Bootstrapped Newtonian gravity
from stars to black holes

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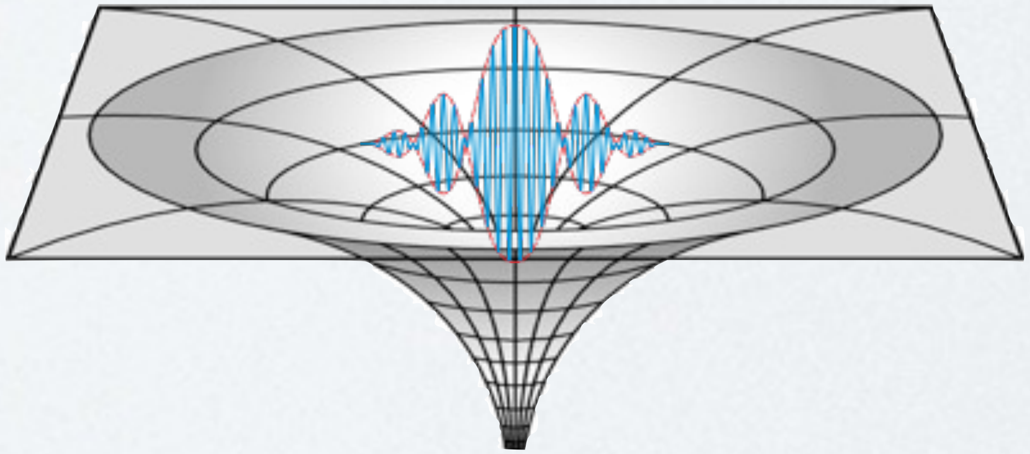
Pisa



Gravitational collapse = Singularities

Theorem: GR predicts (infinitely dense) singularity at the end of the collapse.

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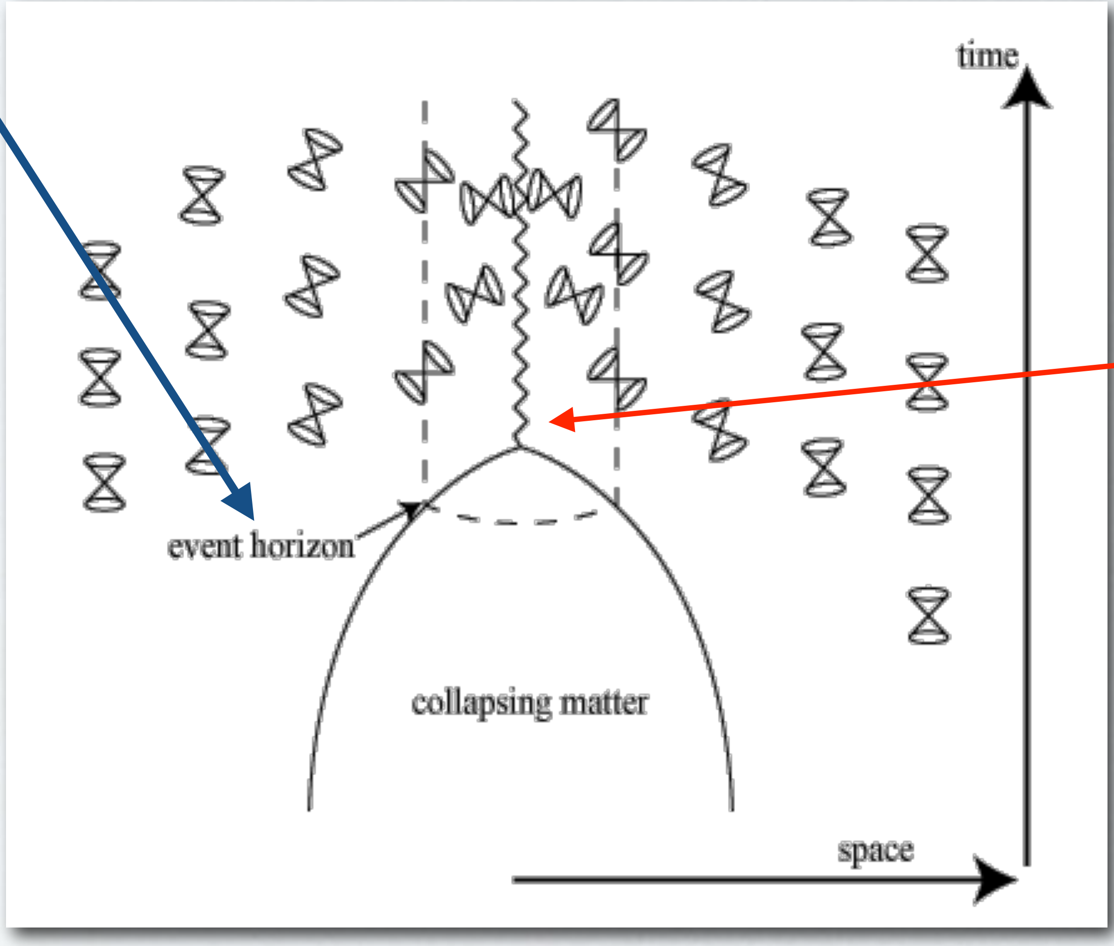
Gravitational Radius



