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# The holographic QCD axion

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Based on

- FB, A. Caddeo, A.L. Cotrone, P. Di Vecchia, A. Marzolla, 1906.12117
- FB, A.L. Cotrone, M. Järvinen, E. Kiritsis, 1906.12132

# Plan

- Holography
- Holographic QCD and axion
- Axion couplings to nucleons
- Finite temperature

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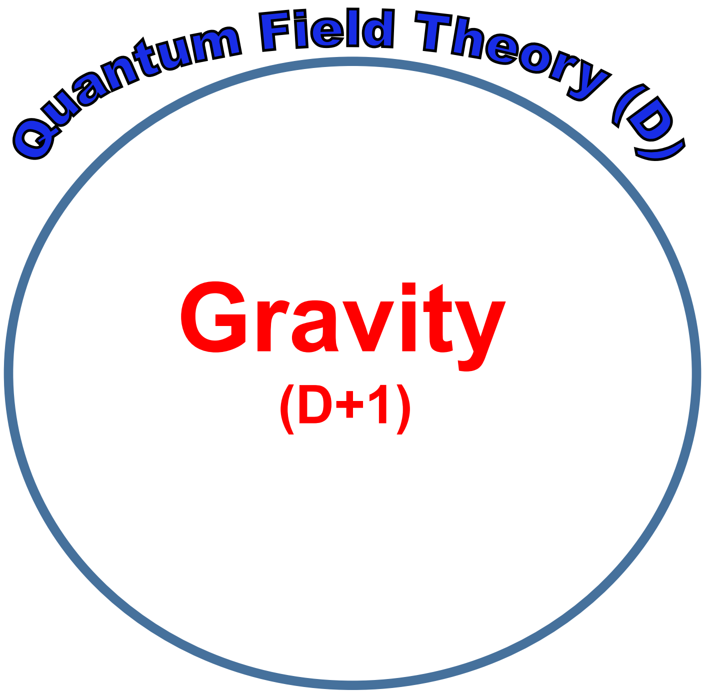
# The holographic conjecture

| Quantum Gravity in D+1 dimensions | = | Quantum Field Theory in D dimensions |
| --- | --- | --- |

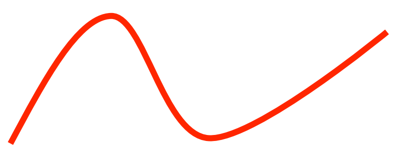
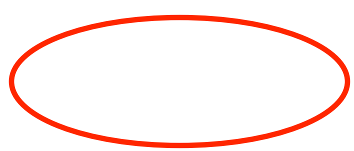



$$Z_{QFT} = Z_{QG} \approx e^{-S_{GR}}$$

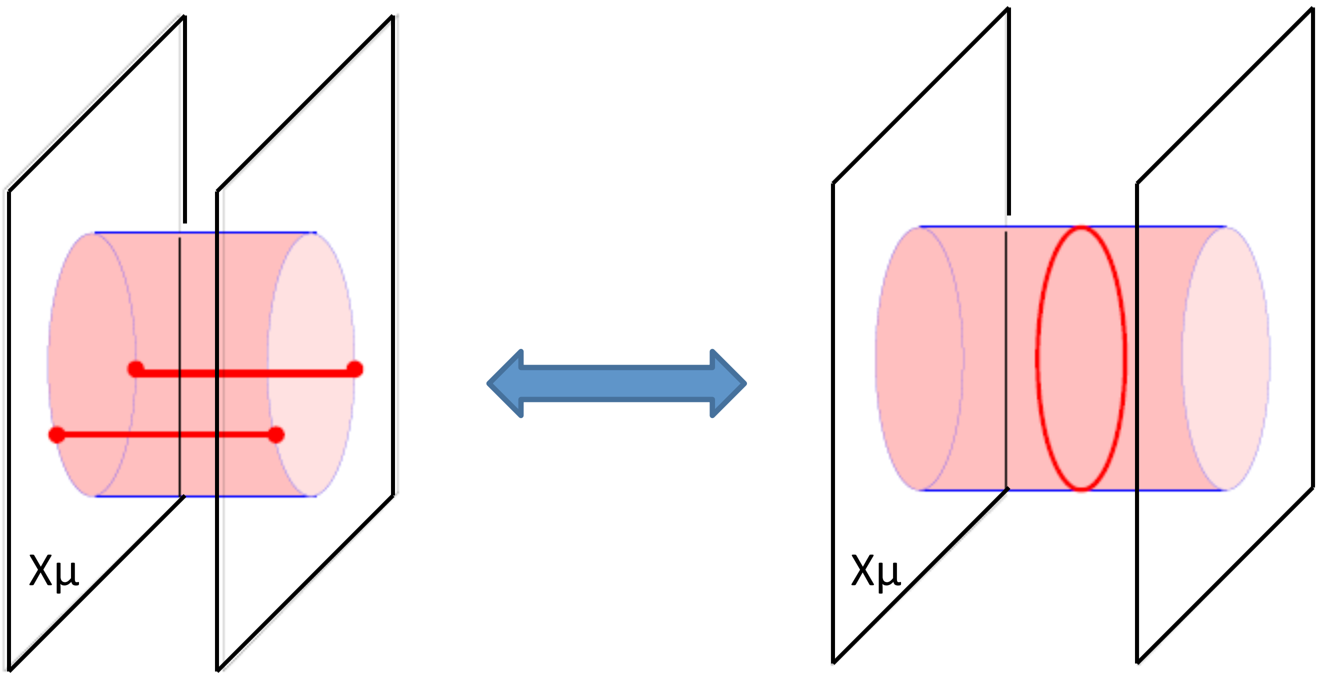
[Maldacena, 97. Gubser, Klebanov, Polyakov; Witten, 98]

