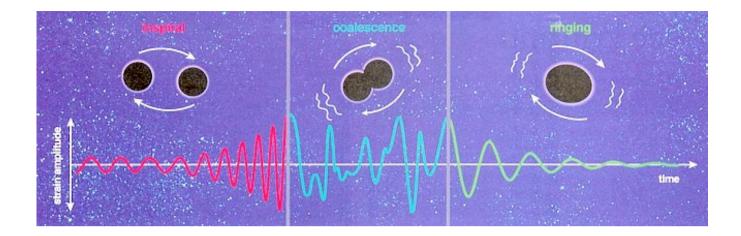
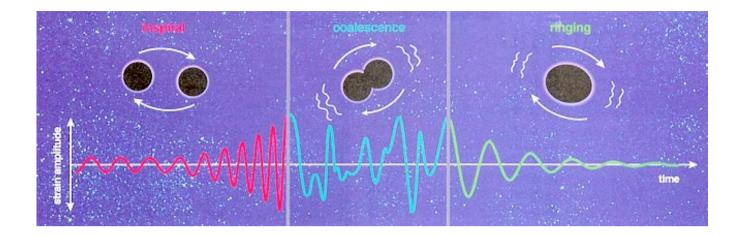
# EFT for Black Hole Quasi Normal Modes: testing extensions to GR with GW

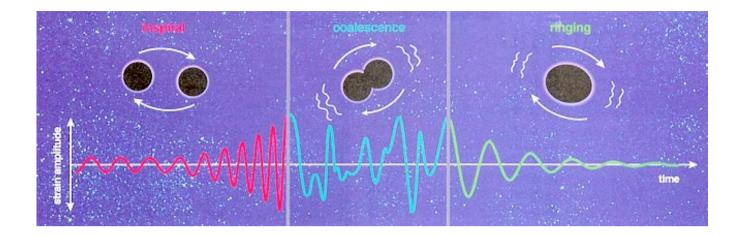
Enrico Trincherini (SNS & INFN, Pisa)

with G. Franciolini, L. Hui, R. Penco & L. Santoni arXiv:1810.07706





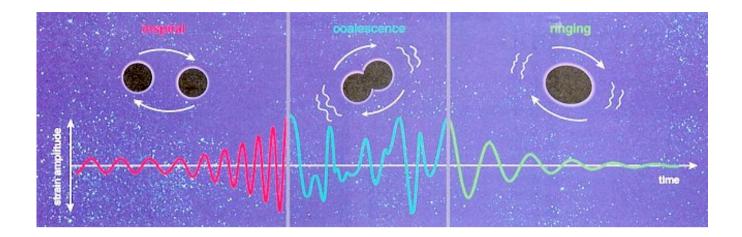
We can detect the presence of new light DOF even if they are NOT coupled to the SM



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DARK ENERGY

DARK MATTER

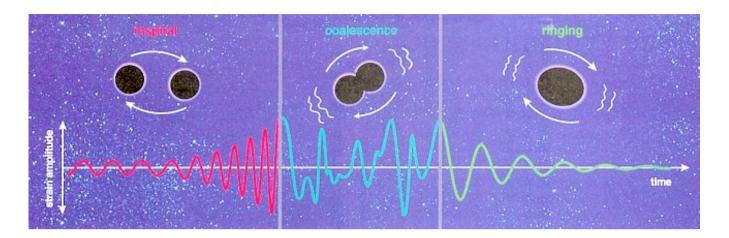


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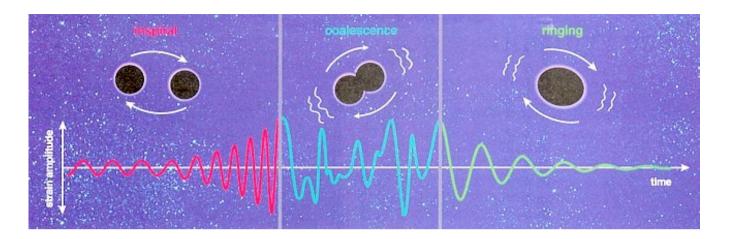
DARK ENERGY

DARK MATTER

Today's perspective: put aside our prejudices in favor of empirical verification

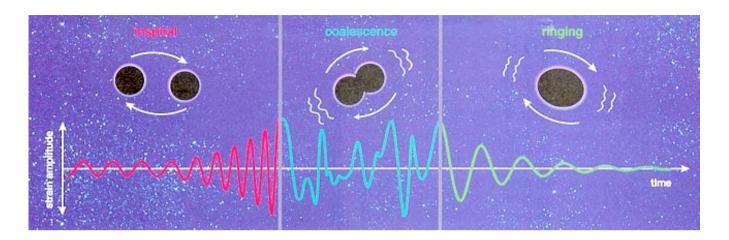


Measure observables and compare with GR



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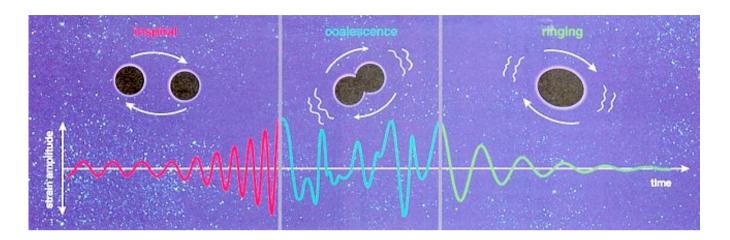
Choose a model BGR and constrain its parameters



Measure observables<br/>and compare with GRPut constraints on the<br/>operators of an EFTChoose a model BGR and<br/>constrain its parameters

With no single 'best- motivated' proposal at hand, useful to resort to the maximally model-independent EFT approach

$$\mathcal{L} = \sum_{n} c_n \frac{\Lambda^4}{g_*^2} \hat{\mathcal{L}} \left( \frac{\partial}{\Lambda}, \frac{g_* \phi}{\Lambda} \right)$$

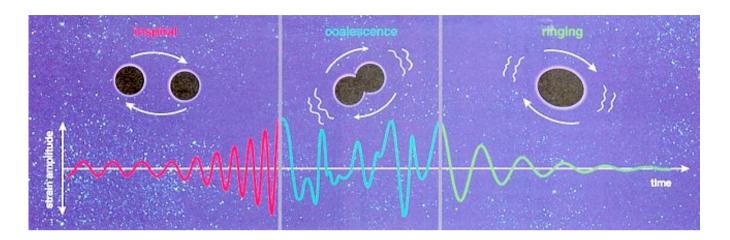


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What are the light degrees of freedom?