Central Singularity

“John’s interest in relativity was triggered in January 1951, when he studied the 1938-39 work of Robert Oppenheimer and …” (*J. A. Wheeler - 1911-2008*, by Kip S. Thorne)

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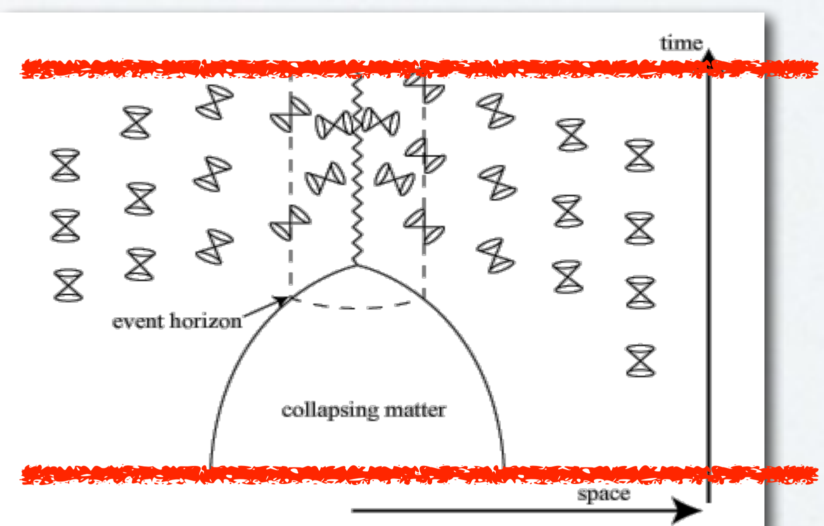
GR must fail (in some regimes)

Lesson from semiclassical theory: new effects already at the horizon scale



$$|0; t = +\infty\rangle$$

$$\uparrow \; e^{-\frac{i}{\hbar}\int \hat{H}\, dt}$$

$$|0; t = -\infty\rangle$$

$|0; t = +\infty\rangle = \sum$ excitations $=$ Hawking radiation

Solvay Congress, June 1958: "... no escape is apparent except to assume that the nucleons at the center of a highly compressed mass must necessarily dissolve away into radiation at such a rate as to keep the total number of nucleons from exceeding a certain critical number." (*J. A. Wheeler - 1911-2008*, by Kip S. Thorne)

Bootstrapped Newtonian Gravity

Newtonian gravity: linear interaction

$$\triangle V_{\rm N} = 4\,\pi\,\frac{\ell_{\rm p}}{m_{\rm p}}\,\rho$$

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Perturbative GR: gravitons self-interact

Einstein-Hilbert action: $$S_{\rm EH} = \int d^4x\,\sqrt{-g}\left(\frac{m_{\rm p}}{16\,\pi\,\ell_{\rm p}}\,\mathcal{R} + \mathcal{L}_{\rm M}\right) \qquad \mathcal{L}_{\rm M} = \rho$$

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1) Weak field
2) Static non-relativistic motion
3) De Donder gauge
4) Fierz-Pauli and some guessing …

$$g_{\mu\nu} = \eta_{\mu\nu} + \epsilon\,h_{\mu\nu}$$

$$h_{\mu\nu} \simeq h_{00} = -2\,V$$

$$0 = 2\,\partial^\mu h_{\mu\nu} - \partial_\nu h \simeq \partial_t V$$

$$S[V] = 4\,\pi \int \epsilon\,dt \int_0^\infty r^2\,dr \left\{\frac{m_{\rm p}}{8\,\pi\,\ell_{\rm p}}\,V\,\triangle V - \rho\,V + \frac{\epsilon}{2}\left[\frac{m_{\rm p}}{4\,\pi\,\ell_{\rm p}}\,(V')^2 + V\,\rho\right] V\right\}$$

