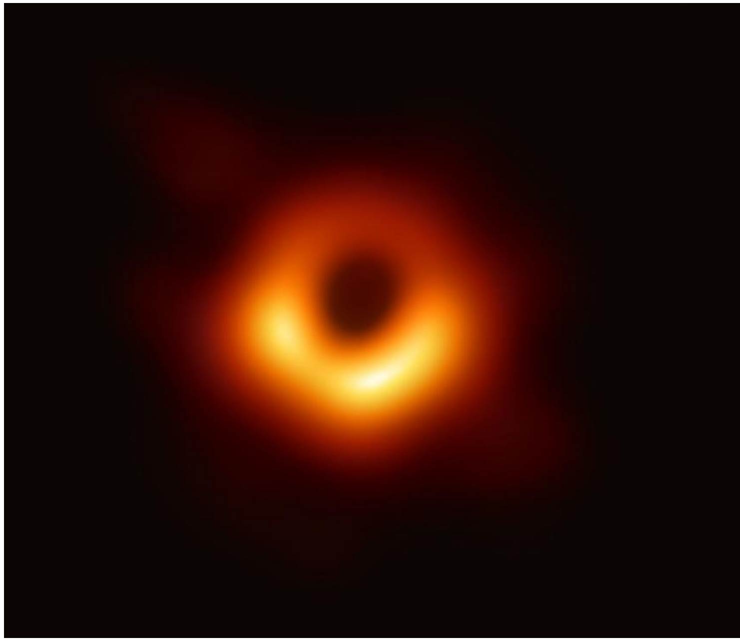
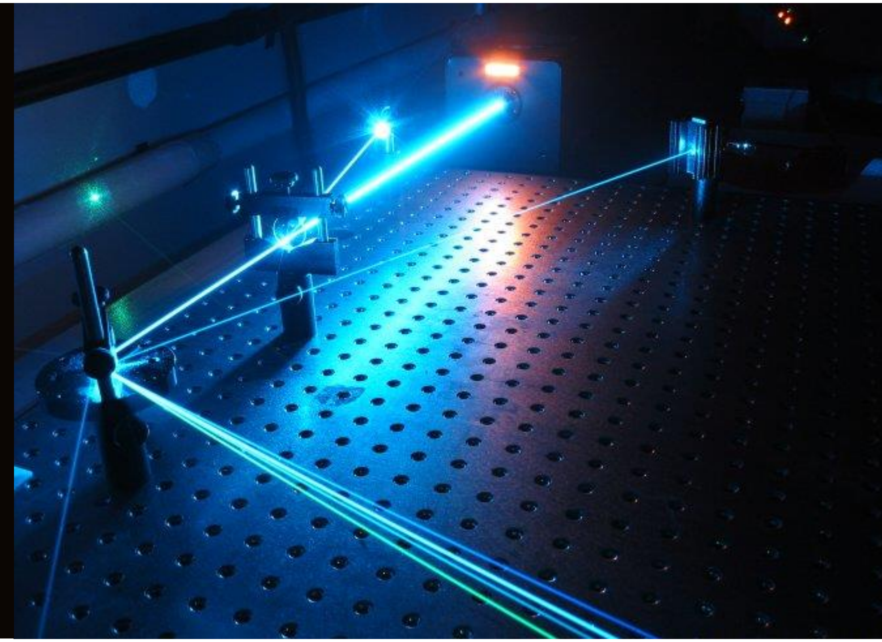


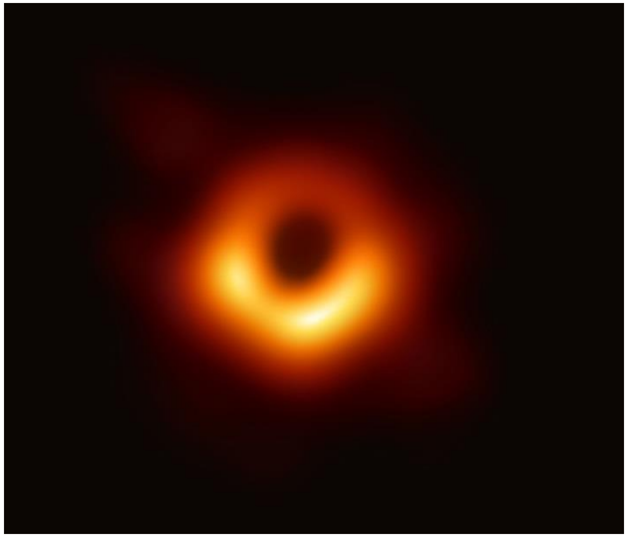


MEASUREMENT OF THE EMISSION OF HAWKING RADIATION WITH THE HAWKING TEMPERATURE IN AN ANALOGUE BLACK HOLE

Juan Ramón Muñoz de Nova
In collaboration with V. I. Kolobov , K. Golubkov &
J. Steinhauer

THEORY

Black hole



-Solution of Einstein equations:

$$ds^2 = -\left(1 - \frac{2GM}{rc^2}\right) dt^2 + \frac{dr^2}{(1 - \frac{2GM}{rc^2})} + r^2 d\Omega^2$$

-Event horizon at $r_S = \frac{2GM}{rc^2}$ Region inside is causally disconnected!!!!

-Semiclassical calculation (classical gravity, quantum field) → Spontaneous emission of radiation with a Planck distribution

$$F(\omega) = \frac{\Gamma(\omega)}{e^{\frac{\hbar\omega}{k_B T_H}} - 1} \qquad k_B T_H = \frac{\hbar\kappa}{2\pi c} = \frac{\hbar c^3}{8\pi GM}, \ \kappa = \frac{c^4}{4GM}$$

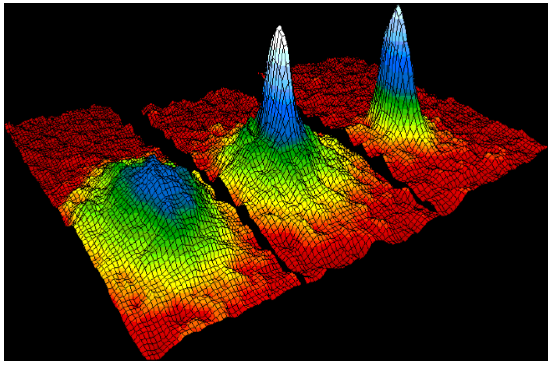
Problems

- Experimental problems.
- Low effective temperature $T_H \simeq 60 \times \frac{M_\odot}{M} \mathrm{nK}$

(CMB=2.7 K)

- Theoretical problems: information paradox, trans-Planckian problem…NO quantum gravity
- Possible solution: use of laboratory analogues!
- Bose-Einstein condensates are great!

E. Cornell, C. E. Wieman JILA (1995)



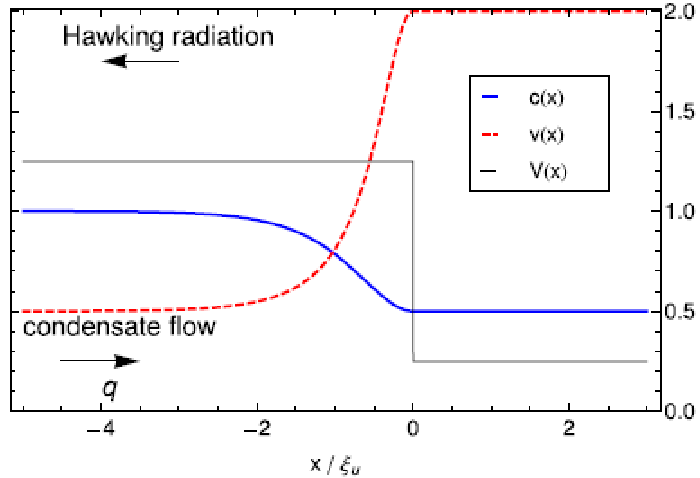
Intuitive picture

-Light cannot escape from the interior of a black hole

- Sound waves cannot travel upstream in a supersonic flow

-Subsonic/supersonic interface = Outside/inside black hole

-Transition between subsonic and supersonic=analog event horizon



Rigorous derivation

-Flow of an ideal fluid, irrotational and barotropic

$$\partial_t \rho + \nabla(\rho \mathbf{v}) = 0 \qquad \partial_t \mathbf{v} + (\mathbf{v}\nabla)\mathbf{v} = -\frac{\nabla P}{\rho} - \frac{\nabla V}{m} \qquad \mathbf{v} = \nabla\phi,\ P = P(\rho)$$

