Synthesis of Majorana mass terms in low-energy quantum systems

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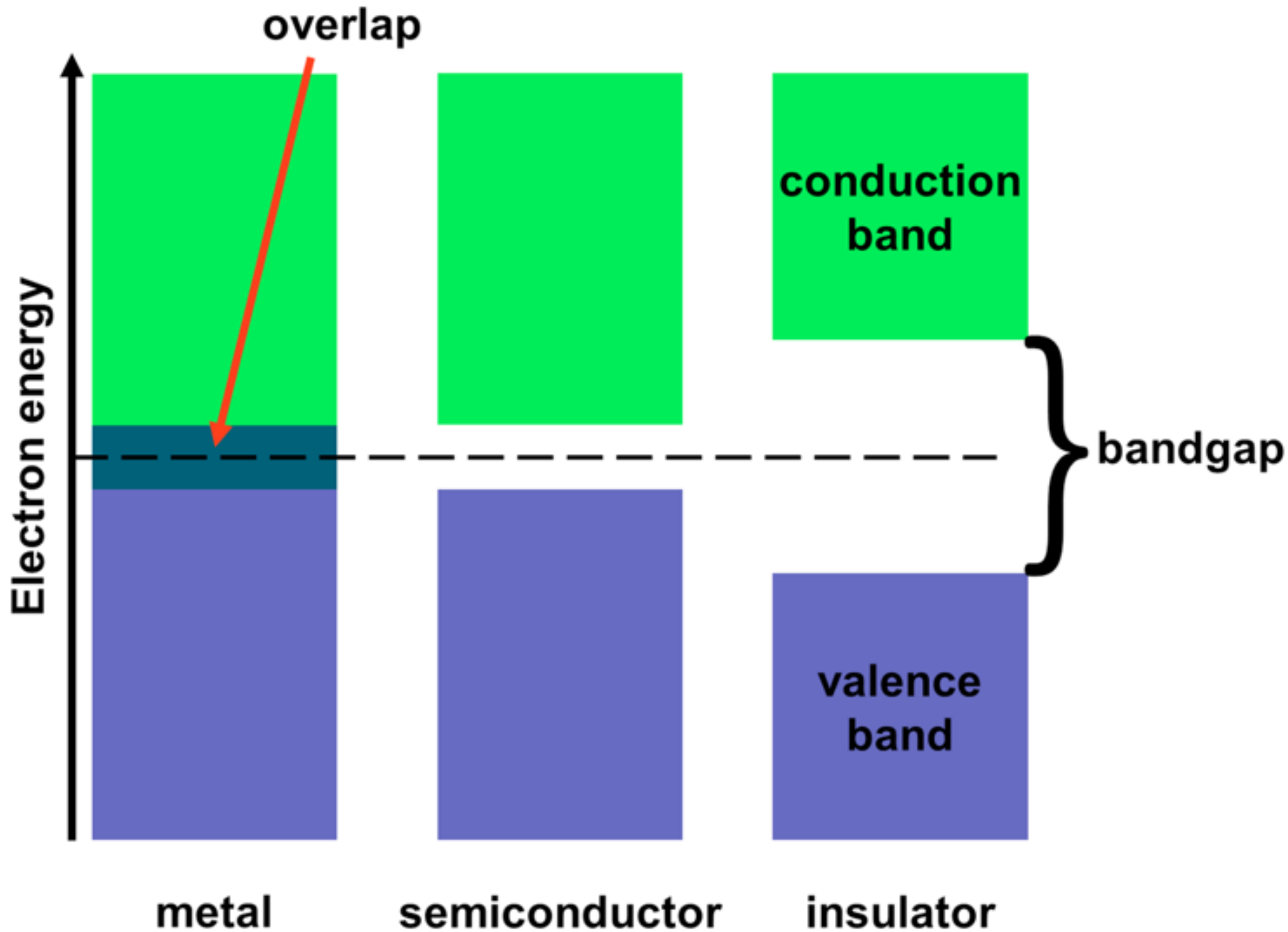
Standard band theory