# A hint : String theory

- Assumption: fundamental constituents are string-like
- Point particles are different modes of a vibrating string

| String | | Particle |
|---|---|---|
| Open |  | photon (gluon) +… |
| Closed |  | graviton +….. |

- Open/closed string duality (or: 2 ways of drawing a cylinder)




Open string loop (quantum)
Quantum Field Theory
Xμ, u (RG scale)

⟺

Closed string propagation (classical)
Theory of gravity
Xμ, r (extra dimension)

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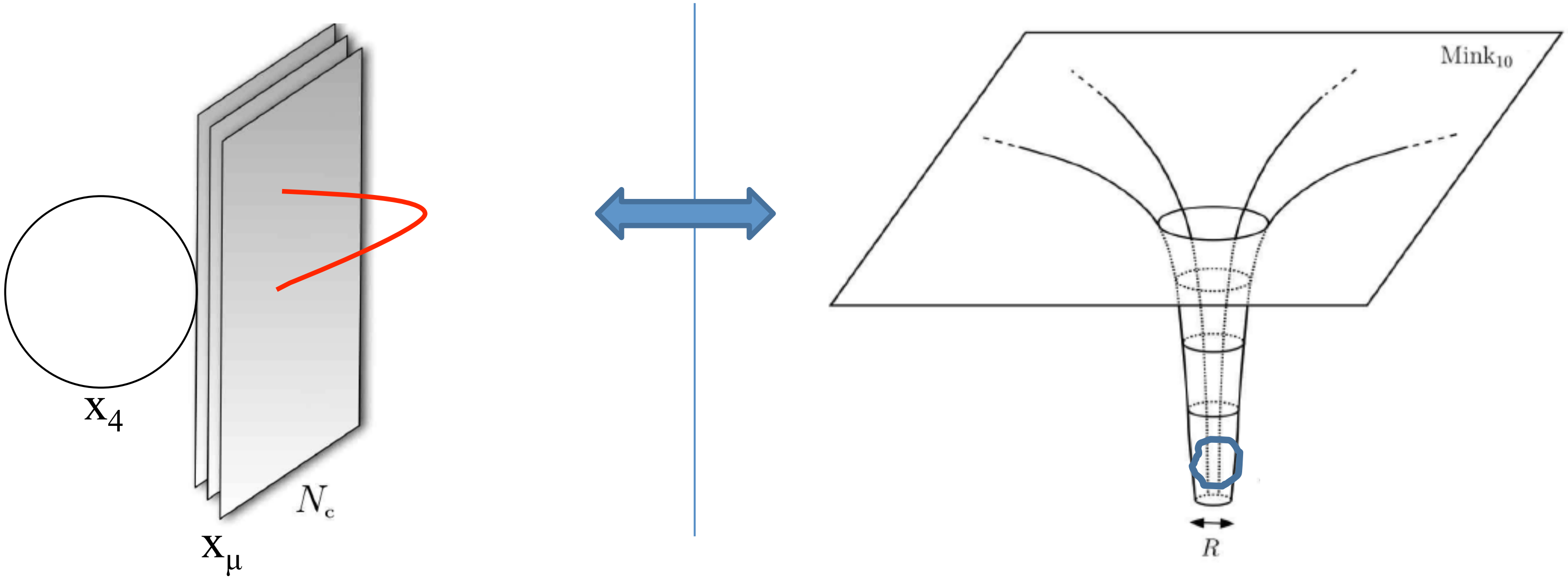
# Holographic Yang-Mills [Witten 1998]

SU($N_c$) Yang-Mills in 3+1 dimensions + massive adjoint KK fields

- $N_c$ D4-branes on circle $S^1{}_{x4}$, radius $R_4 = 1/M_{KK}$, antiperiodic fermions.
- Low energy: 4d non-susy SU($N_c$) Yang-Mills + massive adjoint KK modes
- Dual description: gravity solution sourced by wrapped D4-branes



# Holographic Yang-Mills [Witten 1998]

- Gravity action (closed string description)

$$S = \frac{1}{2k_0^2}\int d^{10}x\sqrt{-g}\left[e^{-2\phi}\left(\mathcal{R}+4(\partial\phi)^2\right)-\frac{1}{2}|F_4|^2-\frac{1}{2}|F_2|^2\right]$$

- Gauge theory action (open string description, IR limit of D4-brane action)

$$S = \frac{1}{8\pi g_s l_s M_{KK}}\int d^4x\ \mathrm{Tr}F_{\mu\nu}F^{\mu\nu} - \frac{i}{32\pi^2 l_s}\int_{S_{x^4}} C_1 \int d^4x\ \epsilon^{\mu\nu\rho\sigma}\mathrm{Tr}F_{\mu\nu}F_{\rho\sigma}$$

$$\boxed{F_2 = d\,C_1} \qquad \boxed{\theta + 2\pi k = \frac{1}{l_s}\int_{S_{x_4}} C_1}$$