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Newtonian Lagrangian

R.C., A. Giugno, A. Giusti, M. Lenzi, PRD 96 (2017) 044010

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Self-interaction

R.C., A. Giugno, A. Giusti, M. Lenzi, PRD 96 (2017) 044010

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R.C., M. Lenzi, O. Micu, arXiv:1904.06752

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$$L_{\mathrm{N}}[V] = -\,4\,\pi \int_0^\infty r^2\,dr \left[\frac{(V')^2}{8\,\pi\,G_{\mathrm{N}}} + \rho\,V\right] \longrightarrow J_V \simeq 4\,\frac{\delta U_{\mathrm{N}}}{\delta \mathscr{V}} = -\,\frac{\left[V'(r)\right]^2}{2\,\pi\,G_{\mathrm{N}}}$$

Pressure contribution $\qquad p \simeq -\,\dfrac{\delta U_{\mathrm{B}}}{\delta \mathscr{V}} = J_{\mathrm{B}}$

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$$\begin{aligned} L[V] &= L_{\rm N}[V] - 4\,\pi\int_0^\infty r^2\,dr\left[q_V\,J_V\,V + q_{\rm B}\,J_{\rm B}\,V + q_\rho\,J_\rho\,(\rho+p)\right] \\ &= -\,4\,\pi\int_0^\infty r^2\,dr\left[\frac{(V')^2}{8\,\pi\,G_{\rm N}}\left(1-4\,q_V\,V\right)+V\left(\rho+q_{\rm B}\,p\right)-2\,q_\rho V^2\,\left(\rho+p\right)\right] \end{aligned}$$

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Couplings (=1)

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Pressure contribution

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Modified Poisson equation:

$$\triangle\,V = 4\,\pi\,G_{\mathrm{N}}\left(\rho + p\right) + \frac{2\,(V')^2}{1 - 4\,V}$$

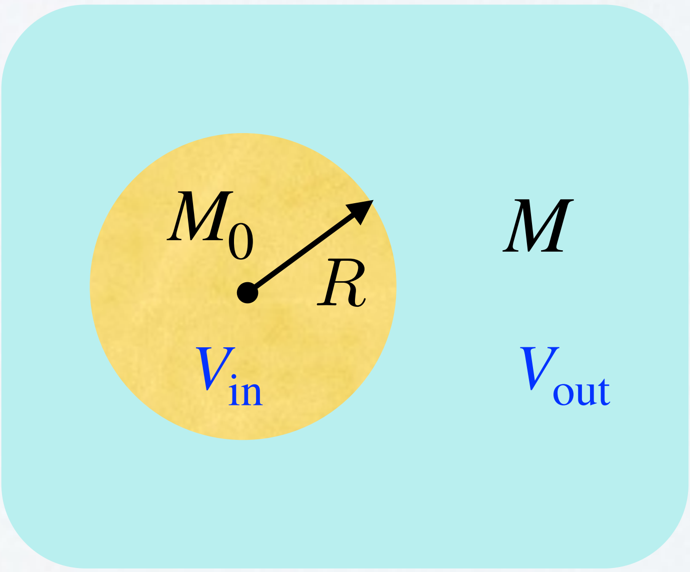
Conservation equation:

$$p' = -\,V'\left(\rho + p\right)$$

R.C., M. Lenzi, O. Micu, arXiv:1904.06752

Bootstrapped Newtonian Stars

Uniform star:



Density: $$\rho = \rho_0 \equiv \frac{3\,M_0}{4\,\pi\,R^3}\,\Theta(R - r)$$

Exact outer potential: $$V_{\text{out}} = \frac{1}{4}\left[1 - \left(1 + \frac{6\,G_{\text{N}}\,M}{r}\right)^{2/3}\right]$$

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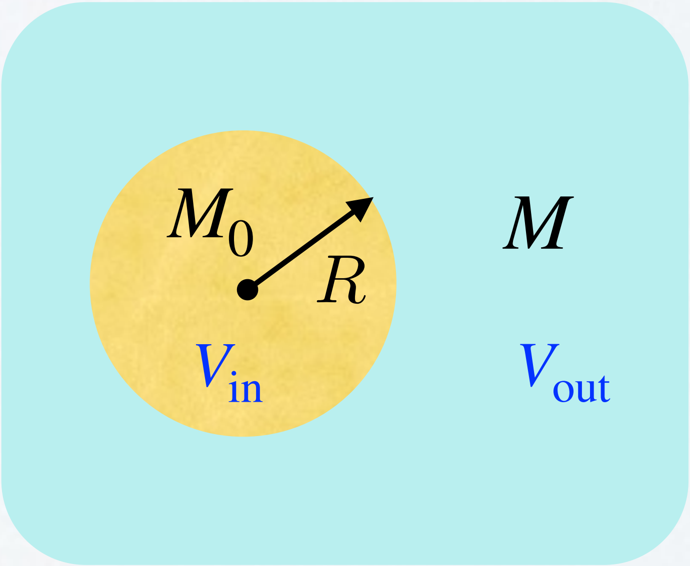