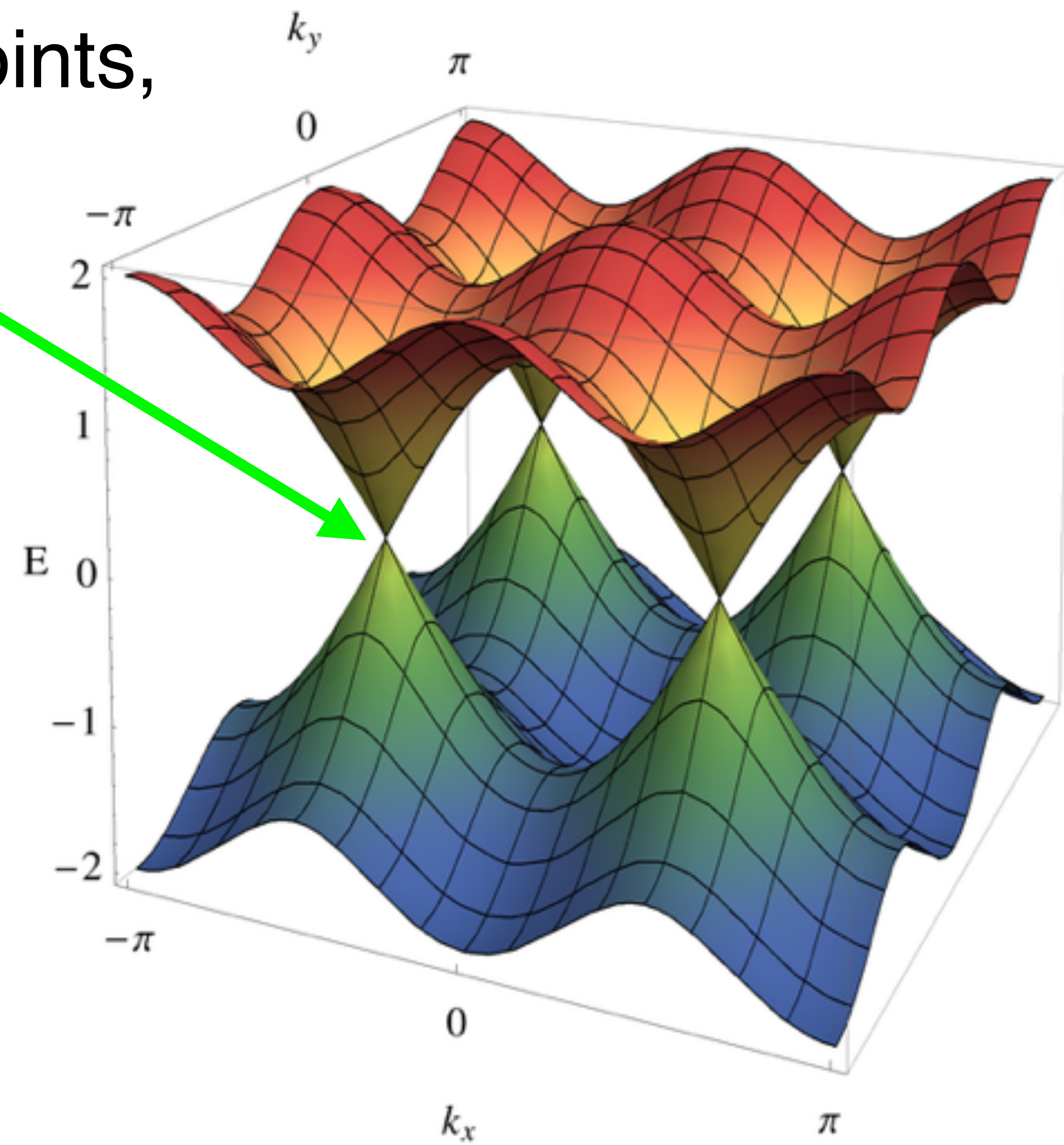
Fermi level



Typical energy scale: ev

Another possibility: semimetals

Band touching at single points,
0-D Fermi surface



Always in pairs
("chirality")

H.B. Nielsen, M. Ninomiya,
Phys. Lett. B 130 (1983) 389

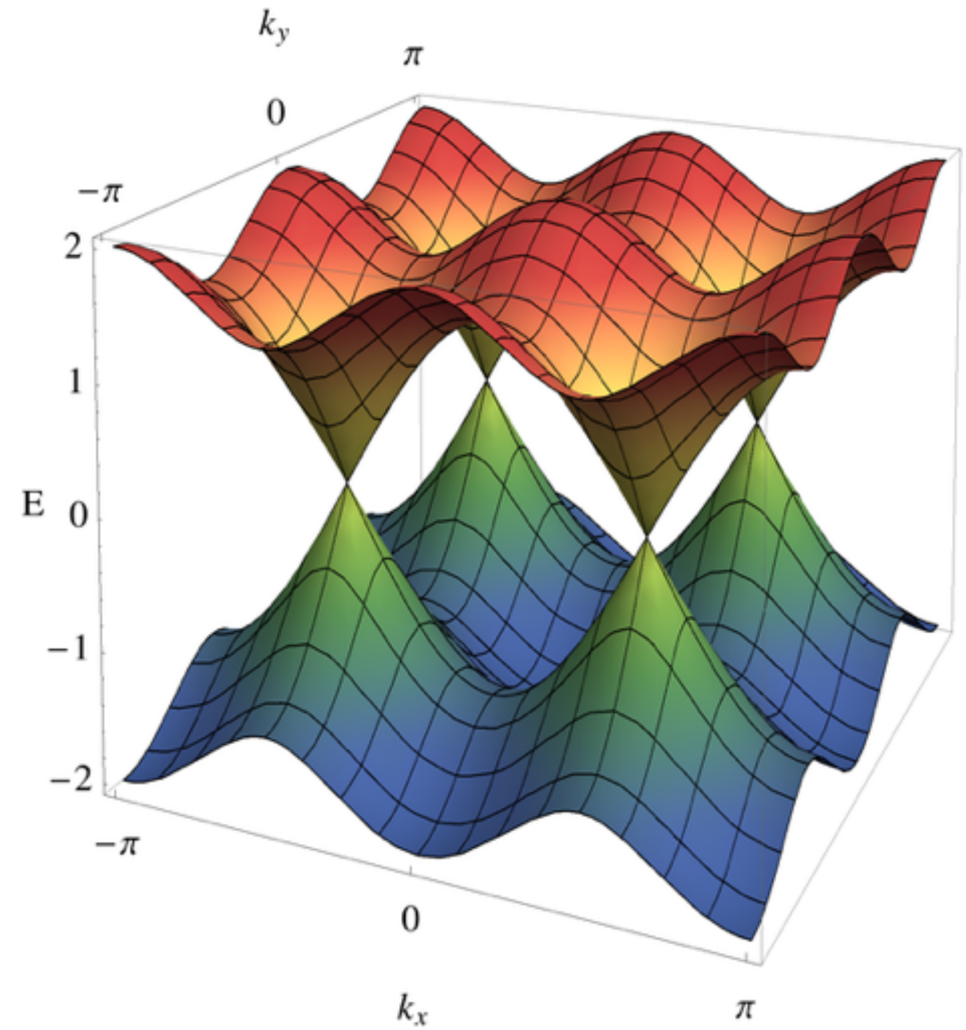
Vanishing
density of states

$\rho(E) \sim E^2$ (isotropic 3D)

$\rho(E) \sim E$ (isotropic 2D, graphene)

Another possibility: semimetals

Band touching at a single point,
0-D Fermi surface

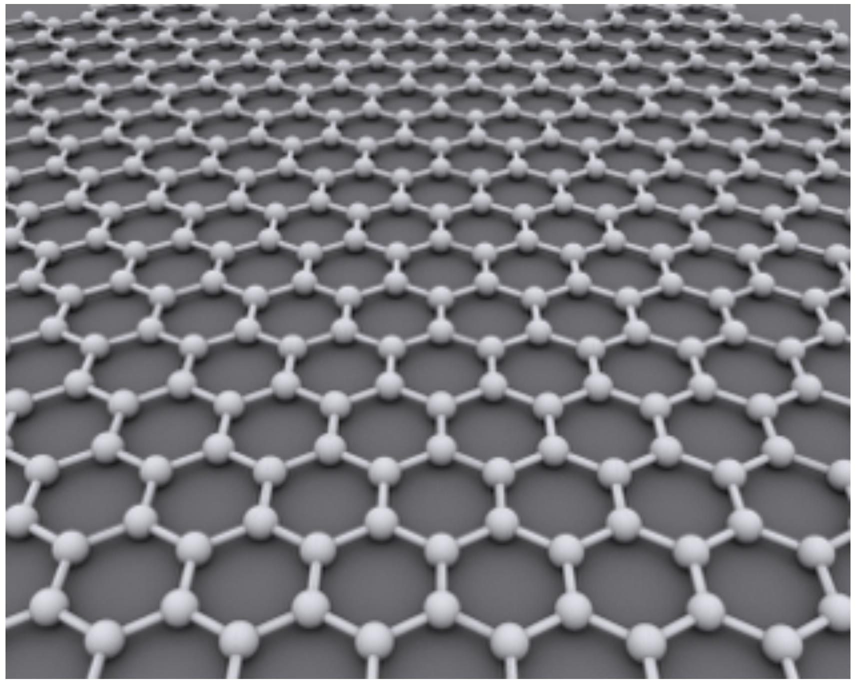


Examples:

3D:
TaAs and NbAs compounds
Science **349**, 613 (2015);
Nat. Phys. (2015)

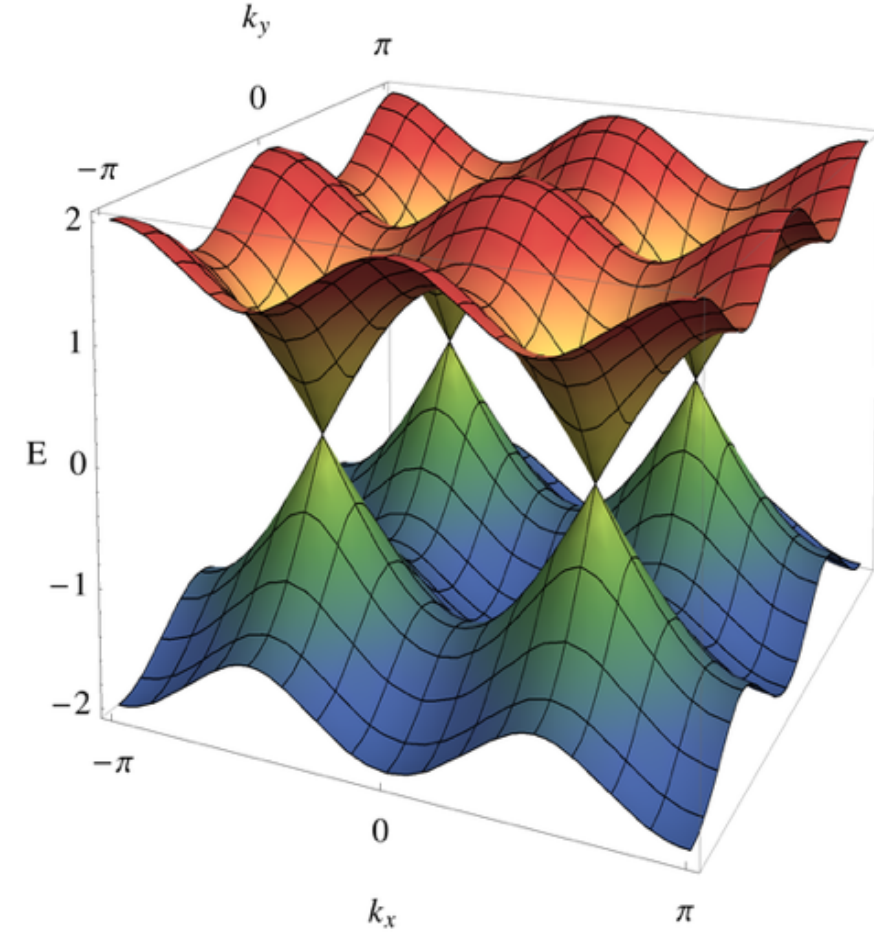
Photonic crystals
Science **349**, 622 (2015)

2D: graphene



Band touching
“mostly” linear

In the vicinity
of the touching points
(isotropic case) :



$$H\vec{\Psi} = \begin{pmatrix} \vec{k}\cdot\vec{\sigma} & 0 \\ 0 & -\vec{k}\cdot\vec{\sigma} \end{pmatrix} \begin{pmatrix} \vec{\psi}_R \\ \vec{\psi}_L \end{pmatrix}$$

Weyl equation for chiral bulk modes

Chiralities in momentum space

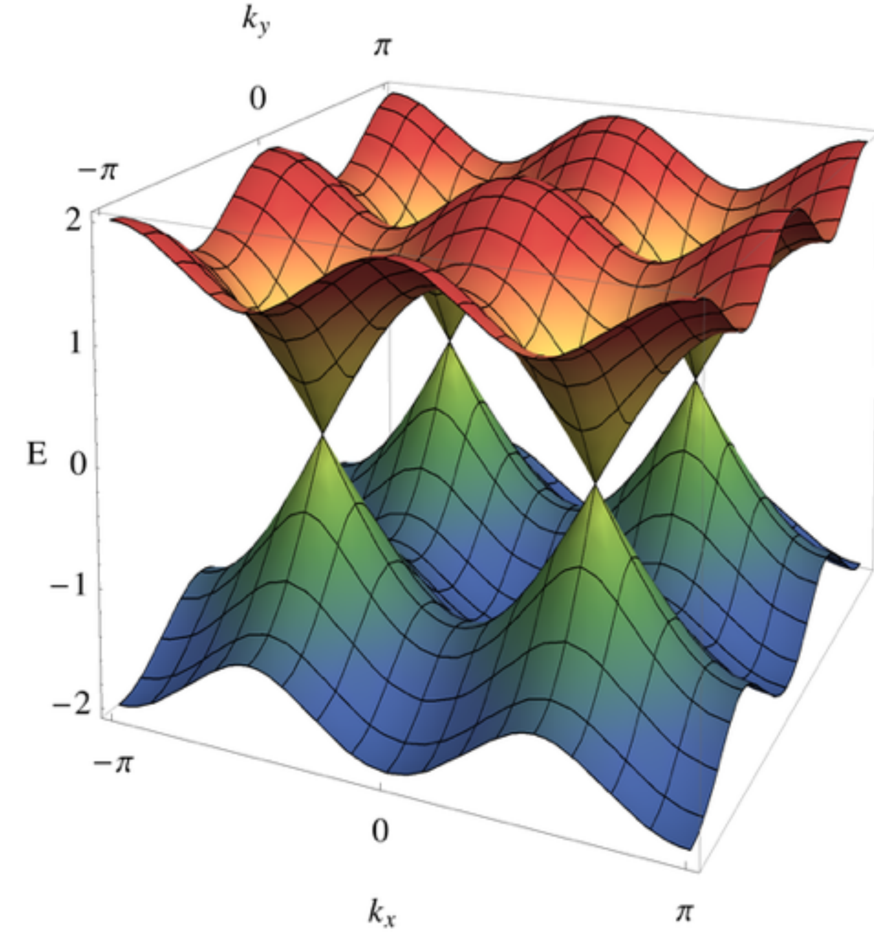
Emergent
Lorentz invariance

→

Chirality for Weyl spinors labels
massles representation of
Lorentz group

with $\vec{S} = \frac{1}{2}$

Band touching
"mostly" linear



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Weyl equation for chiral bulk modes