-Linear perturbations around a given equilibrium solution $\rho + \delta\rho,\ \phi + \delta\phi$

$$\nabla^2 \delta\phi + \frac{\nabla\rho}{\rho}\nabla\delta\phi - \frac{1}{c^2}D_t^2\delta\phi - \frac{1}{\rho}D_t\left[\delta\phi D_t \frac{\rho}{c^2}\right] - \frac{\nabla \mathbf{v}}{c^2}D_t\delta\phi = 0$$

-Equivalent to massless scalar field equation with effective metric

$$\Box\theta \equiv \frac{1}{\sqrt{-g}}\partial_\mu(\sqrt{-g}g^{\mu\nu}\partial_\nu\theta) = 0 \quad g_{\mu\nu}(x) = \frac{\rho(x)}{c(x)}\begin{bmatrix} -[c^2(x) - v^2(x)] & -\mathbf{v}^T(x) \\ -\mathbf{v}(x) & \delta_{ij} \end{bmatrix},\ x \equiv (t, \mathbf{x})$$

-Acoustic horizons at $c(\mathbf{x}) = v(\mathbf{x})$! Gravity surface $\kappa = \frac{1}{2}\frac{d(c^2 - v^2)}{dn}$

-Subsonic/supersonic $\longleftrightarrow$ Exterior/Interior

W. G. Unruh, PRL 46, 1351 (1981)

General idea

-The condensate wave-function $\Psi(\mathbf{x},t)$ follows the Gross-Pitaevskii (GP) equation:

$$i\hbar\partial_t\Psi(\mathbf{x},t) = \left[-\frac{\hbar^2\nabla^2}{2m} + V(\mathbf{x},t) + g|\Psi(\mathbf{x},t)|^2\right]\Psi(\mathbf{x},t)$$

-Decomposing in amplitude and phase $\Psi(\mathbf{x},t) = \sqrt{\rho(\mathbf{x},t)}e^{i\phi(\mathbf{x},t)}$

$$\partial_t\rho + \nabla(\rho\mathbf{v}) = 0 \qquad \mathbf{v}(\mathbf{x},t) = \frac{\hbar\nabla\phi(\mathbf{x},t)}{m}$$ → Potential flow!

$$\hbar\partial_t\phi(\mathbf{x},t) = \frac{\hbar^2}{2m\sqrt{\rho(\mathbf{x},t)}}\nabla^2\sqrt{\rho(\mathbf{x},t)} - \frac{1}{2}mv^2(\mathbf{x},t) - g\rho(\mathbf{x},t) - V(\mathbf{x},t)$$

-Quantum physics only enters through the first term at r.h.s. (“quantum pressure”)

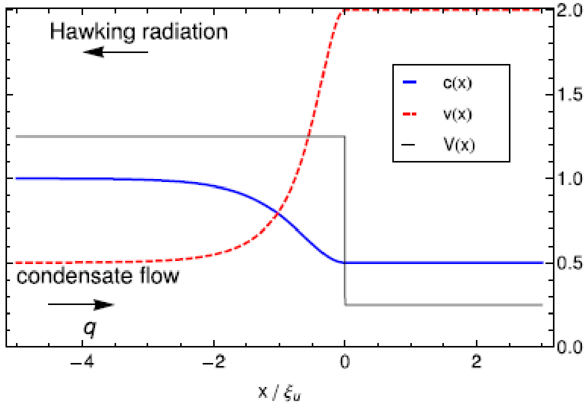
-Quantum fluctuations of density and phase $\delta\hat{\rho}, \delta\hat{\phi}$ + hydrodynamic regime→

Same result as Unruh but for the quantum fluctuations of the phase!!

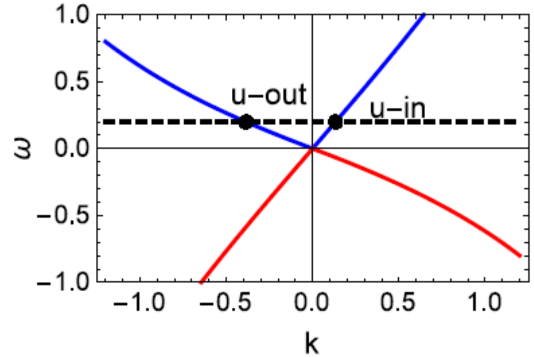
L. J. Garay, J. R. Anglin, J. I. Cirac, & P. Zoller, PRL 85, 4643 (2000)

Black-Hole Configurations

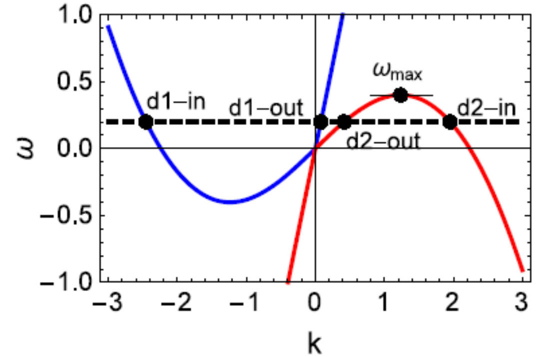
- Stationary 1D GP black-hole solution:



-Asymptotic BdG dispersion relation (superluminal!)



Subsonic (v<c)



Supersonic (v>c)

$$[\omega - vk]^2 = \Omega^2(k) = c^2k^2 + \frac{\hbar^2 k^4}{4m^2}, \; c = \sqrt{gn/m}, \; v = \frac{\hbar q}{m}$$

-Scattering states are solutions of BdG:

$$\hat{\Phi}(x) = \int_0^\infty d\omega \sum_{a=\mathrm{u-in,d1-in}} [z_{a,\omega}(x)\hat{\gamma}_a(\omega) + \bar{z}_{a,\omega}(x)\hat{\gamma}_a^\dagger(\omega)] + \int_0^{\omega_{\max}} d\omega [z_{\mathrm{d2-in},\omega}(x)\hat{\gamma}^\dagger_{\mathrm{d2-in}}(\omega) + \bar{z}_{\mathrm{d2-in},\omega}(x)\hat{\gamma}_{\mathrm{d2-in}}(\omega)]$$

$$\begin{bmatrix} \hat{\gamma}_{u-\mathrm{out}} \\ \hat{\gamma}_{d1-\mathrm{out}} \\ \hat{\gamma}^\dagger_{d2-\mathrm{out}} \end{bmatrix} = \begin{bmatrix} S_{uu} & S_{ud1} & S_{ud2} \\ S_{d1u} & S_{d1d1} & S_{d1d2} \\ S_{d2u} & S_{d2d1} & S_{d2d2} \end{bmatrix} \begin{bmatrix} \hat{\gamma}_{u-\mathrm{in}} \\ \hat{\gamma}_{d1-\mathrm{in}} \\ \hat{\gamma}^\dagger_{d2-\mathrm{in}} \end{bmatrix}$$

-Flux of outgoing particles in vacuum: $\langle 0_{\mathrm{in}} | \hat{\gamma}^\dagger_{u-\mathrm{out}}(\omega)\hat{\gamma}_{u-\mathrm{out}}(\omega') | 0_{\mathrm{in}} \rangle = \delta(\omega - \omega')|S_{ud2}(\omega)|^2 \neq 0$

