Report on the Transverse Filter Concept and its Simulations in Kassiopeia

Wonyong Chung

Princeton University

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- Goal is to give a more visual and conceptual overview of filter development process

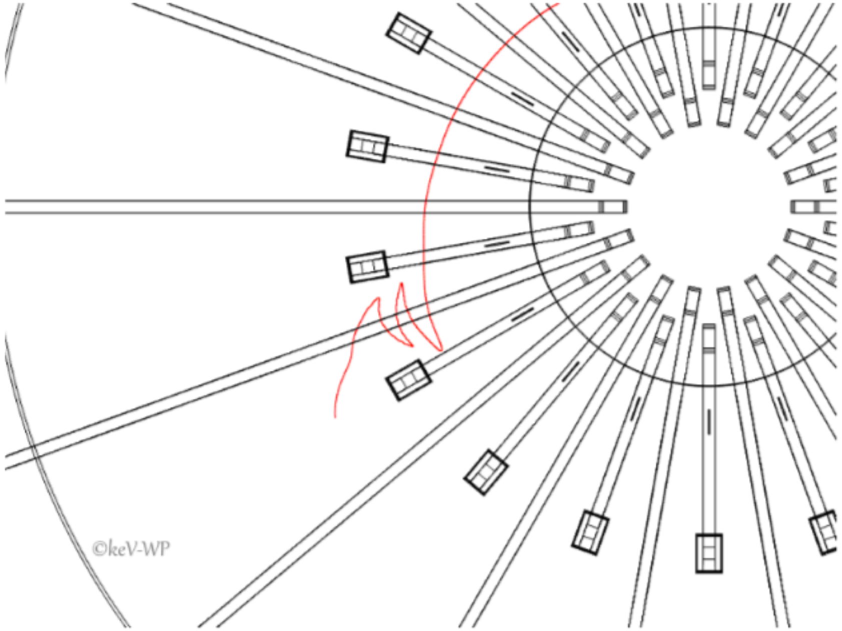
Introduction to Kassiopeia simulation package

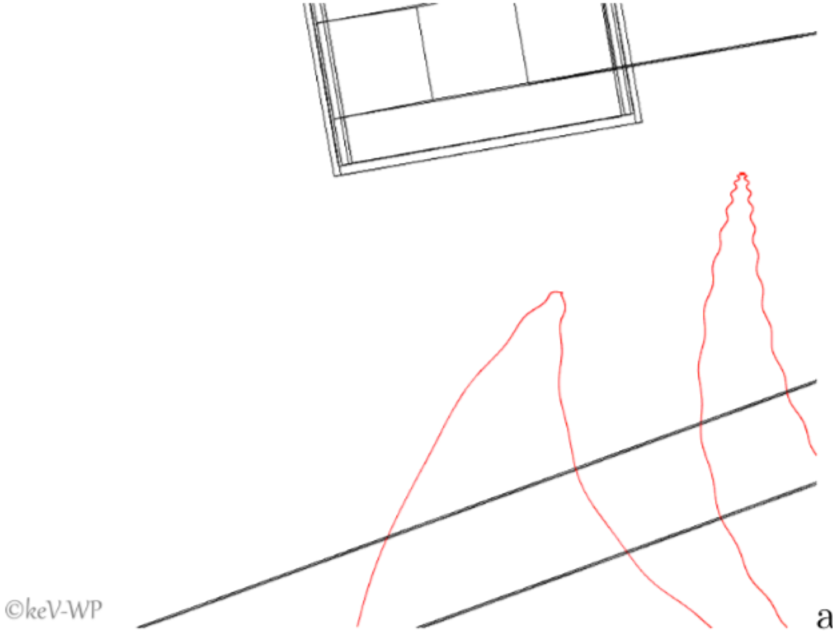
- Physical geometry of charged electrodes and currents specified in XML file
- E and B fields computed (various field solvers available)
- Electron trajectory then simulated through the fields
  - Using Lorentz force law at each step
  - 64 steps per cyclotron revolution (configurable)
- Output in ROOT

- Developed by KATRIN, modified by PTOLEMY to use analytic field expressions to skip the computation step

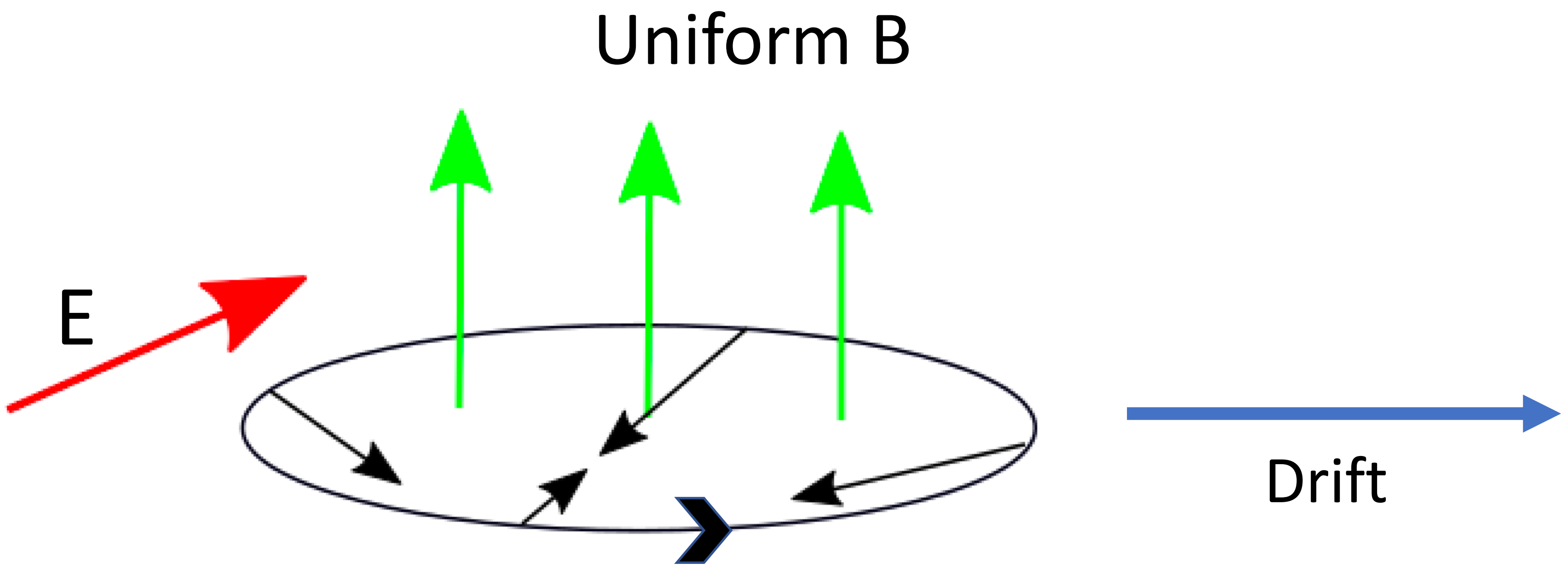
Preliminary Filter Design

- Radial design with filter elements extending radially outward from inside superconducting solenoid
- Tritium source placed at center of solenoid, electrons guided outward into filter elements



