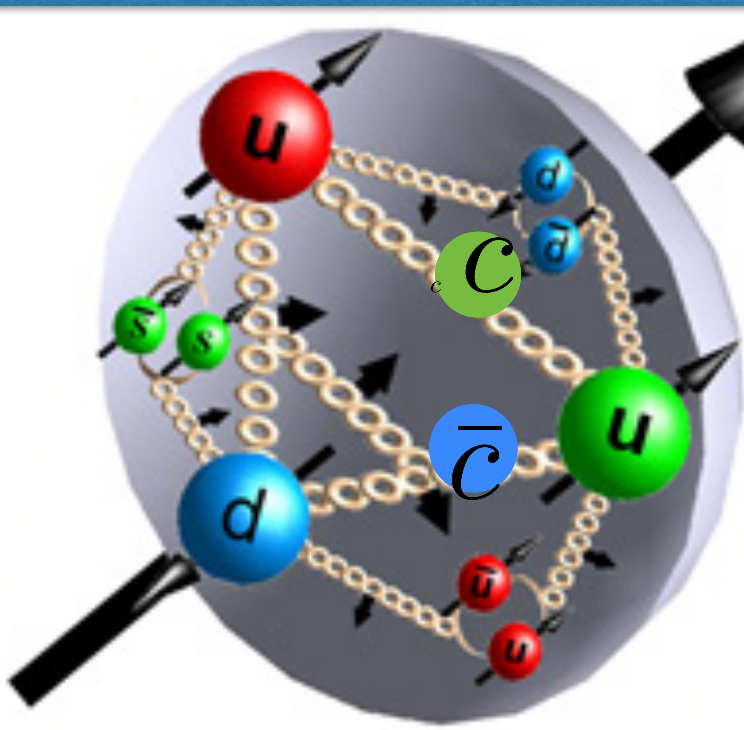
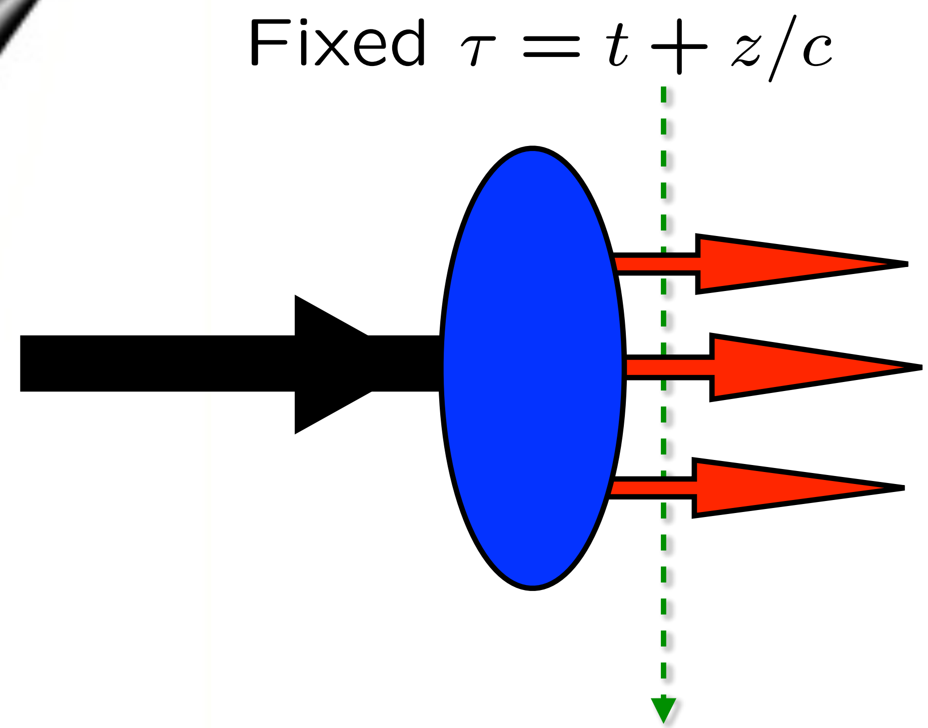
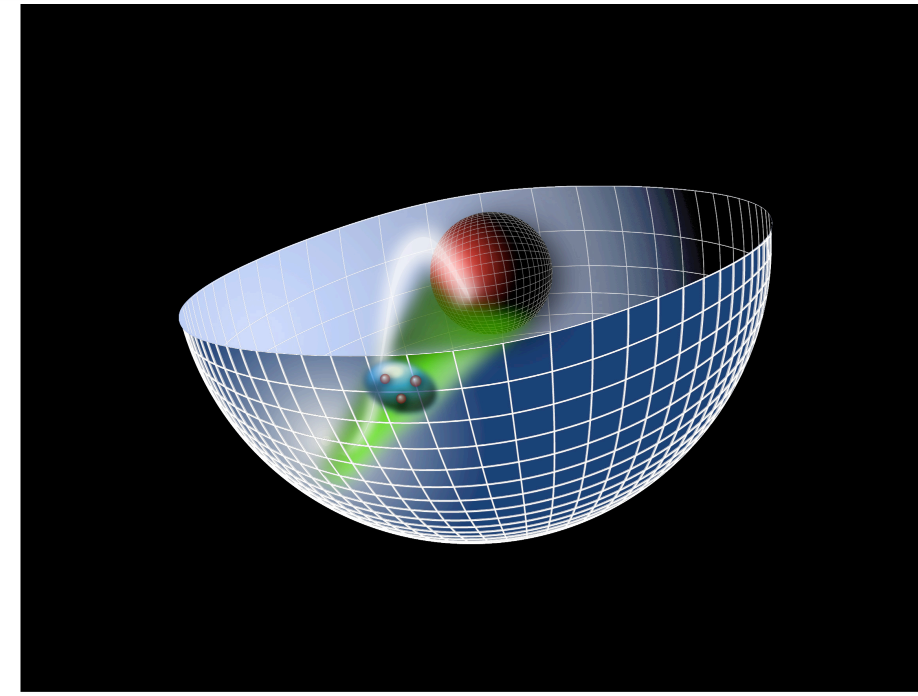
Supersymmetric Properties of Hadron Spectroscopy and Predictions for Hadron Dynamics from Light-Front Holography and Superconformal Algebra







