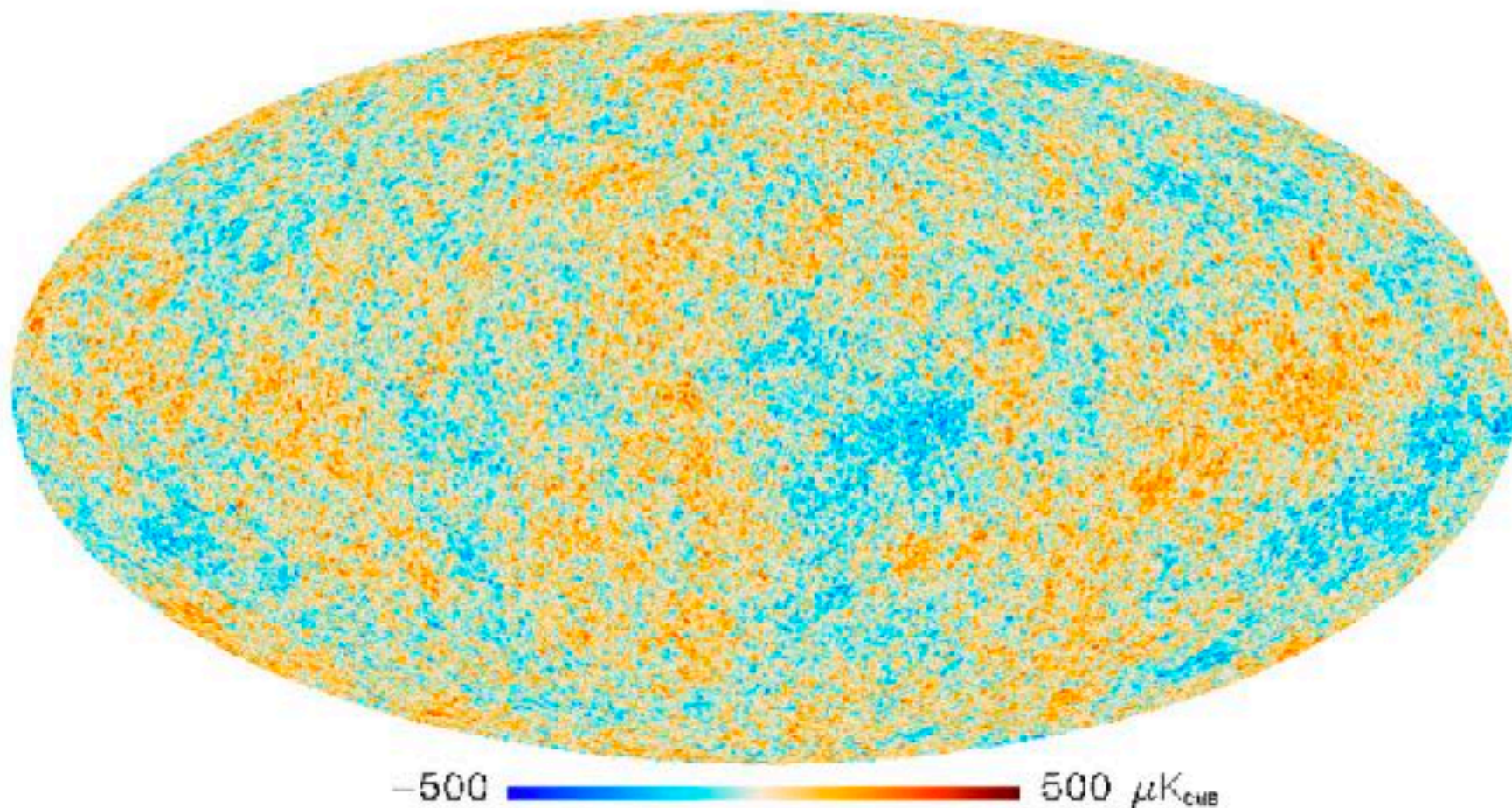
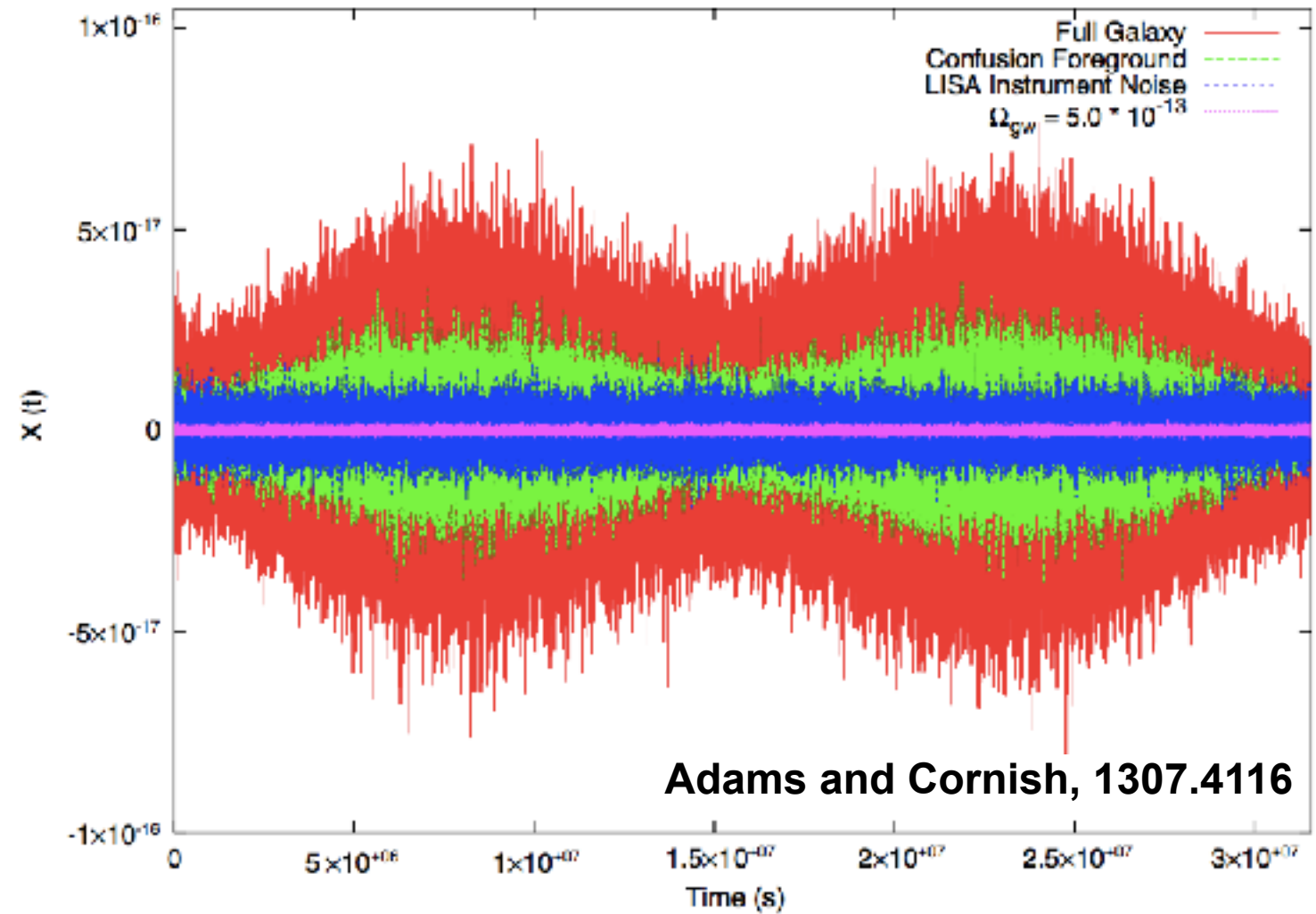


# Cosmological backgrounds

Chiara Caprini  
CNRS (APC Paris)



the questions (hopefully)  
answered by this seminar

- what are cosmological backgrounds? why are they formed?
- what can we use them for? what information do they bring to us?
- in a nutshell, the physics of the Cosmic Microwave Background
- in more detail, the physics of the Stochastic Gravitational Wave Background

the questions that remain open

# what are CBs and why are they formed

description of the universe in the context of General Relativity

$$G_{\mu\nu} = 8\pi G T_{\mu\nu} - \Lambda g_{\mu\nu} \qquad \nabla_{\mu} T^{\mu\nu} = 0$$

COSMOLOGICAL PRINCIPLE:  
the universe is homogenous and isotopic

FLRW metric

$$ds^2 = a^2(\eta)(-d\eta^2 + \delta_{ij}dx_i dx_j)$$

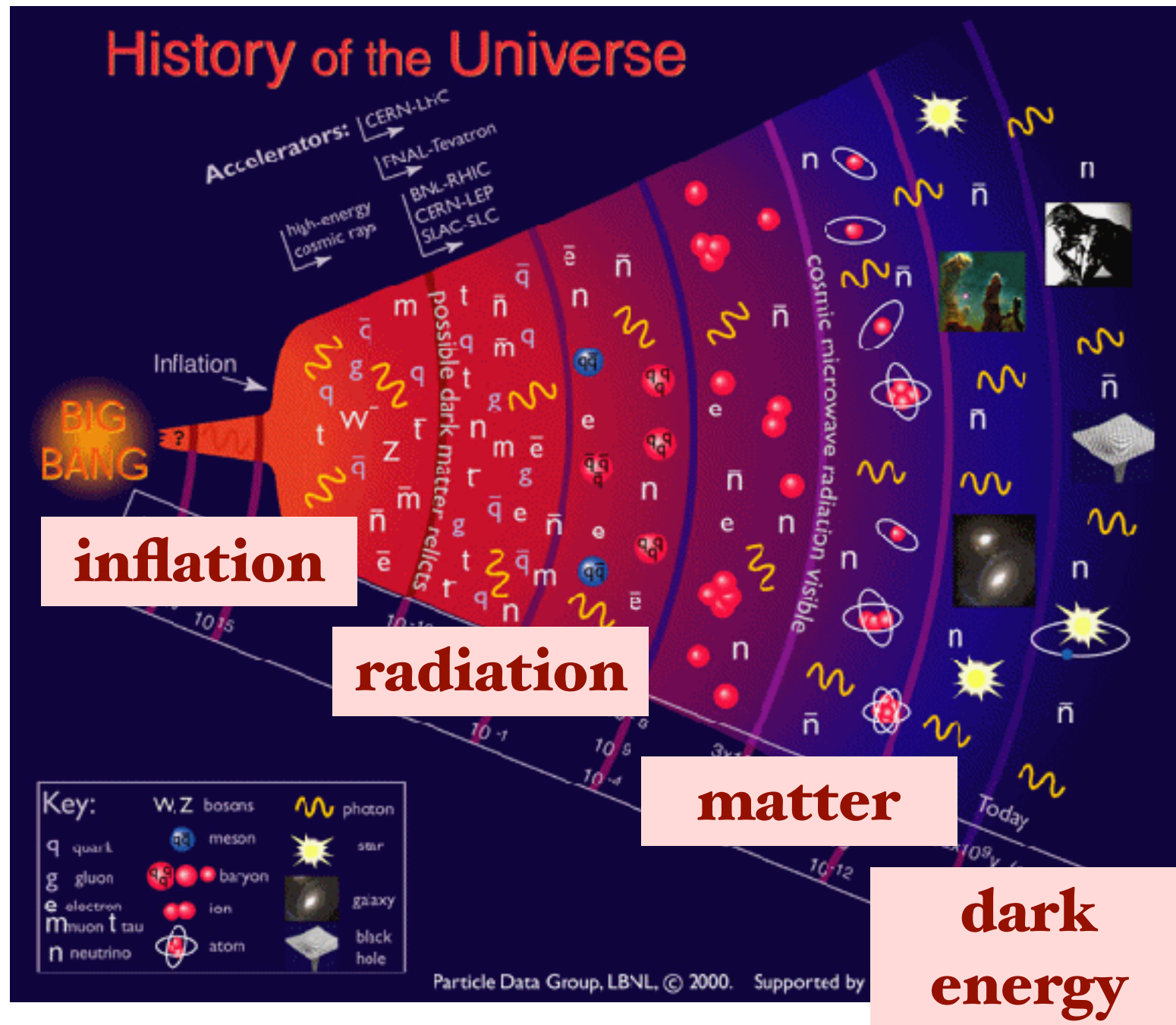
$$T_{\mu\nu} = (\rho + p)u_{\mu}u_{\nu} + pg_{\mu\nu} \qquad p = w \rho$$

linear equation  
of state

- ➡ phase of inflation (scalar field)
- ➡ phase of radiation domination
- ➡ phase of matter domination
- ➡ phase of cosmological constant domination (?)



