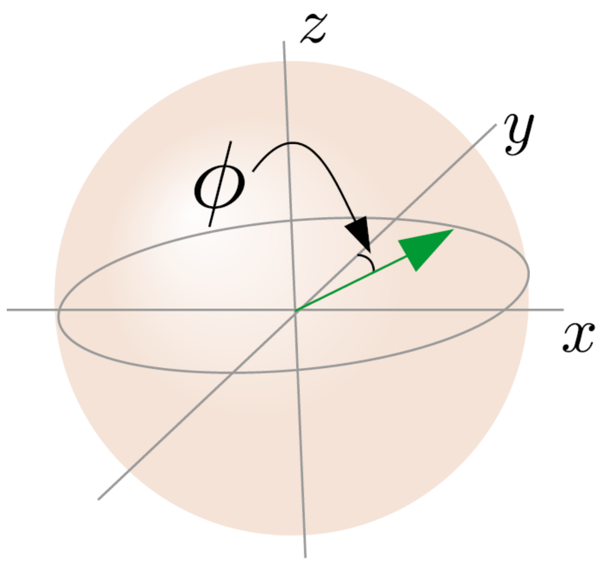
Continuous Real-Time Tracking of a Quantum Phase Below the Standard Quantum Limit

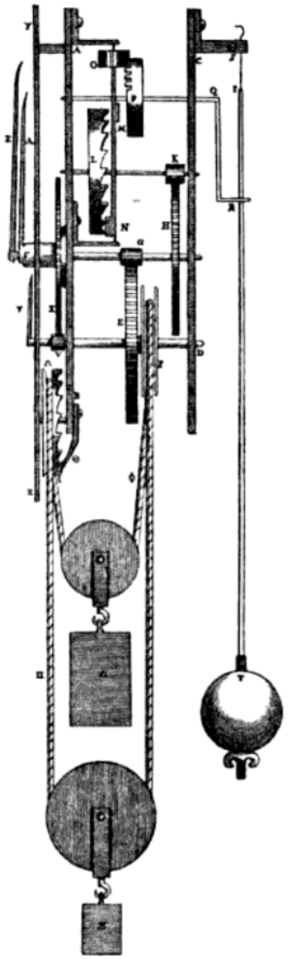


Murray Holland

JILA, University of Colorado Boulder

A story of two competing superradiance processes…

Shankar, Greve, Wu, Thompson, Holland, PRL (2018)