Chiralities in momentum space

Graphene

$$\vec{\psi}_{R/L}(\mathbf{k}) = \left(c_A(\mathbf{k}_{R/L} + \mathbf{k}), c_B(\mathbf{k}_{R/L} + \mathbf{k})\right)$$

$A\,,B$ sublattice - spinor indices

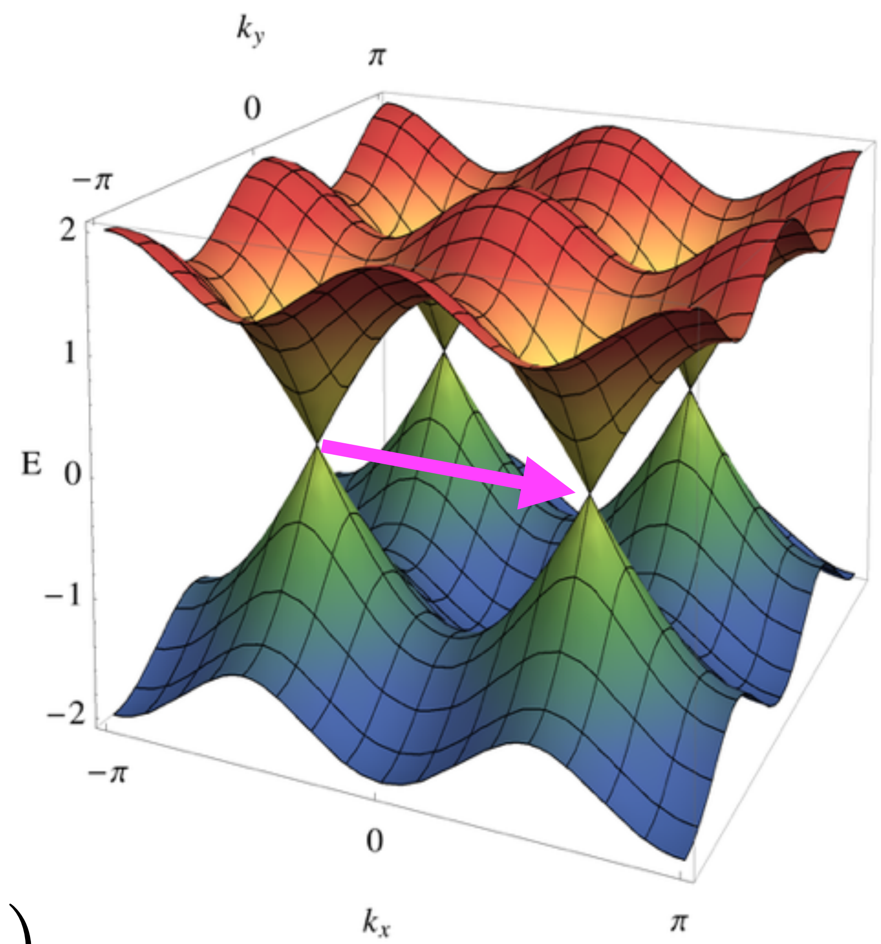
Dirac mass term

$$m_D \left(\psi_R^\dagger \psi_L + \text{H.c.}\right)$$

Lorentz-invariant

Connecting Weyl points,
adding momentum

e.g. via a Bragg pulse
(in cold-atoms exp.)



$$H_B = V_0\left(\sum_{\vec{n}} a_n^\dagger a_n \, e^{i\vec{k}\cdot\vec{n}} e^{-i\omega t} + \text{h.c.}\right)$$

Suitable periodicities in the B.Z.
must be exploited

L. Lepori, G. Mussardo, and A. Trombettoni,
Europhys. Lett. 92 50003 (2010).

Free spinors: what else ?

Emergence of Majorana spinors

L. Lepori, A. Celi, A. Trombettoni, and M. Mannarelli,

”Synthesis of Majorana mass terms in low-energy quantum systems”,

New J. Phys. 20 063032 (2018).

Alessio Celi
IQOQI, Innsbruck
UAB, Barcelona

Massimo Mannarelli
LNGS, L'Aquila

Andrea Trombettoni
CNR/SISSA, Trieste

Majorana spinors (modes)

Teoria simmetrica dell'elettrone e del positrone, in "Il Nuovo Cimento", a. XIV, 1937, pp. 171–184

