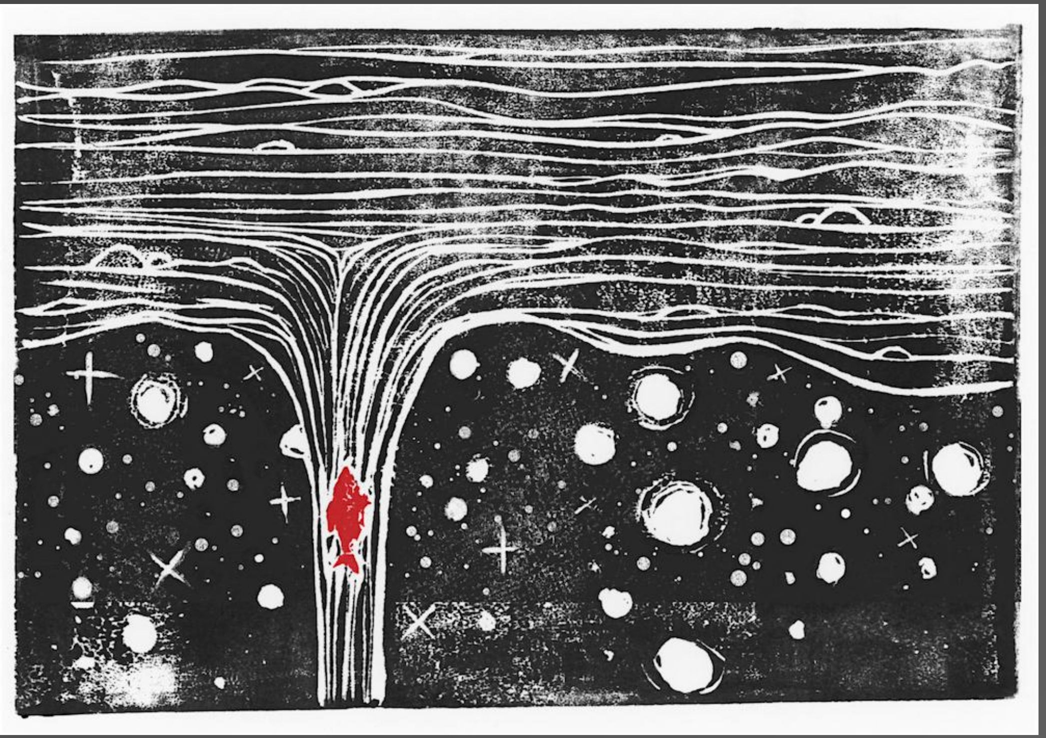
Three small lessons from Analogue Gravity

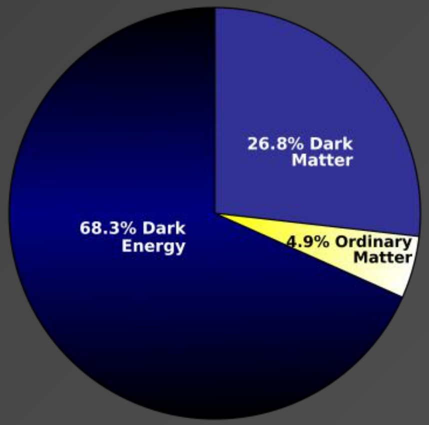
Stefano Liberati



GR, a beautiful but weird theory...

Albeit we "use" GR everyday (e.g. GPS) still it has some tantalising features and it has resisted so far any attempt to be quantised...

- Singularities
- Critical phenomena in gravitational collapse
- Horizon thermodynamics
- The "dark ingredients" of our universe?
- Spacetime thermodynamics: Einstein equations as equations of state.
- Thermodynamics interpretation of Einstein equations
- Faster than light and Time travel solutions
- AdS/CFT duality, holographic behaviour
- Gravity/fluid duality



Gravity as an emergent phenomenon?

Emergent gravity idea: quantizing the metric or the connections does not help because perhaps these are not fundamental objects but collective variables of more fundamental structures.

- GR ⇒ Hydrodynamics
- Metric as a collective variable
- All the sub-Planckian physics is low energy physics
- Spacetime as a condensate of some more fundamental objects
- Spacetime symmetries as emergent symmetries
- Singularities as phase transitions (Big Bang as geometrogenesis)
- Cosmological constant as deviation from the real ground state

- Many models are nowadays resorting to emergent gravity scenarios
  - Causal sets
  - Quantum graphity models
  - Group field theories condensates scenarios
  - AdS/CFT scenarios where the CFT is considered primary
  - Gravity as an entropic force ideas
  - **Condensed matter analogues of gravity**

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Analogue gravity:
a toy model of emergent spacetimes

Analogue models of gravity

An analogue system of gravity is a generic dynamical system where the propagation of linearised perturbations can be described via hyperbolic equations of motion on some curved spacetime possibly characterized be one single metric element for all the perturbations.

analogue models

- Dielectric media
- Acoustic in moving fluids
- Gravity waves
- High-refractive index dielectric fluids: “slow light”
- Optic Fibers analogues
- Quasi-particle excitations: fermionic or bosonic quasi-particles in He3
- Non-linear electrodynamics
- “Solid states black holes”
- Perturbation in Bose-Einstein condensates
- Graphene

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Hydrodynamical models
Theorem: linearised perturbations on a inviscid, irrotational fluid with barotropic EOS move like fields on a curved spacetime

Unruh '81,
Visser '98
But see also White '73

A paradigmatic example: Acoustic Gravity

Continuity $\frac{\partial\rho}{\partial t}+\vec{\nabla}\cdot(\rho\vec{v})=0$ Euler $\rho\frac{d\vec{v}}{dt}\equiv\rho\left[\frac{\partial\vec{v}}{\partial t}+(\vec{v}\cdot\nabla)\vec{v}\right]=-\vec{\nabla}p-\rho\vec{\nabla}\Phi+\vec{f}_{\text{viscosity}}$

External Forces $\vec{f}_{\text{viscosity}}=+\eta\nabla^2\vec{v}+\left(\zeta+\frac{1}{3}\eta\right)\vec{\nabla}\left(\vec{\nabla}\cdot\vec{v}\right)$

$p=$ pressure, $\eta=$ dynamic viscosity, $\zeta=$ bulk viscosity,

$\Phi=$ potential of external driving force (gravity included)

Basic Assumptions

$\vec{\nabla}\times\vec{v}=\vec{0}$ $\vec{v}=\vec{\nabla}\psi$ $\rho=\rho(p)$ $c_s^2=\frac{dp}{d\rho}$

IDEAL PERFECT FLUID
Irrotational Flow
Barotropic
Viscosity free flow

Linearize the above Eq.s around some background

$$\rho(t,x)=\rho_0(t,x)+\varepsilon\rho_1(t,x)$$
$$p(t,x)=p_0(t,x)+\varepsilon p_1(t,x)$$
$$\psi(t,x)=\psi_0(t,x)+\varepsilon\psi_1(t,x)$$

And combine then so to get a second order field equation

$$\frac{\partial}{\partial t}\left(c_s^{-2}\rho_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right)=\nabla\cdot\left(\rho_0\nabla\psi_1-c_s^{-2}\rho_0\vec{v}_0\left(\frac{\partial\psi_1}{\partial t}+\vec{v}_0\cdot\nabla\psi_1\right)\right)$$

This looks messy but if we introduce the "acoustic metric"

$$g_{\mu\nu}\equiv\frac{\rho_0}{c_s}\begin{bmatrix}-\left(c_s^2-v_0^2\right) & -v_0^j\\ -v_0^i & \delta_{ij}\end{bmatrix}$$

We get

$$\Delta\psi_1\equiv\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi_1\right)=0$$

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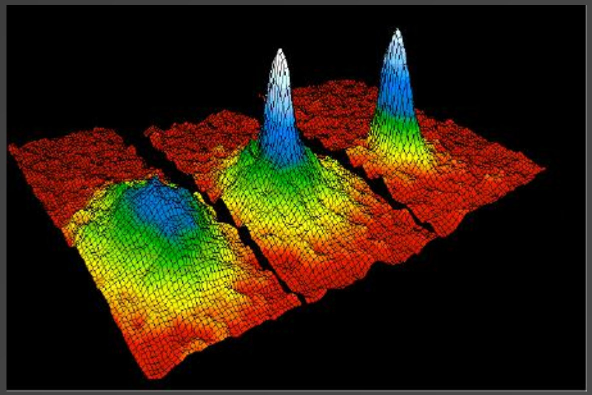
$$\Delta\psi_1\equiv\frac{1}{\sqrt{-g}}\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\partial_\nu\psi_1\right)=0$$

This is the same equation as for a scalar field moving in curved spacetime, possibility to simulate FRW and Black Holes!
Analysis can be generalised to relativistic fluids=>Disformal geometries

A concrete example: BEC analogue gravity

A BEC is quantum system of N interacting bosons in which most of them lie in the same single-particle quantum state
($T<T_c \sim 100$ nK, $N_{atoms} \sim 10^5 \div 10^6$)



It is described by a many-body Hamiltonian which in the limit of dilute condensates gives a non-linear Schrödinger equation

$$i\hbar \frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi}.$$

(a=s-wave scattering length)

$$\kappa(a) = \frac{4\pi a\hbar^2}{m}.$$

This is still a very complicate system, so let's adopt a mean field approximation

Mean field approximation: $\hat{\Psi}(t,\mathbf{x}) = \psi(t,\mathbf{x}) + \hat{\chi}(t,\mathbf{x})$ where $|\psi(t,\mathbf{x})|^2 = n_c(t,\mathbf{x}) = N/V$

$\psi(t,\mathbf{x}) = \langle \hat{\Psi}(t,\mathbf{x}) \rangle$ = classical wave function of the BEC , $\hat{\chi}(t,\mathbf{x})$ = excited atoms

Note that: $\hat{\Psi}|0\rangle = 0$ (atomic Fock vacuum) $\qquad \hat{\Psi}|\Omega\rangle \neq 0$ (ground state)

The ground state is the vacuum for the collective excitations of the condensate (quasi-particles) but this an inequivalent state w.r.t. the atomic vacuum. They are linked by Bogoliubov transformations.

Bose-Einstein condensate: an example of analogue emergent spacetime

By direct substitution of the mean field ansatz in the non-linear Schrödinger equation gives

$$i\hbar\,\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + \kappa|\psi|^2\right)\psi + 2\kappa\left(\tilde{n}\psi + \tilde{m}\psi^*\right)$$

Background dynamics

$$i\hbar\,\frac{\partial}{\partial t}\widehat{\chi} = \left(-\frac{\hbar^2}{2m}\nabla^2 - \mu + 2\kappa n_T\right)\widehat{\chi} + \kappa m_T\,\widehat{\chi}^\dagger$$

Excitations dynamics

$$n_c \equiv |\psi(t,\mathbf{x})|^2\,; \qquad m_c \equiv \psi^2(t,\mathbf{x});$$
$$\tilde{n} \equiv \langle\widehat{\chi}^\dagger\widehat{\chi}\rangle; \qquad \tilde{m} \equiv \langle\widehat{\chi}\widehat{\chi}\rangle;$$
$$n_T = n_c + \tilde{n}; \qquad m_T = m_c + \tilde{m}.$$

These are the so called Bogoliubov-de Gennes equations
The first one encodes the BEC background dynamics
The second one encodes the dynamics for the quantum excitations

The equations are coupled via the so called anomalous mass m̃ and density ñ. Which we shall neglect for the moment...

Let's consider quantum perturbations over the BEC background and adopt the "quantum acoustic representation" (Bogoliubov transformation)

$$\widehat{\chi}(t,\mathbf{x}) = e^{-i\theta/\hbar}\left(\frac{1}{2\sqrt{n_c}}\,\widehat{n}_1 - i\frac{\sqrt{n_c}}{\hbar}\,\widehat{\theta}_1\right)$$

For the perturbations one gets the system of equations

$$\partial_t\widehat{n}_1 + \frac{1}{m}\nabla\cdot\left(\widehat{n}_1\,\nabla\theta + n_c\,\nabla\widehat{\theta}_1\right) = 0,$$
$$\partial_t\widehat{\theta}_1 + \frac{1}{m}\nabla\theta\cdot\nabla\widehat{\theta}_1 + \kappa(a)\,n_1 - \frac{\hbar^2}{2m}\,D_2\widehat{n}_1 = 0.$$

Where D_2 is a represents a second-order differential operator: the linearized quantum potential

$$D_2\,\widehat{n}_1 \equiv -\frac{1}{2}n_c^{-3/2}\,[\nabla^2(n_c^{+1/2})]\,\widehat{n}_1 + \frac{1}{2}n_c^{-1/2}\,\nabla^2(n_c^{-1/2}\,\widehat{n}_1).$$

Acoustic metric and the fate of Lorentz invariance

For very long wavelengths the terms coming from the linearized quantum potential D_2 can be neglected.

$$\Delta\theta_1 \equiv \frac{1}{\sqrt{-g}}\,\partial_\mu\left(\sqrt{-g}\,g^{\mu\nu}\,\partial_\nu\right)\widehat{\theta}_1 = 0,$$

The so obtained metric is again the acoustic metric

$$c_s = \frac{\hbar}{m}\sqrt{4\pi\rho a}$$

$$g_{\mu\nu}(t,\mathbf{x}) \equiv \frac{c_s}{\lambda}\begin{bmatrix} -\left(c_s^{\;2} - v_0^2\right) & \vdots & -(v_0)_j \\ \cdots & \cdot & \cdots \\ -(v_0)_i & \vdots & \delta_{ij} \end{bmatrix} = \frac{n_0}{c_s m}\begin{bmatrix} -\left(c_s^{\;2} - v_0^2\right) & \vdots & -(v_0)_j \\ \cdots & \cdot & \cdots \\ -(v_0)_i & \vdots & \delta_{ij} \end{bmatrix}$$

If instead of neglecting the quantum potential we adopt the eikonal approximation (high-momentum approximation) we find, as expected, deviations from the Lorentz invariant physics of the low energy phonons.

E.g. the dispersion relation for the BEC quasi-particles is

$$\omega^2 = c_s^2 k^2 + \left(\frac{\hbar}{2m}\right)^2 k^4$$

This (Bogoliubov) dispersion relation (experimentally observed) actually interpolates between two different regimes depending on the value of the fluctuations wavelength
$\lambda=2\pi/|k|$ with respect to
the "acoustic Planck wavelength"
$\lambda_C=h/(2mc_s)=\pi\xi$ with ξ=healing length of BEC$=1/(8\pi\rho a)1/2$

For $\lambda \gg \lambda_C$ one gets the standard phonon dispersion relation $\omega \approx c|k|$

For $\lambda \ll \lambda_C$ one gets instead the dispersion relation for an individual gas particle $\omega \approx (\hbar^2 k^2)/(2m)$ (breakdown of the continuous medium approximation)

Spin-off, Quantum gravity phenomenology: Planck scale Lorentz violation as a topic example

Suggestions for Lorentz violation searches came from several QG models and are tight to the presence of a fundamental length scale

- String theory tensor VEVs (Kostelecky-Samuel 1989, ...)
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A common prediction of these models is that the Lorentz breaking in the UV leads to a Planck suppressed modified dispersion relation.

$$E^2 = c^2 p^2 \left(1 + \frac{m^2 c^2}{p^2} + \eta \frac{p^{n-2}}{M^{n-2}}\right)$$

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But how can we test Planck scales?

$$\frac{m^2}{p^2} \approx \frac{p^{n-2}}{M^{n-2}} \Rightarrow p_{crit} \approx \sqrt[n]{m^2 M^{n-2}}$$

n	p_{crit} for ν_e	p_{crit} for e^-	p_{crit} for p^+
2	$p \approx m_\nu$~1 eV	$p \approx m_e$=0.5 MeV	$p \approx m_p$=0.938 GeV
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Phenomenology? One needs a dynamical framework ⇒

Model via Effective Field Theory with Lorentz breaking
(Standard Model Extension to LIV operators, CPT even or Odd)

$$E_\gamma^2 = k^2 + \xi_\pm^{(n)} \frac{k^n}{M_{pl}^{n-2}} \quad \text{photons}$$

$$E_{matter}^2 = m^2 + p^2 + \eta_\pm^{(n)} \frac{p^n}{M_{pl}^{n-2}} \quad \text{leptons/hadrons}\,,$$

where, in EFT, $\xi^{(n)} \equiv \xi_+^{(n)} = (-)^n \xi_-^{(n)}$ and $\eta^{(n)} \equiv \eta_+^{(n)} = (-)^n \eta_-^{(n)}$.

