Galactic Rotation Curves vs. Ultra-Light Dark Matter

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1805.00122: Bar, Blas, KB, Sibiryakov
1903.03402: Bar, KB, Sato, Eby

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What is dark matter?
Light (pseudo-)scalar fields are featured in many UV models, where they arise as PNGBs of spontaneously broken symmetries.

The PQ- or QCD-axion is an example.

Such scalar field, initially displaced from a minimum of its potential during the early cosmological history, begins to oscillate around the minimum when $H \sim m$.

Correct cosmological equation of state for dark matter.
Natural initial condition: \( \phi \sim f \) = range of Goldstone boson.

Assuming SSB before end of inflation, contribution to energy density today:

\[
\Omega_m \sim 0.1 \left( \frac{m}{10^{-22} \text{eV}} \right)^{\frac{1}{2}} \left( \frac{f}{10^{17} \text{GeV}} \right)^2
\]

Ultra-light dark matter (ULDM)
On scales much larger than de Broglie wavelength, ULDM behaves like WIMP DM.

dB length \( \sim 100 \text{ pc} \) for \( m \sim 10^{-22} \text{ eV} \)
On scales of order de Broglie wavelength, **ULDM is markedly different than WIMPs.**

$dB \text{ length} \sim 100 \text{ pc for } m \sim 10^{-22} \text{ eV}$
Numerical simulations: inner part of galaxies develops a core ("soliton")
On scales of order de Broglie wavelength, equations of motion of ULDM are simple enough to solve

\[ \partial_r^2 (r \chi) = 2r (\Phi - \gamma) \chi, \]
\[ \partial_r^2 (r \Phi) = r \chi^2. \]
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On scales of order de Broglie wavelength, equations of motion of ULDM are simple enough to solve

\[
\frac{\partial^2}{\partial r^2} (r \chi) = 2r (\Phi - \gamma) \chi, \\
\frac{\partial^2}{\partial r^2} (r \Phi) = r \chi^2.
\]

...nothing like this for WIMPs!
Numerical simulations: soliton — halo relation

\[ M_c \approx \alpha \left( \frac{|E_h|}{M_h} \right)^{\frac{1}{2}} \frac{M_{pl}^2}{m} \]
\[ \alpha = 1 \]
Numerical simulations: soliton — halo relation

**What it says:**

\[
\frac{K}{M} \bigg|_{\text{soliton}} = \frac{K}{M} \bigg|_{\text{halo}}
\]

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M_c \approx \alpha \left( \left| \frac{E_h}{M_h} \right| \right)^{\frac{1}{2}} \frac{M_{pl}^2}{m}
\]

\[
\alpha = 1
\]
Rotation curves from simulations: velocity bump in inner galaxy

- Refs. [9] (Schive 2014) and [29] (Chan 2017).
- FIG. 4. Comparison of the prediction of Eq. (49) (dashed lines) to the numerical simulation results (solid lines), use it instead of max of the numerically extracted halo rotation curves (solid lines). It gives the correct soliton peak rotation.
- We should roughly equal the peak circular velocity in the DM-dominated galaxies, the soliton peak circular velocity.
- In other works, we do not pursue it further, one reason being that this range of small details is not enough to resolve the discrepancy highlighted by the simulations of [9, 10] and above the minimal contribution predicted by Eq. (49), for which high-resolution kinematical data is available.
- In Fig. 5 we show the rotation curves of four dwarf galaxies taken from Ref. [24] (see Ref. [25] for a recent file). Building on Eq. (35), we expect in general that for a host halo. This limit, where the galaxies are entirely composed of a single giant soliton, was considered.
- We choose to do so by examining the rotation for deriving the soliton profile would suffer from the detailed halo shape, but it relieves the soliton
- We ignore any details of the shape of the host halo. As we have learned from the NFW analysis, this prescription is correct, then ULDM in the mass range of about a factor of two in their Eq. (34) between simulations of the range of the measurement; this means that our soliton bump, derived from the peak velocity actually seen in the data, underestimate the range of the measurement.
$m = 1 \times 10^{-22} \text{ eV}$