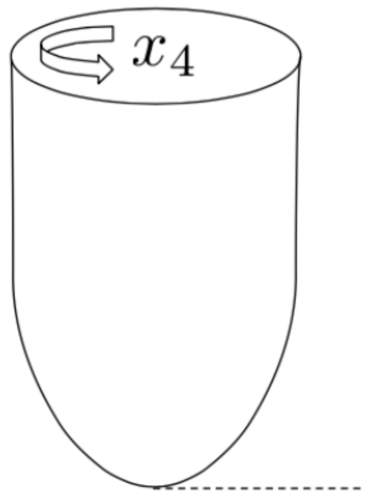
- **Classical gravity** picture **dual to gauge theory at** $\lambda_4 >> 1$, $N_c >> 1$

$$\lambda_4 = g_{YM}^2 N_c = 2\pi g_s l_s M_{KK} N_c$$

# Holographic Yang-Mills [Witten 1998]

$$ds^2 = \left(\frac{u}{R}\right)^{3/2}\left[dx_\mu dx^\mu + f(u)dx_4^2\right] + \left(\frac{u}{R}\right)^{-3/2}\left[\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right]$$

$$e^\phi = g_s\left(\frac{u}{R}\right)^{3/4}, \qquad F_4 = \frac{3N_c}{4\pi}\omega_4\,, \qquad f(u) = 1 - \frac{u_0^3}{u^3}\,. \qquad R = (\pi g_s N_c)^{1/3} l_s$$




- $x_4 \sim x_4 + 2\pi/M_{KK}$
- $(u,x_4)$ subspace is a cigar
- $g_{00}(u_0) \neq 0$ (regular metric)
- ✓ IR: Confinement and mass gap
- UV: very different from pure YM

$$u_0 = \frac{4R^3}{9}M_{KK}^2$$

- To leading order in θ/Nc treat $C_1$ as a probe [Witten 1998]:

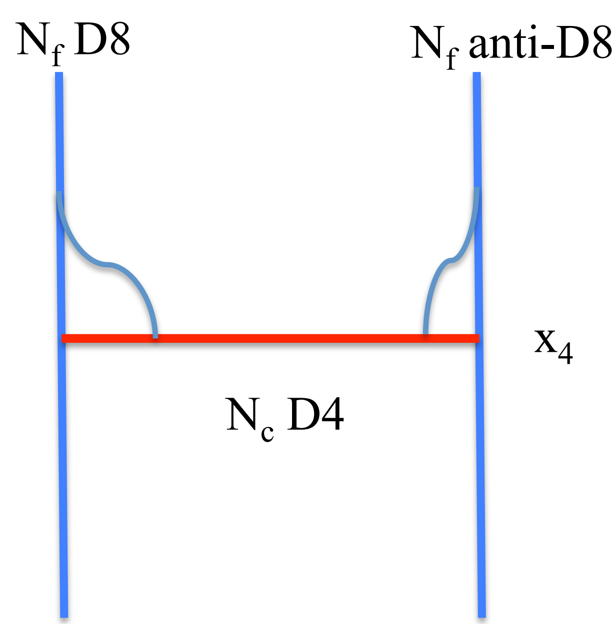
$$C_1 = \frac{\Theta}{l_s g_s} f(U) dx_4\,, \quad \text{where} \quad \Theta \equiv \frac{\lambda}{4\pi^2}\left(\frac{\theta + 2\pi k}{N_c}\right)$$

- Theta dependence of free energy from $S_\theta \sim \int d^{10}x |F_2|^2\,, \quad F_2 = dC_1$

# Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + $N_f$ massless fundamental quarks

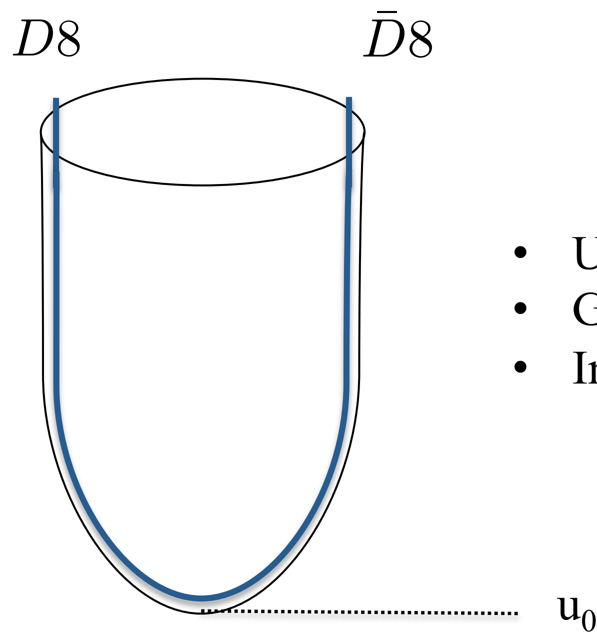
- Nf massless flavors from extra D4-D8 open strings
- $U(Nf)_L$ x $U(Nf)_R$ gauge symmetry on D8 dual to classical QFT chiral symm.



