Flavor Mixing, Neutrino Oscillations and Neutrino Masses

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Abstract

We discuss mass matrices with four texture zeros for the quarks and leptons. The three flavor mixing angles for the quarks are functions of the quark masses and can be calculated. The results agree with the experimental data. The texture zero mass matrices for the leptons and the see-saw mechanism are used to calculate the matrix elements of the lepton mixing matrix as functions of the lepton masses. The neutrino masses are calculated: $m_1 \approx 1.4$ meV, $m_2 \approx 9$ meV, $m_3 \approx 51$ meV. The neutrinoless double beta decay is discussed. The effective Majorana neutrino mass, describing the double beta decay, can be calculated - it is about 5 meV. The present experimental limit is 140 meV.

The flavor mixing of the quarks is described by the CKM matrix:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} . \tag{1}$$

The absolute values of the nine matrix elements have been measured in many experiments:

$$U_{CKM} => \begin{pmatrix} 0.974 & 0.224 & 0.004 \\ 0.218 & 0.997 & 0.042 \\ 0.008 & 0.040 & 1.019 \end{pmatrix} . \tag{2}$$

There are several ways to describe the CKM-matrix in terms of three angles and one phase parameter. I prefer the parametrization, which Z. Xing and I introduced years ago (ref.(1)), given by the angles θ_u, θ_d, θ and a phase parameter ϕ, which describes CP violation:

$$V_{CKM} = \begin{pmatrix} c_u & s_u & 0 \\ -s_u & c_u & 0 \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} e^{-i\phi} & 0 & 0 \\ 0 & c & s \\ 0 & -s & c \end{pmatrix} \times \begin{pmatrix} c_d & -s_d & 0 \\ s_d & c_d & 0 \\ 0 & 0 & 1 \end{pmatrix} . \tag{3}$$

Here we used the short notation: $c_{u,d} \sim \cos\theta_{u,d}$, $s_{u,d} \sim \sin\theta_{u,d}$, $c \sim \cos\theta$ and $s \sim \sin\theta$.

Relations between the quark masses and the mixing angles can be derived, if the quark mass matrices have "texture zeros", as shown by S. Weinberg and me in 1977 (ref.(2)). For six quarks the mass matrices have four "texture zeros":

$$M = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & B \\ 0 & B^* & C \end{pmatrix} . \tag{4}$$

We can now calculate the angles θ_u and θ_d as functions of the mass eigenvalues:

$$\theta_d \simeq \sqrt{m_d/m_s}, \qquad \theta_u \simeq \sqrt{m_u/m_c}. \tag{5}$$

Using the observed masses for the quarks, we find for these angles:

$$\theta_d \simeq (13.0 \pm 0.4)^\circ, \qquad \theta_u \simeq (5.0 \pm 0.7)^\circ. \tag{6}$$

The experimental values agree with the theoretical results:

$$\theta_d \simeq (11.7 \pm 2.6)^\circ, \qquad \theta_u \simeq (5.4 \pm 1.1)^\circ. \tag{7}$$

There is a relation between the four heavy quark masses and V_{cb}:

$$V_{cb} \cong \sqrt{m_s/m_b} - \sqrt{m_c/m_t}. \tag{8}$$

We use the following values for the quark masses:

$$m_s \simeq 0.08\ GeV,\ m_b \simeq 4.7\ GeV,\ m_c \simeq 1.3\ GeV,\ m_t \simeq 172\ GeV. \tag{9}$$

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In this case we find $V_{cb} \cong 0.043$. This value agrees with the experimental result: $0.039 < V_{cb} < 0.043$.

In the Standard Theory of particle physics the neutrinos do not have a mass. But a mass term can be introduced analogous to the mass term for the electrons. Nevertheless the masses of the neutrinos must be very small, much smaller than the mass of the electron. According to the limit from cosmology the sum of the neutrino masses must be less than 0.23 eV.