# what are CBs and why are they formed



- the universe is in *thermal equilibrium* and undergoes *adiabatic expansion*

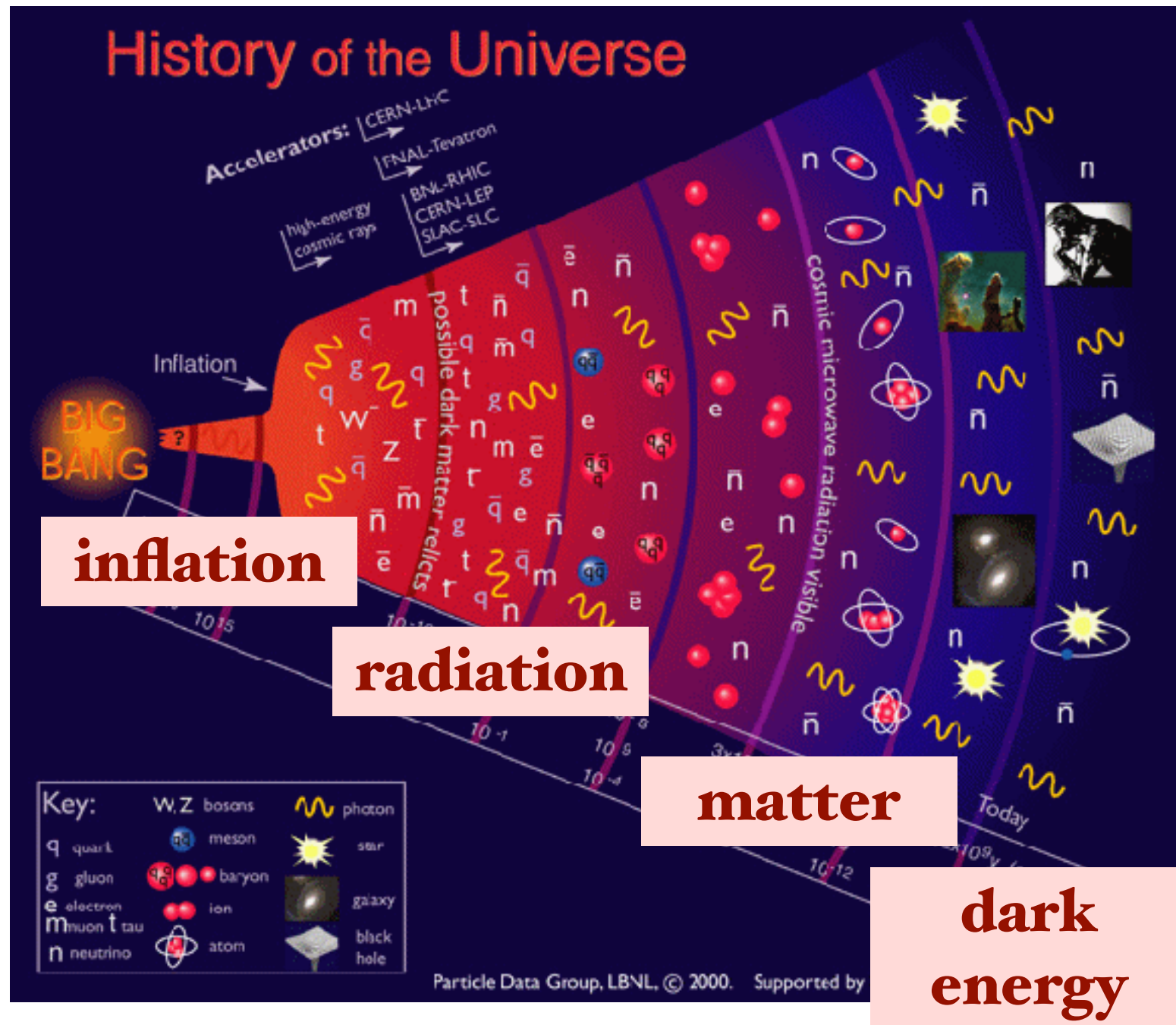


time



temperature

# what are CBs and why are they formed



rate of the interaction  
maintaining thermal  
equilibrium

$$\Gamma = n \sigma v$$

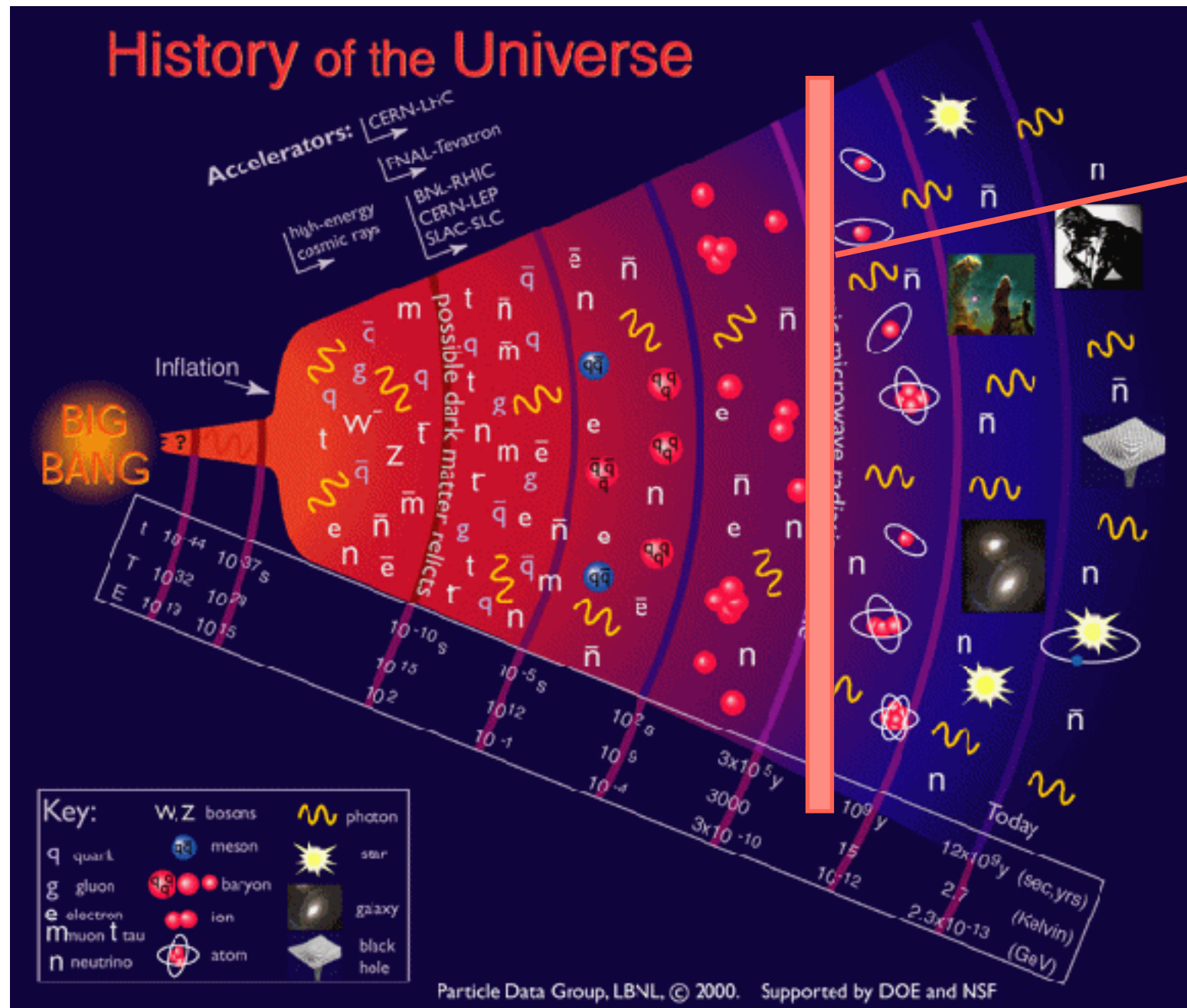
- the universe is in *thermal equilibrium* and undergoes *adiabatic expansion*
- as the universe expands particles can get out of thermal equilibrium: *freeze-out*

$$\frac{\Gamma(T)}{H(T)} < 1$$

rate of expansion  
of the universe



# what are CBs and why are they formed



$$T_{\text{dec}} \simeq 0.25 \text{ eV}$$

freeze-out of

$$e + p \leftrightarrow H + \gamma$$

$$e + \gamma \rightarrow e + \gamma$$

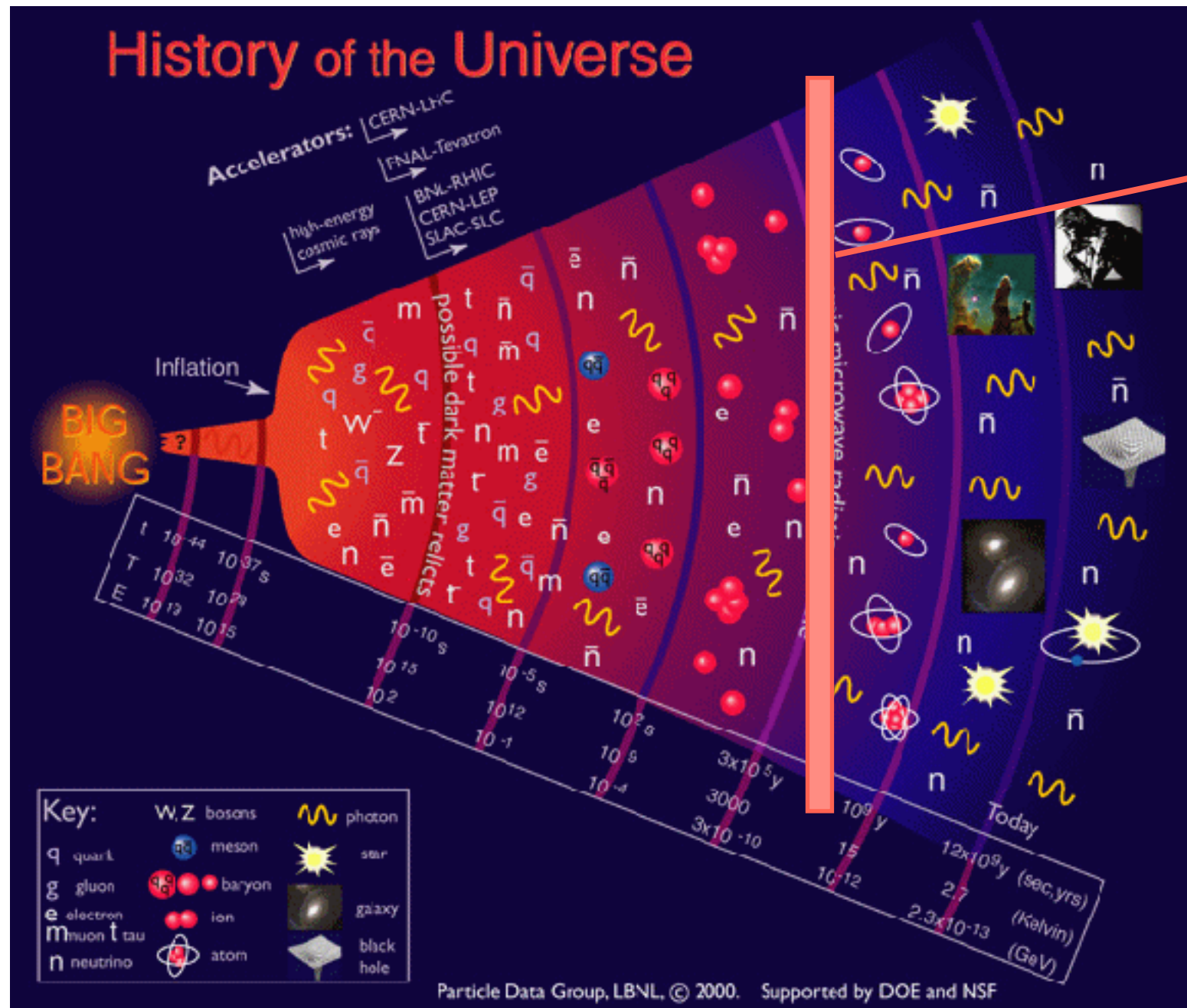
photons are decoupled thereafter:

**COSMIC MICROWAVE BACKGROUND**

today:  $T_0 \simeq 2 \cdot 10^{-4} \text{ eV}$

recombination:  $T_{\text{rec}} \simeq 0.3 \text{ eV}$

# what are CBs and why are they formed



$$T_{\text{dec}} \simeq 0.25 \text{ eV}$$

freeze-out of

$$e + p \leftrightarrow H + \gamma$$

$$e + \gamma \rightarrow e + \gamma$$

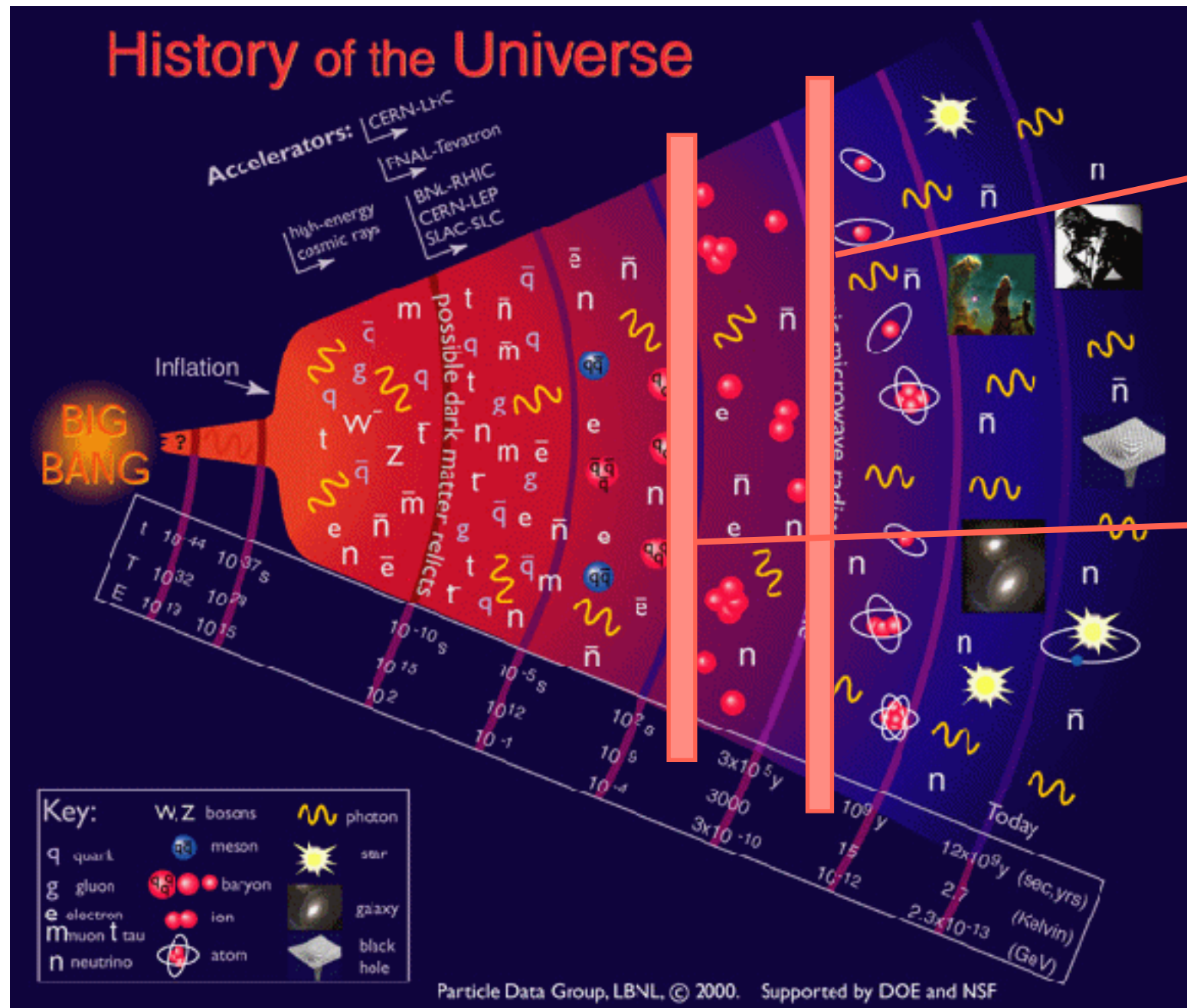
photons are decoupled  
thereafter:

**COSMIC MICROWAVE  
BACKGROUND**

**particles that decouple at temperature  $T_{\text{dec}}$  carry direct information about the status of the universe at that temperature**



# what are CBs and why are they formed



$T_{\text{dec}} \simeq 0.25 \text{ eV}$   
photons decouple:  
CMB

$T_{\text{dec}} \simeq 1.4 \text{ MeV}$   
freeze-out of weak  
interactions

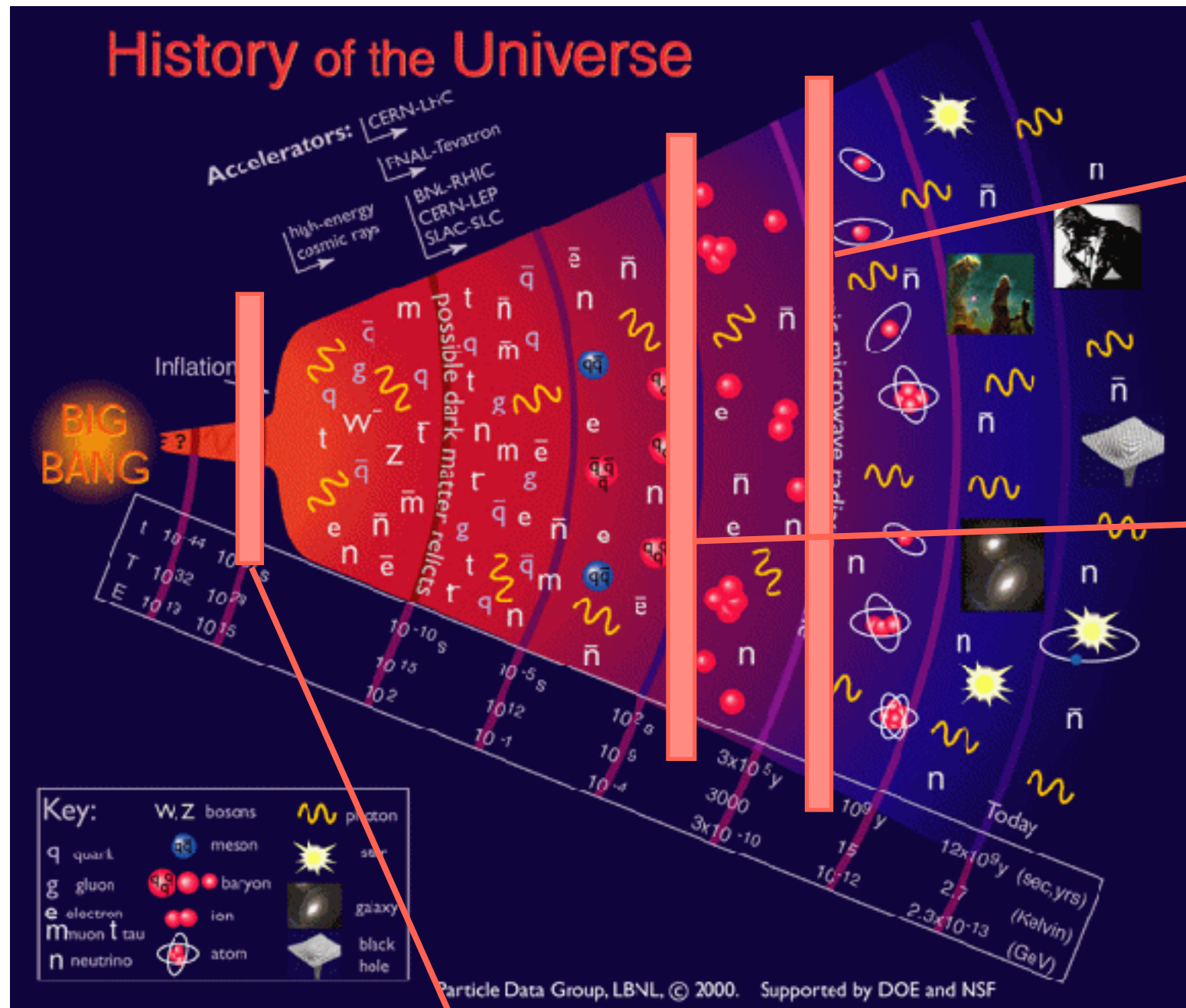
$$e + \bar{\nu} \leftrightarrow e + \bar{\nu}$$