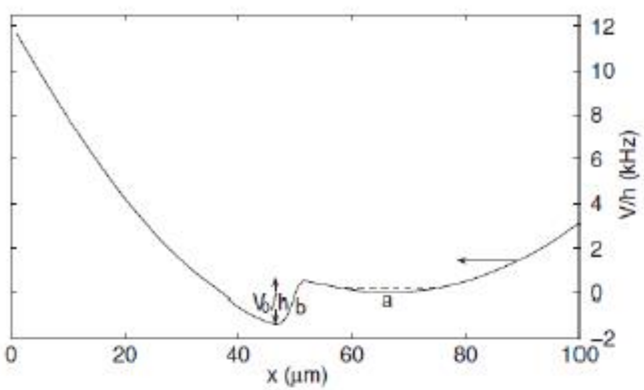
Hydrodynamic regime → $|S_{ud2}(\omega)|^2 = \frac{1}{e^{\frac{\hbar\omega}{k_B T_H}} - 1}, \; k_B T_H = \frac{\hbar}{2\pi}|c'(x_H) - v'(x_H)|$

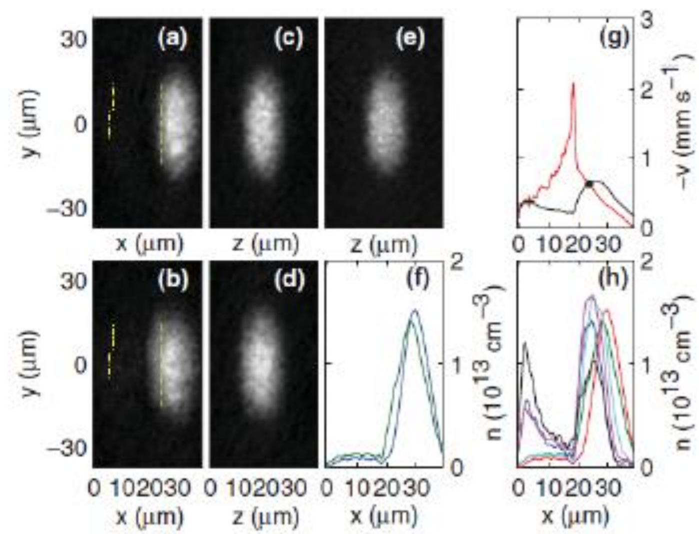
A. Recati et al., PRA 80, 043603 (2009)   J. Macher et al., PRD 79, 124008 (2009)

EXPERIMENT

Realization of a Sonic Black Hole Analog in a Bose-Einstein Condensate

Oren Lahav, Amir Itah, Alex Blumkin, Carmit Gordon, Shahar Rinott, Alona Zayats, and Jeff Steinhauer
Technion—Israel Institute of Technology, Haifa, Israel





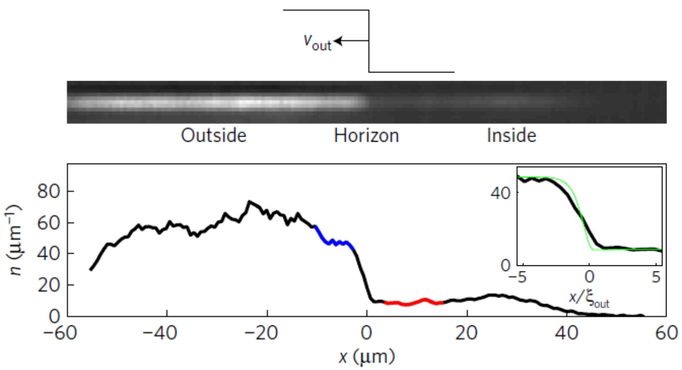
nature physics

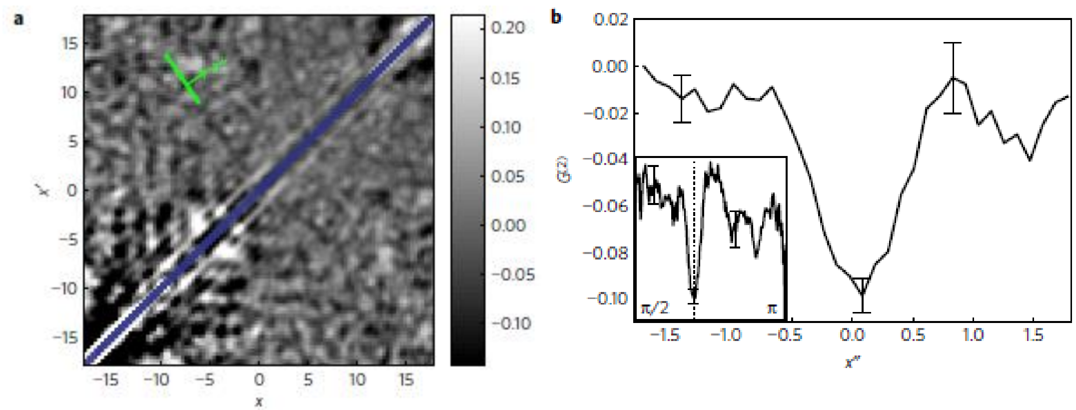
ARTICLES

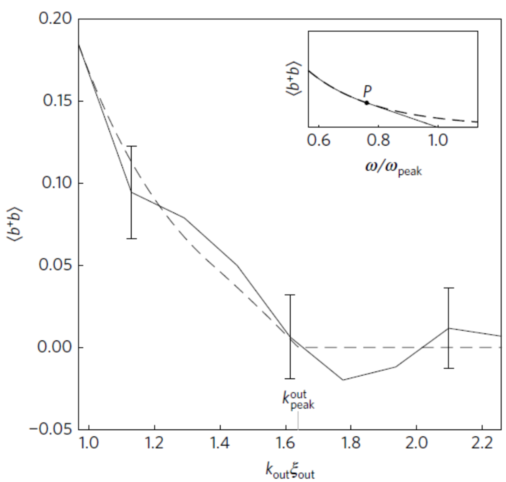
PUBLISHED ONLINE: 15 AUGUST 2016 | DOI: 10.1038/NPHYS3863

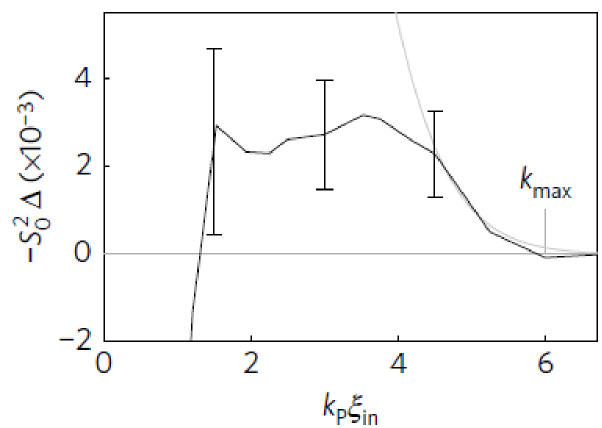
Observation of quantum Hawking radiation and its entanglement in an analogue black hole

Jeff Steinhauer









LETTER

https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

JRMdN, V. I. Kolobov, K. Golubkov & J. Steinhauer Nature 569, 688 (2019)

LETTER

https://doi.org/10.1038/s41586-019-1241-0

Observation of thermal Hawking radiation and its temperature in an analogue black hole

Juan Ramón Muñoz de Nova[1], Katrine Golubkov[1], Victor I. Kolobov[1] & Jeff Steinhauer[1]*

MANY IMPROVEMENTS WITH RESPECT TO PREVIOUS EXPERIMENTS:

-REDUCED NOISE MAGNETIC FIELD

-CORRECTION OF THE CENTER OF THE TRAP EVERY 5 RUNS

-REDESIGNED OPTICS FOR THE WATERFALL

-MIRROR FOR TRANSLATING THE WATERFALL

-IMPROVED MECHANICAL AND OPTICAL STABILITY

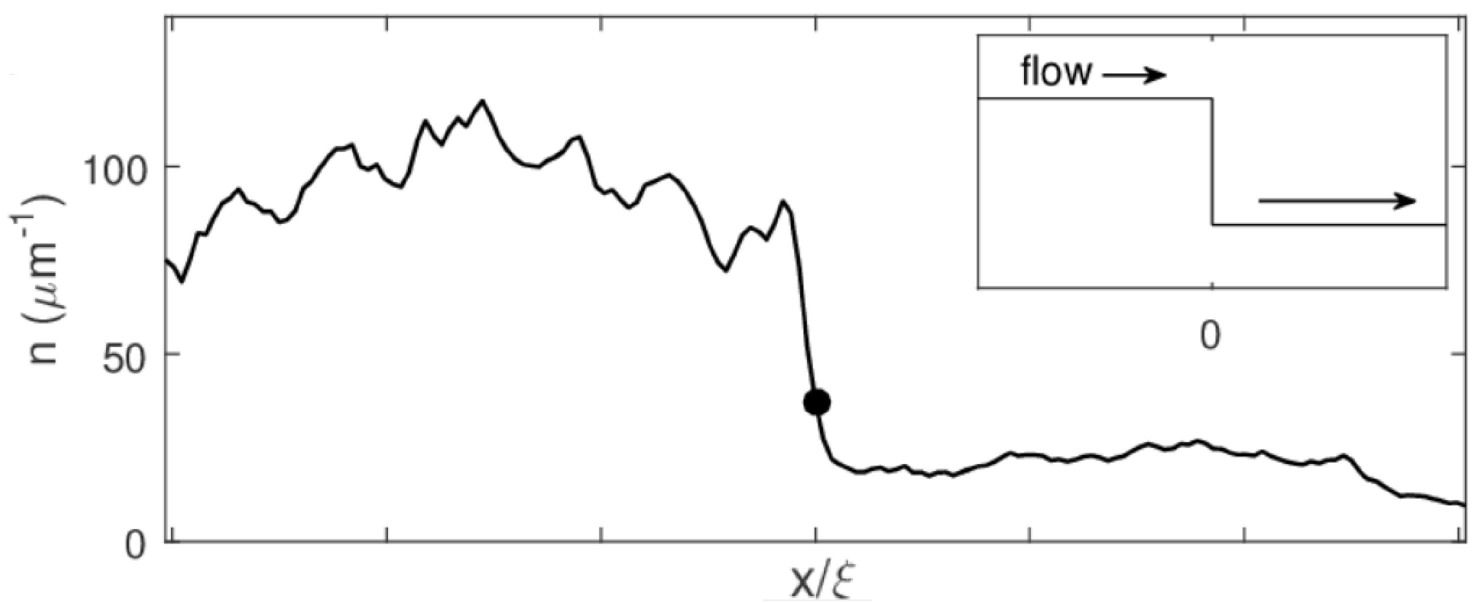
JRMdN, V. I. Kolobov, K. Golubkov & J. Steinhauer Nature 569, 688 (2019)

Background condensate

-CONDENSATE ACCELERATED BY A WATERFALL POTENTIAL TO SUPERSONIC SPEEDS!

-DENSITY IS MEASURED THROUGH IN-SITU IMAGING WITH CAMERA

-AVERAGE DENSITY OVER AN ENSEMBLE OF 7400 REPETITIONS



Background condensate

-WE NEED TO MEASURE THE PREDICTED HAWKING TEMPERATURE

$$k_B T_H = \frac{\hbar}{2\pi}\left[\frac{dv}{dx} - \frac{dc}{dx}\right]_{x=0}$$

-HOWEVER, COMPUTING THE ACTUAL FLOW VELOCITY IS VERY DEMANDING…

-USING CONTINUITY EQUATION +QUASI STATIONARITY ->

$$k_B T_H = \frac{\hbar}{2\pi}\left[\frac{dv}{dx} - \frac{dc}{dx}\right]_{x=0} = -\frac{\hbar}{2\pi}\left[\frac{v}{n}\frac{dn}{dx} + \frac{dc}{dx}\right]_{x=0}$$

-ONLY DENSITY MEASUREMENTS ARE NEEDED!

Background condensate

-ACCURATE MEASUREMENT -> ACCURATE KNOWLEDGE OF THE RELATION BETWEEN DENSITY AND SPEED OF SOUND.

-ADAPTED FORMALISM OF NON-POLYNOMIAL GPE [*L. Salasnich et al. PRA 69, 045601 (2004)]*

$$\mathfrak{L}[\Psi] = \int \mathrm{d}^3\mathrm{x}\ i\hbar\Psi^*(\mathrm{x},t)\partial_t\Psi(\mathrm{x},t) - \frac{\hbar^2}{2m}|\nabla\Psi(\mathrm{x},t)|^2 - V(\mathrm{x},t)|\Psi(\mathrm{x},t)|^2 - \frac{g_{3\mathrm{D}}}{2}|\Psi(\mathrm{x},t)|^4$$

-GAUSSIAN ANSATZ FOR TRANSVERSE DOF $\Psi(\mathrm{x},t) = \psi(x,t)\frac{e^{-\frac{\rho^2}{2\sigma^2(x,t)}}}{\sqrt{\pi}\sigma(x,t)},\ \rho = \sqrt{y^2+z^2}$

-NEW MODIFIED GPE->MODIFIED SOUND SPEED->NON-HARMONIC TRAP (ODT)!

$$\begin{aligned} i\hbar\partial_t\psi(x,t) &= -\frac{\hbar^2}{2m}\partial_x^2\psi(x,t) + \left[V(x,t) - \frac{\bar{U}_0}{1+\left(\frac{x}{x_R}\right)^2}\right]\psi(x,t) \\ &+ \frac{g_{1\mathrm{D}}(x)\left[1-\alpha_0\sqrt{1+2|\psi(x,t)|^2 a}\right]}{\sqrt{1+2|\psi(x,t)|^2 a}}|\psi(x,t)|^2\psi(x,t) \\ &+ \frac{\hbar\omega_r(x)}{2}\left[\sqrt{1+2|\psi(x,t)|^2 a} + \frac{1}{\sqrt{1+2|\psi(x,t)|^2 a}} - 2\right]\psi(x,t) \end{aligned}$$

$$V_{\mathrm{trap}}(\mathrm{x}) = -U_0\frac{w_0^2}{w^2(x)}e^{-2\frac{\rho^2}{w^2(x)}},\ w(x) = w_0\sqrt{1+\left(\frac{x}{x_R}\right)^2}$$

$$c(x) = \sqrt{\frac{2\hbar\omega_r(x)n(x)a}{m}}\sqrt{\frac{1+\frac{3n(x)a}{2}}{(1+2n(x)a)^{\frac{3}{2}}} - \frac{\hbar\omega_r(x)}{2U_0}}$$

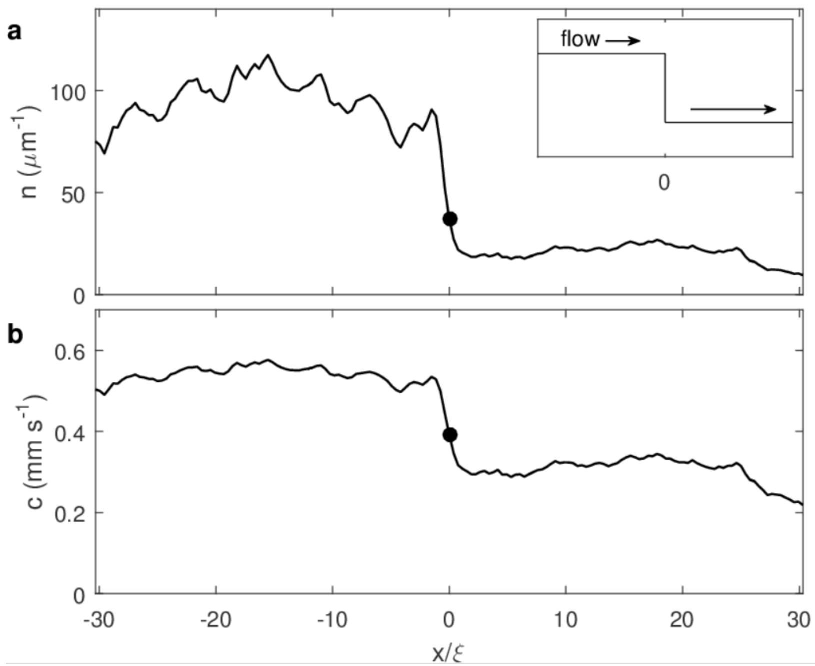
GREAT AGREEMENT WITH 3D SIMULATIONS!