Assume the light degrees of freedom are the graviton + 1 extra scalar

On cosmological scales, FRW universes are characterized by a "medium" with a homogeneous and isotropic stress energy tensor

This medium, at variance with a CC, breaks spontaneously Lorentz invariance

The simplest example: in single field Inflation a scalar with a time-dependent expectation value breaks time translations and Lorentz boosts to *ISO(3)* 

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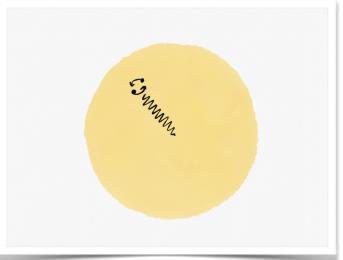
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Lorentz is spontaneously broken: no a priori reason to expect luminal speed



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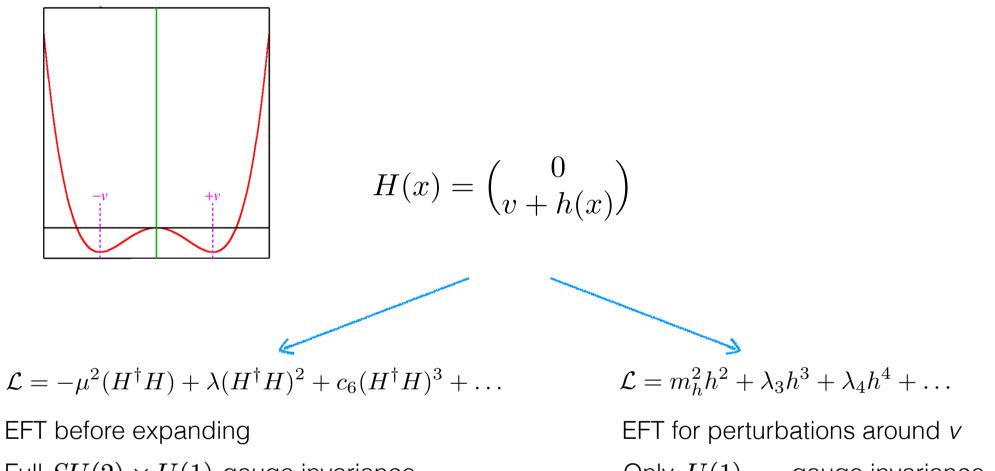
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Two different approaches

"covariant" EFT (trivial background) EFT for perturbations around the relevant solution

