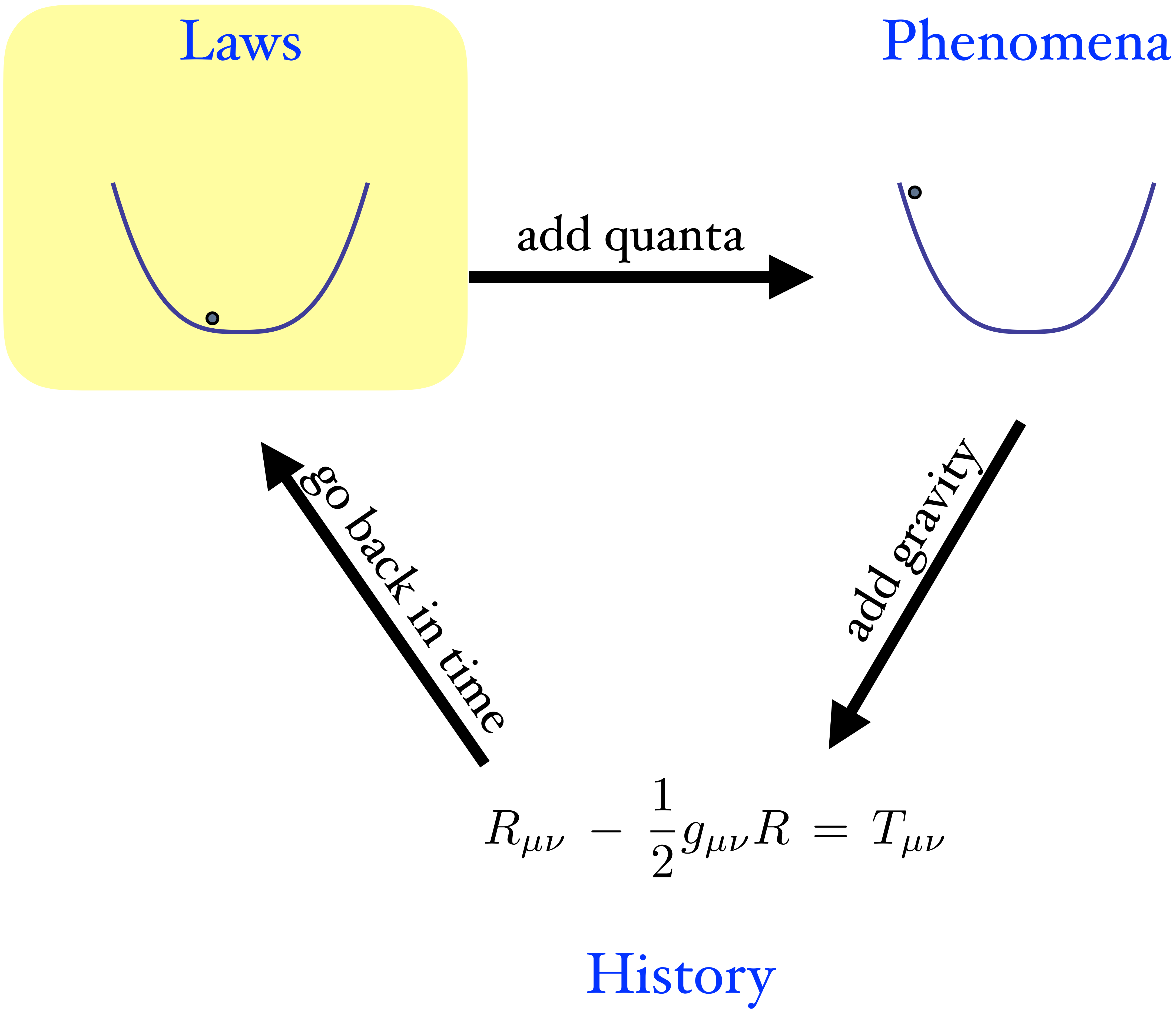
Laws, Phenomena & History

Riccardo Rattazzi, EPFL

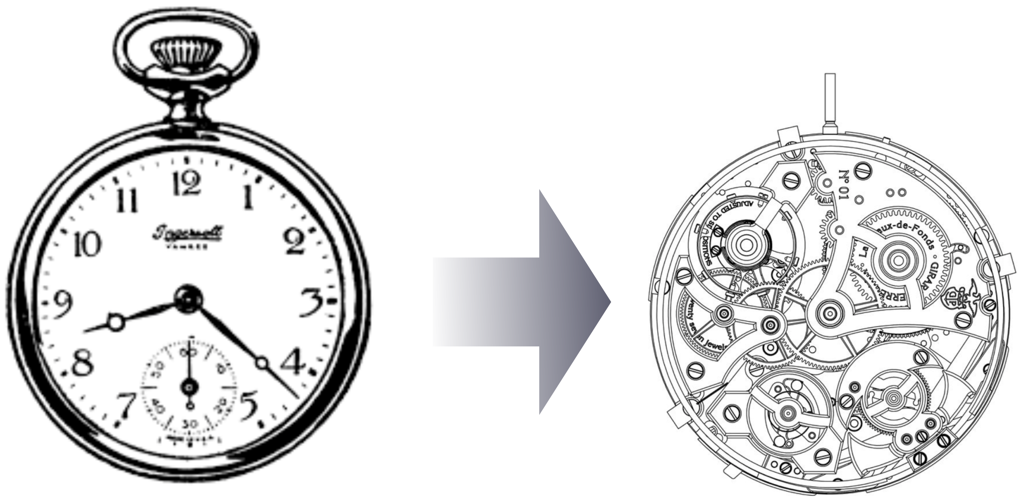


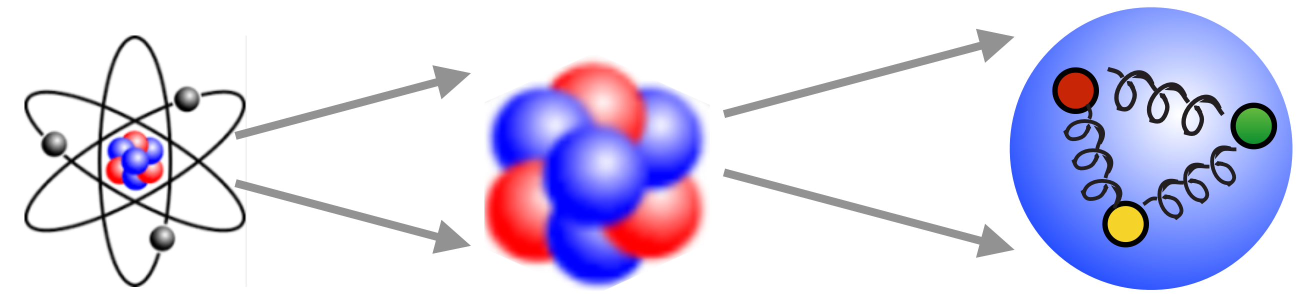
Laws
Phenomena
add quanta
add gravity
go back in time
$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}$
History