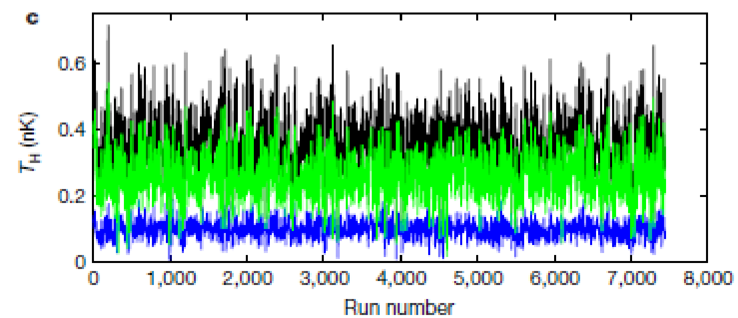
Background condensate

AVERAGE OVER ENSEMBLE

7400 REPETITIONS!

$$k_B T_H = \frac{\hbar}{2\pi}\left[\frac{dv}{dx} - \frac{dc}{dx}\right]_{x=0} = -\frac{\hbar}{2\pi}\left[\frac{v}{n}\frac{dn}{dx} + \frac{dc}{dx}\right]_{x=0}$$





$$T_H = 0.351(4)\ \mathrm{nK}$$

~200 Solar Masses!

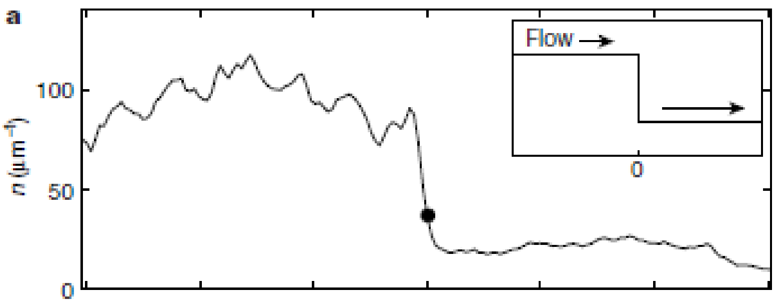
$$c(x) = \sqrt{\frac{2\hbar\omega_r(x)n(x)a}{m}}\sqrt{\frac{1+\frac{3n(x)a}{2}}{(1+2n(x)a)^{\frac{3}{2}}} - \frac{\hbar\omega_r(x)}{2U_0}}$$

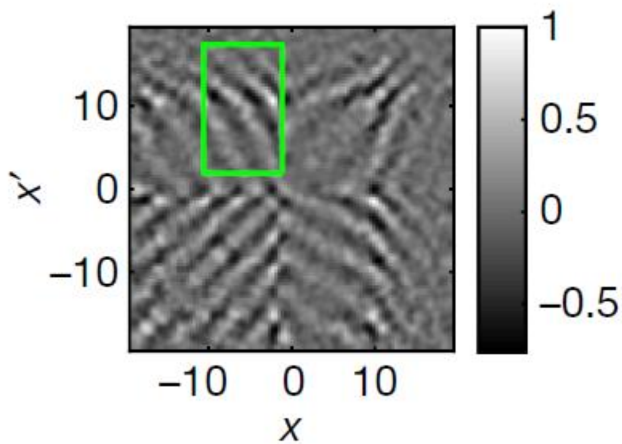
Oscillating horizon

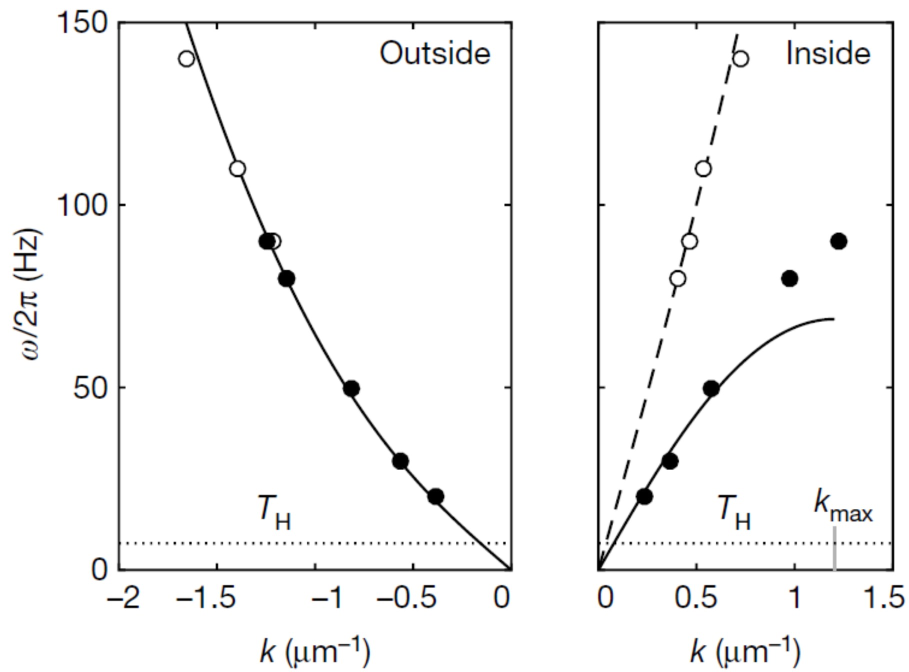
POSITION OF THE HORIZON OSCILLATES WITH GIVEN FREQUENCY

-> OUTGOING WAVES ARE GENERATED

EACH REPETITION WITH A RANDOM PHASE-> DENSITY-DENSITY CORRELATIONS-> FOURIER TRANSFORM-> OUTGOING DISPERSION RELATION





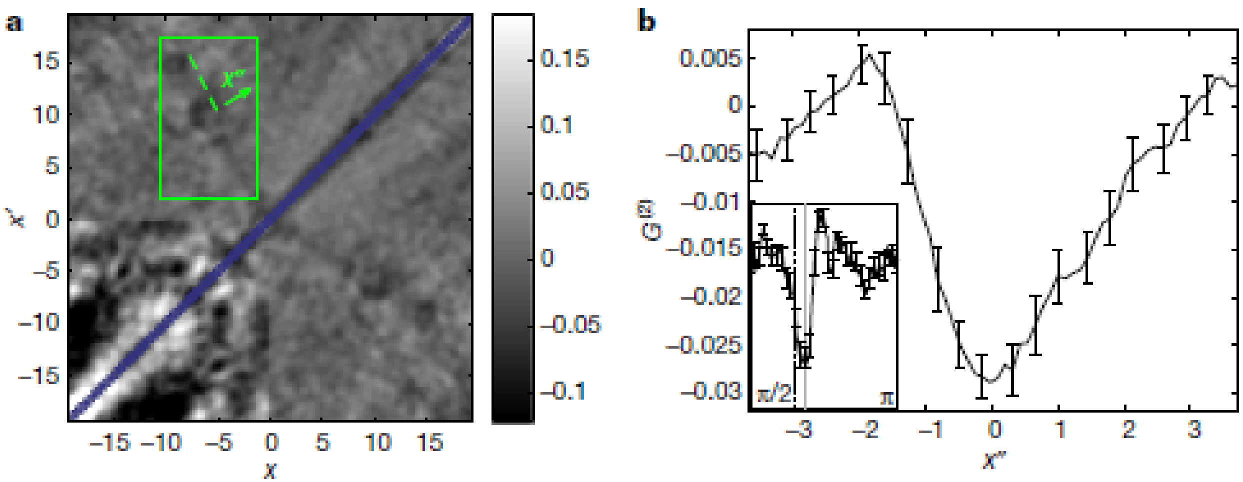


Density-density correlations

-Density fluctuations->Density-density correlation $G^{(2)}(x,x') = \sqrt{\frac{\xi_u \xi_d}{n_u n_d}} \langle \delta n(x) \delta n(x') \rangle$

-Signal averaged along the correlation angle.

