: if $\boldsymbol{P(t_i|S_*)}$ is exponential then this is consistent with the idea that events occur at random uncorrelated times, i.e. $\boldsymbol{t_i}$ is not an interesting quantity.

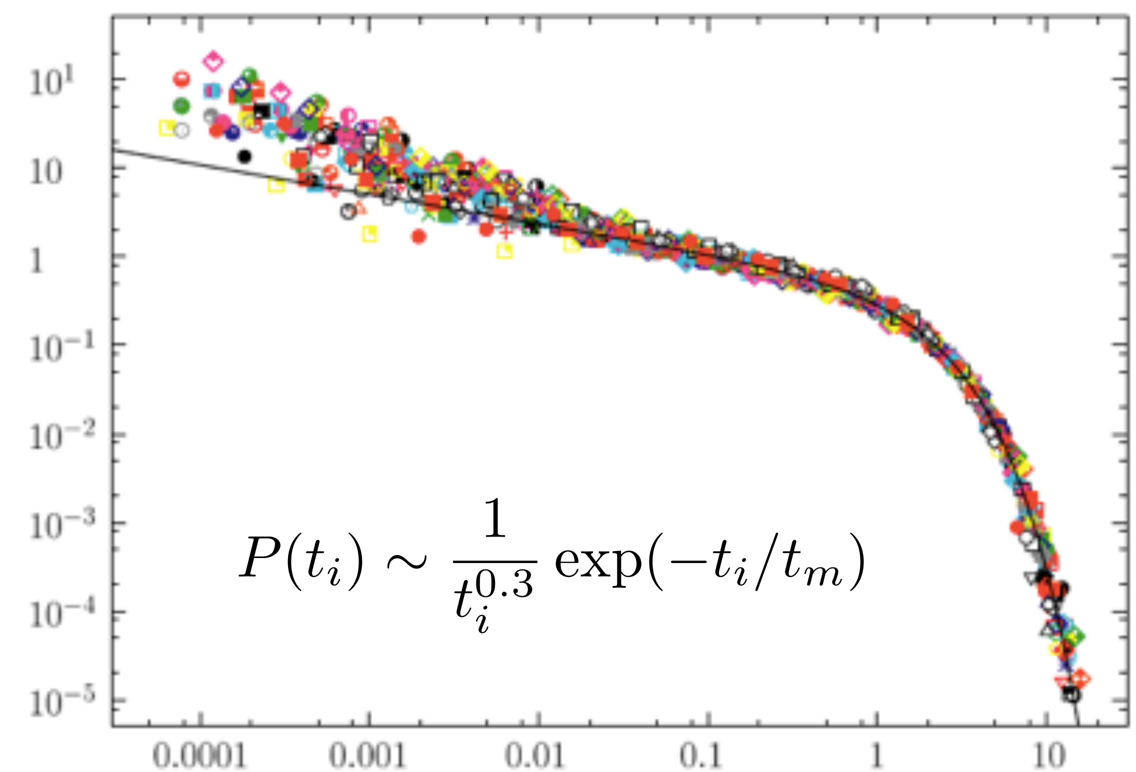
Mean field theories (deppining transition, SOC, …) assume an exponential distribution for $P(t_I)$. For this reason, most experiments and/or numerical simulations do not report information on $P(t_I)$. The situation is more complicated for sismic events (earthquakes).

Motivation

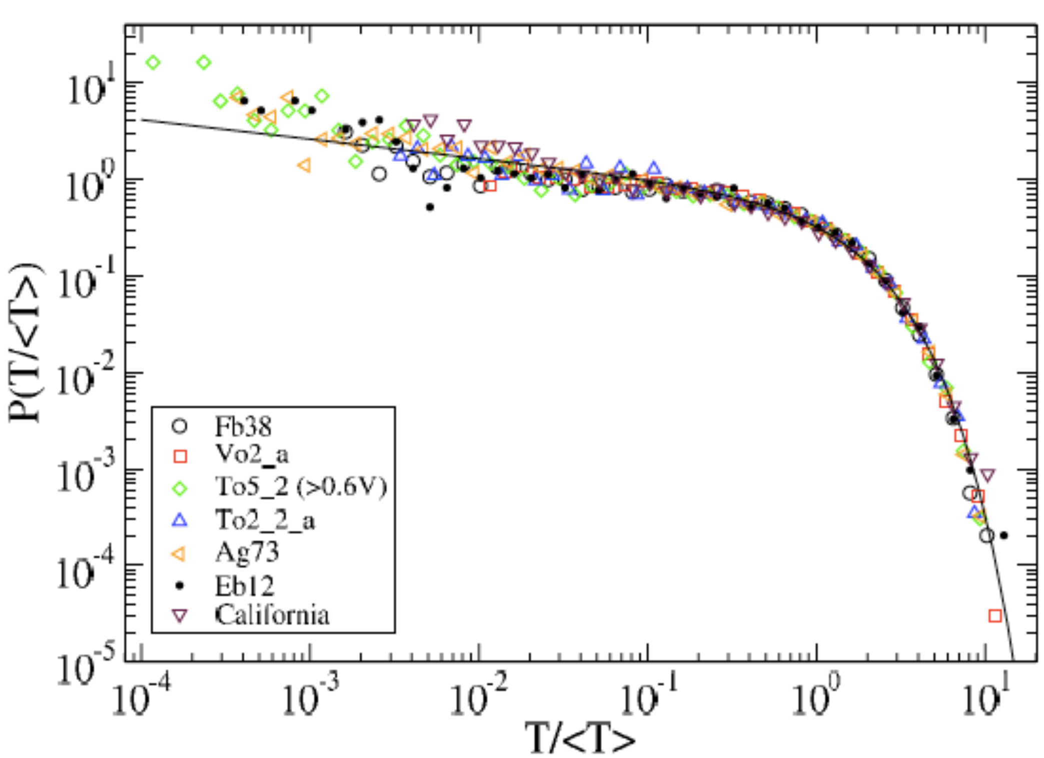
Since the original paper by Bak, Christensen, Danon Scanlon (PRL 2002), **$P(t_i|S_*)$** has been the subject of many investigations related to the inter event time distribution for earthquake events.

Here we focus on Corral results (PRL 2004) who showed that $P(t_I)$ for earthquakes is not exponential looking at earthquake in two different ways:

- **single fault;**
- **on the whole Earth, independently of earthquake location.**



A long debate on this issue is still going on



However there are some experimental results.
Inter event time for acoustic emission in Rock Fracture
Davidsen, Stanchits, Dresen
Prl 2007