Particle Physics in a Nutshell

The Hierarchy Paradox

Reductionism

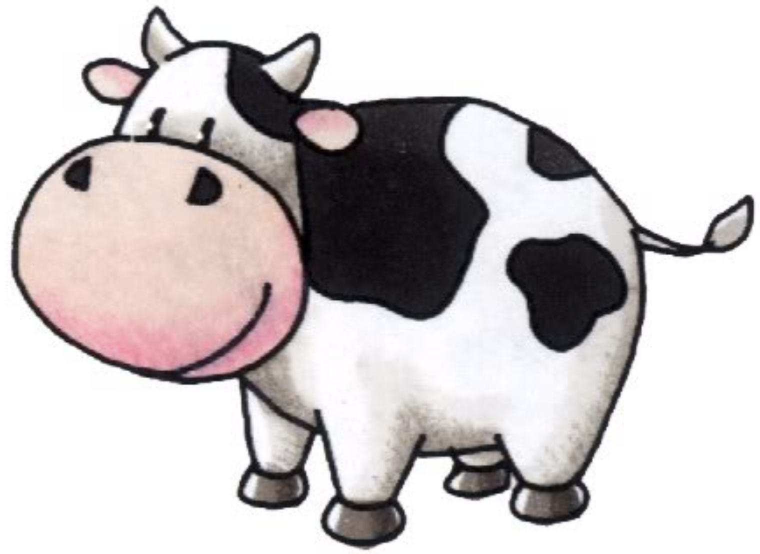




Effective Long Distance Description

- *Multipole expansion*
- *Effective Field Theory*
- …

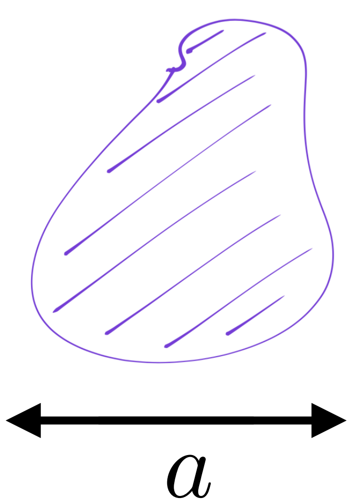
Long Distance Physics: Simplicity & Accidental Symmetries



accidental

$SO(3)$

Ex.: electrostatic potential at large distance



$R \gg a$ $\longrightarrow$

$1/R \qquad a/R^2 \qquad a^2/R^3$

$$\Phi(R) = \frac{Q_0}{R} + \frac{\vec{Q}_1 \cdot \vec{R}}{R^3} + \frac{Q_2^{ij} R_i R_j}{R^5} + \dots$$

$SO(3) \supset SO(2) \supset \varnothing$

Modern view

Standard Model is just an effective field theory valid below a physical energy cut-off $\Lambda_{UV} = 1/a$

$(E \ll \Lambda_{UV}$ or $\lambda \gg \Lambda_{UV}^{-1})$

$$\mathcal{L}_{SM} = \mathcal{L}^{d\leq 4} + \frac{1}{\Lambda_{UV}}\mathcal{L}^{d=5} + \frac{1}{\Lambda_{UV}^2}\mathcal{L}^{d=6} + \ldots$$

the three problems

$$+\Lambda_{UV}^4 \sqrt{g}$$ d=0

$$+c\Lambda_{UV}^2 H^\dagger H$$ d=2

$$+\theta\, \tilde{G}_{\mu\nu}\tilde{G}^{\mu\nu}$$ d=4

$$\mathcal{L}_{SM} = \mathcal{L}_{kin} + gA_\mu \bar{F}\gamma_\mu F + Y_{ij}\bar{F}_i H F_j + \lambda(H^\dagger H)^2$$ d=4

$$+ \frac{b_{ij}}{\Lambda_{UV}} L_i L_j H H$$
$$+ \frac{c_{ijkl}}{\Lambda_{UV}^2} \bar{F}_i F_j \bar{F}_k F_\ell + \frac{c_{ij}}{\Lambda_{UV}} \bar{F}_i \sigma_{\mu\nu} F_j G^{\mu\nu} + \ldots$$
$$+ \ldots$$ d>4

$\Lambda_{UV} \gg \mathrm{TeV}$ (pointlike limit) nicely accounts for 'what we see'

The Hierarchy Paradox

Observations speak for Simplicity] $\Lambda_{UV} \gg m_{weak}$ [

- $\mathcal{L}_{SM} \to \mathcal{L}^{d\leq 4}$ B, L, "GIM suppression", custodial symm, ...
- $m_\nu \ll m_{weak}$ beautifully explained

Theory expects Naturalness] $\delta m_h^2 \sim \frac{y_t^2}{4\pi^2}\Lambda_{UV}^2 + \dots$ ⇒ $\Lambda_{UV} \lesssim 500\,\text{GeV}$

Clash between Simplicity and Naturalness

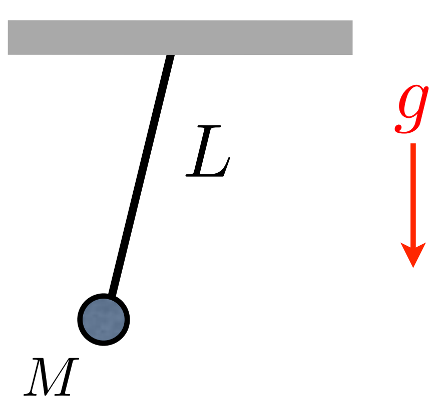
Made concrete by all available Natural models (SUSY, Comp Higgs,...)