F. Wilczek, Nature Physics 5, 614 - 618 (2009)

$$\psi^C = C\,\psi^* = \psi \sim \psi^* = \psi$$

$E = 0$ or additive $\{q_i\} = 0$

M. spinors characterized by a M. mass term, the other Lorentz-invariant one

So far only massless Majorana modes

$$E = 0$$

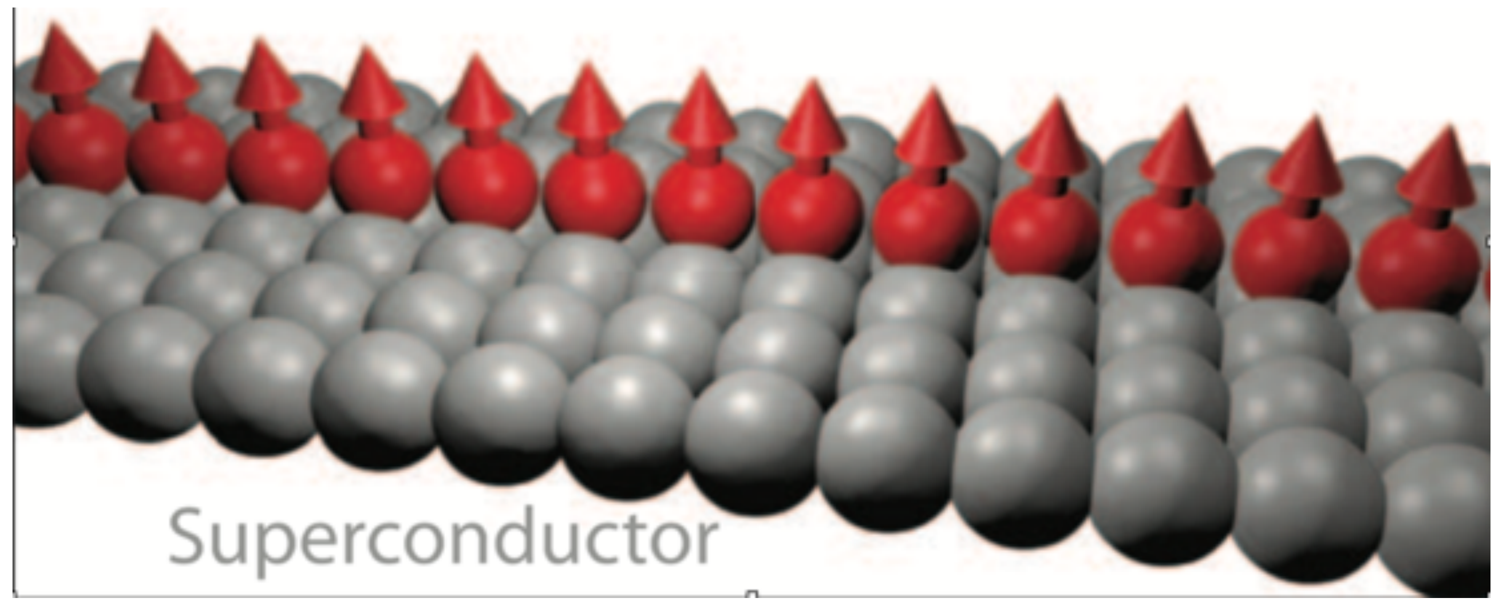
Majorana modes

Quite recently synthesized:

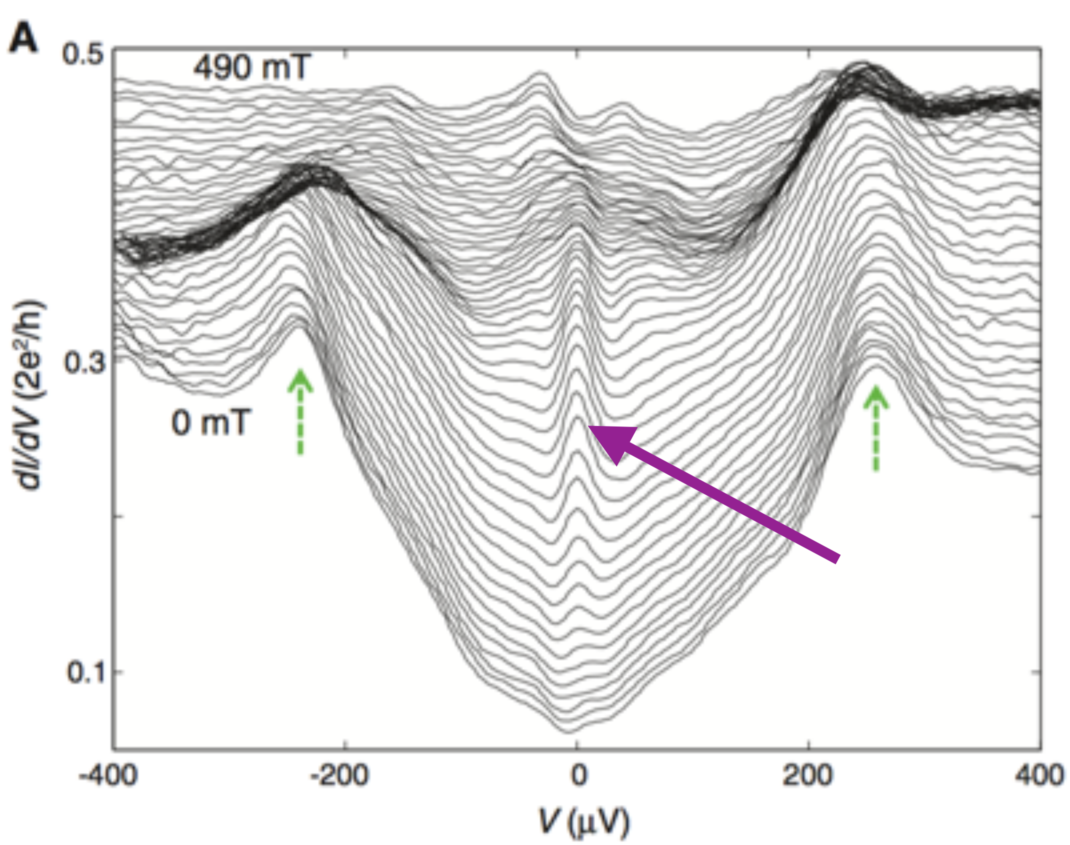
V. Mourik et al., Science 336, 6084 (2012)