Motivation

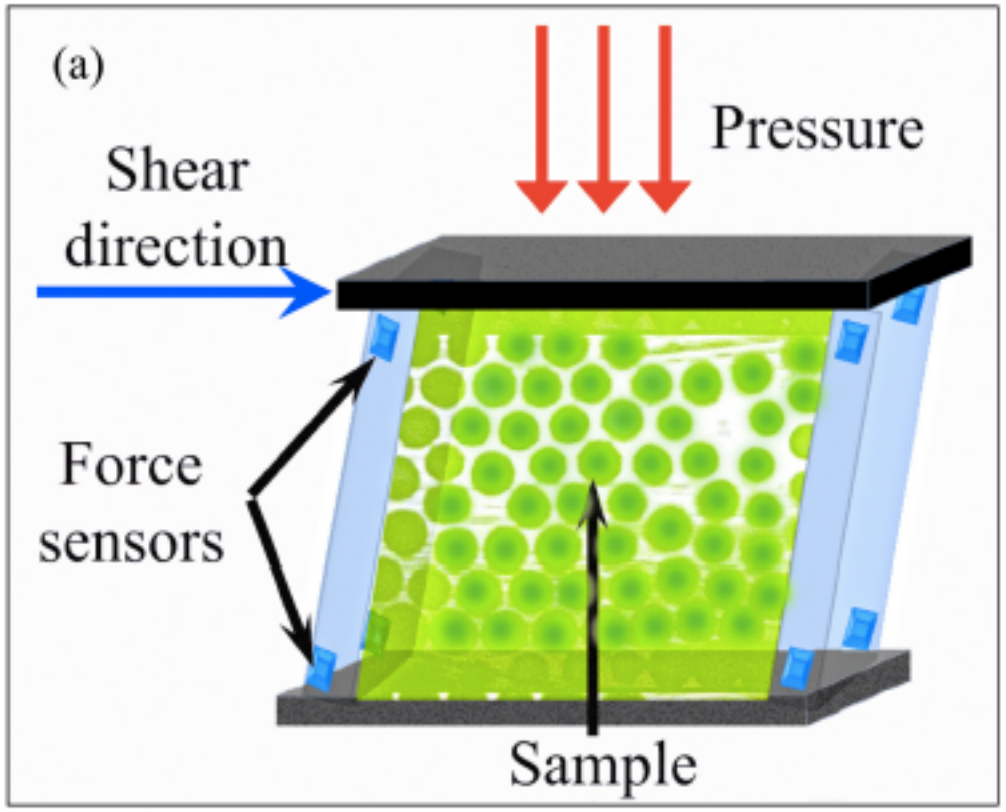
We may reasonably assume that earthquakes refers to systems where the packing ratio **φ** is extremely large. There is no experiment and/or numerical investigation which shows how $\mathbf{P(t_i|S_*)}$ depends on **φ**.

The above observations lead to two different questions:

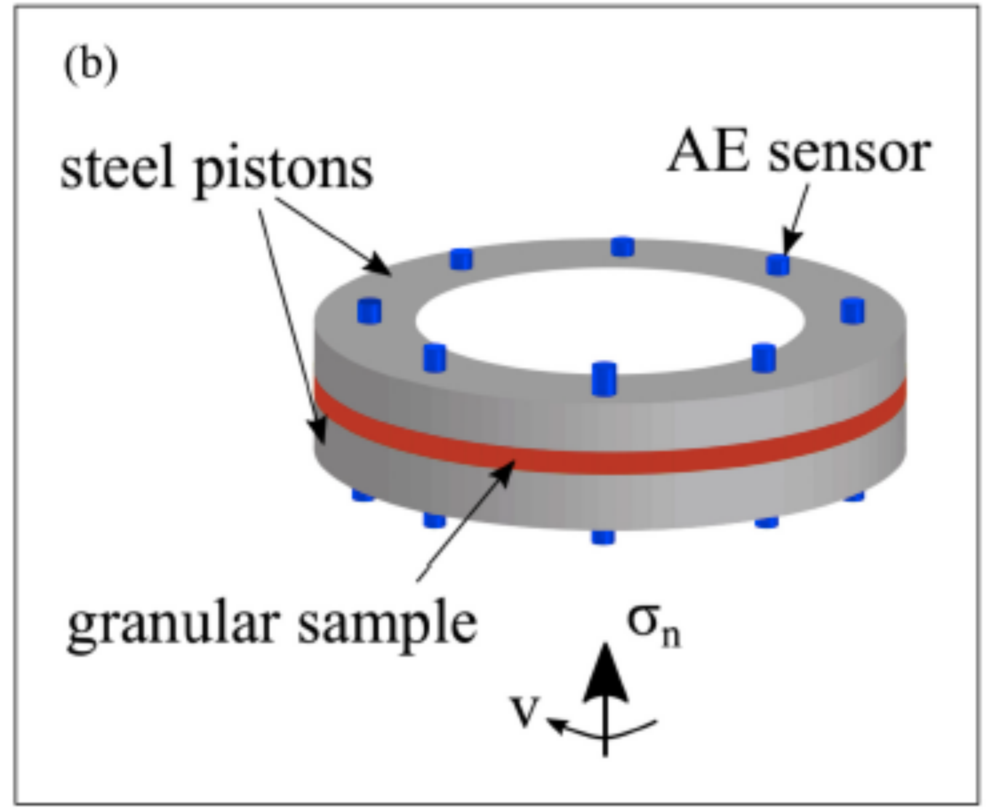
- is there any evidence that $\mathbf{P(t_i|S_*)}$ changes upon increasing **φ** ?
- if $\mathbf{P(t_i|S_*)}$ is not exponential, is that true that scale invariance is observed?

We want to answer these questions and for this purpose we consider 3 different systems: 2 different experiments with granular systems and 1 numerical simulations of emulsion

