Universal laws of Quantum Information

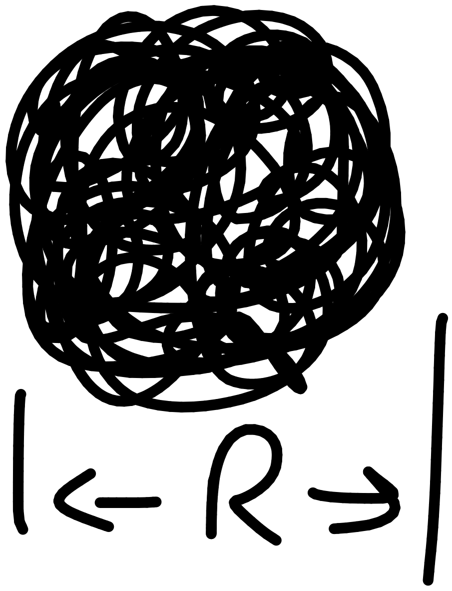
(Black holes, Universe, Solitons, Baryons, Instantons, ... = critical many body)

Gia Dvali

QFC 2019

Pisa, Oct. 2019

Bekenstein-Bremermann Bound



$$S \leq M R$$

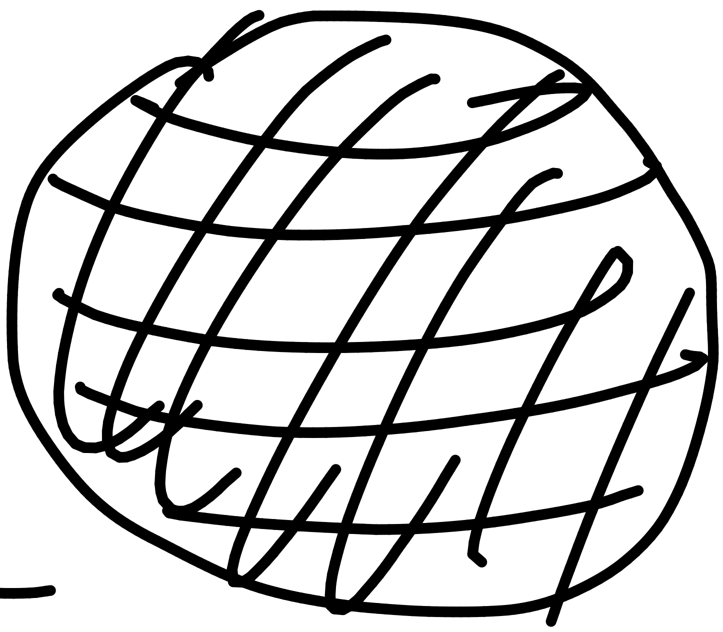
In gravity is saturated by black holes.

Bekenstein entropy:

$$S_{BH} = M_{BH} R = (R M_P)^2$$

* What is physical meaning of bound?

* What is physical meaning of Area-Law?



$$S = (R M_p)^2$$

The bound has (mostly) been discussed in gravity.

What happens beyond gravity?

In renormalizable theories?

Our main results:

* Bekenstein bound is saturated when theory saturates the bound on unitarity.

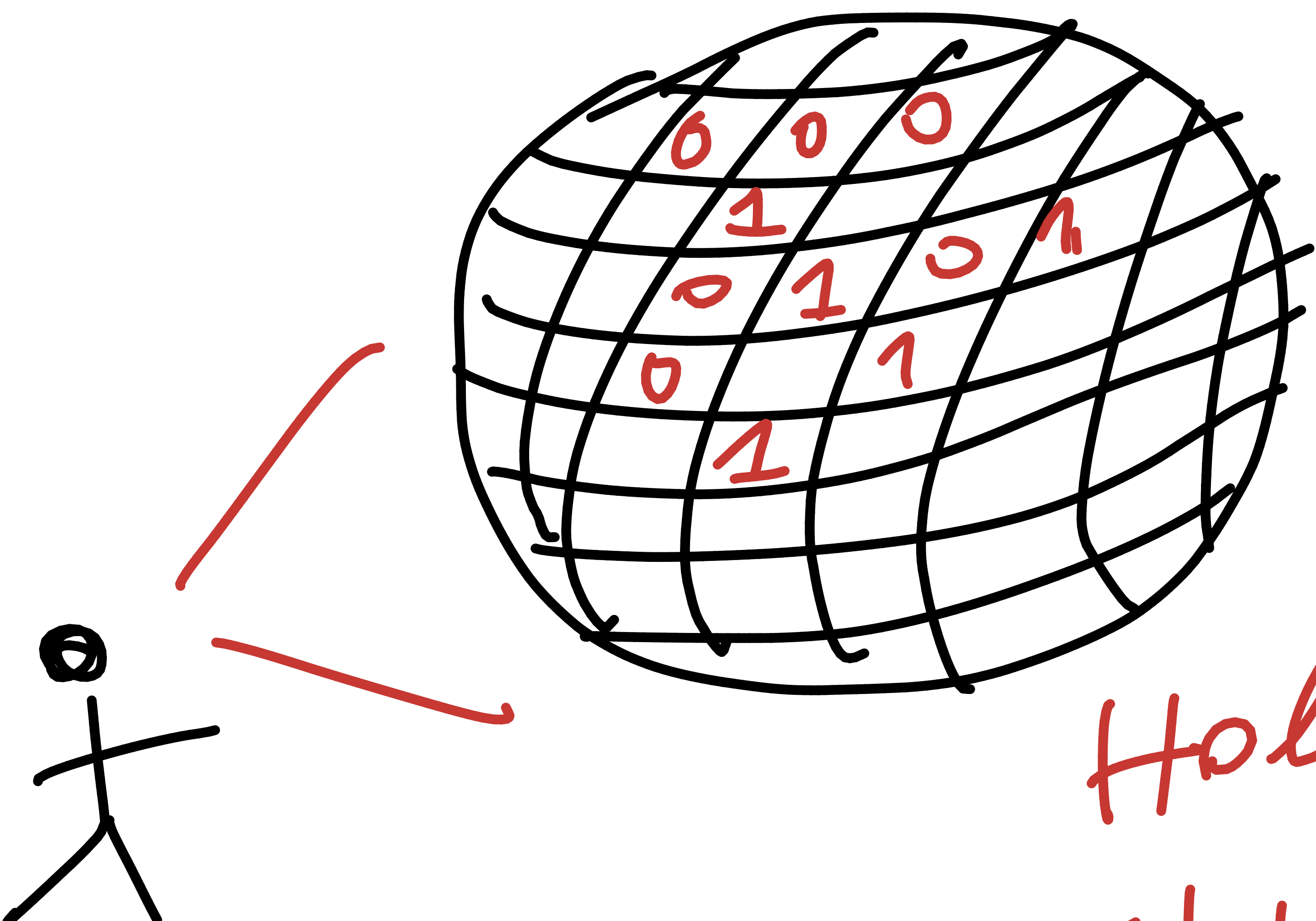
* Simultaneously the entropy assumes the area law:

$$S = MR = (Rf)^2$$

scale

Black hole Bekenstein-Hawking entropy

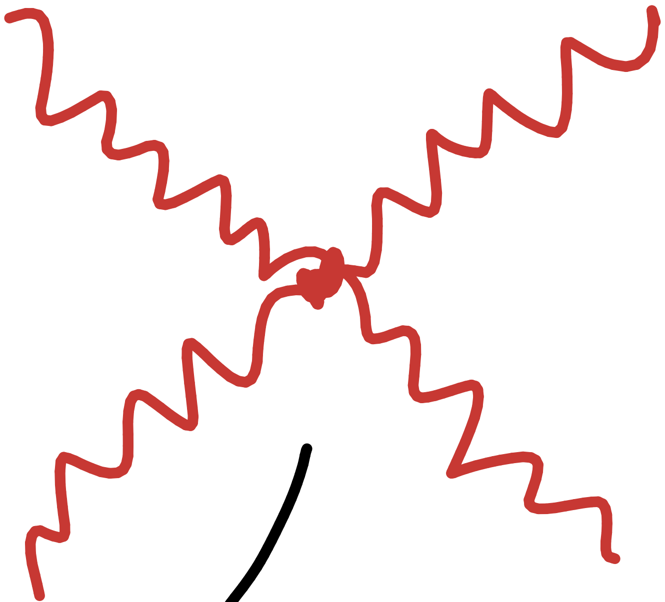
$$S_{BH} = \frac{\text{Area}}{4G_N} \sim (R M_p)^{d-2}$$



Holography
't Hooft, ...

What is gravity?

QFT of graviton!



graviton-graviton 4-point coupling for wavelength R

$$\alpha_g = \frac{1}{(R M_p)^{d-2}} = \frac{G_N}{\text{Area}} \; !$$

Thus, BH entropy is

$$S_{BH} = \frac{1}{\alpha_g} !$$

What is more fundamental?

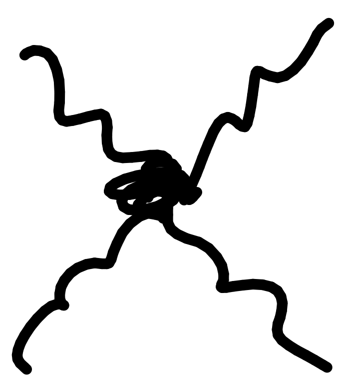
Thus,

$$S_{BH} = \frac{1}{\alpha_g} = \frac{\text{Area}}{L_p^2}$$

What about other theories?

e.g. Gauge theory

$$\alpha = g^2 \rightarrow$$



Bekenstein = Unitarity = Area

$$S_{max} = \frac{1}{g^2} = \text{Area}$$

Quantum Coupling

We shall demonstrate for:

① 't Hooft - Polyakov monopole:

$$S = M_m R_m = \frac{1}{g^2_{gauge}} = (R_m v)^2$$

② Baryon:

$$S \approx M_B R_B = \frac{1}{g^2_{QCD}} = (R_B f_\pi)^2$$

③ QCD-Instanton

$$S_{inst} = \frac{1}{g^2_{QCD}} = (R_{ins} f)^2$$

In all cases at the saturation point

$$S = \frac{\text{Area}}{4G}$$

Goldstone coupling!

Monopole. SO(3) gauge symmetry Higgsed by

$$\phi^a \quad (a=1,2,3)$$

$$\mathcal{L} = \mathcal{D}_\mu \phi^a \mathcal{D}^\mu \phi^a$$

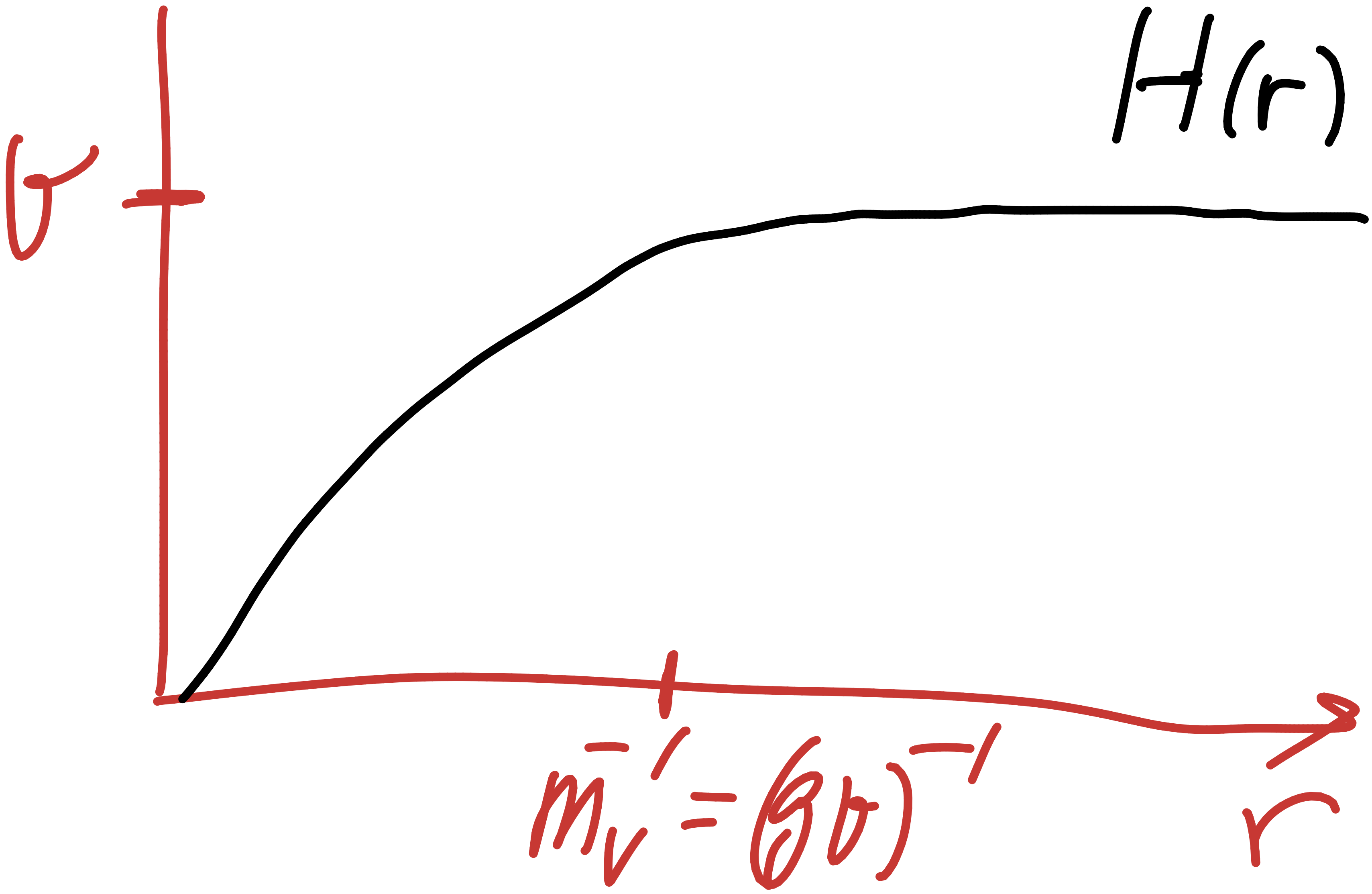
$$- \lambda^2 (\phi^a \phi^a - v^2)^2$$

$$- \quad F_{\mu\nu}^a F^{a\mu\nu}$$

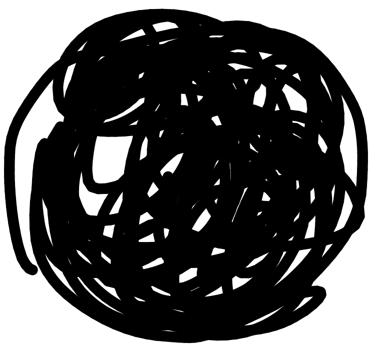
Monopole:

$$\phi^a = \frac{x^a}{r} H(r)$$

$$A^a_\mu = \varepsilon_{0a\mu\nu} \frac{x^\nu}{g r^2} F(r)$$



Monopole mass and size



$|\leftarrow R_m \rightarrow|$

$$R_m = (m_v)^{-1} = (g v)^{-1}$$

$$M_m = \frac{m_v}{g^2}$$

Entropy bound on monopole:

$$S \leq M_m R_m = \frac{1}{g^2}$$

Can it be saturated?

Entropy from Goldstones

σ_α $\alpha = 1, 2, \ldots N$

$\leftarrow$ scalar

$SO(N)$ - global symmetry

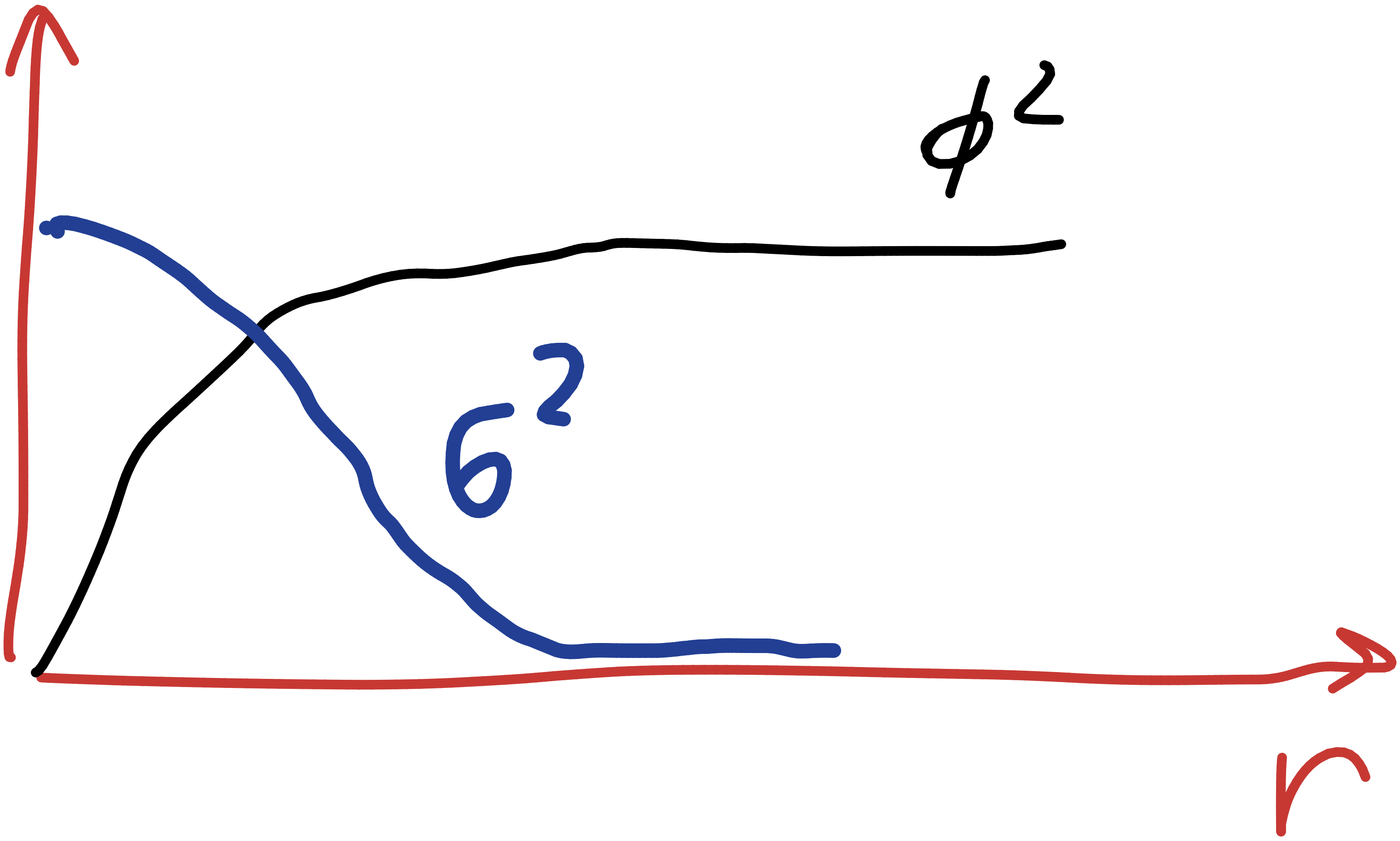
spontaneous breaking

in monopole:

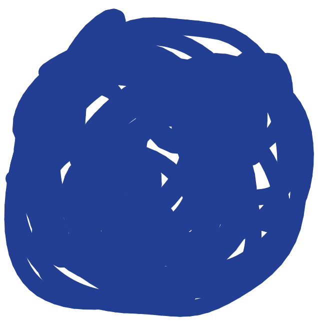
$$\mathcal{L} = \partial_\mu \sigma_\alpha \partial^\mu \sigma_\alpha$$

$$- (g^2 \phi^2 - m^2) \sigma_\alpha \sigma_\alpha$$

$$- g_\sigma^2 (\sigma_\alpha \sigma_\alpha)^2$$



$\langle \sigma \rangle \neq 0$



$SO(N) \to SO(N-1)$

$\sim N$ localized Goldstones!

Number of degenerate micro-states

$$n_{st} \sim \binom{2N}{N} \sim 2^{2N}$$

Monopole entropy:

$$S_{mon} \simeq \ln(n_{st}) \sim N$$

Unitarity bound:

$$g^2 N \leq 1$$

For $N = \frac{1}{g^2}$ entropy bound is saturated by area-law!

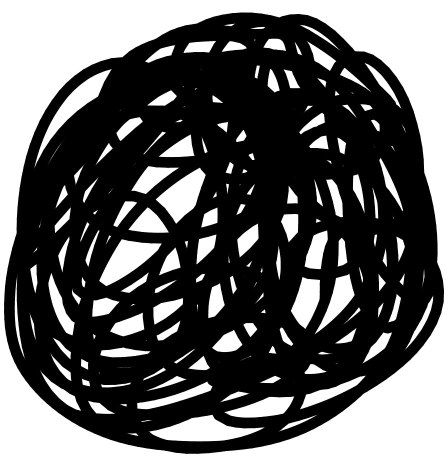
$$S_{mon} = N = \frac{1}{g^2} = M_m R_m = (R_m v)^2$$

The same is achieved by coupling to fermion flavors:

$$\phi^a \psi^b_\alpha \lambda^c_\alpha \varepsilon_{abc}$$

$\alpha = 1, 2 \, .. \, N$

$SO(N)$



$\sim N$ localized fermion zero modes!

Again:

$$n_{st} \sim 2^N$$

Entropy: $S_{mon} \sim N$

Unitarity bound

$$g^2 N \leq 1$$

$$S_{mon} = N = \frac{1}{g^2} = M_m R_m = (R_m v)^2$$

Baryons in $SU(N_c)$ QCD with N-flavors of quarks.

't Hooft limit

$N_c \to \infty$

$g^2 N_c = \text{fixed}$

$\Lambda = \text{fixed}$

Spontaneous chiral symmetry breaking

$$U(N)_L \otimes U(N)_R \to U(N)_F$$

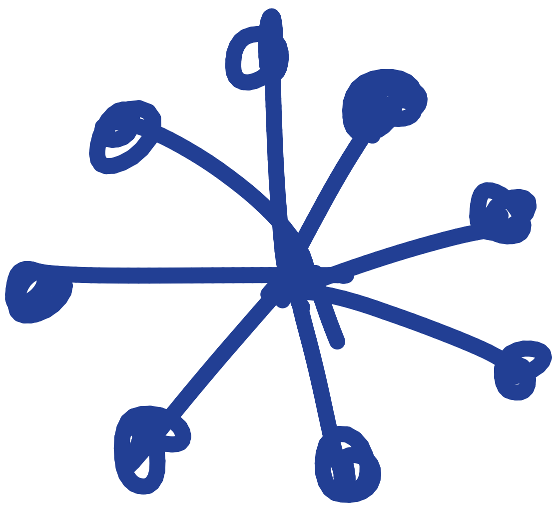
N^2-1 Goldstones (pions)

+ η'-meson

Pion decay constant:

$$f_\pi = \sqrt{N_c}\,\Lambda$$

Baryons (Witten):



N_c quarks

$|\leftarrow R_B \sim \Lambda^{-1} \rightarrow|$

Mass: $M_B \sim N_c \Lambda$

Entropy bound:

$$S_{MAX} = M_B R_B = N_c$$

Is saturated at the unitarity bound:

$$N_c \sim N \sim \frac{1}{g^2}$$

Baryon entropy:

$$S_B = \ln \binom{N_c + N}{N_c} \sim N$$

Thus, at the unitarity bound we have:

$$S_B = M_B R_B = \frac{1}{g^2} = N =$$

$$= \left(R_B f_\pi\right)^2$$

Area law!

Exactly same results for Instantons in

4D QCD:

$$\alpha_5 = \frac{R}{g^2} \longleftrightarrow \alpha_{QCD} = \frac{1}{g^2(R)}$$

Instanton entropy saturates bound for $\lambda_t = 1$

$$S_{ins.1} = \frac{64\pi^2}{g^2} \ln(2)$$

And at the saturation point we get the Area!

$$S_{\mathrm{Inst}} = \frac{A}{4G} = \frac{(4\pi R^2)}{4\bar{f}^{-2}}$$

$$G = \frac{g^2 R^2}{64\pi \ln(2)} = \bar{f}^{-2}$$

Goldstone coupling!