# Holographic QCD [Sakai,Sugimoto 2004]

Witten's SU(Nc) Yang-Mills + $N_f$ massless fundamental quarks

- At strong coupling, replace D4s by dual background.
- If $N_f << N_c$ treat D8-branes as probes.




- $U(N_f)$ gauge theory on D8
- Gauge field fluctuations = **mesons**
- Instanton solutions = **baryons**

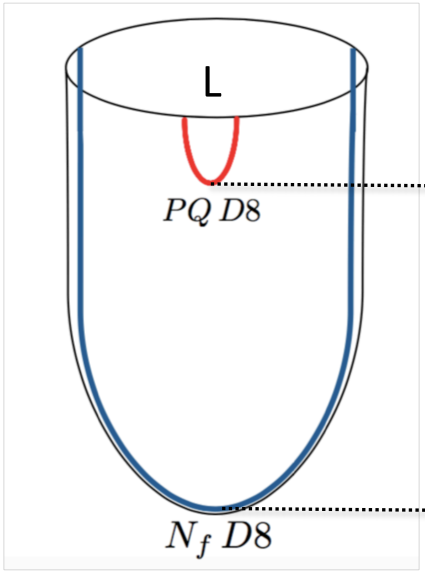
- Chiral symmetry breaking = joining of the two branches
- Pion coupling $f_\pi \sim u_0$

# The Holographic QCD axion

[ FB, Caddeo, Cotrone, Di Vecchia, Marzolla, ‘19]

Witten’s SU(Nc) Yang-Mills + $N_f$ massless fundamental quarks + **1 extra quark**

- Extra massless quark flavor = extra (Peccei-Quinn) non antipodal D8-anti D8
- Spontaneous breaking of PQ symmetry = joining of extra D8




$u_J \sim f_a$

$u_0 \sim f_\pi$

- Extra parameter $L < \pi R_4$ (or $u_J - u_0$) : related to NJL coupling of extra quark
  [Antonyan, Harvey, Jensen, Kutasov, 06]

- If $L \ll \pi R_4$, axion coupling $f_a \gg f_\pi$

# Low energy effective action

$$S_{tot} = S_{WSS} + S_{PQ} + S_\theta + S_{mass}$$

- At low energy and integrating over u and $S^4$: **chiral Lagrangian + axion**
- Same as in [Di Vecchia, Rossi, Veneziano, Yankielowicz 2017]

$$\boxed{\mathcal{L}_{\rm eff} = -\frac{f_\pi^2}{4}{\rm Tr}\left[\partial_\mu U \partial^\mu U^\dagger\right] - \frac{1}{2}\partial_\mu a \partial^\mu a + c{\rm Tr}\left[MU^\dagger + {\rm h.c.}\right] - \frac{\chi_{\rm WYM}}{2}\left(\theta + \frac{\sqrt{2N_f}}{f_\pi}\eta' + \frac{\sqrt{2}}{f_a}a\right)^2}$$

- Mass term from worldsheet instantons. Topological term from Chern-Simons.
- $U = \mathcal{P}e^{i\int dz \mathcal{A}_z^{WSS}} = e^{\frac{2i}{f_\pi}\left(\pi^a T^a + \frac{\eta'}{\sqrt{2N_f}}\right)}$ , $\boxed{\sqrt{2}\frac{a}{f_a} = \int A_z^{PQ} dz}$
- Axion mass: $m_a^2 = \frac{2}{f_a^2}\chi\,, \quad \chi = \frac{4c\,\chi_{WYM}}{4c + 2\chi_{WYM}{\rm Tr}[M^{-1}]}$
- All parameters in terms of $\lambda$, $N_c$, $N_f$, $R_4 = 1/M_{KK}$, L, M = diag ($m_u$, $m_d$, …)

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# Axion couplings to nucleons

[FB, Cotrone, Kiritsis, Yarvinen, '19]

- Focus on $N_f$=2 case. M=diag ($m_u$ , $m_d$ ). N=(p, n) nucleon field.

- Compute CP even ($c_N$) and CP odd ($\bar{c}_N$) axion-nucleon couplings

$$\delta\mathcal{L}_{aNN,\text{der}} = -\frac{\partial_\mu a}{\sqrt{2}f_a} c_N \bar{N}\gamma^\mu\gamma^5 N\,, \quad \delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$$

- Ingredient 1: in WSS model, **nucleons = instanton** solutions for $\mathcal{F}_{WSS}$
- Nucleon mass scales like $N_c$ : a non relativistic limit can be taken

- Ingredient 2: neglecting η'- π mixing, mass eigenstates to O(1/$f_a$) are

$$\hat{\eta}' = \eta' + \frac{\chi f_\pi}{4c}\text{Tr}[M^{-1}]\frac{\sqrt{2}a}{f_a}\,,$$
$$\hat{\pi}^a = \pi^a + \frac{\chi f_\pi}{4c}\text{Tr}[\tau^a M^{-1}]\frac{\sqrt{2}a}{f_a}\,,$$
$$\hat{a} = a - \frac{\chi f_\pi}{4c}\text{Tr}[M^{-1}]\frac{\sqrt{2}\eta'}{f_a} - \frac{\chi f_\pi}{4c}\text{Tr}[\tau^a M^{-1}]\frac{\sqrt{2}\pi^a}{f_a}$$