$$m_h^2 = c\,\frac{y_t^2}{4\pi^2}\Lambda_{UV}^2 + \ldots$$

high spin symm

dilatation symm

As good as dimensional analys in mechanics



$$\omega = c\sqrt{\frac{g}{L}}$$

Fine Tuning: violation of expectations from symmetry and dim. analysis

$$\epsilon_T \equiv \frac{m_H^2|_{observed}}{m_H^2|_{expected}}$$

Criticality ⟷ ***Fine Tuning*** ⟷ ***Landscape***

Ex. Quantum criticality in anti-ferromagnet

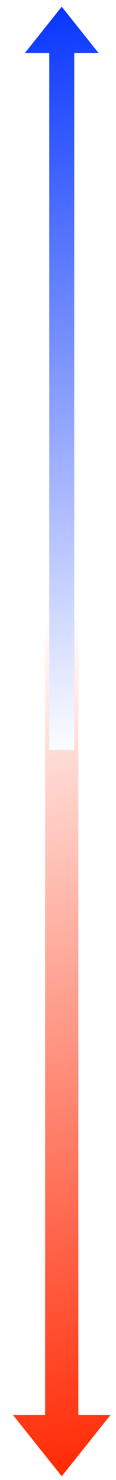
Sachdev '09

$$V(\vec{S}) = m^2(P)\vec{S}\cdot\vec{S} + \lambda(\vec{S}\cdot\vec{S})^2 \qquad m^2(P) = m_0^2\left(1 - \frac{P}{P_c}\right)$$

Can undo ***natural*** expectation from atomic physics by ***tuning*** the pressure at a ***critical*** value in a ***landscape*** of possibilities

The two Chief Systems

Simplicity



I. The SM is valid up to $\Lambda_{UV} \gg TeV$

- B, L and Flavor: beautifully in accord with observation
- Higgs mass & C.C. hierarchy point beyond naturalness
  - anthropic selection
  - failure of EFT ideology (UV/IR connection)
  - ...

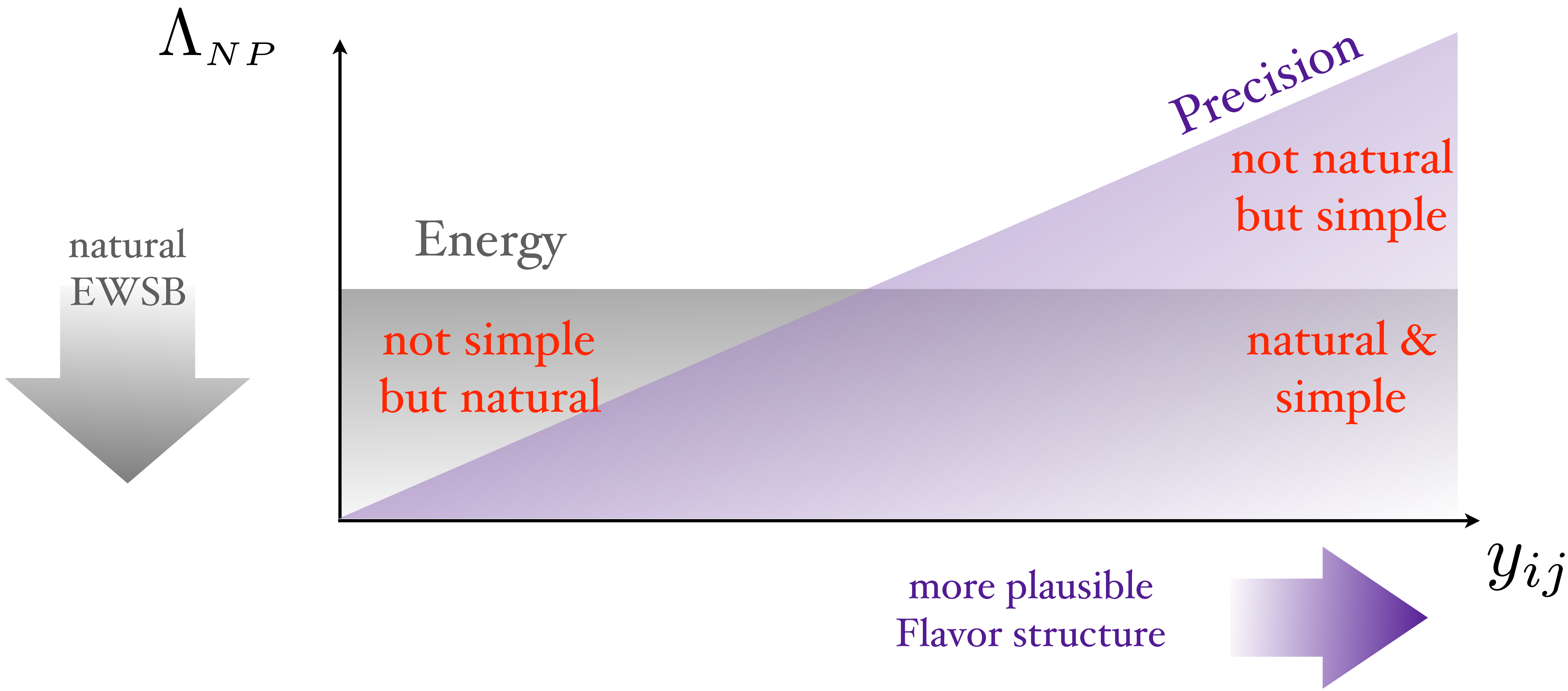
II. Naturalizing New Physics appears at $\Lambda_{UV} \sim 1\,\mathrm{TeV}$