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

with Guy de Tèramond, Hans Günter Dosch, and Alexandre Deur

GGI School | Frontiers in Nuclear and Hadronic Physics | February 25 - March 8, 2019

Need a First Approximation to QCD

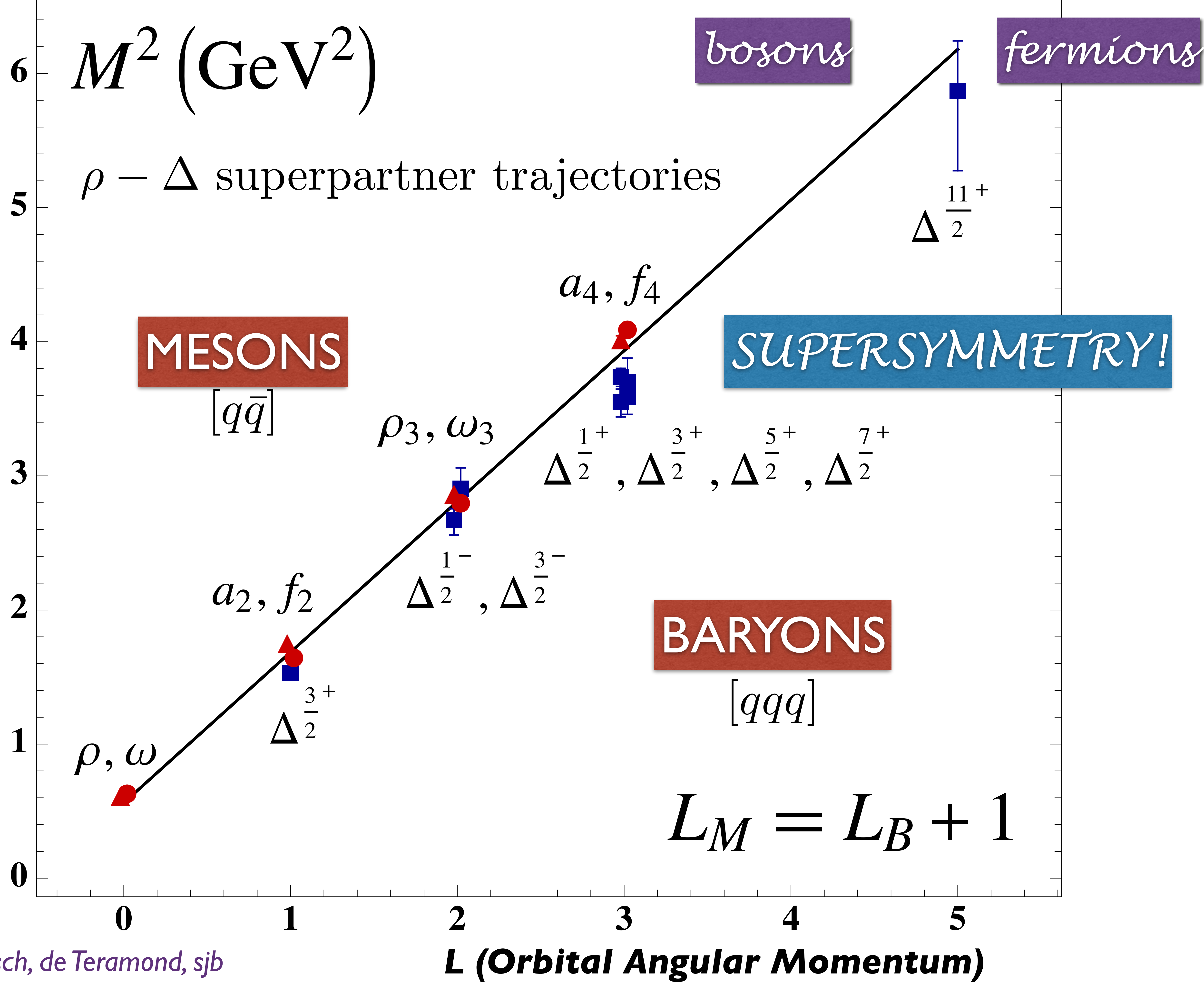
Comparable in simplicity to Schrödinger Theory in Atomic Physics