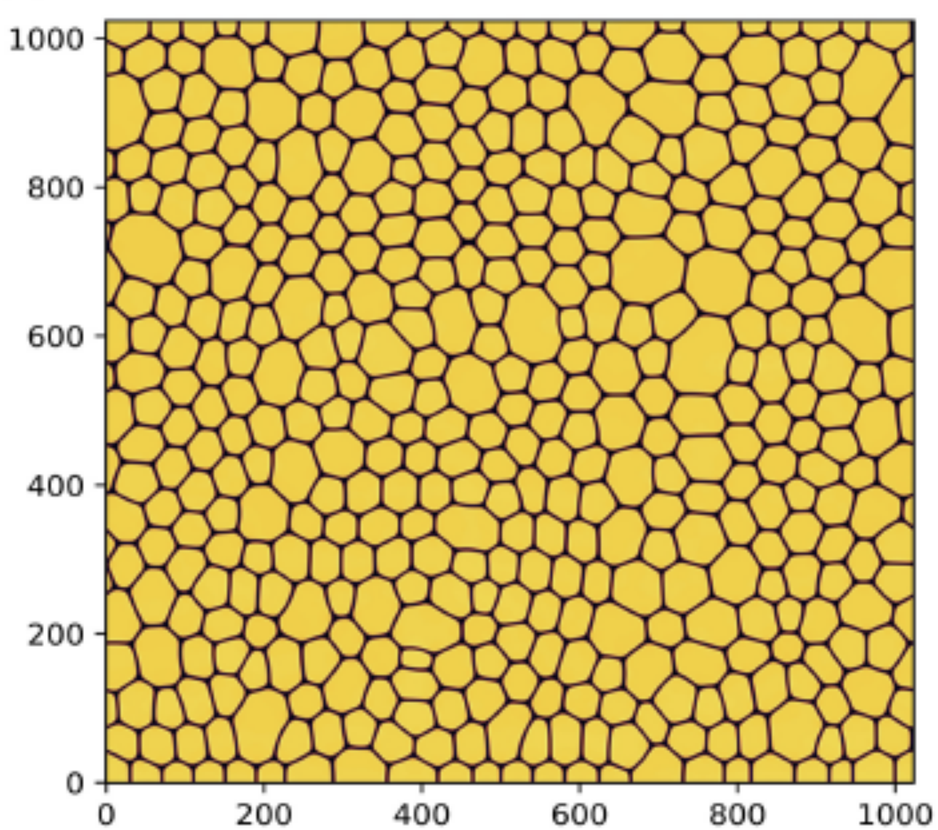
Granular experiment



Granular rotary experiment



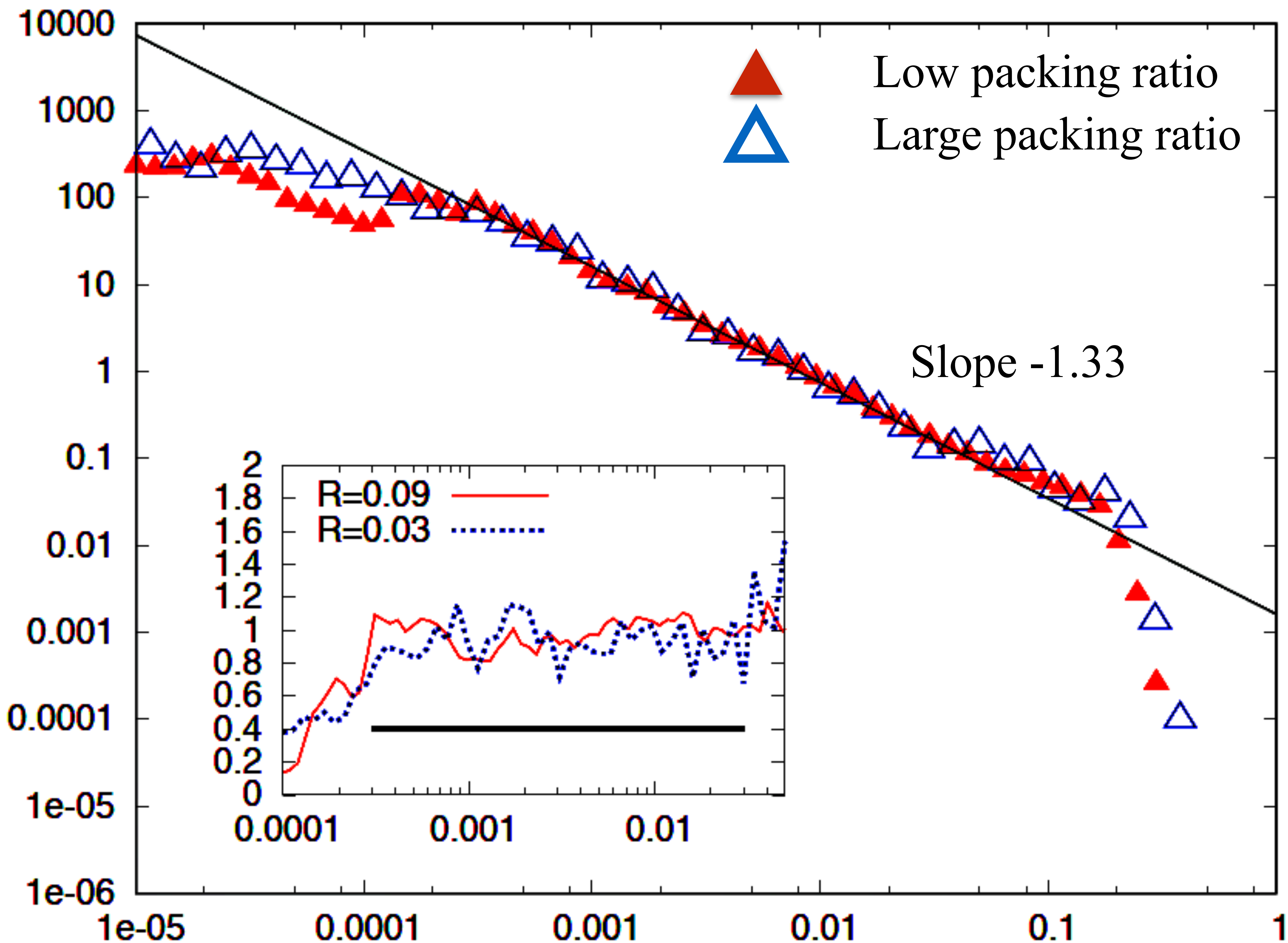
(a)



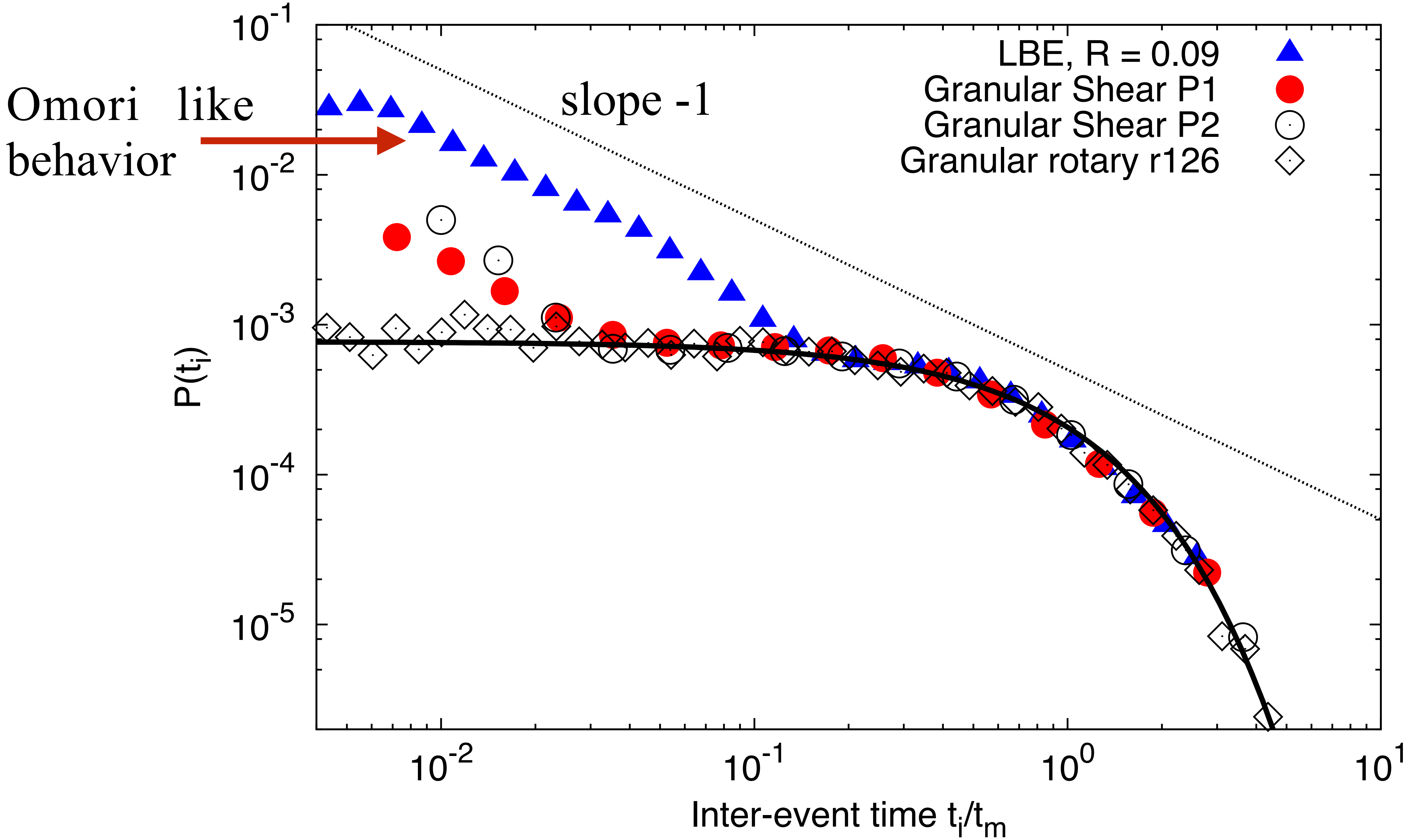
Lattice Boltzmann Simulations for emulsion.