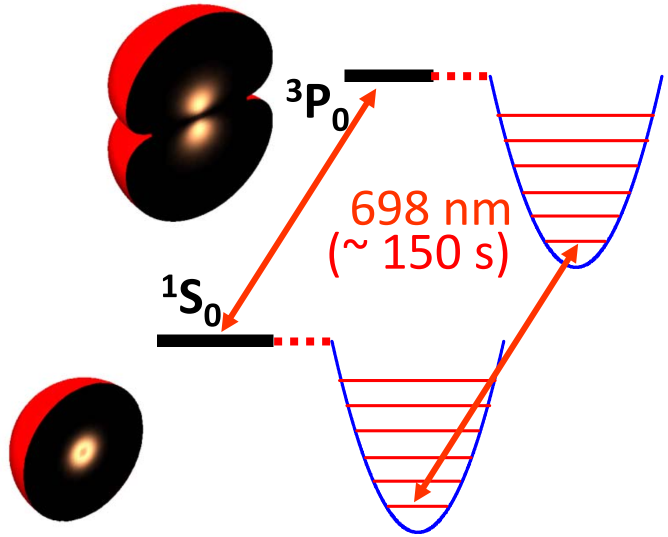
First Pendulum Clock 1656

Mechanical

Two components

- High Q oscillator
- Counter

Atomic

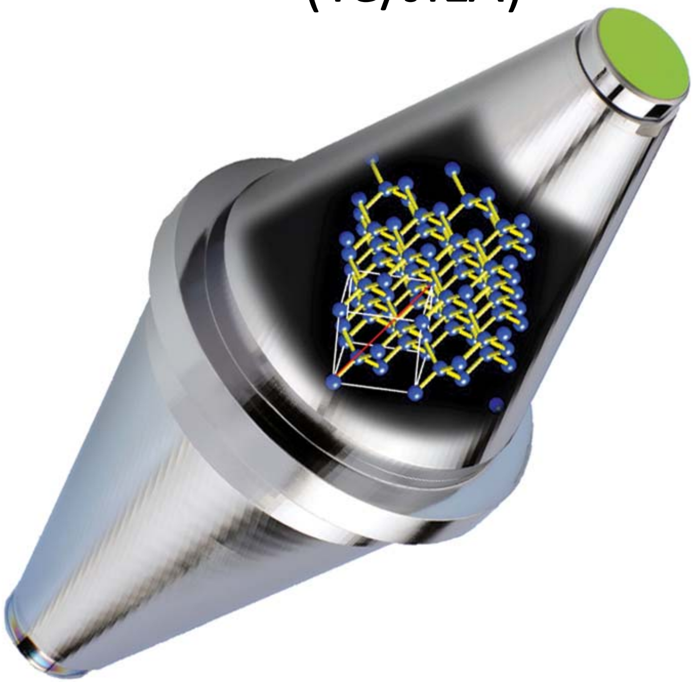


Quantum mechanics

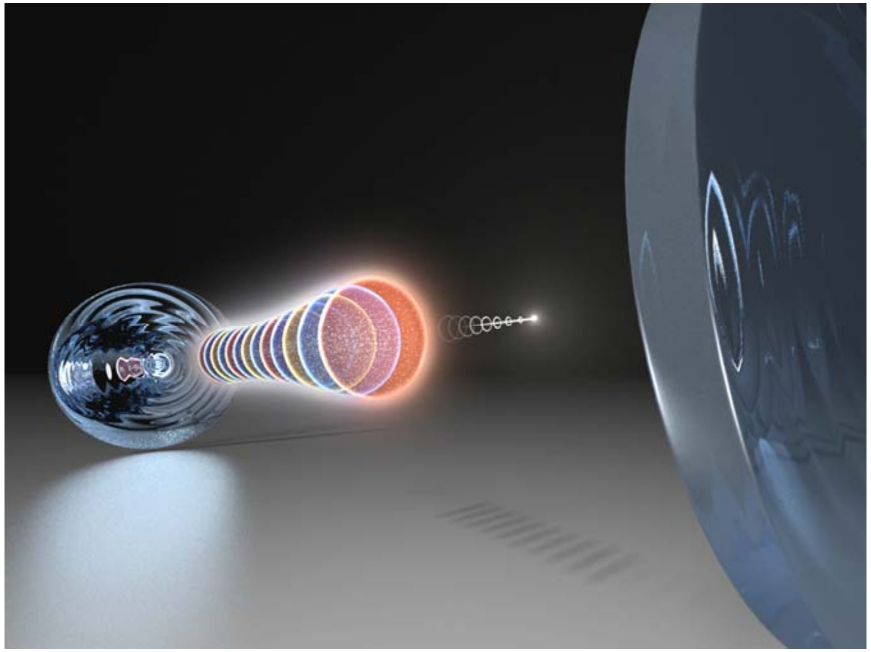
- Superposition
- Amplitude cannot gradually decay
- High Q~10^{17}

Highest quality oscillators: light and matter

Light: Crystalline Cavities
(Ye/JILA)



Matter: No-Photon Laser

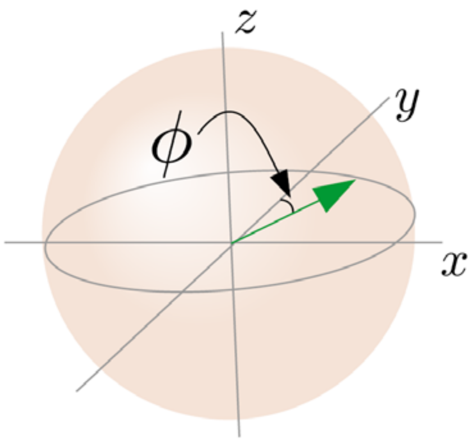


Two approaches: same goal (extreme optical coherence)