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Small compactness:

$R \gtrsim G_{\rm N}\,M$

$$V_{\rm out} \simeq -\frac{G_{\rm N}\,M}{r}+\frac{G_{\rm N}^2\,M^2}{r^2}$$

Approximate inner potential: $$V_{\rm in} \simeq \frac{R^3\left[\left(1+6\,G_{\rm N}\,M/R\right)^{1/3}-1\right]+2\,G_{\rm N}\,M\left(r^2-4\,R^2\right)}{4\,R^3\left(1+6\,G_{\rm N}\,M/R\right)^{1/3}}$$

Approximate mass relation: $$M_0 \simeq M\left(1-\frac{5\,G_{\rm N}\,M}{R}\right)$$

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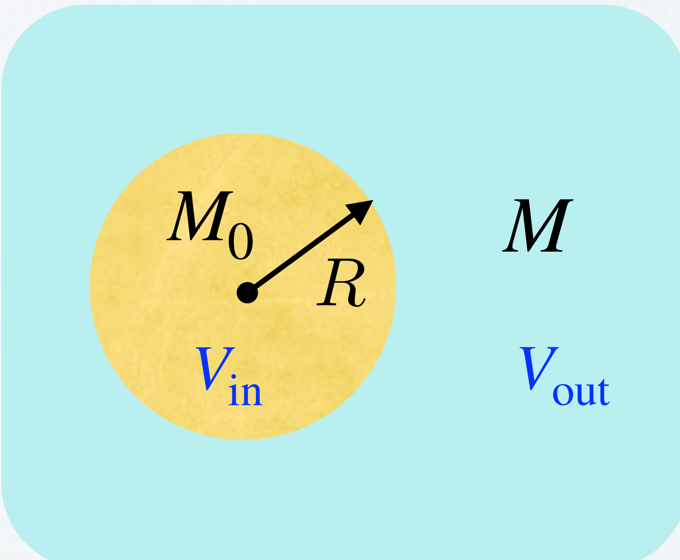