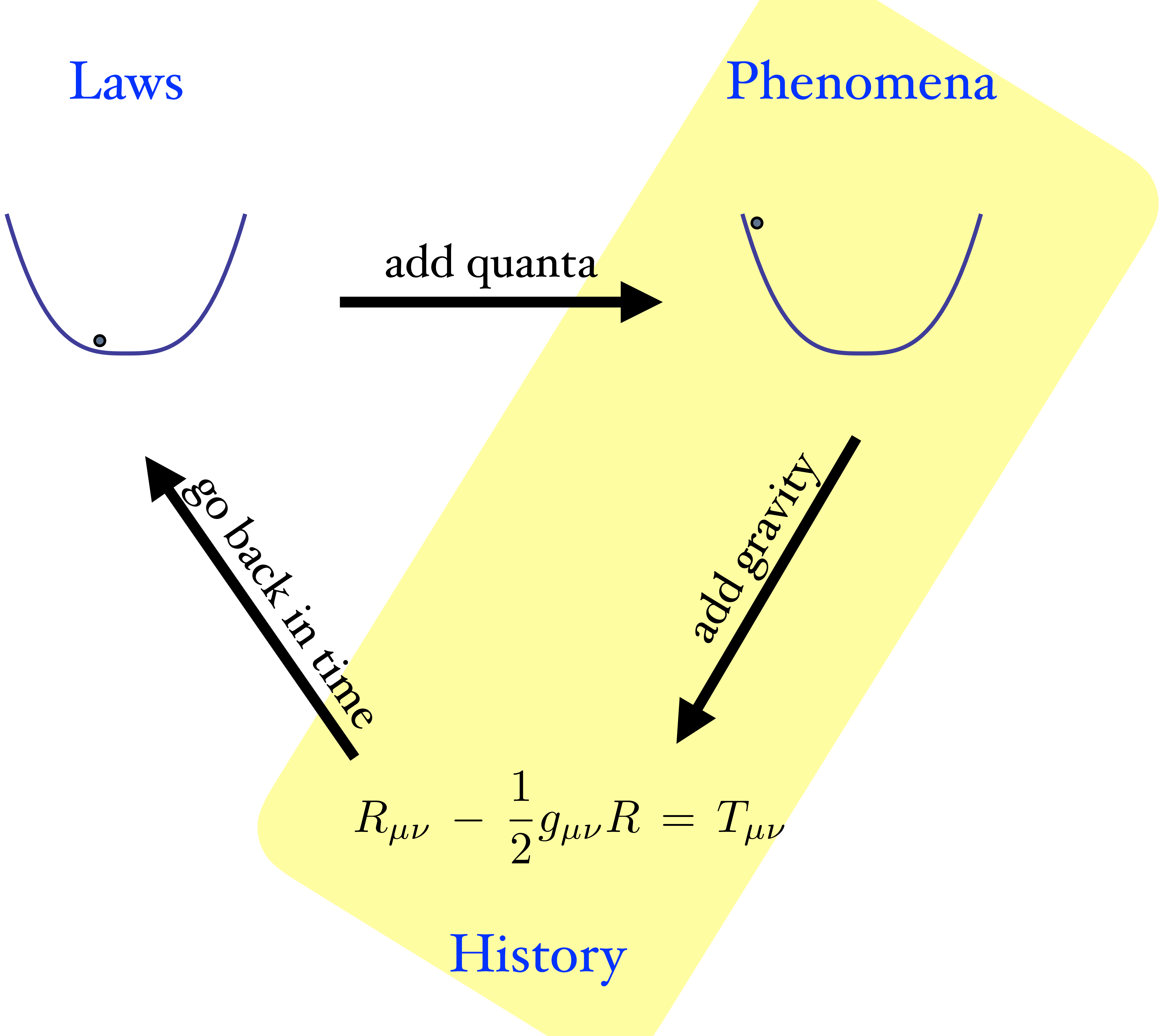
- Constraints on B, L, Flavor & CP only met by clever model building

Naturalness

Complementarity of Energy and Precision

$$\mathcal{L}_{eff} = \frac{y_{ijk\ell}}{\Lambda_{NP}^2}\,\bar{q}_i q_j \bar{q}_k q_\ell + m_i \frac{y_{ij}}{\Lambda_{NP}^2}\,\bar{q}_i \sigma_{\mu\nu} q_j F^{\mu\nu} + \ldots$$

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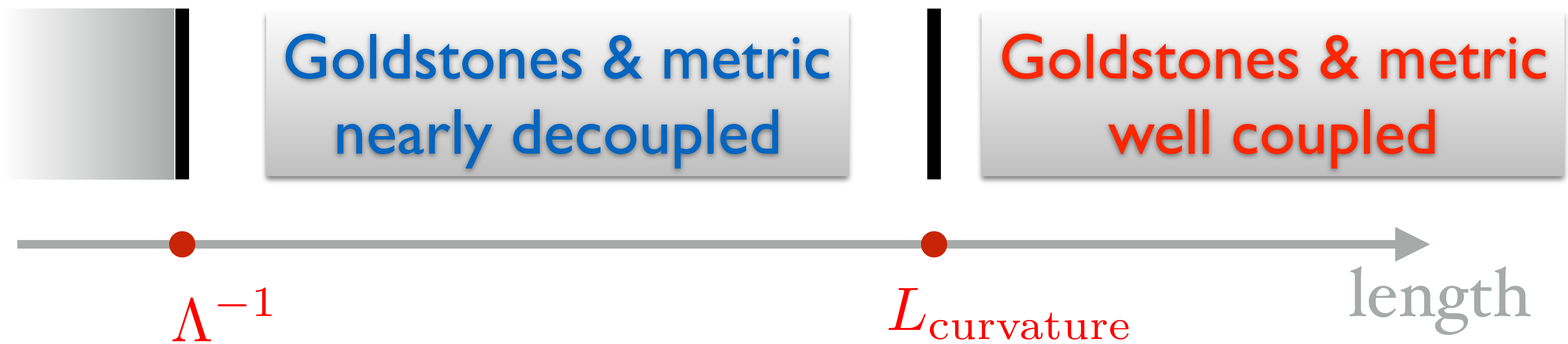
- FRW → isotropy and homogeneity at large scales

$T_{\mu\nu}$ ⟺ finite density phase spontaneously breaking spacetime symmetry down to euclidean group $ISO(3)$

- macroscopic dynamics universally described by hydrodynamics modes (Goldstone bosons) long distance dynamics modified