# Axion couplings to nucleons

- CP even axion couplings from meson-nucleon ones. In chiral limit:

$$c_p \approx -\frac{1}{2}\hat{g}_A - \frac{1}{2}g_A\frac{m_d - m_u}{m_d + m_u}$$
$$c_n \approx -\frac{1}{2}\hat{g}_A + \frac{1}{2}g_A\frac{m_d - m_u}{m_d + m_u}$$

$$\hat{g}_A \approx \frac{27}{2\lambda}$$
$$g_A \approx \frac{2}{\pi}\sqrt{\frac{2}{15}}N_c$$

- $g_{\eta' NN} = \frac{m_N}{f_\pi}\hat{g}_A\,, \quad g_{\pi NN} = \frac{m_N}{f_\pi}g_A$ [Goldberger-Treiman relations]
- Just as in KSVZ class.

# Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- [If there is any residual θ angle]. In chiral limit, from mixing, get

$$\bar{c}_p \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} - \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$
$$\bar{c}_n \approx -\frac{1}{2}\bar{g}_{\eta' NN}\frac{f_\pi}{f_a} + \frac{1}{2}\bar{g}_{\pi NN}\frac{f_\pi}{f_a}\frac{m_d - m_u}{m_d + m_u}$$

- Computing mass term on instanton solution get, to first order in $m_d$-$m_u$

$$\bar{c}_p \approx \frac{1}{4}\sigma_N\frac{\theta}{f_a} - \frac{1}{8}(M_n - M_p)_{\text{str.}}\frac{\theta}{f_a}\,,$$
$$\bar{c}_n \approx \frac{1}{4}\sigma_N\frac{\theta}{f_a} + \frac{1}{8}(M_n - M_p)_{\text{str.}}\frac{\theta}{f_a}$$

- $\sigma_N$ : pion-nucleon sigma term (quark mass contribution to nucleon mass)
- Cfr [Moody, Wilczek 1984]

# Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- Computed in holoQCD and in Skyrme as functions of model parameters.
- Compare with estimates from previous (CL) formulae, using value of sigma term from lattice [ETCM coll. 2019] or from pionic atoms [pheno: Meissner 2017]. $(M_n\text{-}M_p)_{\text{str.}}$ from [Borsanyi et al 2015]

| | CL + lattice | CL + pheno | Skyrme | Holography |
|---|---|---|---|---|
| $\bar{c}_n$ | 9.4(9) | 13.0(1.1) | 27(5) | 21(4) |
| $\bar{c}_p$ | 8.8(9) | 12.4(1.1) | 27(5) | 20(4) |

[in MeV $\theta/f_a$ units]

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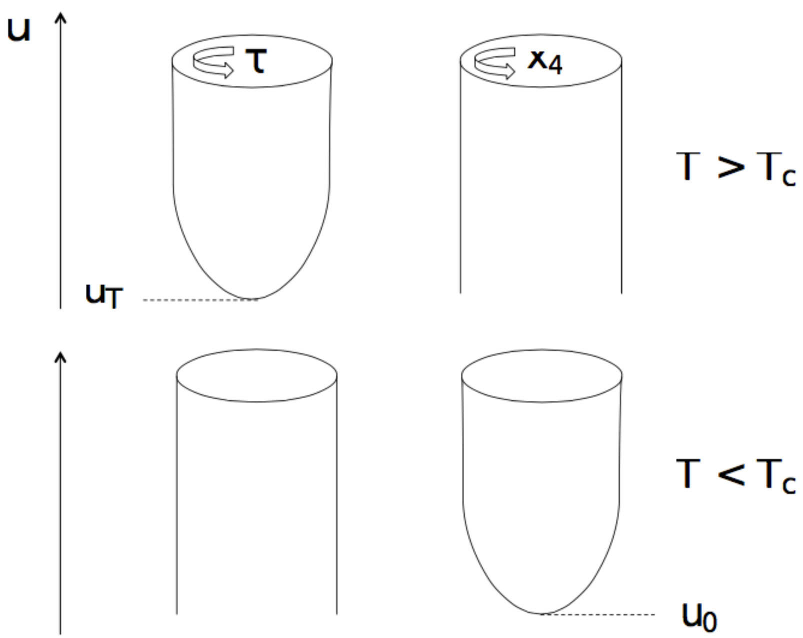
# Holographic YM at Finite temperature

Two possible gravity solutions, with Euclidean time circle of length 1/T




- black hole solution
- $g_{00}(u_T) = 0$ : deconfinement
- no theta dependence to leading order