COSMIC NEUTRINO  
BACKGROUND

- indirect evidence: CMB, structure formation, Big Bang Nucleosynthesis



# what are CBs and why are they formed



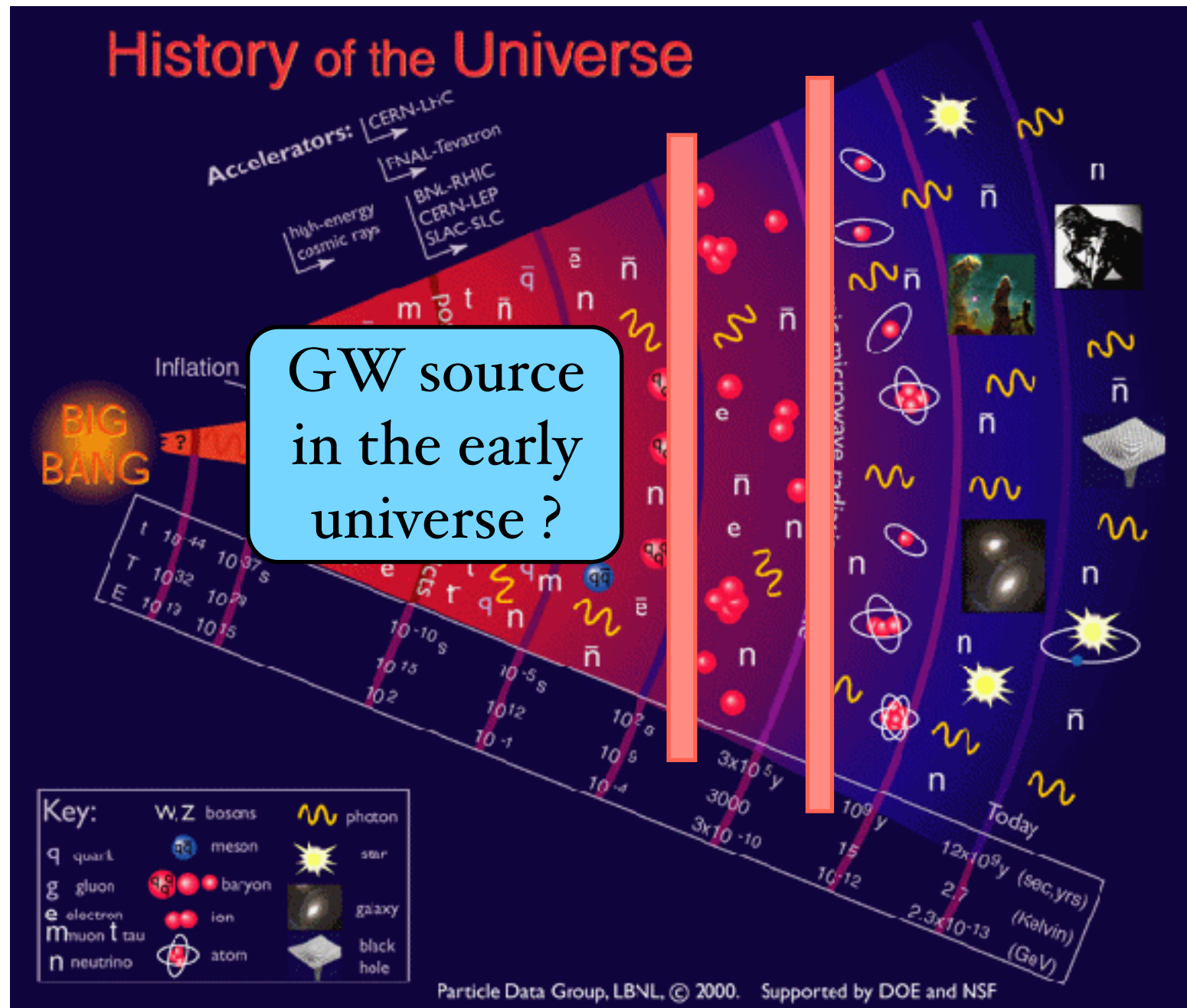
$T_{\text{dec}} \simeq 0.25 \text{ eV}$   
photons decouple:  
CMB

$T_{\text{dec}} \simeq 1.4 \text{ MeV}$   
neutrinos decouple:  
CNB

for gravitons the decoupling temperature would be

$$\frac{\Gamma(T)}{H(T)} \sim \frac{G^2 T^5}{T^2/M_{Pl}} \sim \left( \frac{T}{M_{Pl}} \right)^3 < 1$$

# what are CBs and why are they formed



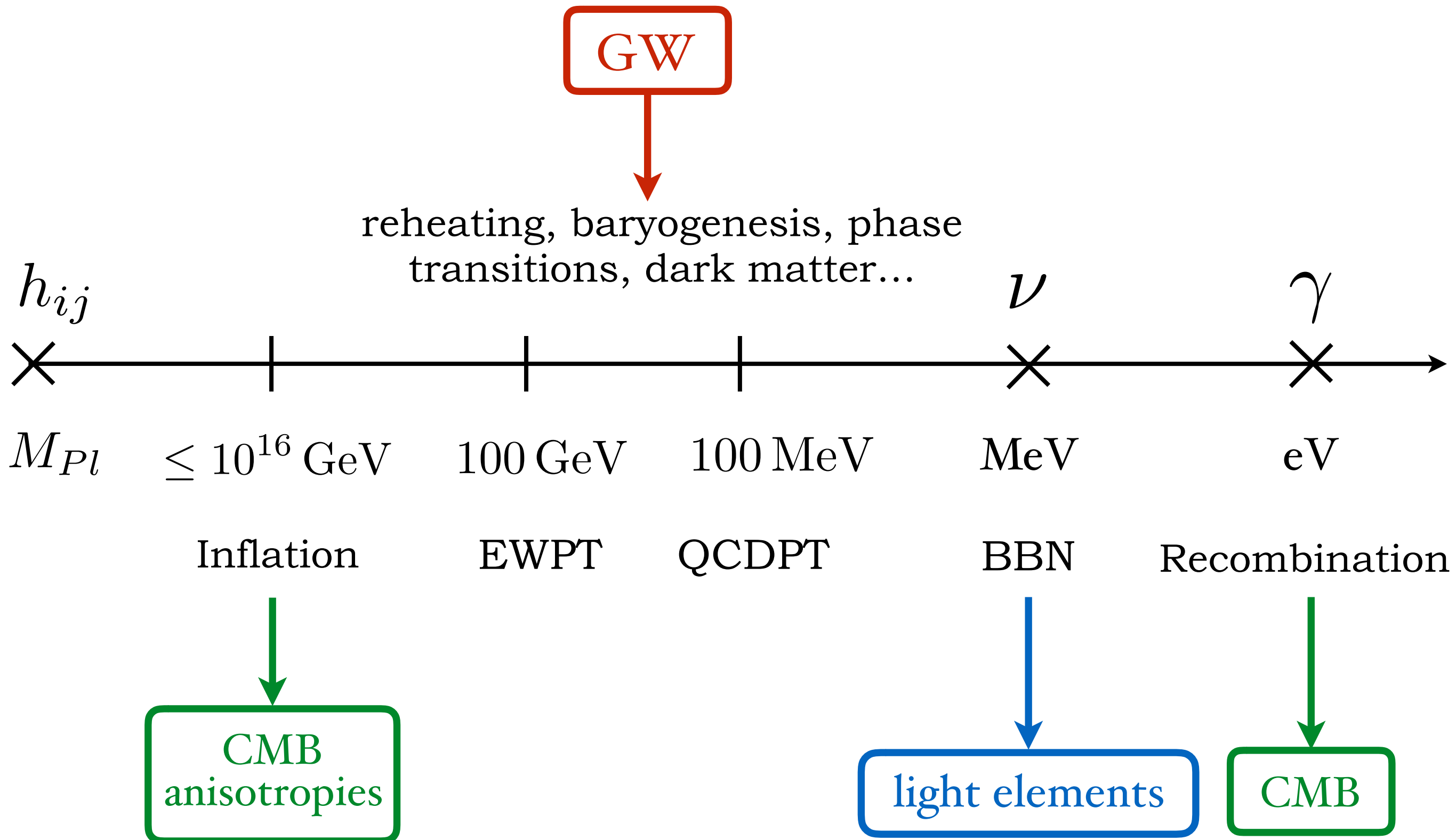
because of the weakness of the gravitational interaction  
GW propagate freely in the early universe

processes in the early universe could have created a

**STOCHASTIC GW BACKGROUND**

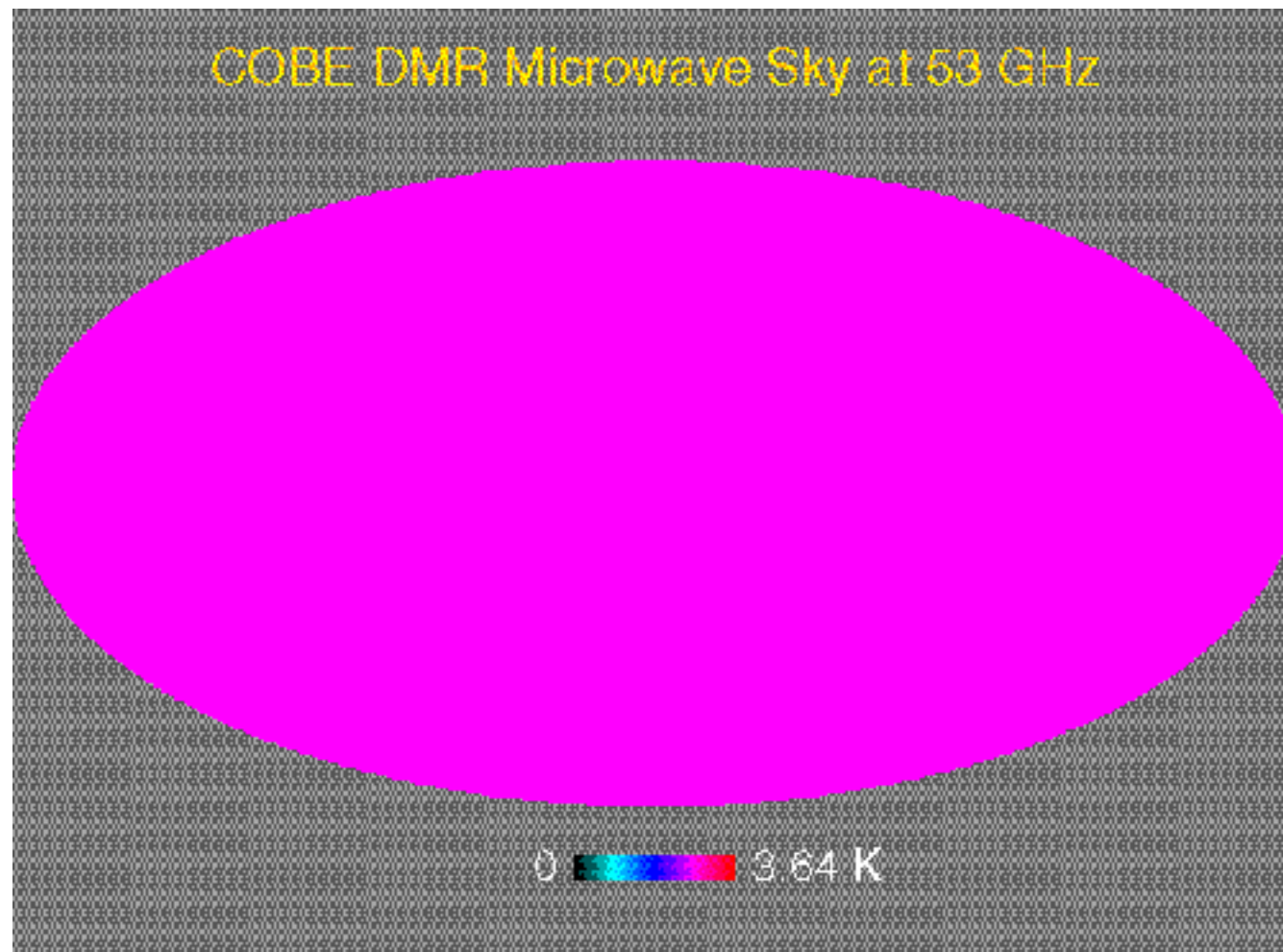
**the detection of the SGWB would be a big step forward in our knowledge of the very early universe**

CB: what can we use them for?  
what information do they bring to us?





# CMB in a nutshell



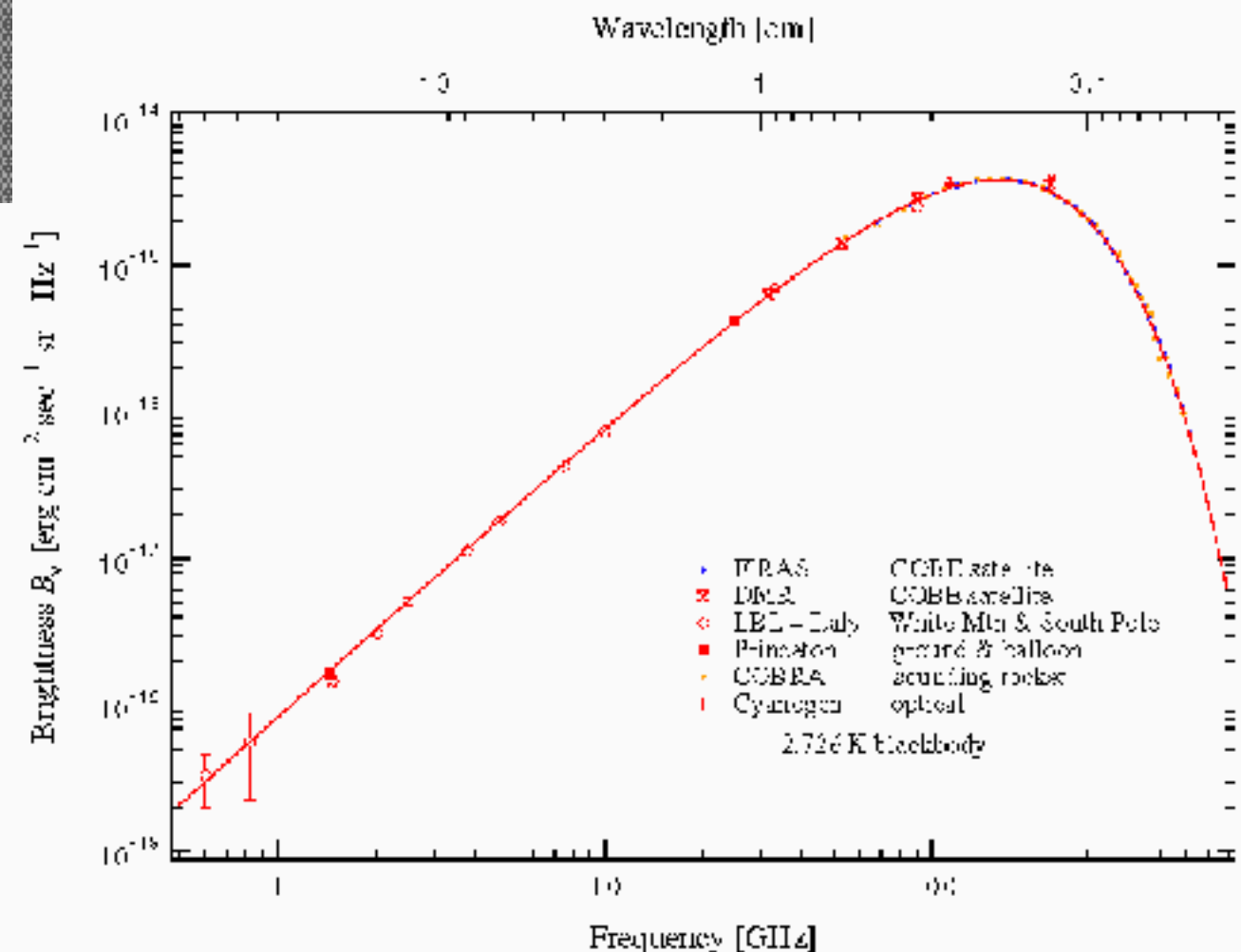
marked the birth of modern cosmology

spectacular confirmation of the Big Bang theory

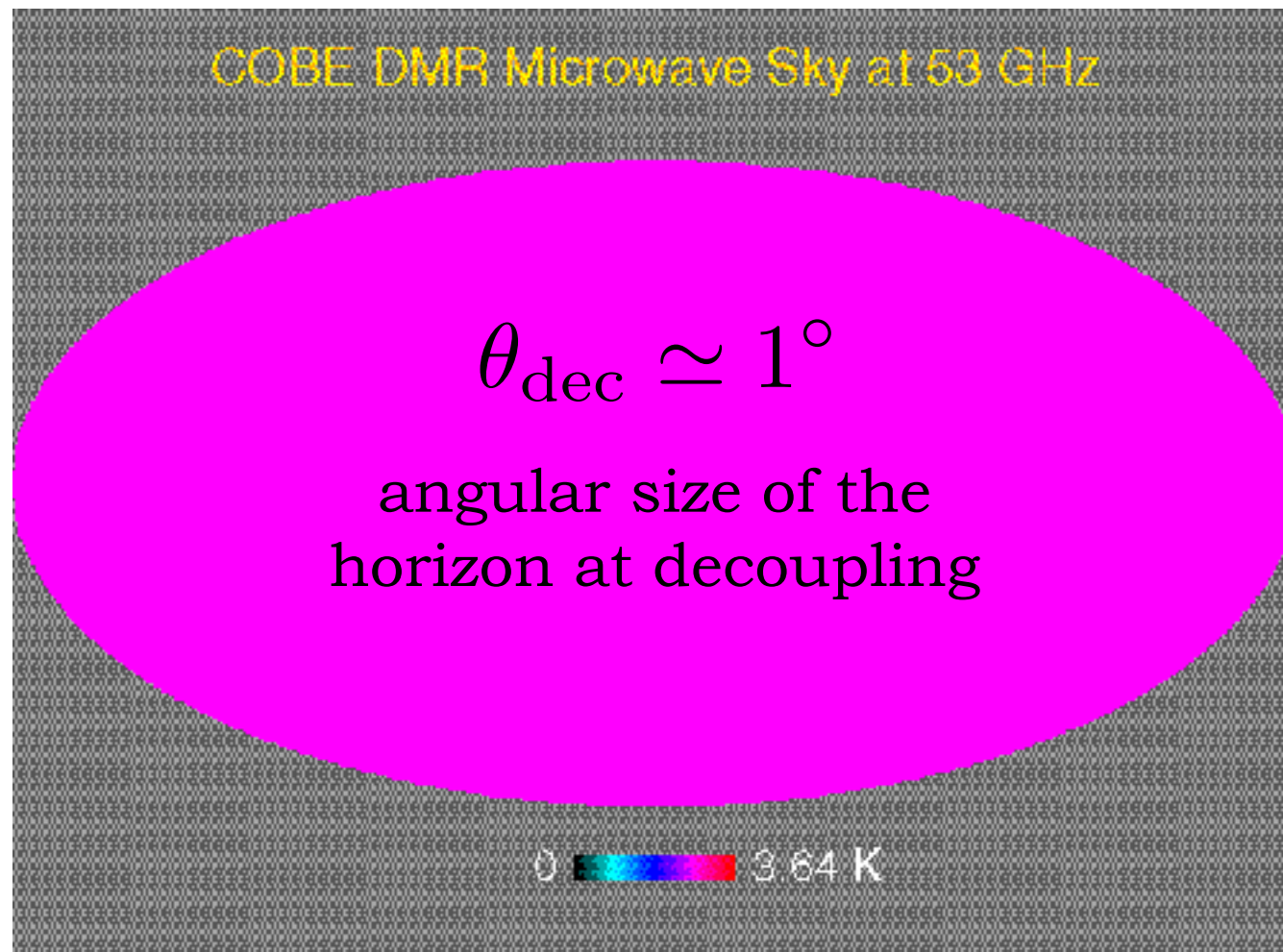
<http://cosmology.berkeley.edu>

all interactions changing the photon number froze-out at 1keV before photon decoupling