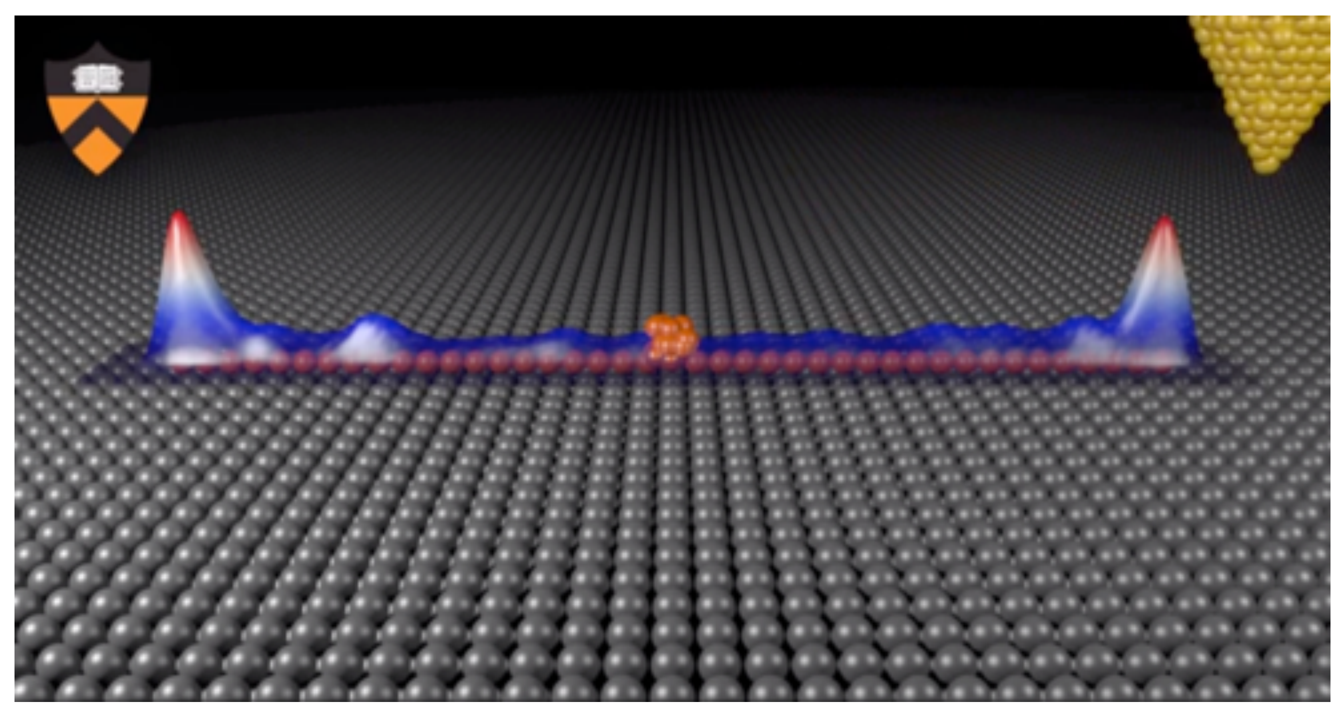
S. Nadj-Perge et al., Science 346, 602 (2014).

InSb nanow. ,
Fe atoms.



strong S.O. required





Majorana modes (E = 0)

No internal (spinor) structure, no Fermi statistics

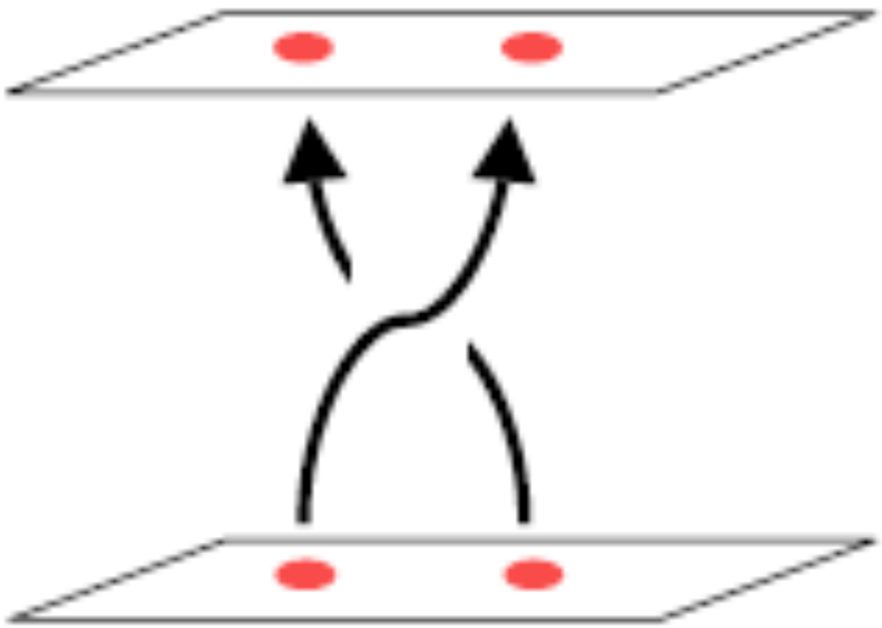
(abelian anyons)

$$\{\psi_i, \psi_j^\dagger\} = \delta_{ij}$$ (canonical quant.)

Majorana cond. (and no spinor structure)

$$\{\psi_i, \psi_j\} = \delta_{ij}$$!!!

A. Stern, Annals of Physics
Volume 323, Issue 1, January 2008



Majorana equation

$$i\,\gamma^\mu \partial_\mu \psi = m\,\psi^C$$

J. Casanova et al.,
Phys. Rev. X 1, 021018 (2011).

Majorana spinors ??

Defined from Weyl spinors by Majorana mass terms

$$m_M \left(\psi_R^t\, i\,\sigma_2 \psi_R - \psi_L^t\, i\sigma_2\, \psi_L + \text{H.c.}\right)$$

to be compared with Dirac mass terms

$$m_D \left(\psi_R^\dagger \psi_L + \text{H.c.}\right)$$

“superfluid pairing” between the same chiralities

If both are present :

- no phase redefinition for $\psi_{R/L}$ → CP violation
- see-saw mechanism, as for neutrinos

S. M. Bilenky and S. T. Petcov, Rev. Mod. Phys. 59, 671 (1987)

$$m_\pm = \frac{1}{2}\left|m_L + m_R \pm \sqrt{(m_L - m_R)^2 + 4m_D^2}\right|$$

Majorana spinors ??

Defined from Weyl spinors by Majorana mass terms

$$m_M\left(\psi_R^t\, i\,\sigma_2\psi_R - \psi_L^t\, i\sigma_2\,\psi_L + \text{H.c.}\right)$$

"superfluid pairing" between the same chiralities

to be compared with Dirac mass terms

$$m_D\left(\psi_R^\dagger\psi_L + \text{H.c.}\right)$$

Common lore: Majorana spinors from Bogoliubov quasiparticles

$$\mathcal{H}_{\text{BCS}}(\mathbf{k}) = \Delta\Big(\Phi^\dagger(\mathbf{k})\, i\sigma_2\,\Phi^*(-\mathbf{k}) + (\mathbf{k}\to-\mathbf{k})\Big) + \text{H.c.}$$

$$\Phi(\mathbf{k}) = \big(c_\uparrow(\mathbf{k}), c_\downarrow(\mathbf{k})\big)$$

$$\Psi(\mathbf{k}) = \Big(\Phi(\mathbf{k}), -i\sigma_2\, \Phi^*(-\mathbf{k})\Big)^T$$

Charge conjugation

$$\mathcal{H}_{\mathrm{BCS}}(\mathbf{k}) = -\,C^{-1}\,\mathcal{H}^*_{\mathrm{BCS}}(-\mathbf{k})\,C \qquad C = \sigma_2 \otimes \sigma_2$$

However........