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Astrophysical tests

- Cosmological variation of couplings, CMB
- Cumulative effects in astrophysics
- Anomalous threshold reactions
- Shift of standard thresholds reactions with new threshold phenomenology
- LV induced decays not characterised by a threshold
- Reactions affected by “speeds limits”: synchrotron radiation
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Constraints

Table 2 Summary of typical strengths of the available constrains on the SME at different orders.

Order	photon	e^-/e^+	Hadrons	Neutrinos[a]
n=2	N.A.	$O(10^{-13})$	$O(10^{-27})$	$O(10^{-8})$
n=3	$O(10^{-14})$ (GRB)	$O(10^{-16})$ (CR)	$O(10^{-14})$ (CR)	$O(30)$
n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-4})$* (CR)

GRB=gamma rays burst, CR=cosmic rays
[a] From neutrino oscillations we have constraints on the difference of LV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. * Expected constraint from future experiments.

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Feedback on QG models

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$$E^2 = c^2 p^2 \left(1 + \frac{m^2 c^2}{p^2} + \eta \frac{p^{n-2}}{M^{n-2}}\right)$$

But how can we test Planck scales?

$$\frac{m^2}{p^2} \approx \frac{p^{n-2}}{M^{n-2}} \Rightarrow p_{crit} \approx \sqrt[n]{m^2 M^{n-2}}$$

n	p_{crit} for ν_e	p_{crit} for e^-	p_{crit} for p^+
2	$p \approx m_\nu$~1 eV	$p \approx m_e$=0.5 MeV	$p \approx m_p$=0.938 GeV
3	~1 GeV	~10 TeV	~1 PeV
4	~100 TeV	~100 PeV	~3 EeV

Phenomenology? One needs a dynamical framework ⇒

Model via Effective Field Theory with Lorentz breaking
(Standard Model Extension to LIV operators, CPT even or Odd)

$$E_\gamma^2 = k^2 + \xi_\pm^{(n)} \frac{k^n}{M_{pl}^{n-2}} \quad \text{photons}$$

$$E_{matter}^2 = m^2 + p^2 + \eta_\pm^{(n)} \frac{p^n}{M_{pl}^{n-2}} \quad \text{leptons/hadrons},$$

where, in EFT, $\xi^{(n)} \equiv \xi_+^{(n)} = (-)^n \xi_-^{(n)}$ and $\eta^{(n)} \equiv \eta_+^{(n)} = (-)^n \eta_-^{(n)}$.

SL, CQG Topic Review 2013

Astrophysical tests

- Cosmological variation of couplings, CMB
- Cumulative effects in astrophysics
- Anomalous threshold reactions
- Shift of standard thresholds reactions with new threshold phenomenology
- LV induced decays not characterised by a threshold
- Reactions affected by “speeds limits”: synchrotron radiation
- GW waves (but only low energy tests n=2)

Constraints

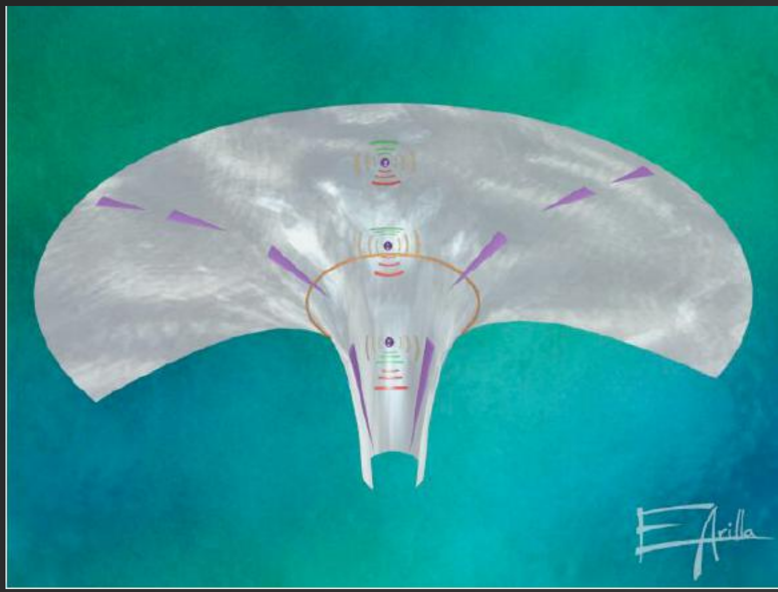
Table 2 Summary of typical strengths of the available constrains on the SME at different orders.

Order	photon	e^-/e^+	Hadrons	Neutrinos[a]
n=2	N.A.	$O(10^{-13})$	$O(10^{-27})$	$O(10^{-8})$
n=3	$O(10^{-14})$ (GRB)	$O(10^{-16})$ (CR)	$O(10^{-14})$ (CR)	$O(30)$
n=4	$O(10^{-8})$ (CR)	$O(10^{-8})$ (CR)	$O(10^{-6})$ (CR)	$O(10^{-4})$* (CR)

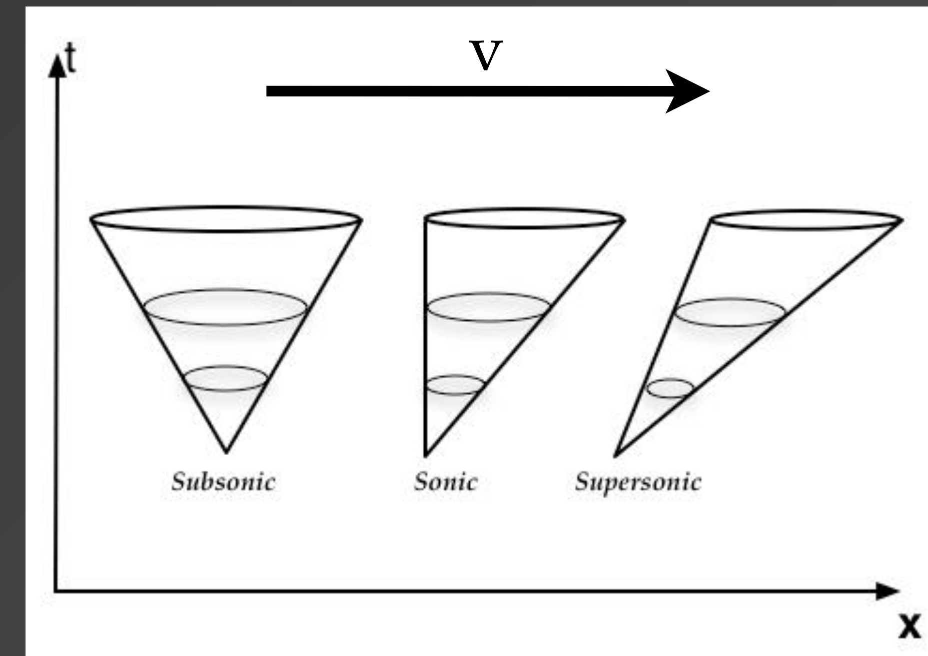
GRB=gamma rays burst, CR=cosmic rays
[a] From neutrino oscillations we have constraints on the difference of LV coefficients of different flavors up to $O(10^{-28})$ on dim 4, $O(10^{-8})$ and expected up to $O(10^{-14})$ on dim 5 (ICE3), expected up to $O(10^{-4})$ on dim 6 op. * Expected constraint from future experiments.

Lesson 1
Analogue Black Holes and the Robustness of Hawking radiation

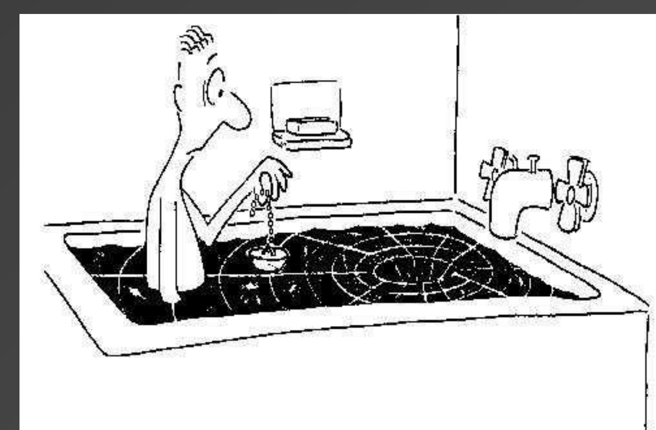
Analogue Black holes



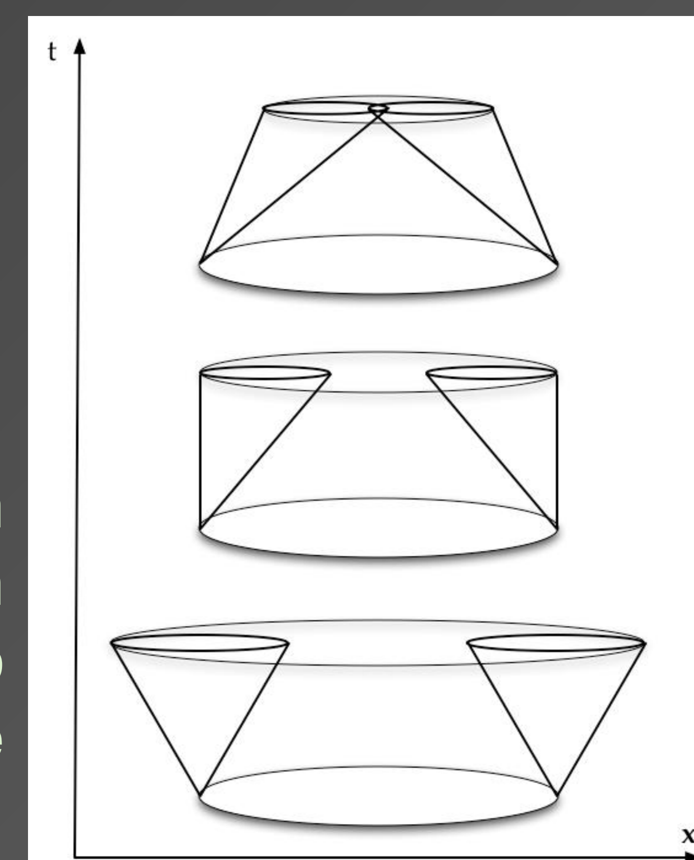
A moving fluid will tip the “sound cones” as it moves. Supersonic flow will tip the cone past the vertical.



GR	Hydrodynamics
Ergoregion	Any region of supersonic flow Es: steady flow $g_{\mu\nu}(\partial/\partial t)^{\mu}(\partial/\partial t)^{\nu} \Rightarrow$ $g_{tt} = -[c_s^2 - v^2]$
Trapped Surface	Any closed two-surface where the fluid velocity is everywhere inward-pointing and the normal component of the fluid velocity is always greater than the speed of sound
Future Event Horizon	Boundary of the region from which null geodesics (phonons) cannot escape.
Surface Gravity	$g_H = \frac{1}{2}\left\lvert\frac{\partial}{\partial n}\left(c_s^2 - v_{\perp}^2\right)\right\rvert$

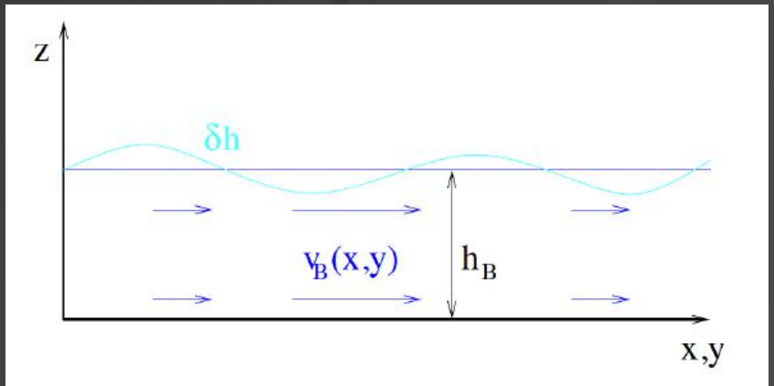


A moving fluid can form “trapped regions” when supersonic flow will tip the cone past the vertical.



Classical analogues: Gravity waves

Schutzhold, Unruh. Phys.Rev.D66:044019,2002.



Let's consider gravity waves on an inviscid, irrotational flow of a barotropic fluid under the influence of gravity. The Bernoulli's and continuity equations imply that in the long wavelength limit (shallow basin) surface wave propagate on an effective geometry

$$\mathrm{d}s^2 = \frac{1}{c^2}\left[-(c^2 - v_B^{\|2})\,\mathrm{d}t^2 - 2\mathbf{v}_B^{\|}\cdot\mathbf{dx}\,\mathrm{d}t + \mathbf{dx}\cdot\mathbf{dx}\right]$$

where $c \equiv \sqrt{g h_B}$.

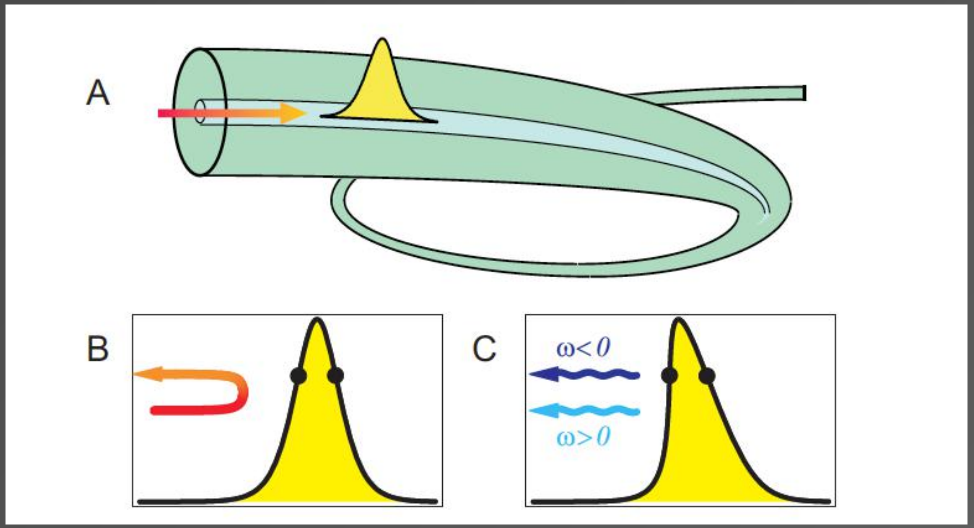
$$\omega = \mathbf{v}\cdot\mathbf{k} \pm \sqrt{\left(gk + \frac{\sigma}{\rho}k^3\right)\tanh(kh)}$$

Badulin (1983)

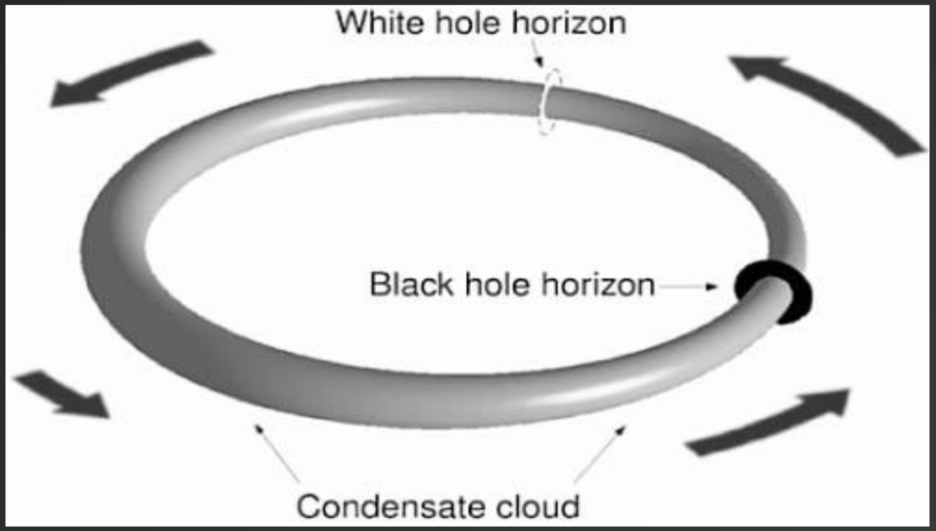
For arbitrary wavelength the dispersion relation is non-relativistic and goes from linear to "subluminal" to "superluminal".