![Graph showing the distribution of UGC 1281 with $V_{\text{circ}}$ in km/s on the y-axis and $x$ in kpc on the x-axis. The graph indicates a linear increase in $V_{\text{circ}}$ with $x$.](image-url)
Of the 175 galaxies in [25], 160 pass the $m_{\text{gal}}$ cut for $m = 10^{-22}$ eV, and 174 pass it for $m = 10^{-21}$ eV. Next, for each galaxy we determine the observed maximal halo rotation velocity $V_{\text{circ}}$, and use it to compute the soliton prediction from Eq. (49). Our first pass on the data includes only galaxies for which the predicted soliton is resolved, namely, $x_{\text{peak}}$, from Eq. (50), with $V_{\text{circ}} = V_{\text{circ}}$, $h$. For these galaxies, we compute from data the ratio $V_{\text{circ}}$, $\text{obs}$ ($x_{\text{peak}}$) / $V_{\text{circ}}$, $h$. Here, $V_{\text{circ}}$, $\text{obs}$ ($x_{\text{peak}}$) is the measured velocity at the expected soliton peak position. We compute it by averaging the two data points corresponding to measured velocities at $x_{\text{peak}}$. The results of this first pass on the data are shown in Fig. 14. 46 galaxies passed the resolved soliton cut for $m = 10^{-22}$ eV, and 4 galaxies pass it for $m = 10^{-21}$ eV. Including only galaxies with a resolved soliton causes us to lose many relevant rotation curves, with discriminative power. To overcome this, yet maintain a simple analysis, we perform a second pass on the data. Here, we allow galaxies with unresolved soliton, as long as the lowest radius data point is located not farther than $3 \times x_{\text{peak}}$. We need to correct for the fact that the soliton peak velocity is outside of the measurement resolution. To do this, we modify the velocity observable. Bar et al. 1805.00122.
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m=1e-22 eV
m < 1e-21 eV  in tension with observations.
(excludes ULDM from addressing small-scale puzzles of DM.)
Future prospects: e.g., Milky Way

![Graph showing the enclosed mass [M_☉] as a function of radius [r [pc]] with data points and a NFW fit.]

- Ghez 2003
- McGinn 1989
- Fritz 2016
- Lindqvist 1992
- Schodel 2014
- Sofue 2009
- Sofue 2012
- Sofue 2013
- Chatzopoulos 2015
- Deguchi 2004
- Oh 2009
- Trippe 2008
- Gilessen 2008

NFW fit
Future prospects: e.g., Milky Way
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Future前景：例如，银河系
Summary

* ULDM exhibits wave dynamics on scales ~ de Broglie wavelength.
* Lends itself to analytic understanding (*nothing like this for WIMPs*).
* Predicts features in inner kinematics of galaxies.
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Comparable independent constraints from Ly-alpha Forest
Armengaud (1703.09126), Irsic (1703.04683), Zhang (1708.04389), Kobayashi (1708.00015)
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Open questions / work in progress:
Is the soliton—host halo relation correct? (or artefact of numerical simulations?)
If yes, what is the dynamical reason for it?

More observational tests of particle nature of dark matter, based on gravity alone?
Xtra
$m = 1 \times 10^{-22} \text{ eV}$
$m = 1 \text{e-22 eV}$
Rotation curves from simulations

Velocity bump in inner galaxy: implied by soliton—halo relation
**SPARC** Lelli et al, 1606.09251

175 rotation curves

* 3.6um
* HI + Halpha rotation curves

- $\max \frac{V_{\text{bar}}}{V_{\text{DM}}} < 1$
- $\max \frac{V_{\text{bar}}}{V_{\text{DM}}} < 0.5$
- $\max \frac{V_{\text{bar}}}{V_{\text{DM}}} < 0.3$
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The Milky Way: nuclear bulge vs. soliton

- Enclosed mass $M$ vs. radius $r$ in parsecs (pc)
- Logarithmic scale for mass and radius
- Different colors represent different masses: $m=10^{-19}$ eV, $m=10^{-20}$ eV, $m=10^{-21}$ eV, $m=10^{-22}$ eV
- Data points from various studies:
  - Ghez 2003
  - McGinn 1989
  - Fritz 2016
  - Lindqvist 1992
  - Schodel 2014
  - Sofue 2009
  - Sofue 2012
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  - Chatzopoulos 2015
  - Deguchi 2004
  - Oh 2009
  - Trippe 2008
  - Gilesen 2008

NFW fit, Piffl (2015)
The Milky Way: nuclear bulge vs. soliton

there are about $10^9$ stars in there…

The graph shows the enclosed mass versus the radius (r [pc]) for different masses (m) in eV. The data points are labeled with the respective authors and years, including:

- Ghez 2003
- McGinn 1989
- Fritz 2016
- Lindqvist 1992
- Schodel 2014
- Sofue 2009
- Sofue 2012
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A Nuclear Bulge (disc+star cluster) from photometry, Launhardt (2002) is indicated. The NFW fit, Piffl (2015), is also shown.
In our 100 simulations of virialized multi-body mergers, essentially characterised by a single parameter \( \Xi \equiv \frac{|E|}{M^3}/\left(\frac{Gm}{\hbar}\right)^2 \) set by the initial mass and energy (we have assumed no net angular momentum), we do find a fundamental relation between core mass \( M_c \) and \( \Xi \).

\[
M_c/M \approx 2.6 \Xi^{1/3} = 2.6 \left( \frac{|E|}{M^3 (Gm/\hbar)^2} \right)^{1/3},
\]

which reproduces our simulations spanning two orders of magnitude in \( E \), as shown in Fig. 4. More precisely, a nu-
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\[
E = \frac{1}{2} \int d^3x \left( \frac{1}{m^2} |\nabla \psi|^2 + \Phi |\psi|^2 \right)
\]

\[
E_\lambda \approx -0.476 \lambda^3 \frac{M_{pl}^2}{m},
\]

\[
M_\lambda \approx 2.06 \lambda \frac{M_{pl}^2}{m}.
\]

\[
\frac{M_\lambda}{(M_{pl}^2/m)} \approx 2.64 \left| \frac{E_\lambda}{(M_{pl}^2/m)} \right|^{1/3}
\]

\[\Xi = |E|/M^3/(Gm/\hbar)^2\]
Mocz et al 1705.05845

This means that the total energy in the simulation box was eaten up by 1 soliton.

Should not apply to real galaxies above ~1e8 Msol

(initial conditions?)

\[ \frac{M_\lambda}{(M_{pl}^2/m)} \approx 2.64 \left| \frac{E_\lambda}{(M_{pl}^2/m)} \right|^{1/3} \]

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conditions in each of the 100 simulation runs in Ref. [3] contained a number \( N \) of randomly placed initial solitons, with \( N \) ranging between 4 to 32. Each initial soliton was characterised by a core radius \( x_c \) randomly selected (we assume from a uniform distribution) in the range 8 to 50 kpc. Since the mass (energy) of a given soliton is inversely proportional to its core radius (radius cubed), a uniform distribution of \( x_c \) corresponds to a non-uniform distribution of initial soliton masses and energies.

In Fig. 3 we copy the information from Fig. 4 of Ref. [3]: black diamonds show the 100 converged simulation results of [3]. The black dashed line shows the soliton mass-halo energy relation noted in [3]. Now, the x-axis of Fig. 3 is \( \frac{|E_{\text{tot}}|}{M_{\text{tot}}^3} \), so, since \( E_{\text{tot}} \) and \( M_{\text{tot}} \) are conserved quantities, the x-axis location of each simulation run stays fixed between the initial conditions at time \( t = 0 \) and the final state at large \( t \). What moves in time in the simulation is the y-axis coordinate, which shows the central soliton mass normalised to the total conserved mass \( \frac{M_c}{M_{\text{tot}}} \). This y-coordinate grows as the central soliton grows in mass by absorbing ULDM from the surrounding. We can estimate how much vertical movement is actually happening in the simulation, as follows. Given the number of initial solitons \( N \), and assuming that the initial core radii are drawn from a uniform distribution in the range stated by [3], we can produce random sets of \( M_{\text{tot}}, E_{\text{tot}} \) with the corresponding statistics. In Fig. 3 we show the results of such random set generation, for \( N = 4, 6, 12, 20, 30 \) (blue, green, cyan, magenta, red circles). The x-axis locations of the random sets of initial solitons is well defined. On the y-axis, we characterised the initial conditions by taking the mass of the most massive soliton in the set.