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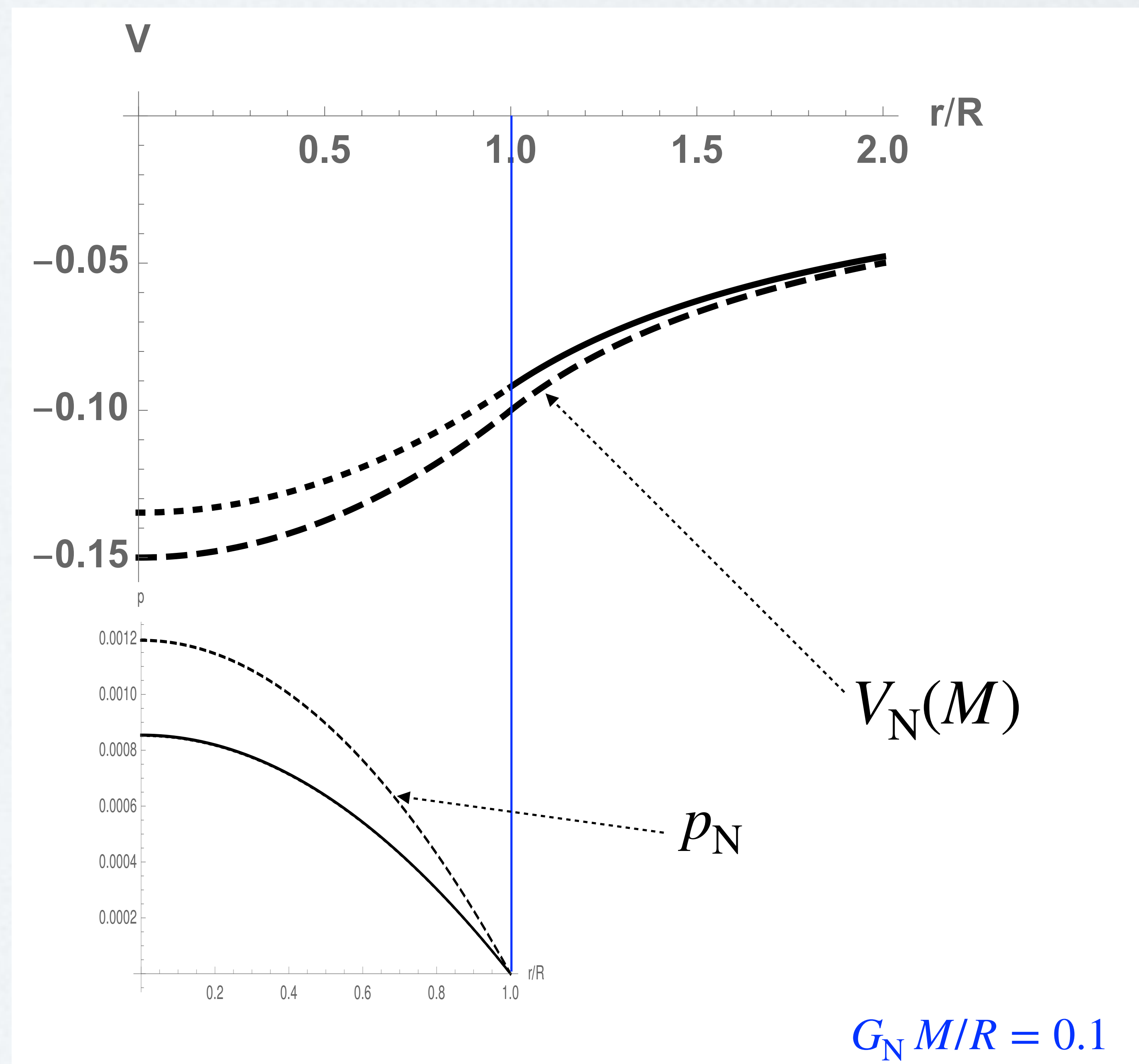

Bootstrapped Newtonian Black Holes

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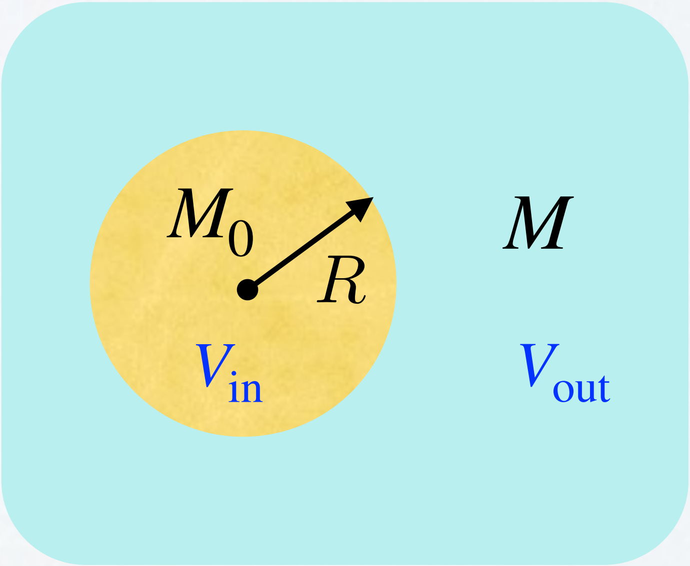

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Large compactness (no Buchdahl limit!):

$$R \ll G_{\rm N}\,M$$

Approximate inner potential: $$V_{\rm in} \simeq V_R + V'_R\,(r-R)$$

Approximate mass relation: $$\frac{M_0}{M} \sim \left(\frac{G_{\rm N}\,M}{R}\right)^{-1/3}$$

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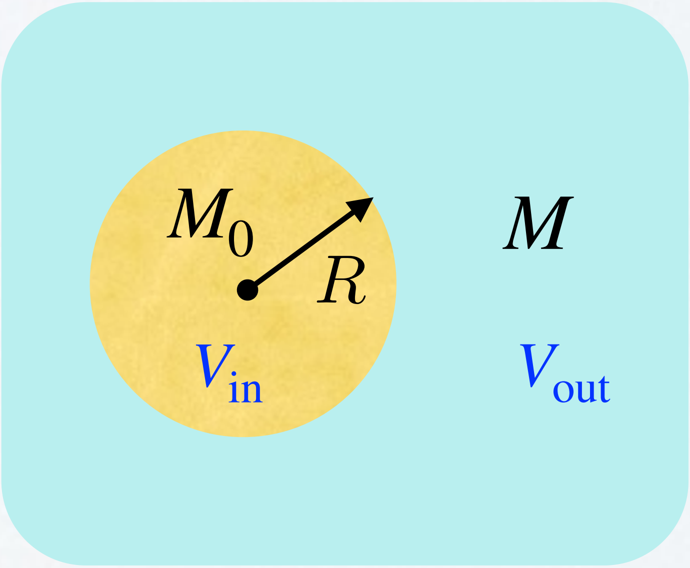

