Current transport properties and phase diagram of a Kitaev chain with long-range pairing

Domenico Giuliano, Simone Paganelli, and Luca Lepori

Università della Calabria and INFN - Gruppo Collegato di Cosenza

domenico.giuliano@fis.unical.it

Bari, December 2019

Kitaev model Hamiltonian with short-range pairing

$$H_{SR} = -\sum_{j=1}^{\ell-1} \left\{ w\, c_j^\dagger c_{j+1} - \Delta\, c_{j+1} c_j + \mathrm{h.c.} \right\} - \mu \sum_{j=1}^{\ell} c_j^\dagger c_j$$

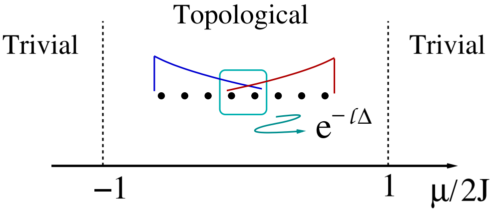
- Lattice model for a p-wave superconductor in 1d
- (For $w = \Delta$ Fermionic description of the 1d Quantum Ising chain (via JW)
- Single-particle dispersion relation

$$\epsilon = \sqrt{(2w\cos(k) + \mu)^2 + 4\Delta^2 \sin^2(k)}\ , \ (-\pi < k < \pi)$$

- Gap closure at $2w = \pm\mu \Rightarrow$ **topological** phase transition
  $2w > |\mu| \rightarrow$ topological phase, $2w < |\mu|$, "trivial" phase

Emerging Majorana modes

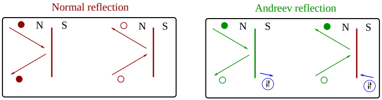
- Emerging Majorana modes only in the topological phase



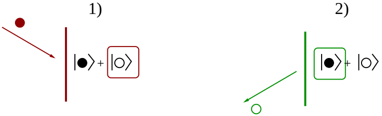
- Within the TP the wavefunctions of the two MMs localized at the endpoints of the chain have an overlap $\sim e^{-\ell \Delta_{\mathrm{Eff}}}$, $\Delta_{\mathrm{Eff}} = |2w - |\mu||$ They become true ZMO's as $\ell \to \infty$

Emerging MM $\Rightarrow$ resonant Andreev reflection as $\epsilon \rightarrow 0$

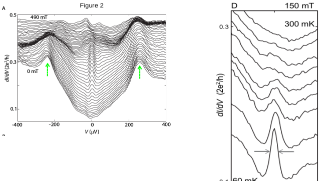
- Subgap energies at the SN-interface: normal, or Andreev reflection



- MZM $\Rightarrow$ enhancement of Andreev reflection as $\epsilon \rightarrow 0$



- Zero-bias peak [V. Mourik *et. al.*, Science 25, 336 (2012)]



Kitaev model Hamiltonian with long-range pairing

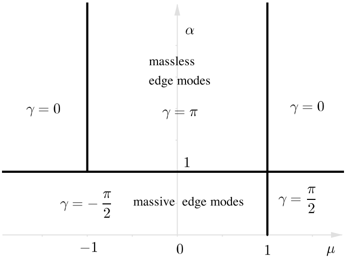
- Model Hamiltonian

$$H_{LR}^{\alpha} = -\sum_{j=1}^{\ell-1} \left\{ w\, c_j^{\dagger} c_{j+1} - \Delta \sum_{r} \delta_{r,j}^{-\alpha}\, c_{j+r} c_j + \text{h.c.} \right\} - \mu \sum_{j=1}^{\ell} c_j^{\dagger} c_j$$

- $\delta_{r,j} = |r|\theta(\ell/2 - |r|) + (L - |r|)\theta(|r| - \ell/2)$
- $\alpha \to \infty \to$ only pairing at $r \sim 1$ matters $\to H_{LR}^{\alpha} \to H_{SR}$
- $\epsilon_k = \sqrt{(\mu + 2w\cos(k))^2 + \Delta^2 f_\alpha^2(k)}$. Diverging at $k = 0$ for $\alpha < 1 (\to_{\ell\to\infty} \; -\frac{i}{2}\{\text{Li}_\alpha(e^{ik}) - \text{Li}_\alpha(e^{-ik})\})$.