The quantum phase

$$|\psi\rangle \propto |\downarrow\rangle + e^{i\phi(t)}\,|\uparrow\rangle$$

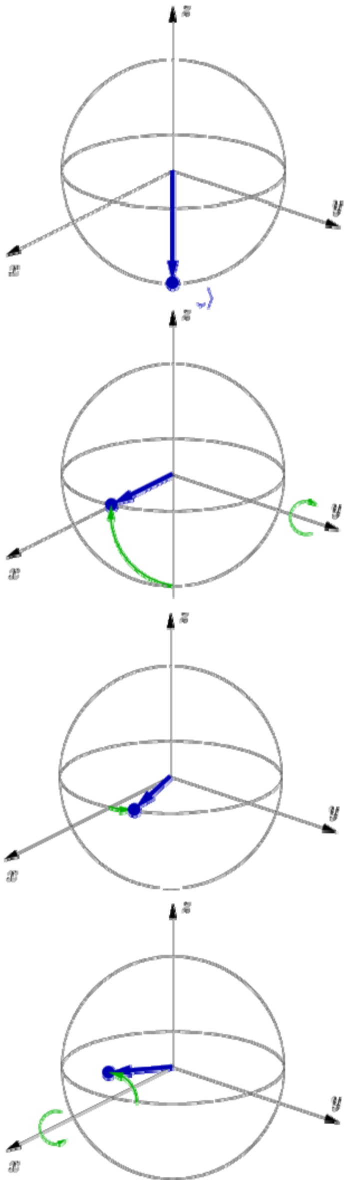
N spins

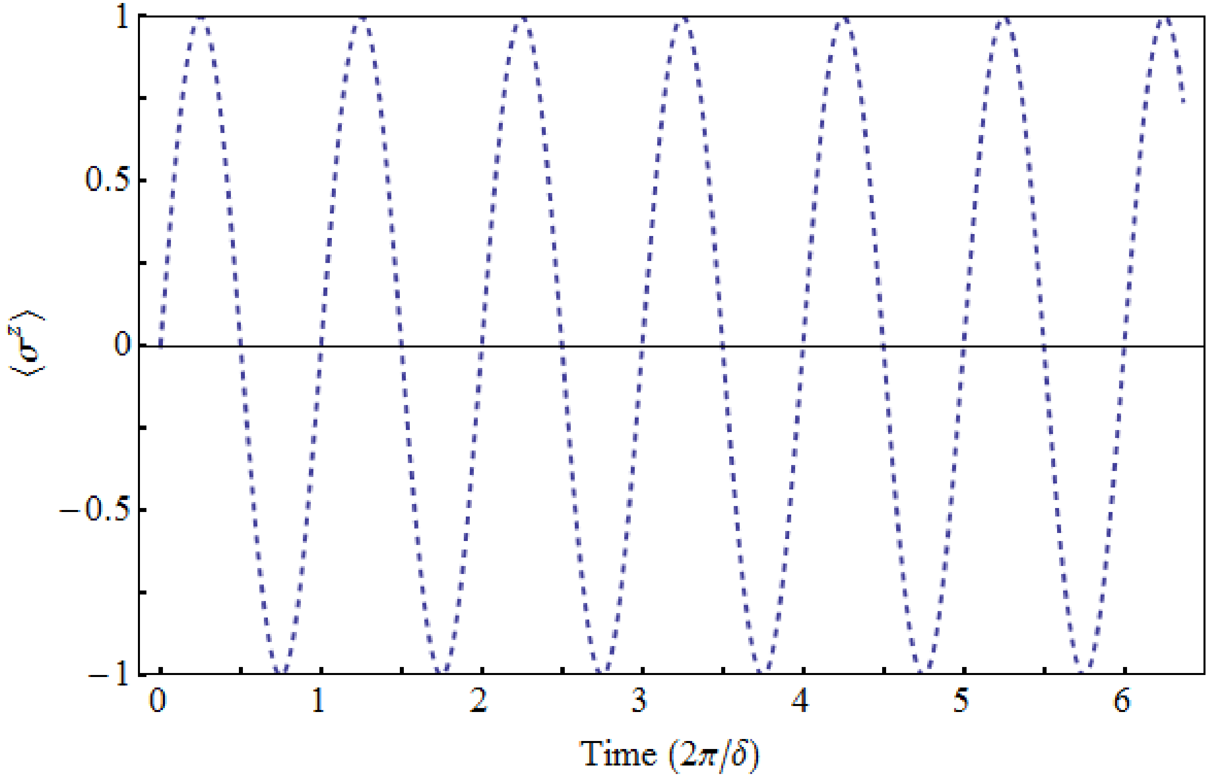


Populations are simply projections or components

- Cannot construct a true phase operator conjugate to number
- Alternative: phase-localized states, operationally defined …
- Useful concepts: Spin coherent state, Standard Quantum Limit