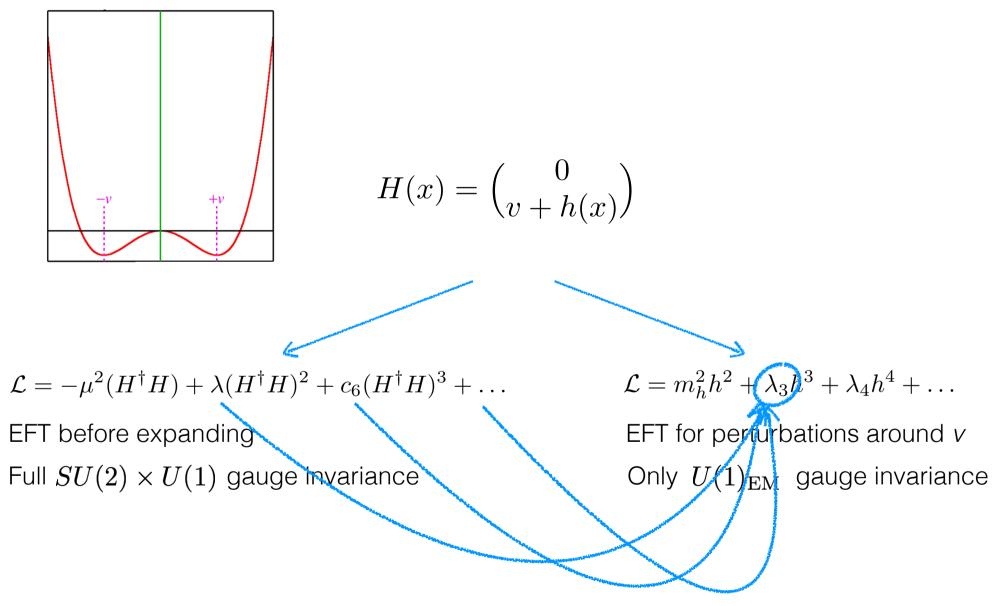
#### The Standard Model EFT



Full  $SU(2) \times U(1)$  gauge invariance

Only  $U(1)_{\rm EM}$  gauge invariance

#### The Standard Model EFT

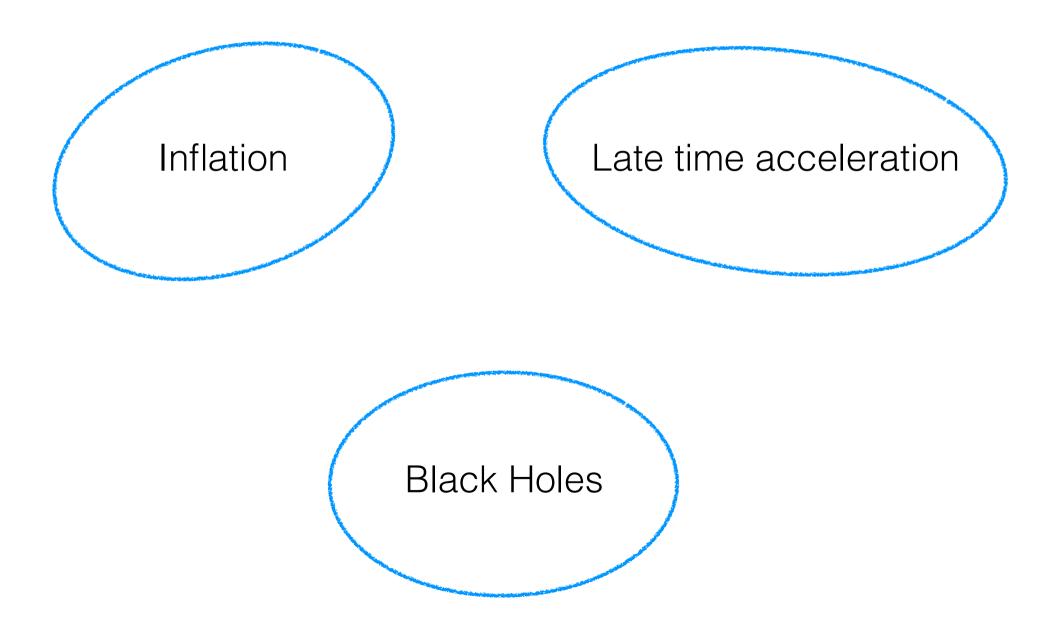


Resum the contribution of many operators if non-linearities are large

EFT around space-time dependent backgrounds



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# The EFT of quasi de Sitter

Start from a background solution  $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ 

Construct an EFT for perturbations

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Start from a background solution  $ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$ 

Construct an EFT for perturbations

Choose a foliation of spacetime (unitary gauge) such that  $\phi = \phi_0(t)$ 

Write down in a derivative expansions all the operators that are invariant under the residual symmetries (spatial diffs)  $x_i \rightarrow x_i + \xi_i(t, \vec{x})$ 

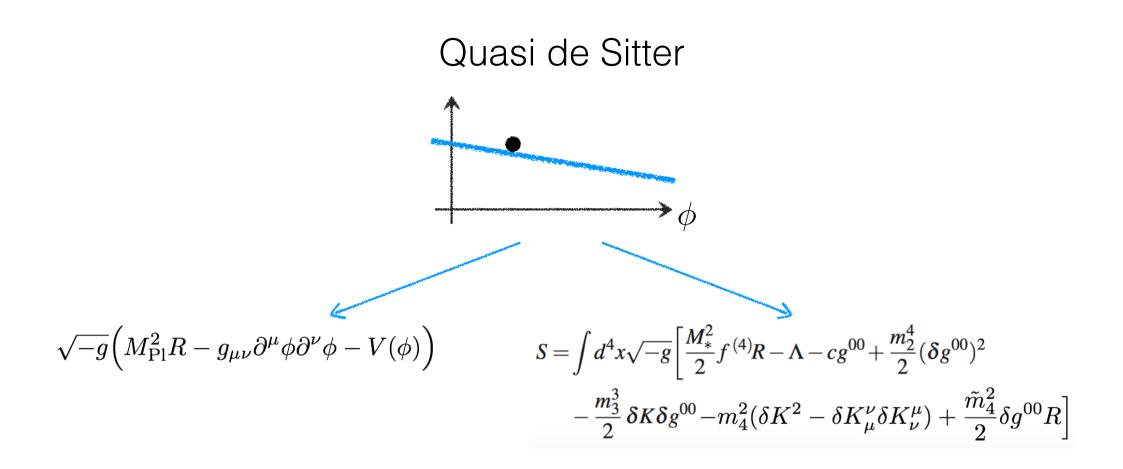
The EFT can contain:

- generic functions of time
- free 0 indices, like  $g^{00}, R^{00}$
- geometric objects of the 3d spatial slices such as  $K^{\mu
  u},\ R^{(3)}$

Write the action already expanded in perturbations, e.g.  $\delta K_{\mu\nu} = K_{\mu\nu} - Hh_{\mu\nu}$ 

$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 (\delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu) + \frac{\tilde{m}_4^2}{2} \delta g^{00} R + \dots \right]$$

Cheung, Creminelli, Fitzpatrick, Kaplan, Senatore '07

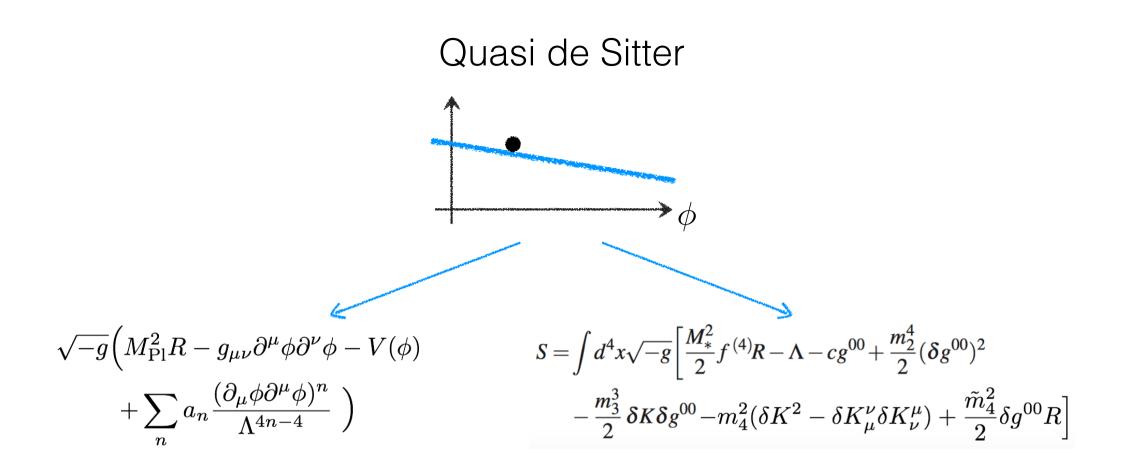


Solve the equation of motion to compute the background

$$\phi(x) = \phi_0(t)$$
$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Expand in small perturbations

 $\mathcal{L} = (\partial \varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$ 