- gravity + hydrodynamics modes

- similar to (photon + Cooper pair) in superconductor



Ex: hot plasma

$$\Lambda \sim T$$

$$L_{\text{curvature}} = H^{-1} \sim \frac{M_P}{T^2}$$

The picture of the connubium dates back to pre-Higgs days

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Plasmons, Gauge Invariance, and Mass

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Bell Telephone Laboratories, Murray Hill, New Jersey
(Received 8 November 1962)

Schwinger has pointed out that the Yang-Mills vector boson implied by associating a generalized gauge transformation with a conservation law (of baryonic charge, for instance) does not necessarily have zero mass, if a certain criterion on the vacuum fluctuations of the generalized current is satisfied. We show that the theory of plasma oscillations is a simple nonrelativistic example exhibiting all of the features of Schwinger's idea. It is also shown that Schwinger's criterion that the vector field $m \neq 0$ implies that the matter spectrum before including the Yang-Mills interaction contains $m=0$, but that the example of superconductivity illustrates that the physical spectrum need not. Some comments on the relationship between these ideas and the zero-mass difficulty in theories with broken symmetries are given.

It is noteworthy that in most of these cases, upon closer examination, the Goldstone bosons do indeed become tangled up with Yang-Mills gauge bosons and, thus, do not in any true sense really have zero mass. Superconductivity is a familiar example, but a similar phenomenon happens with phonons; when the phonon frequency is as low as the gravitational plasma frequency, $(4\pi G\rho)^{1/2}$ (wavelength$\sim 10^4$ km in normal matter) there is a phonon-graviton interaction: in that case, because of the peculiar sign of the gravitational interaction, leading to instability rather than finite mass.[12] Utiyama[13] and Feynman have pointed out that gravity is also a Yang-Mills field. It is an amusing observation that the three phonons plus two gravitons are just enough components to make up the appropriate tensor particle which would be required for a finite-mass graviton.

Ex 1: relativistic superfluid

$\not{P}_0, \not{K}_i, \not{Q}$ broken

$$\bar{P}_0 = P_0 - \mu Q\,, \quad \bar{P}_i = P_i\,, \quad \bar{J}_i = J_i$$

$$Q: \phi \to \phi + c \quad \Longrightarrow \quad \mathcal{L} \equiv \mathcal{L}(\partial\phi)$$

$$\phi = \mu t + \pi \longrightarrow \text{phonon}$$

- Add small explicit Q breaking: $V(\phi)$ $\quad V(\phi)'' \ll H^2$

π
- Goldstone of $\not{P}_0$
- pseudo-Goldstone of $\not{Q}$

Effective Field Theory of Inflation

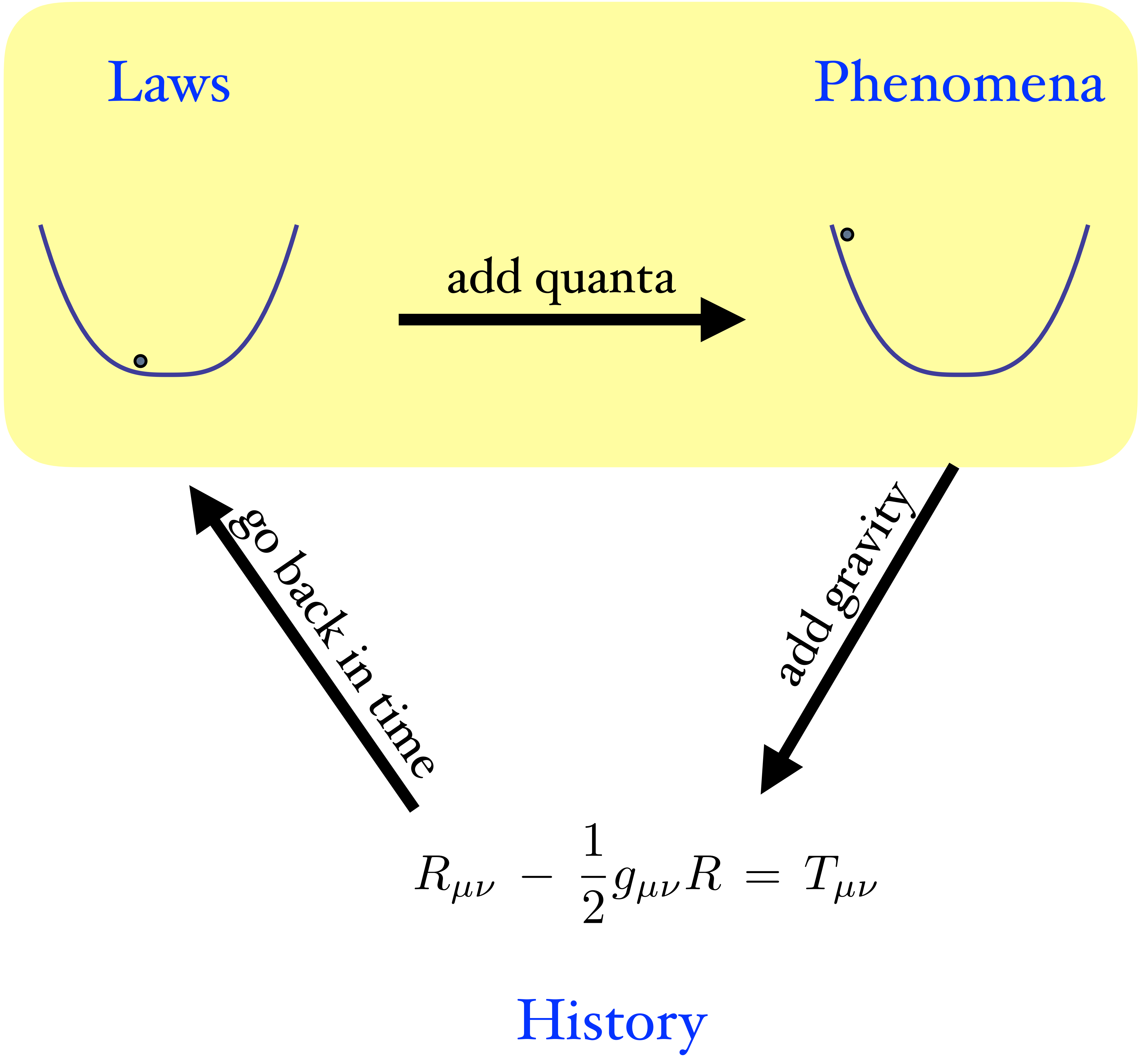
Ex 2:

Fierz-Pauli massive gravity

⇕

relativistic super-solid coupled to gravity

Arkani-Hamed, Georgi, Schwartz '02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Feynman diagrams through superfluids

with Gil Badel, Gabriel Cuomo, Alexander Monin, arXiv:1909.01269

{Weak vs Strong} & {Classical vs Quantum}

- Weak coupling: loop expansion around leading trajectory $\gamma_{c\ell}$

$$e^{-W} = e^{-[S_0+S_1+S_2+\dots]}$$

- Strong coupling: PI cannot be described by leading trajectory

Common practice: few legs in weakly coupled QFT

= small fluctuations around trivial trajectory

However when the number of legs grows expansion breaks down

see old review by Rubakov, arXiv:9511236, 1995

How do we describe physics in this regime?

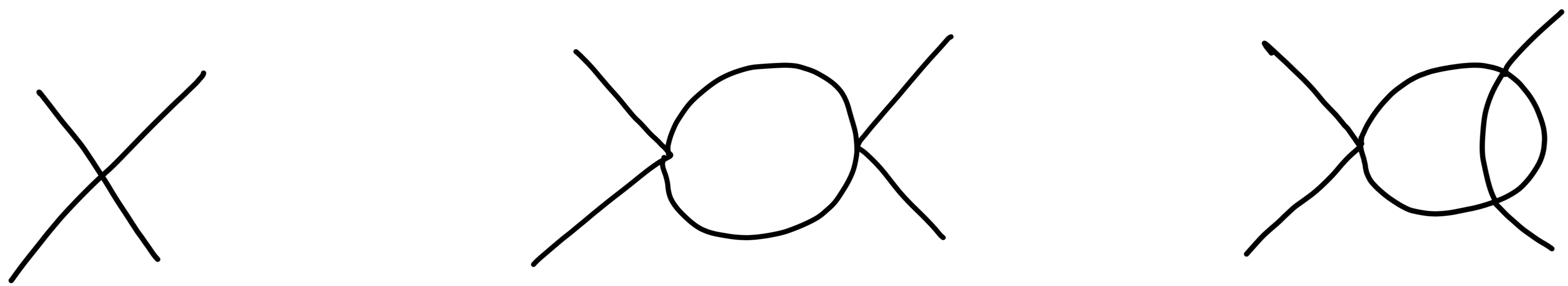
▲ Charged ϕ^4 $D = 4 - \epsilon$ dimension

$$\mathcal{L} = \partial_\mu \overline{\phi}\, \partial^\mu \phi + \frac{\lambda}{4}(\overline{\phi}\phi)^2$$

▲ Conformal invariant at Wilson-Fisher fixed point

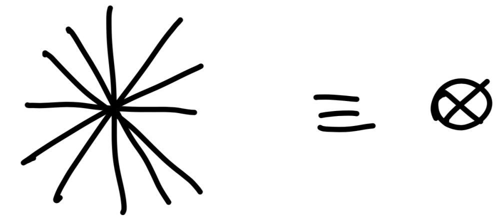
$$\frac{\lambda_*}{(4\pi)^2} = \frac{\epsilon}{5} + \frac{3\epsilon^2}{25} + O(\epsilon^3)$$

Few Legs

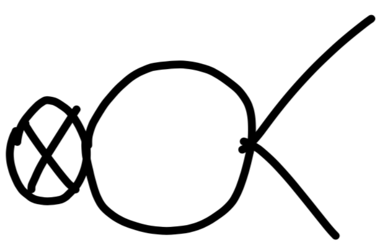


λ $\qquad$ $\frac{\lambda^2}{16\pi^2}$ $\qquad$ $\frac{\lambda^3}{(16\pi^2)^2}$

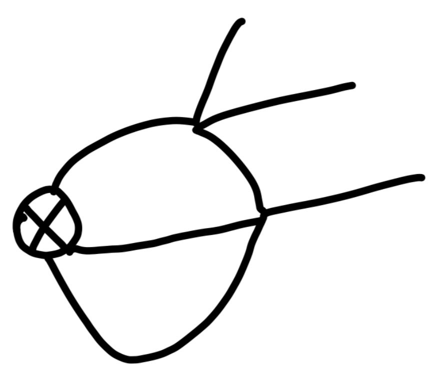
Many Legs: ϕ^n $\quad n \gg 1$



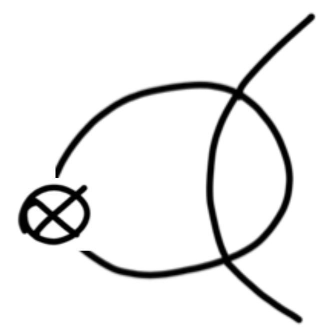
$$\langle \phi^n(x)\bar{\phi}^n(0)\rangle \propto \frac{1}{x^{2\Delta_n}} \qquad \Delta_n \equiv \frac{D-2}{2} + \gamma_n$$



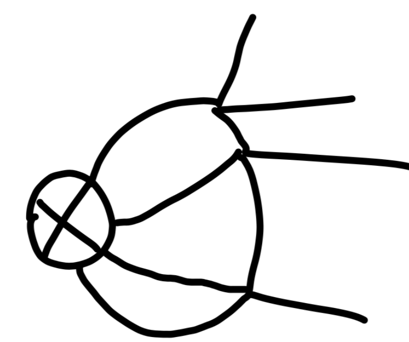
$\lambda n(n-1)$



$\lambda^2 n(n-1)(n-2)$



$\lambda^2 n(n-1)$



$\lambda^3 n(n-1)(n-2)(n-3)$

perturbation theory breaks down at $\quad \dfrac{\lambda n}{16\pi^2} \gtrsim 1$

series can be organized as a double expansion

$$\frac{\gamma_n}{n} = P_0(\lambda n) + \lambda P_1(\lambda n) + \lambda^2 P_2(\lambda n) + \ldots$$

similar to RG $F_0(\lambda\, Log) + \lambda F_1(\lambda\, Log) + \ldots$

or to 't Hooft large-N expansion

$$\frac{\gamma_n}{n} = P_0(\lambda n) + \frac{1}{n}\bar{P}_1(\lambda n) + \frac{1}{n^2}\bar{P}_2(\lambda n) + \ldots$$