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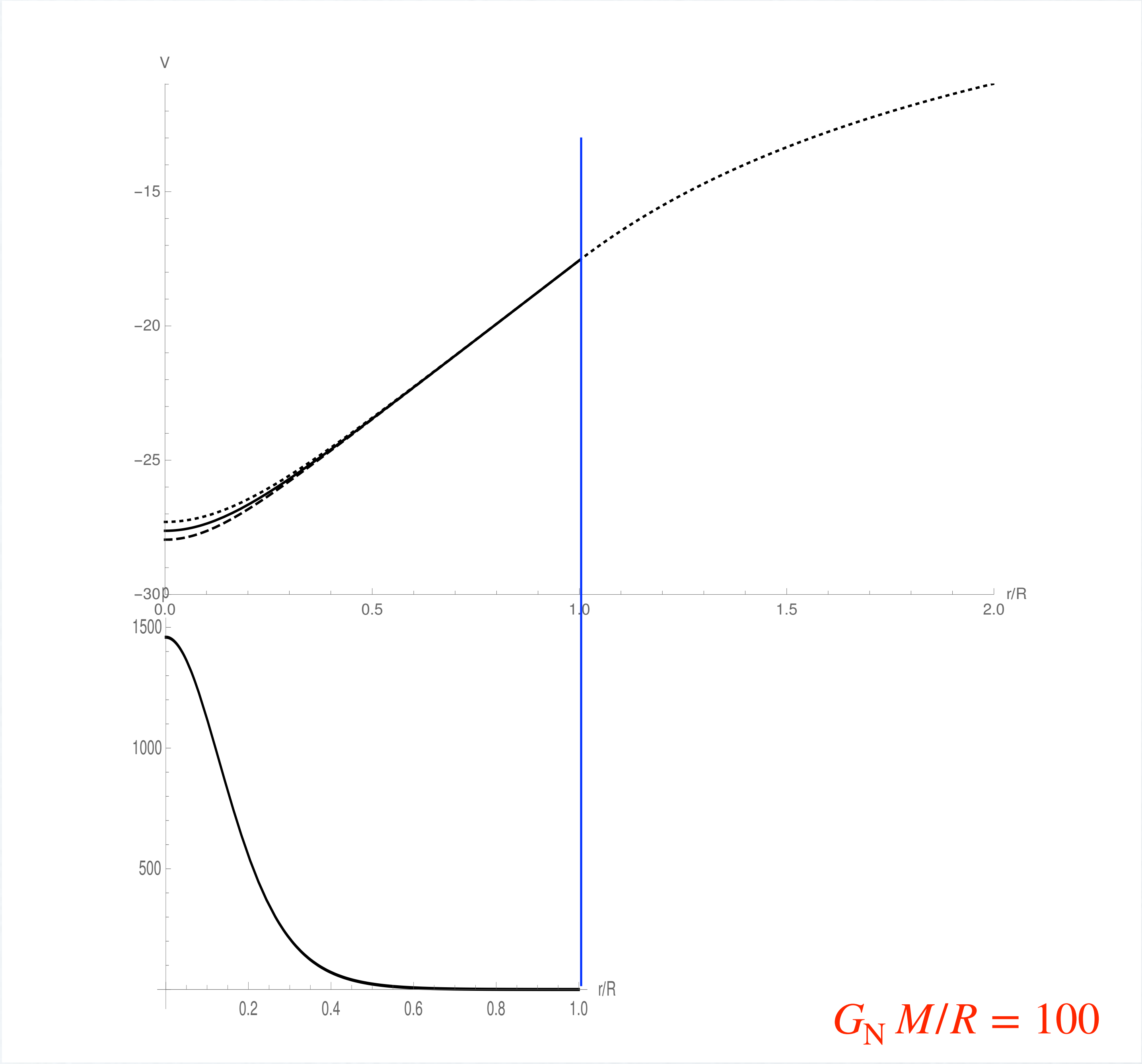