Relativistic, Frame-Independent, Color-Confining

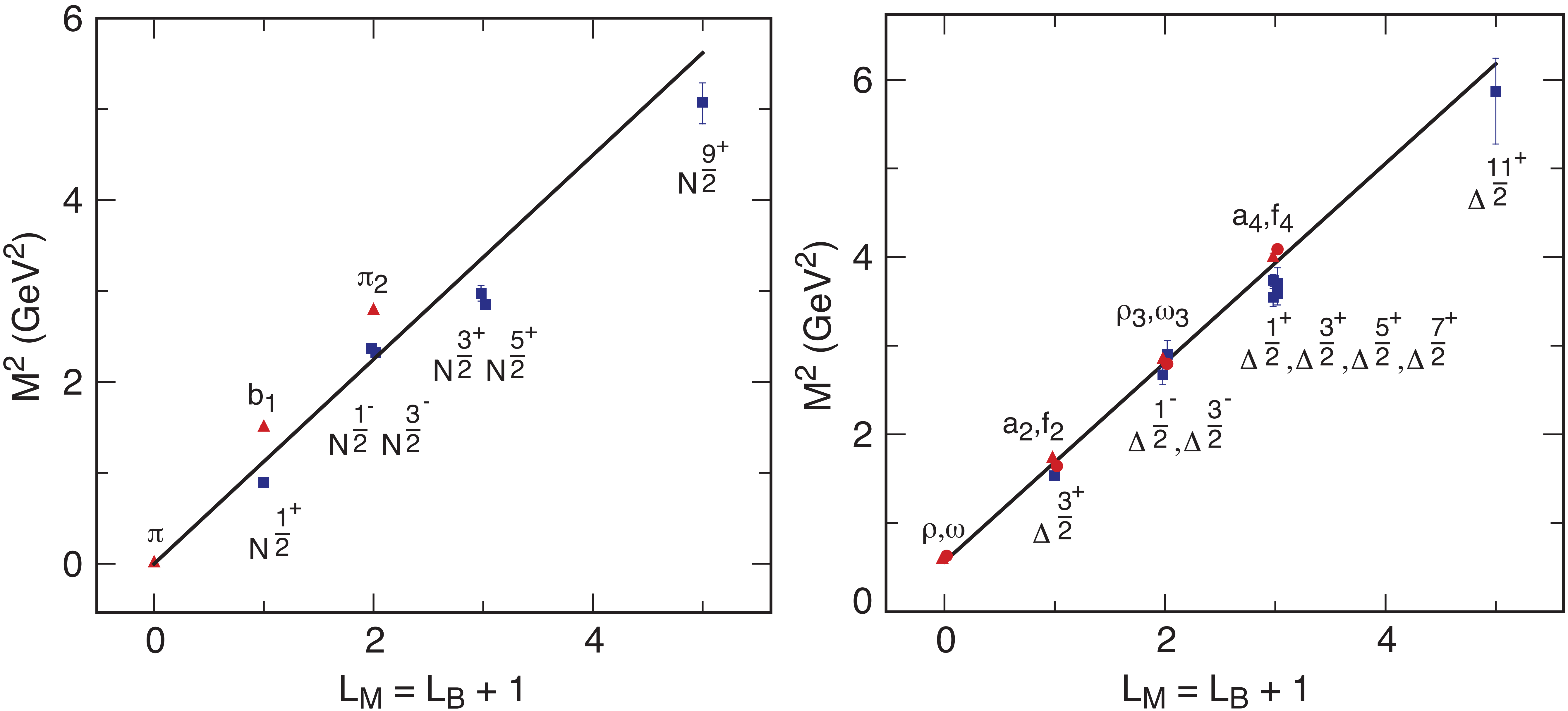
Origin of hadronic mass scale

AdS/QCD
Light-Front Holography
Superconformal Algebra

Spectroscopy and Dynamics



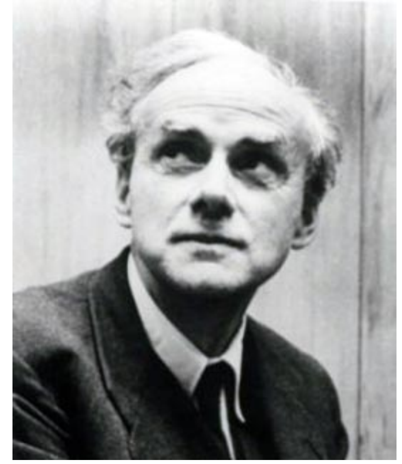
Dosch, de Teramond, sjb



Superconformal meson-nucleon partners

de Tèramond, Dosch, Lorce, sjb

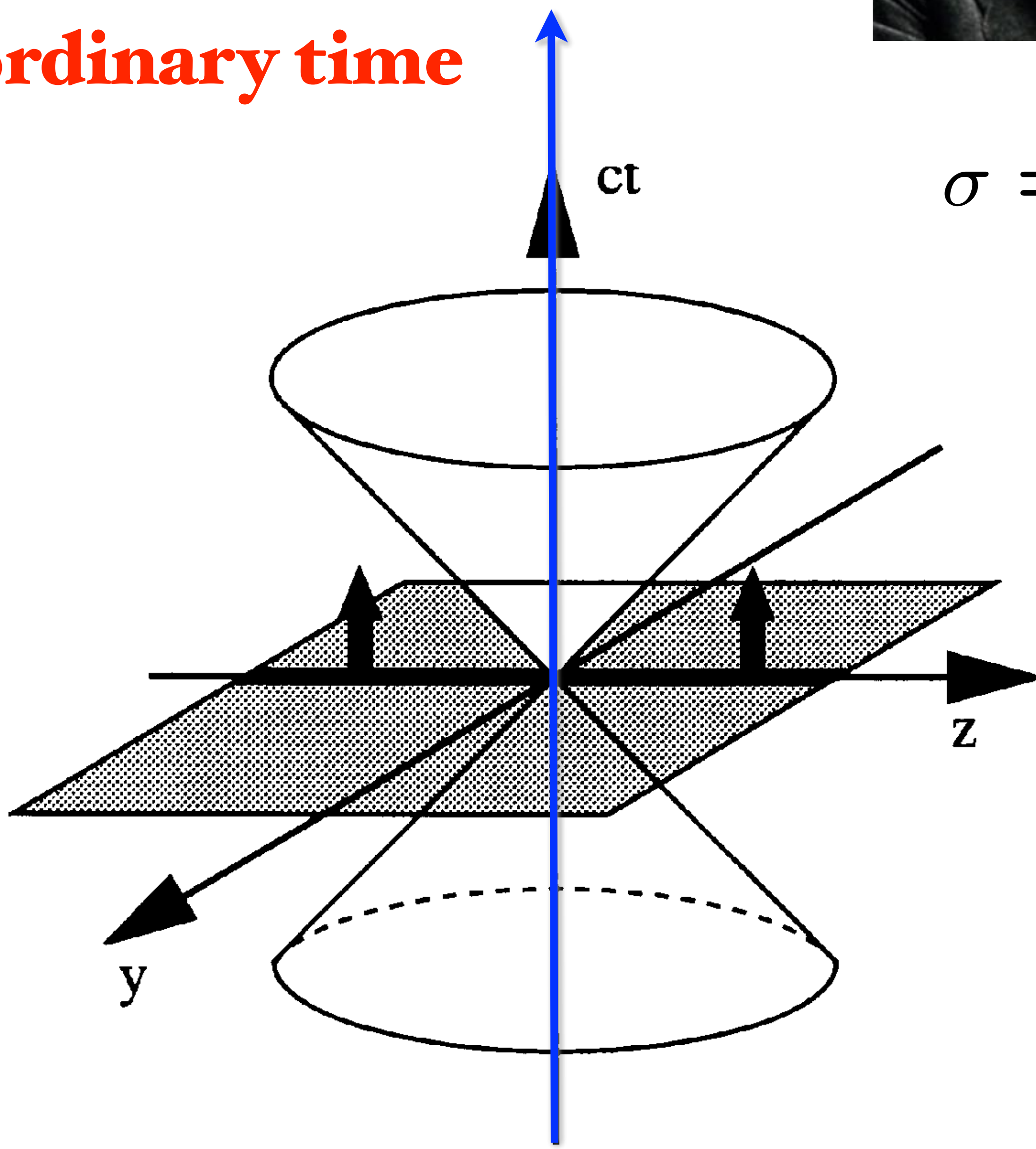
Dirac's Amazing Idea:
The "Front Form"