$$T = (2.725 \pm 0.001) K$$



# CMB in a nutshell



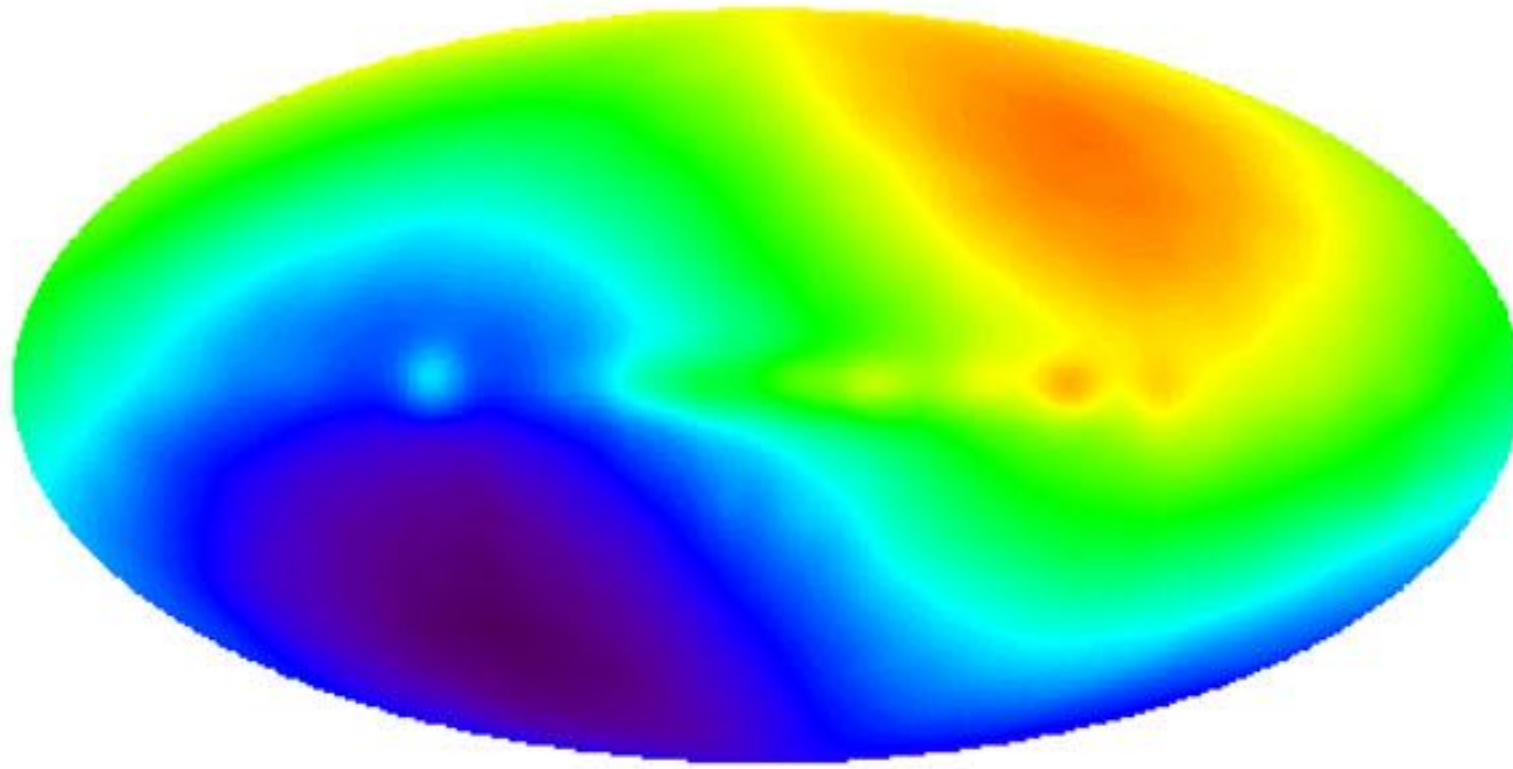
marked the birth of modern cosmology

spectacular confirmation of the Big Bang theory

**why is this radiation so isotropic?**

the CMB monopole contains  $10^4$  causally disconnected regions all at the same temperature

# CMB in a nutshell



COBE satellite

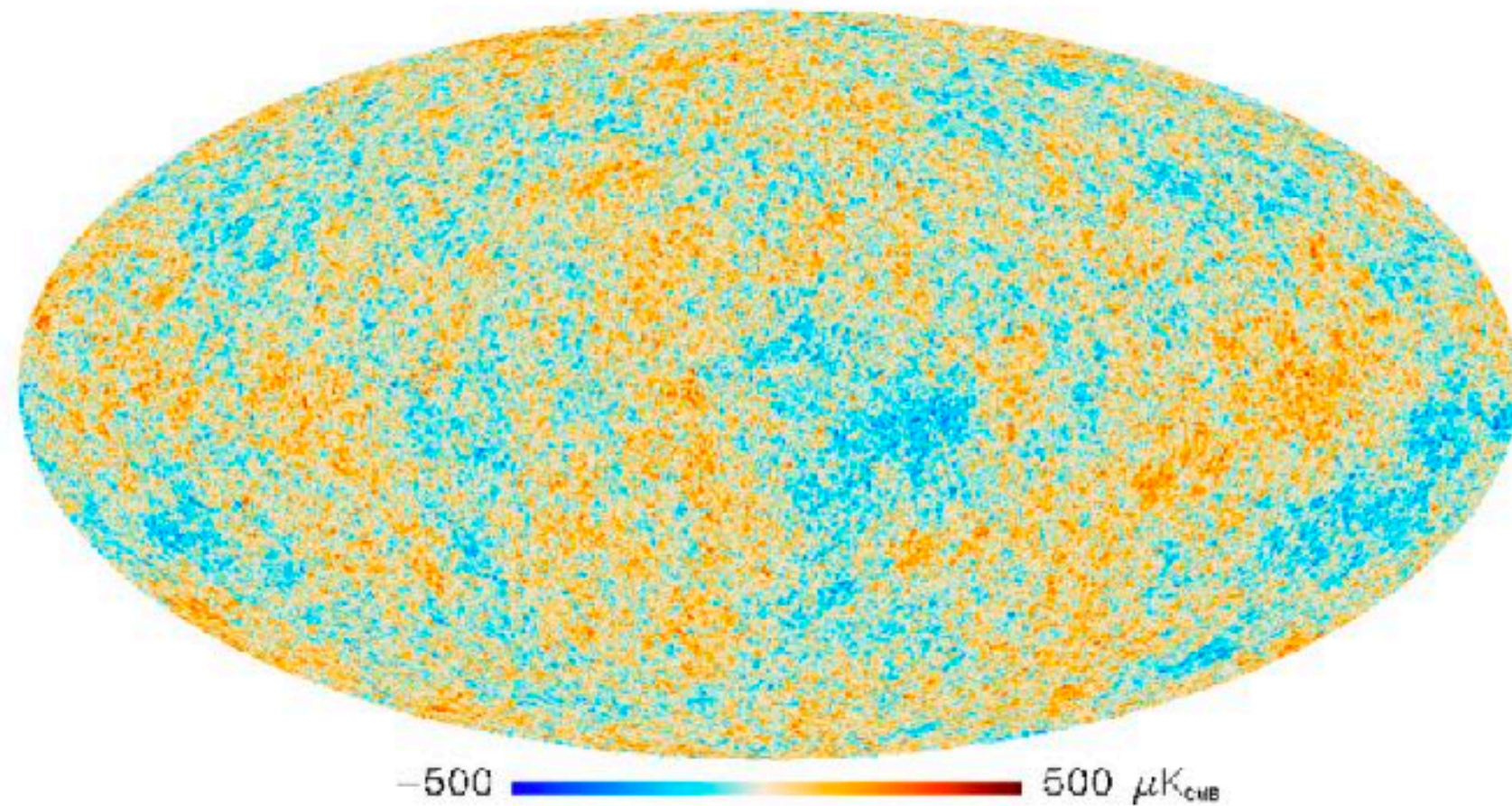
<https://apod.nasa.gov/apod/ap010128.html>

dipole due to Doppler effect of Earth's movement

$$\frac{\delta T}{T} = |\mathbf{v}| = 1.23 \times 10^{-3}$$



# CMB in a nutshell

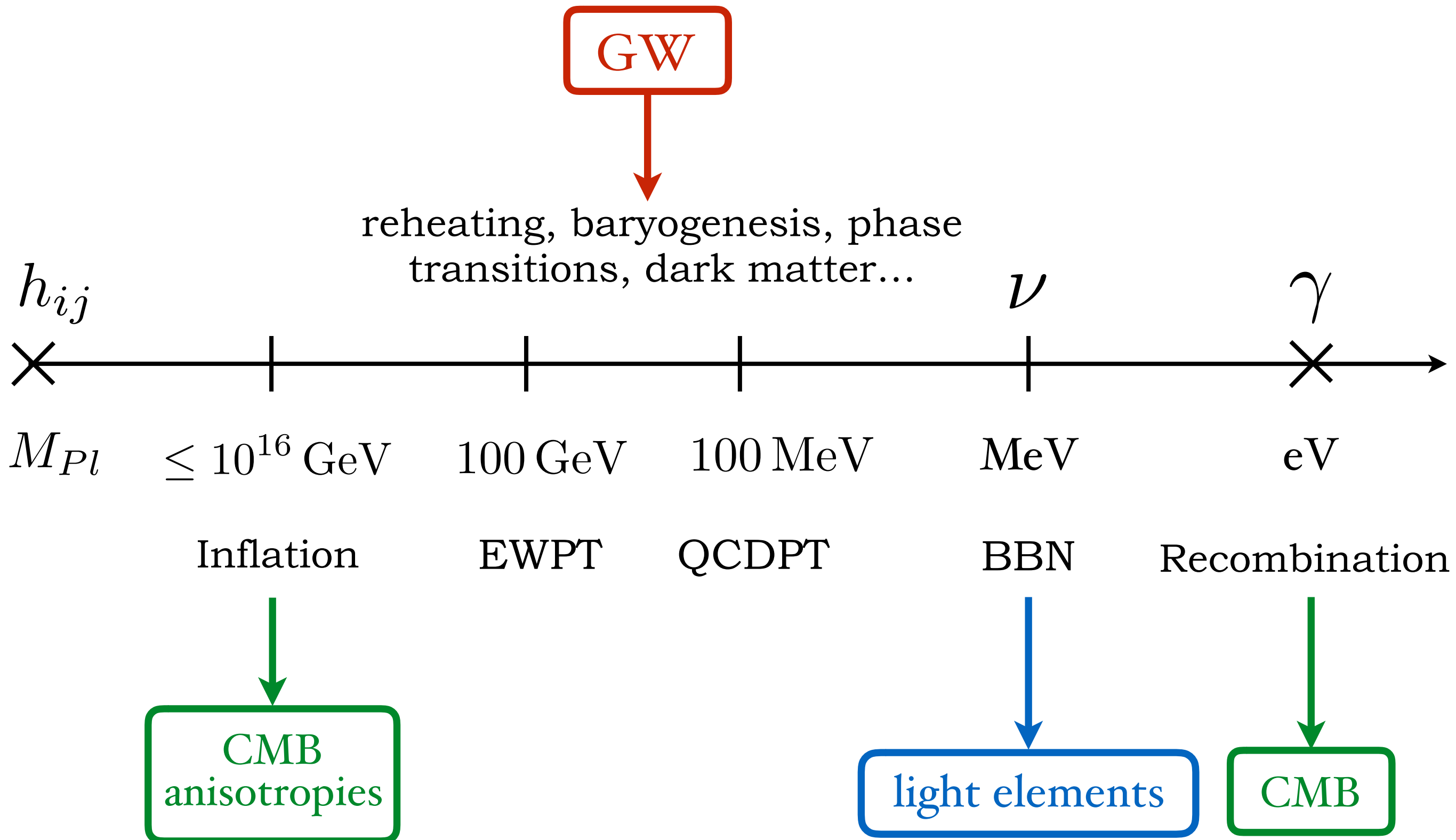


Planck satellite  
arXiv:1303.5062

multipoles  $> 2$  all with amplitude

$$\frac{\delta T}{T} \sim 10^{-5}$$

CB: what can we use them for?  
what information do they bring to us?



# CMB in a nutshell

**Inflation** provides a model for the “initial conditions” of the observed universe  
it is a phase of **accelerated expansion** isotropising the universe

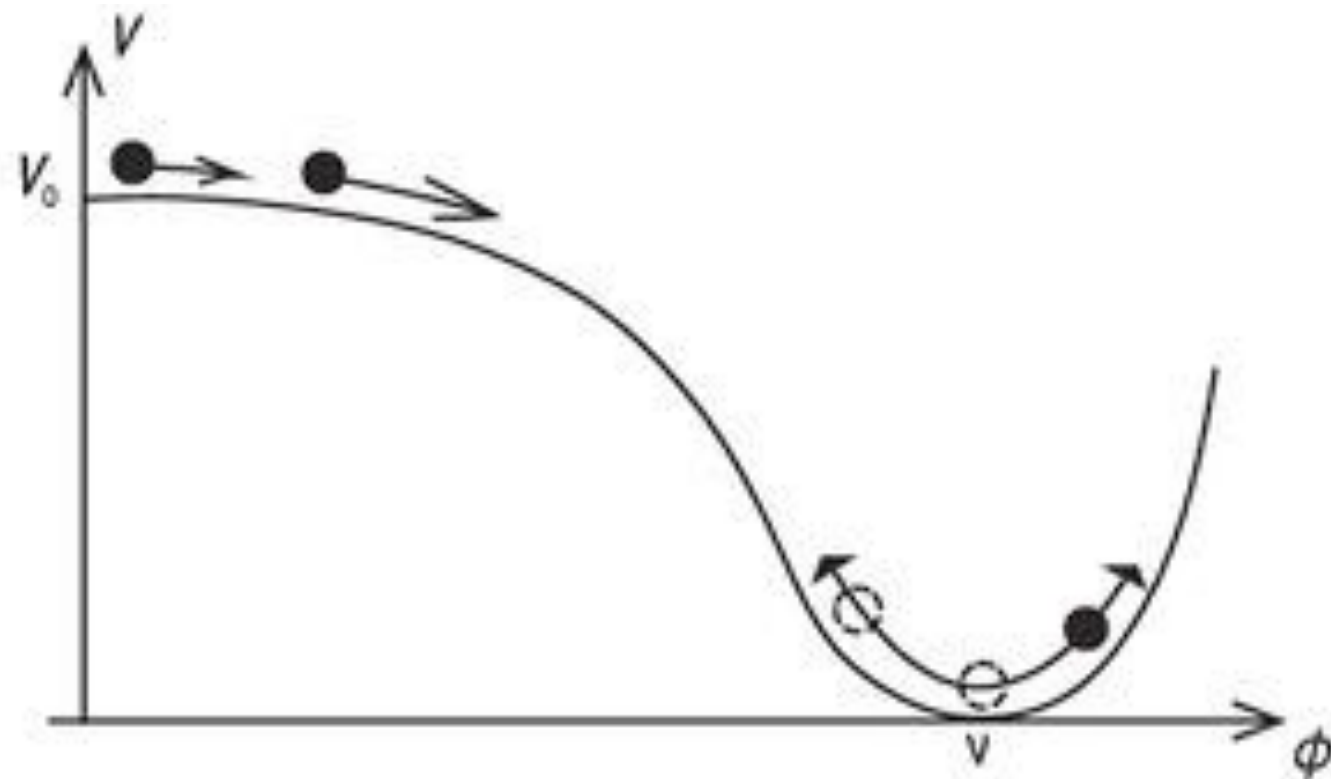
$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) > 0$$

Phase dominated by a scalar field in a sufficiently flat potential

$$\rho_\phi = \cancel{\frac{1}{2}\dot{\phi}^2} + \cancel{\frac{1}{2}(\nabla\phi)^2} + V(\phi)$$

$$p_\phi = \cancel{\frac{1}{2}\dot{\phi}^2} - \cancel{\frac{1}{6}(\nabla\phi)^2} - V(\phi)$$