-Good agreement with hydrodynamic angle!



-Background substracted due to experimental reasons (filtering).

-Theory?

Theoretical model

-Quantum fluctuations are modeled using the Truncated Wigner method:

Classical field+stochastic initial condition $$\Psi_W(x,0)=\Psi_0(x)+\sum_{n=1}^{M}\left[\gamma_n u_n(x)+\gamma_n^* v_n(x)\right]$$

Numerical integration in time: $$i\hbar\partial_t\Psi_W(x,t)=\left[-\frac{\hbar^2\partial_x^2}{2m}+V(x,t)+g|\Psi_W(x,t)|^2\right]\Psi_W(x,t)$$

Amplitudes of the modes sampled from the equilibrium Wigner distribution $\langle\gamma_m^*\gamma_n\rangle=\frac{\delta_{nm}}{2},\ \langle\gamma_m\gamma_n\rangle=0$
(Independent Gaussian variables)

Average over ensemble=Quantum expectation values of symmetric ordered operators!

$$\langle\Psi^*(x')\Psi(x)\rangle_W=\frac{\langle\hat\Psi^\dagger(x')\hat\Psi(x)\rangle+\langle\hat\Psi(x)\hat\Psi^\dagger(x')\rangle}{2}=\langle\hat\Psi^\dagger(x')\hat\Psi(x)\rangle+\frac{\delta(x-x')}{2}$$

For the second-order correlation function:

$$\begin{aligned}\langle\hat\Psi^\dagger(x)\hat\Psi^\dagger(x')\hat\Psi(x)\hat\Psi(x')\rangle-\langle\hat\Psi^\dagger(x)\hat\Psi(x)\rangle\langle\hat\Psi^\dagger(x')\hat\Psi(x')\rangle &= \langle\Psi^*(x')\Psi^*(x')\Psi(x)\Psi(x')\rangle_W-\langle\Psi^*(x)\Psi(x)\rangle_W\langle\Psi^*(x')\Psi(x')\rangle_W\\ &-\ \delta(x-x')\frac{\langle\Psi^*(x')\Psi(x)\rangle_W+\langle\Psi^*(x)\Psi(x')\rangle_W}{2}+\frac{\delta^2(x-x')}{4}\end{aligned}$$

Numerical procedure

1-Computation of the ground state using imaginary time GP equation.

2-Computation of the BdG modes (cutoff M~700 modes)

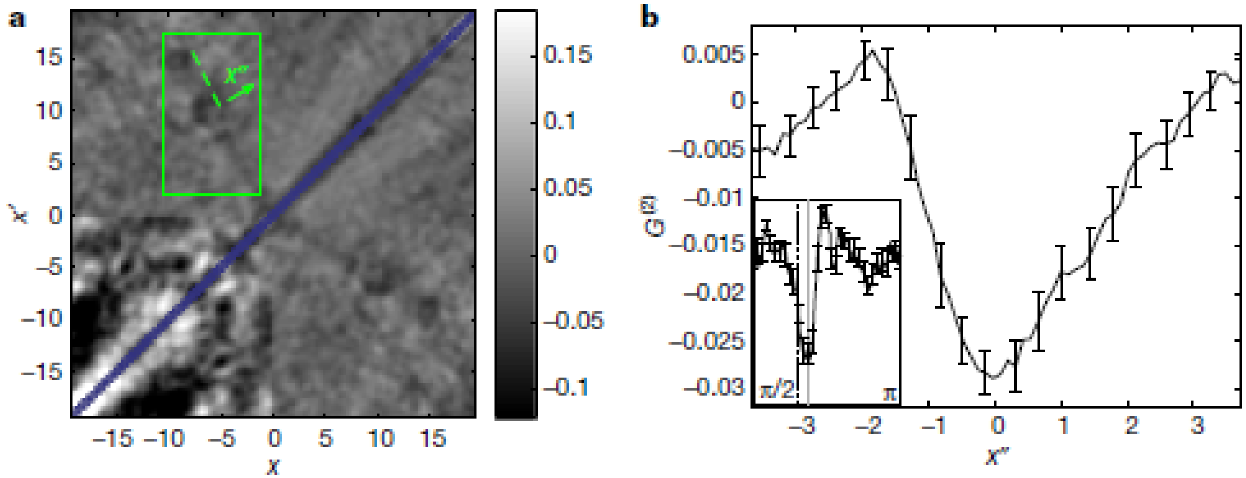
3-Initial stochastic condition sampled from the equilibrium Wigner distribution

4-Numerical integration of the time-dependent GP equation.

5-Computation of the correlation by averaging over the ensemble
(typically 10000 realizations)→Focus on non-diagonal elements!