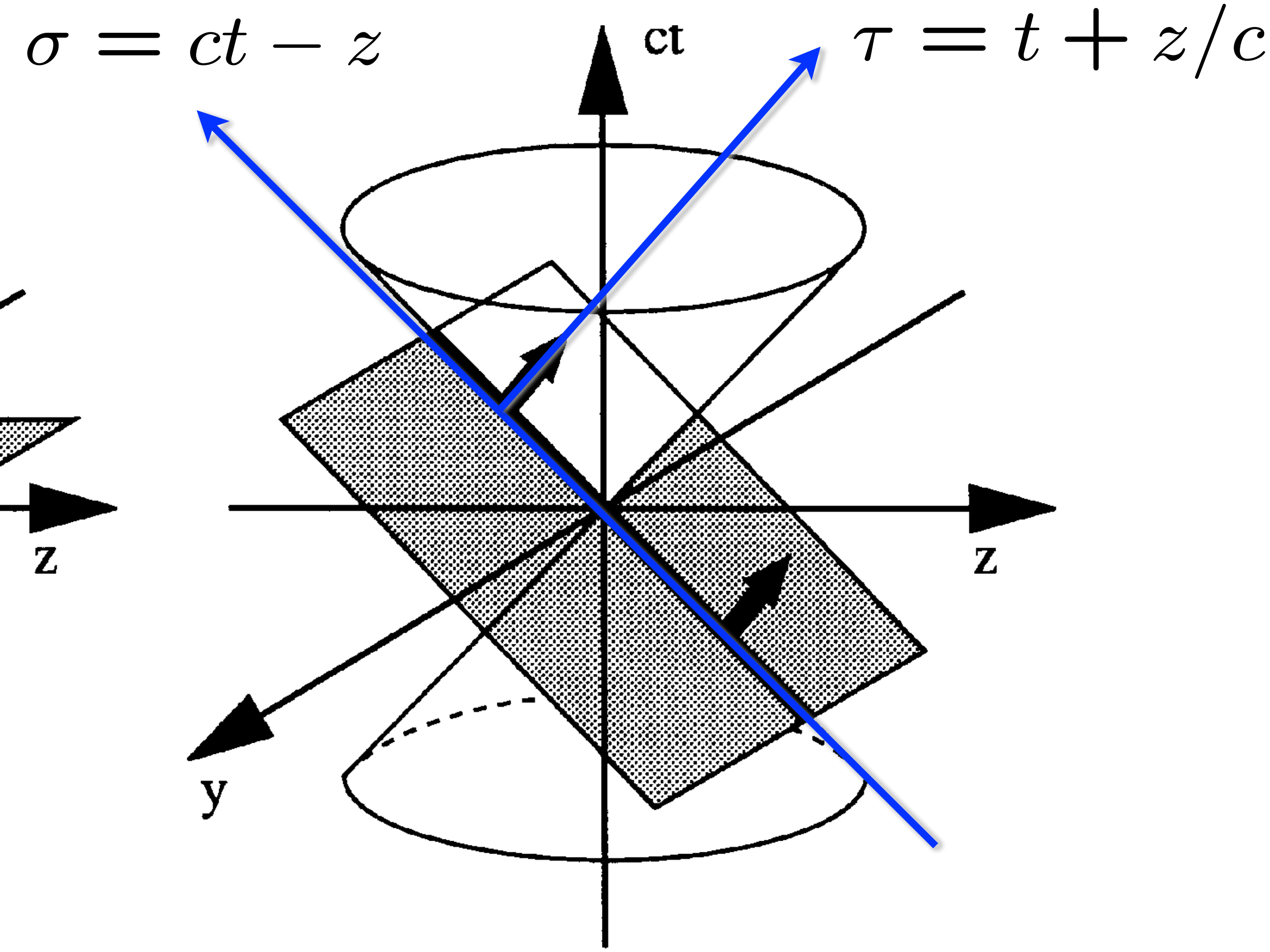
P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time

Evolve in light-front time!



Instant Form



Front Form

Causal, Boost Invariant!