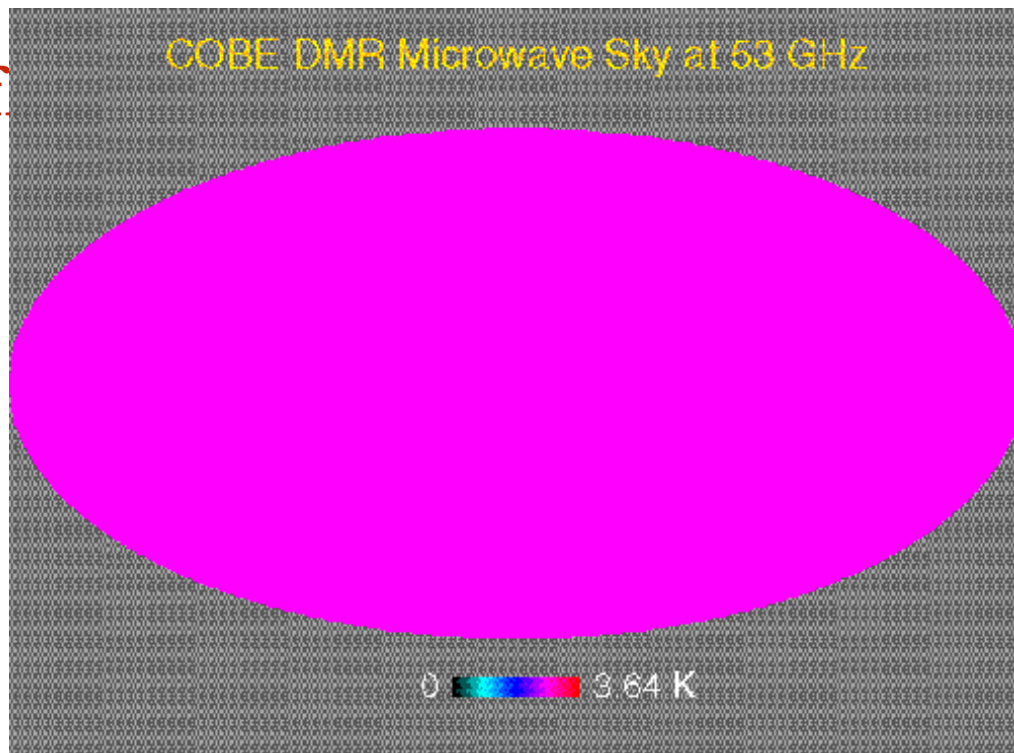
$$p_\phi \simeq -\rho_\phi$$





# CMB in a nutshell

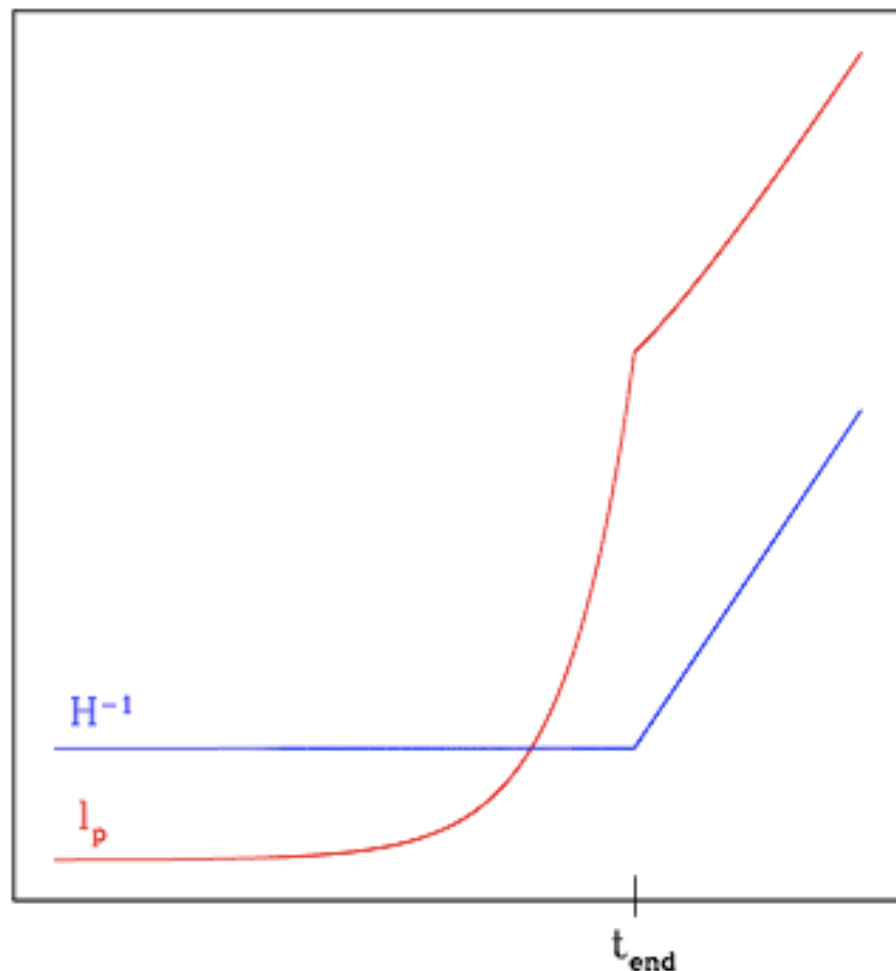
Infl



the “initial conditions” of the observed universe  
and **expansion** isotropising the universe

$$\frac{4\pi G}{3}(\rho + 3p) > 0$$

scalar field in a sufficiently flat potential

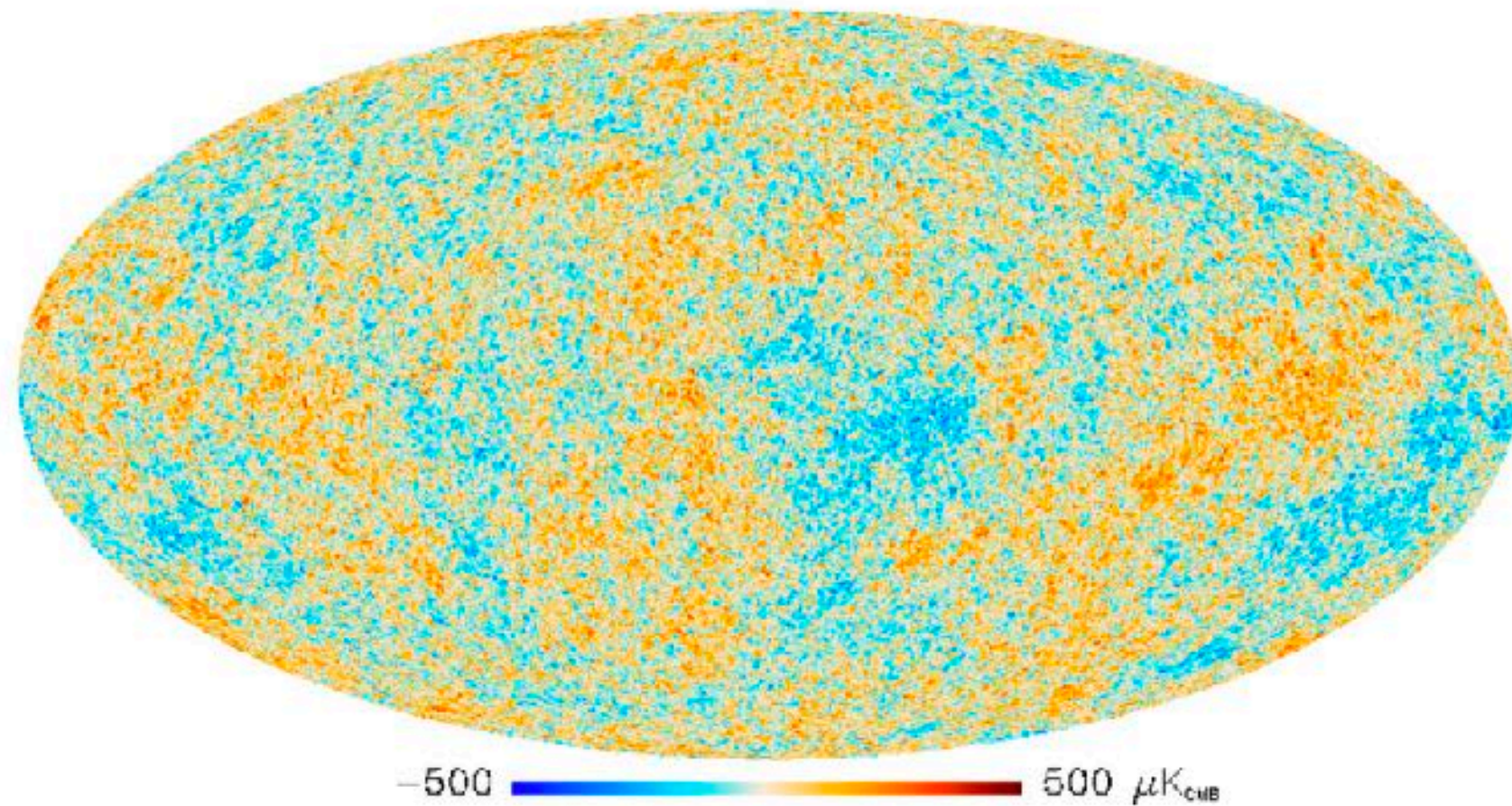


during a phase which is  
not accelerating:

$$\ell_p \simeq 1/H$$

during an accelerating phase the  
particle horizon grows  
exponentially  
at the end it is much larger than  
if inflation had not been there

# CMB in a nutshell



Planck satellite  
arXiv:1303.5062

**Inflation** also provides an explanation for the small temperature anisotropies in the CMB

# CMB in a nutshell

the phase of accelerated expansion amplifies and stretches  
out of the horizon quantum fluctuations

- scalar modes: quanta of the inflaton field  $\phi = \bar{\phi} + \delta\phi$
- tensor modes: quanta of the gravitational field  $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$



# CMB in a nutshell

the phase of accelerated expansion amplifies and stretches out of the horizon quantum fluctuations

- scalar modes: quanta of the inflaton field  $\phi = \bar{\phi} + \delta\phi$
- tensor modes: quanta of the gravitational field  $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$

scalar modes: field fluctuations perturb the metric  
photons and baryons evolve in the fluctuating potential

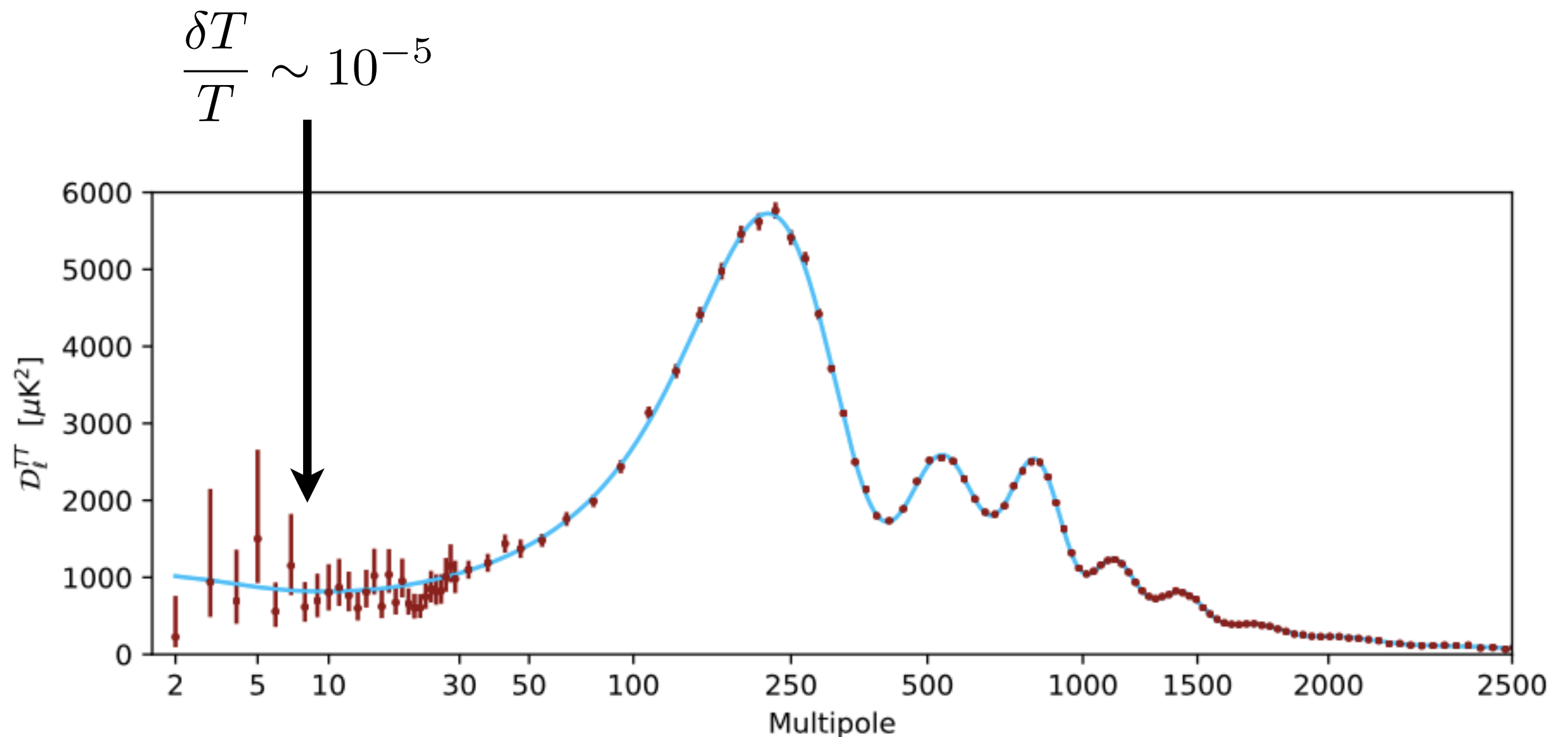
$$ds^2 = a^2(\eta) [-(1 + \Phi)d\eta^2 + (1 - \Psi)d\mathbf{x}^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

$$\delta\phi \rightarrow (\Psi, \Phi) \rightarrow \left( \frac{\delta\rho}{\rho}, \mathbf{v} \right) \rightarrow \frac{\delta T}{T}$$

# CMB in a nutshell

the phase of accelerated expansion amplifies and stretches out of the horizon quantum fluctuations

- scalar modes: quanta of the inflaton field  $\phi = \bar{\phi} + \delta\phi$



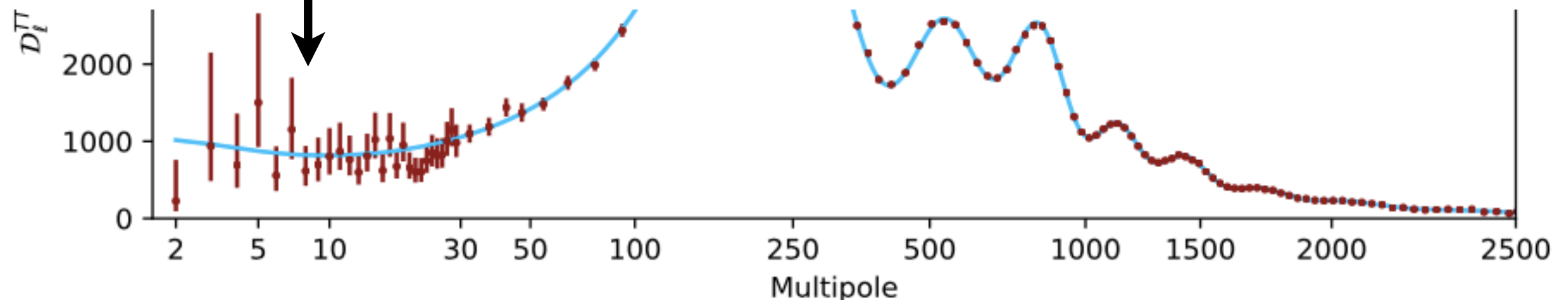
# CMB in a nutshell

the phase of accelerated expansion amplifies and stretches out of the horizon quantum fluctuations

- scalar modes: quanta of the inflaton field  $\phi = \bar{\phi} + \delta\phi$

$$\frac{\delta T}{T} \sim 10^{-5}$$

- ➡ content of the universe ( $\Lambda$ CDM)
- ➡ confirmation that inflation works
- ➡ information on structure formation
- ➡ constraints on  $H_0$ , neutrinos, dark energy...