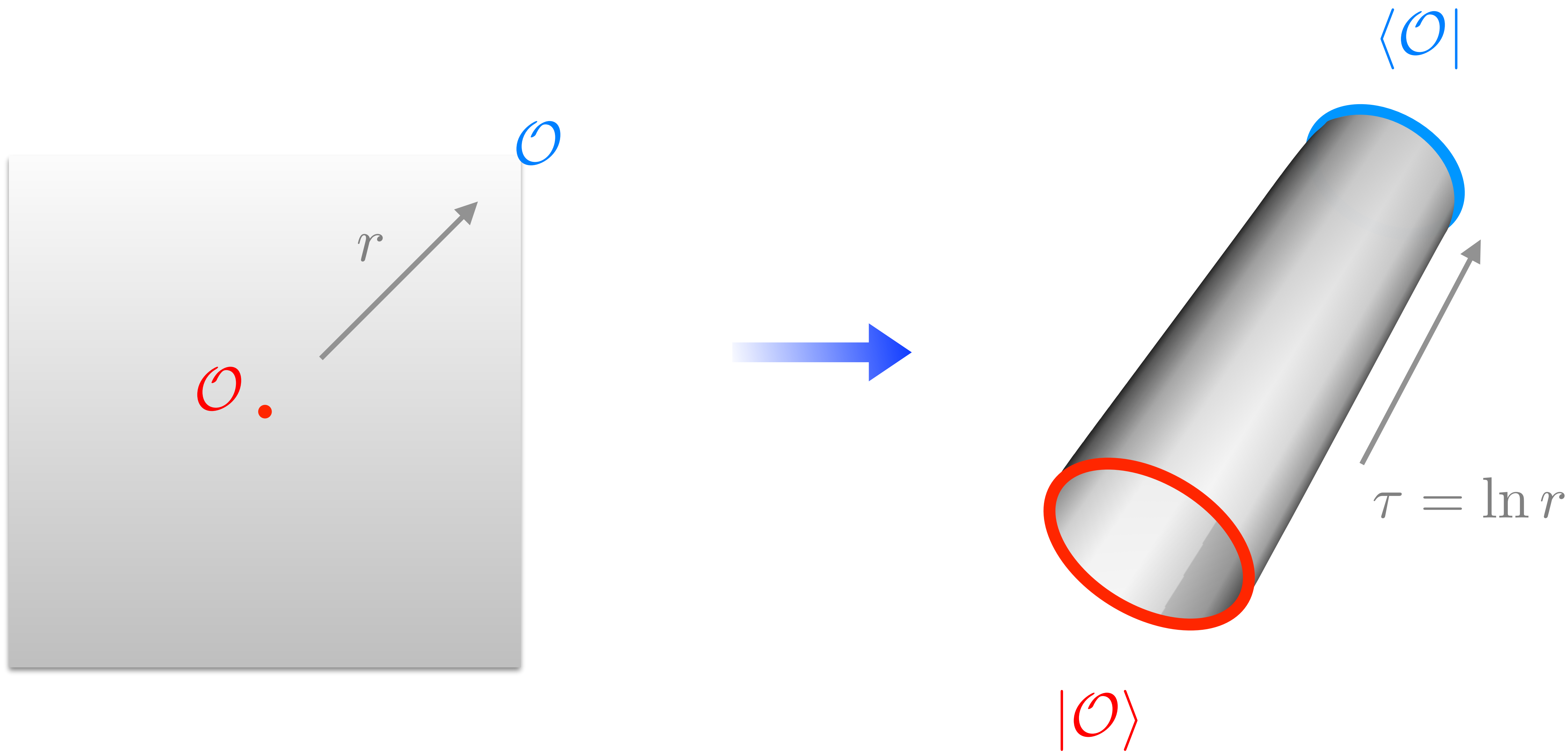
▲ What is the physics behind this?

▲ Can one compute the λn series?

Common answer:

Semiclassical expansion around non-trivial trajectory

Mapping to the cylinder & operator/state correspondence



$$\langle\, \mathcal{O}(r)\, \mathcal{O}(0)\, \rangle \;=\; \frac{1}{r^{2\Delta}} \quad \longrightarrow \quad \langle \mathcal{O} | e^{-H\tau} | \mathcal{O} \rangle \;=\; e^{-\Delta\tau}$$

- path integral dominated by superfluid configuration

$$\phi_{c\ell} = \rho e^{i\chi} \qquad \begin{aligned} \rho &= \text{const} \\ \chi &= -i\mu\tau \end{aligned}$$

- plug back into action and perform systematic loop expansion around classical trajectory

$$\Delta_{\phi^n} = \frac{1}{\lambda_*}\Delta_{-1}(\lambda_* n) + \Delta_0(\lambda_* n) + \lambda_* \Delta_1(\lambda_* n) + \ldots .$$

Leading order

$$\frac{1}{\lambda_* n}\Delta_{-1} = \frac{3\left[9x-\sqrt{81x^2-3}\right]^{1/3} + 3^{2/3}\left[9x-\sqrt{81x^2-3}\right]}{\left[\left(9x-\sqrt{81x^2-3}\right)^{2/3} + 3^{1/3}\right]^2}$$

$$+ \frac{9 \times 3^{1/3} x \left[9x-\sqrt{81x^2-3}\right]^{2/3}}{2\left[\left(9x-\sqrt{81x^2-3}\right)^{2/3} + 3^{1/3}\right]^2}$$