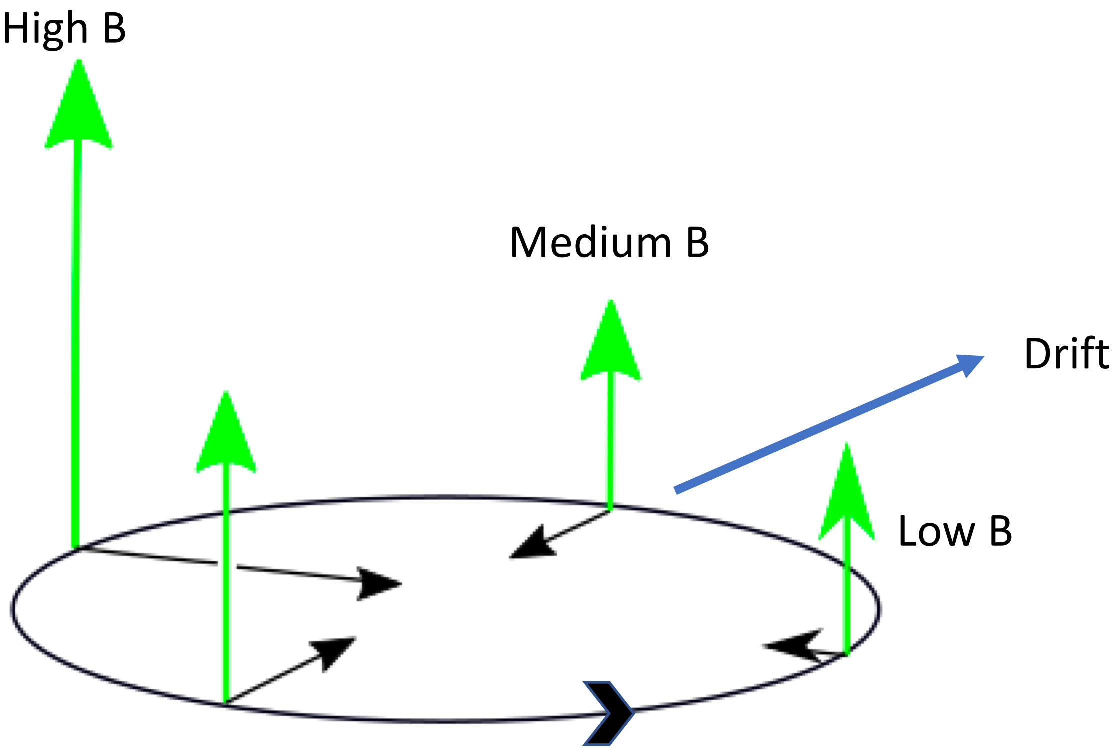


ExB Drift



F = q(E + v x B)

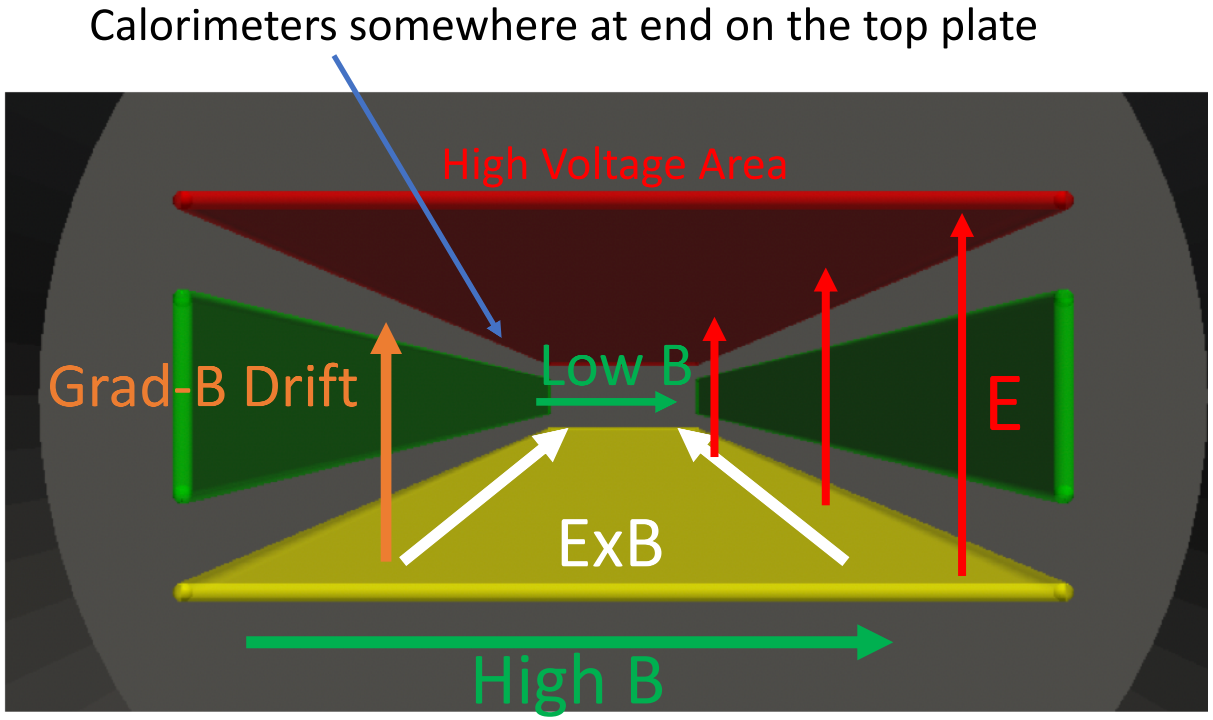
Gradient-B Drift



F = qv x B

Fundamental Filtering (Selecting) Process

- ExB drift on electron's guiding center motion pushes electron down the length of the filter
- Gradient-B drift pushes electron into high voltage area to exchange transverse KE for voltage potential
- Works because orbital magnetic moment (u = T/B) is adiabatically invariant (decrease B -> decrease transverse KE)