- pairing in spin (not spinor) space
- BCS pairing does not require Weyl fermions (?!)
- BCS pairing in semimetals generally couple opposite chiralities

BAD !

Majorana spinors

- requires to start from Weyl spinors
- intravalley pairings (not at zero momentum)
- pairing in spinor (sublattice) space
- nearest-neighbour attraction required

$$\Delta_{ij} = \langle c_{iA\uparrow} c_{jB\downarrow} \pm c_{iA\downarrow} c_{jB\uparrow} \rangle \sim \Delta(\mathbf{k}_R)\, e^{i\mathbf{k}_R\cdot(\mathbf{i}+\mathbf{j})} \pm \Delta(\mathbf{k}_L)\, e^{i\mathbf{k}_L\cdot(\mathbf{i}+\mathbf{j})}$$

$\langle i, j \rangle$

Spin triplet (+): Kekulè pattern
(also spinless)

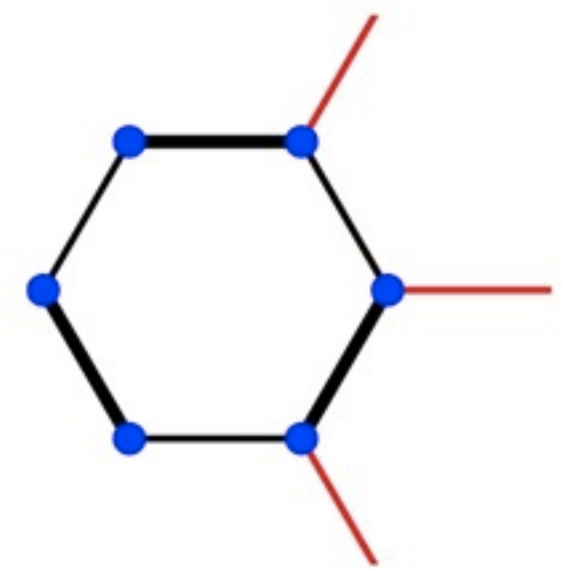


FIG. 1. (Color online) The unit cell of the Kekule lattice of superconducting bond order parameters. The red line corresponds to $\Delta \cos\alpha$, the bold line to $\Delta \cos(\alpha+2\pi/3)$, and the thin line to $\Delta \cos(\alpha-2\pi/3)$. The unit cell contains six sites (blue points) and

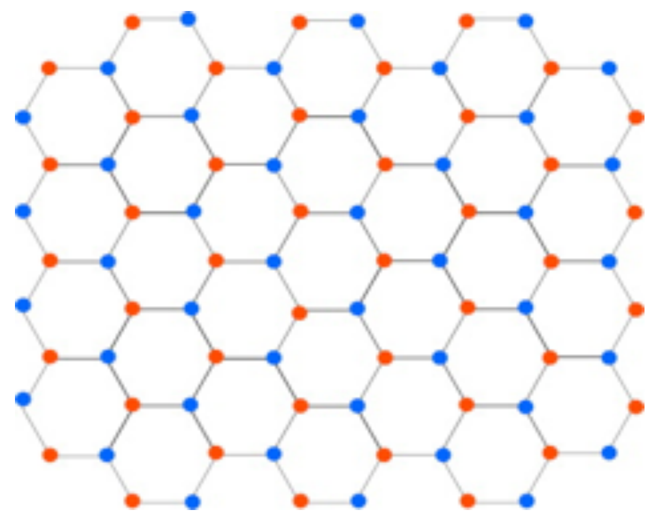
B. Roy and I. F. Herbut,
Phys. Rev. B 82, 035429 (2010).

Spin triplet: Kekulè pattern

+

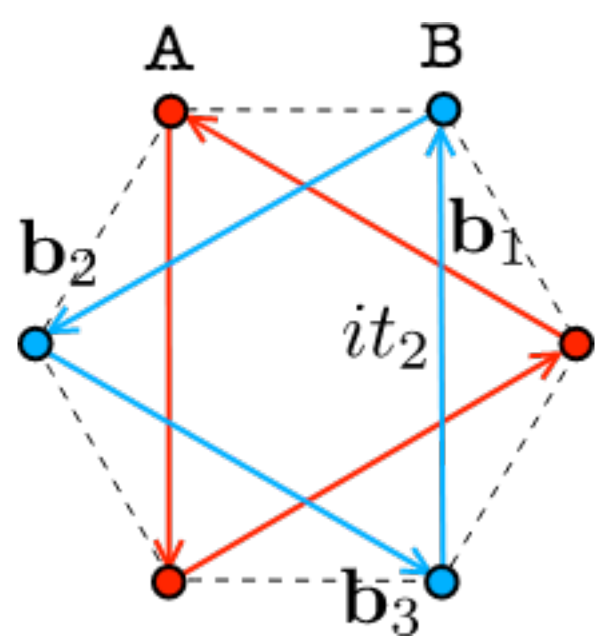
$$\Delta_{ij} = \langle c_{iA\uparrow}c_{jB\downarrow} \pm c_{iA\downarrow}c_{jB\uparrow}\rangle \sim \Delta(\mathbf{k}_R)\, e^{i\mathbf{k}_R\cdot(\mathbf{i}+\mathbf{j})} \pm \Delta(\mathbf{k}_L)\, e^{i\mathbf{k}_L\cdot(\mathbf{i}+\mathbf{j})}$$

Honeycomb



B. Roy and I. F. Herbut,
Phys. Rev. B 82, 035429 (2010).

Kane-Mele
model (2 Haldane)



S. Tsuchiya, J. Goryo, E. Arahata, and M. Sigrist,
Phys. Rev. B 94, 104508 (2016).

K. Lee, et al.,
Phys. Rev. B 99, 184514 (2019)

$$\hat{H} = \sum_{\langle ij\rangle} t_{ij}\hat{c}_i^\dagger \hat{c}_j + \sum_{\langle\langle ij\rangle\rangle} e^{i\Phi_{ij}} t'_{ij}\hat{c}_i^\dagger \hat{c}_j + \Delta_{\mathrm{AB}} \sum_{i\in A} \hat{c}_i^\dagger \hat{c}_i$$

F. D. M. Haldane,
Phys. Rev. Lett. 61, 2015–2018 (1988)

$$\mathcal{H}_M = \int \mathrm{d}\mathbf{p} \left(\Delta(\mathbf{k}_R)\, \psi^\dagger_{\uparrow,R}(\mathbf{p})\, i\sigma_2\, \psi^*_{\downarrow,R}(-\mathbf{p}) - (R \to L) \right) + \mathrm{H.c.} \Big)$$

$$\psi_{R,\alpha}(\mathbf{p}) \equiv \big(c_{A,\alpha}(\mathbf{p}), c_{B,\alpha}(\mathbf{p})\big) \qquad \alpha = \{\uparrow, \downarrow\}$$

$i\,\sigma_2$ in sub-lattice (spinor) indices !