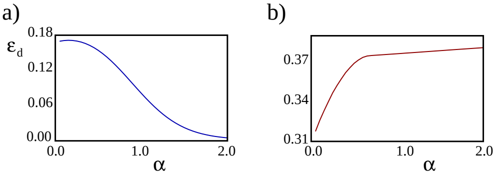
Phase diagram of H_{LR}: (1) – The winding number

- For $\alpha \leq 1$ the Polylogarithms yield divergencies in both ϵ_k and $\partial_k \epsilon_k$, for $1 < \alpha \leq \frac{3}{2}$ ϵ_k is no more divergent at $k \sim 0$, but $\partial_k \epsilon_k$ still is.
- For $\alpha \leq 1$ W is ill-defined, due to the singularities on the integration path in k-space, and so is the Berry's phase $\gamma = i \int_{-\pi}^{\pi} dk \, \langle k; -|\partial_k|k, -\rangle$ $\Rightarrow$ pertinent regularization
- Phase diagram ($|\Delta\gamma| = \pi$ across a topological PT)



Phase diagram of H_{LR}: (2) – Edge modes and $\mathbf{Z}_2$-symmetry

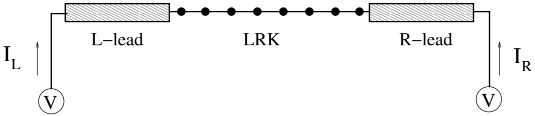
- $\alpha > 1 \leftrightarrow$ von Neumann area law, phases of the SR-model with $\gamma = \pi(0)$ in the topological (trivial) phase, gapless real-fermion modes at the boundaries, breaking of $\mathbf{Z}_2$ fermion-parity symmetry
- $0 < \alpha < 1 \leftrightarrow$ violation of von Neumann area law, new phases with $\gamma = \pm\pi/2$, **massive** real-fermion modes at the boundaries, $\mathbf{Z}_2$ fermion-parity symmetry unbroken
- Boundary-mode (a)) and bulk (b)) mass and phase transition for $w = \Delta = 1$, $\mu = 0.25$, $\ell = 26$



Contacting the chain with metallic leads

- DC-transport properties across the Kitaev chain are extremely sensitive to the behavior (and to the emergence, of course) of the subgap modes ⇒ DC-transport as a tool to probe the quantum phase transition in the LR Kitaev chain
- Model Hamiltonian = H_{LR} + lead + contact Hamiltonian:

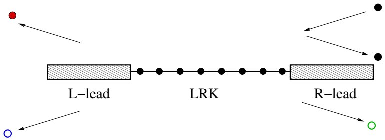
$$
\begin{aligned}
\mathcal{H}_L &= -J \sum_{j \leq -1} \{c^\dagger_{j,L} c_{j+1,L} + \mathrm{h.c.}\} - (\mu - eV) \sum_{j \leq 0} c^\dagger_{j,L} c_{j,L} \\
\mathcal{H}_R &= -J \sum_{j \geq \ell+1} \{c^\dagger_{j,R} c_{j+1,R} + \mathrm{h.c.}\} - (\mu - eV) \sum_{j \geq \ell+1} c^\dagger_{j,R} c_{j,R} \\
\mathcal{H}_T &= -t\{c^\dagger_{0,L} c_1 + c^\dagger_{R,\ell+1} c_\ell + \mathrm{h.c.}\}
\end{aligned}
$$



Scattering amplitudes and scattering coefficients

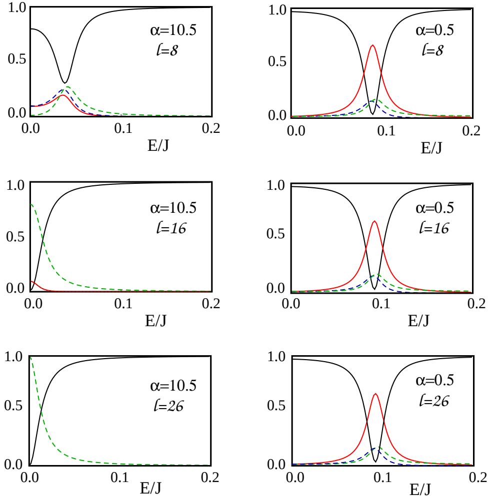
- $$S(E) = \left[\begin{array}{cccc} r_{L,L}(E) & \tilde{a}_{L,L}(E) & t_{L,R}(E) & \tilde{c}_{L,R}(E) \\ a_{L,L}(E) & \tilde{r}_{L,L}(E) & c_{L,R}(E) & \tilde{t}_{L,R}(E) \\ t_{R,L}(E) & \tilde{c}_{R,L}(E) & r_{R,R}(E) & \tilde{a}_{R,R}(E) \\ c_{R,L}(E) & \tilde{t}_{R,L}(E) & a_{R,R}(E) & \tilde{r}_{R,R}(E) \end{array} \right]$$
- $L \leftrightarrow R$- and $p - h$-symmetry close to the Fermi level $\Rightarrow$

$$|r_{L,L}|^2 = |\tilde{r}_{L,L}|^2 = |r_{R,R}|^2 = |\tilde{r}_{R,R}|^2 \equiv R \text{ [Normal reflection]}$$
$$|t_{L,R}|^2 = |\tilde{t}_{L,R}|^2 = |t_{R,L}|^2 = |\tilde{t}_{R,L}|^2 \equiv T \text{ [Normal transmission]}$$
$$|\tilde{a}_{L,L}|^2 = |a_{L,L}|^2 = |\tilde{a}_{R,R}|^2 = |a_{R,R}|^2 \equiv A \text{ [Andreev reflection]}$$
$$|\tilde{c}_{L,R}|^2 = |c_{L,R}|^2 = |\tilde{c}_{R,L}|^2 = |c_{R,L}|^2 \equiv C \text{ [Crossed Andreev reflection]}$$