Two main points can be seen in Fig. 3. First, the initially most massive solitons in the simulations typically start their life with a mass that is already not far below the mass of the final state evolved configuration. For simulations starting with 4 initial solitons (blue points), the most massive soliton typically needs to grow by a mere factor of 1.5 or less, to achieve its final mass; for 30 initial solitons (red points), the typical growth is a factor of 2 or less. This is a small mass adjustment: the evolved central solitons of [3] must be the result of only mild processing of the initial state. Second, the global properties of the simulation runs depend on the initial number of solitons in the box. Few-soliton systems populate large \( \frac{|E_{\text{tot}}|}{M_{\text{tot}}^3} \), while many-soliton systems populate small \( \frac{|E_{\text{tot}}|}{M_{\text{tot}}^3} \) in the plot. This is not a huge surprise: if [3] had taken a single soliton initial condition, than that simulation would start and end its life at \( \frac{M}{M_{\text{tot}}} = 1 \) and \( \frac{|E_{\text{tot}}|}{M_{\text{tot}}^3} = \frac{1}{2} \). We conclude that the simulations of [3] were constructed such that one (or a small few) initial state soliton – the soliton of initially largest mass – grew to absorb essentially the entire (negative) total energy of the system. To do so, the most massive soliton needed only to grow in mass by a factor of 1.5-2. Fig. 1 in Ref. [3], showing a rendering of one simulation run, appears qualitatively consistent with this picture. This result is qualitatively consistent with the assumption we made in Sec. III B, namely that the soliton will grow to suck in all available negative energy while maintaining mass conservation together with the soliton mass-energy relation. However, the simulations of [3] describe the evolution of a dominant soliton assembling an energetically sub-dominant halo around it. We, on the other hand, are more interested in the scenario of a massive, energetically dominant, MW-like halo, assembling a soliton inside
Non-gravitational interactions

\[ \delta V(\phi) = \frac{\kappa \phi^4}{4} \]

\[ |\kappa| < \frac{2m^2}{x_c^2 \rho_c \phi} \]

\[ |\kappa| < 4 \times 10^{-93} \left( \frac{m}{10^{-22}\text{eV}} \right)^2 \left( \frac{M_h}{10^{12}M_\odot} \right)^{-\frac{2}{3}} \]

\[ V(\phi) = m^2 f^2 \left( 1 - \cos(\phi/f) \right) \]

\[ \kappa = - \frac{m^2}{6f^2} \]

\[ \approx -1.7 \times 10^{-97} \left( \frac{m}{10^{-22}\text{eV}} \right)^2 \left( \frac{f}{10^{17}\text{GeV}} \right)^{-2} \]
Some facts about solitons

Real, free, KG field doing the job of DM

\[ i\partial_t \psi = -\frac{1}{2m} \nabla^2 \psi + m\Phi \psi, \]
\[ \nabla^2 \Phi = 4\pi G|\psi|^2. \]

\[ \phi(x, t) = \frac{1}{\sqrt{2m}} e^{-imt} \psi(x, t) + cc \]
Some facts about solitons

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\[ \nabla^2 \Phi = 4\pi G|\psi|^2. \]

On scales of order de Broglie wavelength:

\[ \psi(x, t) = \left( \frac{mM_{pl}}{\sqrt{4\pi}} \right) e^{-i\gamma mt} \chi(x) \]

\[ \partial_r^2 (r\chi) = 2r (\Phi - \gamma) \chi, \]

\[ \partial_r^2 (r\Phi) = r\chi^2. \]
Some facts about solitons

Continuous family of ground state solutions, characterised by one parameter

Let $\chi_1(r)$ be defined to satisfy $\chi(0) = 1$, vanishing at infinity with no nodes.

$$M_1 = \frac{M_{pl}^2}{m} \int_0^\infty dr r^2 \chi_1^2(r)$$

$$\approx 2.79 \times 10^{12} \left( \frac{m}{10^{-22} \text{eV}} \right)^{-1} M_\odot$$
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\approx 2.79 \times 10^{12} \left( \frac{m}{10^{-22} \text{eV}} \right)^{-1} M_\odot
\]

Other solutions obtained by scaling

\[
\chi_\lambda(r) = \lambda^2 \chi_1(\lambda r), \\
\Phi_\lambda(r) = \lambda^2 \Phi_1(\lambda r), \\
\gamma_\lambda = \lambda^2 \gamma_1,
\]

\[
M_\lambda = \lambda M_1, \\
x_{c\lambda} = \lambda^{-1} x_{c1}
\]
Numerical simulations: soliton — halo relation

What it says:

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M_c \approx \alpha \left( \frac{|E_h|}{M_h} \right)^{\frac{1}{2}} \frac{M_{pl}^2}{m}
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