Conclusions:

The universal phenomenon:

B.B. = unitarity = Area

$\downarrow$

$$\text{Bound} = \frac{1}{\text{coupling}} = \text{Area}$$

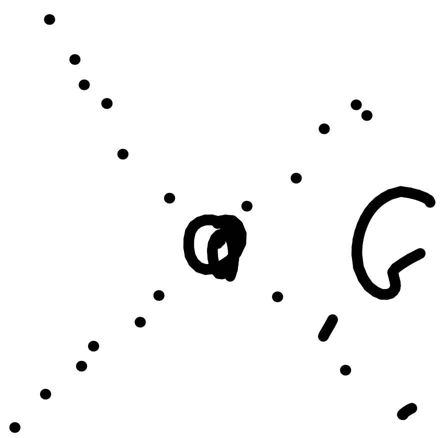
$$S_{mon} = \frac{\hbar}{4G}$$

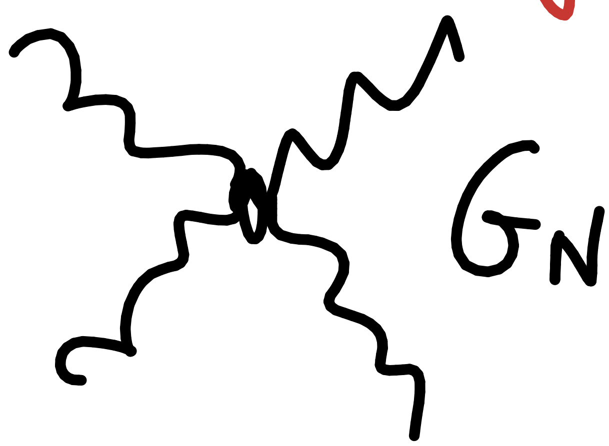
Exactly as for Black Hole!

$$G \longleftrightarrow G_{\text{Newton}}$$

$\downarrow$ Goldstone coupling

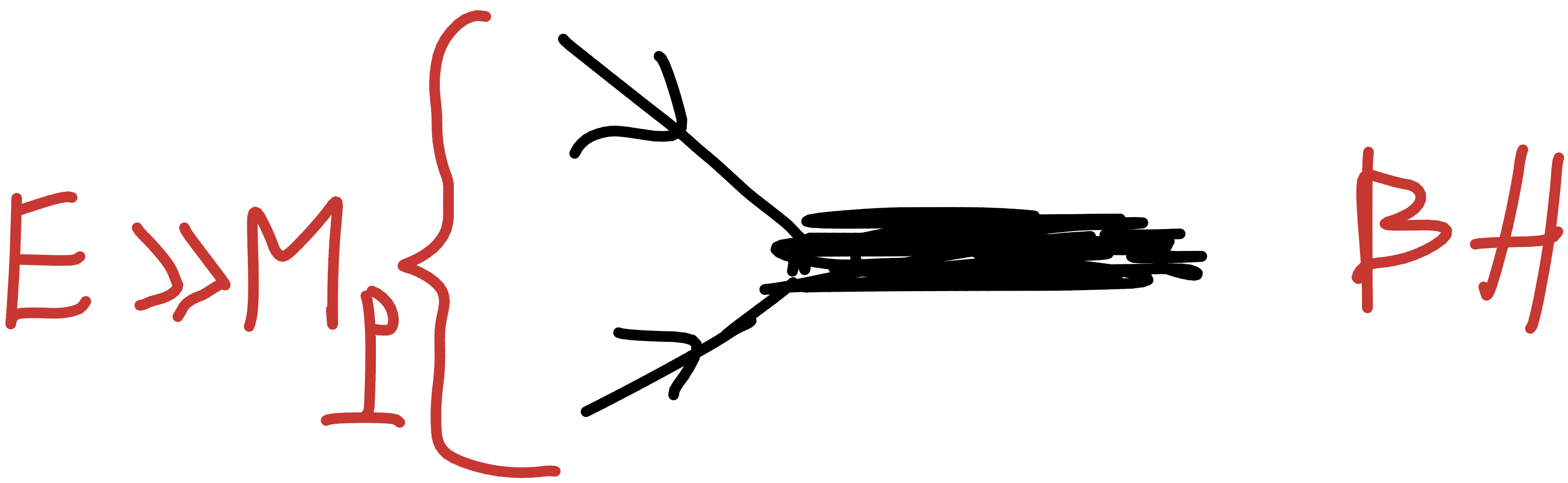
$\downarrow$ Graviton coupling



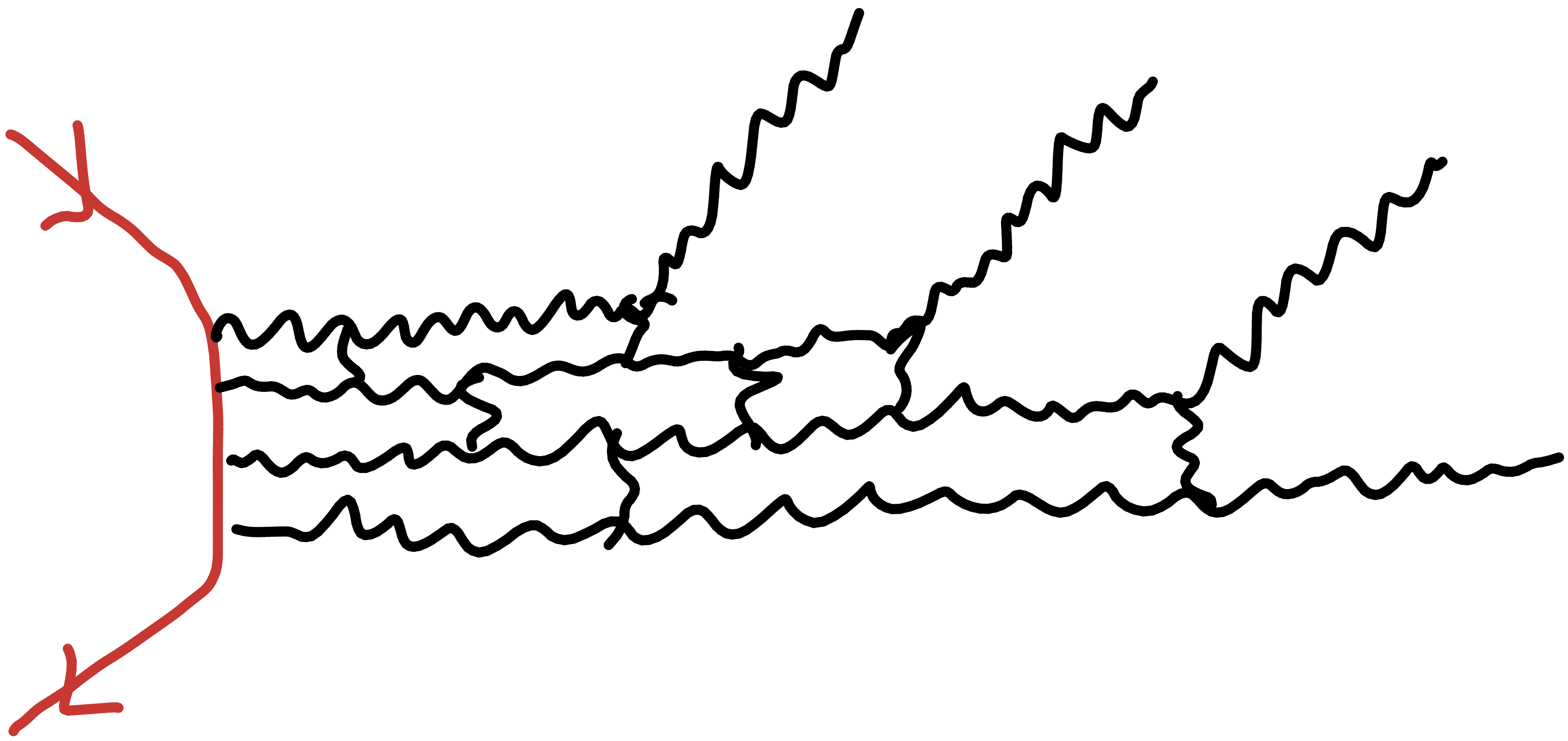


Old idea ('87): Black holes dominate scattering at $E \gg M_P$

't Hooft; Gross, Mende; Amati, Ciafaloni, Veneziano



Understanding as $2 \to N$ process:



$$2 \to BH \to N$$

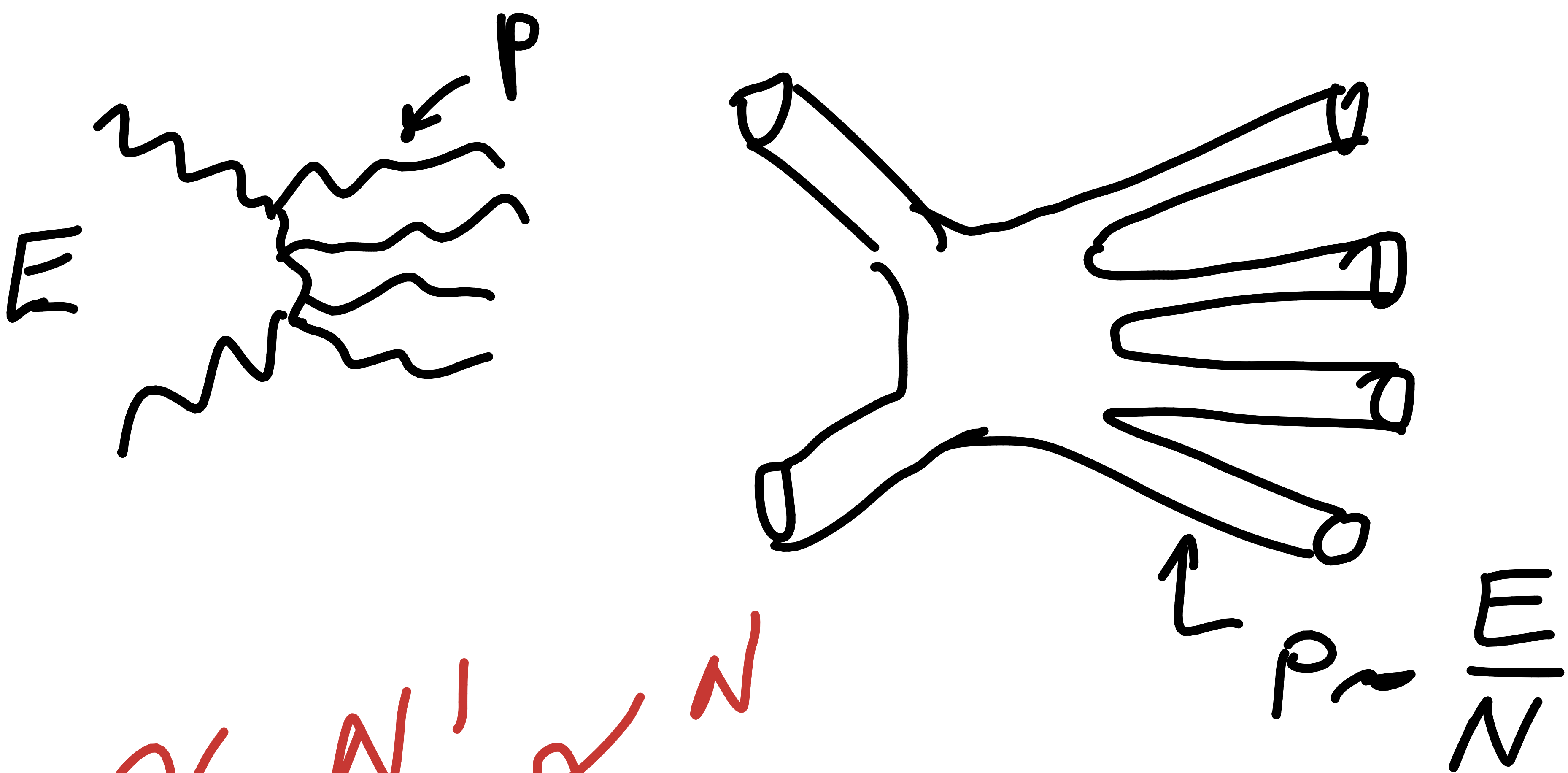
where $N = (M_P R)^2$

and

$$R = \frac{E}{M_P{}^2} = T_H^{-1}$$

$2 \to N$ computation

G.D., Gomez, Isermann, Lüst, Stieberger '14;
Addazi, Bianchi, Veneziano '16



$\sim N! \, \alpha_g^N$

$$\alpha_g = \left(\frac{p}{M_P}\right)^2$$

Saturates unitarity when: $P = \frac{M_P^2}{E} = T_H$

$$\alpha_g = \frac{1}{N} = \frac{1}{S} = (M_P R)^{-2}\ !$$

by the way:

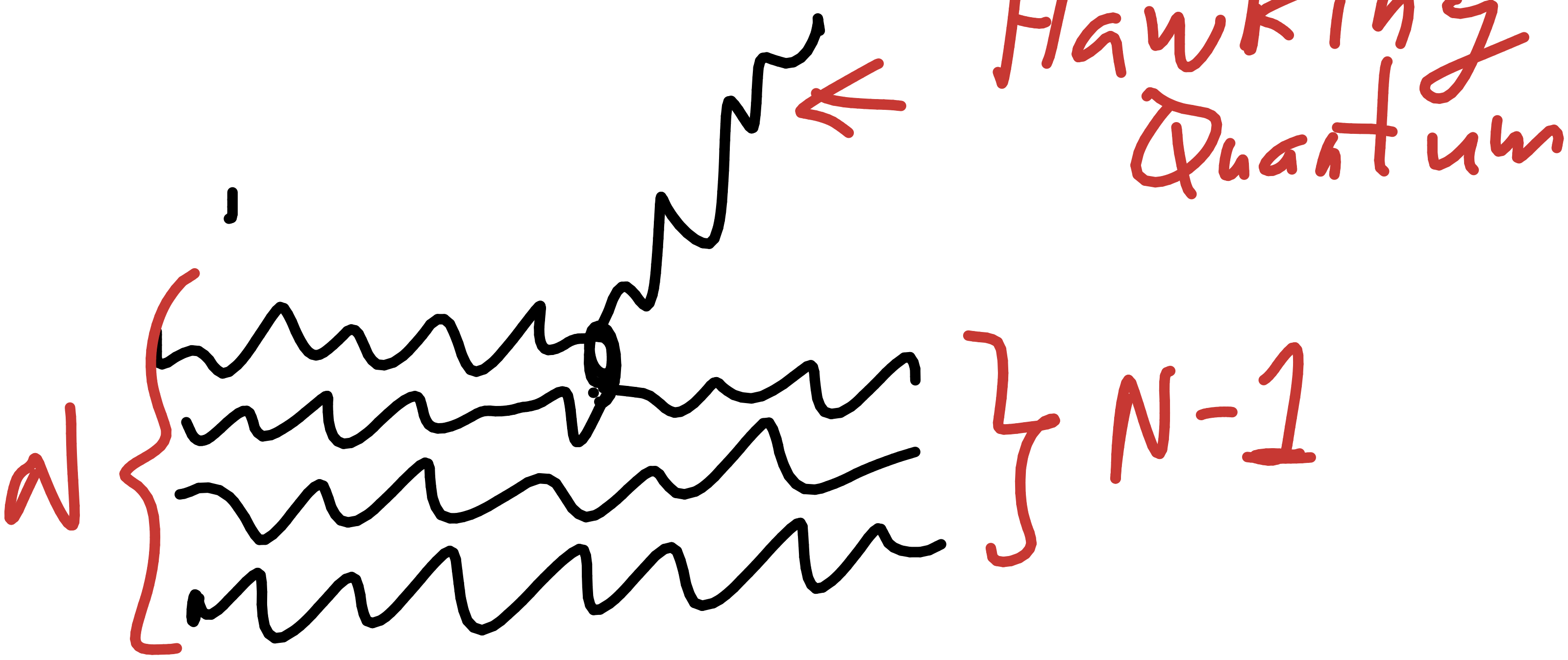
$$N!\,\alpha_g^N \longrightarrow e^{-S}\ !$$