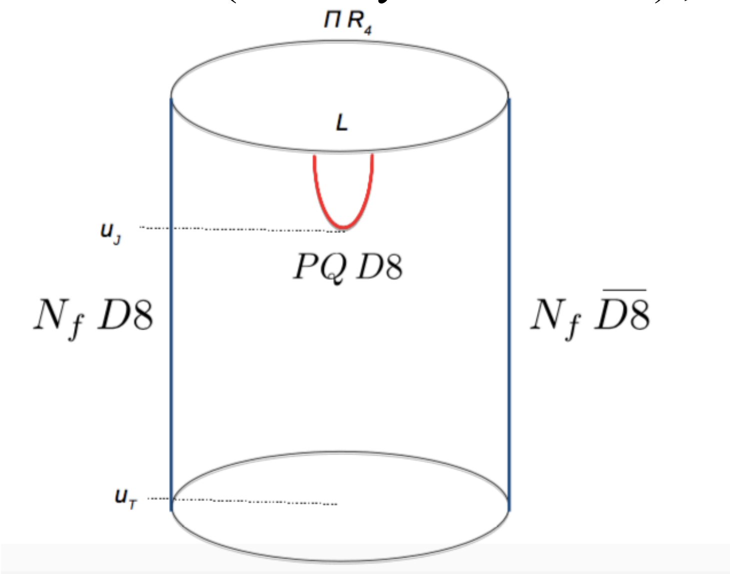
$$C_1 \sim \theta dx_4, \;\; F_2 = 0 \quad [(u, x_4) : \text{cylinder}]$$

- Euclidean version of T=0 one
- Theta-dependence
- Confinement

- At $T_c = \frac{M_{KK}}{2\pi}$ first order phase transition

# Holographic axion at Finite temperature

- If $T_c < T \ll f_a$ QCD mesons melt (chiral symm. restored) , while our axion survives




- We can study the temperature dependence of the axion mass in the unflavored case
- Hence we can compute $\chi_{WYM}$ (T)

- Since $F_2=0$ at $T>T_c$, classical gravity approximation gives $\chi_{WYM}(T)=0$
- Need stringy instanton corrections to gravity action [Green, Gutperle, Vanhove 97]

# Axion mass at T > $T_c$

[FB, Caddeo, Cotrone, Di Vecchia, Marzolla, 19]

- Topological susceptibility and hence the axion mass
- $$\boxed{\chi_{\rm WYM}(T) = \frac{3285\pi^{3/2}}{42}\left(\frac{4\pi}{3}\right)^4 \frac{\sqrt{N_c}}{\sqrt{\lambda_{\rm eff}(T)}}\, T^4 e^{-\frac{8\pi^2}{g^2_{\rm YM}}}}$$ $$\lambda_{\rm eff}(T) \equiv g^2_{\rm YM} N_c \frac{T^2}{M^2_{\rm KK}} \equiv \lambda_{\rm YM}\frac{T^2}{M^2_{\rm KK}}$$
- $m_a^2(T) f_a^2 = 2\chi_{WYM}(T) \sim M_{KK} T^3$

- Axion mass increases with T
- Strong difference with Yang-Mills instanton gas.

- Aside result: also computed for $N$=4 SYM: $\chi_{\rm SYM}(T) = \frac{15}{128}\pi^{3/2}\sqrt{N_c}\, T^4 e^{-\frac{8\pi^2}{g^2_{\rm YM}}}$

# Final comments

- Our model: one extra massless quark flavor added to (holographic) QCD.
- It is NOT a standard extra flavor: it condenses at a scale $M_a \sim f_a >> \Lambda\chi_{SB}$
- Thus extra hadrons decouple.

- Exception, the axion: (pseudo) Goldstone boson of extra chiral symmetry breaking.
- Anomalous $U(1)_A$ symmetry of extra flavor = $U(1)_{PQ}$ broken symmetry.

- In our model $f_a/f_\pi \approx R_4/L$ can be made parametrically large.
- At weak coupling [Antonyan, Harvey, Jensen, Kutasov 06]: NJL coupling.

- UV completion under control, higher dimensional model in string theory.

- The model has a dual holographic description at large $N_c$.
- It provides analytic predictions on the (strongly coupled) IR physics.
- Derivative and non-derivative axion-nucleon couplings computed.

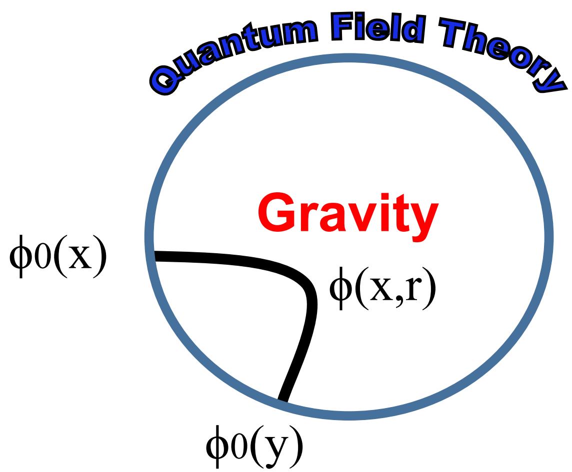
- UV completion is higher dimensional: very different behavior from QCD.
- We have shown that explicitly by studying the high temperature behavior.