- *Satisfies Poincarè Invariance*

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

$$P^{\pm} = P^0 \pm P^z$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

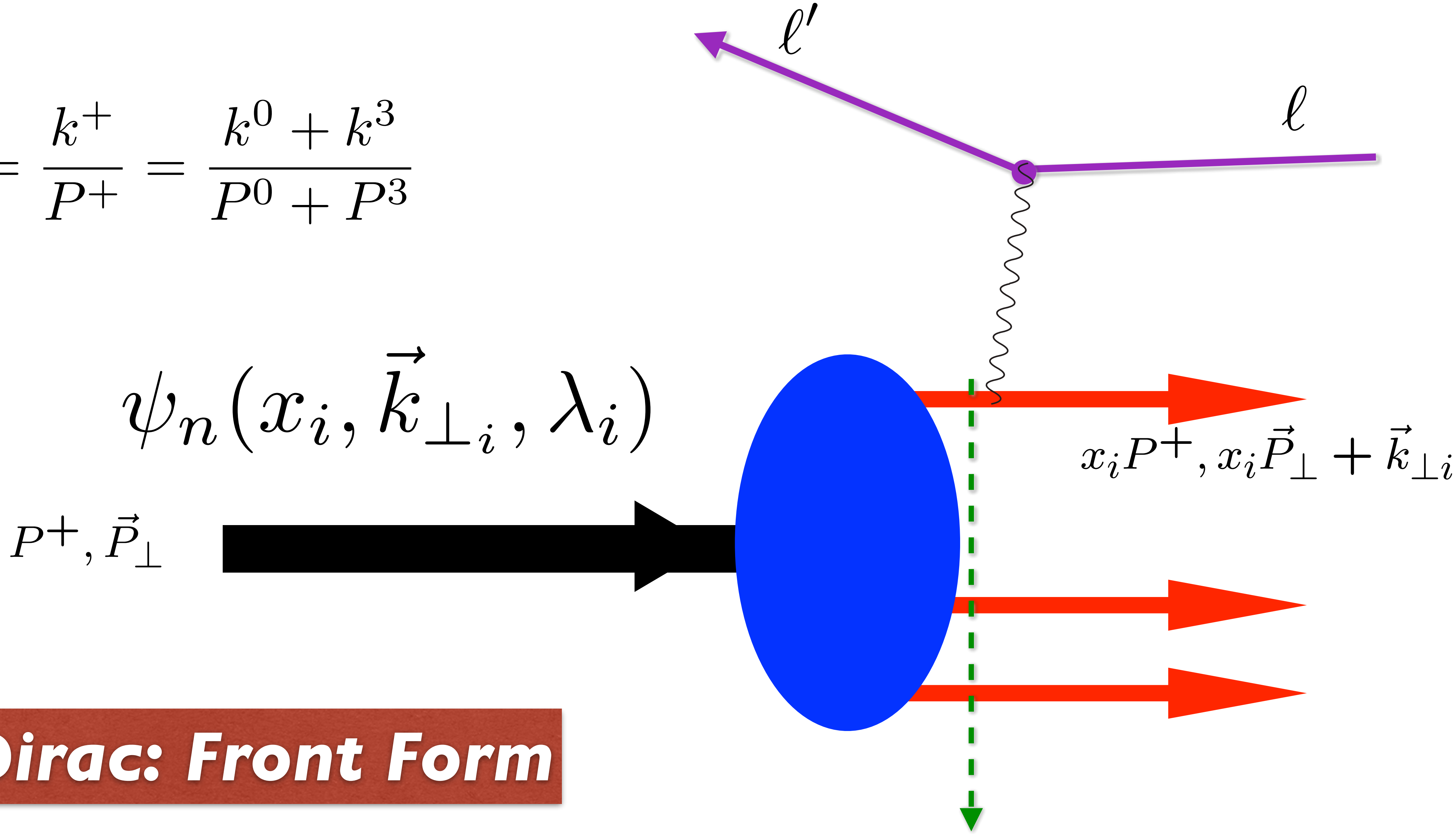
Eigenvalue $P^- = \frac{\mathcal{M}^2+\vec{P}^2_\perp}{P^+}$

$$H_{LF} = P^+P^- - \vec{P}^2_\perp$$

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$



Dirac: Front Form

Measurements of hadron LF wavefunction are at fixed LF time

Fixed $\tau = t + z/c$

Like a flash photograph

$$x_{bj} = x = \frac{k^+}{P^+}$$

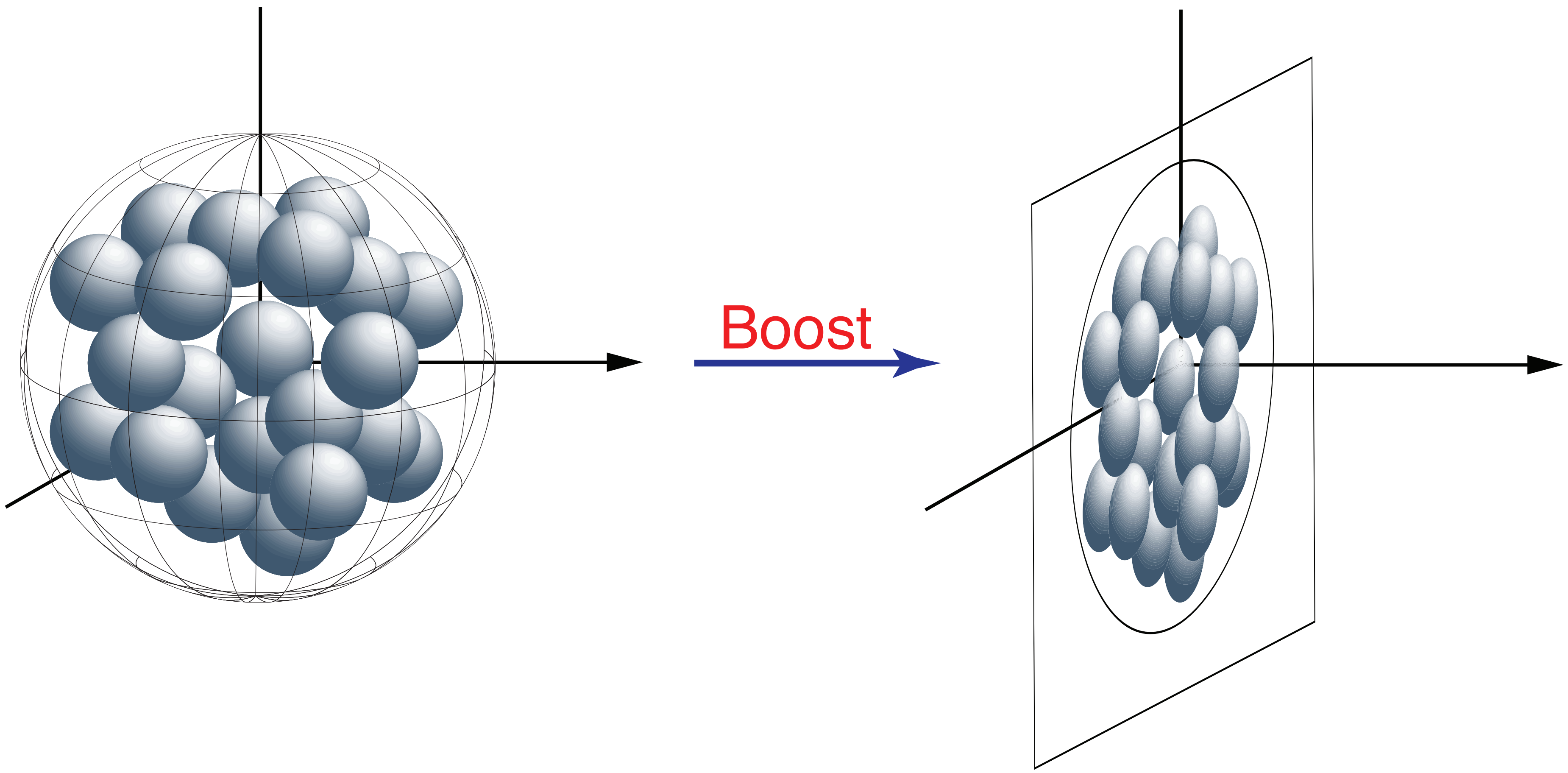
Invariant under boosts! Independent of P^μ