Interface between two fluids stabilized by frustration which introduces a disjoining pressure

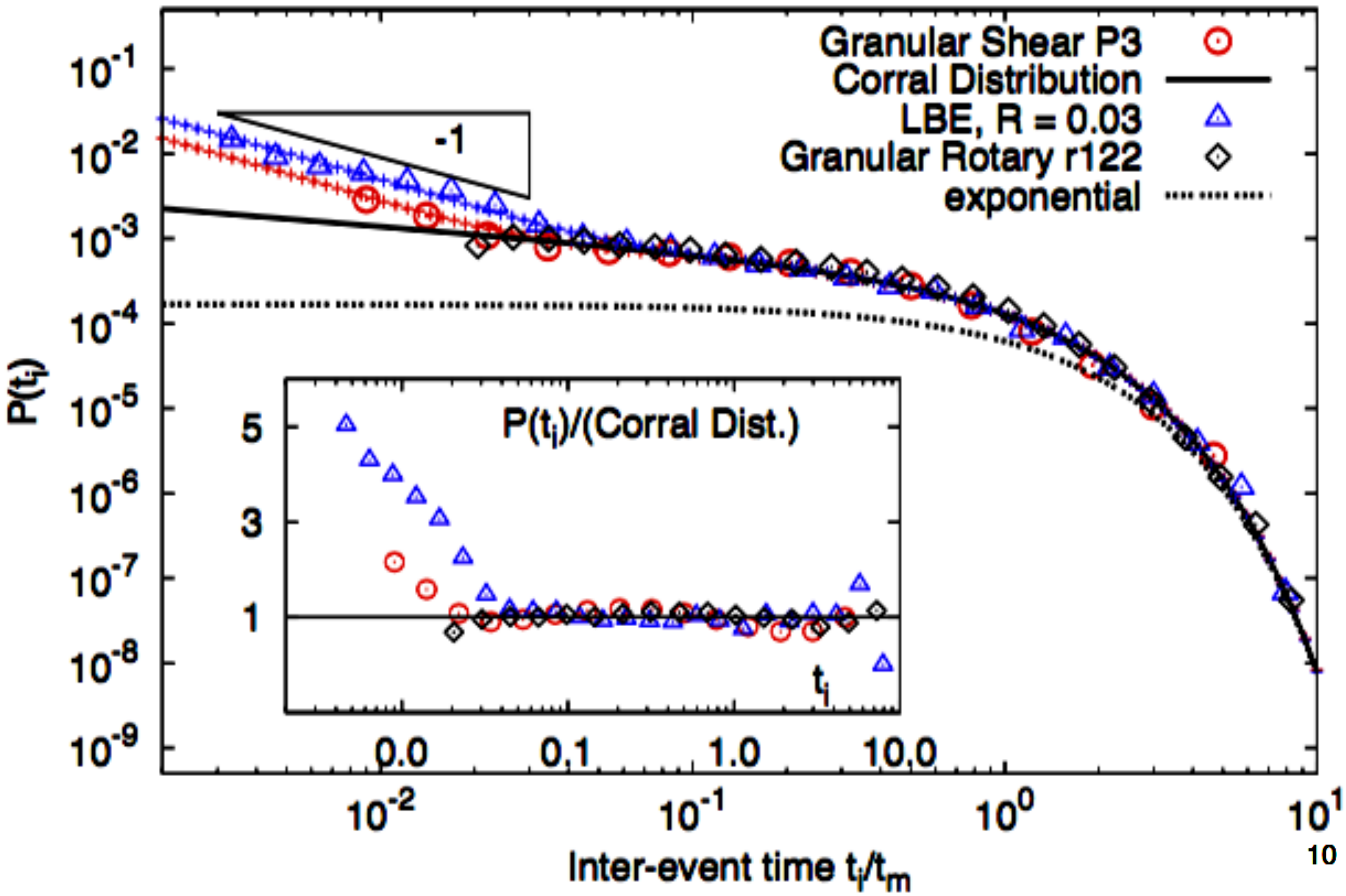
A closer look of the avalanche scaling law from the LBE simulations at "low" and "large" packing ratio



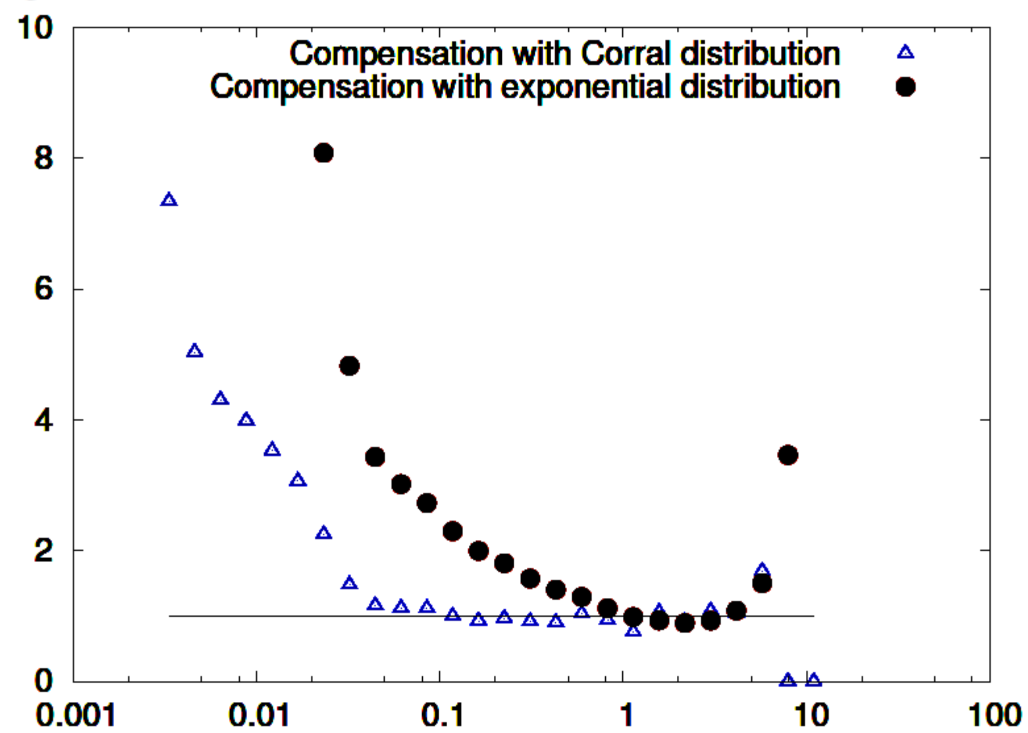
Interevent time distribution for “low” packing ratio