If the neutrinos have a small mass and if they are superpositions of mass eigenstates, there would be also a flavor mixing of the leptons. An electron neutrino, emitted from a nucleus, can turn into a muon neutrino after travelling a certain distance. Afterwards it would again become an electron neutrino, etc. Thus neutrinos oscillate (see ref.(3)).

The flavor mixing of the leptons is described by a unitary 3x3-matrix, which is similar to the CKM-matrix for the quarks:

$$U = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 2} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix}. \tag{10}$$

This matrix can be described by three angles and a phase parameter. Here we use the standard parametrization, given by the three angles θ_{12}, θ_{13} and θ_{23}.

In the nuclear fusion on the sun many electron neutrinos are produced. In 1963 John Bahcall calculated the flux of the solar neutrinos. He concluded that this flux could be measured by experiments. Raymond Davis prepared such an experiment. It was placed in the Homestake Gold Mine in Lead, South Dakota and took data from 1970 until 1994. One observed only about 1/3 of the flux, calculated by Bahcall. Thus there were problems with the solar neutrinos, or the calculation of the flux by Bahcall was wrong. Today we know that the reduction of the solar neutrino flux is due to neutrino oscillations.

In the Japanese Alps, near the small village “Kamioka”, a big detector was built in 1982. It is located about 1000 m underground. This detector “Kamiokande” was built in order to find the hypothetical decay of a proton. Thus far no proton decay has been observed, but the detector can also be used to study neutrinos, in particular the atmospheric neutrinos, produced by the decay of pions in the upper atmosphere.

In 1996 a new detector “Superkamiokande” started to investigate these neutrinos. This detector consists of a water tank, containing 50 000 liters of purified water, surrounded by about 11 000 photo multipliers. With this detector one could measure the flux of the neutrinos. The flux of neutrinos, coming from the atmosphere above Kamioka, was as high as expected, but the flux of the neutrinos, coming from the other side of the earth, was only about 50% of the expected rate.

Afterwards a neutrino beam, sent from the KEK laboratory near Tsukuba towards Kamioka, was investigated. Again the flux of muon neutrinos was less than expected. Oscillations between the muon neutrinos and the tau neutrinos could explain the observed reduction of the flux. These oscillations are described by the angle θ_{23}. According to the experiments this angle is very large:

$$40,3^\circ \leq \theta_{23} \leq 52,4^\circ. \tag{11}$$

In Canada a neutrino detector was built near Sudbury (Ontario), the

Sudbury Neutrino Observatory (SNO). With this detector one could observe the solar neutrinos. An solar neutrino hits a deuteron, which splits up into two protons and an electron - this process can be observed. Furthermore it was possible to observe the neutral current interaction of the neutrinos. If a solar neutrino collides with a deuteron, it splits up into a proton and a neutron. Also this reaction can be observed.

The neutral current interaction is not affected by oscillations, since all neutrinos have the same neutral current interaction. However oscillations can be observed for the charged current interaction. An electron neutrino, which becomes a muon neutrino, will not produce an electron after colliding with a nucleus. By comparing the interaction rates for the neutral and for the charged current interactions one has observed the oscillations of the solar neutrinos. For the corresponding mixing angle θ_{12} one finds:

$$31,6^\circ \leq \theta_{12} \leq 36,3^\circ. \tag{12}$$

Nuclear reactors emit electron antineutrinos. These neutrinos have been investigated at a few nuclear reactors, e,g, at the CHOOZ reactor in Belgium and afterwards at the Daia Bay reactors in China. Here neutrino oscillations have been observed, and one could measure the mixing angle θ_{13}:

$$8,2^\circ \leq \theta_{13} \leq 9,0^\circ. \tag{13}$$

Also the two small mass differences between the three neutrinos have been measured. The mass difference between the first and the second neutrino is about 0.0086 eV, and the mass difference between the second and the third neutrino is about 0.05 eV.