# Thank you

# Holography at work

[Maldacena, 97; Witten; Gubser, Klebanov, Polyakov, 98]

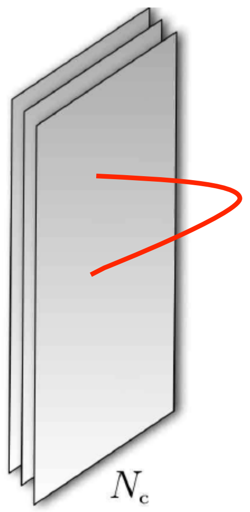
- Large N, strongly correlated QFT ⟷ Classical Gravity

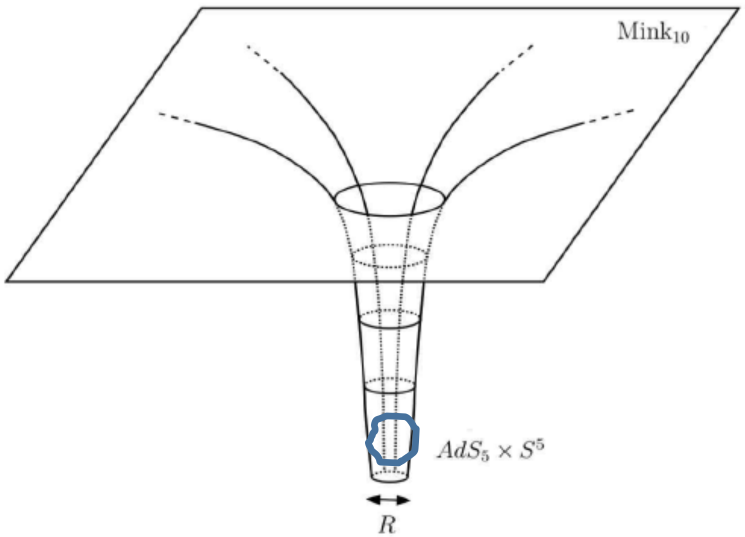



$$\boxed{Z_{QFT}[\phi_0] = Z_{QG/String} \approx e^{iS_{gravity}[\phi_0]}|_{\text{“}\phi(x,r)\to\phi_0(x)\text{”}}}$$

- QFT phase (finite T, $\mu$, …) ⟷ Gravity background (black hole, charged bh, …)
- QFT operator ($T_{\mu\nu}$, $J_\mu$, Tr $F^2$, …) ⟷ Gravity field ($g_{\mu\nu}$, $A_\mu$ , $\phi$, …)
- Global symmetry (e.g. Chiral) ⟷ Gauge symmetry

- The dual nature of Dp-branes [Polchinski, 95]




- Taking low energy limit on both sides: two interacting theories [J.M. Maldacena, 97]:

- Left: 4d SU($N_c$) susy Yang-Mills **(CFT)**. From open strings on Nc D3-branes
- Right: closed IIB strings (gravity) on **Anti-de-Sitter** 5d background (times $S^5$)

- $N_c >> 1$, $\lambda = g^2_{YM} N_c >> 1$ in QFT $\Longleftrightarrow$ Classical theory of gravity

- Reminder. Strong CP problem: why is QCD $\theta$ angle so tiny?
- Peccei-Quinn: if extra axial symmetry, $\theta$ can be rotated away.
- We do not see PQ symmetry: it must be (spontaneously) broken.

- Here, PQ symmetry = axial U(1) acting on extra quark, NOT on SM quarks.
- Extra quark condensation (U-shaped PQ brane) spontaneously breaks it.
- Pseudo-Goldstone boson: axion. Gets mass due to anomaly, like η'.

- PQ and WSS branes are distant: quarks and extra quark interact only through gauge sector. Since we work at large N interactions are suppressed.
- Hence our axion model fits in the KSVZ class [Kim; Shifman, Vainshtein, Zakharov, '80]
- Thus axion coupling to nucleons only receive IR contributions.

- Electromagnetic current in WSS: weakly gauging of U(1) in U($N_f$)
- As such it pertains to WSS D-branes: the PQ quarks are uncharged.
- Hence electromagnetic interactions of the axion just come from mixing with neutral pion and η', with no UV contribution.

# NJL model from D4-D8 setup

[Antonyan, Harvey, Jensen, Kutasov, 06]

In the D4-D8 setup $q_L$ and $q_R$ are separated in the $x_4$ direction.
They live in the 3+1 dimensional D4-D8 intersection but the
gauge field they interchange is 4+1 dimensional (before compactifying x4)

$$\mathcal{S} = \int d^5x \left[ -\frac{1}{4g_5^2} F_{MN}^2 + \delta(x^4 + \frac{L}{2}) q_L^\dagger \bar{\sigma}^\mu (i\partial_\mu + A_\mu) q_L + \delta(x^4 - \frac{L}{2}) q_R^\dagger \sigma^\mu (i\partial_\mu + A_\mu) q_R \right]$$

At weak coupling $q_L$ and $q_R$ interact via a (non local) one (five dimensional)
gluon exchange. Integrating out the 5d gauge field in the single gluon exchange approx

$$\mathcal{S}_{\text{eff}} = i \int d^4x \left( q_L^\dagger \bar{\sigma}^\mu \partial_\mu q_L + q_R^\dagger \sigma^\mu \partial_\mu q_R \right) + \frac{g_5^2}{4\pi^2} \int d^4x d^4y G(x-y, L) \left[ q_L^\dagger(x) \cdot q_R(y) \right] \left[ q_R^\dagger(y) \cdot q_L(x) \right]$$

$G(x, x_4)$ 5d scalar propagator

A (non local) NJL model: $\mathcal{L}_{\text{int}} = G q_L^\dagger \cdot q_R q_R^\dagger \cdot q_L$

NJL: for $G>G_c$ chiral symmetry breaking i.e. $\langle q_L^\dagger \cdot q_R \rangle$ condensate.