Two big limitations of preliminary filter

- ExB magnitude is equal to E/B
  - As B decreases exponentially, ExB magnitude blows up (since E is constant)
  - Unable to capture electron as it speeds away

- Since E is constant, as electron is pushed into high voltage, if the voltage difference within one diameter of a cyclotron revolution exceeds the electron’s transverse KE, it will immediately bounce in the opposite direction

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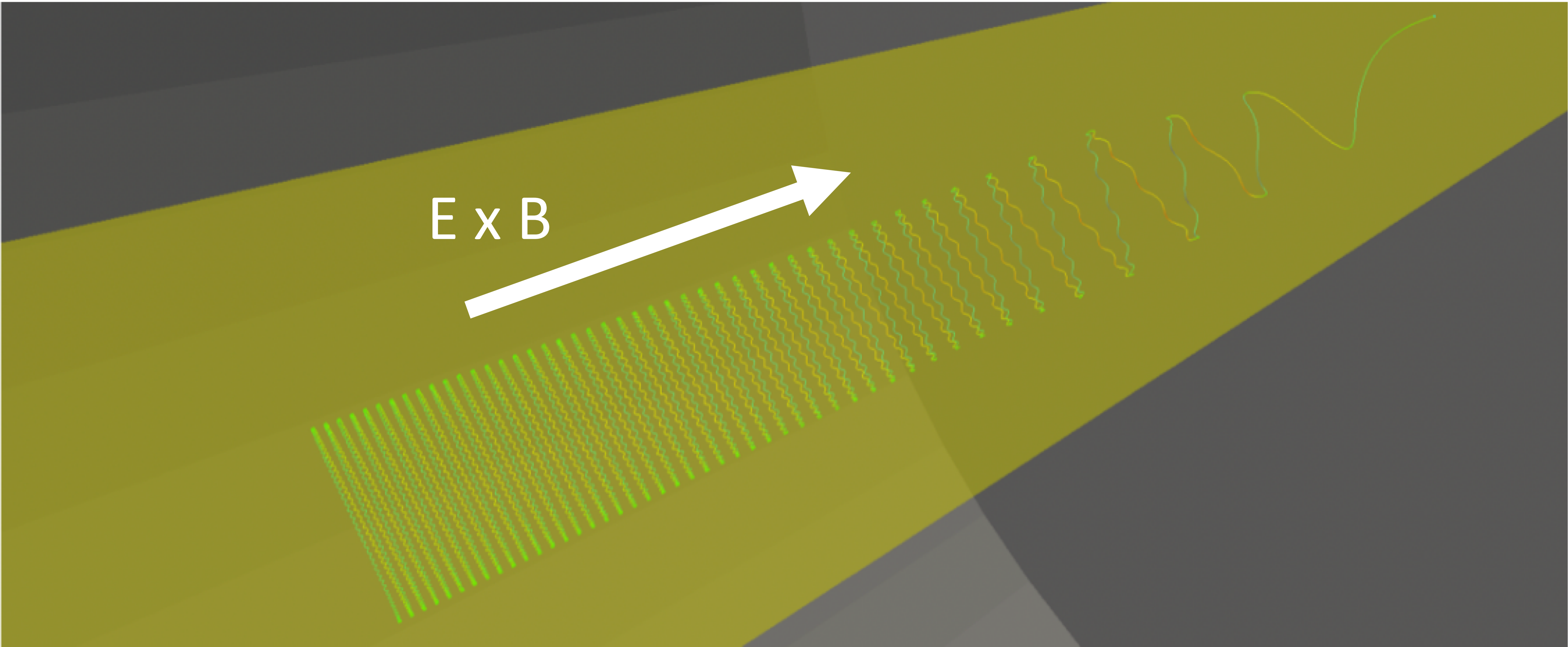
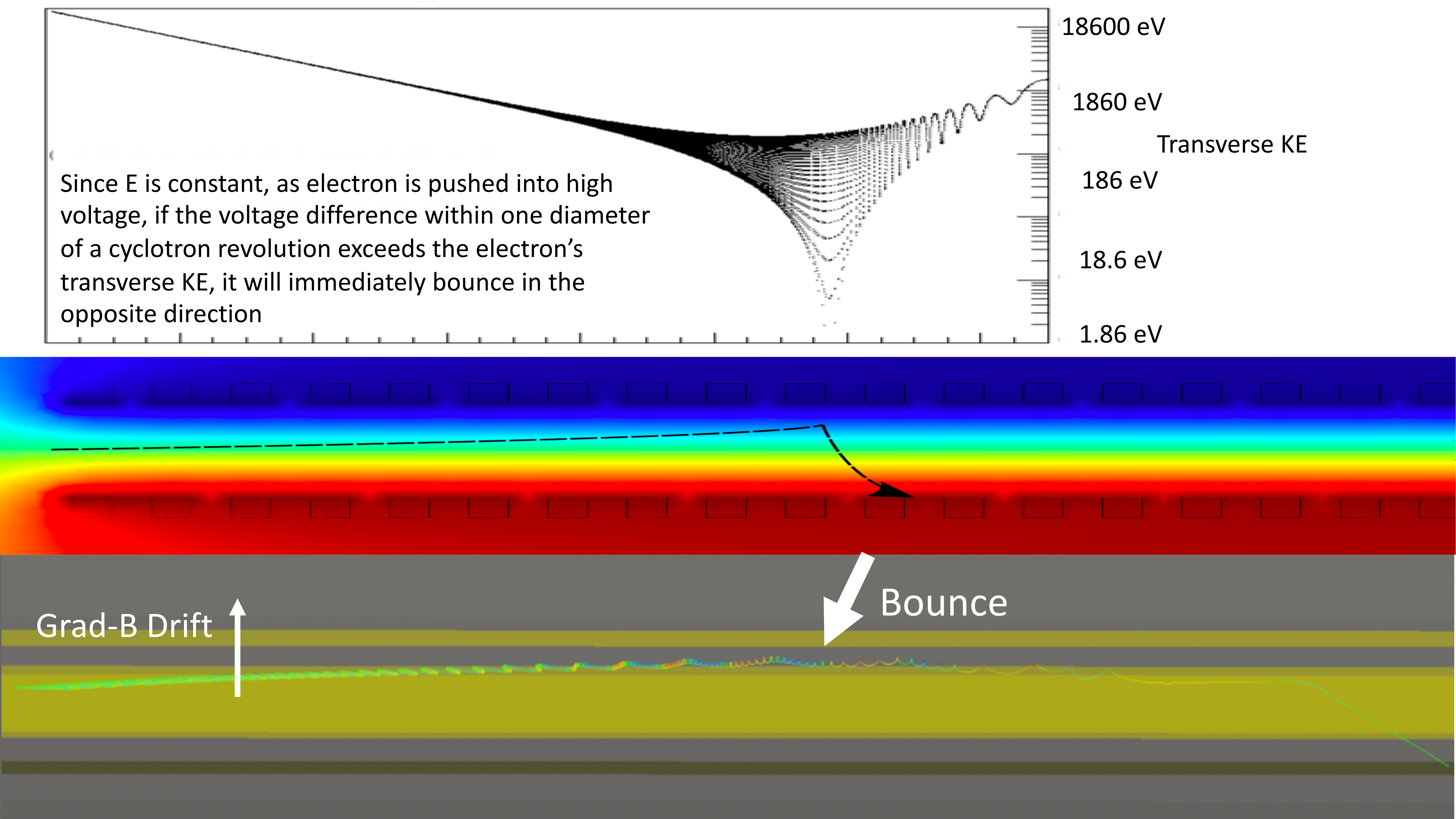