The neutrino masses are very small, and the question arises, if the neutrino masses are different from the Dirac masses of the charged leptons. Since the neutrinos are neutral, the neutrino masses might be Majorana masses. The smallness of the neutrino masses can be understood by the "seesaw"-mechanism. The mass matrix of the neutrinos is a matrix with one "texture zero" in the (1,1)-position. The two off-diagonal terms are given by the Dirac mass term D - a large Majorana mass term is in the (2,2)-position:

$$M_\nu = \begin{pmatrix} 0 & D \\ D & M \end{pmatrix} . \tag{14}$$

After diagonalization one obtains a large Majorana mass M and a small neutrino mass:

$$m_\nu \simeq D^2/M. \tag{15}$$

Now we assume that the Dirac mass matrices of the leptons also have four texture zeros:

$$M_D = \begin{pmatrix} 0 & A & 0 \\ A^* & 0 & C \\ 0 & C^* & D \end{pmatrix} . \tag{16}$$

In the seesaw formula we replace the Dirac mass by the texture zero mass matrix M_D and the Majorana mass by a Majorana mass matrix M_D:

$$M_\nu = M_{\rm D}^T M_{\rm R}^{-1} M_D. \tag{17}$$

Since the Majorana masses are much larger than the masses of the leptons and quarks, we assume, that the Majorana mass matrix is proportional to the unit matrix. In this case the mixing angles are functions of the ratios of the charged lepton masses and of the neutrino masses.

But the mass ratios of the charged leptons are very small and cannot give large mixing angles. These angles must be related to large ratios of the neutrino masses (ref.(4,5,6,7)). In first approximation we can neglect the mass ratios of the charged leptons and can calculate the matrix elements of the mixing matrix, in the particular those matrix elements, mentioned below:

$$\begin{aligned} |U_{e2}| &\cong \left(\frac{m_1}{m_2}\right)^{1/4} , \\ |U_{\mu 3}| &\cong \left(\frac{m_2}{m_3}\right)^{1/4} , \\ |U_{e3}| &\cong \left(\frac{m_2}{m_3}\right)^{1/2} \left(\frac{m_1}{m_3}\right)^{1/2} . \end{aligned} \tag{18}$$

We use these relations and the experimental results for the mixing angles to determine the three neutrino masses:

$$\begin{aligned} m_1 &\simeq 1.4\ meV, \\ m_2 &\simeq 9\ meV, \\ m_3 &\simeq 51\ meV. \end{aligned} \tag{19}$$

One expects that the Dirac term D is similar to the corresponding charged lepton mass. For example, let us consider the tau lepton and its neutrino. If D is given by the tau lepton mass and the corresponding neutrino mass is 51 meV, we obtain for the heavy Majorana mass M:

$$M \simeq 6.3 \times 10^{10}\ GeV. \tag{20}$$

The only way to test the nature of the neutrino masses is to study the neutrinoless double beta decay, which violates lepton number conservation. Two neutrons inside an atomic nucleus decay by emitting two electrons and two neutrinos. The two Majorana neutrinos annihilate - only two electrons are emitted. The annihilation rate is a function of the Majorana mass of the neutrino.

If neutrinos mix, all three neutrino masses will contribute to the decay rate. Their contributions are given by the masses of the neutrinos and by the mixing angles. Using the neutrino masses and the observed mixing angles, one finds for the effective neutrino mass, relevant for the neutrinoless double beta decay:

$$\widetilde{m} \simeq 5\ meV. \tag{21}$$

In various experiments one has searched for the neutrinoless double beta decay, for example for the decay of tellurium. Thus far the decay has not been observed. Here is the present limit for this effective mass, given by the Cuore and the Gerda experiments in the Gran Sasso Laboratory:

$$\widetilde{m} < 140\ meV. \tag{22}$$

This limit is about thirty times larger than the expected value - thus it will be very difficult to find the neutrinoless double beta decay.

Conclusions: Using 4 texture zeros for the mass matrices of the quarks and leptons, we derived relatons between the flavor mixing angles and the mass ratios of the leptons and quarks. For the quarks the results agree with the experimental values. Using the observed mixing angles of the leptons, we calculated the three neutrino masses. The effective neutrino mass, describing the neutrinoless double beta decay, is much smaller than the present limit of the experiments.

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