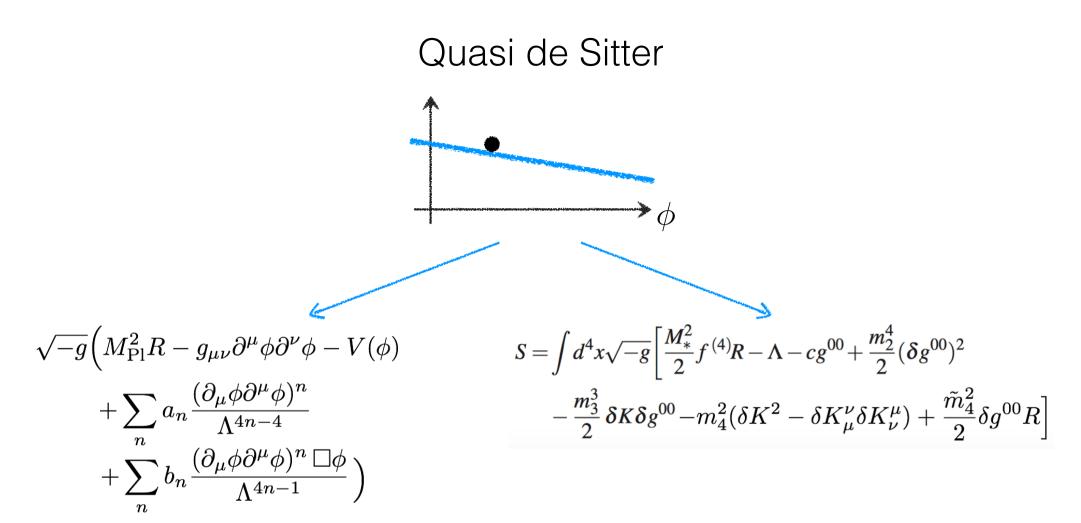


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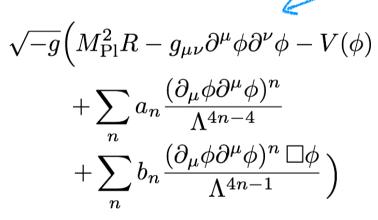
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# Quasi de Sitter



Solve the equation of motion to compute the background

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$$ds^2 = -dt^2 + a(t)^2 d\vec{x}^2$$

Expand in small perturbations

$$\mathcal{L} = (\partial \varphi)^2 + (\partial h)^2 + \mathcal{O}(\varphi^3, h^3, \dots)$$

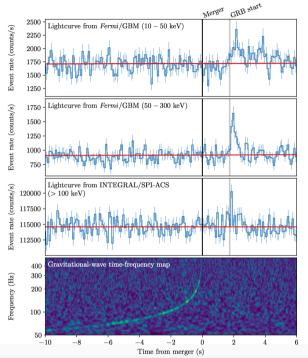
$$\begin{split} S = & \int d^4x \sqrt{-\xi} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{(0)} + \frac{m_2^4}{2} (\delta g^{00})^2 \right] \\ & - \frac{m_3^3}{2} \, \delta K \delta g^{00} - m_4^2 (\delta K^2 - \delta K_\mu^\nu \delta K_\nu^\mu) + \frac{\tilde{m}_4^2}{2} \delta g^{00} R \right] \end{split}$$

 $(\mathcal{D})$ 

These terms are fixed by the background solution Only 3 indep. operators because of FRW symm.

These 4 give linear equations with 2 derivatives for the propagating DOF

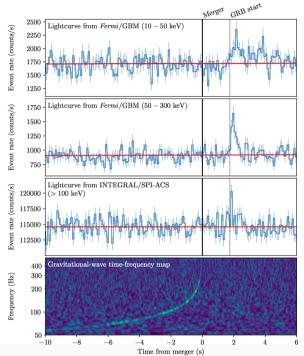




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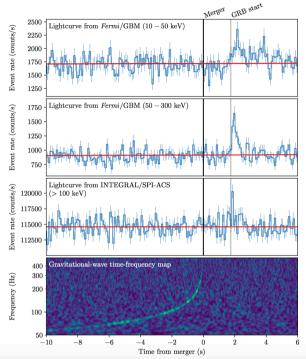
$$S = \int d^4x \sqrt{-g} \left[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 - \frac{m_3^3}{2} \delta K \delta g^{00} - m_4^2 \delta \mathscr{K}_2 + \frac{\tilde{m}_4^2}{2} \delta g^{00} R \right]$$

$$\delta \mathscr{K}_2 \equiv \delta K^2 - \delta K^{\nu}_{\mu} \delta K^{\mu}_{\nu}$$

$$m_4^2 \sim 0$$

Creminelli, Vernizzi '17 Baker *et al '17* 





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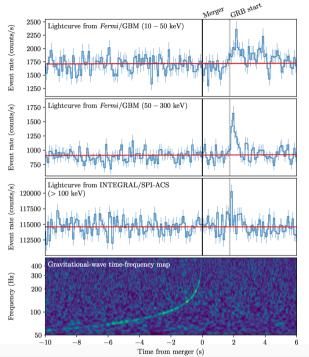
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Depends on the background (dark matter abundance,...): robustly set it to zero!

Creminelli, Vernizzi '17 Baker *et al '17* 





$$\begin{aligned} c_T^2 - 1 &\lesssim 10^{-15} \qquad c_T^2 - 1 = -2m_4^2/M^2 \\ S &= \int d^4 x \sqrt{-g} \Big[ \frac{M_*^2}{2} f^{(4)} R - \Lambda - cg^{00} + \frac{m_2^4}{2} (\delta g^{00})^2 \\ &- \frac{m_3^3}{2} \,\delta K \delta g^{00} - m_4^2 \delta \mathscr{K}_2 + \frac{\tilde{m}_4^2}{2} \,\delta g^{00} R - \frac{m_5^2}{2} \delta g^{00} \delta \mathscr{K}_2 \\ &- \frac{m_6}{3} \,\delta \mathscr{K}_3 - \tilde{m}_6 \delta g^{00} \delta \mathscr{G}_2 - \frac{m_7}{3} \,\delta g^{00} \delta \mathscr{K}_3 \Big] \end{aligned}$$

$$\delta \mathscr{K}_{2} \equiv \delta K^{2} - \delta K_{\mu}^{\nu} \delta K_{\nu}^{\mu} \qquad \delta \mathscr{G}_{2} \equiv \delta K_{\mu}^{\nu} R_{\nu}^{\mu} - \delta K R/2$$
$$\delta \mathscr{K}_{3} \equiv \delta K^{3} - 3\delta K \delta K_{\mu}^{\nu} \delta K_{\nu}^{\mu} + 2\delta K_{\mu}^{\nu} \delta K_{\rho}^{\mu} \delta K_{\nu}^{\rho}$$

$$m_4^2 \sim 0$$
  
 $\tilde{m}_4^2 = m_5^2$   
 $m_6 = \tilde{m}_6 = m_7 = 0$ 

Depends on the background (dark matter abundance,...): robustly set it to zero!