Quantum analogues: Fiber optics analogues

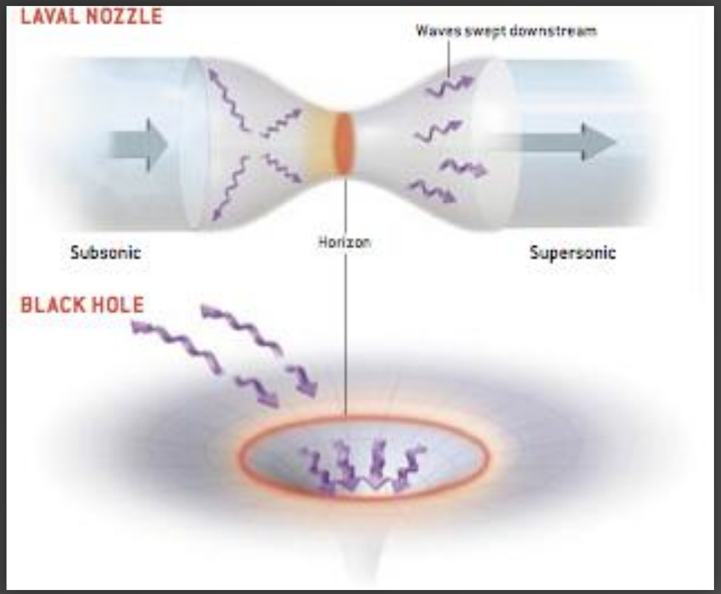
- Original idea: send non-dispersive pulses (solitons) through a optical fiber. Each pulse modifies the optical properties of the fiber due to the Kerr effect:
- the effective refractive index of the fiber, n_0, gains an additional contribution δn that is proportional to the instantaneous pulse intensity I at position z and time t.
- launch a continuous wave of light, a probe, that follows the pulse with slightly higher group velocity, attempting to overtake it
- As the probe approaches the pulse it slows down so much so that for some frequency it cannot "enter" the pulse. The rear front acts like a white horizon.
- Similarly probe light insight the pulse cannot escape from it, so the front of the pulse acts like a black horizon.



BHs in BEC



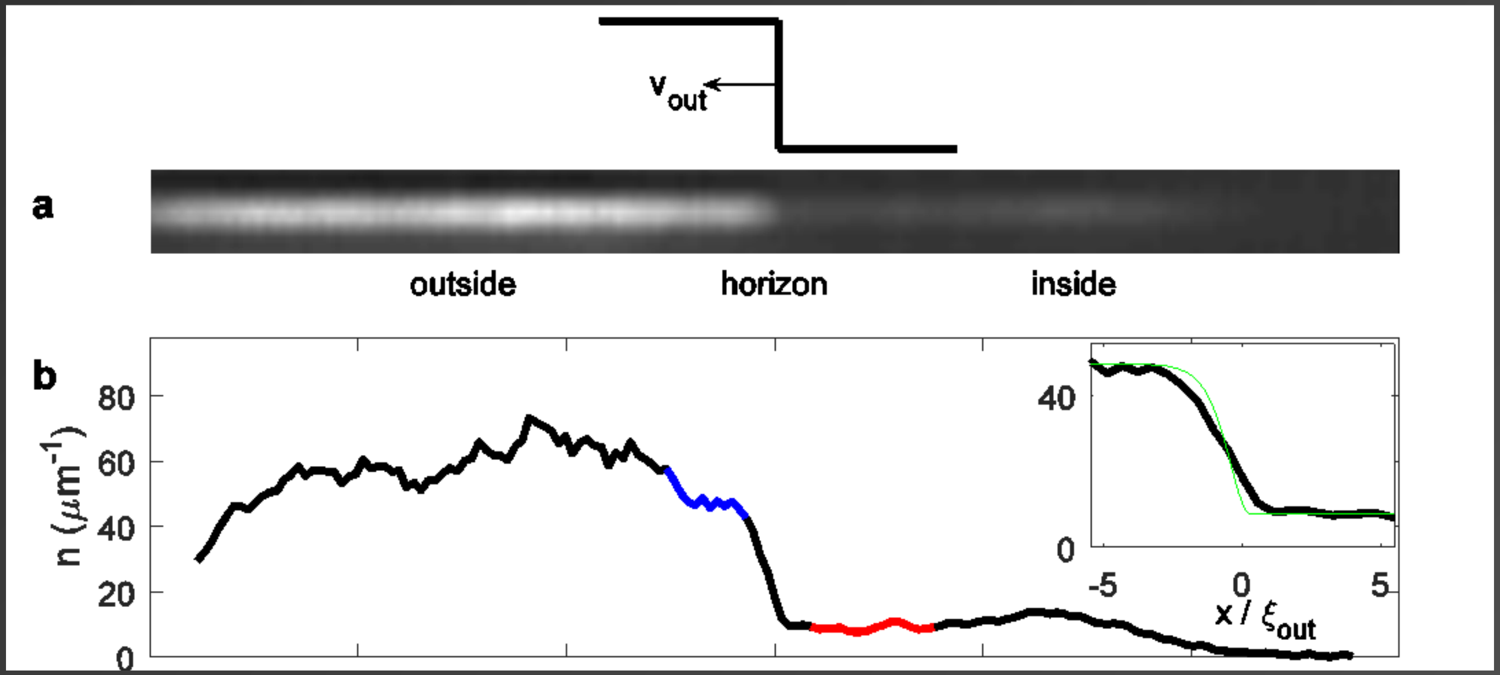
L.J. Garay, J.R. Anglin, J.I. Cirac, P. Zoller.
Phys.Rev.Lett. 85 (2000) 4643-4647
Phys.Rev. A63 (2001) 023611



CARLOS BARCELO, SL, MATT VISSER.
INT.J.MOD.PHYS. A18 (2003) 3735.

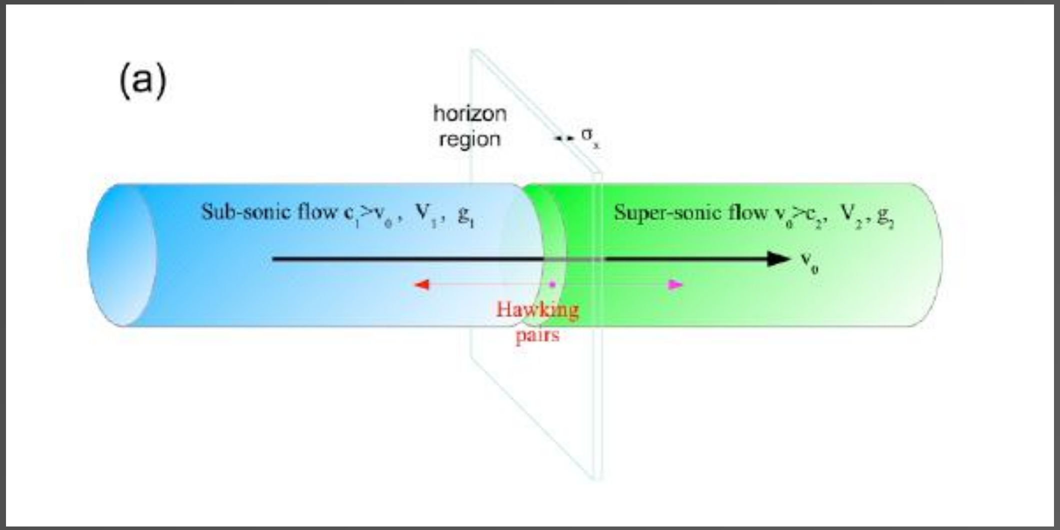
Use a sweeping one step potential to generate a BEC "waterfall".

JEFF STEINHAUER
NATURE PHYS. 12 (2016) 959

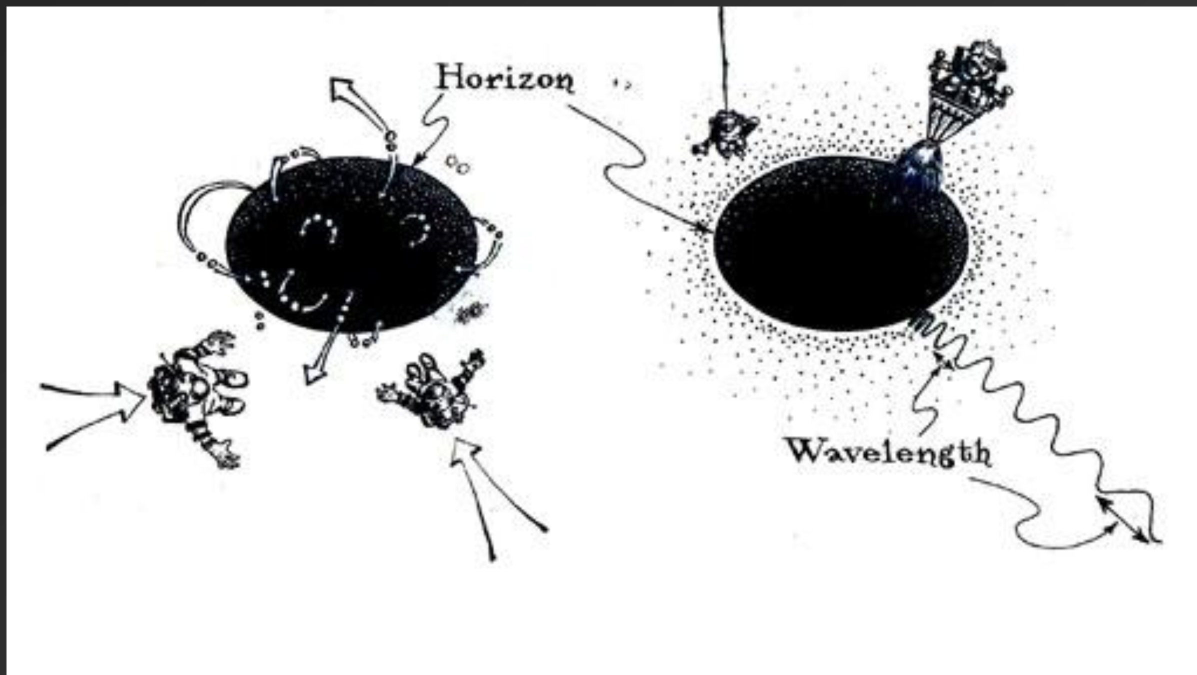


Use a Feshbach resonance to control the scattering length and hence the speed of sound. in order to create an analogue

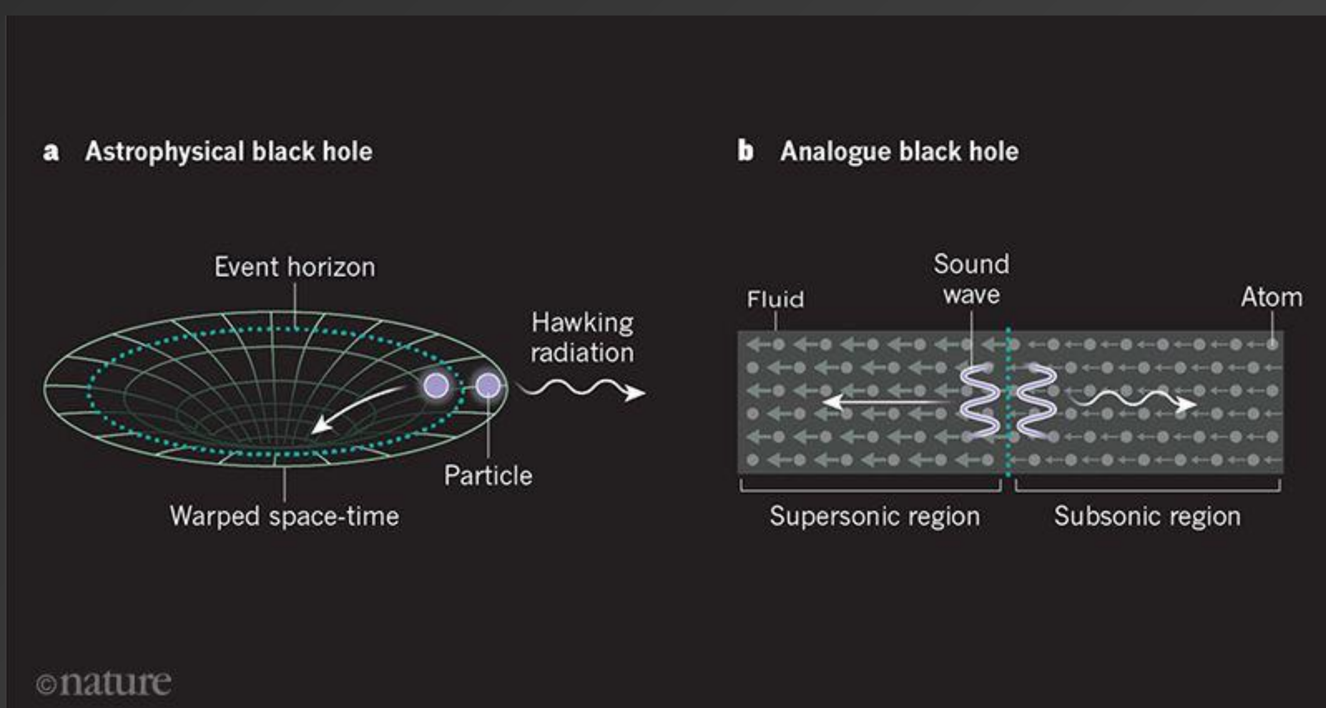
Carusotto, Fagnocchi, Recati, Balbinot, Fabbri.
New J. Phys.10, 103001 (2008)
See also Macher, Parentani: arXiv:0905.3634