# CMB in a nutshell

the phase of accelerated expansion amplifies and stretches out of the horizon quantum fluctuations

- scalar modes: quanta of the inflaton field  $\phi = \bar{\phi} + \delta\phi$
- tensor modes: quanta of the gravitational field  $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$

$$ds^2 = a^2(\eta)[-(1 + \Phi)d\eta^2 + (1 - \Psi)d\mathbf{x}^2 + (\delta_{ij} + h_{ij})dx^i dx^j]$$

# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$|h_{ij}| \ll 1$$

$$h_{\dot{i}}^{\dot{i}} = \partial_j h_{\dot{i}}^{\dot{j}} = 0$$

# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 0$$

WAVE  
EQUATION

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 0$$



# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 0$$

WAVE  
EQUATION

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amplification of metric vacuum fluctuations during inflation

# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$
$$G_{\mu\nu} = 0$$

WAVE  
EQUATION

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 0$$

amplification of metric vacuum fluctuations during inflation

- ✓ canonically normalised free field  $v_{\pm} = a M_{Pl} h_{\pm}$
- ✓ quantisation
- ✓ homogeneous wave equation: harmonic oscillator with *time dependent* frequency

$$v_{\pm}''(t) + (k^2 - a^2 H^2)v_{\pm}(t) = 0$$

# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$
$$G_{\mu\nu} = 0$$

WAVE  
EQUATION

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 0$$

amplification of metric vacuum fluctuations during inflation

$$v''_{\pm}(t) + (k^2 - a^2 H^2)v_{\pm}(t) = 0$$

$k \gg aH$  sub-Hubble modes

$$\omega^2(t) = k^2$$

free field in vacuum  
zero occupation number

$k \ll aH$  super-Hubble modes

$$\omega^2(t) = -a^2 H^2$$

super-Hubble modes have very large  
occupation number



# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

WAVE  
EQUATION

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

source:  $\Pi_{ij}^{TT}$  tensor anisotropic stress

# GW from early universe sources

tensor  
perturbations of  
FRW metric:

$$ds^2 = -dt^2 + a^2(t)[(\delta_{ij} + h_{ij})dx^i dx^j]$$

$$G_{\mu\nu} = 8\pi G T_{\mu\nu}$$

WAVE  
EQUATION

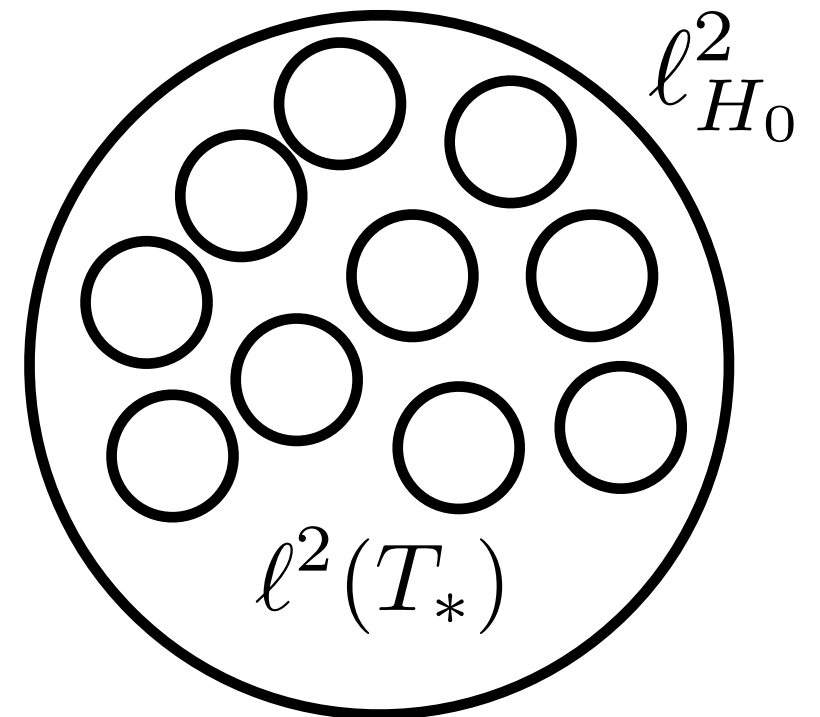
$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

source:  $\Pi_{ij}^{TT}$  tensor anisotropic stress

- fluid  $\Pi_{ij} \sim \gamma^2(\rho + p) v_i v_j$
- electromagnetic field  $\Pi_{ij} \sim (E^2 + B^2) \frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$
- scalar field  $\Pi_{ij} \sim \partial_i \phi \partial_j \phi$

# stochastic GW background

- **inflation**: intrinsic, quantum fluctuations that become classical (stochastic) outside the horizon
- **causal source**: of GW cannot operate beyond the horizon (Hubble scale)  
signal visible today originated from many independent horizon volumes



GW energy density power spectrum

$$\Omega_{\text{GW}} = \frac{\rho_{\text{GW}}}{\rho_c} = \frac{\langle \dot{h}_{ij} \dot{h}_{ij} \rangle}{32\pi G \rho_c} = \int \frac{df}{f} \frac{d\Omega_{\text{GW}}}{d \ln f}$$

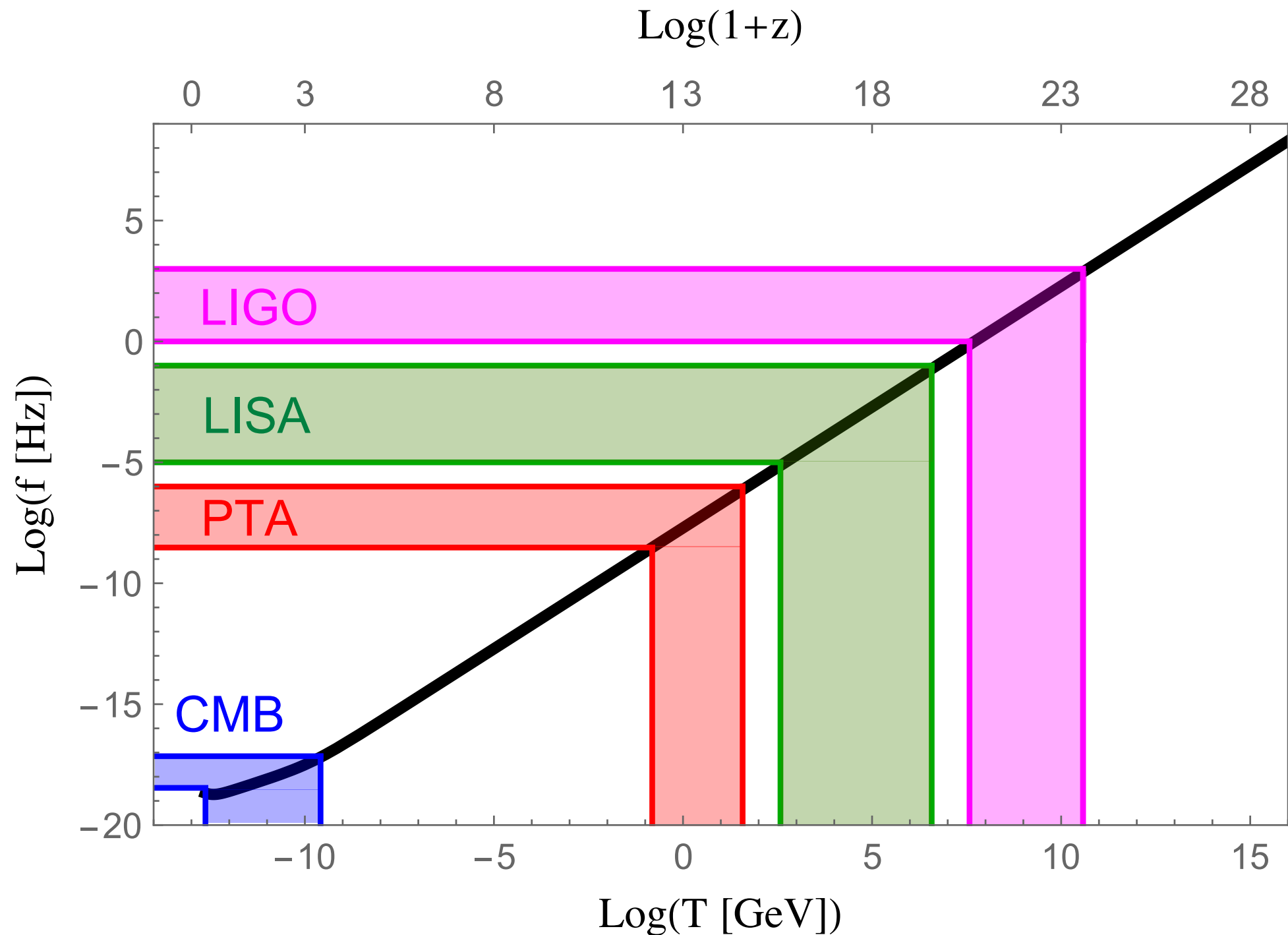
# stochastic GW background

Characteristic frequency for  
*causal sources*

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

$$\epsilon_* = 1$$

parameter  
depending on the  
characteristic  
scale of source





# Advanced LIGO/Virgo interferometers

arm length  $L = 4 \text{ km}$

frequency range of detection:  $10 \text{ Hz} < f < 5 \text{ kHz}$

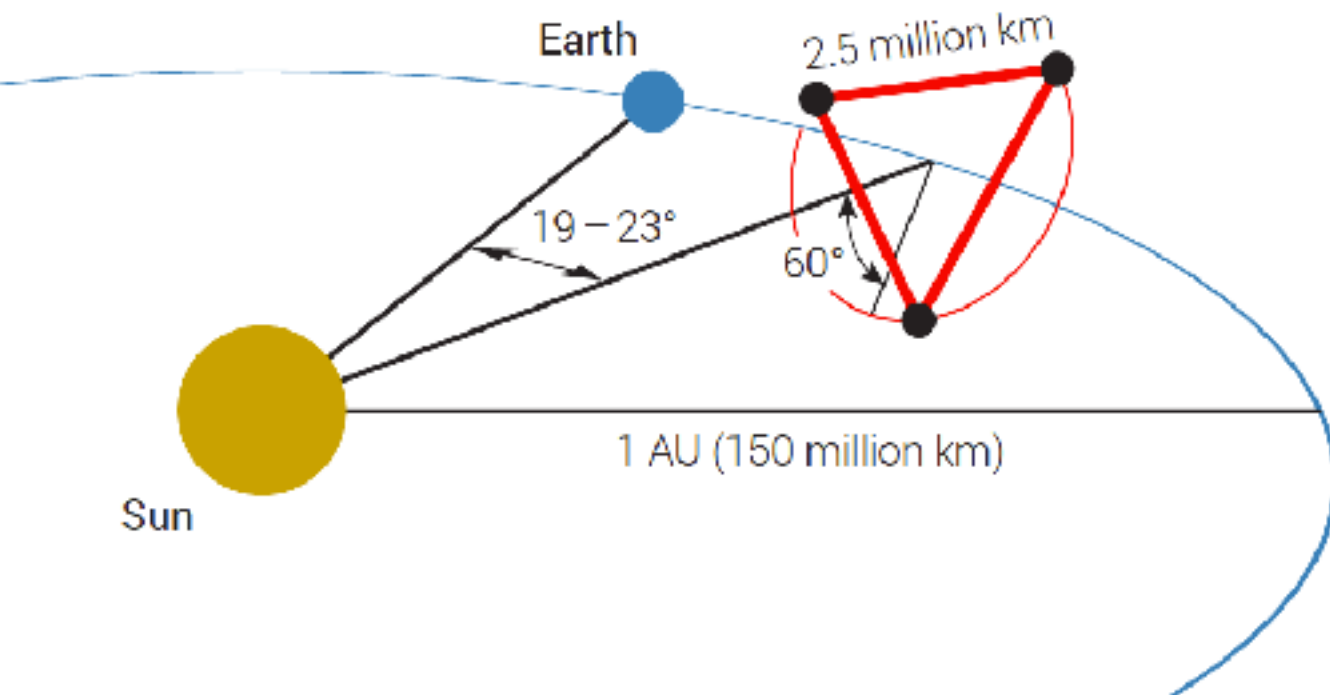
- Black hole coalescing binaries of masses few to dozens solar masses
- Neutron Star and NS-BH binaries
- Stochastic GW background



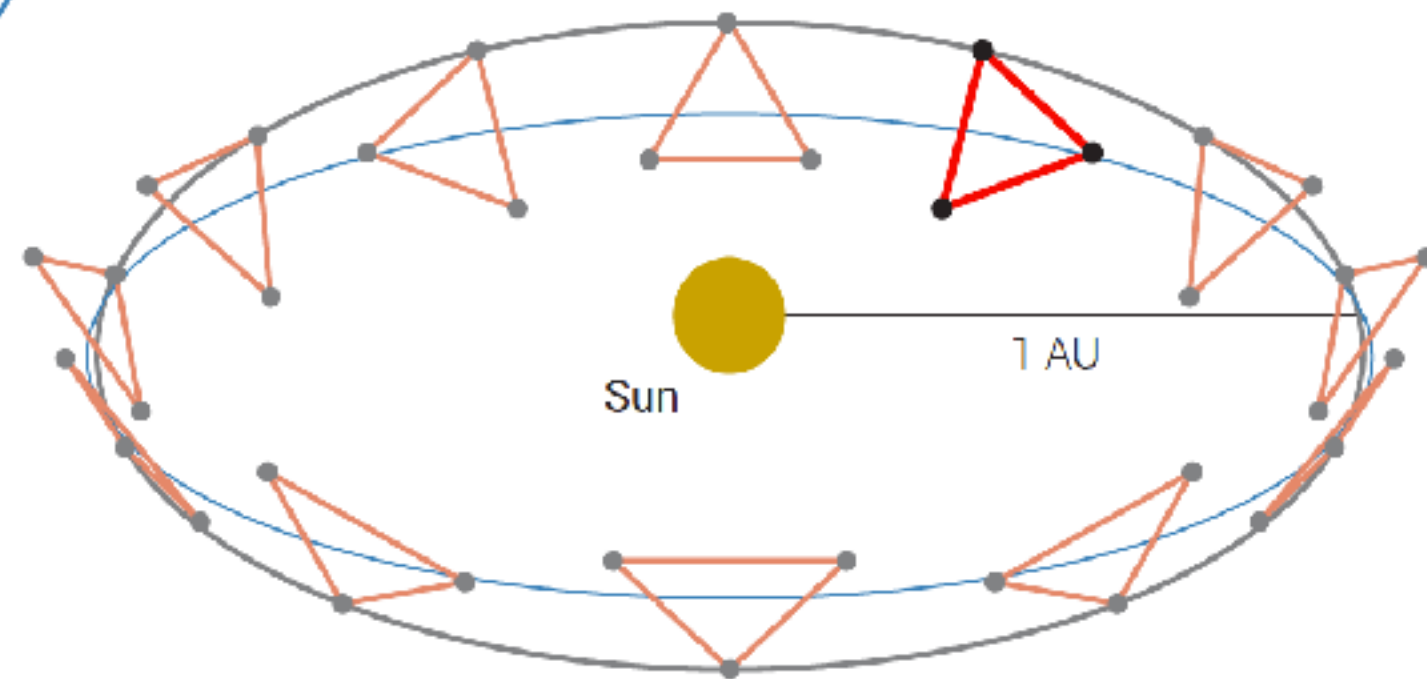
# LISA: Laser Interferometer Space Antenna

- no seismic noise
- much longer arms than on Earth

frequency range of detection:  $10^{-4} \text{ Hz} < f < 1 \text{ Hz}$



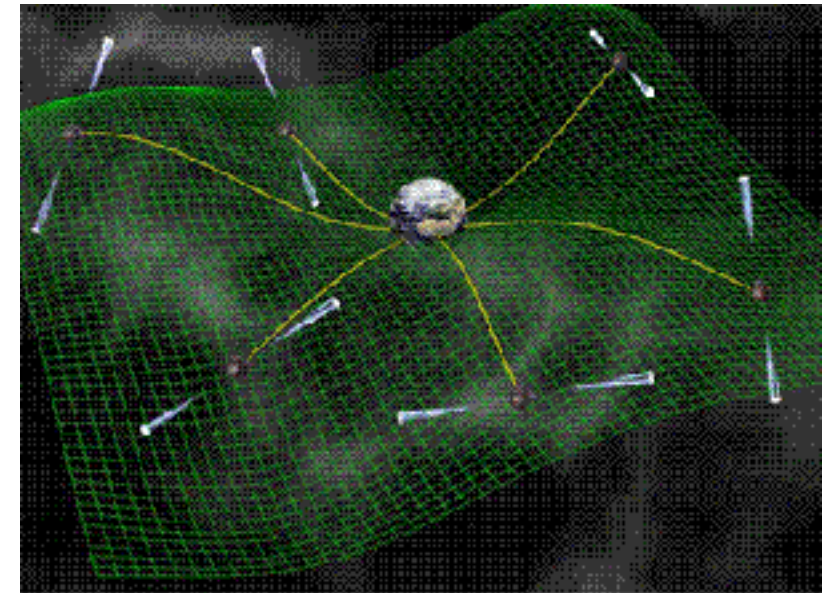
- Launch in ~2032
- two masses in free fall per spacecraft
- 2.5 million km arms
- picometer displacement of masses