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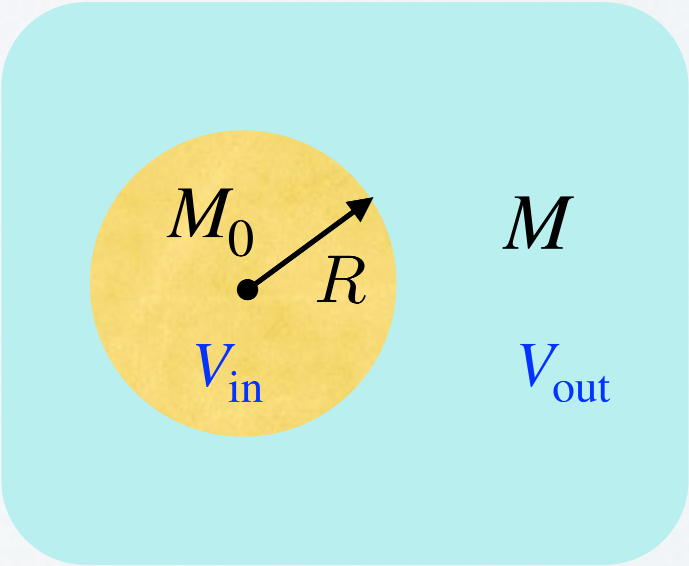

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Horizon (escape velocity=c): $$2\,V(r_{\rm H}) = -1$$

Star $\longrightarrow$
$$\begin{cases} \text{no horizon} & \text{for} \quad G_{\rm N}\,M/R \lesssim 0.46 \\ 0 < r_{\rm H} \le R & \text{for} \quad 0.46 \lesssim G_{\rm N}\,M/R \le 0.69 \\ r_{\rm H} \simeq 1.4\,G_{\rm N}\,M & \text{for} \quad G_{\rm N}\,M/R \gtrsim 0.69 \end{cases}$$

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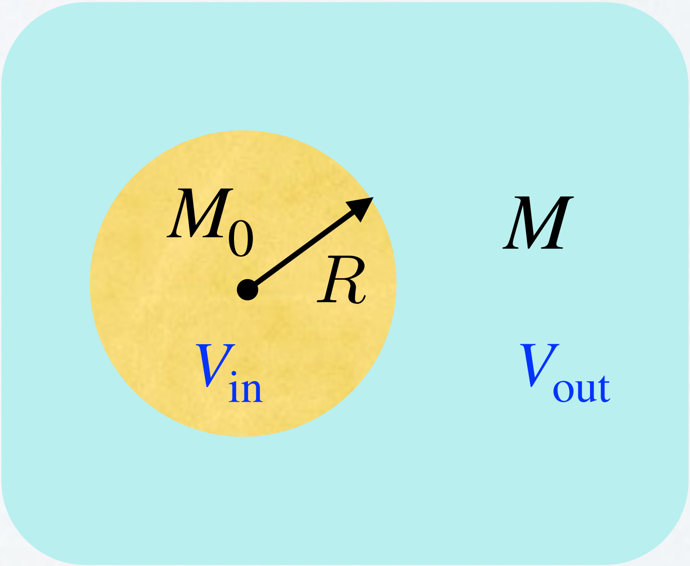

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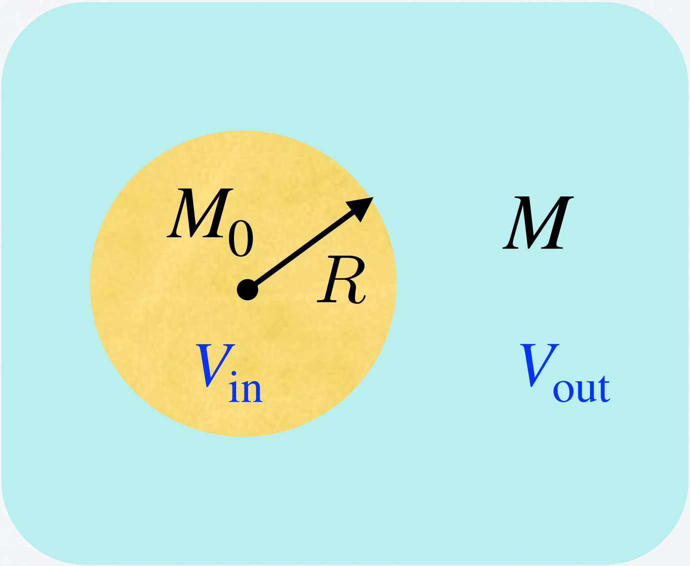