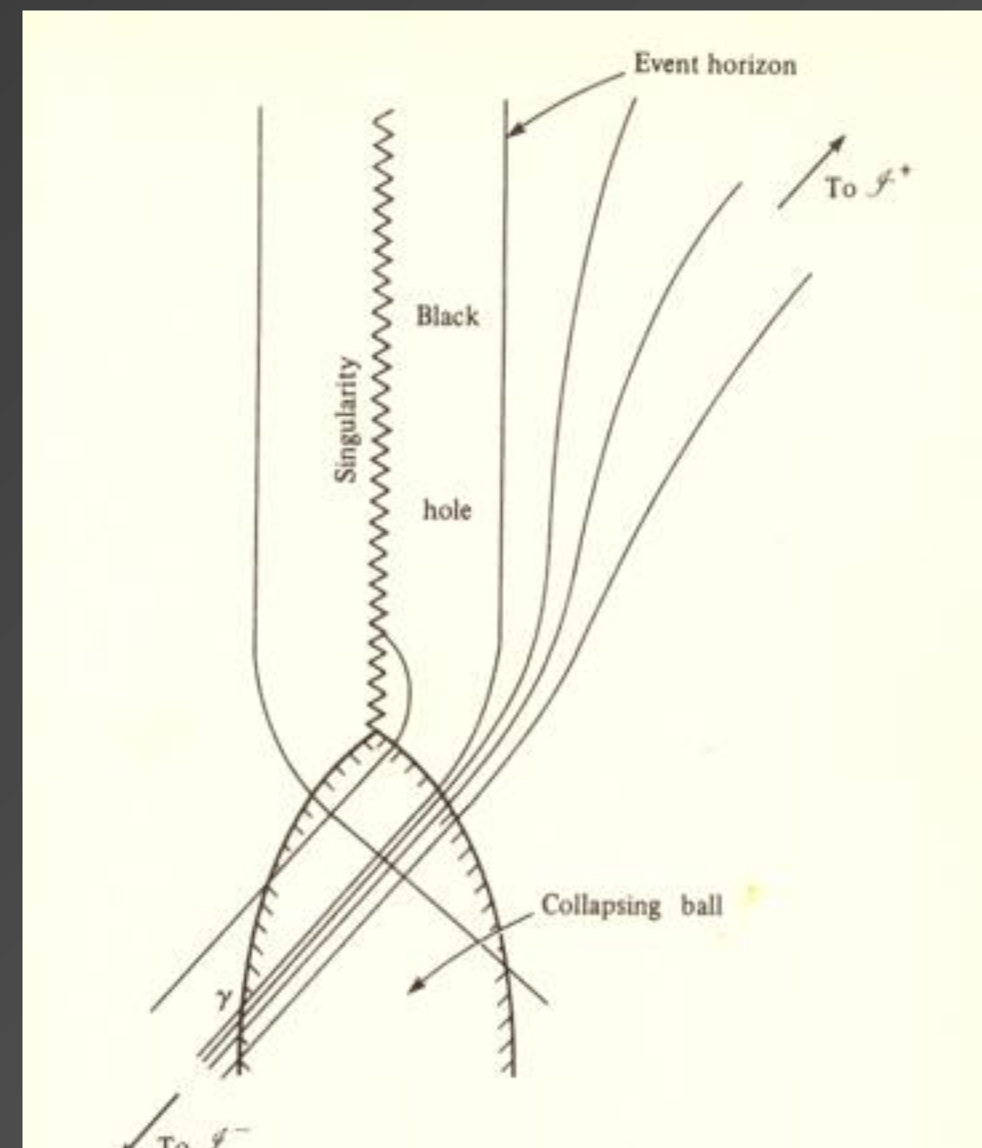


The transplanckian problem



$$kT_H = \frac{\hbar}{2\pi} \left. \frac{\partial |c-v|}{\partial n} \right|_H ,$$





Hawking radiation is based on the extrapolation of standard QFT in CS up to transplanckian frequencies.
It seems to relate far UV physics to IR at infinity.
Question: is HR robust against UV physics features?

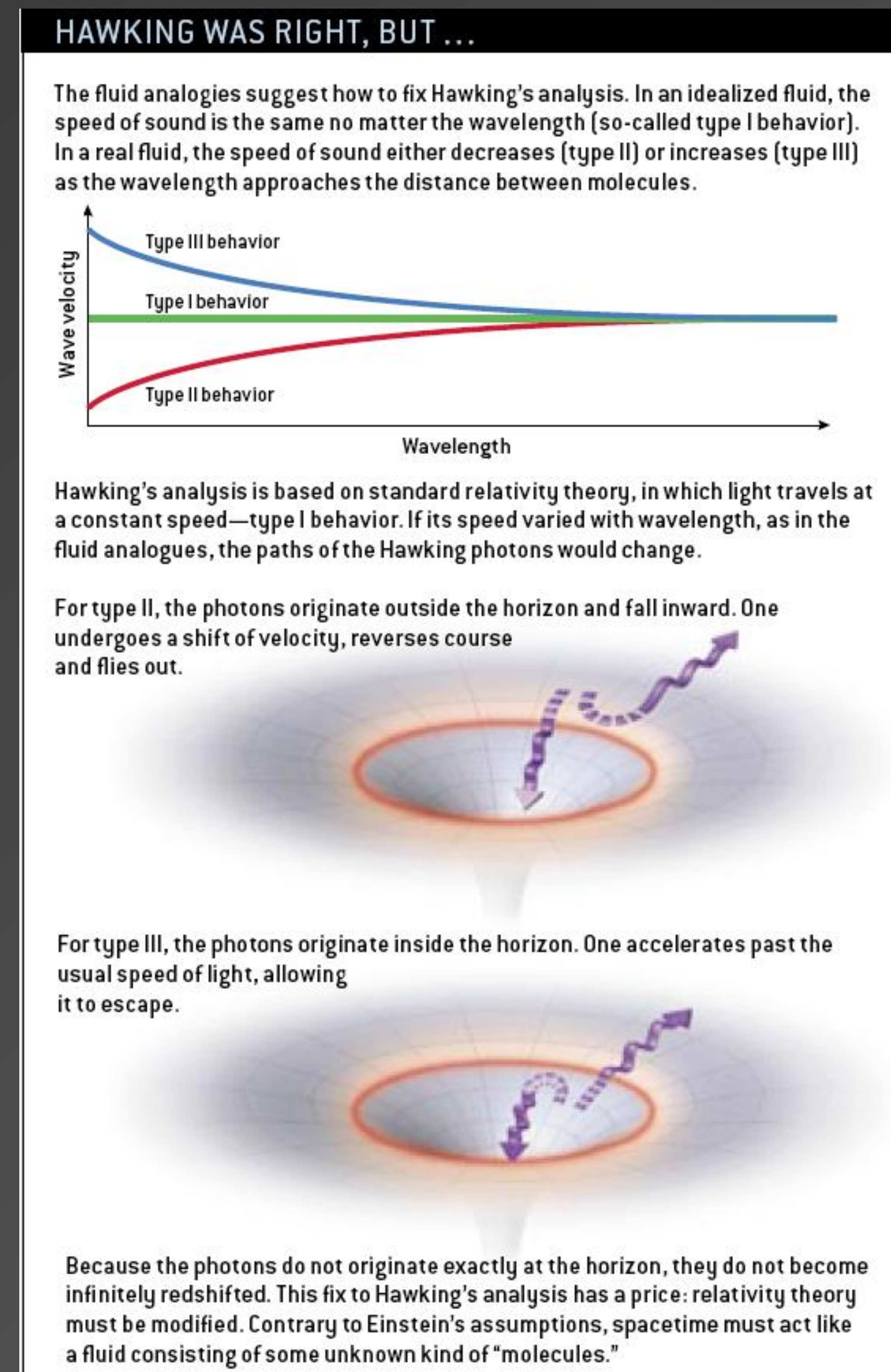
Robustness of Hawking radiation in Black hole analogues: Theory

$$\omega^2 = c_s^2 \left(k^2 \pm \frac{k^4}{K^2} \right)$$

It turned out that Hawking Radiation is robust against LIV (see e.g. Parentani et al. papers), however you also can get (controllable) instabilities such as "black hole laser effect" (superluminal relation in compact supersonic region or vice versa. see e.g. Jacobson-Corley and Parentani-Finazzi. See Jeff's talk tomorrow for observational evidence)

Some facts:

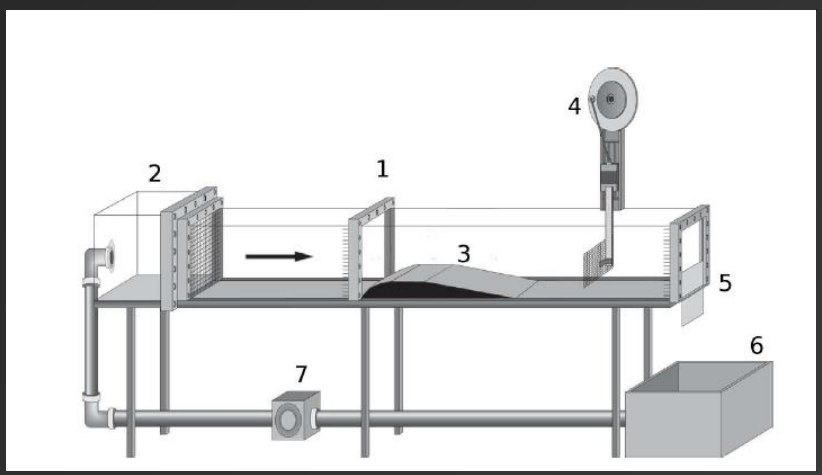
- In static spacetimes Hawking radiation robustness is generally assured if there is a separation of scales: $\kappa_{BH} \ll \Lambda$ where $\Lambda = K * F(v_{int-asym})$ for superluminal disp.rel.
- For subliminal $|v_{int-asym}| < \kappa_{BH}/K$.
- Indeed in this cases κ_{BH} stays in this case constant for a wide range of k in spite of the modified dispersion relation.
- the quantity that really fixes the Hawking temperature is an average of the spatial derivative of the velocity profile on a region across the horizon whose size is related to the UV LIV scale: <u>the horizon becomes thick</u>
- Key point for HR is also vacuum condition at particle creation region for freely falling observers (which are carrying with them the preferred frame associated to LIV)
- White hole-Cauchy horizons UV instabilities are regularised by LIV although at the price of new, slow, IR instabilities (undulation).



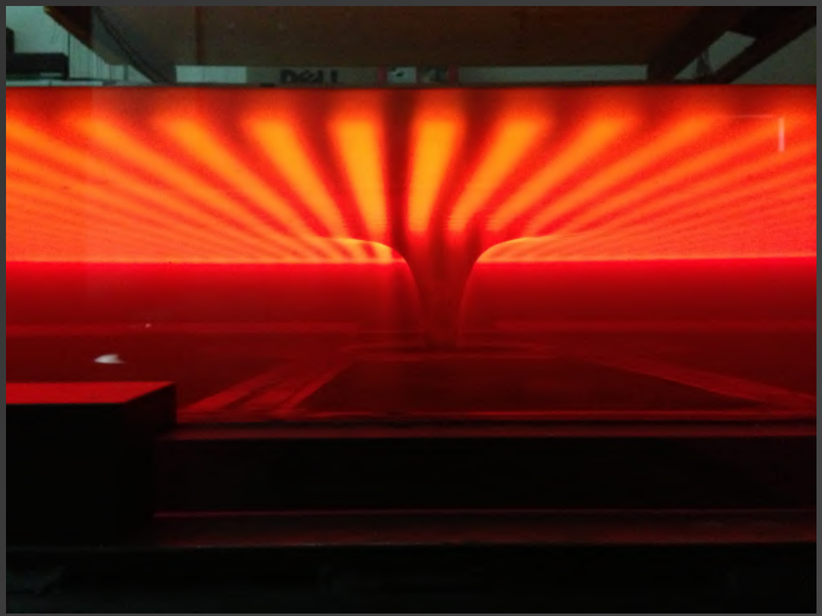
From Jacobson-Parentani: Sci. Am. 2005

Classical analogues: Gravity waves

Analogue system used for detection of classical wave conversion analogue of Hawking effect
G. Rousseaux et al. 2008
S.Weinfurtner, E.W. Tedford, M. C. J. Penrice, W. G. Unruh, and G. A. Lawrence. Phys.Rev.Lett. 106 (2011) 021302
Application to detection of Analogue Superradiance
SISSA-Nottingham experiment: an analogue of superradiant scattering (PI: S. Weinfurtner)

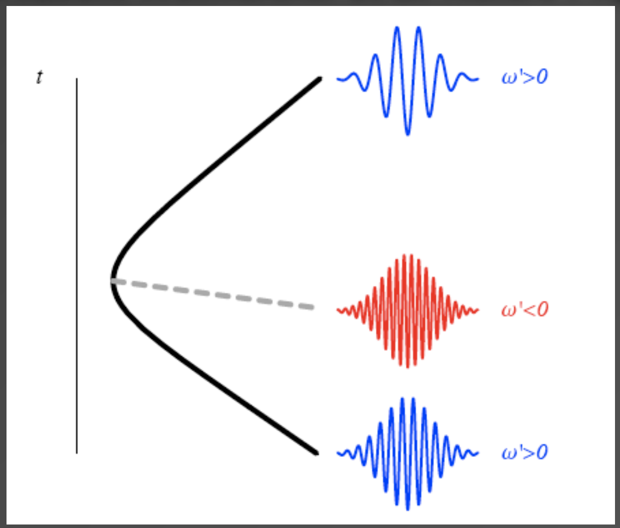


Vancouver Experiment



SISSA-Nottingham Experiment

Mauricio Richartz, Angus Prain, SL, Silke Weinfurt
Class.Quant.Grav. 30 (2013) 085009 and arXiv:1411.1
Observation: Weinfurtner et al. Nature Phys. 13 (20



Quantum analogues: Fiber optics analogues

F. Belgiorno et al, Phys. Rev. Lett. 105, 203901 (2010). reported the use of ultrashort laser pulse filaments to create a traveling RIP in a transparent dielectric medium (fused silica glass) They observed photon emission in the expected energy window. While surely relevant, the interpretation of the result as Hawking radiation is still subject of debate

(see e.g. SL, Prain, Visser, Phys.Rev. D85 (2012) 084014 and Schutzhold, Unruh, Phys.Rev.Lett. 107 (2011) 149401 - Phys.Rev. D86 (2012) 064006)

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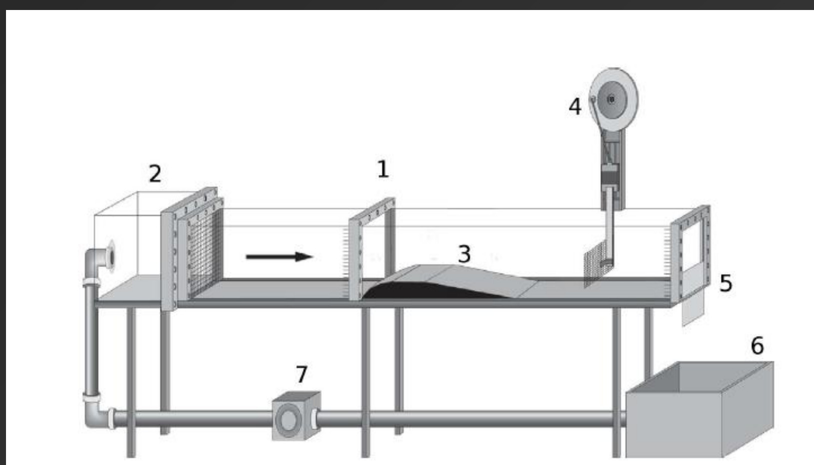

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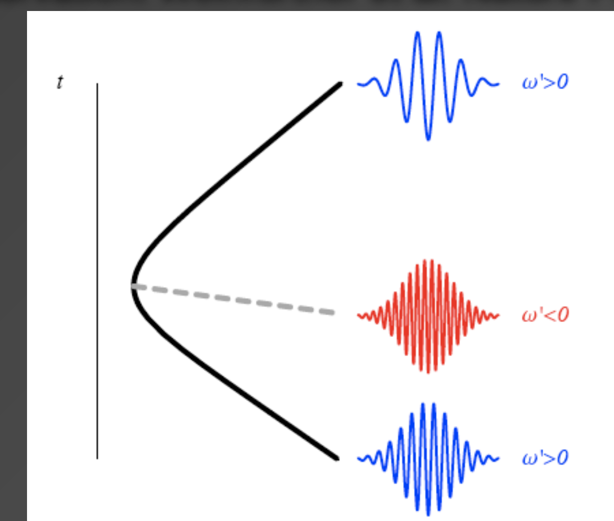

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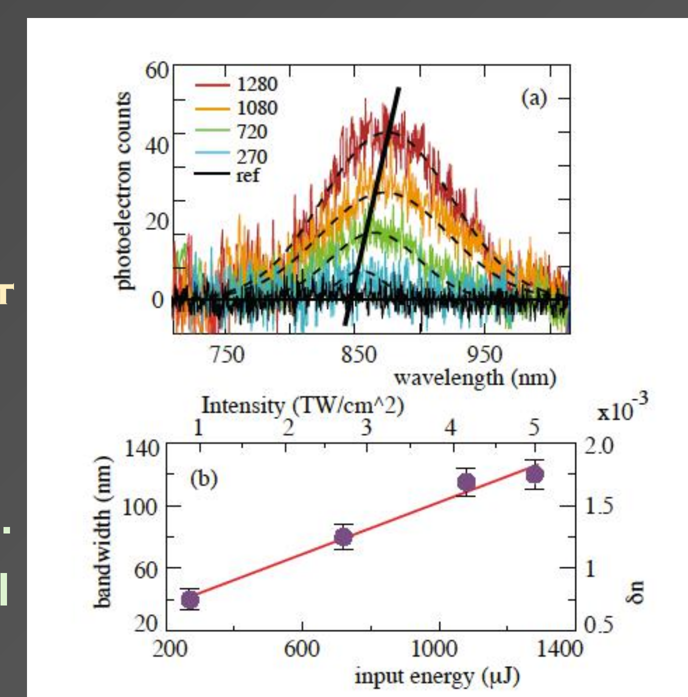


More recently: Leonhardt and collaborators, claim first detection of first stimulated emission.