Measuring frequency: Ramsey fringes



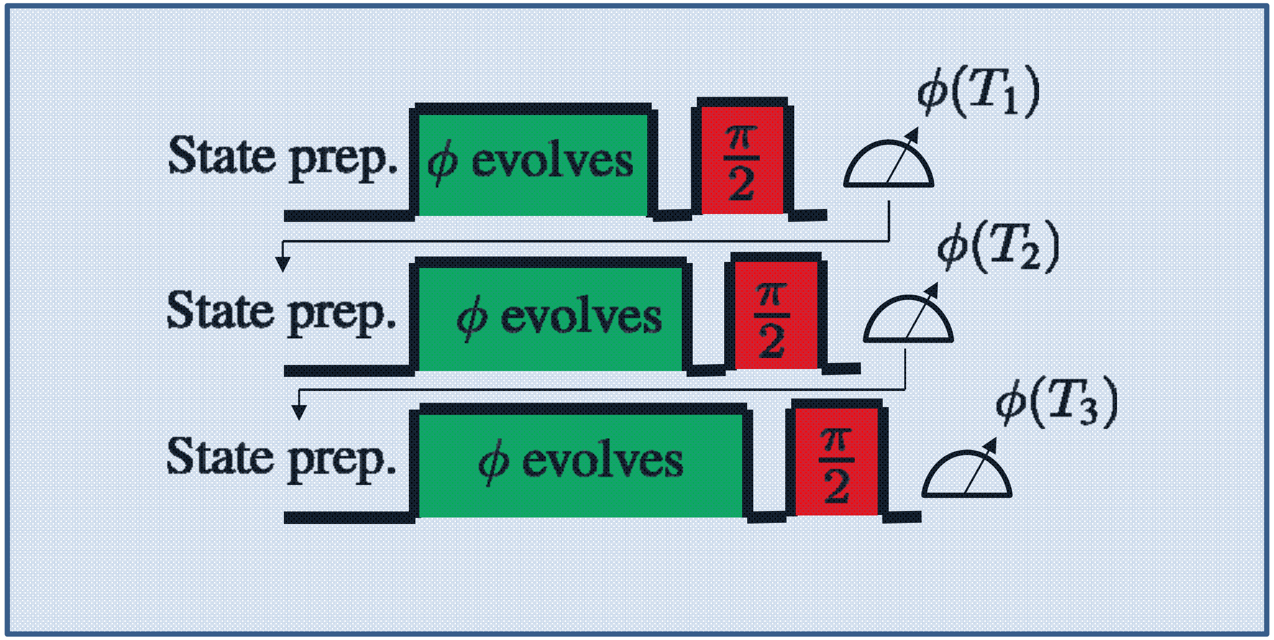


$$\frac{\Delta f}{f} = \frac{1}{2\pi\sqrt{N}n}$$

- *N*=Number of atoms or trials
- *n*=number of cycles

Measuring the quantum phase: Ramsey

$$|\psi\rangle \propto |\downarrow\rangle + e^{i\phi(t)}\,|\uparrow\rangle \qquad \text{Population} \propto \phi(T) = \int_0^T \frac{d\phi}{dt}dt$$



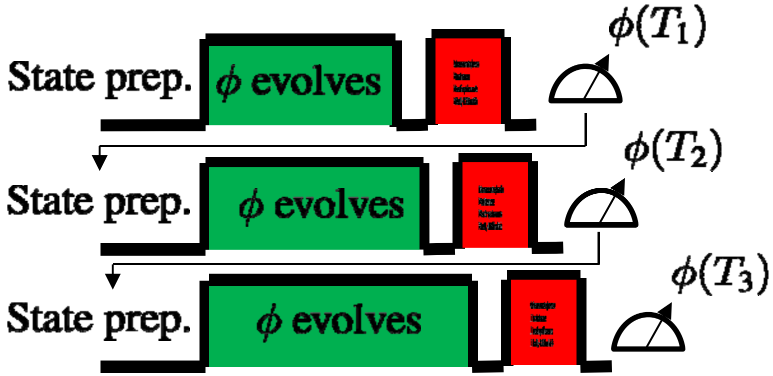
- Rotations to measure populations
- Reinitialize for each measurement
- SQL=Standard Quantum Limit: $1/\sqrt{N}$ rad

Our scheme

$$|\psi\rangle \propto |\downarrow\rangle + e^{i\phi(t)} |\uparrow\rangle$$

Ramsey

$$\text{Population} \propto \phi(T) = \int_0^T \frac{d\phi}{dt} dt$$



- Rotations to measure populations
- Reinitialize for each measurement
- SQL limited: $1/\sqrt{N}$ rad