Inter event time distribution "large" packing ratio



$$P(t_i) \sim \frac{1}{t_i^{0.3}} \exp(-t_i/t_m)$$

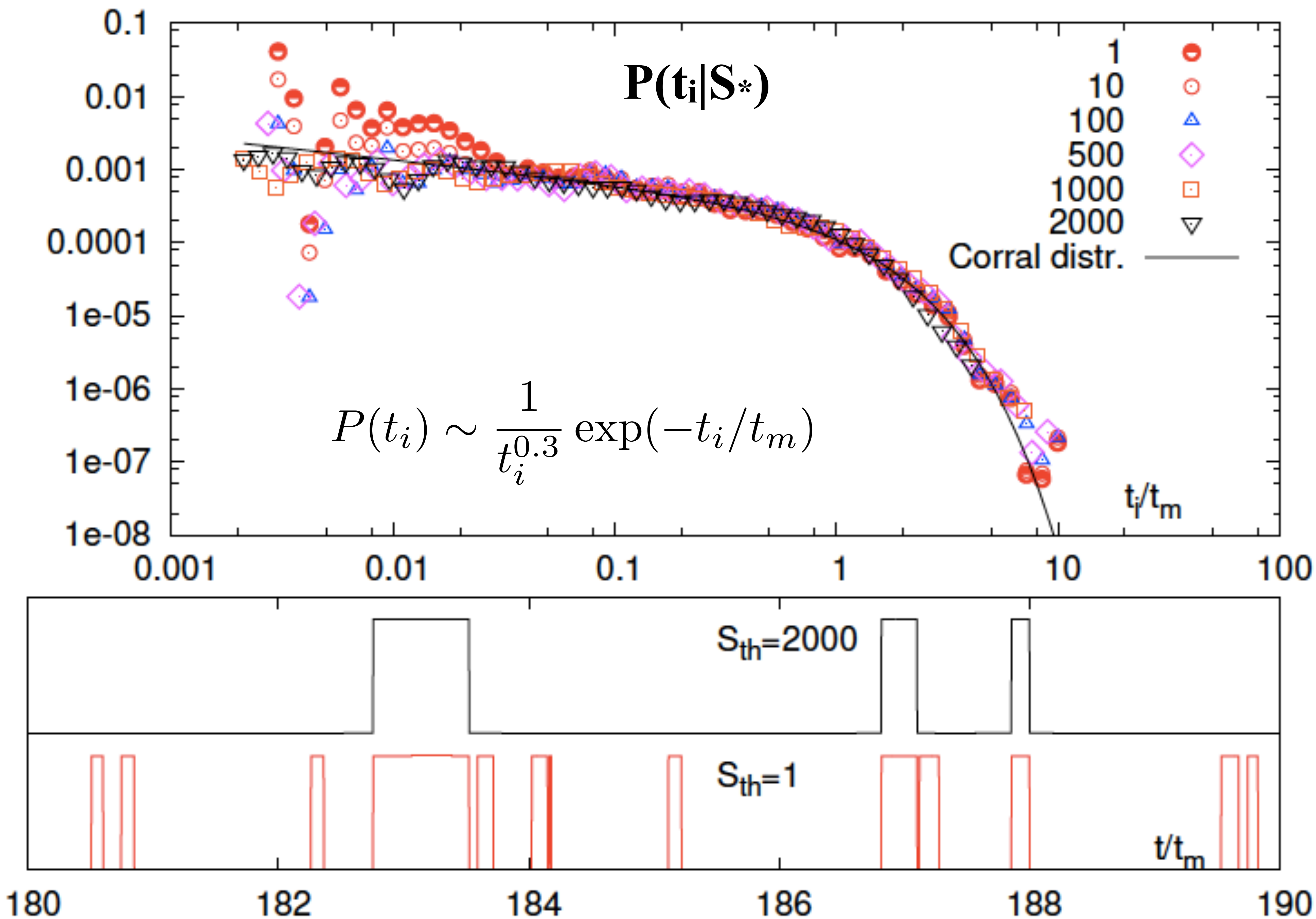


What about scale invariance?

R. Benzi, MIT 2019

What about scale invariance?

Scale invariance for LBE simulation



R. Benzi, MIT 2019

More on scale invariance

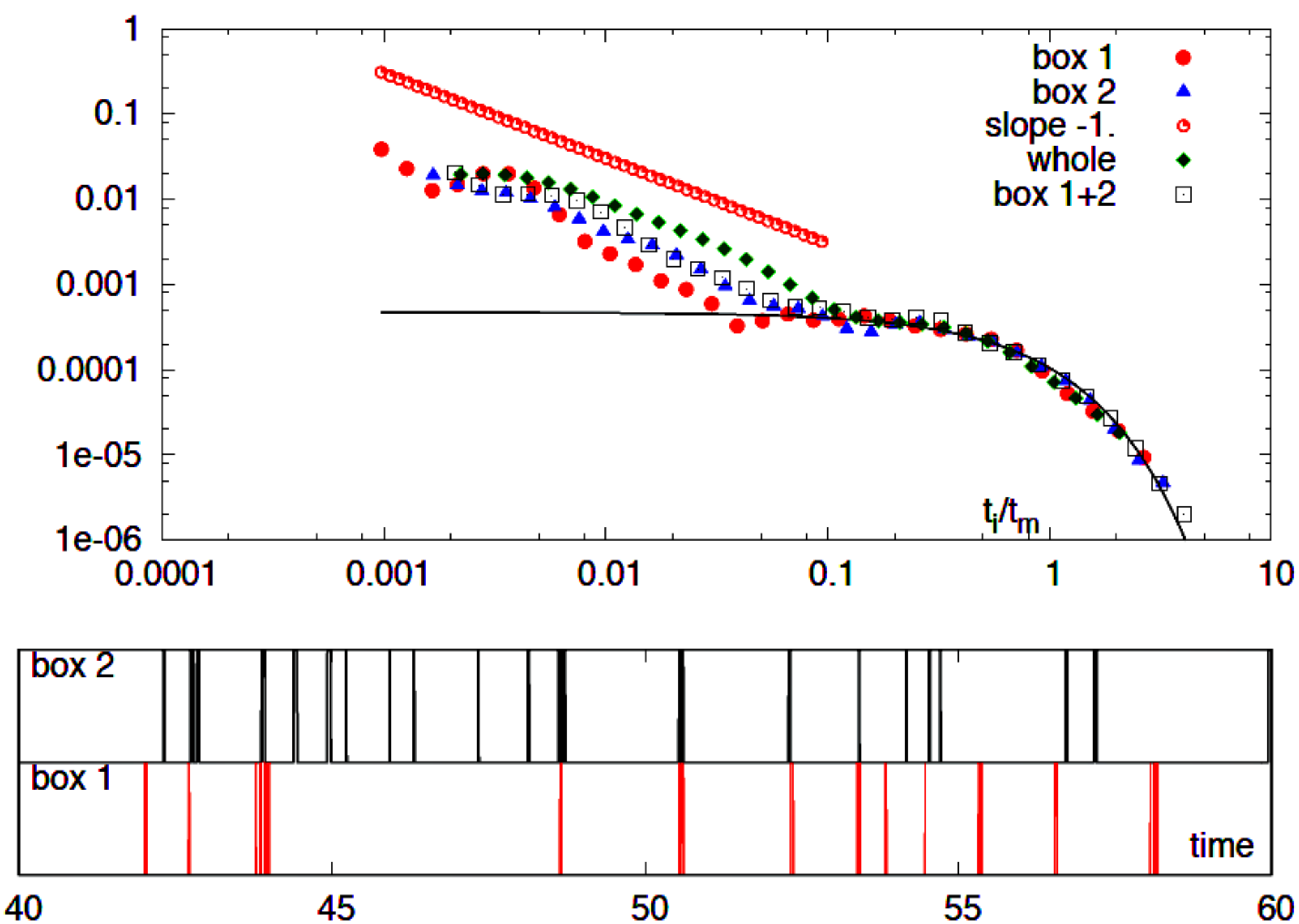
- We consider two regions in space (BOX 1 and BOX 2).
- We chose two regions where events are uncorrelated.