Creminelli, Vernizzi '17 Baker *et al '17 + many others* 

#### Very strong constraints on the Covariant Theory

$$\mathcal{L}=\mathcal{L}(g_{\mu
u},\phi)$$

$$L_{2} \equiv G_{2}(\phi, X) , \qquad L_{3} \equiv G_{3}(\phi, X) \Box \phi ,$$

$$L_{4} \equiv G_{4}(\phi, X)^{(4)}R - 2G_{4,X}(\phi, X)(\Box \phi^{2} - \phi^{\mu\nu}\phi_{\mu\nu})$$

$$+ F_{4}(\phi, X)\varepsilon^{\mu\nu\rho}\sigma \varepsilon^{\mu'\nu'\rho'\sigma}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'} ,$$

$$L_{5} \equiv G_{5}(\phi, X)^{(4)}G_{\mu\nu}\phi^{\mu\nu}$$

$$+ \frac{1}{3}G_{5,X}(\phi, X)(\Box \phi^{3} - 3\Box\phi \phi_{\mu\nu}\phi^{\mu\nu} + 2\phi_{\mu\nu}\phi^{\mu\sigma}\phi^{\nu}\sigma)$$

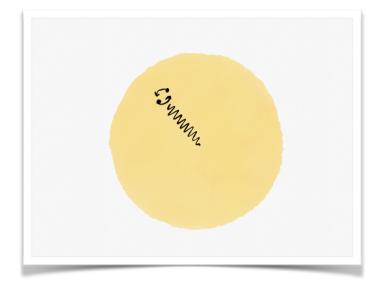
$$+ F_{5}(\phi, X)\varepsilon^{\mu\nu\rho\sigma}\varepsilon^{\mu'\nu'\rho'\sigma'}\phi_{\mu}\phi_{\mu'}\phi_{\nu\nu'}\phi_{\rho\rho'}\phi_{\sigma\sigma'}$$

 $G_{5,X} = 0$ ,  $F_5 = 0$ ,  $2G_{4,X} - XF_4 + G_{5,\phi} = 0$ 

$$X \equiv g_{\mu\nu} \nabla^{\mu} \phi \nabla^{\nu} \phi$$
$$\phi_{\mu} \equiv \nabla_{\mu} \phi$$
$$\phi_{\mu\nu} \equiv \nabla_{\mu} \nabla_{\nu} \phi$$
$$G_{i}(\phi, X) = \sum_{nm} c_{nm} \phi^{n} X^{m}$$

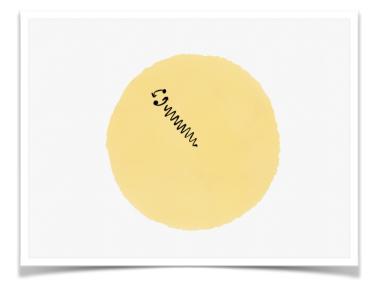
$$L_{c_T=1} = G_2(\phi, X) + G_3(\phi, X) \Box \phi + B_4(\phi, X)^{(4)} R$$
  
$$- \frac{4}{X} B_{4,X}(\phi, X) (\phi^{\mu} \phi^{\nu} \phi_{\mu\nu} \Box \phi - \phi^{\mu} \phi_{\mu\nu} \phi_{\lambda} \phi^{\lambda\nu}) ,$$

 $B_4 \equiv G_4 + XG_{5,\phi}/2.$ 

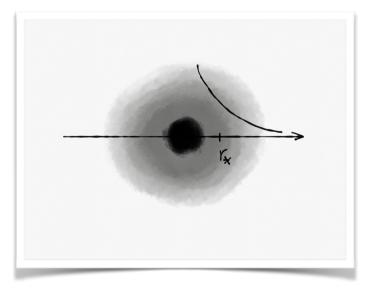




Can we study the propagation around some other background and maybe discover a new field?

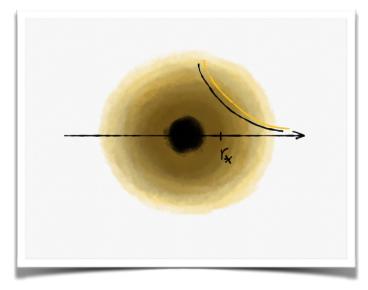


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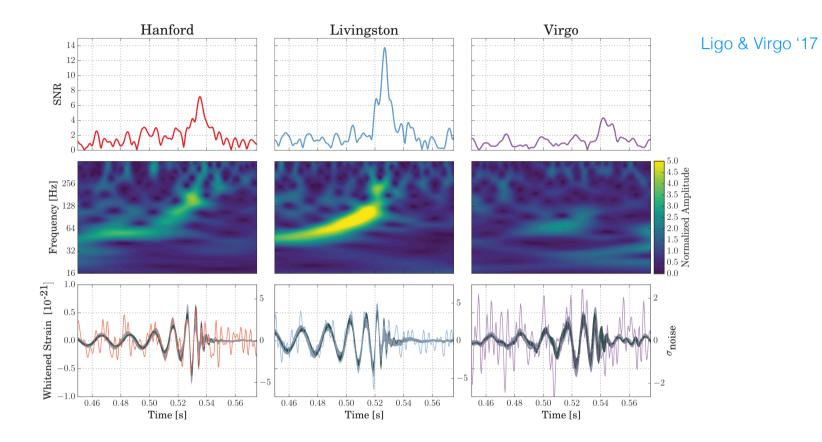