Observation of stimulated Hawking radiation in an optical analogue By Jonathan Drori, Yuval Rosenberg, David Bermudez, Yaron Silberberg, Ulf Leonhardt.
arXiv:1808.09244 [gr-qc].
Physical Review Letters 122, 1 (2019) 010404.

Hawking radiation in Black hole analogues: Observation in BEC

Tentative detection via density-density correlations

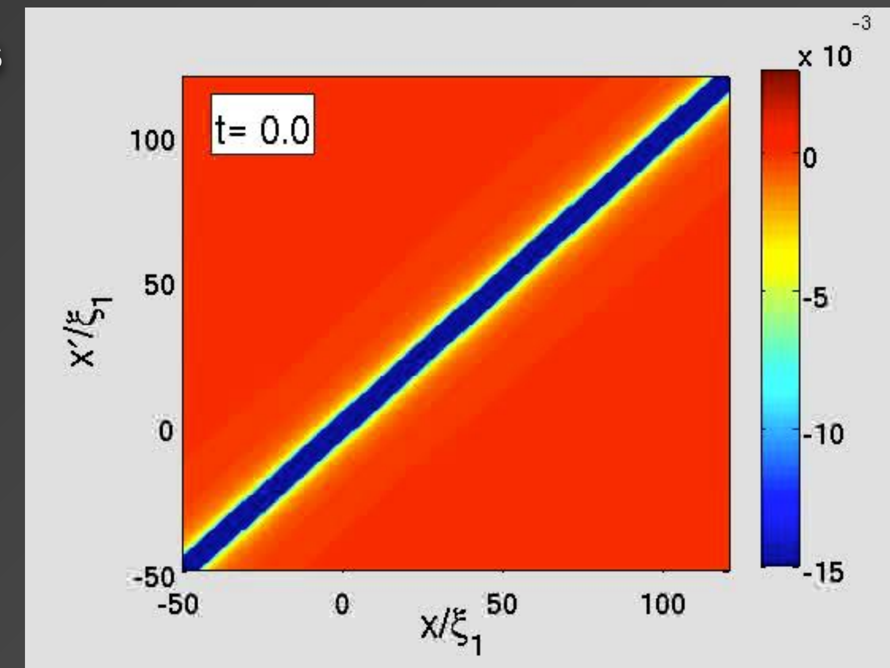
$$S_0\langle \hat{b}_H \hat{b}_P\rangle = \sqrt{\frac{\xi_{out}\xi_{in}}{L_{out}L_{in}}}\int dx\, dx' e^{ik_H x} e^{ik_P x'} G^{(2)}(x,x')$$

Hawking-Partners correlators

$$G^{(2)}(x,x') = \sqrt{\xi_{out}\xi_{in}/n_{out}n_{in}}\,\langle \delta n(x)\delta n(x')\rangle$$

Density-density correlators

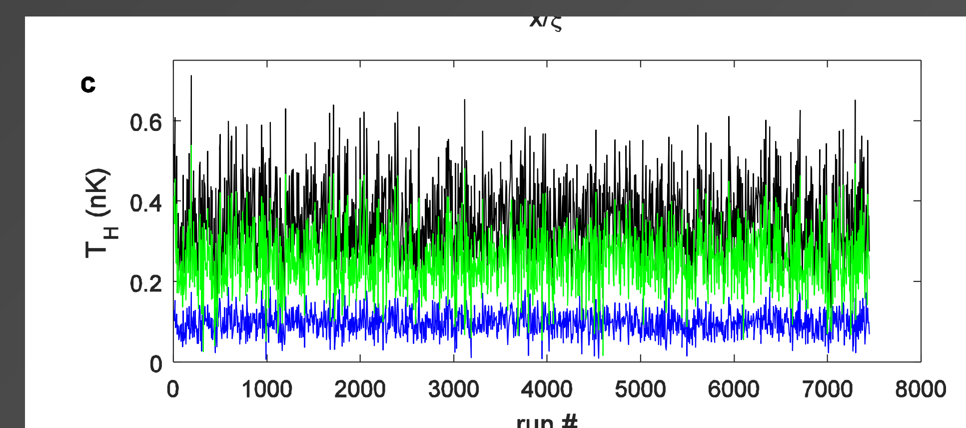
Hawking Radiation signature in density-density correlation. BEC simulation. Carusotto et al.



Recent observation of a characteristic instability for compact ergo regions
J. Steinhauer. Nature Physics (2014).

Even more recently first claim of Hawking detection (J. Steinhauer. 2015) and
Nature 569# 7758 (2019) 688-691

See Jeff's and Muñoz De Novi Talks Tomorrow!



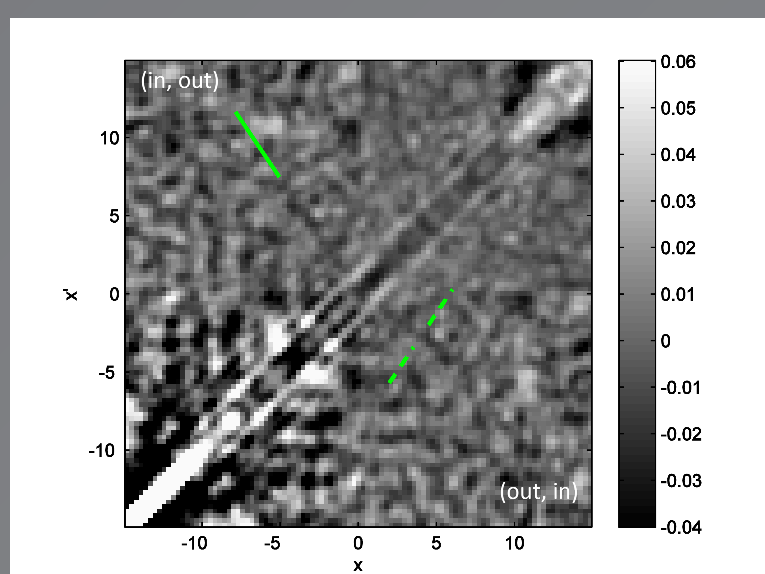


Fig. 3. Observation of Hawking/partner pairs. The horizon is at the origin. The dark bands emanating from the horizon are the correlations between the Hawking and partner particles. The solid line shows the angle of equal times from the horizon, found in Fig. 4. The Fourier transform along the dashed line measures the entanglement of the Hawking pairs.

So does BH thermodynamics survive without Lorentz invariance?
This is interesting even if you do not believe Lorentz invariance can be broken in the UV.
Where thermodynamics comes from in Gravitational Theories?
Interesting stuff from Einstein-Aether, Horava Black holes...

Hawking radiation in Black hole analogues: Observation in BEC

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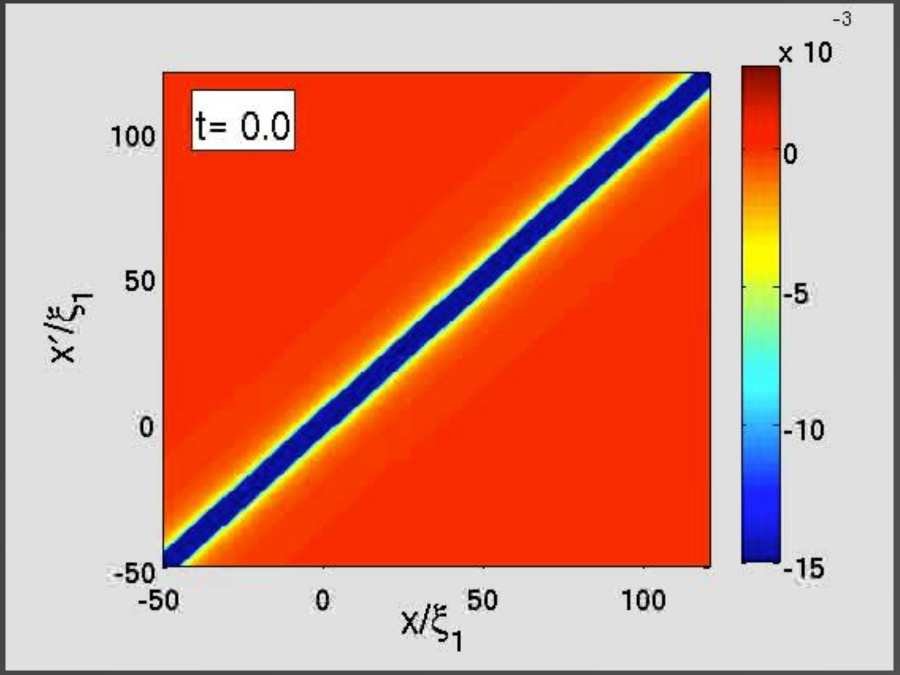
HAWKING-PARTNERS CORRELATORS

$$G^{(2)}(x,x') = \sqrt{\xi_{\mathrm{out}}\xi_{\mathrm{in}}/n_{\mathrm{out}}n_{\mathrm{in}}}\,\langle \delta n(x)\delta n(x')\rangle$$

DENSITY-DENSITY CORRELATORS

HAWKING RADIATION SIGNATURE IN DENSITY-DENSITY CORRELATION. BEC SIMULATION. CARUSOTTO ET AL.



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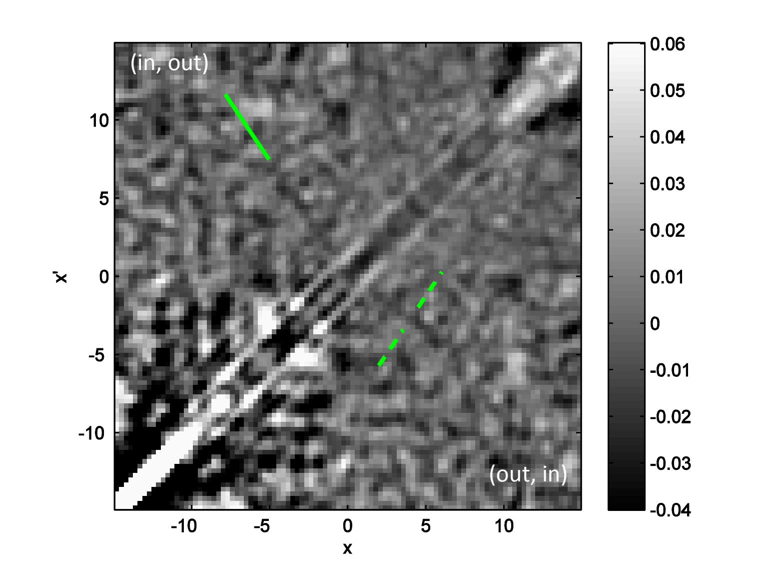

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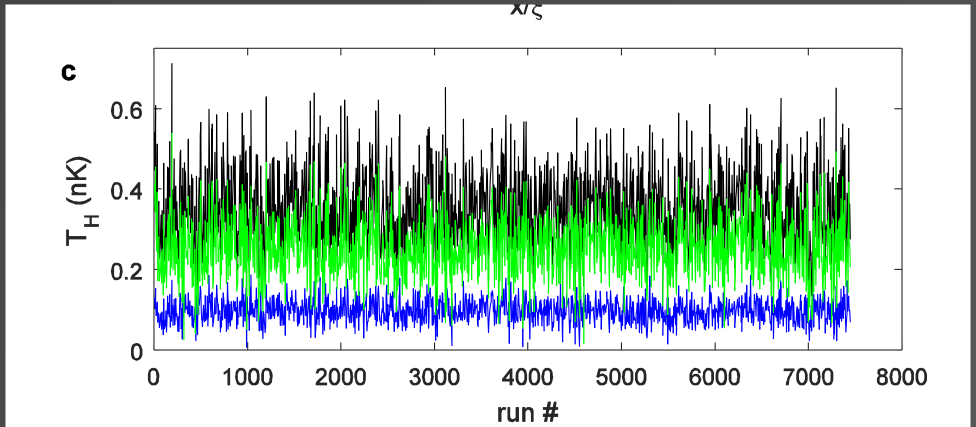