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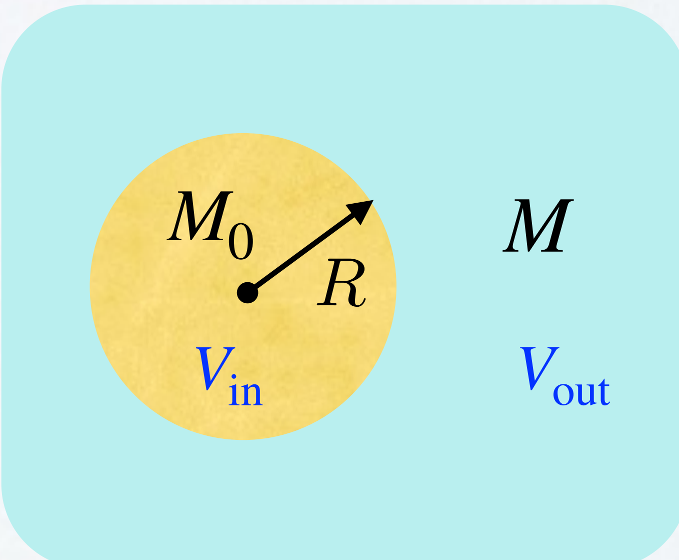

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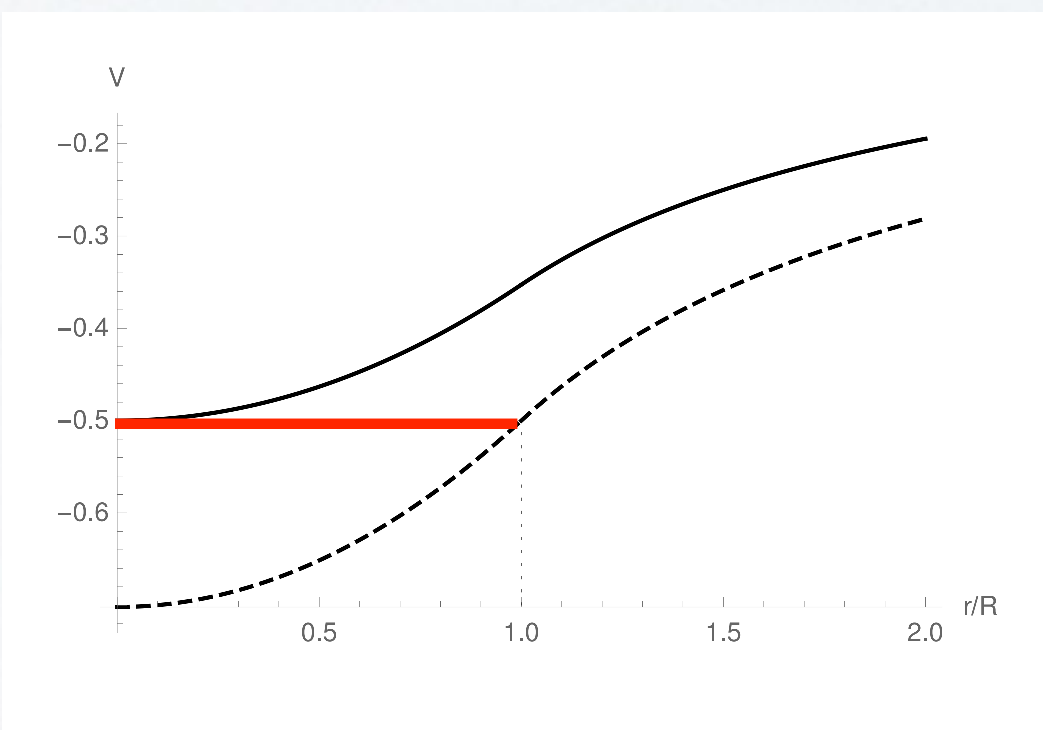

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BH →

Quantum Bootstrapped Newtonian system

Classical picture is effective (~ mean field) ⟶ Quantum state

Quantum Bootstrapped Newtonian system

Classical picture is effective (~ mean field) → Quantum state

Gravitational potential → Coherent state of massless scalar field

$$V = V(r; M_0, M, R)$$

$$\langle g \,|\, \hat{\Phi} \,|\, g \rangle = V$$

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Uniform source → Highly degenerate state of fermions?

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Dvali Gomez Scalings

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$$R^3 \sim M_0 \sim M^3$$

(compactness ~ 1)

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Thank you!

Dvali Gomez
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