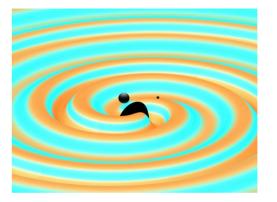


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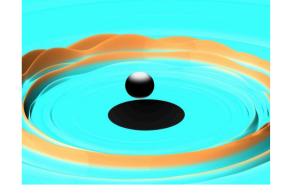
#### Perturbations around Black Holes





Inspiral





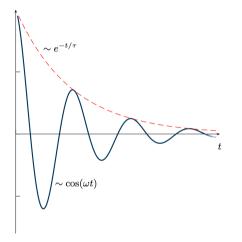
Merger

Ringdown

## Perturbations around Black Holes

Exponentially damped sinusoidal waves: Quasi Normal Modes (QNM)

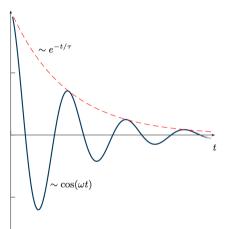
Spectrum of characteristic (complex) frequencies  $\omega_{nlm}$ 



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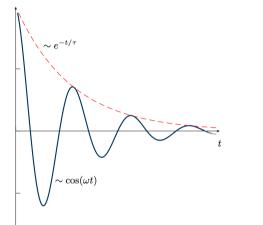
n	$2M_{\bullet}\omega (L=2)$	$2M_{\bullet}\omega \ (L=3)$	$2M_{\bullet}\omega$ ( $L=4$ )
0	0.747 343 + 0.177 925i	1.198 887 + 0.185 406i	1.618 36 + 0.188 32i
1	0.693 422 + 0.547 830i	1.165 288 + 0.562 596i	1.593 26 + 0.568 86i
2	0.602 107 + 0.956 554i	1.103 370 + 0.958 186i	1.545 42 + 0.959 82i
3	0.503 010 + 1.410 296i	1.023 924 + 1.380 674i	1.479 68 + 1.367 84i

In GR black holes are characterized only by 3 parameters: M, J, Q No-hair hypothesis

Nollert '99

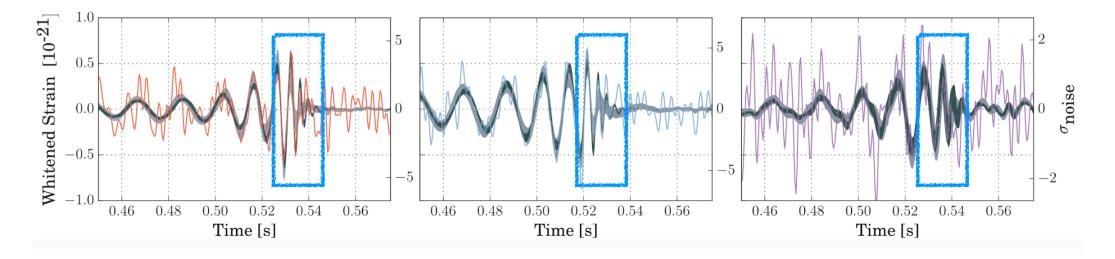
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	-	-	Nollert '99	
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In GR black holes are characterized only by 3 parameters: M, J, Q No-hair hypothesis



$$\begin{split} g_{\mu\nu} &= g_{\mu\nu}^{\rm BH}(r) + h_{\mu\nu} & \text{Schwarzschild: static, spherically symmetric background} \\ h(t,r,\theta,\phi) &= \sum_{lm} h_{lm}(r) Y_{lm}(\theta,\phi) e^{i\omega t} \\ & \text{Classified accordingly to the behavior under parity} \quad (\theta,\phi) \to (\pi - \theta, \phi + \pi) \end{split}$$

Axial (odd) perturbations

Polar (even) perturbations

Regge Wheeler '57

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Axial (odd) perturbations Reage Wheeler '57 Polar (even) perturbations

Fix the gauge + solve for the constraint

One propagating DOF in the odd sector

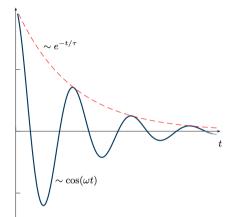
$$\left[\frac{d^2}{dr^2} + \omega^2\right]h(r) = V^{(-)}(r)h(r)$$

$$V^{(-)}(r) = \frac{l(l+1)}{r^2} \left(1 - \frac{r_S}{r}\right) - 3\frac{r_S}{r^3} \left(1 - \frac{r_S}{r}\right)$$

One propagating DOF in the even sector

$$\left[\frac{d^2}{dr^2} + \omega^2\right]h(r) = V^{(+)}(r)h(r)$$

 $V^{(+)}(r) = \dots$ 



—	—	
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In GR quasi-normal modes are isospectral

One propagating DOF in the odd sector

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One propagating DOF in the even sector

Nollert '99

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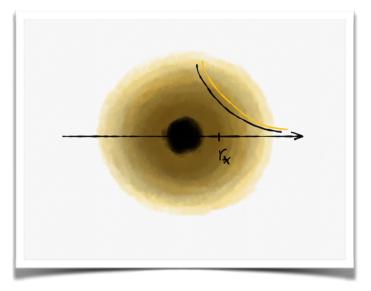
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# EFT for perturbations spher. symm.