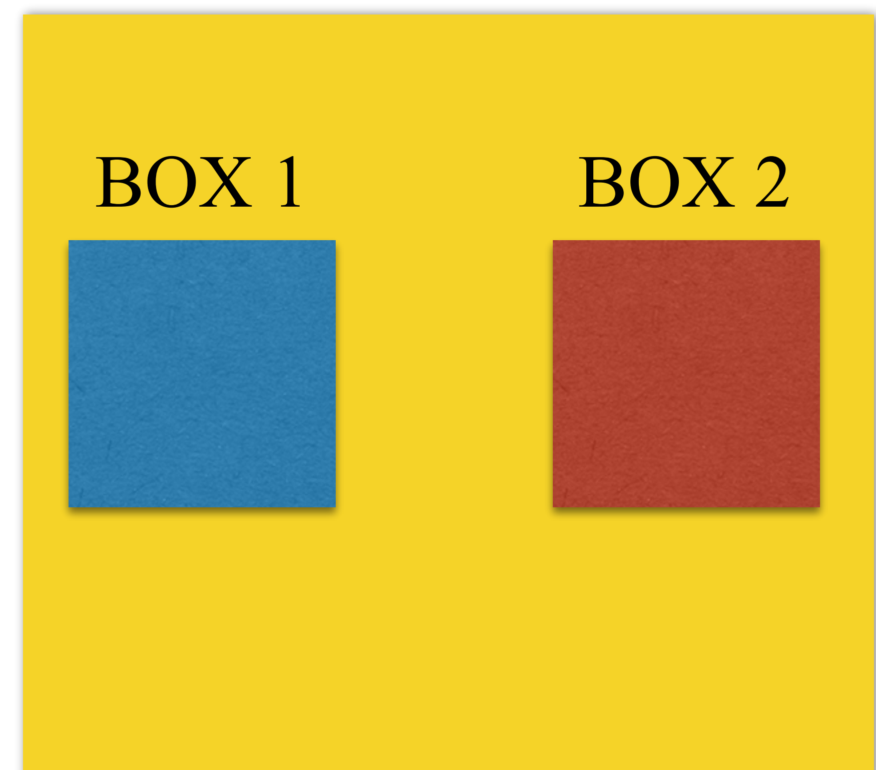
Warning:

<u>uncorrelated avalanche events do not imply short range correlation in the strain</u>