18600 eV
1860 eV
Transverse KE
186 eV
18.6 eV
1.86 eV
Since E is constant, as electron is pushed into high voltage, if the voltage difference within one diameter of a cyclotron revolution exceeds the electron's transverse KE, it will immediately bounce in the opposite direction
Bounce
Grad-B Drift

First Limitation (ExB velocity)

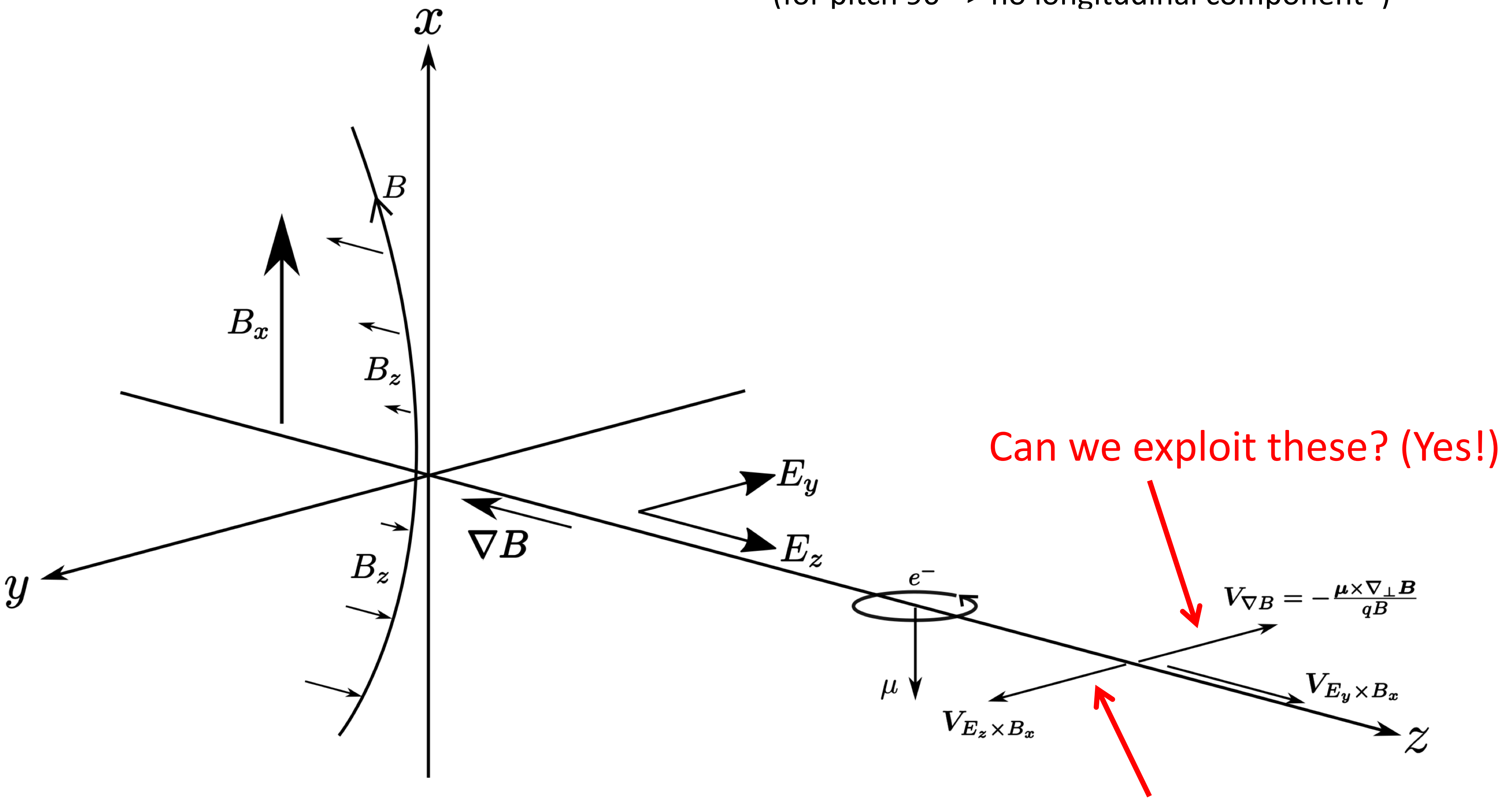
- Need to reduce ExB velocity so the electron can be captured
- Solution: Reduce the E_y field along the length of the filter by splitting the electrode into segments with decreasing voltages
- Consequences: ExB velocity is reduced but an additional E component in the z direction is introduced due to the changing E, making another ExB (E_z x B) drift
  - (But on the other hand, this kind of solves the bouncing problem…)

Second Limitation (Bouncing)

- We will see that this can be solved together with the ExB velocity problem with the right field configuration