The propagation of gravity is different IF BHs have scalar hair

The linearized equations of motion are modified

More information than just the velocity: the whole QNM spectra are modified

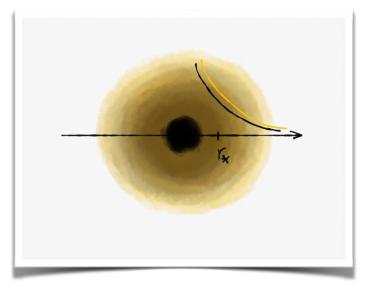


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There are computations of QNM in specific models. Different approach: not case by case instead use an EFT around static and spherically symmetric backgrounds

Assumption: there is a non-trivial r-dependent scalar profile  $\bar{\Phi}(r)$ 

$$ds^{2} = -a^{2}(r)dt^{2} + \frac{dr^{2}}{b^{2}(r)} + c^{2}(r)\left(d\theta^{2} + \sin^{2}\theta d\phi^{2}\right) \qquad \text{Choose unitary gauge } \delta\Phi \equiv 0$$

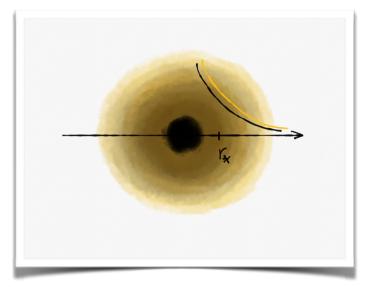
$$S = \int d^4x \sqrt{-g} \mathcal{L}\left(g_{\mu\nu}, R_{\mu\nu\alpha\beta}, g^{rr}, K_{\mu\nu}, \nabla_{\mu}; r\right)$$

# EFT for perturbations spher. symm.

The propagation of gravity is different IF BHs have scalar hair

The linearized equations of motion are modified

More information than just the velocity: the whole QNM spectra are modified



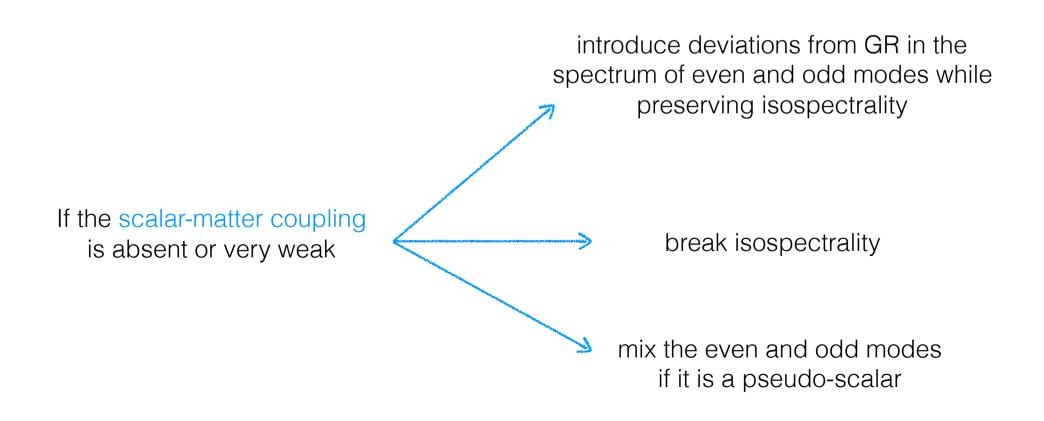
There are computations of QNM in specific models. Different approach: not case by case instead use an EFT around static and spherically symmetric backgrounds

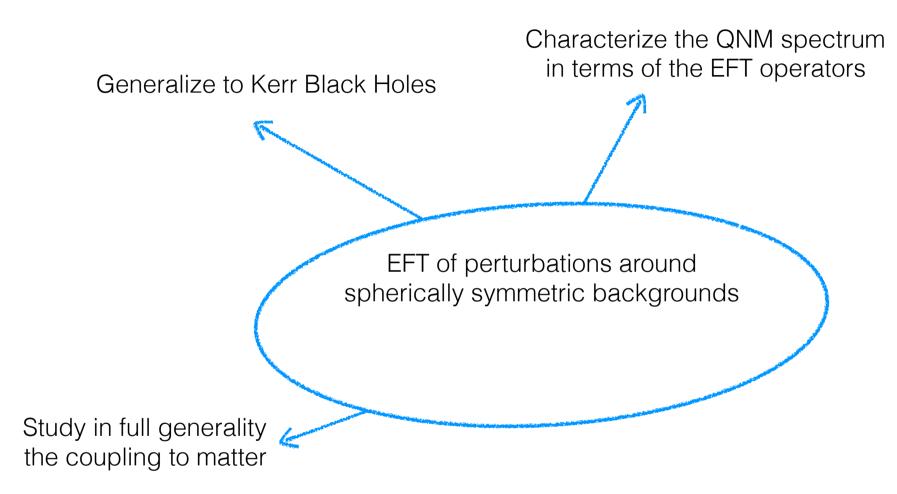
$$\begin{split} S &= \int \mathrm{d}^4 x \, \sqrt{-g} \bigg[ \frac{1}{2} M^2(r) R - \Lambda(r) - f(r) g^{rr} - \alpha(r) \bar{K}_{\mu\nu} K^{\mu\nu} \\ &+ M_2^4(r) (\delta g^{rr})^2 + \hat{M}_1^3(r) \delta g^{rr} \delta K + \hat{M}_2^3(r) \bar{K}_{\mu\nu} \delta g^{rr} \delta K^{\mu\nu} \\ &+ \tilde{M}_1^2(r) (\partial_r \delta g^{rr})^2 + \tilde{M}_2^2(r) (\partial_r \delta g^{rr}) \delta K + \tilde{M}_3^2(r) K_{ab} (\partial_r \delta g^{rr}) \delta K^{\mu\nu} \\ &+ \bar{M}_1^2(r) (\delta K)^2 + \bar{M}_2^2(r) \delta K_{\mu\nu} \delta K^{\mu\nu} + \bar{M}_3^2(r) \bar{K}_{\mu\nu} \delta K \delta K^{\mu\nu} + \bar{M}_4^2(r) \bar{K}_{\mu\nu} \delta K^{\mu\rho} \delta K^{\nu} \rho \\ &+ \bar{M}_5^2(r) \bar{K}_{\mu\rho} \bar{K}_{\nu}^{\rho} \delta K \delta K^{\mu\nu} + \bar{M}_6^2(r) \delta g^{rr} \delta \hat{R} + \bar{M}_7^2(r) \bar{K}_{\mu\nu} \delta g^{rr} \delta \hat{R}^{\mu\nu} + \dots \bigg] \,, \end{split}$$
  