Towards simulation (ongoing…)

- honeycomb-like lattices (two species), stabilize "FFLO"

Experimental realizations

L Tarruell, D Greif, T Uehlinger, G Jotzu, T Esslinger
Nature 483 (7389), 302-305 (2012)

G. Jotzu, et al.,
Nature 515, 237-240 (2014)

- NN interaction, from boson-fermion mixtures or dipolar couplings

required $V \gtrsim 3t$
(below uniform pairing)

L.-K. Lim, A. Lazarides, A. Hemmerich, and C. Morais Smith, Europhys. Lett. 88, 36001 (2009).

P. Massignan, A. Sanpera, and M. Lewenstein, Phys. Rev. A 81, 031607 (2010).

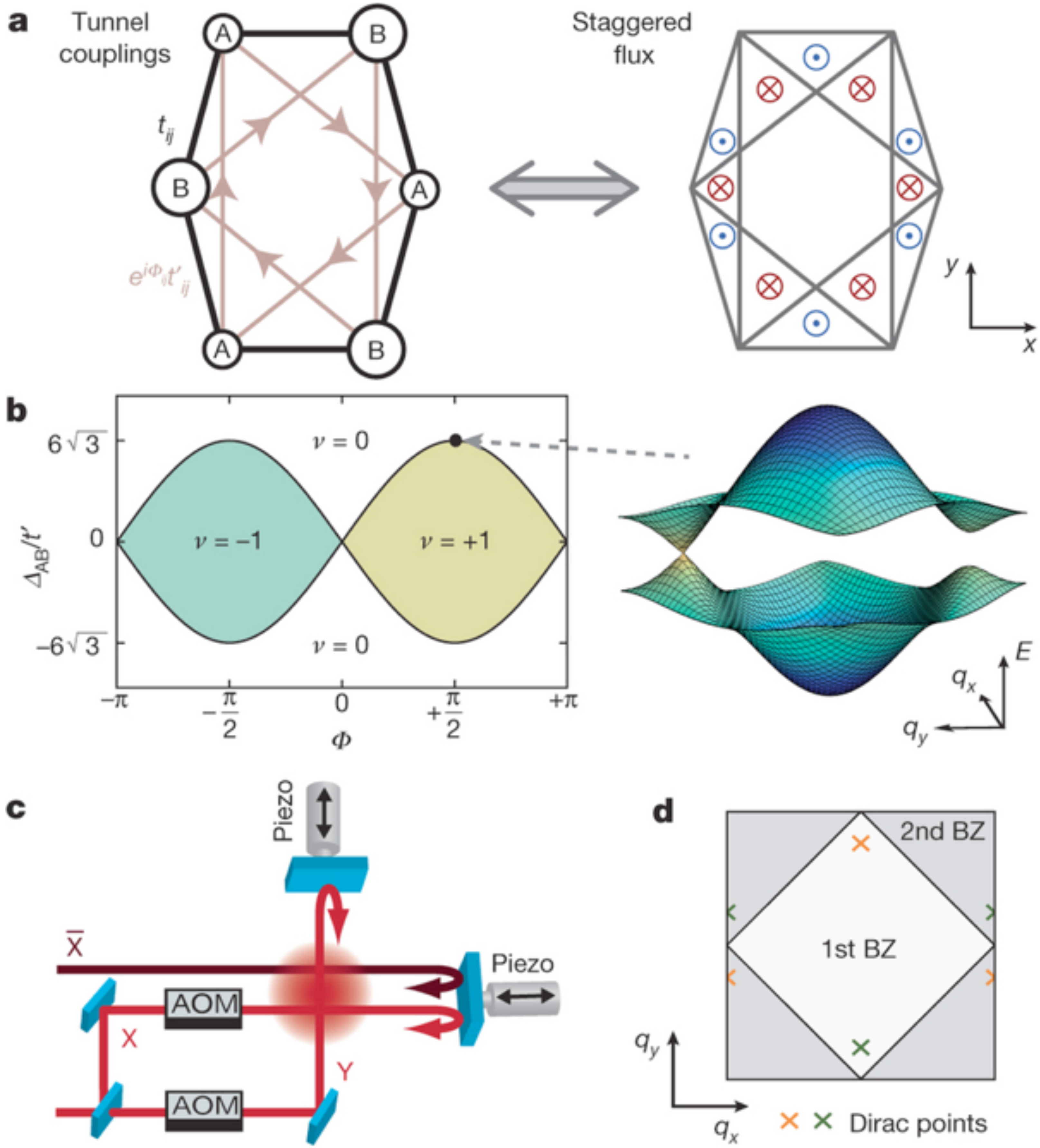
S. Baier, D. Petter, J. H. Becher, A. Patscheider, G. Natale, L. Chomaz, M. J. Mark, and F. Ferlaino, Phys. Rev. Lett. 121, 093602 (2018)

Different Zitterbewegung…….. $\sim \dfrac{t}{m_M} a$ few lattice sites

Next steps…….

- Different Zitterbewegung
- CP violation, see-saw
- Topological crystalline insulator ?

Thank you
for attention


a
Tunnel couplings
Staggered flux
A
B
t_{ij}
$e^{i\Phi_{ij}}t'_{ij}$
y
x
b
$6\sqrt{3}$
0
$-6\sqrt{3}$
Δ_{AB}/t'
$\nu = 0$
$\nu = -1$
$\nu = +1$
$-\pi$
$-\frac{\pi}{2}$
$+\frac{\pi}{2}$
$+\pi$
Φ
E
q_x
q_y
c
Piezo
$\bar{X}$
AOM
X
Y
d
2nd BZ
1st BZ
q_y
q_x
Dirac points