Fields and Velocities Inside New Filter Element

(for pitch 90 -> no longitudinal component*)



*Full diagram with longitudinal components in paper

Set E_z x B drift equal to Grad-B drift*

$$B = B_x(z) = B_0\,\mathrm{e}^{-z/\lambda_b}$$

Grad-B Drift

$$\boldsymbol{V}_{\nabla B} = -\frac{\boldsymbol{\mu} \times \nabla_{\perp} \boldsymbol{B}}{qB} = -\frac{\mu}{qB}\frac{\partial B_x}{\partial z} = \frac{\mu}{q\lambda}\,,$$

E_z x B Drift

$$\frac{E_z}{B} = \frac{\mu}{q\lambda}$$

$$\boldsymbol{\nabla} \times \boldsymbol{B} = 0$$

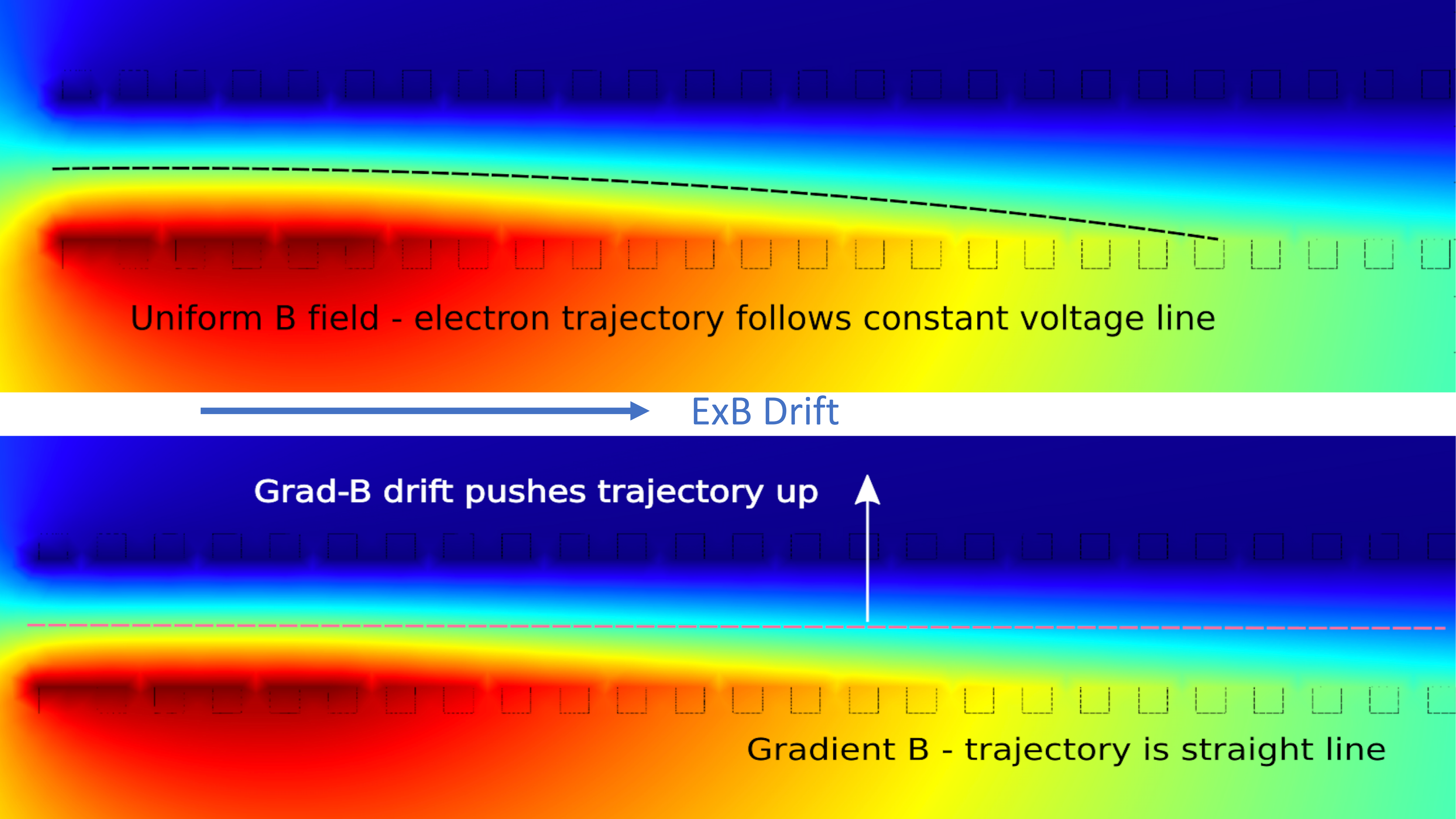
$$B_x = B_0 \cos\left(\frac{x}{\lambda}\right) e^{-z/\lambda}\,,$$
$$B_y = 0\,,$$
$$B_z = -B_0 \sin\left(\frac{x}{\lambda}\right) e^{-z/\lambda}\,.$$

$$E_0 = -\frac{\mu B_0}{q\lambda \sin\left(y_0/\lambda\right)}\,.$$

$$\boldsymbol{\nabla} \times \boldsymbol{E} = 0$$
$$\boldsymbol{\nabla} \cdot \boldsymbol{E} = 0$$