Franciolini, Hui, Penco, Santoni, ET '18

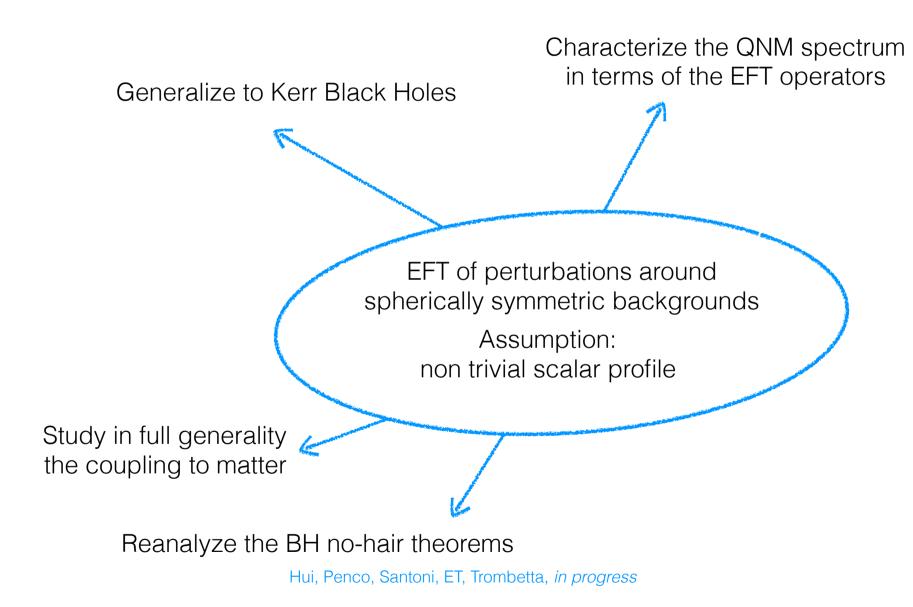
### Phenomenology

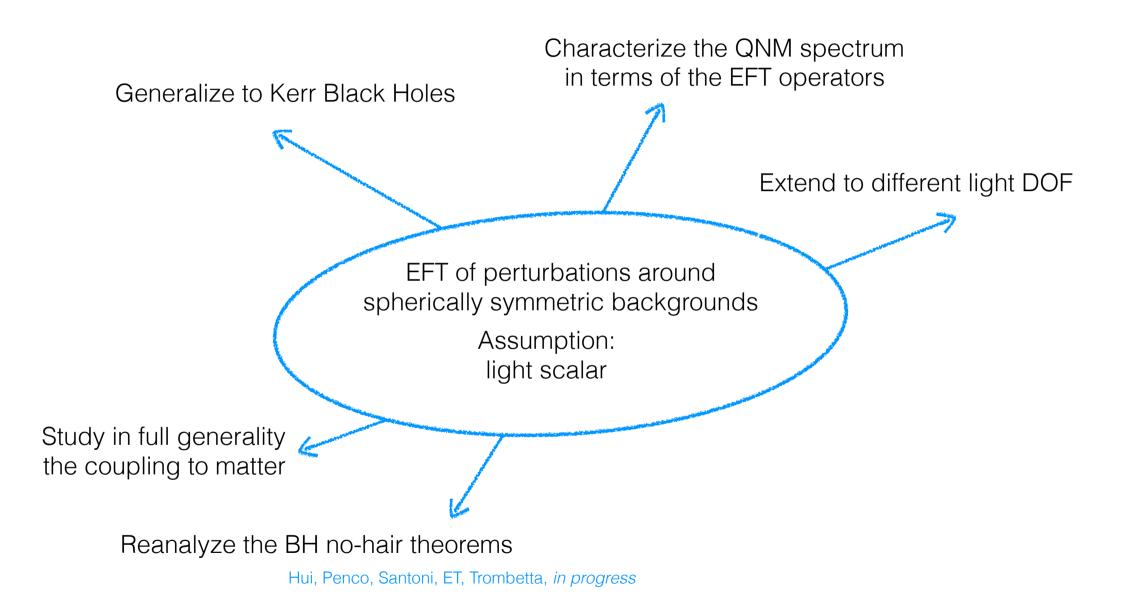
If the strength of the scalar-matter coupling is gravitational or bigger

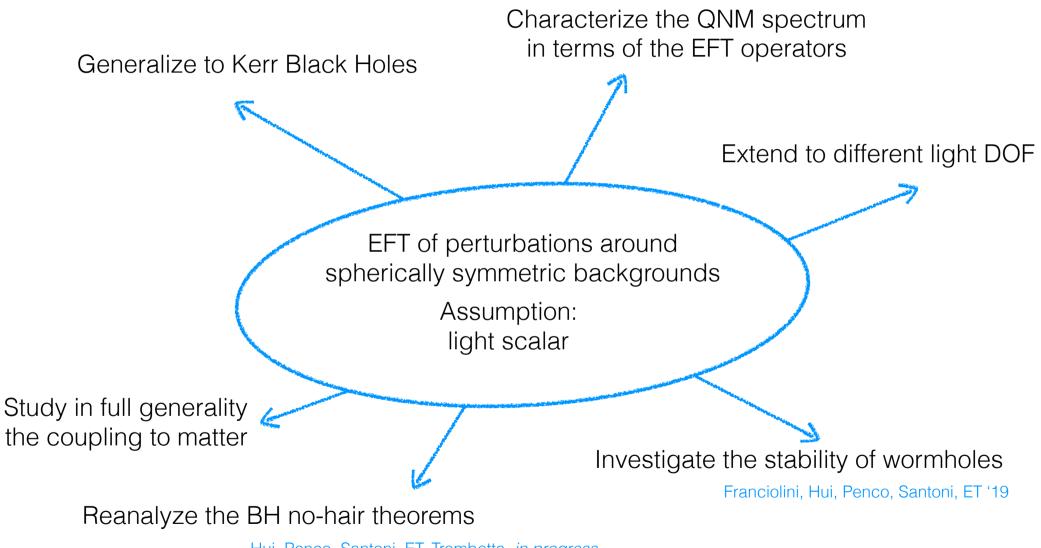
the most prominent observational signal would be the scalar mode itself (the extra mode in the even sector)











Hui, Penco, Santoni, ET, Trombetta, in progress

"[concerning the use of supernovae for cosmology] the optimists were theorists or newcomers who had not worked long in the field, and pessimists (or realists as we prefer to be called) were observers"

Robert Kirshner