SO DOES BH THERMODYNAMICS SURVIVE WITHOUT LORENTZ INVARIANCE?
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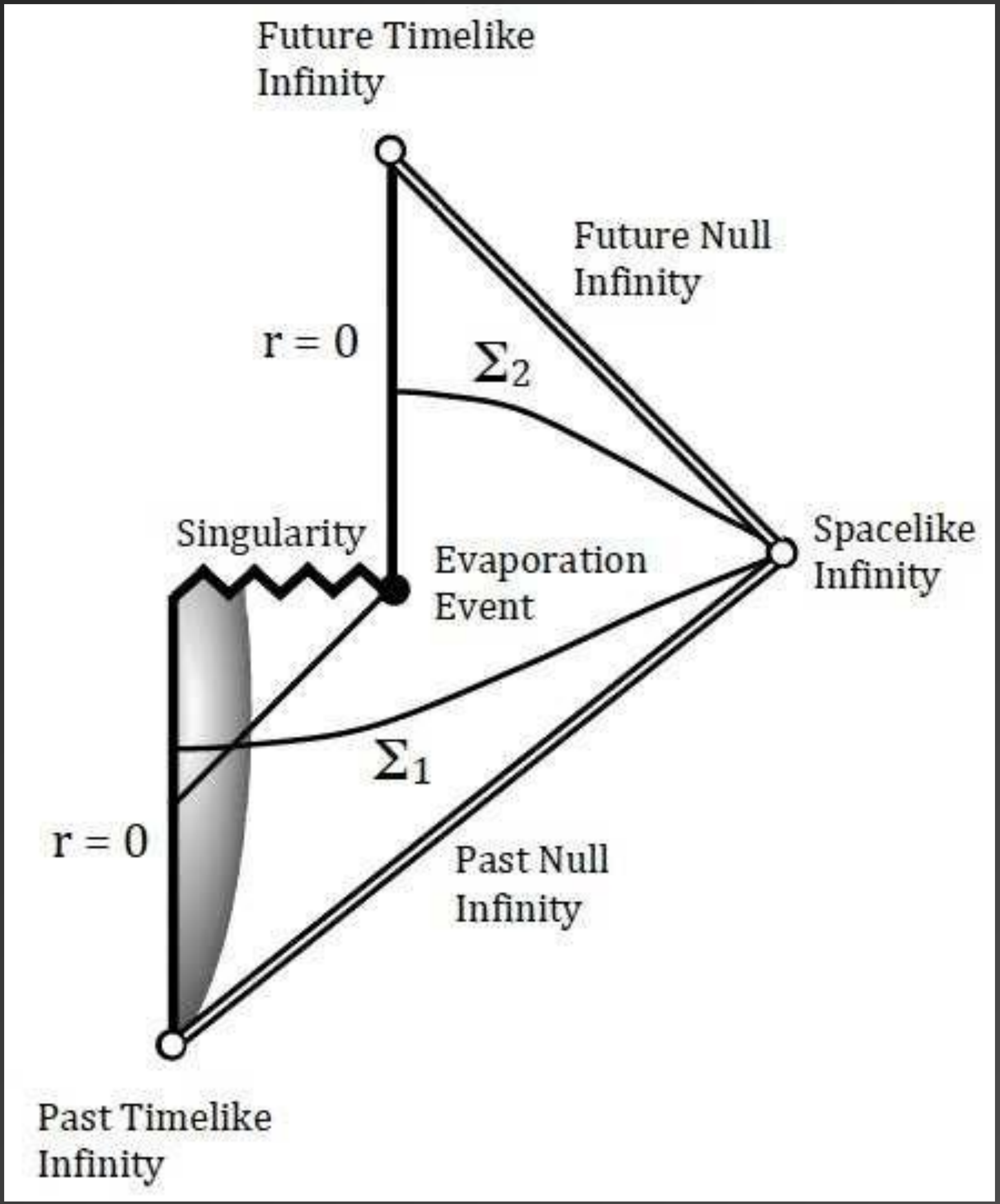
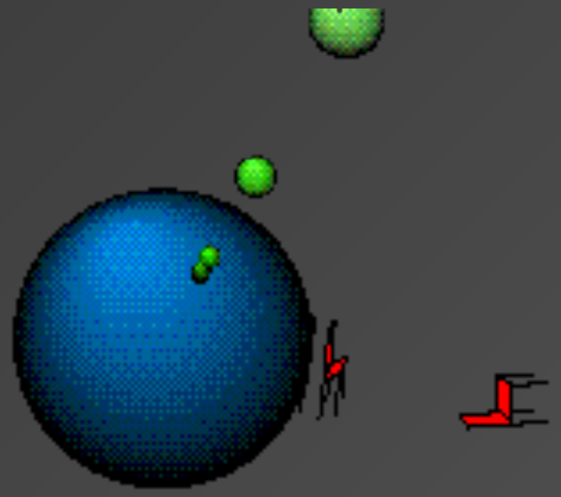
WHERE THERMODYNAMICS COMES FROM IN GRAVITATIONAL THEORIES?
INTERESTING STUFF FROM EINSTEIN-AETHER, HORAVA BLACK HOLES...

Lesson 3
Information loss in analogue black holes

The information loss problem:
an analogue gravity perspective
SL, G. Tricella, A. Trombettoni
Entropy **2019**, 21, 940

The information loss problem

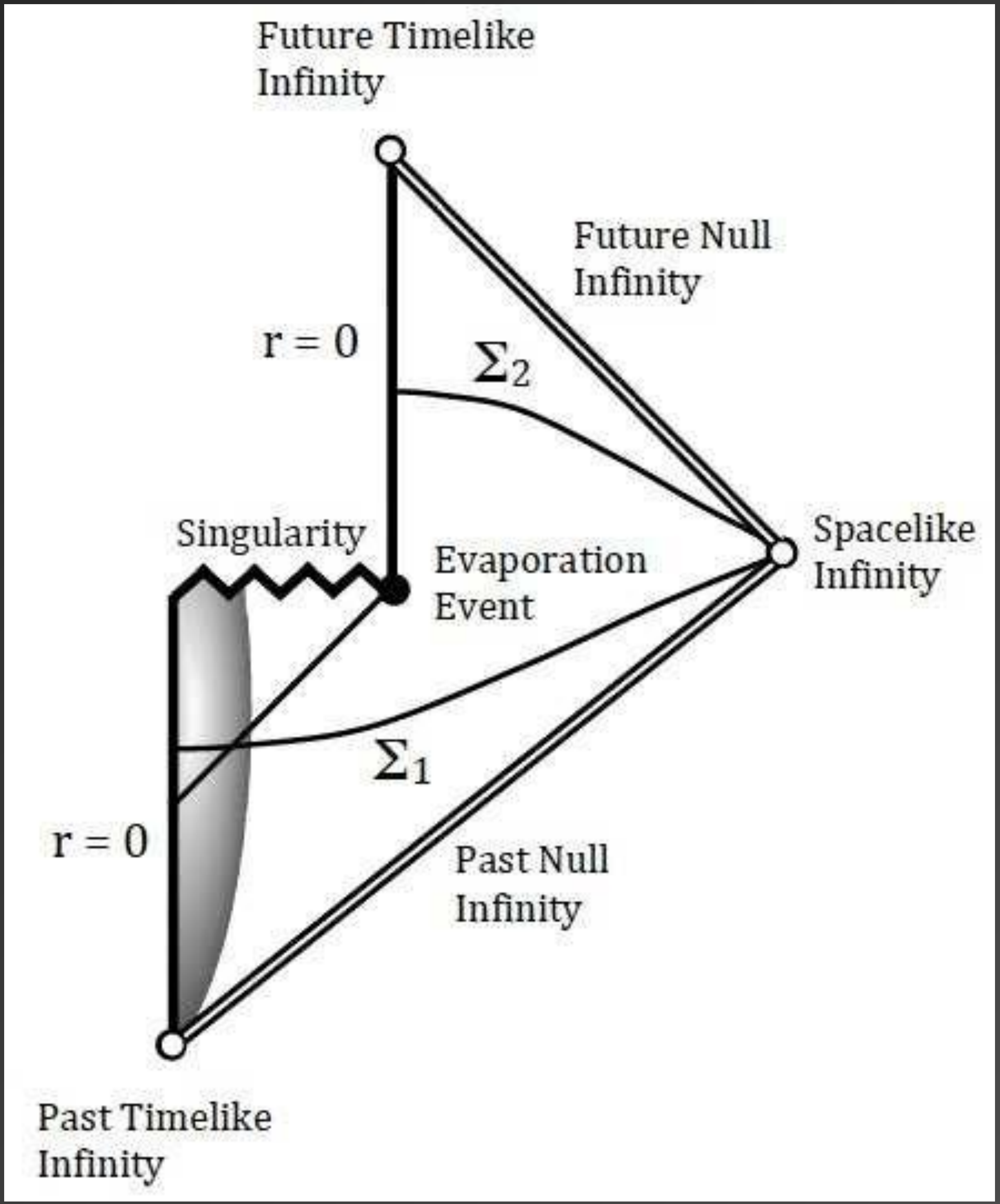


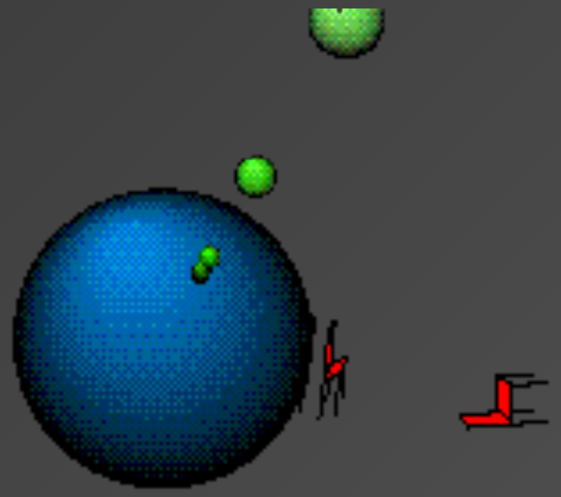


At the core the information loss problem is linked with our picture of BH evaporation.

There is no possible unitary evolution from Σ_1 to Σ_2

The information loss problem





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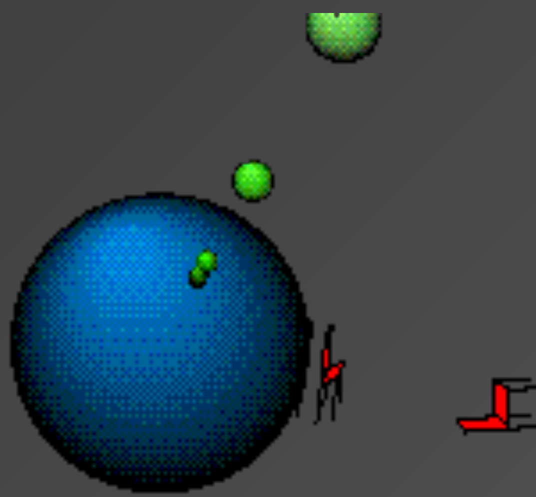
There is no possible unitary evolution from Σ_1 to Σ_2

The Information non-loss in BEC issue

Analogue gravity in BEC introduces a classical background (the wave function of the condensate) and describe the propagation of quantum fields (the quasi-particles) over it.
In this sense it is the analogue of QFT in a curved spacetime.

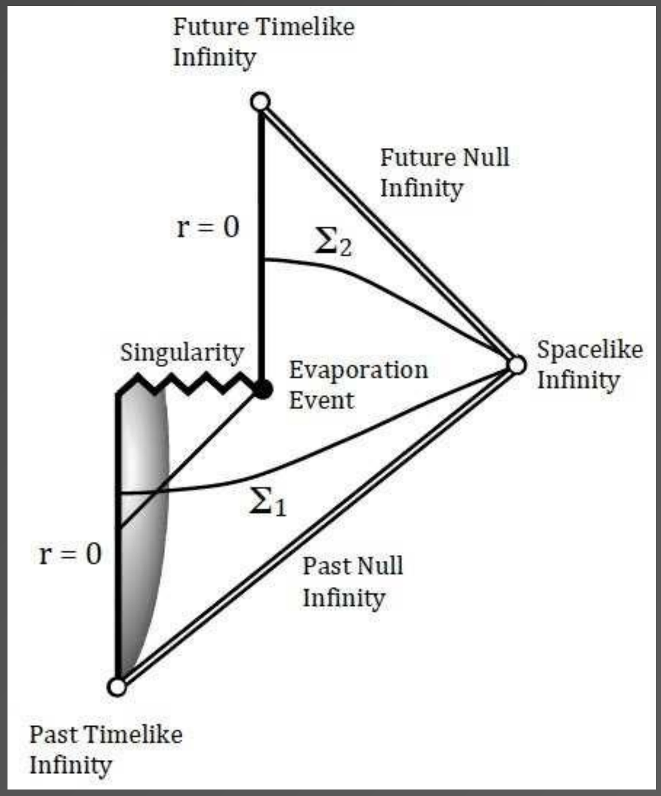


However, nothing forbids in analogue gravity to simulate singularities
(as a regions where the hydrodynamic approximation does not hold)
and even an evaporation process

But from the point of view of BEC the singularity is at most a region where the hydrodynamical limit is not legitimate
(if one does not "kicks out atoms").
Hence the evaporation process must be unitary preserving.

In this cases how the evolution is held unitary?

It seems that a necessary step is to be able to retain a quantum description of the condensate atoms in order to not have an obvious loss of information at the singularity…



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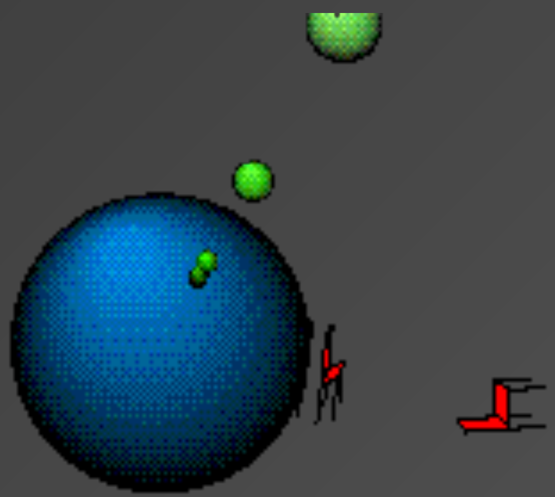

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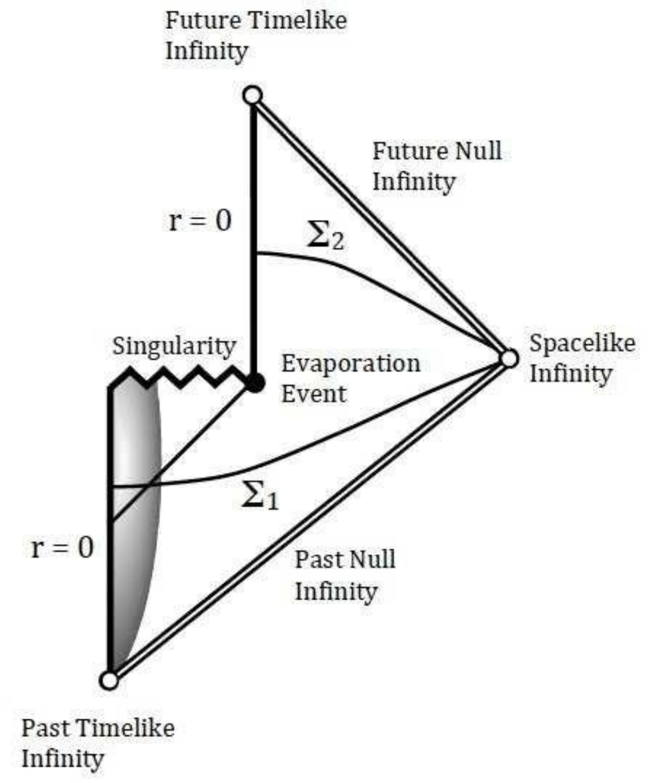

Attend G. Tricella's talk tomorrow for all the details!

Beyond Bogoliubov: Natural Orbitals

The mean field (Bogoliubov) approximation normally used in BEC is not the only way to express the condensation, i.e., the fact that a macroscopic number of particles occupies the same state.

Indeed, the condensation phenomenon can be also defined considering the properties of the 2-point correlation functions: a method allowing to retain the information about the quantum nature of the atoms in the BEC.

The 2-point correlation function is the expectation value on the quantum state of an operator composed of the creation of a particle in a position *x* after the destruction of a particle in a different position *y*, and of course it can be diagonalised as the orthonormal functions f_I, eigenfunctions of the 2-point correlation function, are known as natural orbitals (Penrose-Osanger 1954).

$$\left\langle \phi^\dagger(x)\,\phi(y) \right\rangle = \sum_I \langle N_I \rangle\, \overline{f_I}(x)\, f_I(y)\,,$$

$$\int \mathrm{d}x\, \overline{f_I}(x)\, f_J(x) = \delta_{IJ}\,.$$

The eigenvalues $\langle N_I \rangle$ are the occupation numbers of these wave functions. The sum of these eigenvalues gives the total number of particles in the state.

The natural orbitals, f_I and define a complete basis for the 1-particle Hilbert space and can be used to define destruction and creation operators.

$$a_I = \int \mathrm{d}x\, \overline{f_I}(x)\, \phi(x)\,,$$

$$\left[a_I, a_J^\dagger\right] = \delta_{IJ}\,, \qquad \left[a_I, a_J\right] = 0\,,$$

$$N_I = a_I^\dagger a_I\,.$$

Beyond Bogoliubov: Number preserving formalism

In the mean field approximation the field $\delta\varphi$ describes the quantum fluctuations over the mean-field wave function instead of atoms.

When considering states with fixed number of atoms, and therefore not coherent states, it is better to consider different operators to study the fluctuations.
One can follow the intuition that the fluctuation represents a shift of a single atom from the condensate to the excited fraction and vice versa.