Advantages of the Dirac's Front Form for Hadron Physics
Poincare' Invariant

Physics Independent of Observer's Motion

- **Measurements are made at fixed τ**
- **Causality is automatic**
- **Structure Functions are squares of LFWFs**
- **Form Factors are overlap of LFWFs**
- **LFWFs are frame-independent: no boosts, no pancakes!**
- **Same structure function measured at an e p collider and the proton rest frame**
- **No dependence of hadron structure on observer's frame**
- **LF Holography: Dual to AdS space**
- **LF Vacuum trivial -- no vacuum condensates!**
- **Profound implications for Cosmological Constant**

Terrell, Penrose



A large nucleus before and after an ultra-relativistic boost.

Is this really true? Will an electron-proton collider see different results than a fixed target experiment such as SLAC because the nucleus is squashed to a pancake?

No length contraction — no pancakes!

Penrose
Terrell
Weiskopf

We do not observe the nucleus at one time t!

Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

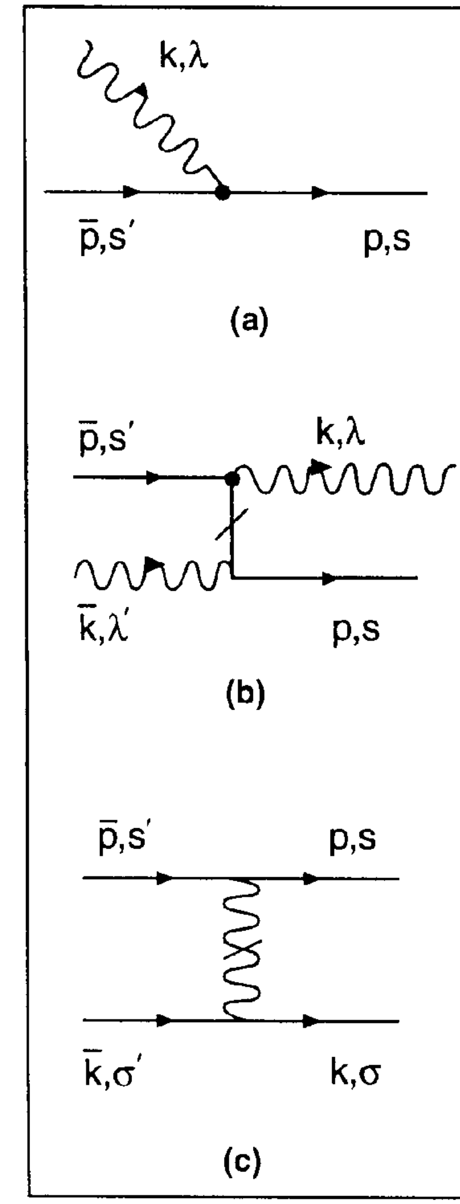
H^{int}_{LF}: Matrix in Fock Space

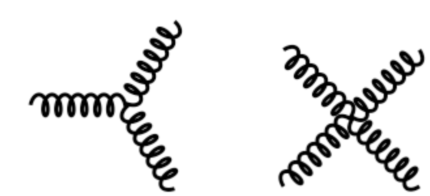
$$\boxed{H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>}$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions

LFWFs: Off-shell in P- and invariant mass





H^{int}_{LF}

Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + \cdots$$

- *"History": Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!*

- *Wick Theorem applies, but few amplitudes since all $k^+ > 0$.*

- *J_z Conservation at every vertex* $\boxed{\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n}$ *at order g^n*