interevent time distribution

low packing ratio





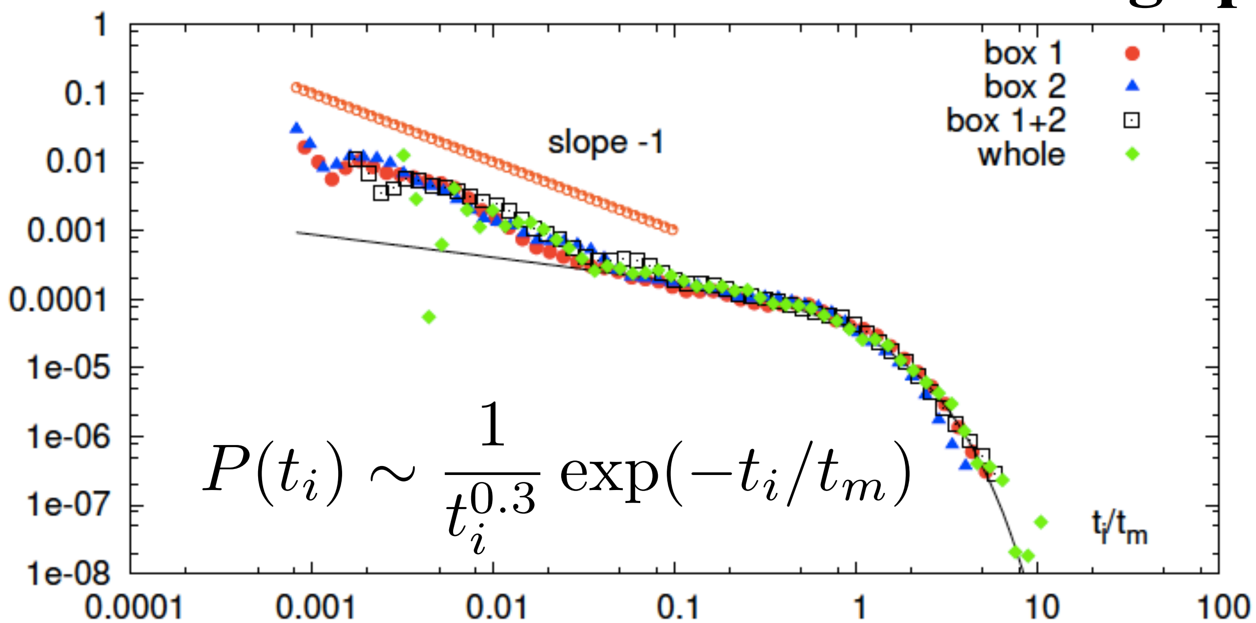
<u>not surprising</u>

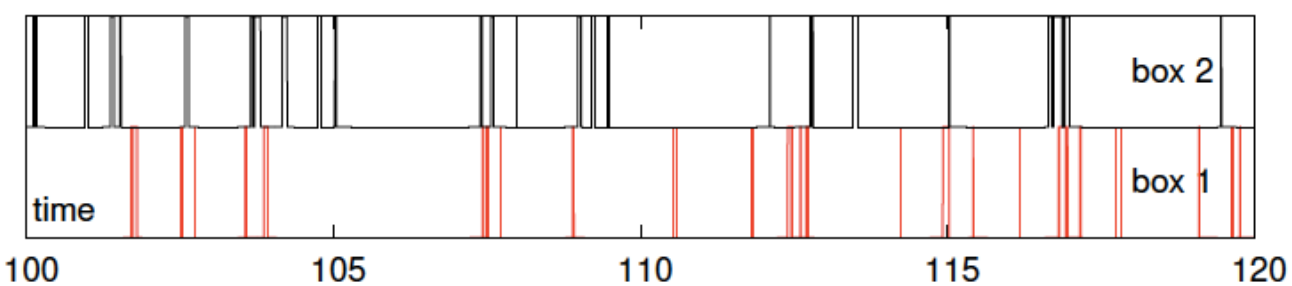
R. Benzi, MIT 2019

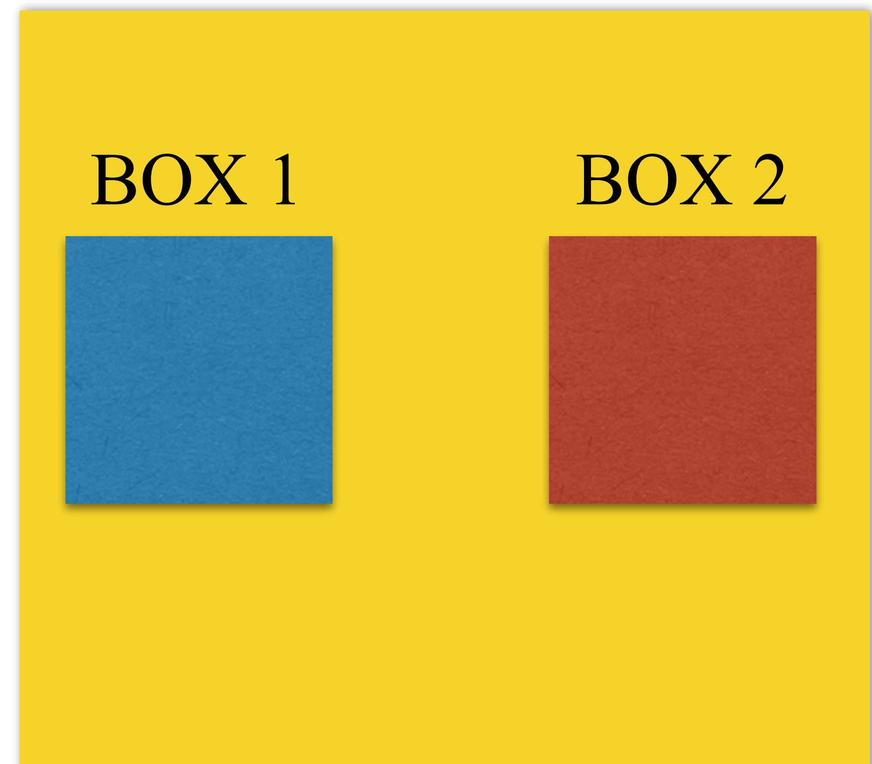
More on scale invariance

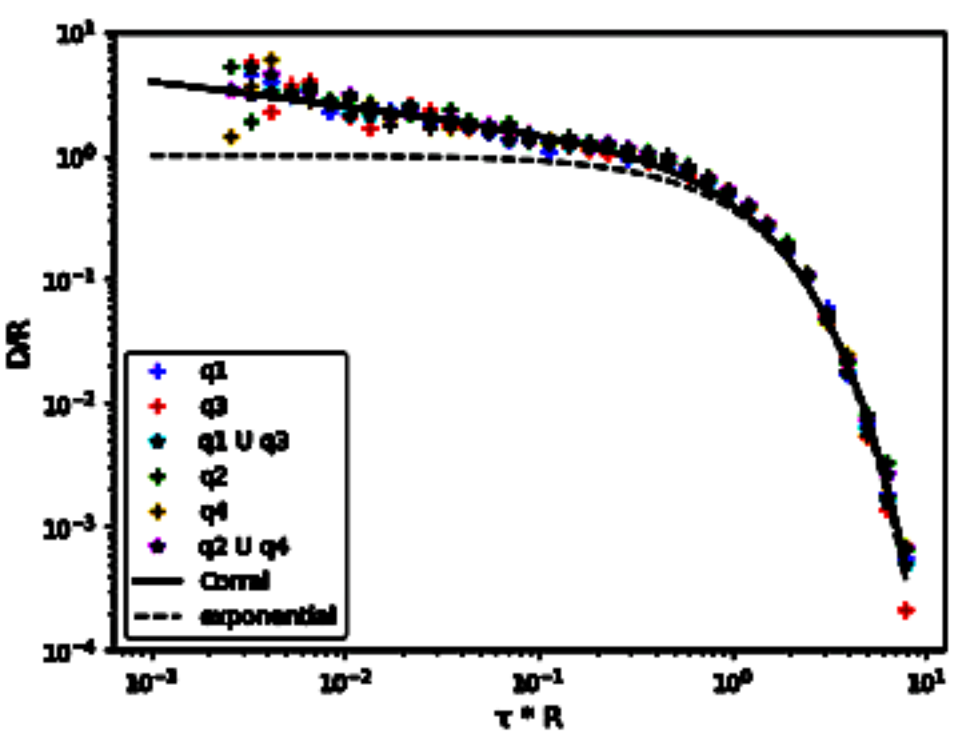
interevent time distribution

large packing ratio





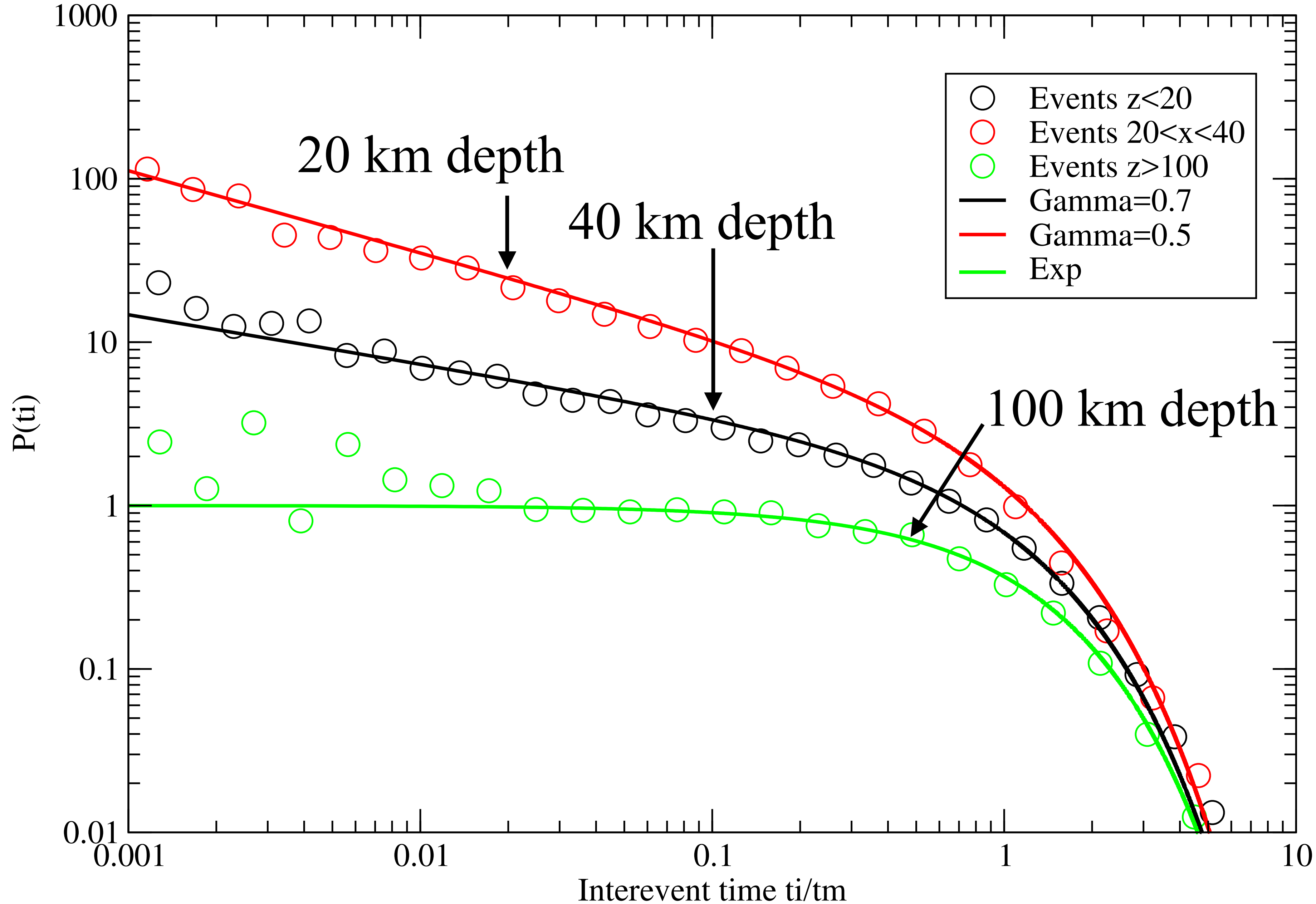




very surprising!!!