Density-density correlations

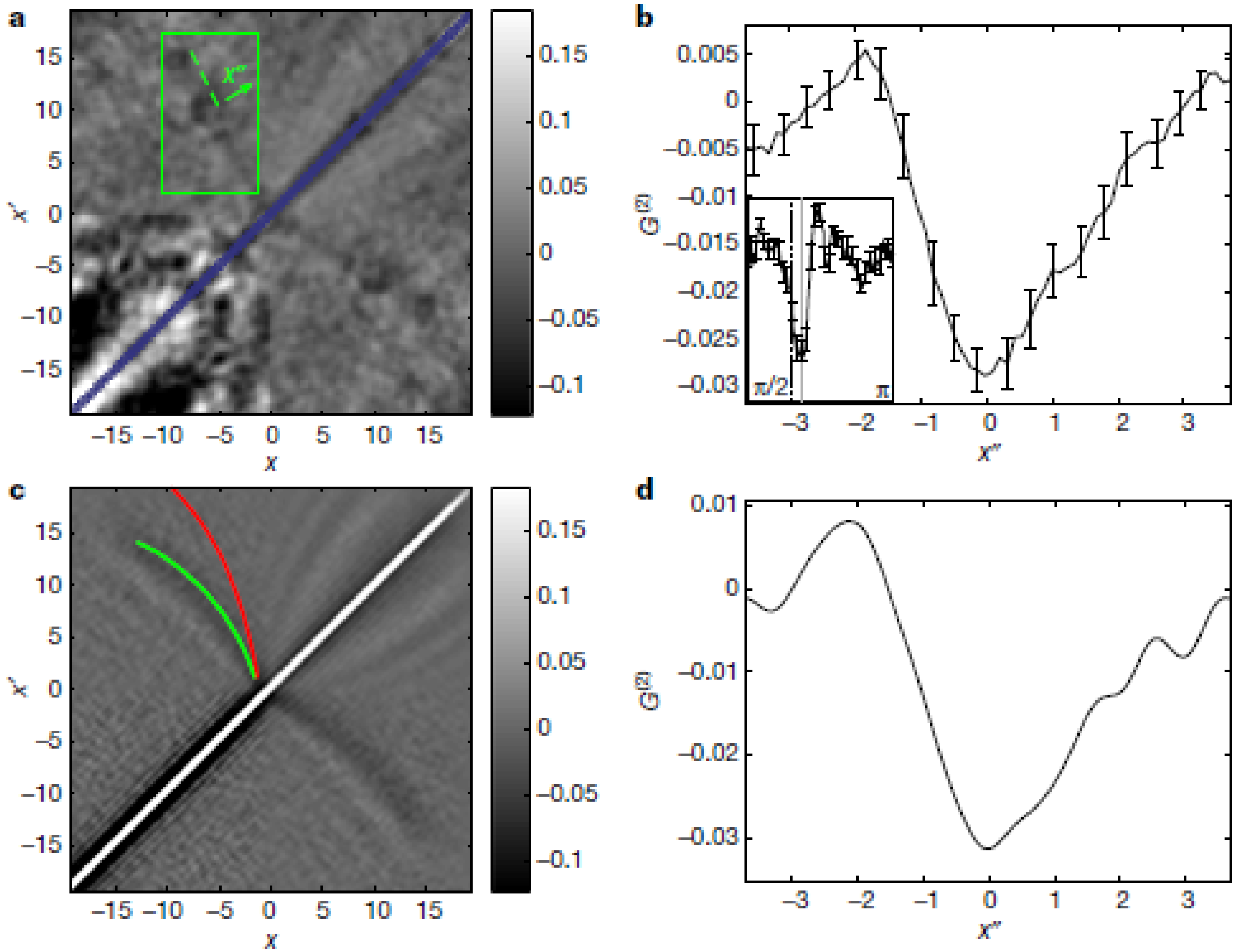
Experiment



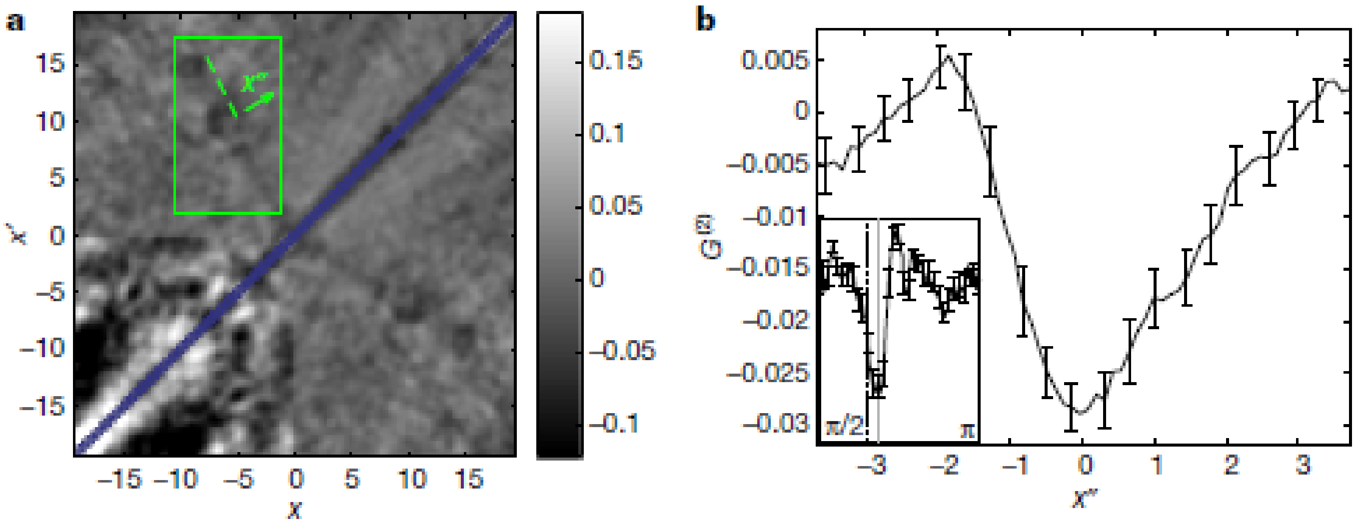
Density-density correlations

Experiment

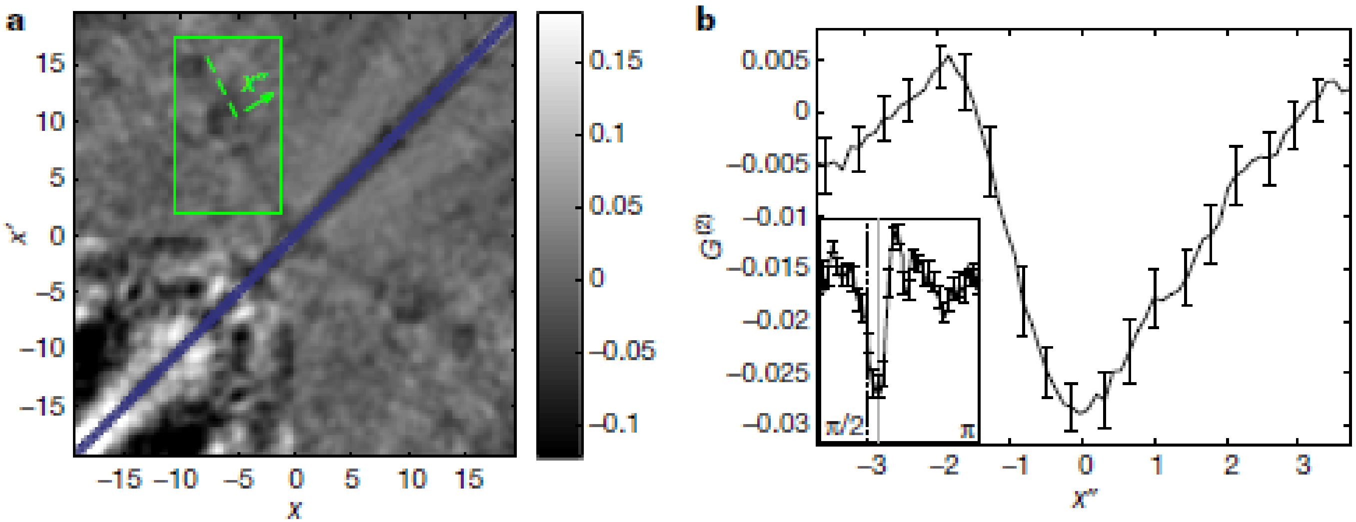
Simulation

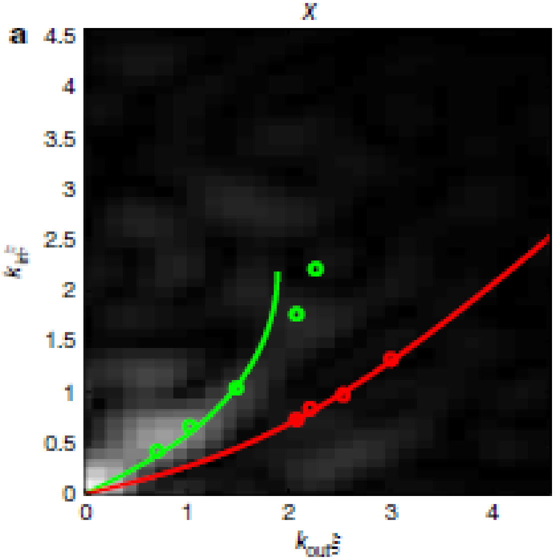


Correlation spectrum



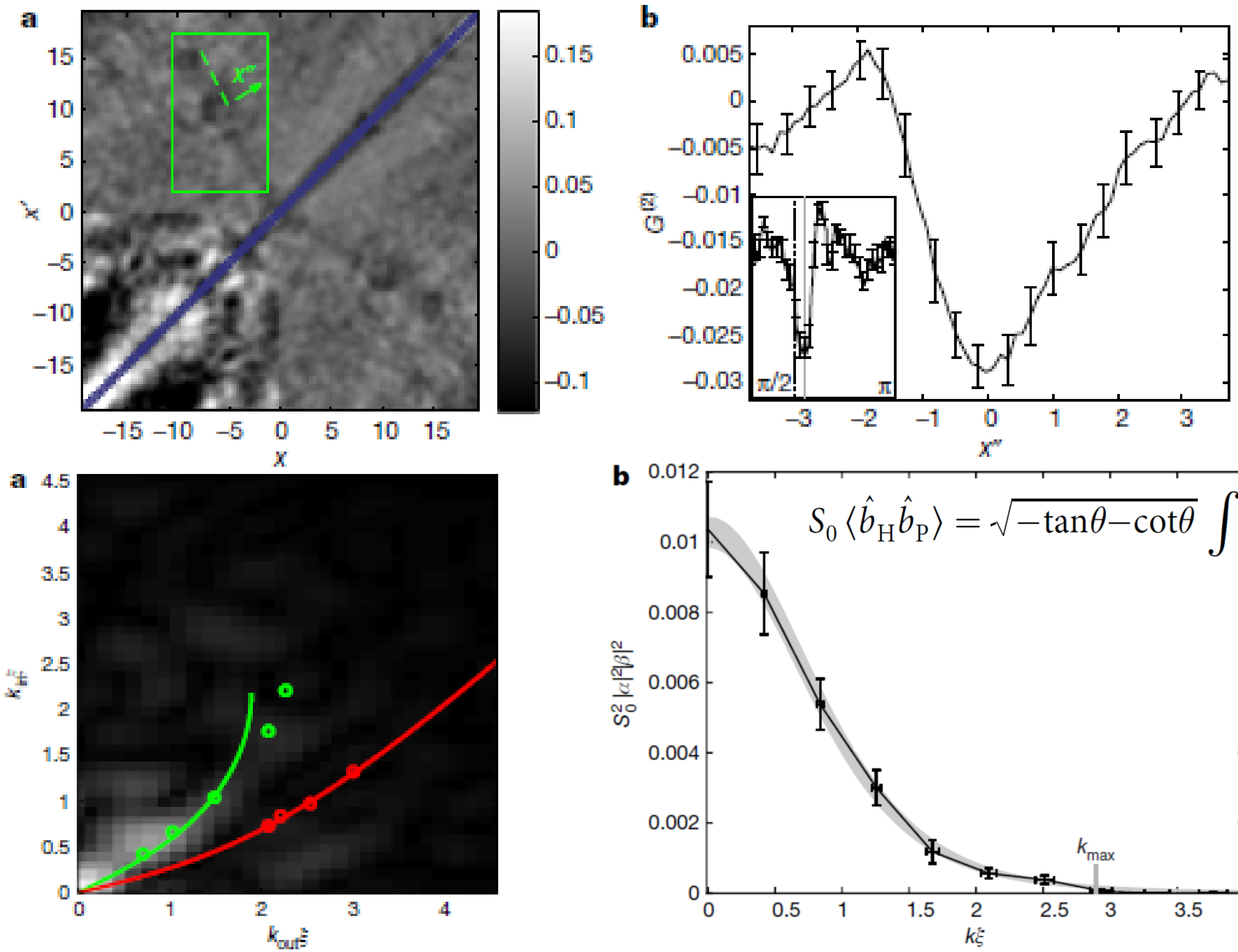
Correlation spectrum





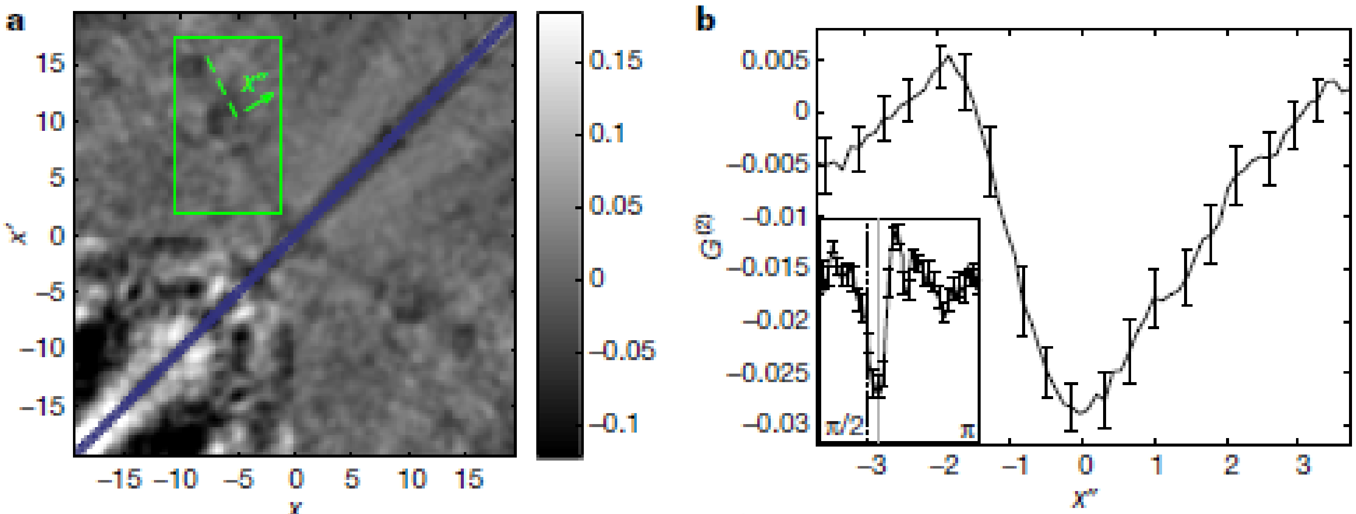
-Hawking radiation so far… is the predicted spontaneous thermal emission?