Black hole N-portrait

G.D, Gomez '11

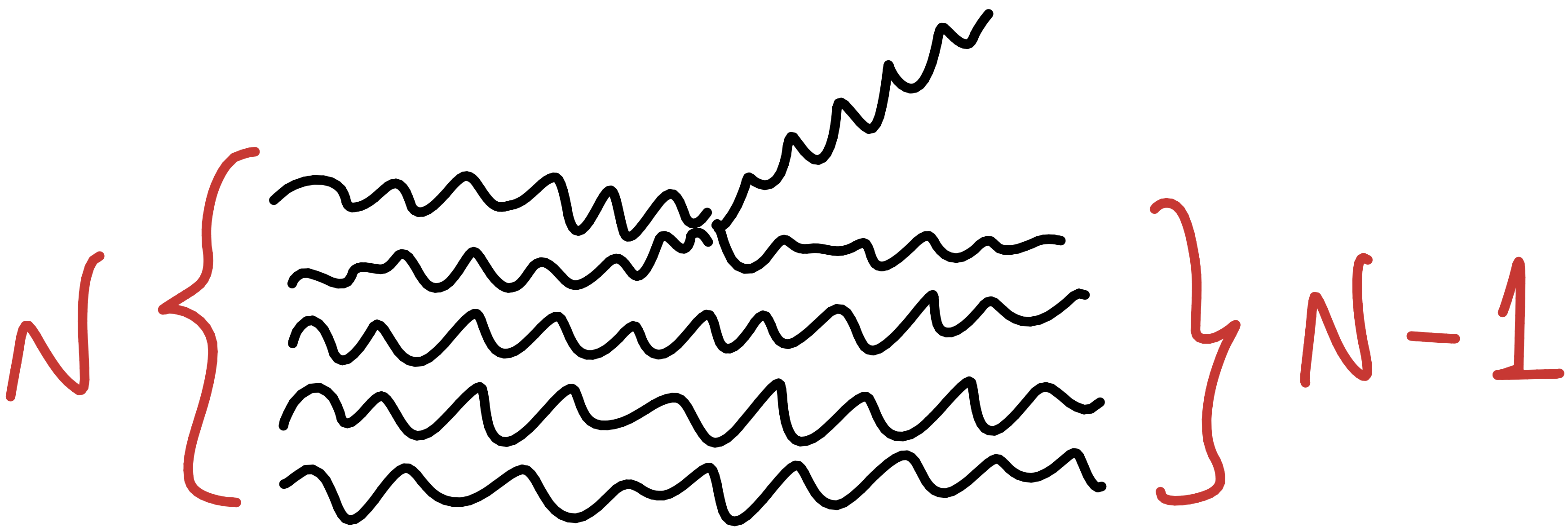
Boundstate of N soft gravitons

$\lambda = R$



$$N = (R M_P)^2 = \frac{1}{\alpha_g} \;!$$

$|dS\rangle =$ Coherent state of N soft gravitons

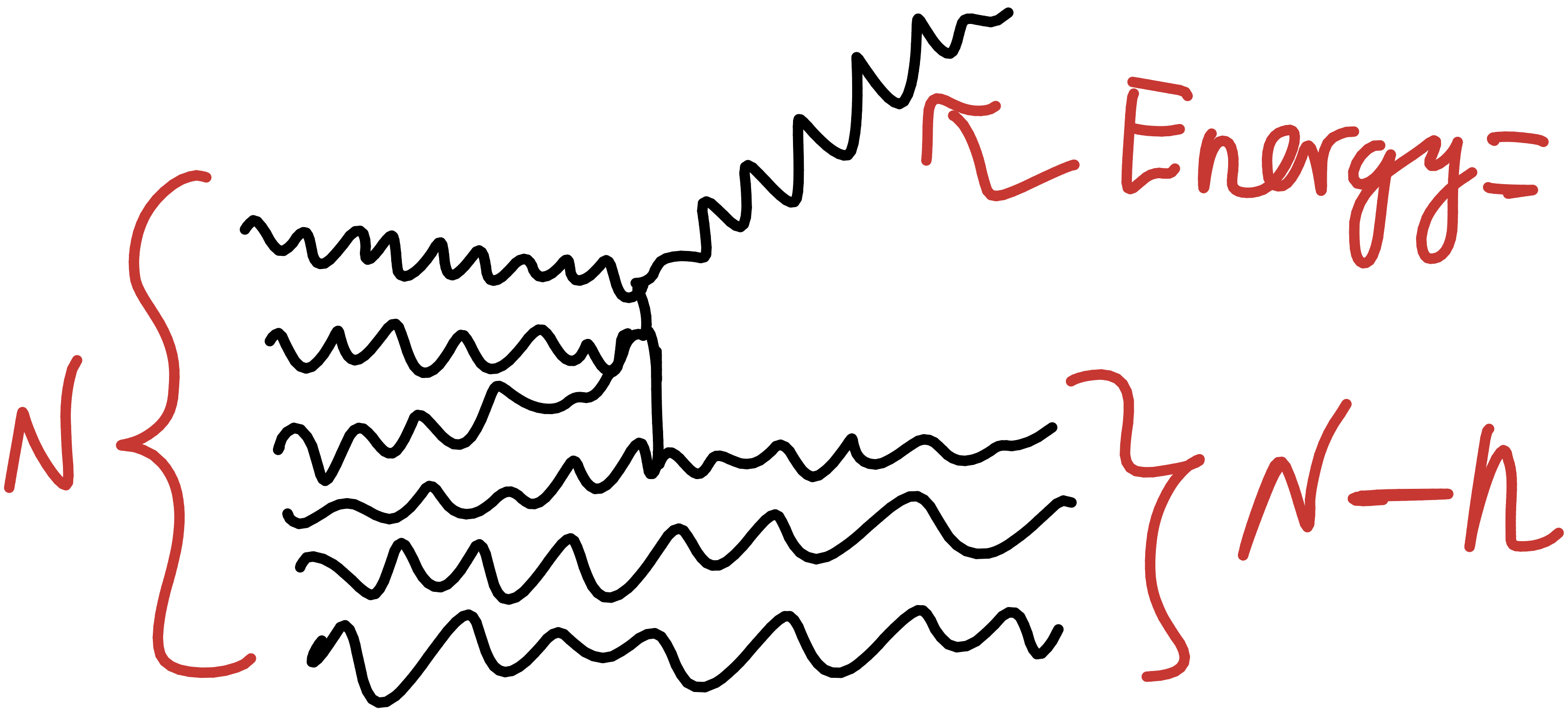


Gibbons-Hawking radiation

$\downarrow$

decay of de Sitter!

Approximate thermality:



Radiation of hard modes is exponentially suppressed!

Half-life time of deSitter:

$$t_Q \sim N_{GH} H^{-1}$$

After

$$t_Q \sim N_{GH} H^{-1}$$

de Sitter Quantum-Breaks!

No classical description available.

Complete loss of coherence.

Prospects for many-body systems (QCD Atoms?).