Continuous phase measurement

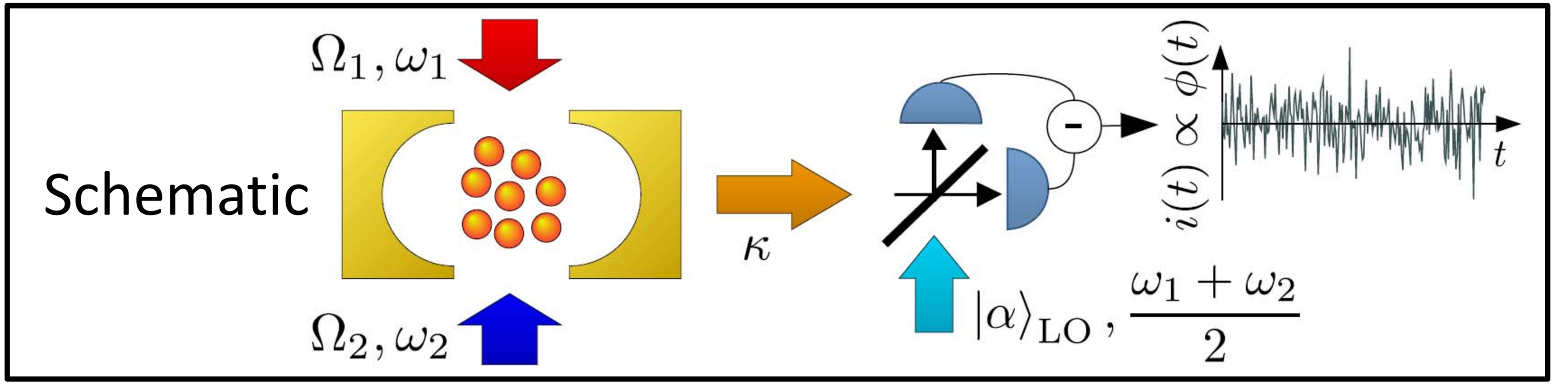
$$i(t) \propto \phi(t)$$

Continuous measurements as phase evolves

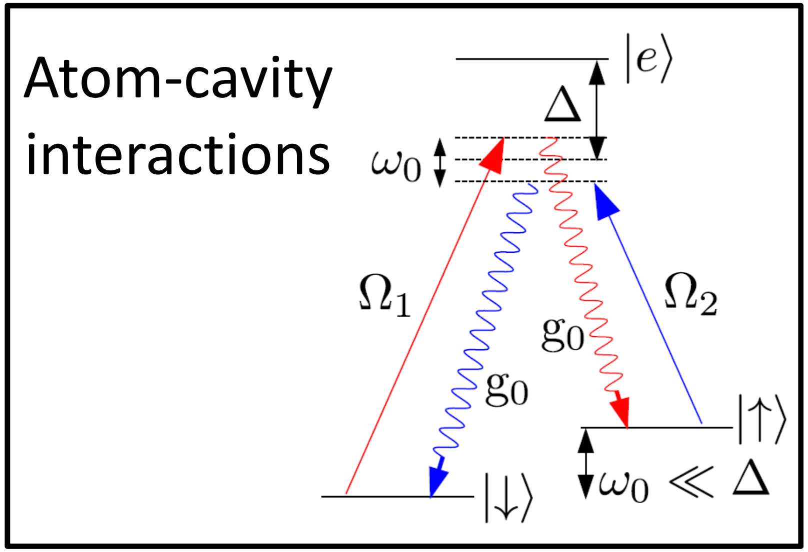
- No rotations to measure
- No reset for repeated measurements
- Below SQL, ~11-12 dB for realistic #'s