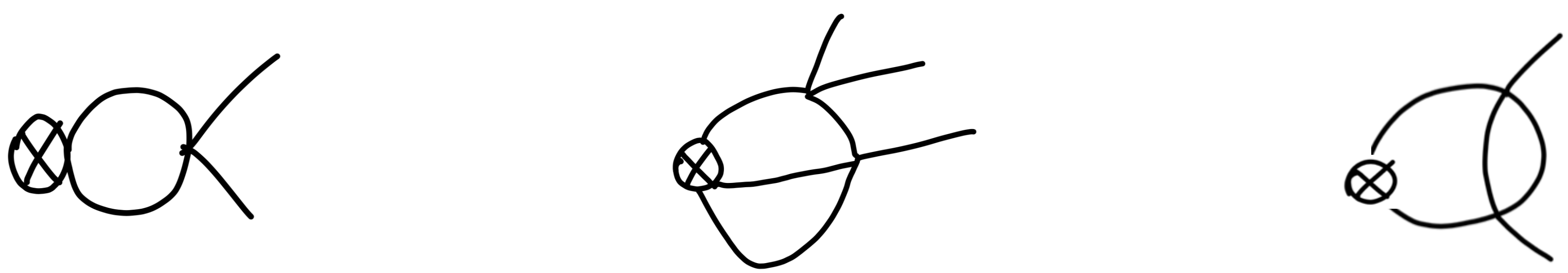
$$x \equiv \frac{\lambda_* n}{16\pi^2}$$

Supposed to resum leading powers of n at all loops!

expanding at small λn

$$\Delta_{\phi^n} = n + \frac{\lambda n^2}{32\pi^2} - \frac{\lambda^2 n^3}{512\pi^4} + \frac{\lambda^3 n^4}{4096\pi^6} + O\left(\lambda^4 n^5\right)$$

and comparing with diagrams



$$\gamma_n = \frac{\lambda n(n-1)}{32\pi^2} - \frac{\lambda^2 n^2(n-1)}{512\pi^4} + \dots$$

they happily agree

- $1/n$ suppressed terms ⟷ Casimir energy of superfluid

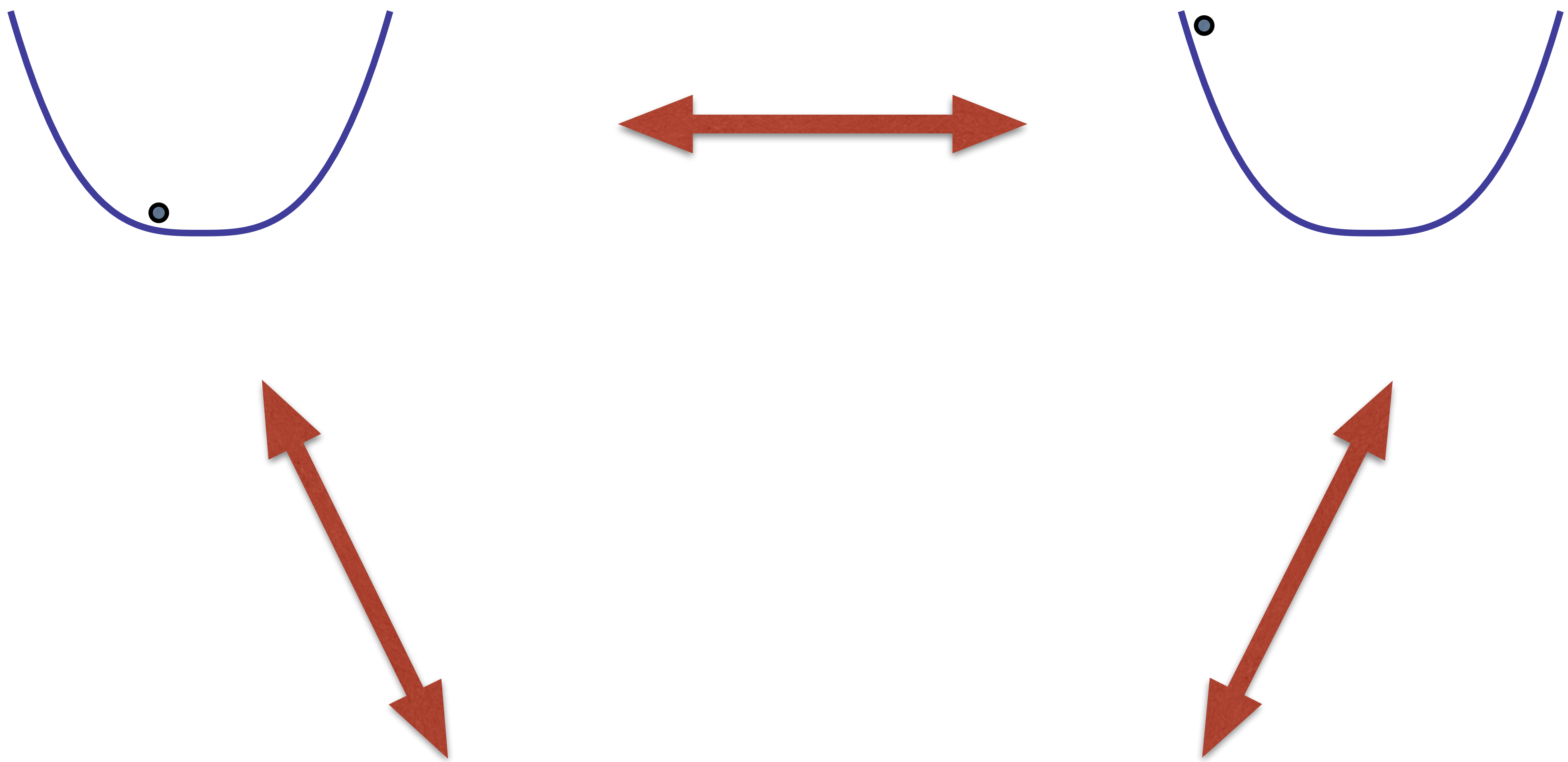
- $\epsilon \to 1$ extrapolation well matches Monte Carlo simulations of U(1) model in D=3

- spectrum of 'nearby' operators described by phonon spectrum

$\phi^{n-2}\partial_\mu\phi\partial_\nu\phi$ ⟷ phonon with $\ell = 2$

Laws

Phenomena



$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = T_{\mu\nu}$$

History