with: Cesar Gomez

Alex Pritzel,

Nico Wintergerst

Daniel Flassig

Andre Franca

Mischa Panchenko

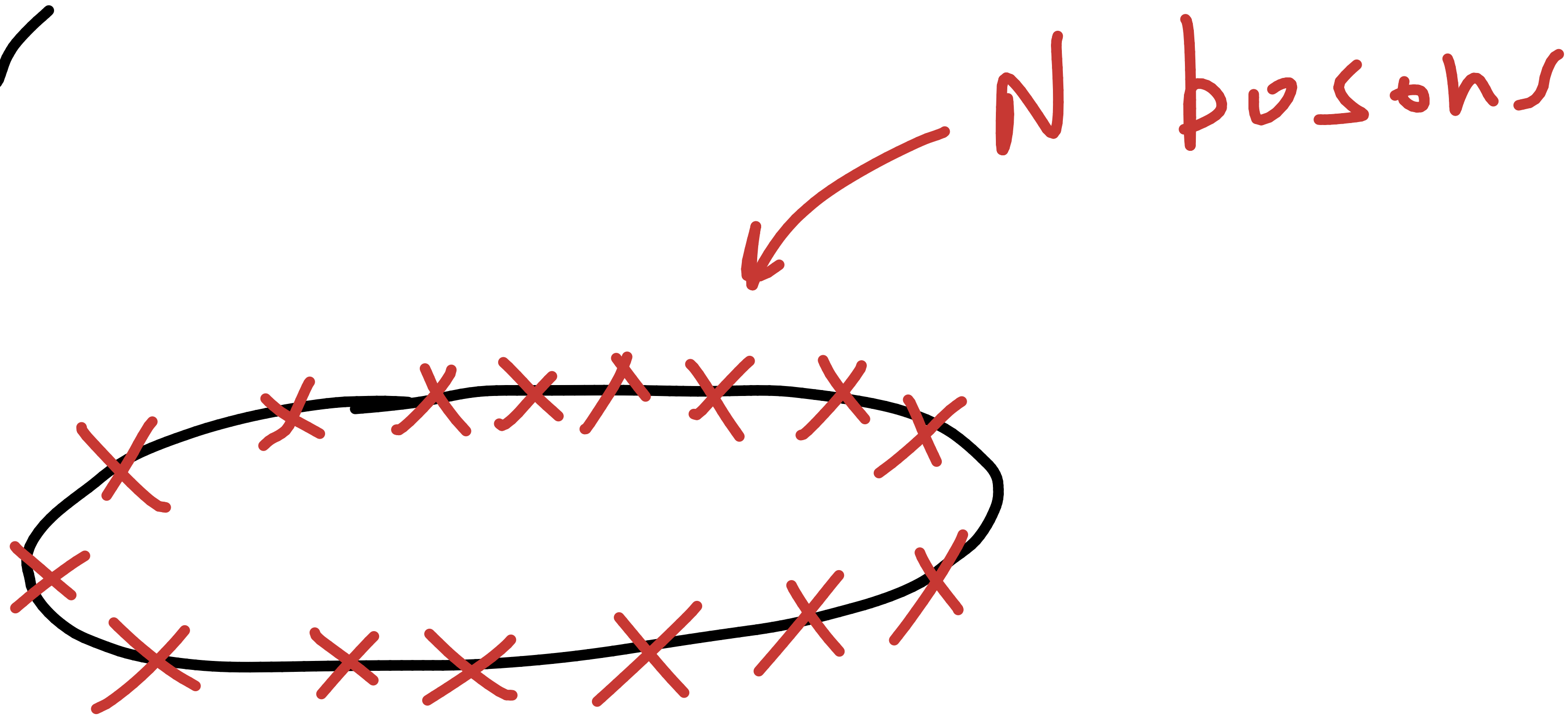
Sebastian Zell

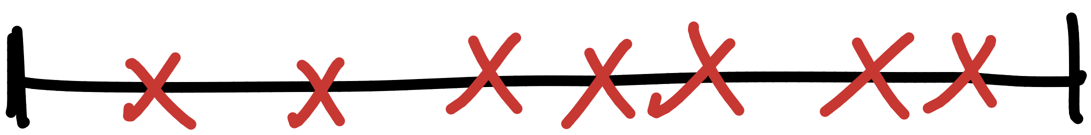
Macro Michel

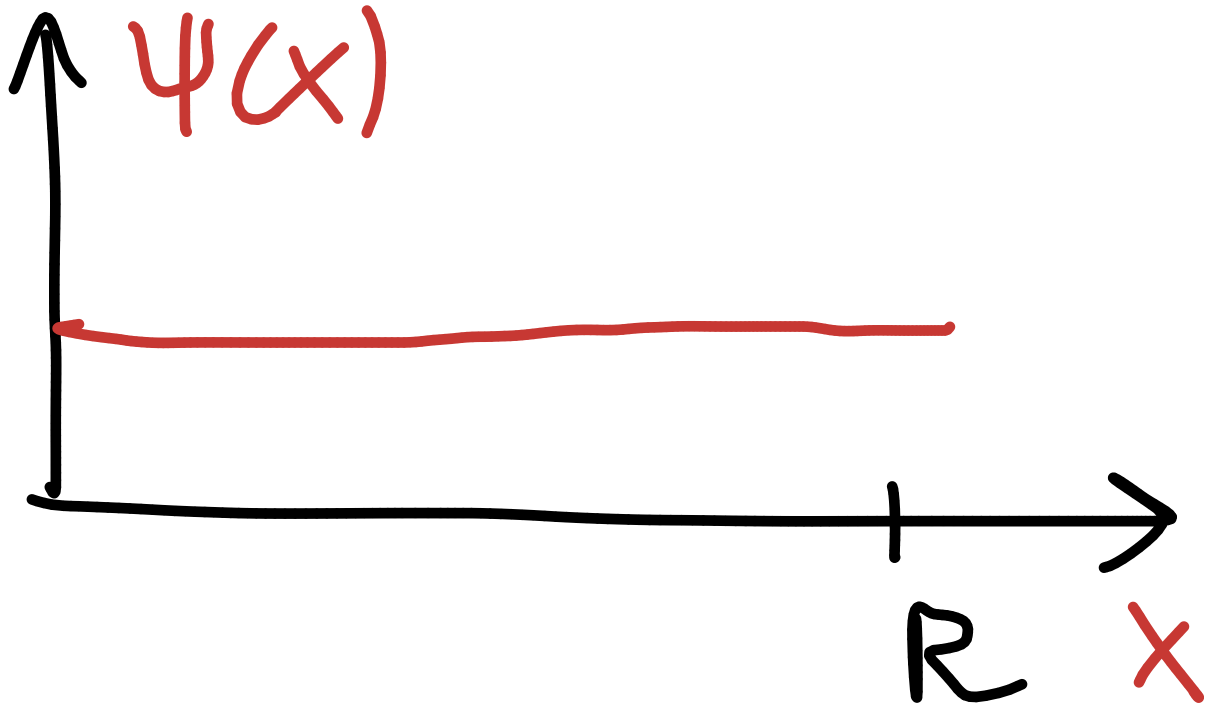
Lukas Eisemann

What we need is N bosons attractive:

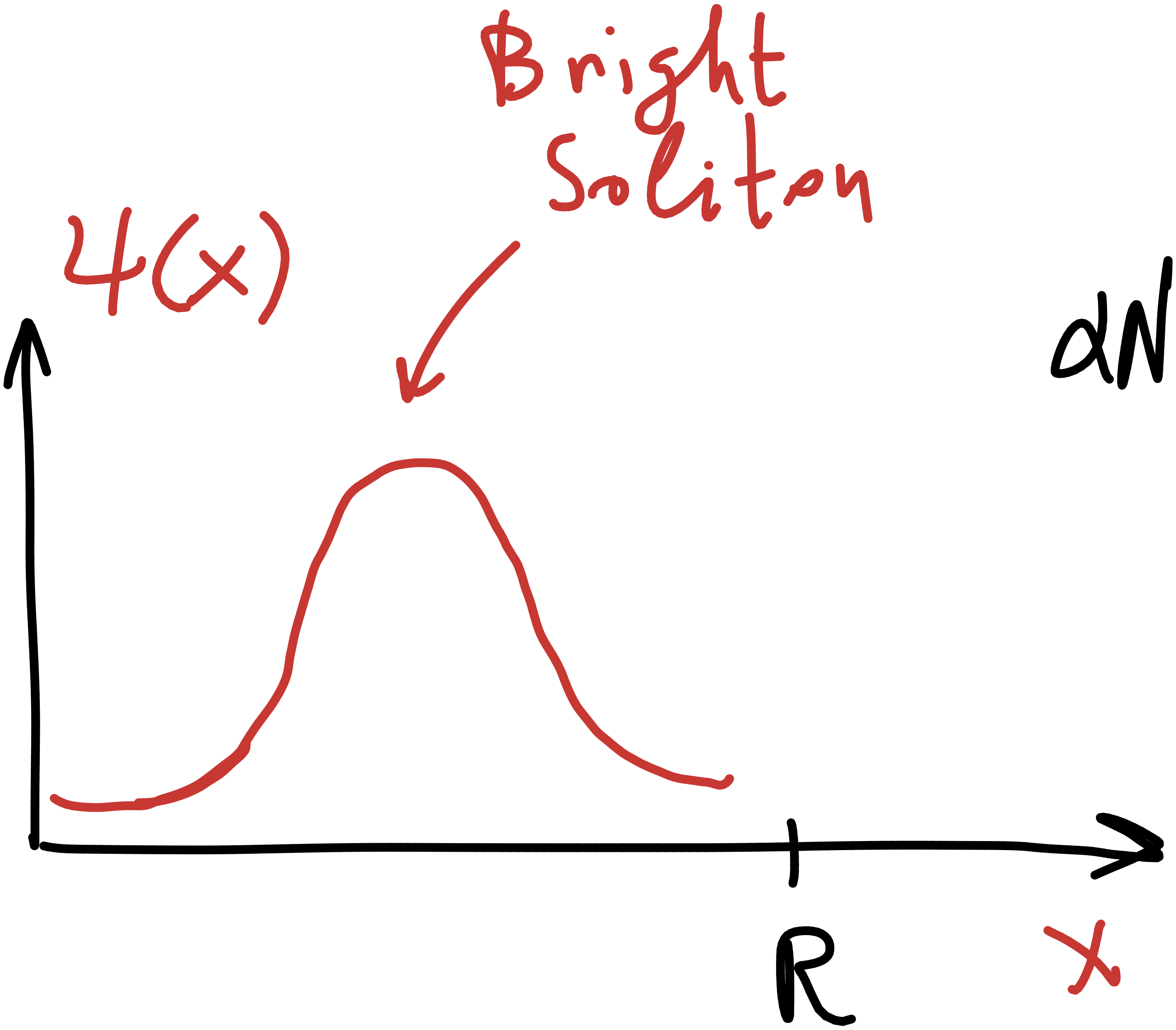
$$\hat{H} = \int_V -\hat{\psi}^\dagger \Delta \hat{\psi} - \alpha \hat{\psi}^\dagger \hat{\psi}^\dagger \hat{\psi} \hat{\psi}$$