Single-shot, precise tracking of irreproducible signals

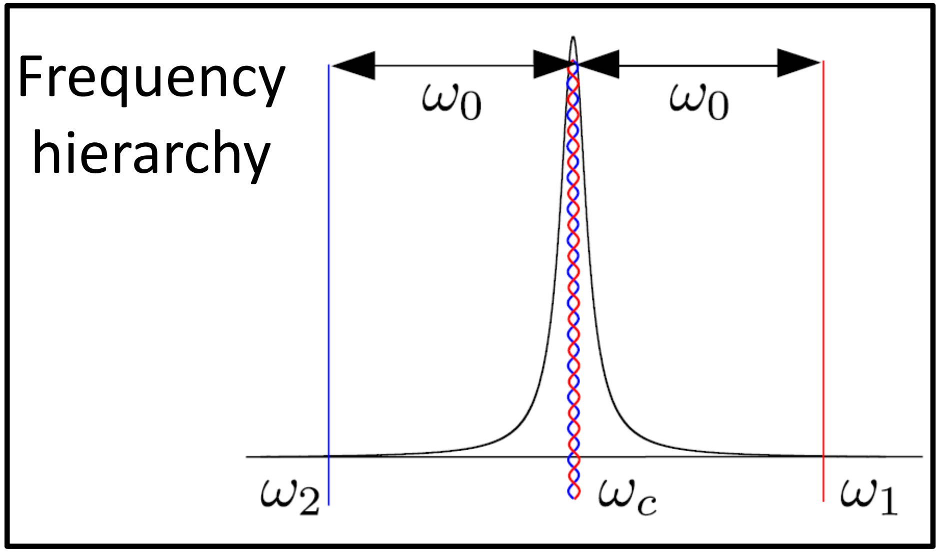
The Model



Schematic
Ω_1, ω_1
Ω_2, ω_2
κ
$|\alpha\rangle_{\mathrm{LO}}, \frac{\omega_1+\omega_2}{2}$
$i(t) \propto \phi(t)$
t

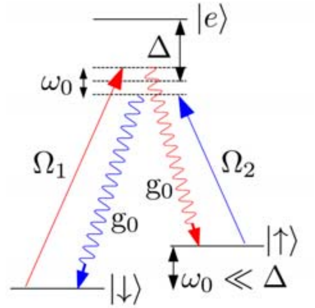


Atom-cavity interactions
$|e\rangle$
Δ
ω_0
Ω_1
Ω_2
g_0
g_0
$|\uparrow\rangle$
$|\downarrow\rangle$
$\omega_0 \ll \Delta$

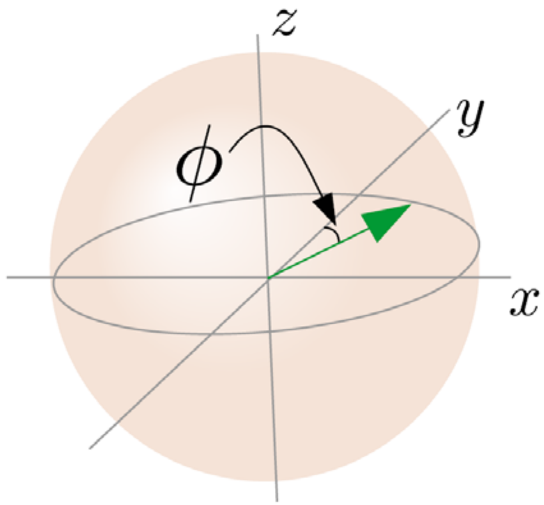


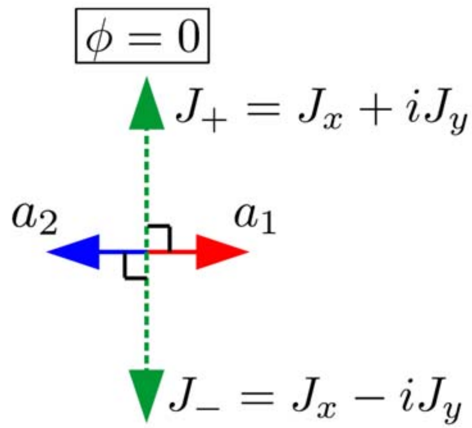
Frequency hierarchy
ω_0
ω_0
ω_2
ω_c
ω_1

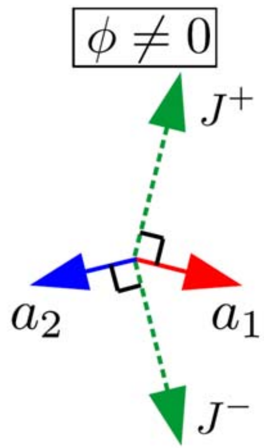
The working principle



Classical understanding: QND measurement





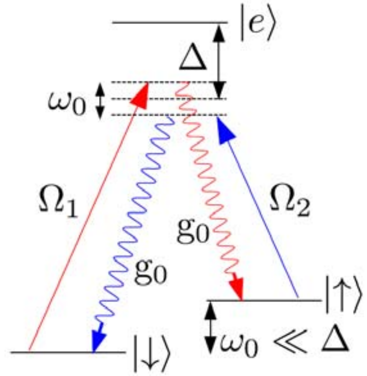


- Mean field: Zero cavity field because of interfering superradiant processes

- Quantum: Projection noise in J_x sources cavity field $\Rightarrow$ QND measurement of J_x