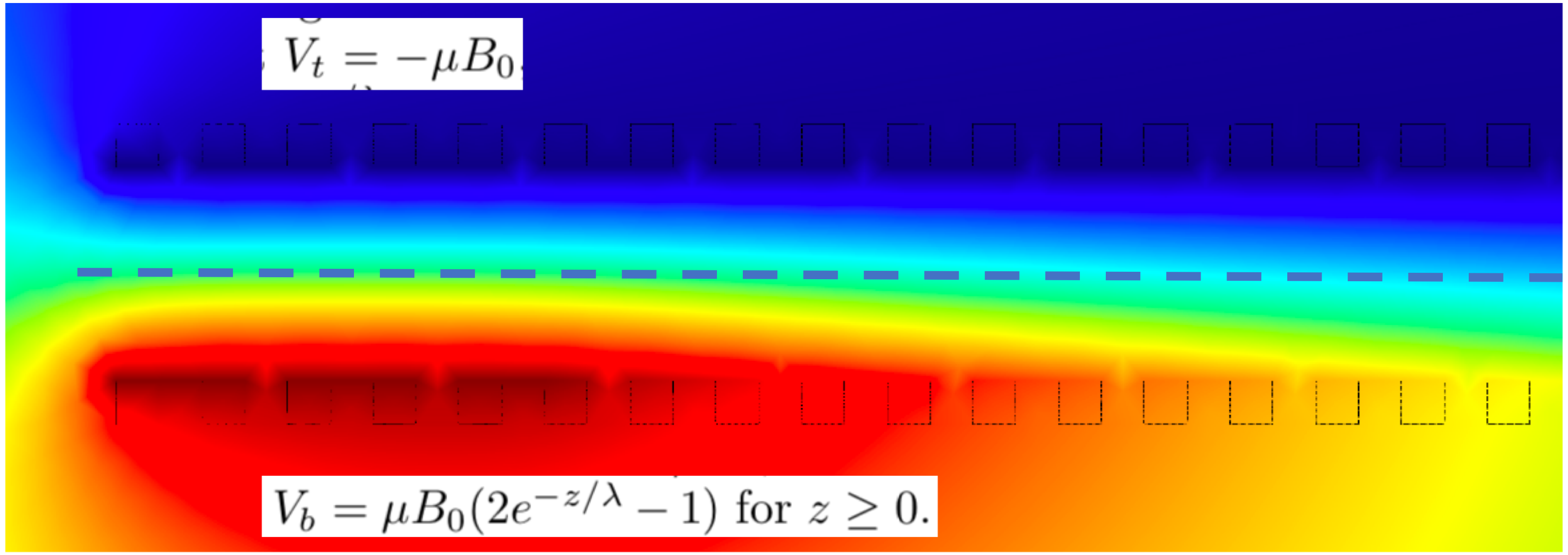
$$E_x = 0\,,$$
$$E_y = E_0 \cos\left(\frac{y}{\lambda}\right) e^{-z/\lambda}\,,$$
$$E_z = -E_0 \sin\left(\frac{y}{\lambda}\right) e^{-z/\lambda}$$

*Full details in paper

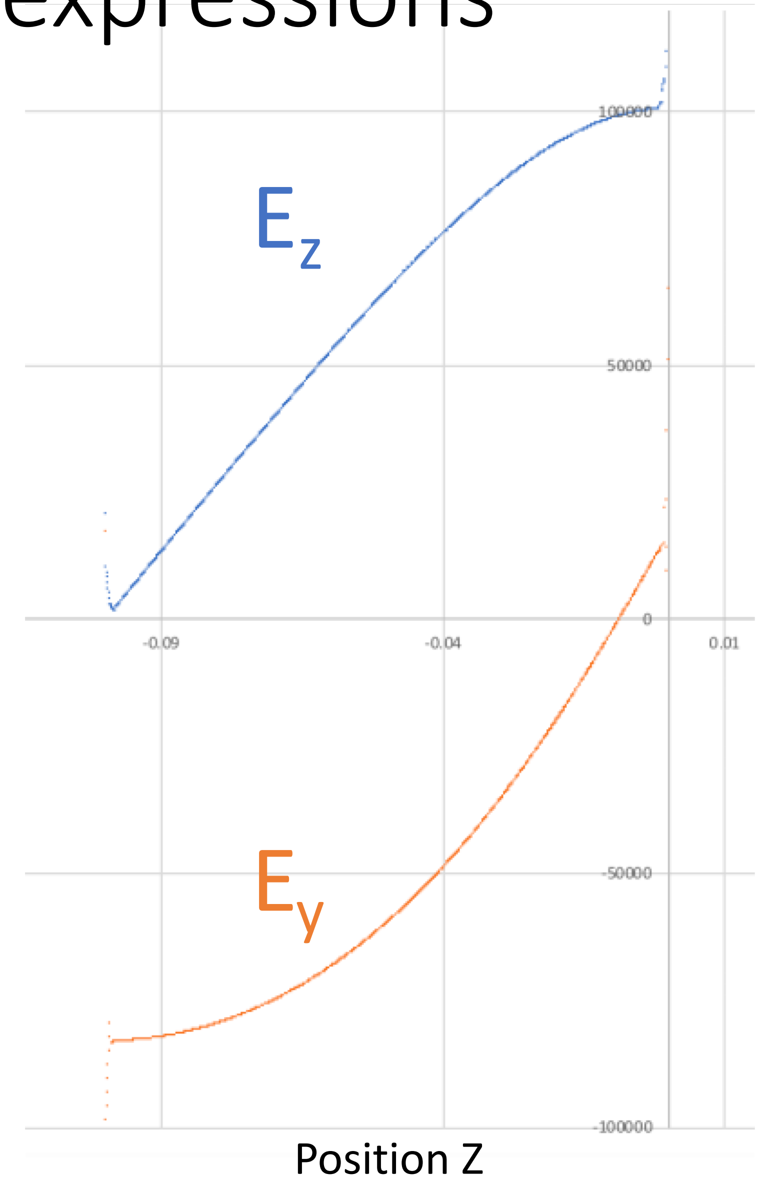


Uniform B field - electron trajectory follows constant voltage line
ExB Drift
Grad-B drift pushes trajectory up
Gradient B - trajectory is straight line

Approximate* the E fields between the plates along the center line, put analytical expressions into Kassiopeia