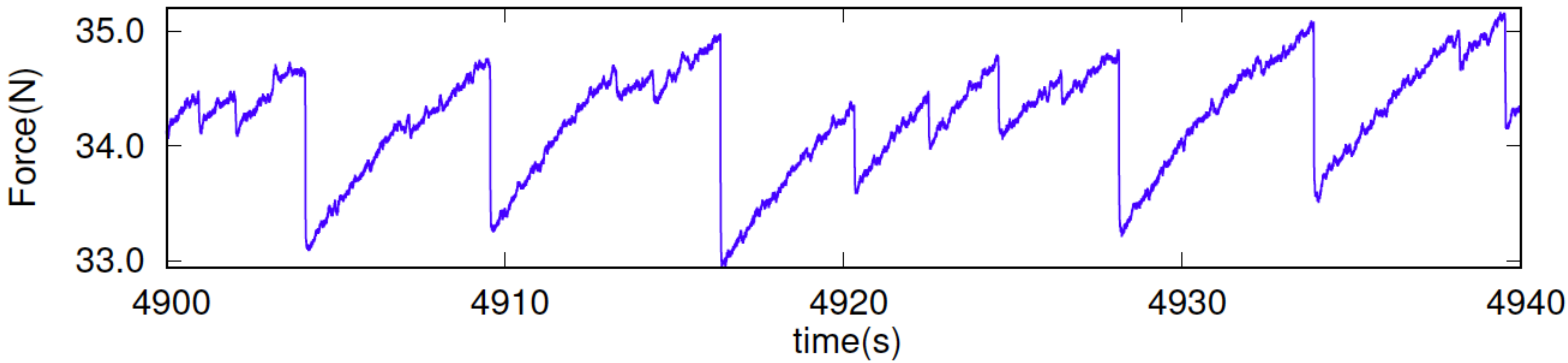
Same results from granular rotary experiments

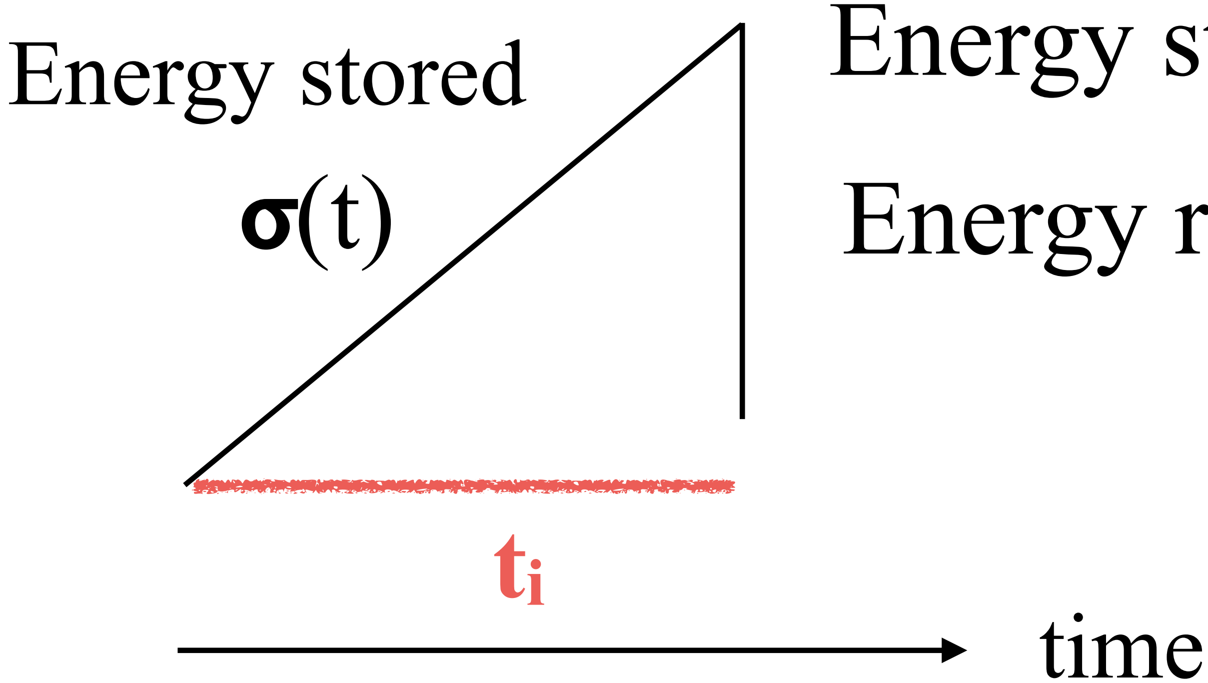
Earthquake inter event time statistics “whole earth”



A theoretical approach

stress versus time (lab measurements)





Energy stored ~ $\boldsymbol{\sigma}^2$ ~ t_i^2

Energy released **= S**

A theoretical approach

t_i and S are statistical independent quantities

Let us consider $P(t_i|S_*)$ with $S_* \equiv S\lambda$ and $\lambda > 1$

Then t_i depends on λ and $t_i(\lambda)$ grows with λ

Now let us consider

$$X(\lambda) \equiv \frac{Energy\ released}{Energy\ stored} \sim \frac{S}{t_i^2}$$

We can write

$$P[X(\lambda)|S]dS = \int dt_i P(t_i)P(S)\delta\left(\frac{S}{t_i} - X\right)$$

Assuming scale invariance

Let us assume $P(t_i) \sim \dfrac{1}{t_i^{\alpha}}$, $P[S] \sim \dfrac{1}{S^{\tau}}$

We obtain $P[X|S]dS = \dfrac{1}{X^{\Gamma}} \dfrac{dS}{S} \dfrac{1}{S^{\tau-\Gamma}}$ $\qquad \Gamma = \dfrac{3}{2} - \dfrac{\alpha}{2}$

The scale transformation $S \to S\lambda$ implies

$$X \to X(\lambda) = X\lambda^{H} \qquad H = \frac{\Gamma - \tau}{\Gamma}$$

If H>0 then for increasing λ we release more energy than stored

If H<0 then for increasing λ we store more energy than released

The physics does not change only if H=0

$$\boxed{\alpha = 3 - 2\tau}$$

Prediction in agreement with experiments, earthquake observations and numerical simulations !

Summary and conclusions:

- inter event time distribution is an interesting quantity to look at in avalanche dynamics.
- scale invariance holds for large enough packing ratio
- scale invariance holds in a "wider" formulation (different regions)
- possible non trivial consequences for earthquake events

Open questions:

- is there any transition?
- is there any theoretical framework?
- how it is possible to compute $\mathbf{P(t_i)}$ from "first principles"?