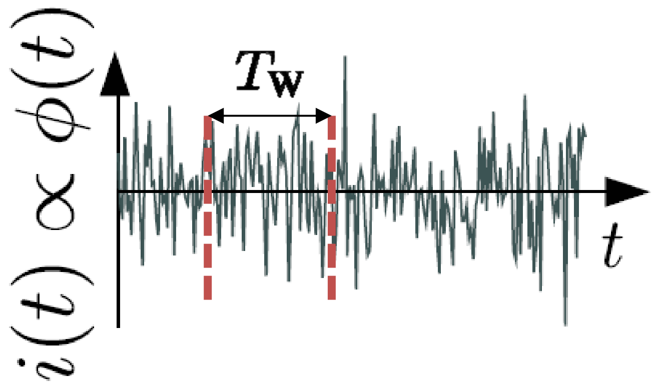
The working principle from the math

$$\hat{H} \sim \frac{\Omega_0 g_0}{\Delta}\hat{a}^\dagger \hat{J}_+ + \frac{\Omega_0 g_0}{\Delta}\hat{a}^\dagger \hat{J}_- + \textbf{h.c.}$$
$$= \frac{\Omega_0 g_0}{\Delta}\hat{a}^\dagger \hat{J}_x + \frac{\Omega_0 g_0}{\Delta}\hat{a}\hat{J}_x$$
$$\equiv \frac{\Omega_0 g_0}{\Delta}\hat{X}\hat{J}_x$$



- Balanced Jaynes-Cummings and anti-Jaynes-Cummings interactions
- Continuously monitor quadrature conjugate to $\hat{X} = \hat{a} + \hat{a}^\dagger$
  $\Rightarrow$ measurement of $\hat{J}_x \propto \phi(t)$

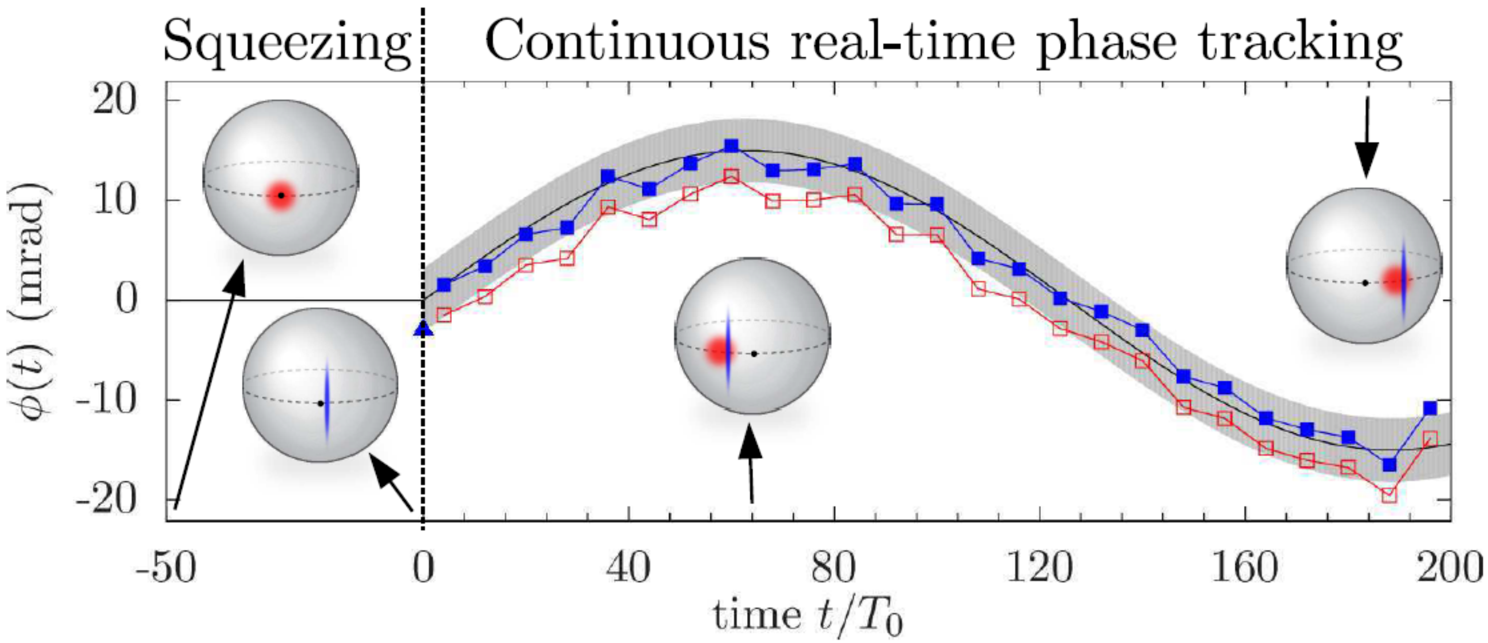
How phase is estimated



Average Photocurrent →	Estimate of $\hat{Y} = i(\hat{a}^\dagger - \hat{a})$ →	Estimate of $\hat{J}_x$ →	Estimate of $\phi(t)$