Scattering coefficients *vs.* E in the two phases

$w = \Delta = 1, \mu = 0.25$



DC-current at zero-bias between the leads $\rightarrow I_L = -I_R \equiv I$

- At finite temperature T use the Fermi function $f(E)$

$$I = \frac{2e}{2\pi} \int_0^\infty dE \{f(-E-eV) - f(-E+eV)\}\{|a_{L,L}(E)|^2 + |c_{L,R}(E)|^2\}$$

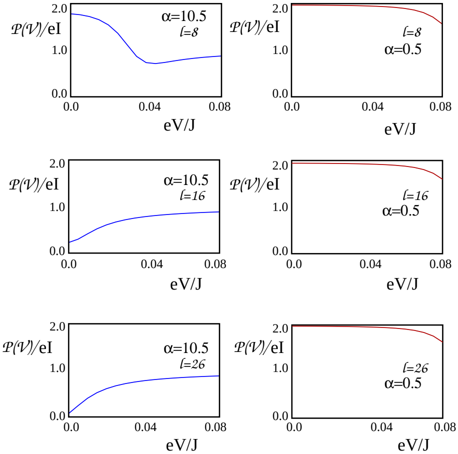
- At $T = 0$

$$I = \frac{2e}{2\pi} \int_0^{eV} dE \{|a_{L,L}(E)|^2 + |c_{L,R}(E)|^2\} = \frac{2e}{2\pi} \int_0^{eV} dE \{A(E) + C(E)\}$$

- I is determined by the sum $A(E) + C(E)$ which encode the subgap current-transport supporting processes

Zero-temperature, zero frequency shot-noise (I)

- Noise $\mathcal{P}(V) = \lim_{\Omega\to 0} \frac{1}{2}\int dt\, e^{i\Omega t}\langle\{\delta J_X(t), \delta J_{X'}(0)\}\rangle$
- Fano factor across the two phases $(w = \Delta = 1, \mu = 0.25)$

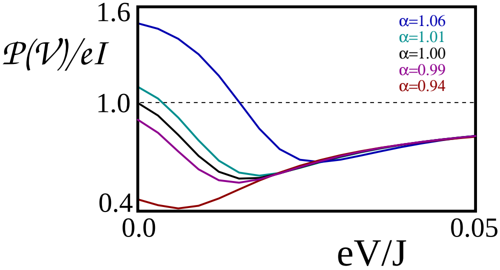


Fano factor to characterize the phases

- SR topological phase ($\alpha > 1$) $\Rightarrow \mathcal{P}(V)/(eI) \to 0$ as $V \to 0$
- Physical interpretation: *All the scattering coefficients but $A(E)$ are suppressed as $E \to 0$. The system behaves as a perfect conductor, there is no fluctuations in the current and, accordingly, $\mathcal{P}(V \to 0) \to 0$, even at a finite I*
- LR topological phase ($0 < \alpha < 1$) $\Rightarrow \mathcal{P}(V)/(eI) \to 2$ as $V \to 0$
- Physical interpretation: *Now, as $E \to 0$, the transport-supporting processes become rare events ($A(E \to 0), C(E \to 0) \to 0$). The shot noise becomes Poissonian and from the Fano factor as $V \to 0$ one reads out the elementary charge tunneling during current transport (in units of e), that is, $e^*/e = 2$*

Fano factor close to the critical line

- Using the Fano factor as a parameter to detect the phase transition ($w = \Delta = 1, \mu = 0,25, \ell = 2000$)



- Interesting (though not yet explored) "e^*-charge fractionalization" at $\alpha = 1$

Conclusions and perspectives

- $\alpha = 1$ appears to mark a phase transition between nontrivial phases in the LR Kitaev model
- This can be detected in a DC-trasport measurement
- Interesting "e^*-charge fractionalization" at the critical line at $\alpha = 1$ possibly deserves further study
- Possible new physics with contacting with interacting leads (see e.g. Ian Affleck and D. G., J. Stat. Mech. P06011 (2013) in the SRK)
- Similar physics in LR 2d p-wave models [L. Lepori, D. G., S. Paganelli, PRB 97, 041109(R)]
- Physical realizations of the Kitaev model with LR pairing in solid state devices: chain states in a chain of magnetic impurities on top of an *s*-wave superconductor [PRB 88, 155420 (2013); PRB 89, 180505 (2014)]