# Pulsar timing array

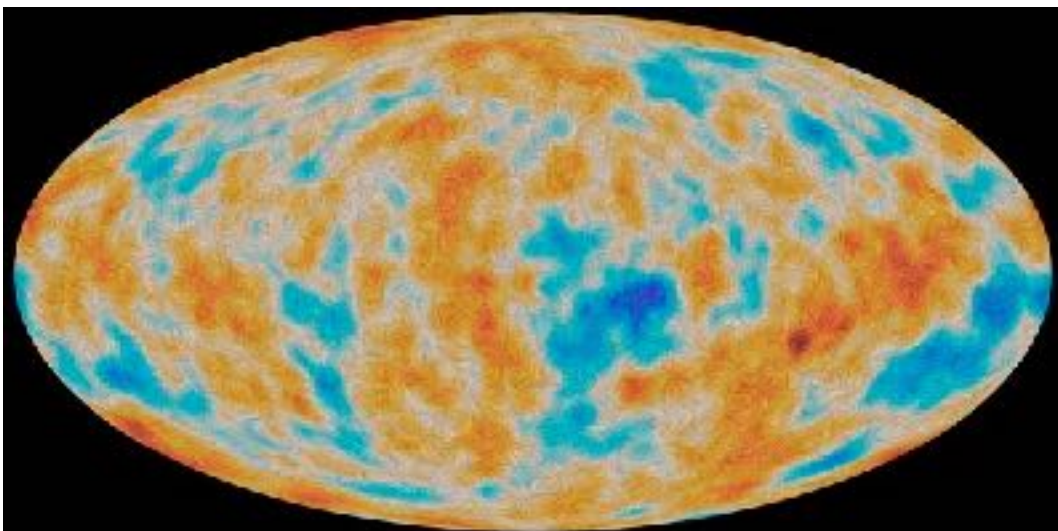
frequency range of detection:  $10^{-9} \text{ Hz} < f < 10^{-7} \text{ Hz}$

- target: stochastic background from inspiralling SMBH binaries (masses of order  $10^9$  solar masses)



# Cosmic microwave background

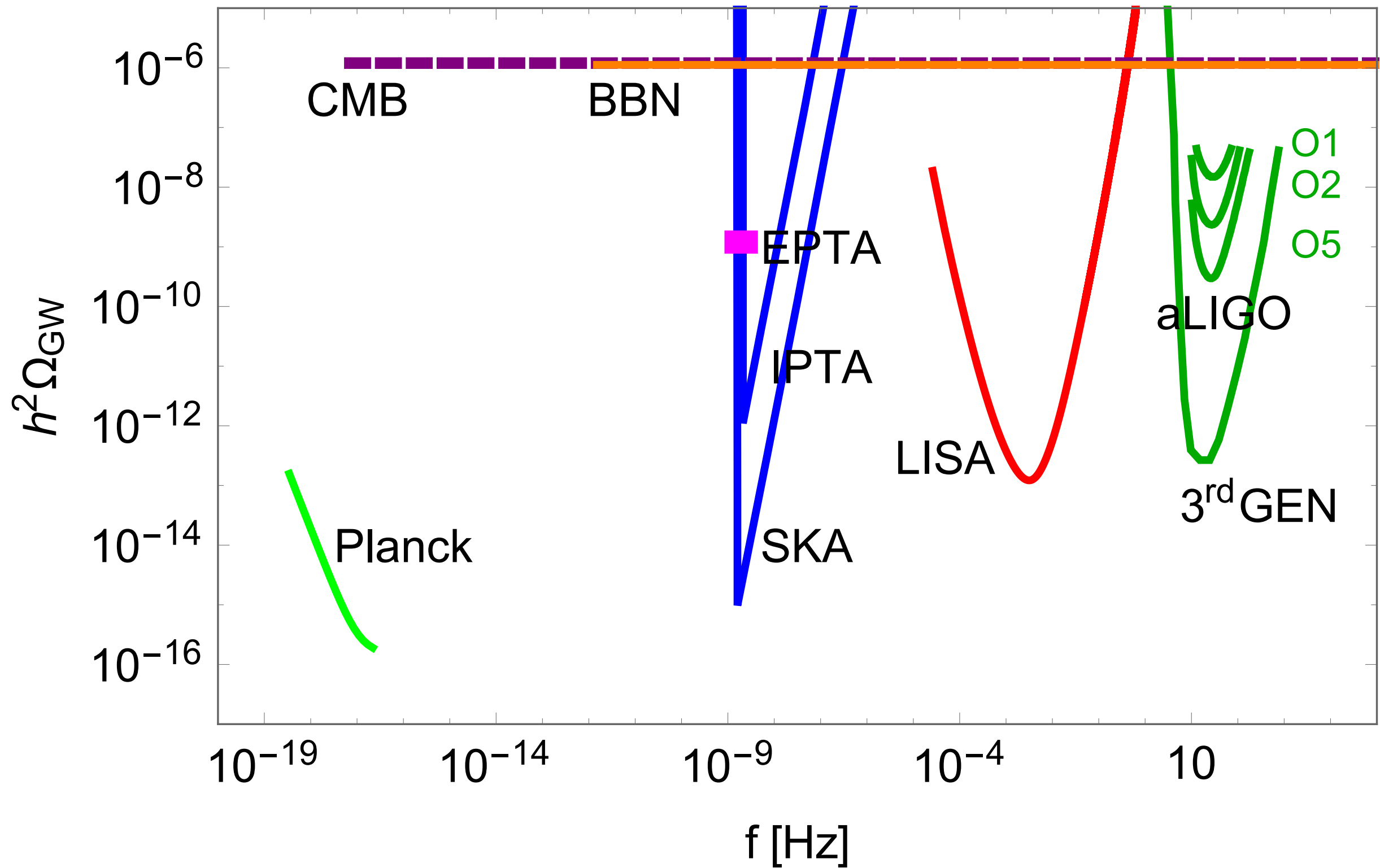
frequency range of detection:  $10^{-18} \text{ Hz} < f < 10^{-16} \text{ Hz}$



- target: temperature fluctuations and B polarisation



# SGWB bounds and detectors



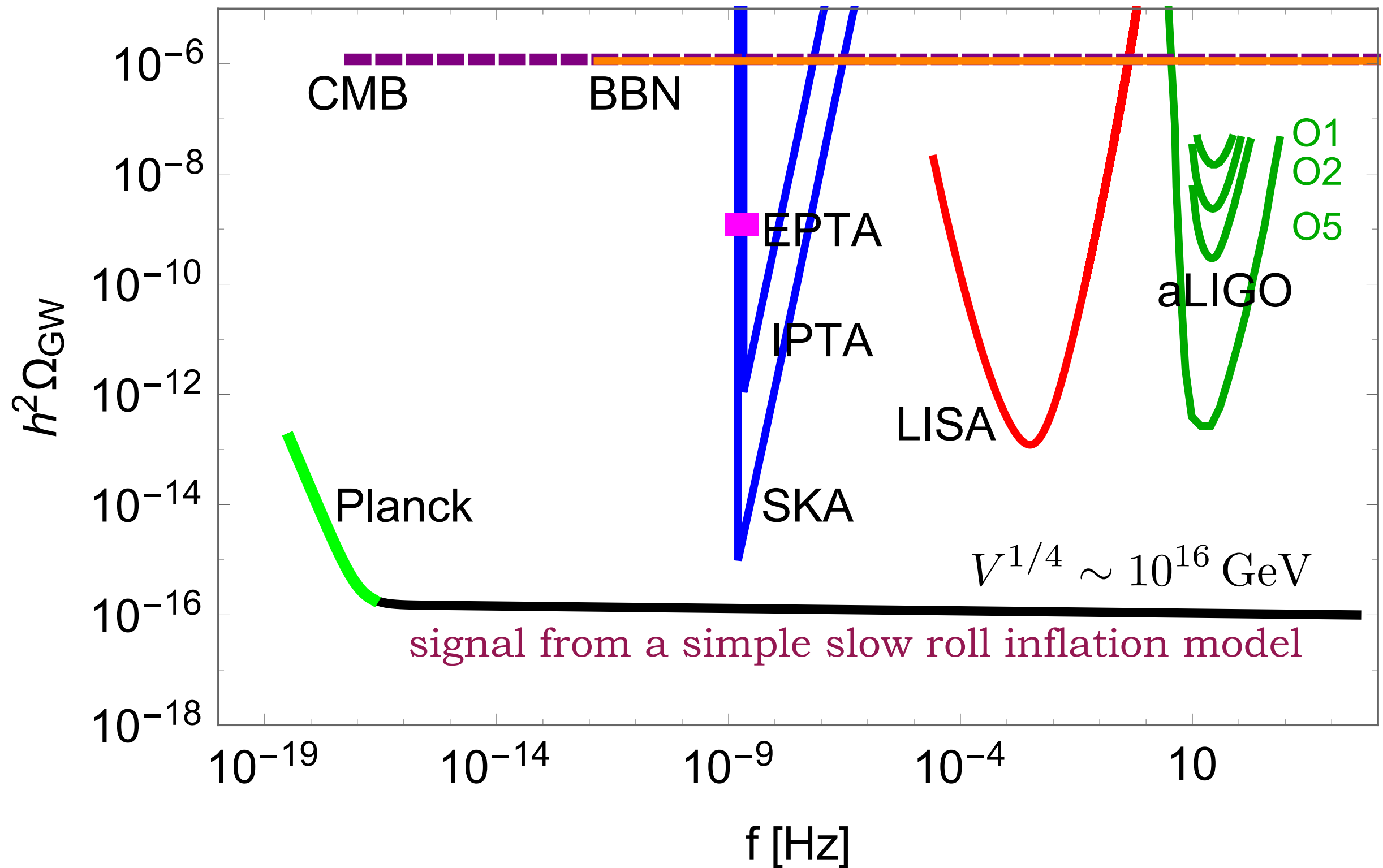


# SGWB from slow roll inflation

$$\Omega_{\text{GW}}(f) = \frac{3}{128} \Omega_{\text{rad}} r \mathcal{P}_{\mathcal{R}}^* \left( \frac{f}{f_*} \right)^{n_T} \left[ \frac{1}{2} \left( \frac{f_{\text{eq}}}{f} \right)^2 + \frac{16}{9} \right]$$

- tensor to scalar ratio  $r = \mathcal{P}_h / \mathcal{P}_{\mathcal{R}}$   $r_* \leq 0.07$
- scalar amplitude at CMB pivot scale  $\mathcal{P}_{\mathcal{R}}^* \simeq 2 \cdot 10^{-9}$   $k_* = \frac{0.05}{\text{Mpc}}$
- tensor spectrum  $\mathcal{P}_h = \frac{2}{\pi} \frac{H^2}{m_{Pl}^2} \left( \frac{k}{aH} \right)^{-2\epsilon}$   $n_T \simeq -2\epsilon$
- transfer function from inflation to today

# SGWB from slow roll inflation



# GW influence CMB photons and leave an imprint in CMB anisotropies

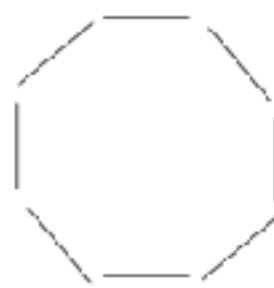
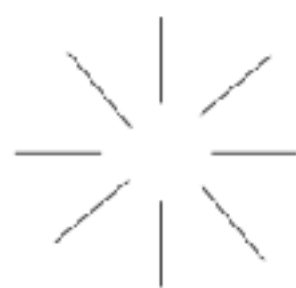
- **temperature** : limit by COBE, WMAP, Planck

$$\frac{\delta T}{T} = - \int_{t_{\text{dec}}}^{t_0} \dot{h}_{ij} n^i n^j dt$$

- **polarisation**: BB spectrum measured by BICEP2 and Planck generated at photon decoupling time, from Thomson scattering of electrons by a **quadrupole temperature anisotropy** in the photons

polarisation patterns

generated by  
primordial scalar  
and tensor  
perturbations



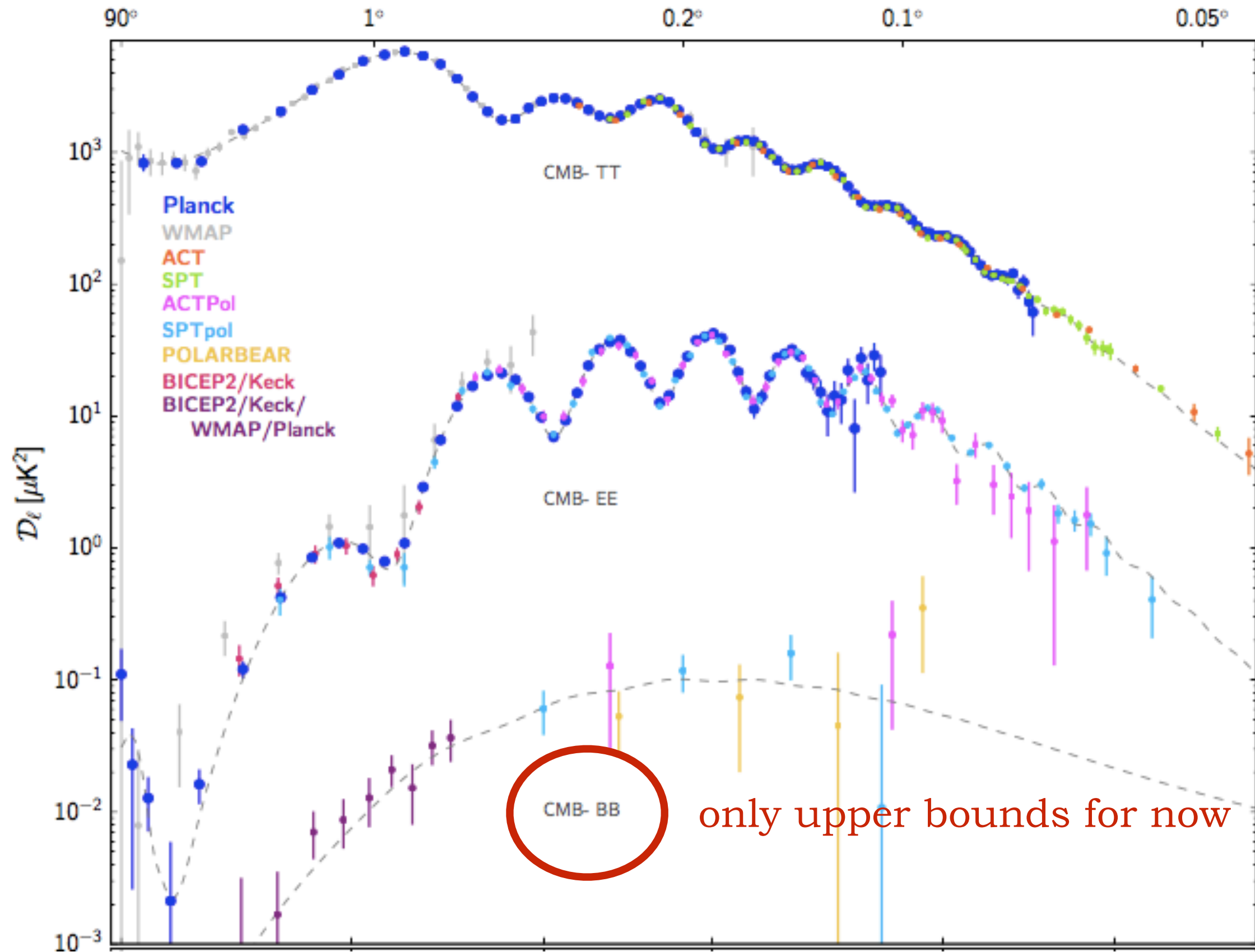
E mode



B mode

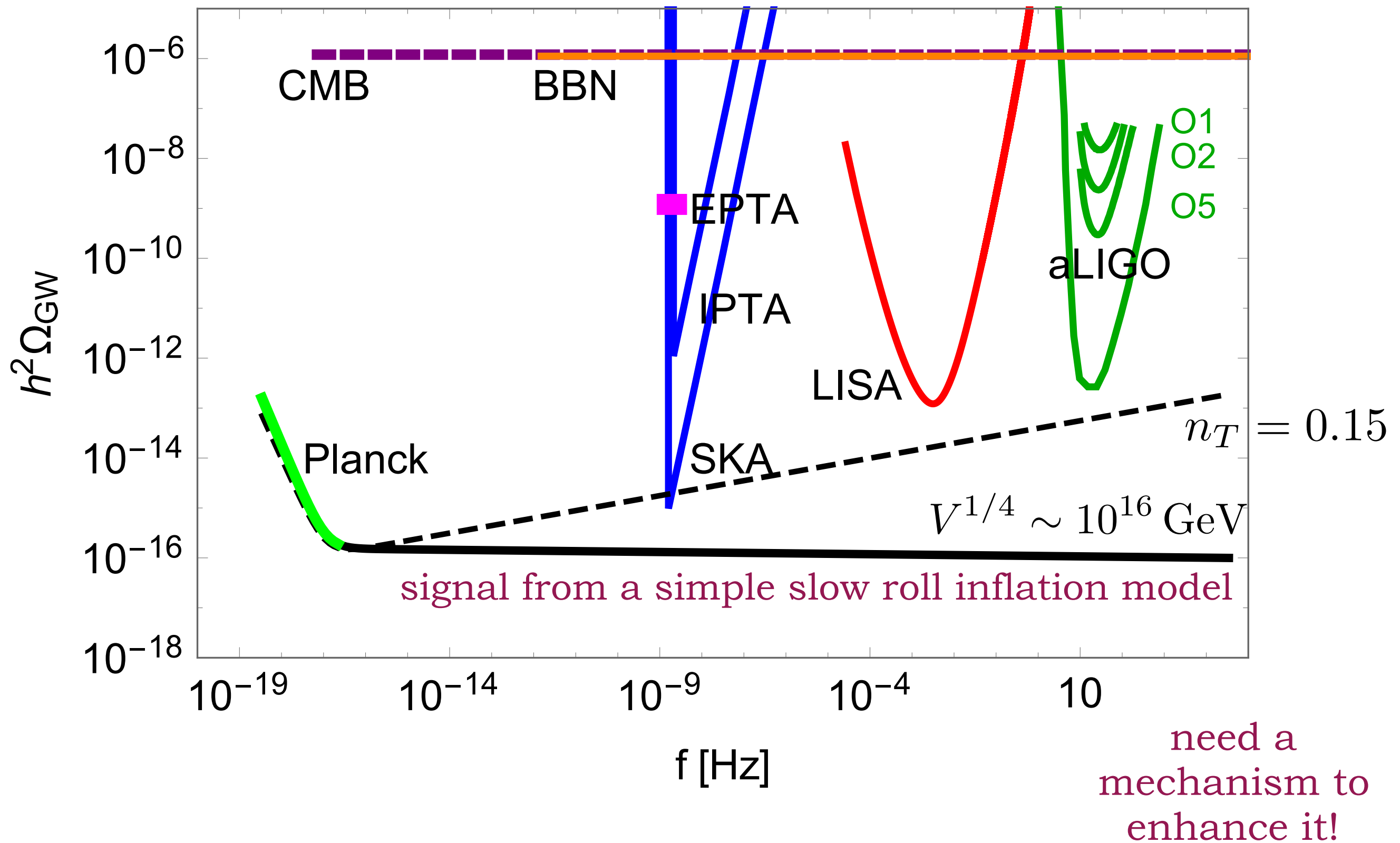
generated only by  
primordial tensor  
perturbations or  
by foregrounds