# The axion coupling $f_a$

$$f_a^2 = \frac{N_c \lambda}{16\pi^3} \frac{J^3(b)}{I(b)} \frac{1}{M_{\rm KK} L^3}\,, \qquad b \equiv \frac{u_0}{u_J}$$

$$J(b) = \frac{2}{3}\sqrt{1-b^3}\int_0^1 dy \frac{y^{\frac{1}{2}}}{(1-b^3 y)\sqrt{1-b^3 y-(1-b^3)y^{\frac{8}{3}}}} \qquad I(b) = \int_0^1 dy\,\frac{y^{-\frac{1}{2}}}{\sqrt{1-b^3 y-(1-b^3)y^{\frac{8}{3}}}}$$

$$L = J(b) R^{\frac{3}{2}} u_J^{-\frac{1}{2}}$$

- Tune b so that $10^9 \lesssim \frac{f_a}{f_\pi} \lesssim 10^{18}$ : $10^{-24} \lesssim b \lesssim 10^{-12}$
- Within this interval $f_a^2 \approx 0.1534 \frac{N_c \lambda}{16\pi^3} \frac{1}{M_{\rm KK} L^3}$

# Axion couplings to nucleons

- CP-odd axion nucleon couplings: $\delta\mathcal{L}_{aNN,\text{non-der}} = \bar{c}_N a\bar{N}N$
- Consider the quark mass contribution to the nucleon mass term

$$\delta\mathcal{L}_M = c\mathrm{Tr}\left[Me^{-i\frac{\theta}{N_f}}\left(U_{cl} - \mathbb{1}_2\right) + h.c.\right]$$

- $U_{cl}$ classical instanton solution describing a nucleon [Hata,Sakai,Sugimoto,Yamato 07]

$$U_{cl} = \exp\left[i\pi\frac{\vec{\tau}\cdot\vec{x}}{|\vec{x}|}\left(1 - \frac{1}{\sqrt{1+\rho^2/|\vec{x}|^2}}\right)\right]$$

- Analogous to Skyrme hedgehog. $\rho$ is the instanton radius, $\rho_{cl}^2 \sim \lambda^{-1}$
- $$\delta M_N(\theta) = -\int d^3x\,\delta\mathcal{L}_M \approx 0.032\frac{m_\pi^2 N_c^{3/2}}{f_\pi}\cos\left(\frac{\theta}{2}\right)$$
- At $\theta=0$ this is the pion-nucleon sigma term

# Holographic YM at Finite temperature

- θ-dependence in deconfined phase can be recovered including instanton corrections
- Instanton = Euclidean D0-brane wrapped on the $x_4$ circle

$$S_{D0} = \frac{1}{l_s}\int e^{-\phi}\sqrt{g_{44}}dx_4 - \frac{i}{l_s}\int C_1 = \frac{8\pi^2}{g_{\rm YM}^2} - i\theta$$

- Instanton corrections to the IIA supergravity action on a circle are known
- They can be deduced from M-theory on a torus [Green, Gutperle, Vanhove 97]

- Notice that WYM background can be obtained from $AdS_7$x $S^4$ black hole solution

$$ds^2 = G_{MN}dx^M dx^N = \frac{y^2}{R^2}\left[-f(y)dt^2 + \sum_{i=1}^{4} dx_i^2 + dx_{10}^2\right] + \frac{4R^2}{f(y)y^2}dy^2 + R^2 d\Omega_4^2; \qquad f(y) = 1 - y_0^6/y^6$$

- torus $S_{x_{10}} \times S_{x_4}$ radii $R_4 = M_{\rm KK}^{-1}$ and $R_{10} = g_s l_s$

# Holographic YM at Finite temperature

- Quartic corrections to 11d sugra action on a torus

$$\delta S = -\frac{1}{\kappa_{11}^{2/3}}\int d^{11}x\sqrt{-G}\,W\left[\frac{2\pi^2}{3}+\mathcal{V}_2^{-3/2}f(\rho,\bar\rho)\right]$$

- W in terms of the Weyl tensor $W = C^{hmnk}C_{pmnq}C_h^{\;rsp}C^q_{\;rsk}+\frac{1}{2}C^{hkmn}C_{pqmn}C_h^{\;rsp}C^q_{\;rsk}$
- $\mathcal{V}_2$ related to the volume of the torus $V_T = \kappa_{11}^{4/9}\mathcal{V}_2 = \int dx_4 dx_{10}\sqrt{G_{(2)}}$
- Modular function up to one-instanton corrections

$$f(\rho,\bar\rho) = 2\zeta(3)\rho_2^{3/2}+\frac{2\pi^2}{3}(\rho_2)^{-1/2}+4\pi(e^{2\pi i\rho}+e^{-2\pi i\bar\rho})+\cdots$$

$$\rho \equiv \rho_1 + i\rho_2 = (2\pi)^{-1}\theta + 4\pi i g_{\mathrm{YM}}^{-2} \sim S_{D_0}$$

- On AdS$_7$xS$^4$ black hole $W = \frac{3285}{64R^8}\frac{y_0^{24}}{y^{24}}$ [Gubser,Klebanov,Tseytlin 98]