- Characteristic measurement time T_0
- Time taken to average photocurrent noise to estimate $\hat{J}_x$ at SQL

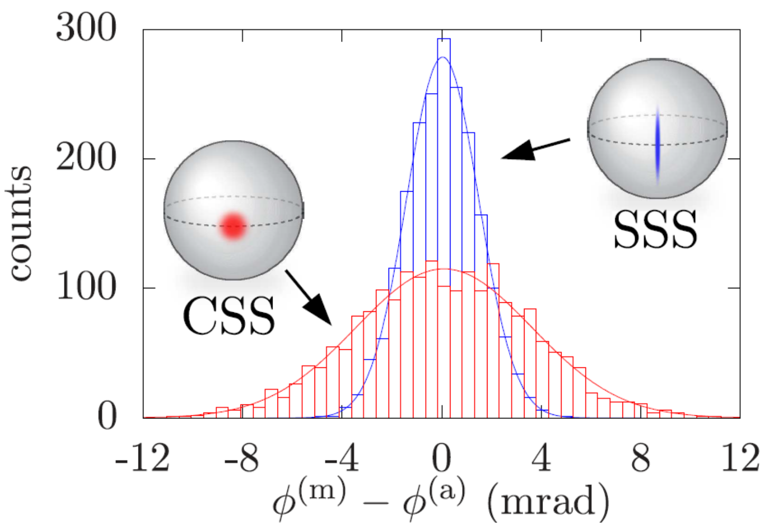
Numerical experiment: Real-time phase tracking



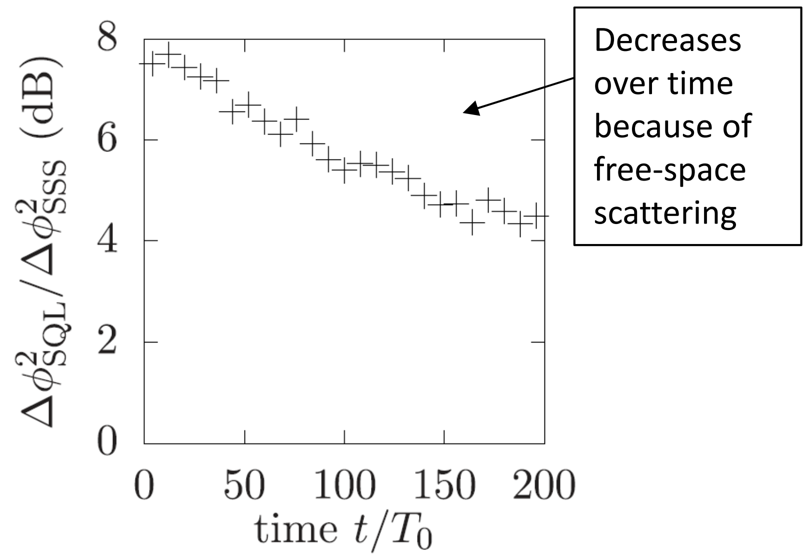
Precision of the initial phase reference

- Not accounting for initial measurement ⇒ phase reference is "zero"
- "Zero" phase of initial state not very precise → at the SQL
- Initial phase measurement more precise → beyond SQL, "squeezed"

Error histogram from 2048 experimental trials

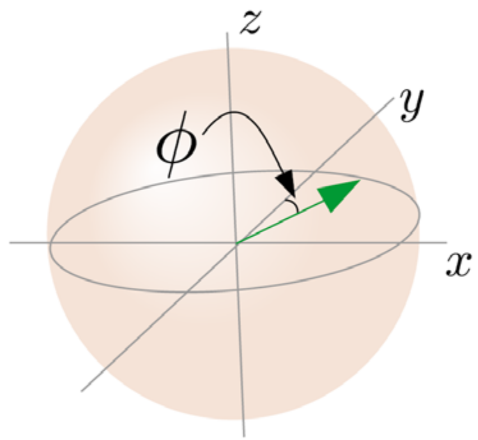


Precision gain in all windows



Demonstration of real-time phase tracking with beyond SQL precision

Future directions



- Modeling and implementing a realistic experiment with Rubidium
- Careful analysis of effects of non-zero ω_0/Δ
- Frequency and intensity stability of drive lasers
- Fluctuating AC Stark shifts
- Motional effects, inhomogeneous coupling, …