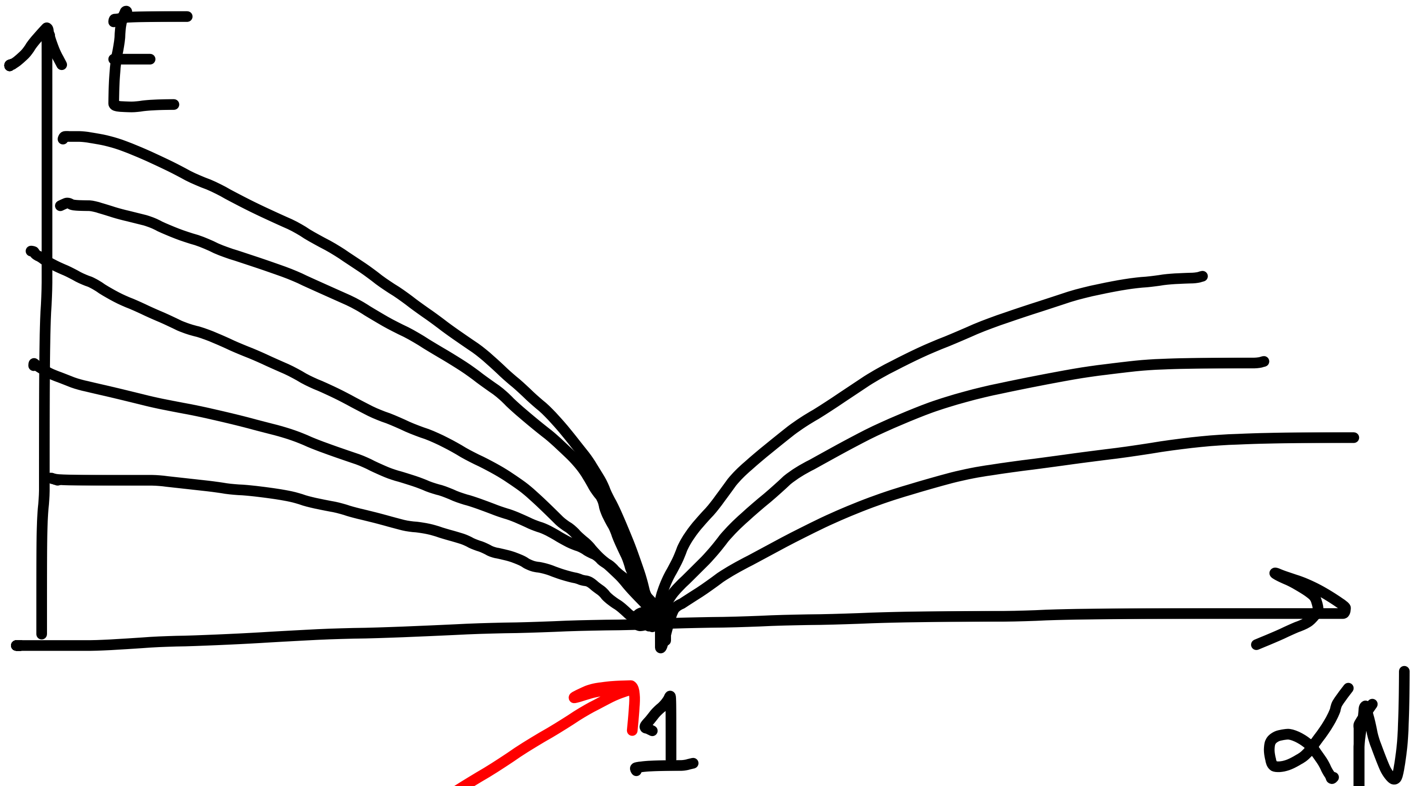


$\alpha N < 1$

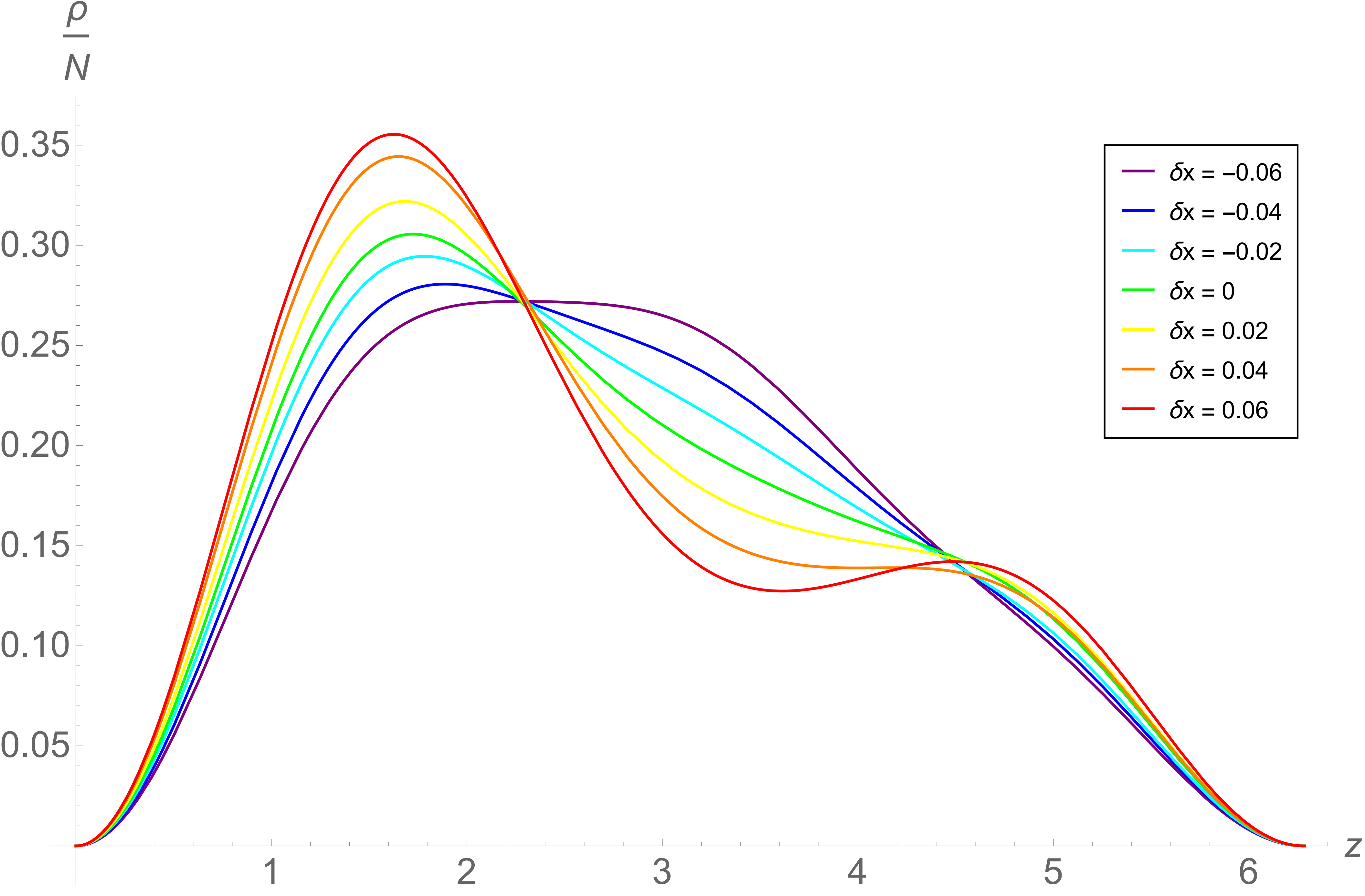


$\alpha N > 1$

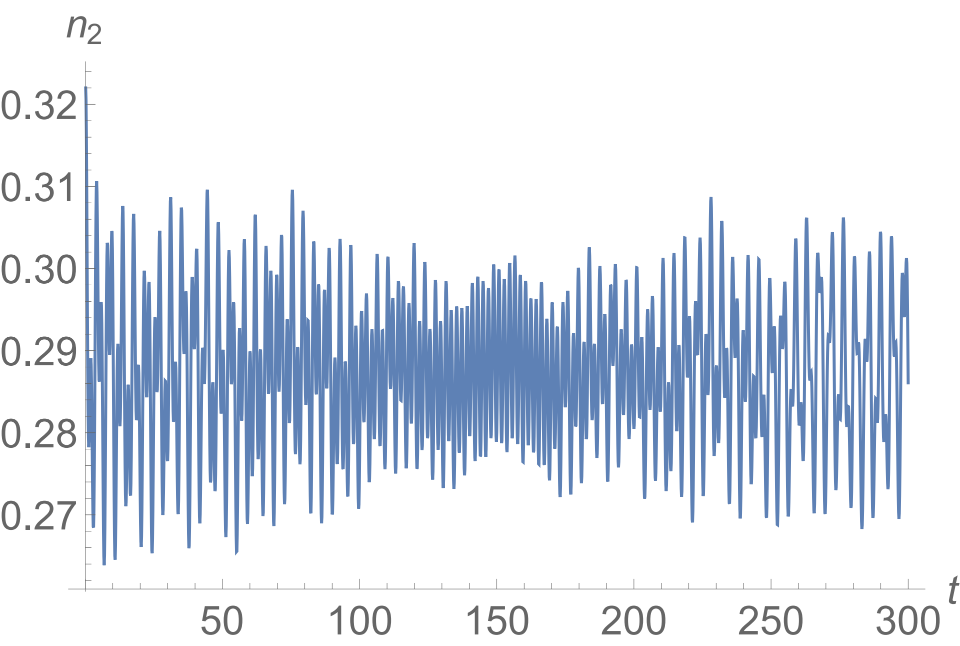
$\alpha N = 1$ ← Critical point of quantum phase transition



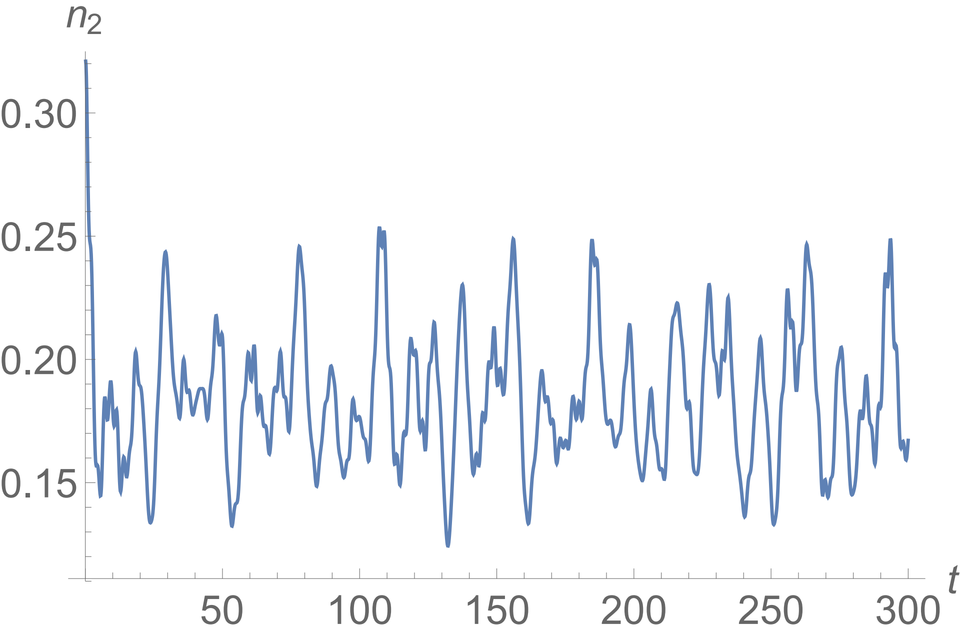
Many cheap (gapless) qubits!

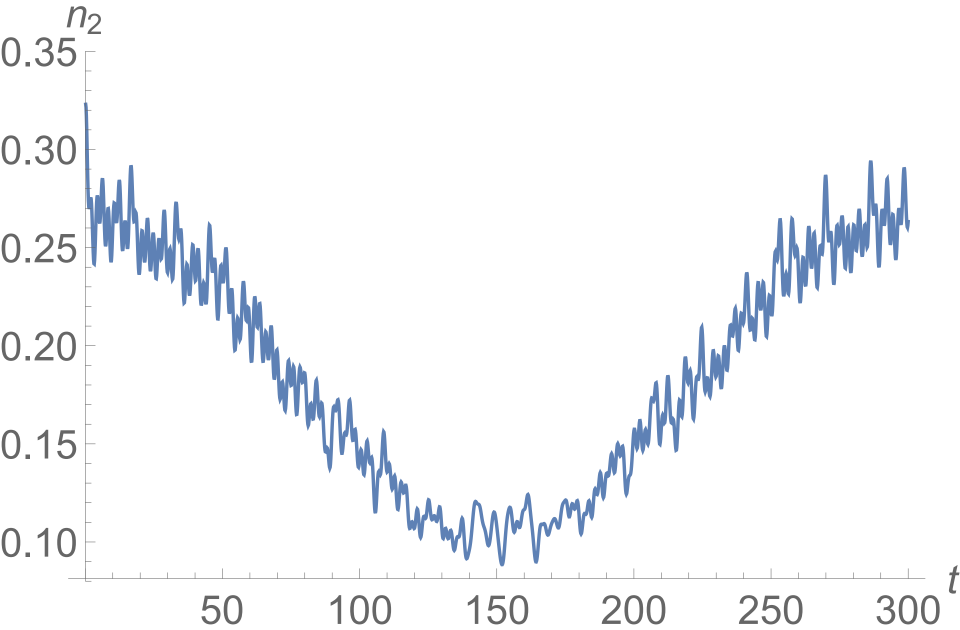


ρ/N
0.35
0.30
0.25
0.20
0.15
0.10
0.05
1
2
3
4
5
6
z
δx = −0.06
δx = −0.04
δx = −0.02
δx = 0
δx = 0.02
δx = 0.04
δx = 0.06

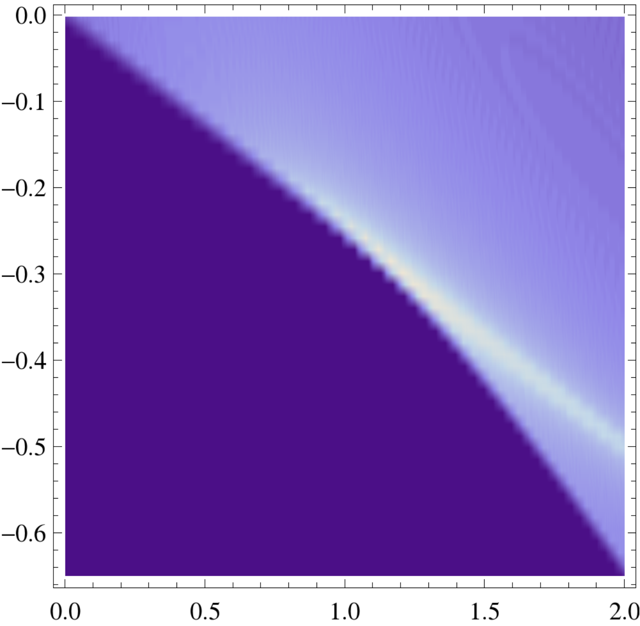


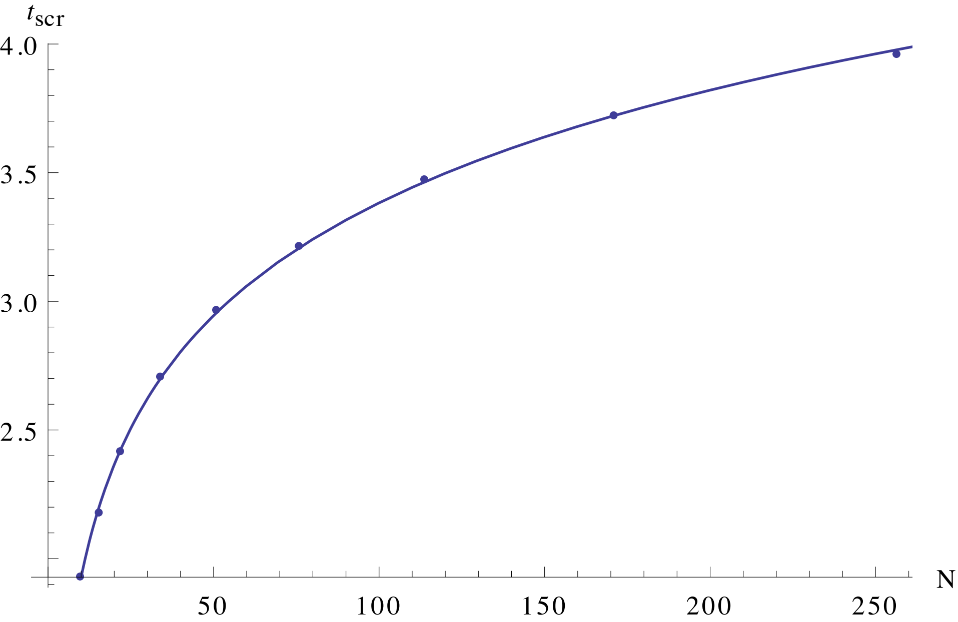
n_2
0.32
0.31
0.30
0.29
0.28
0.27
t
50
100
150
200
250
300


n_2
0.30
0.25
0.20
0.15
t
50
100
150
200
250
300



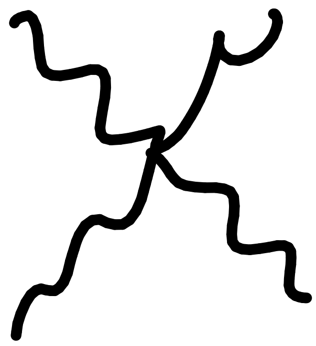
n_2
0.35
0.30
0.25
0.20
0.15
0.10
50
100
150
200
250
300
t


0.0
-0.1
-0.2
-0.3
-0.4
-0.5
-0.6
0.0
0.5
1.0
1.5
2.0

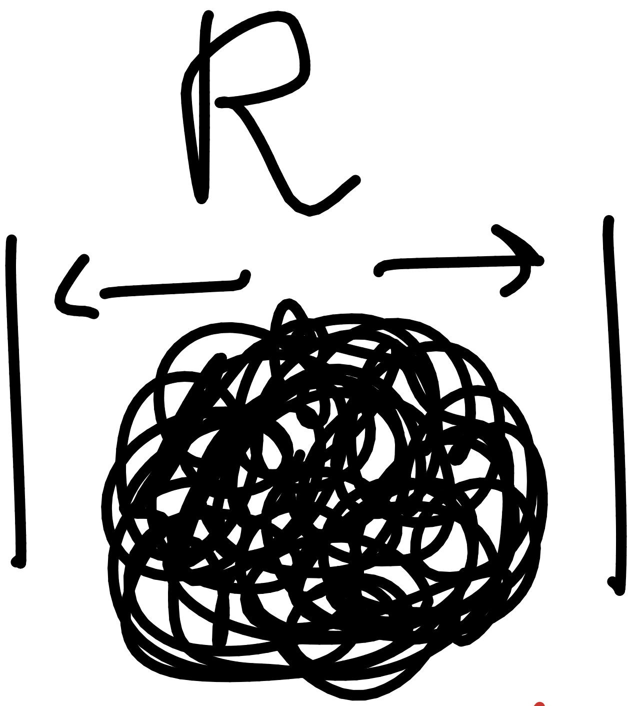


t_{scr}
4.0
3.5
3.0
2.5
N
50
100
150
200
250

Conclusions

In a theory with 4-point coupling $\alpha \rightarrow$  ,

a non-perturbative entity of size



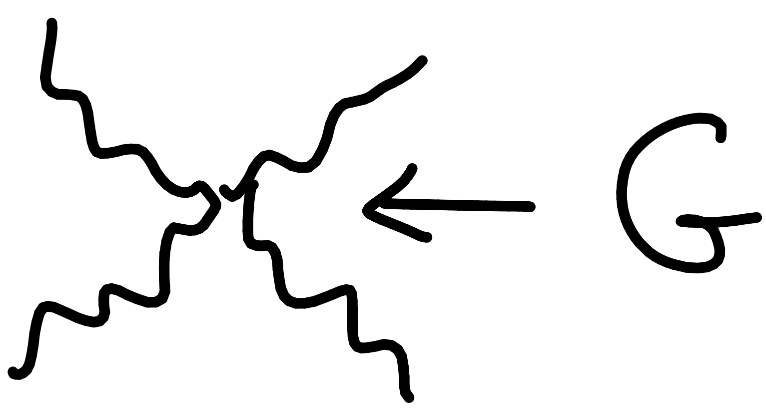
Saturates Bekenstein entropy bound in the following way $\longrightarrow$

* When theory saturates perturbative unitarity.