  K. Chiu, sjb

- *Unitarity is explicit*

- *Loop Integrals are 3-dimensional* $\int_0^1 dx \int d^2 k_\perp$

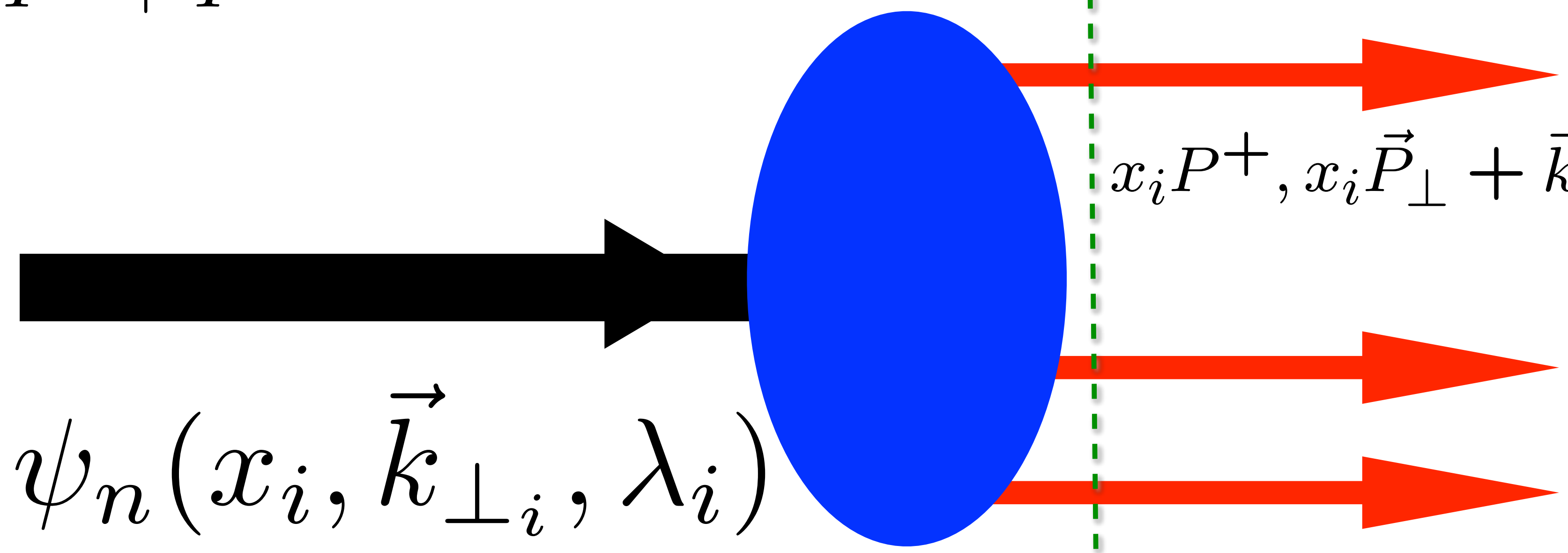
- *hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian $\boxed{H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>}$

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\int \psi_{BS}(p,k)dk^- \to \psi_{LF}$$

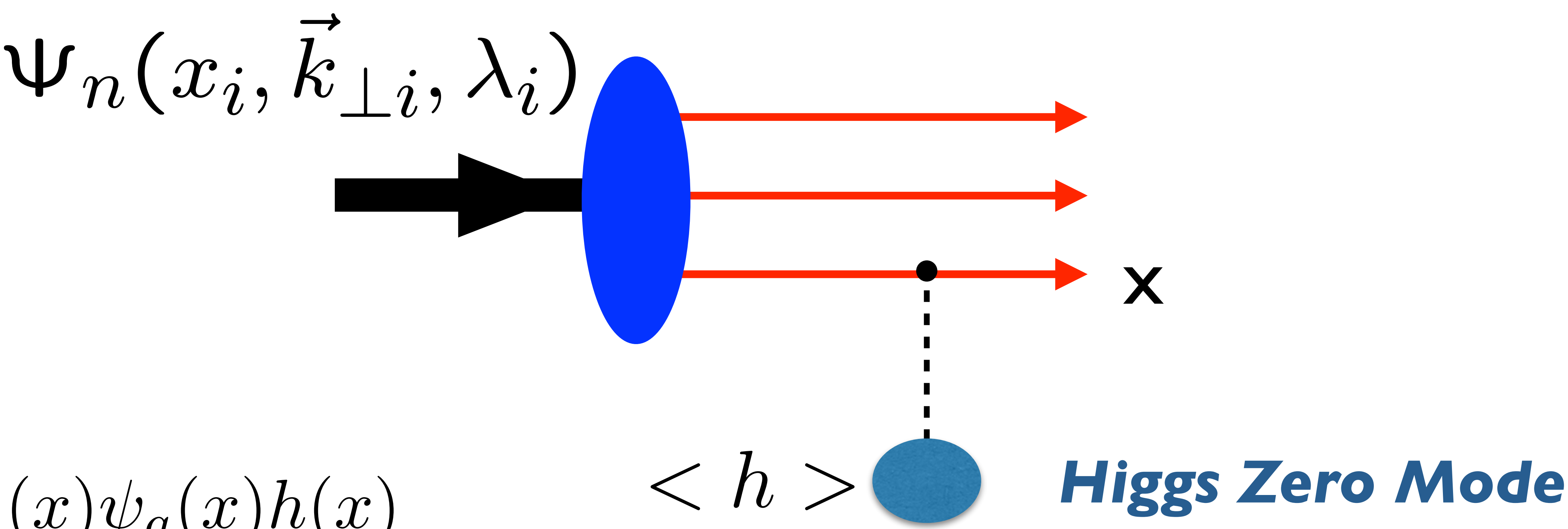
$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