# GW influence CMB photons and leave an imprint in CMB anisotropies



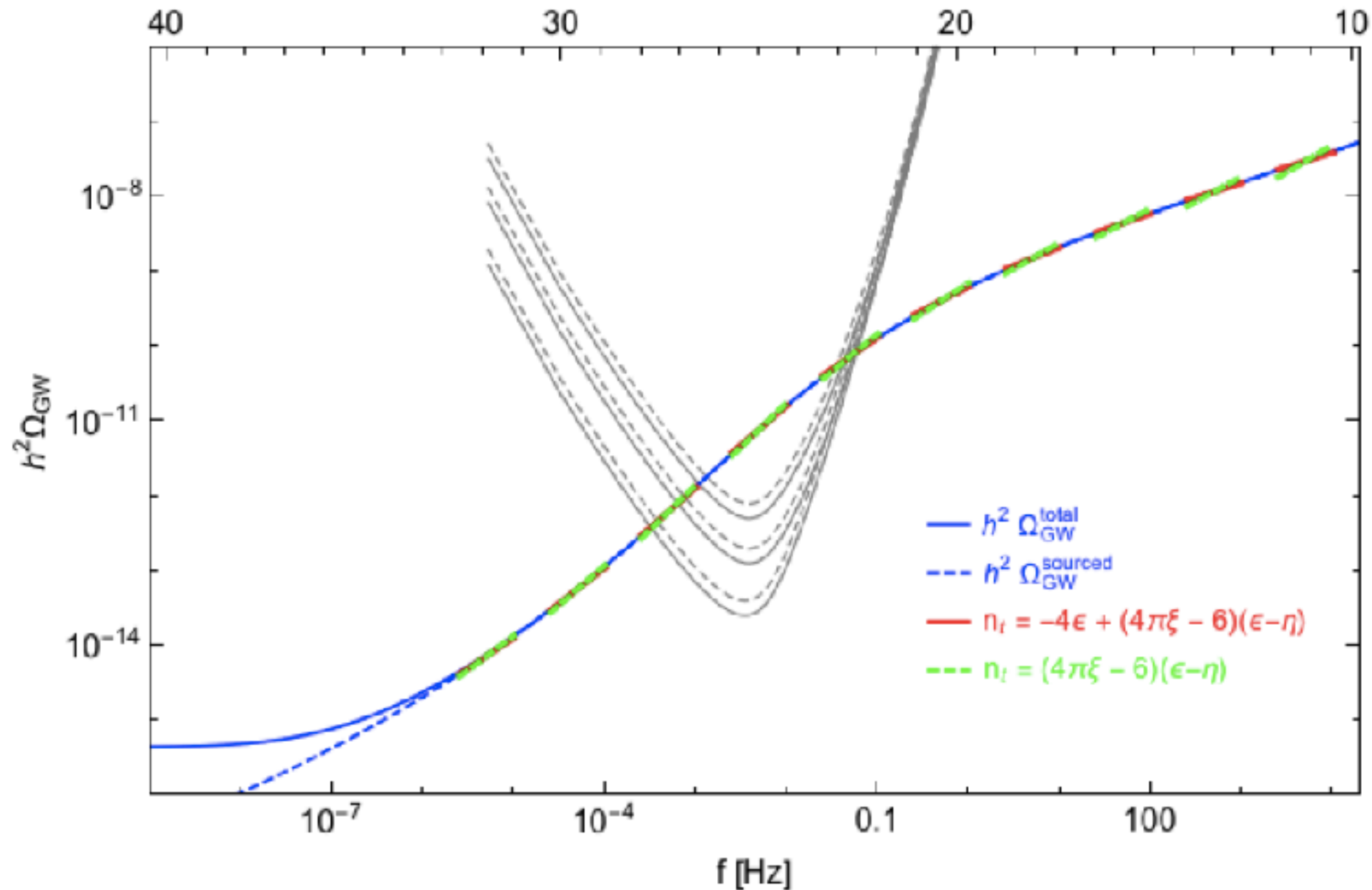


# SGWB from slow roll inflation



# just one example: inflaton-gauge field coupling

$$\Delta\mathcal{L} = -\frac{1}{4\Lambda}\phi F_{\mu\nu}\tilde{F}^{\mu\nu}$$



$$\Lambda = \frac{M_{Pl}}{35}$$

quadratic  
inflaton  
potential

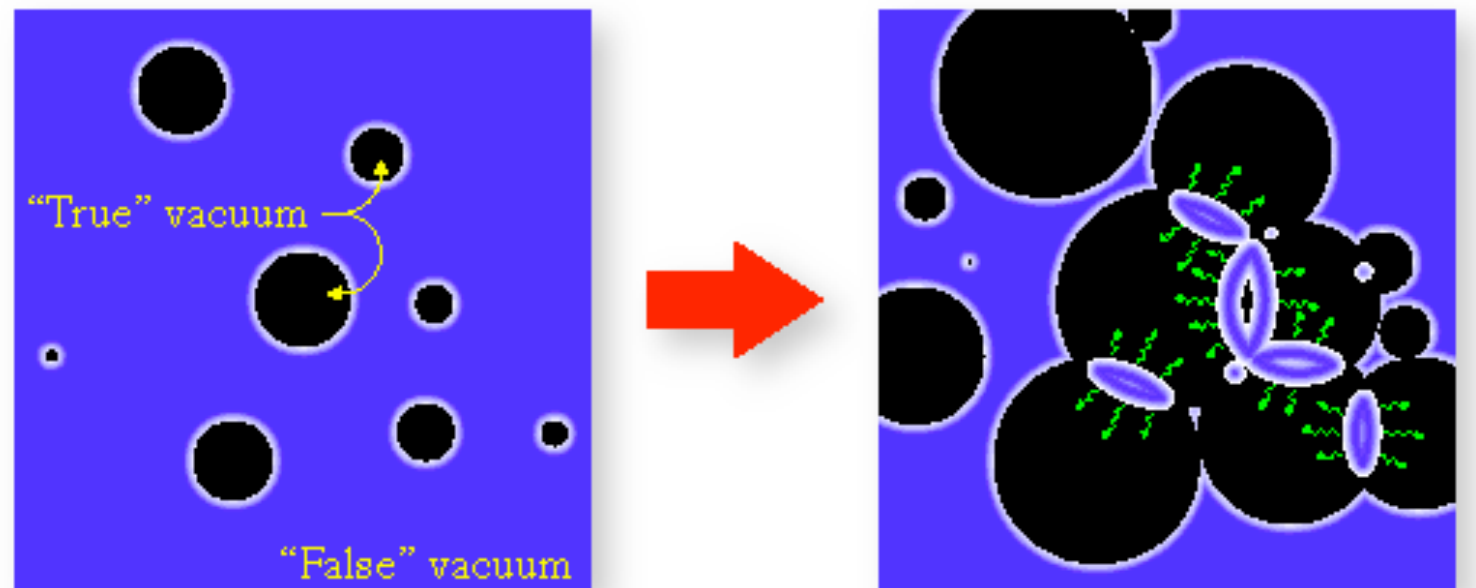
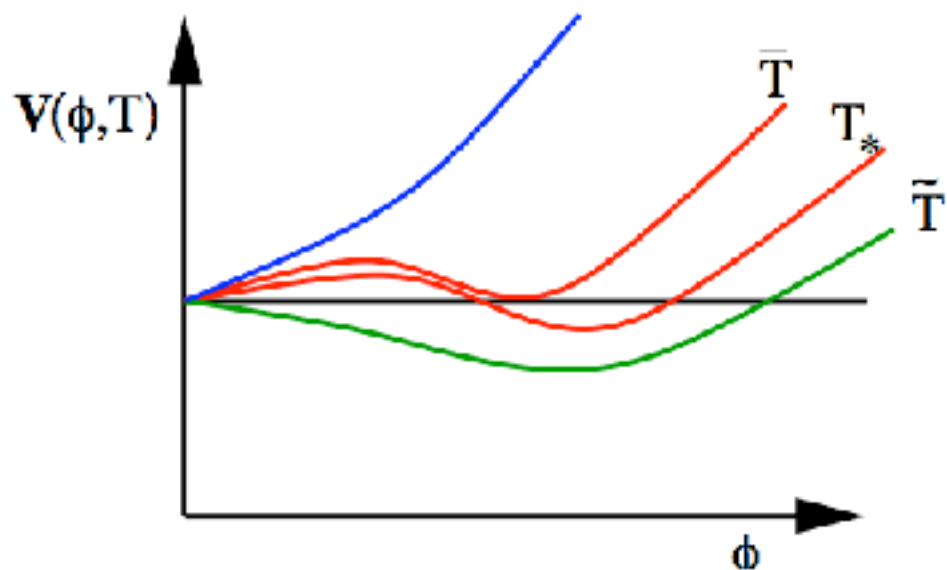
OTHER SIGNATURES:  
non-gaussianity, chirality

# SGWB from first order phase transitions

in the course of its adiabatic expansion, the universe might have undergone several PTs, maybe of first order

potential barrier separates true and false vacua

quantum tunneling across the barrier : nucleation of bubbles of true vacuum



- QCD and EWPT (beyond the standard paradigm)
- higher temperature PTs (extra dimensions, dark matter models...)

# SGWB from first order phase transitions

$$\ddot{h}_{ij} + 3H \dot{h}_{ij} + k^2 h_{ij} = 16\pi G \Pi_{ij}^{TT}$$

- collisions of bubble walls  $\Pi_{ij} \sim \partial_i \phi \partial_j \phi$
- sound waves and turbulence in the fluid  $\Pi_{ij} \sim \gamma^2 (\rho + p) v_i v_j$
- primordial magnetic fields (MHD turbulence)

$$\Pi_{ij} \sim (E^2 + B^2) \frac{\delta_{ij}}{3} - E_i E_j - B_i B_j$$



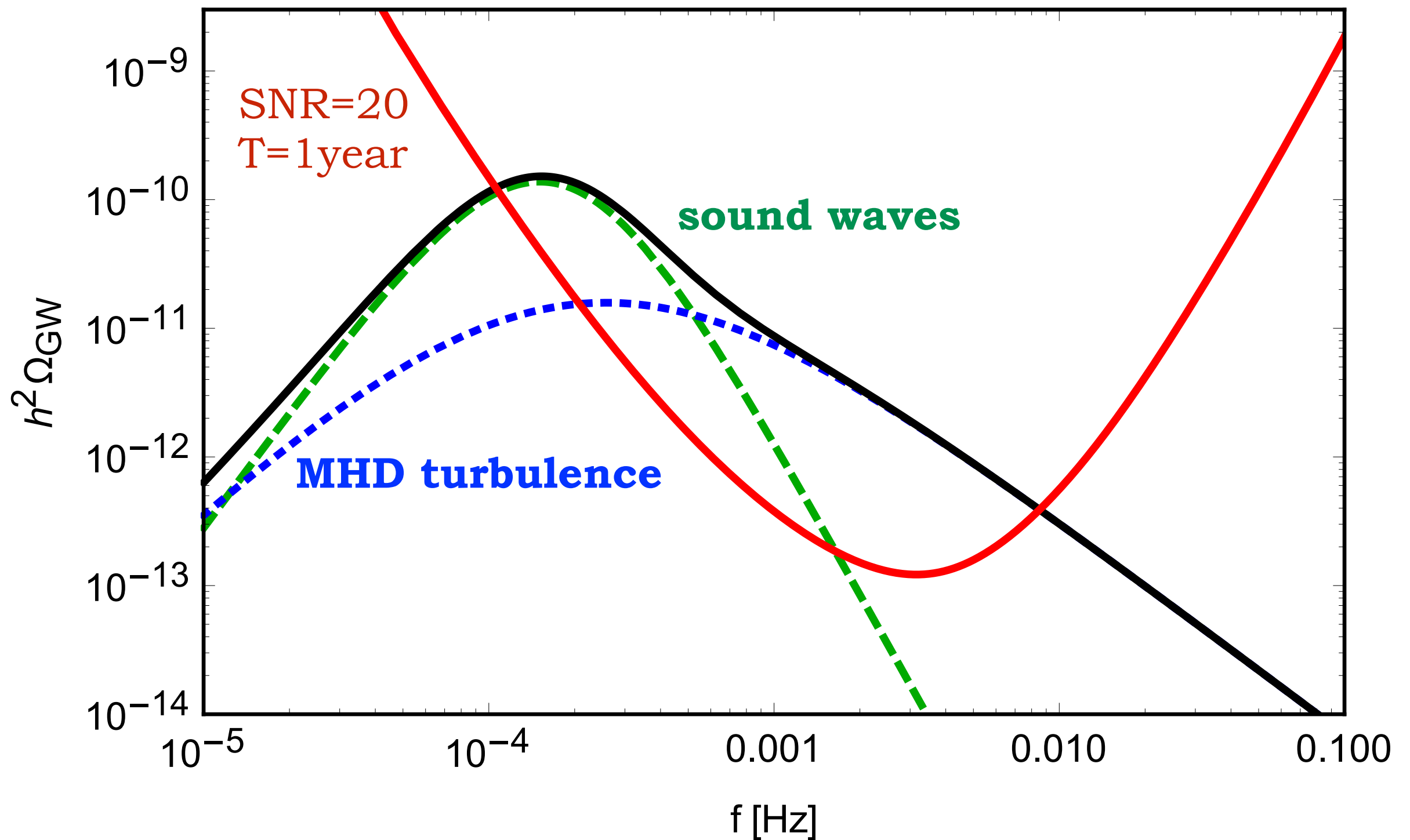
# SGWB from first order phase transitions

$$f_* = \frac{H(T_*)}{\epsilon_*}$$

- **LISA** (mHz) is sensitive to energy scales around the **TeV scale**, so it can probe the EWPT in BSM models and more exotic PTs beyond the EWPT  
  
connections with baryon asymmetry, dark matter : LISA as a probe of BSM physics, complementary to colliders
- **Pulsar Timing Array** (nanoHertz) can probe the **QCDPT scale** at 100 MeV
- **Earth-based detectors** (100 Hz) can probe more exotic PTs possibly occurring around  **$10^5$  GeV**

# Example of signal from FO EWPT in LISA

$$T_*=59.6 \text{ GeV}, \quad \alpha=0.17, \quad \beta/H_*=12.5$$



# Conclusions and open questions

- Cosmological backgrounds are an amazing source of information to understand the early universe and consequently fundamental physics
- the Cosmic Microwave Background is one of the neatest observational probes of the standard cosmological model
- the Stochastic GW Background has the same potential, carrying with itself information on high energy physics beyond the standard model of particle physics

# Conclusions and open questions (SGWB)

- how probable is the existence of early universe GW sources leading to an *observable* signal?
- both for the CMB and for the SGWB, the next observational challenge concerns the removal of “foregrounds”
  - there are expected SGWBs from binaries which are too numerous to be individually detected
  - there are all the other GW sources which need to be subtracted to high enough precision
- once a SGWB is detected, how can we ascertain its origin?
  - can we make sure it is cosmological (and not astrophysical, remember BICEP)?
  - can we understand from which process it comes from?
  - connected challenge: there is the need to make very precise predictions on the expected signals, and it is difficult since the generation mechanisms are based on BSM physics

# GW sources in the early universe : inflation-related

- irreducible SGWB from inflation
  - also sourced by second order scalar perturbations
- beyond the irreducible SGWB from inflation
  - particle production during inflation (scalar, gauge fields... coupled to the inflaton)
  - spectator fields
  - breaking symmetries (space-dependent inflaton, massive graviton...)
  - modified gravity during inflation (massive GWs with  $c \neq 1$ )
  - primordial black holes
- alternatives to inflation
  - pre big-bang, cyclic/ekpyrotic, string gas cosmology...
- preheating and non-perturbative phenomena
  - parametric amplification of bosons/fermions
  - symmetry breaking in hybrid inflation
  - decay of flat directions
  - oscillons



# GW sources in the early universe : phase transition-related

- first order phase transition
  - true vacuum bubble collision
  - sound waves
  - turbulence
- cosmic topological defects
  - irreducible SGWB from topological defect networks
  - decay of cosmic string loops