Let's take as reference state for the condensate a_0, it is a straightforward procedure to define the number-conserving operators α_I, one for each excited wave function, according to the relations

$$\alpha_I = N_0^{-1/2} a_0^\dagger a_I \,,$$
$$\left[\alpha_I, \alpha_J^\dagger\right] = \delta_{IJ} \qquad \forall I, J \neq 0 \,,$$
$$\left[\alpha_I, \alpha_J\right] = 0 \qquad \forall I, J \neq 0 \,.$$

Now the excited part is not given by translation of the field, but number conserving fluctuations + projection, while the condensate one is proportional to f_0

$$\delta\phi(x) = \sum_{I\neq 0} f_I(x)\,\alpha_I$$
$$\rho_0^{1/2} e^{i\theta_0} = \langle N_0\rangle^{1/2} f_0(x)$$

Crucial point: it is possible to show that within reasonable approximations the fluctuations equation of motion have the same functional form of those in the usual mean field approximation. Hence, it can be shown that Analogue gravity continues to hold also in this formalism.

See G. Tricella's talk tomorrow for all the details!

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Analogue Cosmological particle creation

Hawking radiation is harder to get
Cosmological particle creation (dynamical Casimir effect) is much easier but still captures the basic physics.
We simulate that with a tunable interaction (e.g. Feshbach resonance)

$$g_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu = -\mathrm{d}\tau^2 + a^2\delta_{ij}\mathrm{d}x^i\mathrm{d}x^j\,,$$

$$a\left(\tau\left(t\right)\right) = \left(\frac{\rho_0}{m\lambda\left(t\right)}\right)^{1/4}\frac{1}{C}\,,$$

$$\mathrm{d}\tau = \frac{\rho_0}{ma\left(\tau\left(t\right)\right)}\frac{1}{C^2}\mathrm{d}t\,,$$

The initial state is the quasi-particle vacuum
which implies e.g. in terms of the two point correlators of number-conserving operators
(quasi-particle ops and number conserving ops are related via a Bogoliubov transformation)

$$\left\langle \delta\phi_k^\dagger\delta\phi_k\right\rangle = \left\langle a_k^\dagger a_0 N_0^{-1} a_0^\dagger a_k\right\rangle = \left\langle a_k^\dagger a_k\right\rangle = \langle N_k\rangle \approx \frac{1}{4}\sqrt{\frac{2\lambda\rho_0}{\frac{k^2}{2m}}}\delta_{k,k'}$$

$$\left\langle\delta\phi_{-k}\delta\phi_{k'}\right\rangle \approx -e^{2i\theta_0}\left\langle\delta\phi_k^\dagger\delta\phi_{k'}\right\rangle$$

At late times the first one becomes

$$\left\langle \delta\phi_k^\dagger\left(t\right)\delta\phi_k\left(t\right)\right\rangle = \frac{\frac{k^2}{2m}+\lambda'\rho_0}{2\omega_k'}\cosh\left(2\Theta_k\right) - \frac{1}{2} + \frac{\lambda'\rho_0\sinh\left(2\Theta_k\right)}{2\omega_k'}\cos\left(2\omega_k' t - \varphi_k\right)\,. \qquad \omega_k' = \sqrt{\frac{k^2}{2m}\left(\frac{k^2}{2m}+2\lambda'\rho_0\right)}.$$

The last term is oscillating symmetrically around 0 —meaning that the atoms will leave and rejoin the condensate periodically in time—whereas the first two are stationary.
Even assuming that the backreaction of the quasi-particles on the condensate is negligible, the mechanism of extraction of atoms from the condensate fraction is effective and increases the depletion (as also found in the standard Bogoliubov approach).

Main Results

- The Out state and the IN state are related by a squeezing operator that acts on the whole Hilbert space not just on the one of the quasi-particles
- The scattering operator of the quasi-particle couples is particularly simple and takes the peculiar expression that is required for producing squeezed states.

$$S = \exp\left(\frac{1}{2}\sum_{k\neq 0}\left(-e^{-i\varphi_k}c_k c_{-k} + e^{i\varphi_k}c_k^\dagger c_{-k}^\dagger\right)\Theta_k\right) .$$

- This is the general functional expression that is found in cosmological particle creation and in its analogue gravity counterparts, whether they are realized in the usual Bogoliubov framework or in its number-conserving reformulation.
- The time-independent operators c_k depend on the condensate operator a_0 and can be defined as compositions of number-conserving atom operators $\delta\varphi_k(t)$ and $\delta\varphi_{-}{}^{\dagger}{}_{k}(t)$

- The process of particle creation can be seen as an extraction of atoms from the condensate
- This dynamics produces (at any time) non-zero ($1/N_0$ suppressed) correlators between the condensate atoms and the quasi-particles because the operators a_0 and $a_0^\dagger$ do not commute with the creation of coupled quasi-particles $c^\dagger_k c^\dagger_{-k}$, which is described by the combination of the operators $\delta\varphi^\dagger_k\delta\varphi_k$, $\delta\varphi^\dagger_k\delta\varphi^\dagger_{-k}$ and $\delta\varphi_k\delta\varphi_{-k}$

$$\left(\delta\phi_k^\dagger\delta\phi_k\right)a_0^\dagger = a_0^\dagger\left(\delta\phi_k^\dagger\delta\phi_k\right) ,$$

$$\left(\delta\phi_k^\dagger\delta\phi_{-k}^\dagger\right)^n a_0^\dagger = a_0^\dagger\left(\delta\phi_k^\dagger\delta\phi_{-k}^\dagger\right)^n\left(\frac{N_0+1}{N_0+1-2n}\right)^{1/2} ,$$

$$\left(\delta\phi_k\delta\phi_{-k}\right)^n a_0^\dagger = a_0^\dagger\left(\delta\phi_k\delta\phi_{-k}\right)^n\left(\frac{N_0+1}{N_0+1+2n}\right)^{1/2} .$$

SL, G. Tricella, A. Trombettoni
Entropy 2019, 21, 940; doi:10.3390/e21100940

Attend G. Tricella's talk tomorrow for all the details!

Main Lessons

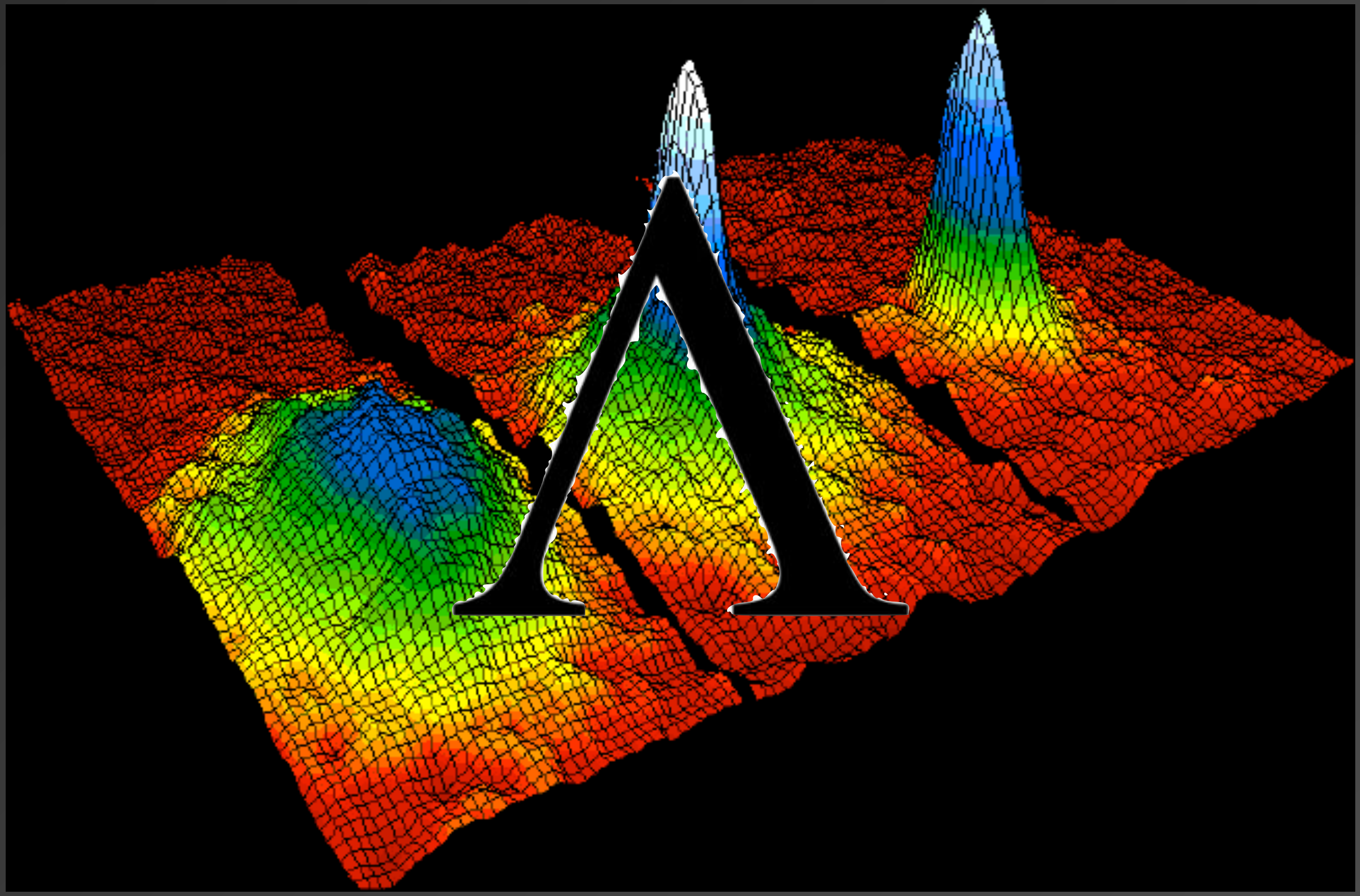
- As expected when describing the particle creation on the full Fock space (condensate+QP), there isn't any unitarity breaking, and the purity of the state is preserved.
- The particle creation unavoidably creates entanglement of the quasi-particles with the atoms in the condensate: even if the initial state factories the final one won't

In
$$\sum_N \frac{e^{-N/2}}{\sqrt{N!}} |N;\varnothing\rangle_{qp} \approx |\langle N\rangle\rangle_{mf} \otimes |\varnothing\rangle_{qp\,Boq},$$

Out
$$\sum_N \frac{e^{-N/2}}{\sqrt{N!}} \sum_{lr} a'_{lr} \left(1+\mathcal{O}\left(N^{-1}\right)\right) |N-l-r,l,r\rangle_a \neq |\langle N\rangle\rangle_{mf} \otimes \sum_{lr} a'_{lr} |l,r\rangle_{a\,Bog}.$$

- In cases such as the cosmological particle creation, where the phenomenon happens on the whole spacetime, *N* is the (large) number of atoms in the whole condensate, and thus the correlations between the substratum and the quasi-particles are negligible.
- In the black hole case, a finite region of spacetime is associated to the particle creation, thus *N* is not only finite but decreases as a consequence of the evaporation making the correlators between geometry and Hawking quanta more and more non-negligible at late stages of the BH evaporation.
- The Bogoliubov limit corresponds to taking the quantum degrees of freedom of the geometry as classical. This is not per se a unitarity violating operation, as it is equivalent to effectively recover the factorization of the above mentioned state. Indeed, the squeezing operator so recovered is unitarity preserving.
- However, the two descriptions are no longer practically equivalent when a region of quantum gravitational evolution is somehow simulated.

This strongly suggests that the information loss can only be addressed in a full quantum gravity description able to keep track of the correlations between quantum matter fields and geometrical quantum degrees of freedom underlying spacetime in QG.
(However AG miss Diffeo invariance which is another important ingredient. See e.g. Jacobson-Nguyen 2019)



Lesson 3

Analogue models of emergent gravity and the Cosmological constant

The cosmological constant problem in analogue/emergent gravity

Let us consider the EFT associated to phonons in BEC

In general $H_{\text{phon}} = \hbar \sum_{k}^{\infty} \omega_k \left(a_k^{\dagger} a_i + \frac{1}{2} \right) \quad E_{vac} = \langle \Omega | H_{\text{phon}} | \Omega \rangle$ → Ground state: no phonons

Not knowing about the physics beyond the "healing scale" we would cut-off the system at ξ=healing length of BEC and hence get a huge cosmological constant=$(1/\xi)^4$ V

However, is this the relevant energy of the vacuum for an analogue gravity system?
We can basically apply an argument firstly proposed by G. Volovik…

$$H_{\text{phon}} = H_{\text{effective}} \quad \text{BUT} \quad H_{\text{tot}} = H_{\text{atoms}}$$

Indeed for a many-body system like BEC the dynamics is generated by the grand-canonical Hamiltonian so it is its phononic vacuum expectation value that should provide the correct vacuum energy density

$$\mathcal{E}_{\text{vac}} = \langle \Omega | \left[H_{\text{atoms}} - \mu N_{\text{atoms}} \right] | \Omega \rangle / V \qquad \mu = \text{chemical potential}$$

Using the Gibbs-Duhem relation $E - TS - \mu N = -pV$

one easily see that at T=0 (no phonons) $\mathcal{E}_{\text{vac}} = -p$

For a finite condensate p≠0 and will in general depend on Volume/Surface terms of order of the energy required to pull atoms out of the condensate ~μ.

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But can we see this at work in a concrete model? Back to BEC…

A toy model for emergent gravity: non-relativistic BEC

So let's go back to the mean field approximation of BEC and focus on the BdG equation for the background:

F. Girelli, S.L.,L.Sindoni
Phys.Rev.D78:084013,2008

$$i\hbar\frac{\partial}{\partial t}\psi(t,\mathbf{x}) = \left(-\frac{\hbar^2}{2m}\nabla^2 + V_{\rm ext}(\mathbf{x}) + \kappa n_c\right)\psi(t,\mathbf{x}) + \kappa\left(2\tilde{n}\psi(t,\mathbf{x}) + \tilde{m}\psi(t,\mathbf{x})\right)$$

- Can this be encoding some form of gravitational dynamics?
- If yes it must be some form of Newtonian gravity (non relativistic equation)
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- But, in order to have any chance to see this, we need to have some massive field

One way to get this is to introduce a soft U(1) breaking term
(i.e. from massless Goldstone bosons to massive pseudo-Goldstone bosons)

Note: this kind of symm breaking is actually experimentally realized in magnon (quantized spin wave) BEC in 3He-B (see e.g. related work by G.Volovik)

$$i\hbar\frac{\partial}{\partial t}\hat{\Psi} = -\frac{\hbar^2}{2m}\nabla^2\hat{\Psi} - \mu\hat{\Psi} + \kappa|\hat{\Psi}|^2\hat{\Psi} - \lambda\hat{\Psi}^\dagger$$

We assume $-\mu<\lambda\ll\mu$ (soft breaking).