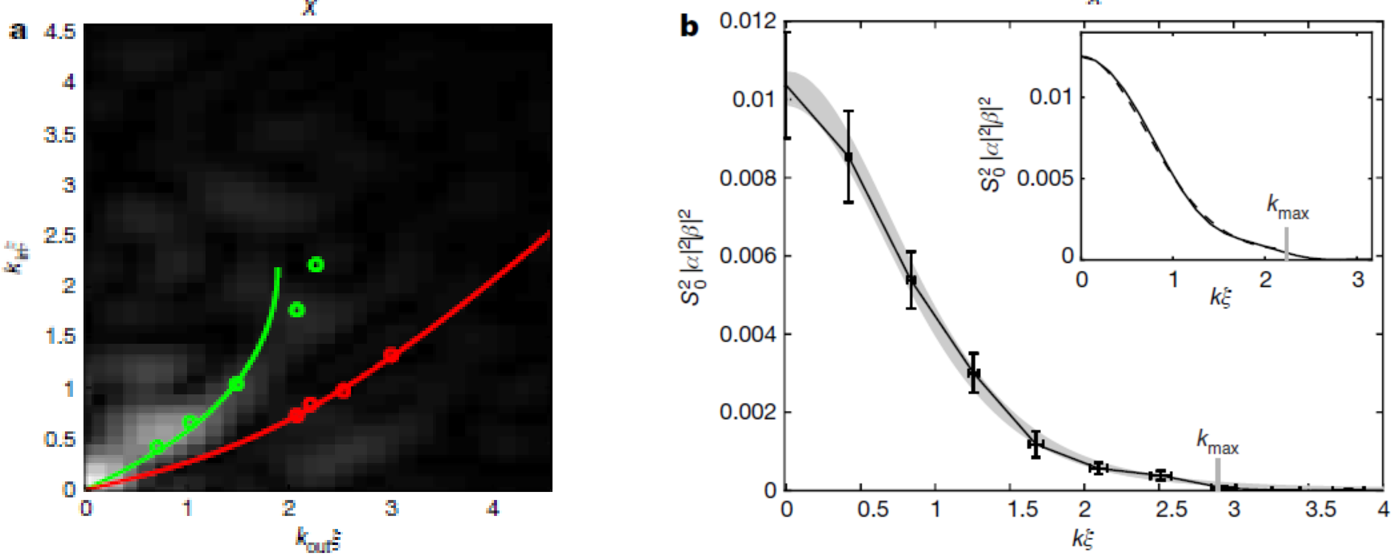
Correlation spectrum



$$S_0 \langle \hat{b}_{\mathrm{H}} \hat{b}_{\mathrm{P}} \rangle = \sqrt{-\tan\theta - \cot\theta} \int \mathrm{d}x'' \mathrm{e}^{ikx''} G^{(2)}(x, x')$$

Correlation spectrum





Conclusions

-Hawking radiation was observed.

-Spectrum thermal -> Very good agreement with the theory!

-Dispersion irrelevant.

-No grey-body factors.