* At saturation point:

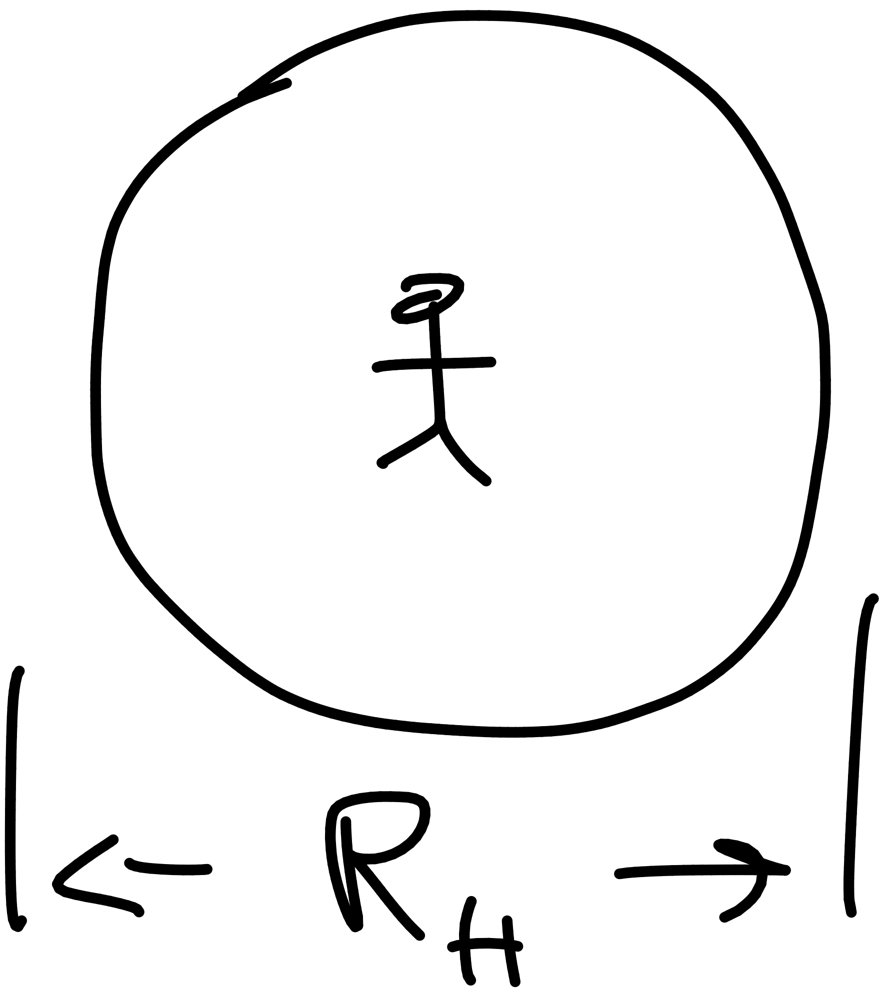
$$S_{max} = \frac{\text{Area}}{4G} = \frac{1}{\alpha(R)}$$

4-point coupling of bosons (pions, gravitons ...)



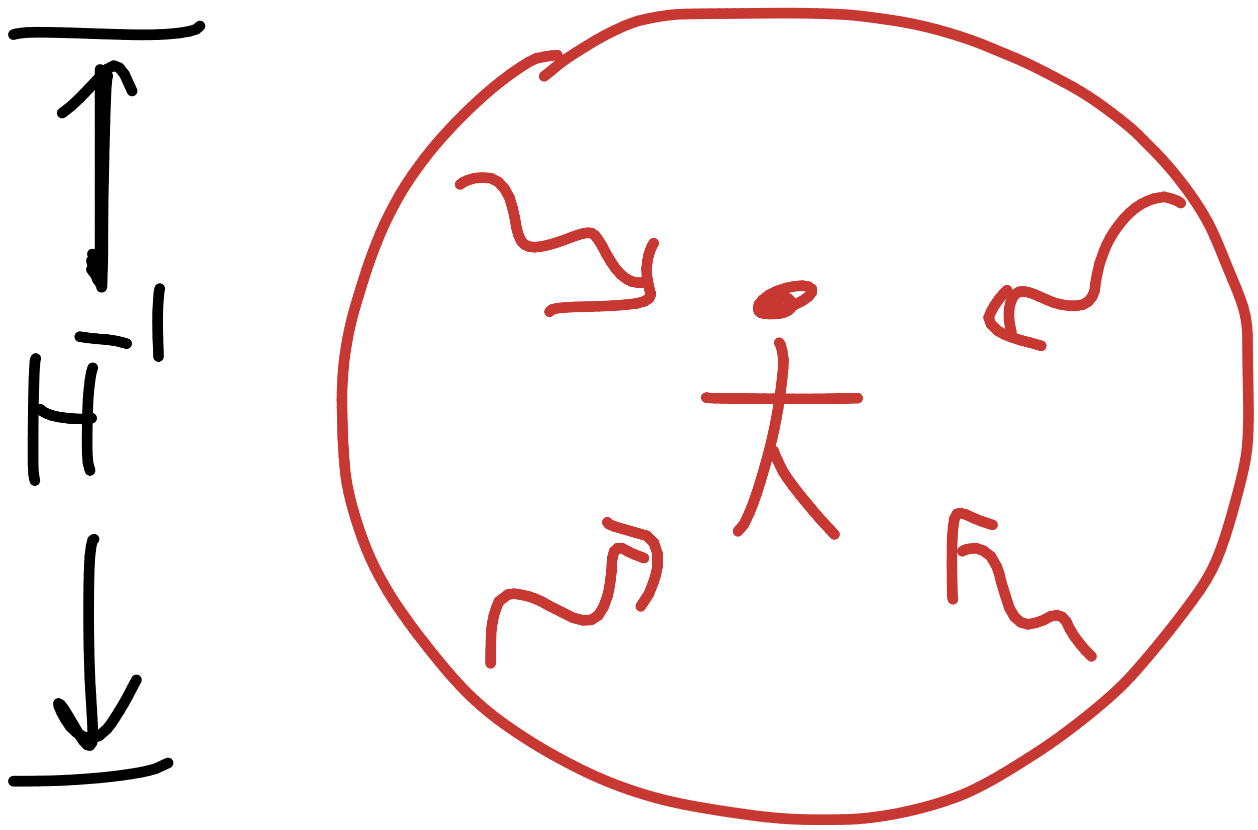
What about deSitter/Inflation?

$$ds^2 = dt^2 - e^{2Ht} d\vec{x}^2$$



$$R_H = H^{-1}$$

Semi-classical de Sitter



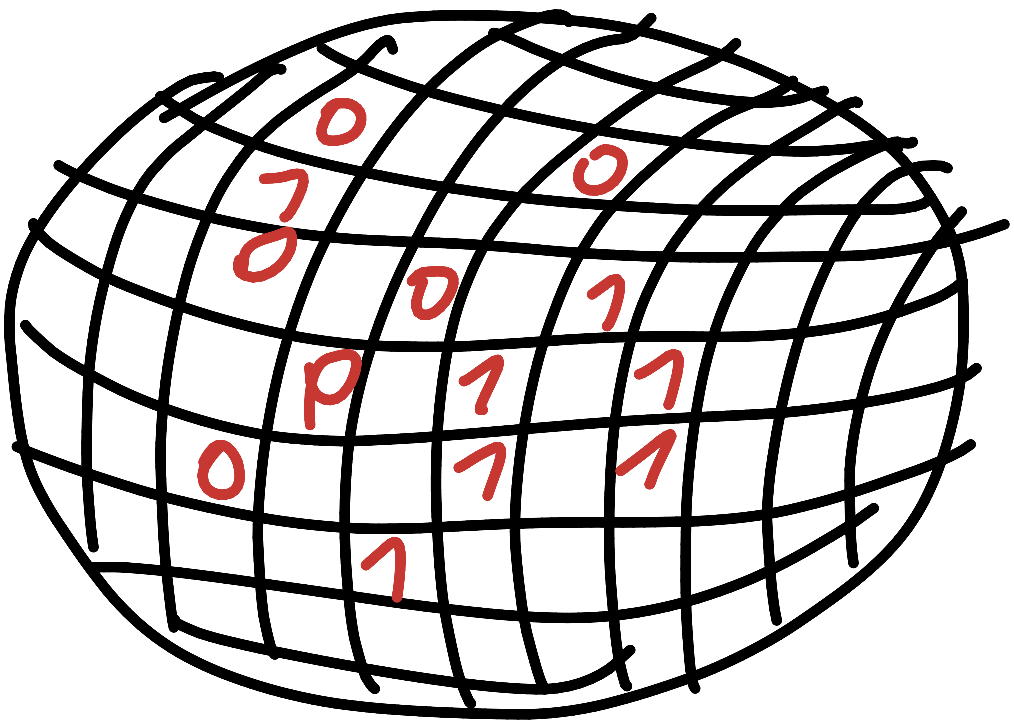
Gibbons-Hawking radiation

$$T_{GH} = H$$

Quantum picture:

Gibbons-Hawking entropy

$$N_{GH} = \frac{R_H^2}{L_P^2} = \frac{M_P^2}{H^2}$$



$$|dS\rangle = |\underbrace{0,1,1,0,\cdots\cdots 1}_{N_{GH}}\rangle$$

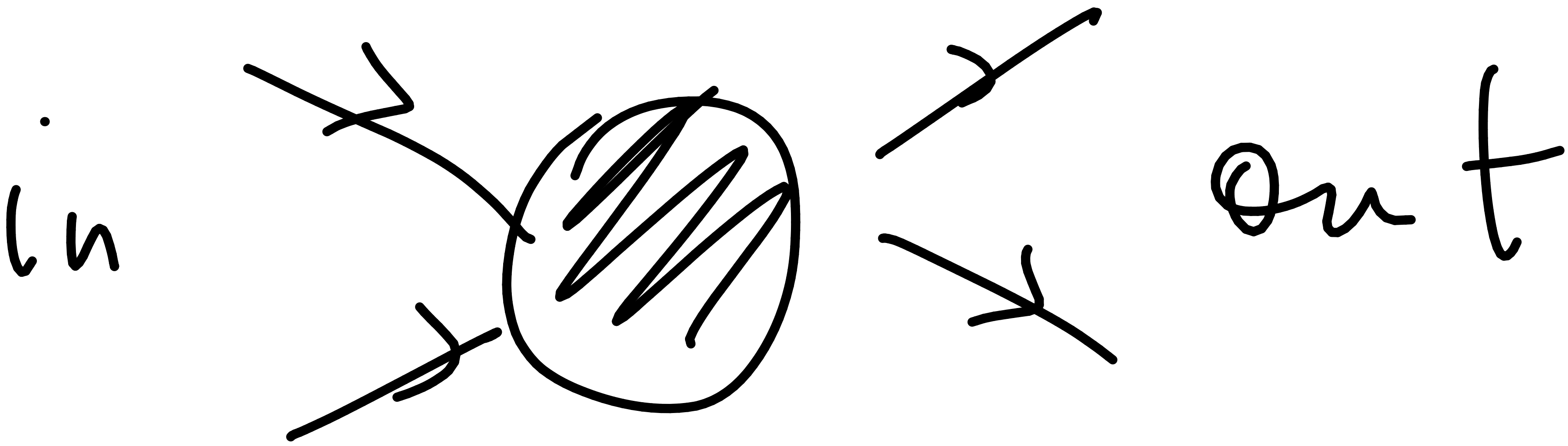
Notice, also here

$$S_{GH} = \frac{1}{\alpha_g} = (R_H M_P)^2$$

4-point coupling of gravitons of wavelength $\sim R_H$!

But, in which sence dS saturated unitarity?

Is it an S-matrix state?



Is $|dS\rangle$ a quantum coherent state of gravitons with $N = \frac{1}{\alpha_g}$?

G.D., Gomez

'12

So in S-matrix theory $|dS\rangle$ cannot be a good vacuum.