It can be checked that the extra term gives massive phonons which at low momenta propagate on the standard acoustic geometry of BEC

$$\mathcal{E}^2(p) = \frac{p^4}{4m^2} + c_s^2p^2 + \mathcal{M}^2c_s^4$$

$$c_s^2 = \frac{\mu+2\lambda}{m}, \qquad \mathcal{M}^2 = 4\frac{\lambda(\mu+\lambda)}{(\mu+2\lambda)^2}m^2$$

$$\mathcal{M} \ll m \quad \text{if} \quad \lambda \ll \mu$$

Non-relativistic BEC gravitational potential

So we would now like to cast the equation for the a stationary, homogeneous, condensate background in a Poisson-like form with the quasi-particles moving accordingly to the analogue gravitational potential.

$$\vec{F} = \vec{a} = -\mathcal{M}\vec{\nabla}\Phi_{\rm grav}$$

$$\left(\nabla^2 - \frac{1}{L^2}\right)\Phi_{\rm grav} = 4\pi G_{\rm N}\rho + \Lambda$$

where L ⇒ range of the gravitational interaction, G_N ⇒ analogue G Newton,

Λ ⇒ analogue cosmological constant

Results

1. It is possible to show by looking at the Newtonian limit of the acoustic geometry that the gravitational potential is encoded in density perturbations

2. By adopting the ansatz $\psi = \left(\frac{\mu+\lambda}{\kappa}\right)^{1/2}(1+u(\mathrm{x}))$ $\eta_{\mu\nu} + h_{\mu\nu}, \qquad h_{00} \propto u(x)$

and looking at the Hamiltonian for the quasi-particles in the non relativistic limit, one can actually show that the analogue of the gravitational potential is

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This is the form the gravitational potential affecting the quasi-particle motion for a slightly inhomogeneous BEC.
We now want to see if it satisfies some modified Poisson equation…

NON-RELATIVISTIC BEC: EMERGENT NEWTONIAN GRAVITY

Let's consider the equation for a static background with a source term.
The latter is given partly by a localized quasi-particle plus a vacuum contribution due to the unavoidable presence/backreaction of excited atoms above the condensate

$$\left(\frac{\hbar^2}{2m}\nabla^2 - 2(\mu+\lambda)\right) u(\mathrm{x}) = 2\kappa\left(\bar{n}(\mathrm{x}) + \frac{1}{2}\bar{m}(\mathrm{x})\right) + 2\kappa\left(\tilde{n}_0 + \frac{1}{2}\tilde{m}_0\right)$$

where $\bar{n}(\mathrm{x}) = \tilde{n}(\mathrm{x}) - \tilde{n}_0, \qquad \bar{m}(\mathrm{x}) = \tilde{m}(\mathrm{x}) - \tilde{m}_0$

and $\tilde{n}_0 = \langle 0|\hat{\chi}^\dagger(\mathrm{x})\hat{\chi}(\mathrm{x})|0\rangle, \qquad \tilde{m}_0 = \langle 0|\hat{\chi}(\mathrm{x})\hat{\chi}(\mathrm{x})|0\rangle$

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$$\rho_{\mathrm{matter}}(\mathrm{x}) = \left(\bar{n}(\mathrm{x}) + \frac{1}{2}\bar{m}(\mathrm{x})\right) \qquad G_N \equiv \frac{\kappa(\mu+4\lambda)(\mu+2\lambda)^2}{4\pi\hbar^2 m\lambda^{3/2}(\mu+\lambda)^{1/2}}$$

$$\Lambda \equiv \frac{2\kappa(\mu+4\lambda)(\mu+2\lambda)}{\hbar^2\lambda}\left(\tilde{n}_0 + \frac{1}{2}\tilde{m}_0\right), \qquad L^2 \equiv \frac{\hbar^2}{4m(\mu+\lambda)}$$

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WEIGHTING LAMBDA...

$$\mathcal{E}_\Lambda = \frac{\Lambda c_s^4}{4\pi G_{\mathrm{N}}}, \quad \mathcal{E}_{\mathrm{P}} = \frac{c_s^7}{\hbar G_{\mathrm{N}}^2}, \quad \Longrightarrow \quad \frac{\mathcal{E}_\Lambda}{\mathcal{E}_{\mathrm{P}}} \propto \rho_0 a^3 \left(\frac{\lambda}{g\rho_0}\right)^{-5/2}.$$

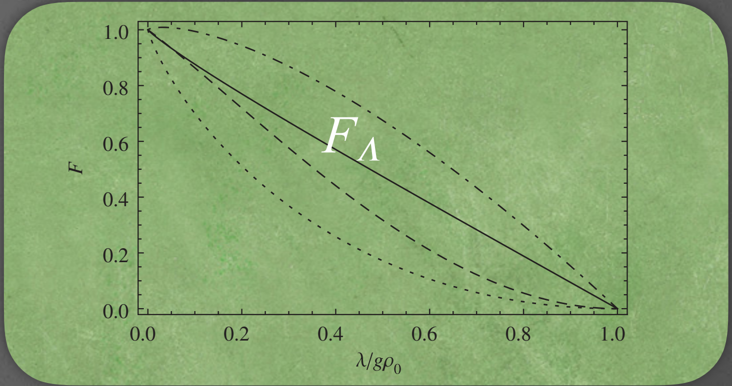
The cosmological constant scale is suppressed by a small number (the dilution factor $\rho a^3 \ll 1$) w.r.t. the analogue/emergent Planck scale!

The (negative) cosmological constant is not the phonons ground state energy, neither it is the atoms grand canonical energy density h, or energy density $\varepsilon = h + \mu\rho$
It is just related to the subdominat second order correction to these latter quantities due to quantum depletion (the part related to the excitations) and its scale is the healing scale.

S. Finazzi, S. Liberati and L. Sindoni,
Phys. Rev. Lett. 108, 071101 (2012)

Cosmological constant in emergent gravity: lessons from BEC

$$\Lambda = -\frac{20 m g \rho_0 (g\rho_0 + 3\lambda)}{3\sqrt{\pi}\hbar^2 \lambda}\sqrt{\rho_0 a^3} F_\Lambda\left(\frac{\lambda}{g\rho_0}\right),$$



This model is too simple to be realistic but still teaches us some lessons

- The analogue cosmological constant that we have discussed cannot be computed as the total zero-point energy of the condensed matter system, even when taking into account the natural cutoff coming from the knowledge of the microphysics
- In fact the value of Λ is related only to the (subleading) part of the zero-point energy proportional to the quantum depletion of the condensate.

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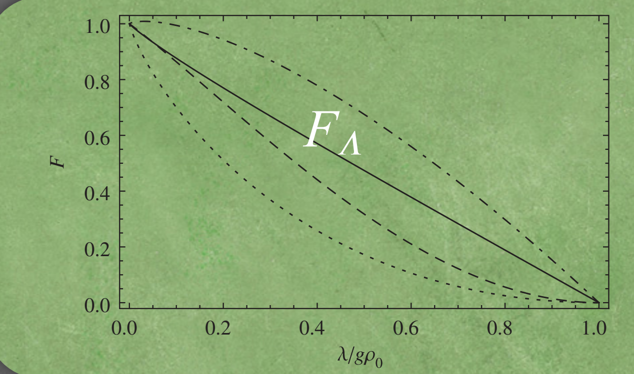

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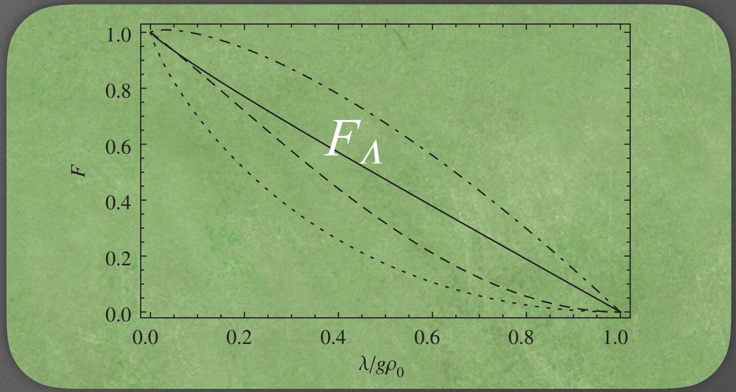

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This implies for emergent gravity scenarios that

- There could be no a priori reason why the cosmological constant should be computed as the zero-point energy of the system: its computation must inevitably pass through the derivation of Einstein equations emerging from the underlying microscopic system.
- the energy scale of Λ can be several orders of magnitude smaller than all the other energy scales for the presence of a very small number, nonperturbative in origin, which cannot be computed within the framework of an EFT dealing only with the emergent degrees of freedom (i.e., semiclassical gravity).

S. Finazzi, S. Liberati and L. Sindoni, Phys. Rev. Lett. 108, 071101 (2012)

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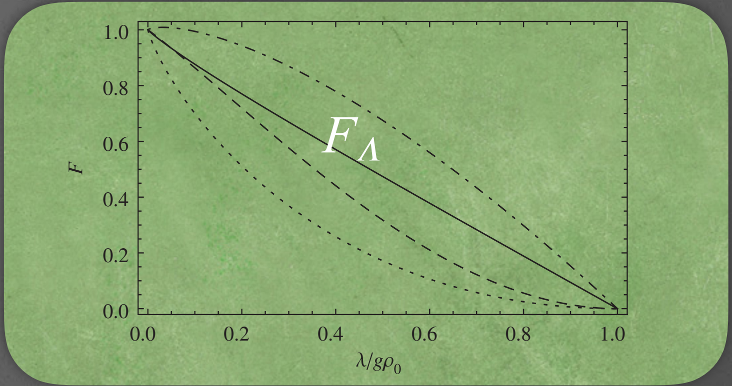

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This implies for emergent gravity scenarios that

- There could be no a priori reason why the cosmological constant should be computed as the zero-point energy of the system: its computation must inevitably pass through the derivation of Einstein equations emerging from the underlying microscopic system.
- the energy scale of Λ can be several orders of magnitude smaller than all the other energy scales for the presence of a very small number, nonperturbative in origin, which cannot be computed within the framework of an EFT dealing only with the emergent degrees of freedom (i.e., semiclassical gravity).

Can we go beyond non-relativist Gravity?

Relativistic BEC and emergent LLI

Of course the BEC model is not Lorentz invariant at all scales as it is fundamentally non relativistic.

What happens if one considers a BEC of Relativistic atoms?

New J.Phys. 12 (2010) 095012

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S.Fagnocchi, S. Finazzi, SL, M. Kormos, A. Trombettoni: New. J. Phys.
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Bose–Einstein condensation, may occur also for relativistic bosons. So far only theoretical model.

$$\hat{\mathcal{L}} = \frac{1}{c^2}\frac{\partial\hat{\phi}^\dagger}{\partial t}\frac{\partial\hat{\phi}}{\partial t} - \nabla\hat{\phi}^\dagger\cdot\nabla\hat{\phi} - \left(\frac{m^2c^2}{\hbar^2} + V(t,\mathbf{x})\right)\hat{\phi}^\dagger\hat{\phi} - U(\hat{\phi}^\dagger\hat{\phi};\lambda_i)\ ,$$

$$\text{with}\quad U(\hat{\phi}^\dagger\hat{\phi};\lambda_i) = \frac{\lambda_2}{2}\hat{\rho}^2 + \frac{\lambda_3}{6}\hat{\rho}^3 + \cdots \quad \text{where}\quad \hat{\rho} = \hat{\phi}^\dagger\hat{\phi}$$

The associated dispersion relation has a gapped and gapless mode

$$\omega_\pm^2 = c^2\left\{k^2 + 2\left(\frac{\mu}{c\hbar}\right)^2\left[1+\left(\frac{mcc_0}{\mu}\right)^2\right] \pm 2\left(\frac{\mu}{c\hbar}\right)\sqrt{k^2 + \left(\frac{\mu}{c\hbar}\right)^2\left[1+\left(\frac{mcc_0}{\mu}\right)^2\right]^2}\right\}.$$

$$\text{where}\quad c_0 = \frac{\hbar^2}{2m}\rho U"(\rho,\lambda)\,,\qquad \mu = \text{relativistic chemical potential.}$$

The gapless/massless mode is the interesting one as it admits an effective metric in the phononic regime

In the limit of very relativistic atoms $b \equiv \dfrac{mcc_0}{\mu} \gg 1$

$$\text{Long wavelengths limit}\quad k \ll \frac{2mc_0}{\hbar} \quad\Longrightarrow\quad \omega_-^2 = c_s^2k^2 \quad \text{with}\quad g_{\mu\nu} = \rho\frac{c}{c_s}\left[\eta_{\mu\nu} + \left(1-\frac{c_s^2}{c^2}\right)\frac{v_\mu v_\nu}{c^2}\right]$$

$$c_s^2 = c^2 b/(1+b)$$

LLI+Gravity ←

SR ← $\text{Short wavelengths limit}\quad k \gg \dfrac{2mc_0}{\hbar} \quad\Longrightarrow\quad \omega_-^2 = c^2k^2 \quad \text{with}\quad g_{\mu\nu} = \eta_{\mu\nu}$

No LLI+Raibow metric/Finsler ? ← $\text{Intermediate}\quad \omega_-^2 = c_s^2\left[k^2 + \dfrac{k^4}{4b\left(\frac{\mu}{\hbar c}\right)^2(1+b)^2}\right]$

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LLI+Gravity ← (long wavelengths limit)

SR ← Short wavelengths limit $\quad k \gg \dfrac{2mc_0}{\hbar} \quad \Longrightarrow \quad \omega_-^2 = c^2k^2 \quad$ with $\quad g_{\mu\nu} = \eta_{\mu\nu}$

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Lesson from rBEC: one can have IR and UV relativistic physics but nonetheless Lorentz violation at intermediate scales.
You have to understand the classical/continuous limit to be sure about LI.

Emergent gravity in relativistic BEC

Belenchia, SL, Mohd: arXiv:1407.7896
Phys.Rev. D90 (2014) 10, 104015

$$\mathcal{L}_{\rm eff} = -\eta^{\mu\nu}\partial_\mu\phi^*\partial_\nu\phi - m^2\phi^*\phi - \lambda(\phi^*\phi)^2 + \mu^2\phi^*\phi + i\mu(\phi^*\partial_t\phi - \phi\partial_t\phi^*)$$

Let us again decompose Φ as $\Phi = \Phi_0(1 + \Psi)$, where Φ_0 is the condensed part of the field ($\langle\Phi\rangle = \Phi_0$) and Ψ is the fractional fluctuation which can be written in terms of its real and imaginary parts $\Psi = \Psi_1 + i\Psi_2$

Crucial point: in some suitable regime (neutral background field, $c_s=c$) you can completely mask the Lorentz breaking. In this regime one finds

Excitations Eq.

$$\Box_g\psi_1 - 4\lambda\psi_1 = 0,$$

$g_{\mu\nu} = \phi^2_0\eta_{\mu\nu}$ $\qquad \Box_g\psi_2 = 0.$

Background Eq.

$$R_g - \frac{m^2}{\phi^2} + \Lambda = \langle T_{\rm qp}\rangle, \qquad R_g = -6\,\frac{\Box\varphi_0}{\varphi_0^3}$$

$$\langle T_{\rm qp}\rangle := -12\lambda\,[\,3\,\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\,],$$

For m->0 (BEC still allowed by non zero chemical potential μ)

Equivalent to Einstein-Fokker equation of Nordström gravity!

$$R + \Lambda = 24\pi G_{\rm N}\,T,$$

Nordström gravity (1913) is the only other theory in 3+1 dimensions which satisfies the Strong Equivalence principle.

However, it is not truly background independent (fixed Minkowski causal structure)

$$\langle T\rangle = -\frac{\Lambda}{6}\left[3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\right] = -2\lambda\left[3\langle\psi_1^2\rangle + \langle\psi_2^2\rangle\right] = 6\langle T_{qp}\rangle. \qquad \Lambda \equiv 12\lambda\frac{\mu^2}{c^2\hbar} = 12\lambda\frac{M_{\rm P}^2c^2}{4\pi\hbar}. \qquad G_{\rm N}^{\rm eft} = G/4\pi = \hbar c^5/(4\pi\mu^2)$$

$$M_{\rm Pl} = \mu\sqrt{4\pi}/c^2 \qquad \frac{\epsilon_\Lambda}{\epsilon_P} \simeq \frac{\lambda\hbar^2c^2}{\mu^2}$$

small

The "bare" Λ is in this case small and positive but it will generically receive a (negative) correction from the fraction of atoms in the non-condensate phase, the depletion factor.

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- This shows their usefulness both towards concrete experimental realisations of QFT in general Spacetimes as well as towards a theoretical understanding.
- Many more issues could be addressed still: e.g. how and where the particle production happens at the atomic level, what is the interpretation of the quasi-particle entanglement at the microscopic scale? etc...
- However, so far the big missing actor is Diffeo invariant. Can we import these ideas into a Diffeo invariant UV completion or can Diffeo invariance be emergent as well?

Analogue gravity lessons seems far from over...

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