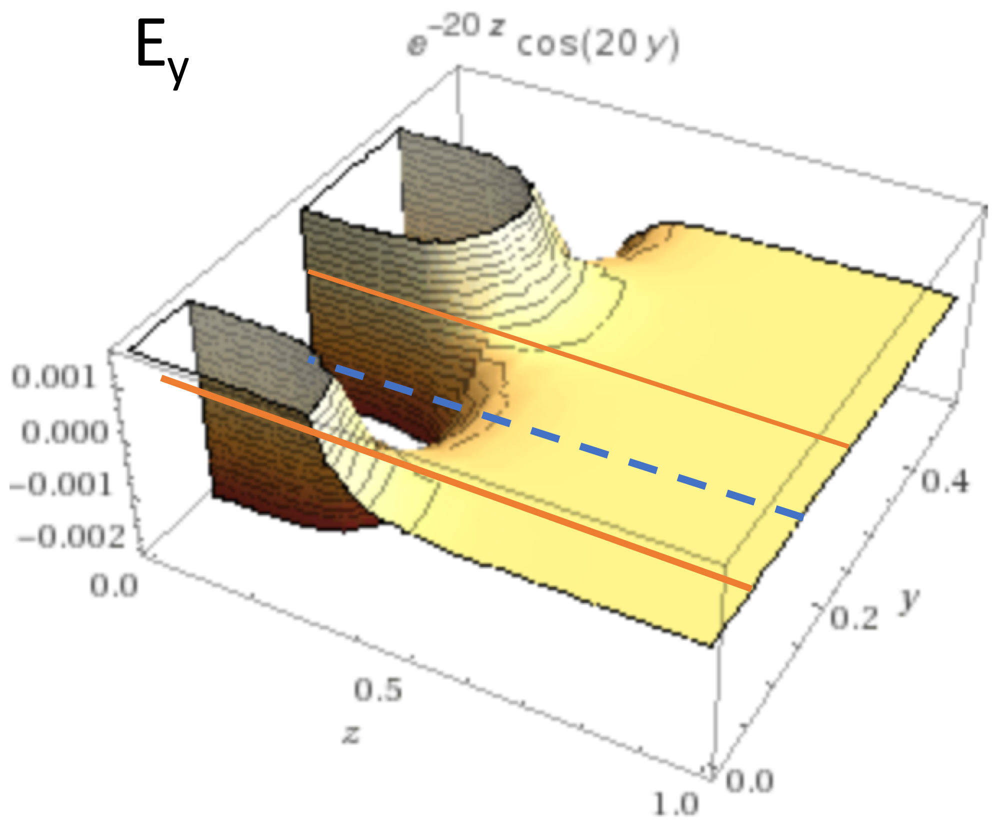


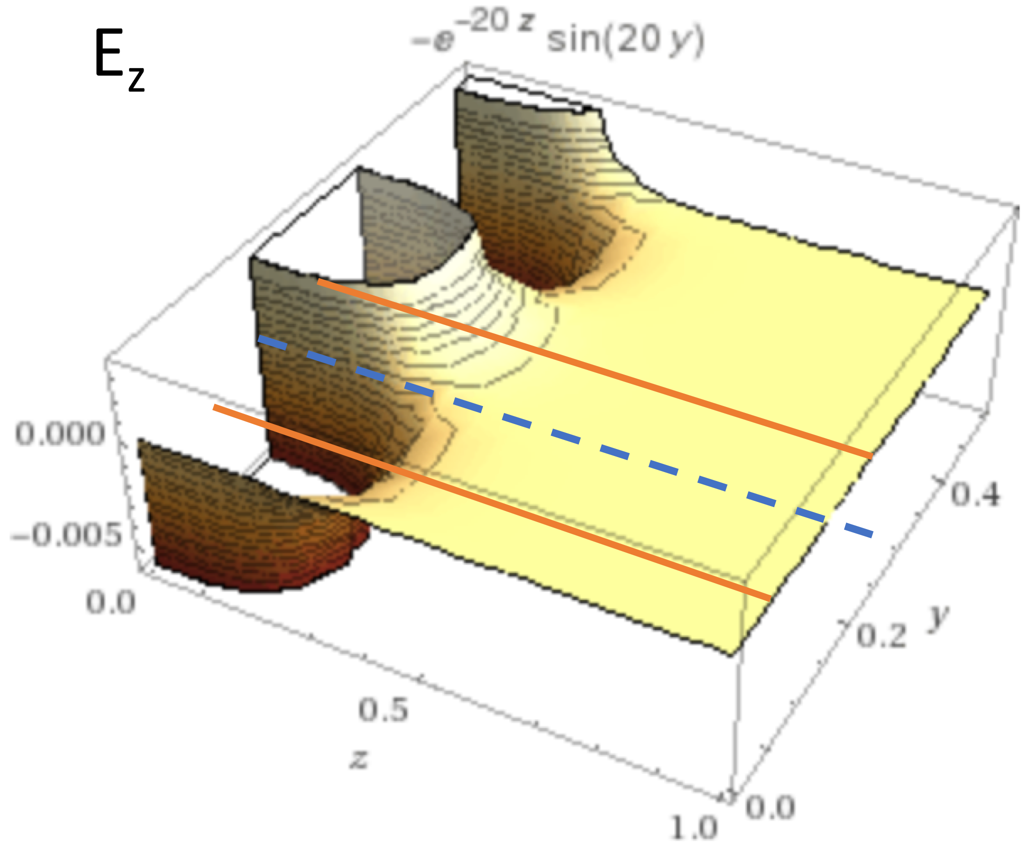
$$V(y,z)|_{y=y_0} = V_0 - \mu B_0 \left(1 - e^{-z/\lambda}\right)$$