It must be viewed as an excited state on good S-matrix vacuum e.g. Minkowski

|de Sitter⟩ as coherent state of gravitons on |Minkowski⟩
(G.D. & Gomez '11, '13, '14 + Zell)

Universal features:

⊛ Number of constituents

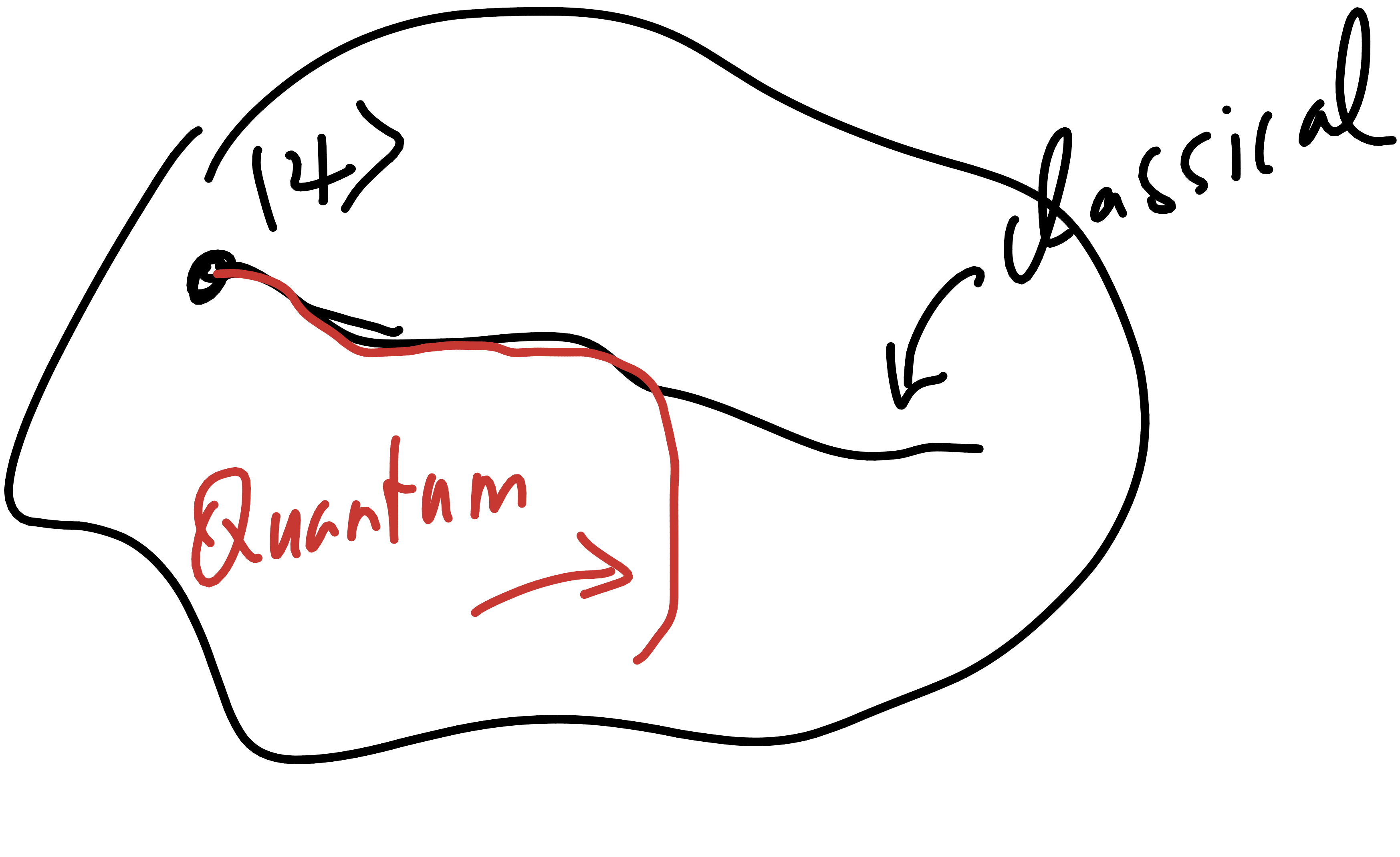
$$N = \frac{M_P^2}{H^2} = N_{GH}$$

⊛ Their frequencies

$$w = H$$

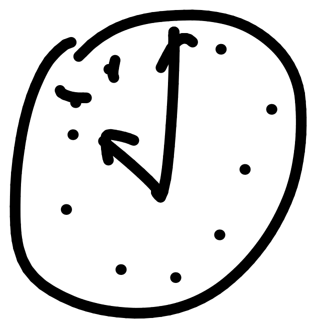
In such a case $|dS\rangle$ undergoes Quantum Breaking after the time-scale

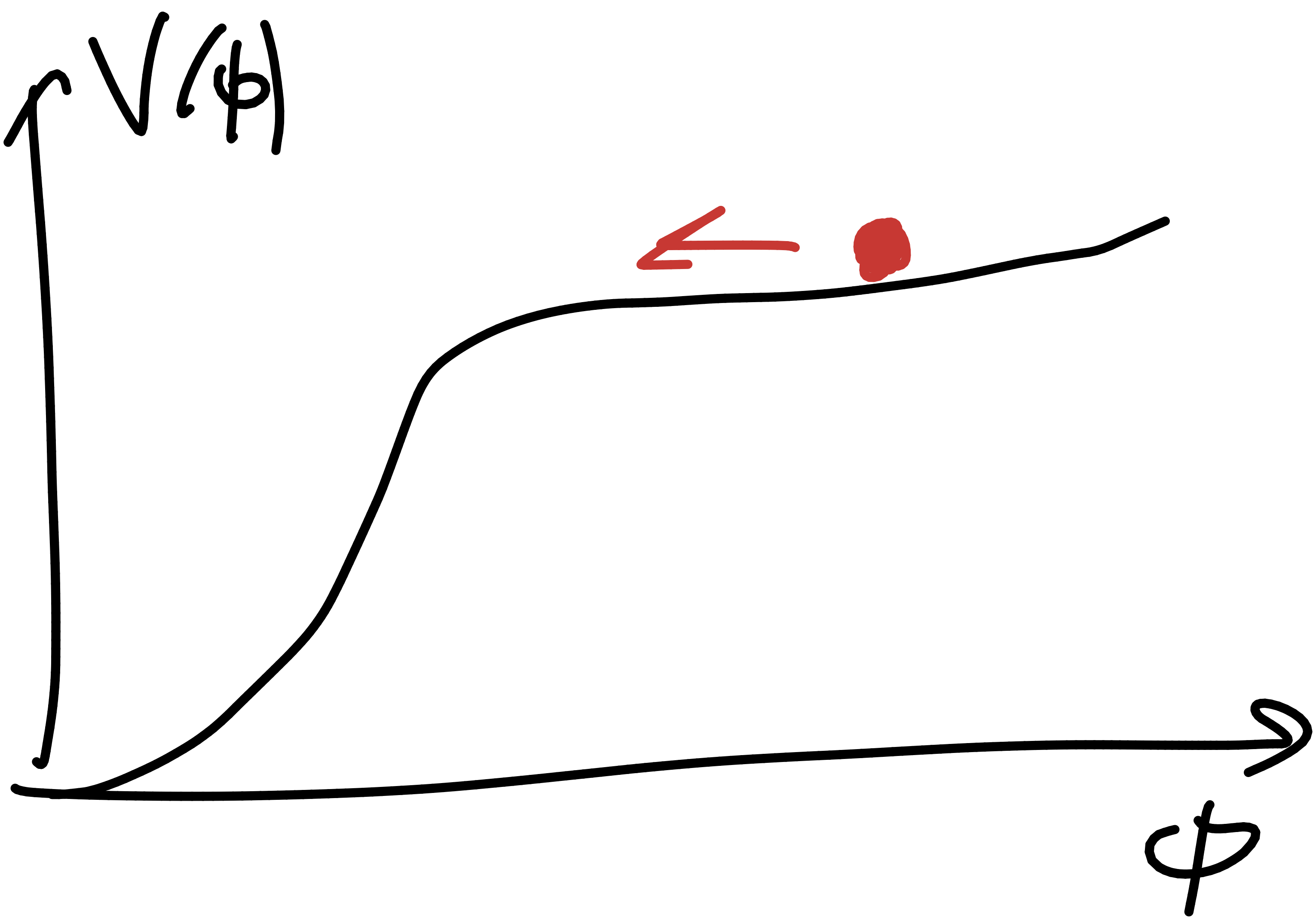
$$t_Q = N_{GH} \cdot H^{-1}$$

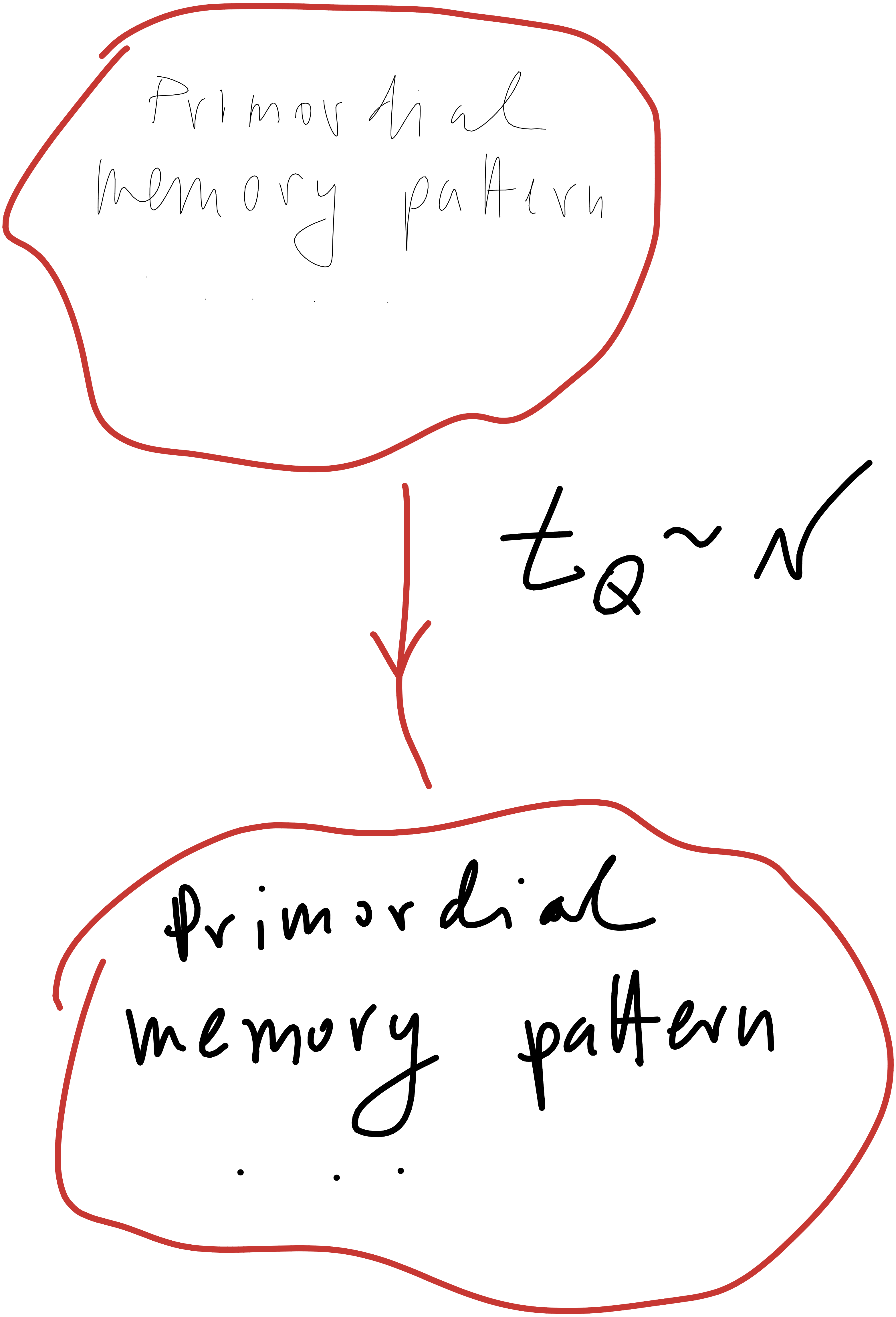


Graceful exit required before t_Q.

Classical clock by inflaton






Primordial memory pattern
...
$t_Q \sim N$
Primordial memory pattern
...

Conclusions:

* In our Hubble inflation lasted

$$N_e < N_{max} = \frac{M_P^2}{H_{inf}^2}.$$

* Can a consistent theory allow $N_e > N_{max}$ in any Hubble patch?

Observational consequence:

If N_e is close to N_{MAX} the effect of quantum breaking must be observable.

How to read out the M-pattern?

$$|M\rangle = |0,1,0,1,1,\ldots\ldots 1,0\rangle$$

I will show that for systems with

$$\alpha N = 1$$

$$\downarrow$$

$$S_{MAX} = \frac{1}{\alpha} = \text{Area}$$

is VERY general.

It follows from UNITARITY!