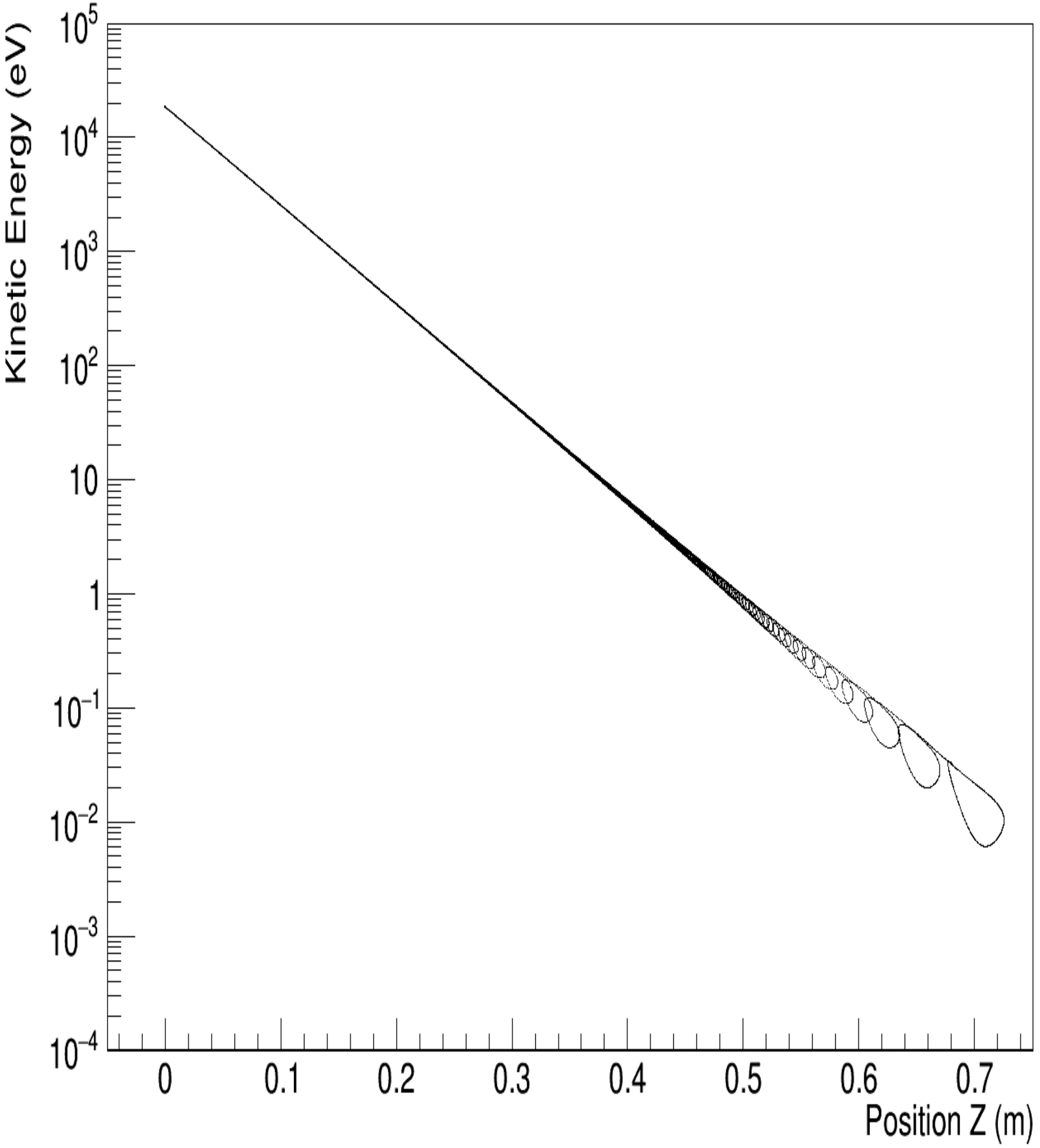
*Field values from COMSOL

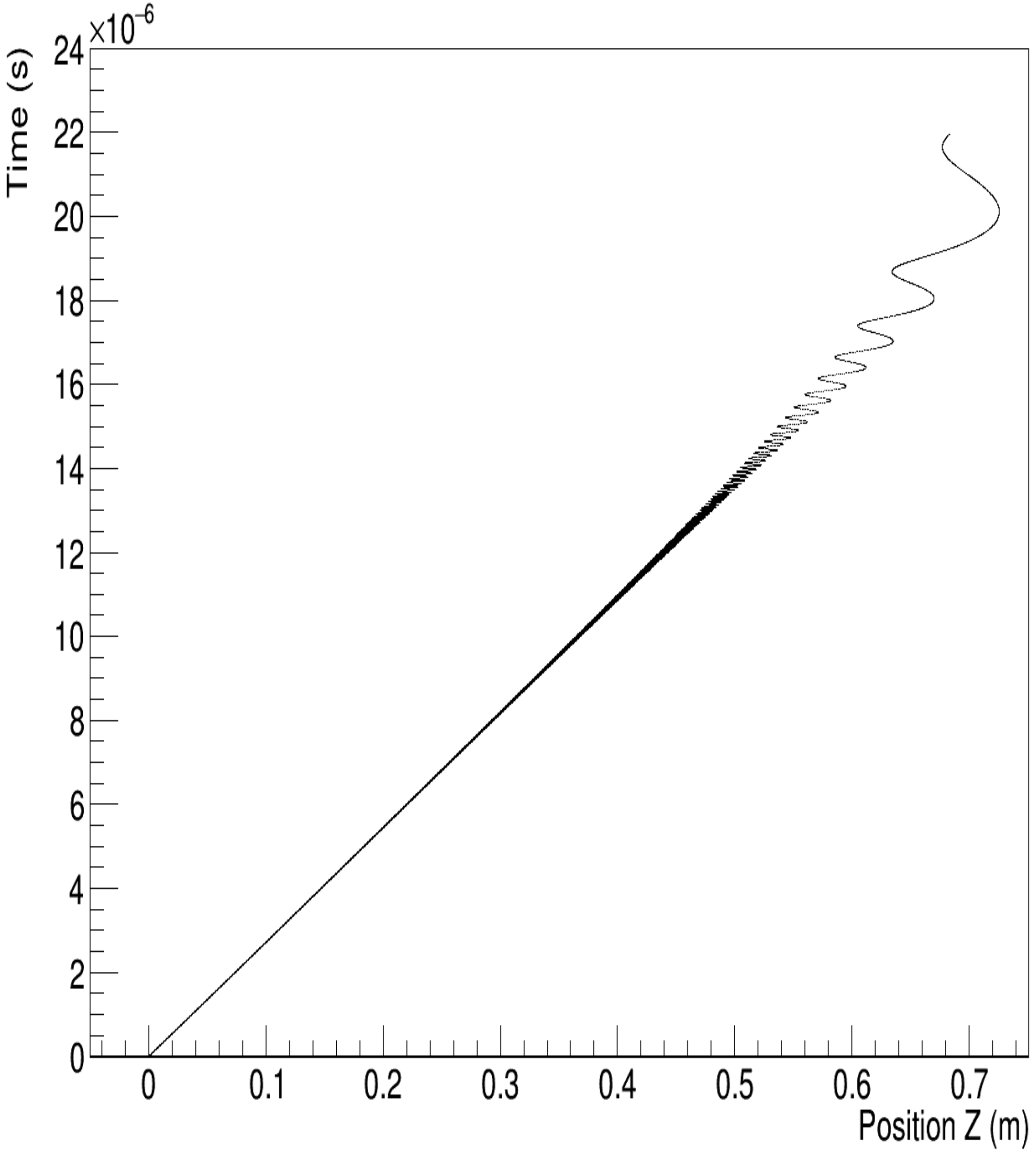
Visualization of where filter would be in analytic fields





B = 1 Tesla   Higher precision possible with higher B -> smaller cyclotron radius





Next studies

- Develop ways to handle a range of incoming kinetic energies for one filter element, at different entrance heights
  - Presented fields are for center line
  - Could stagger calorimeters at end of filter to handle a range of energies
  - Have separate lambdas for B and E fields to fully specify desired final position/height for a certain initial energy

- Study granularity and precision requirements to build physically