Invariant under boosts! Independent of P^μ

Off-shell in P- and invariant mass

- *Poincarè Invariance*

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS



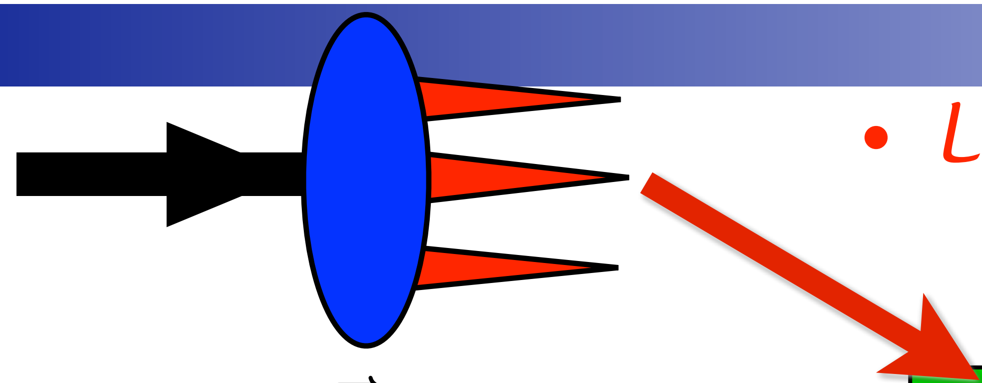
$$g_q \bar{\psi}_q(x) \psi_q(x) h(x)$$

Yukawa Higgs coupling of confined quark to Higgs zero mode gives

$$\bar{u}u \; g_q < h > = \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

$$H_{LF} = \sum_q \frac{k_{\perp q}^2 + m_q^2}{x_q}$$

- *Light Front Wavefunctions:*



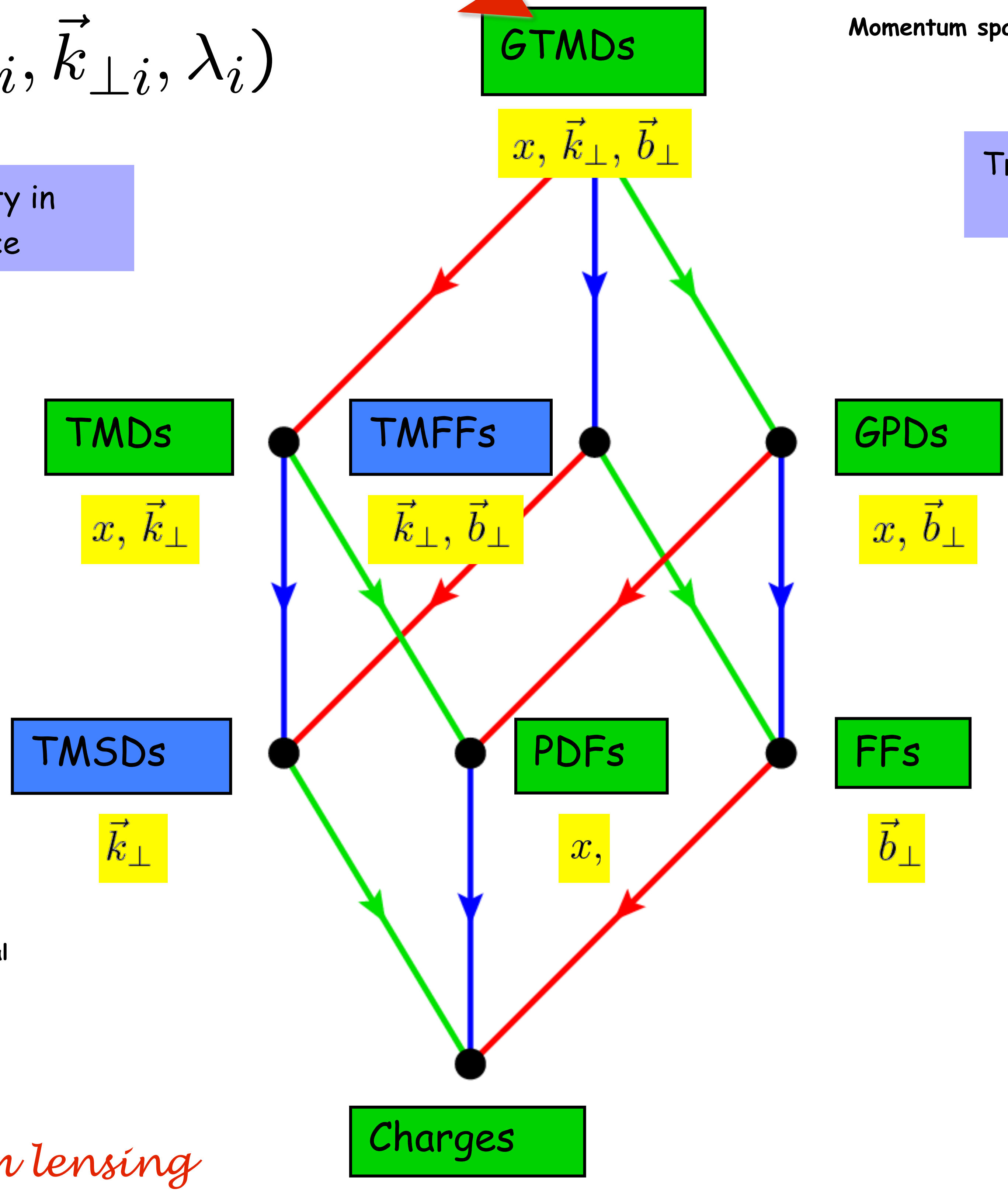
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Transverse density in momentum space

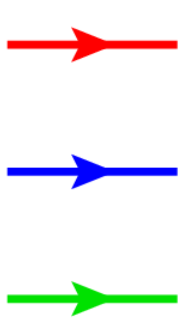
Momentum space $\quad \vec{k}_\perp \leftrightarrow \vec{z}_\perp \quad$ Position space

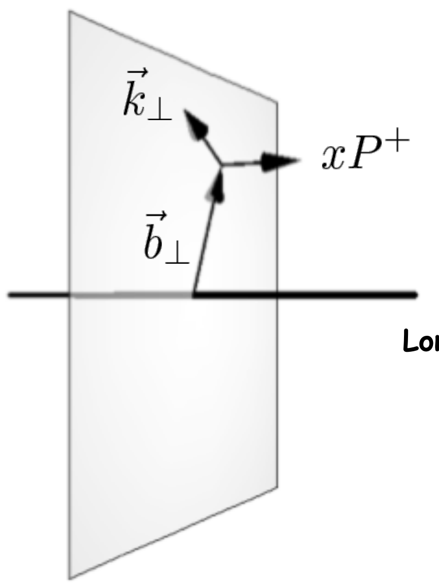
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

Transverse density in position space



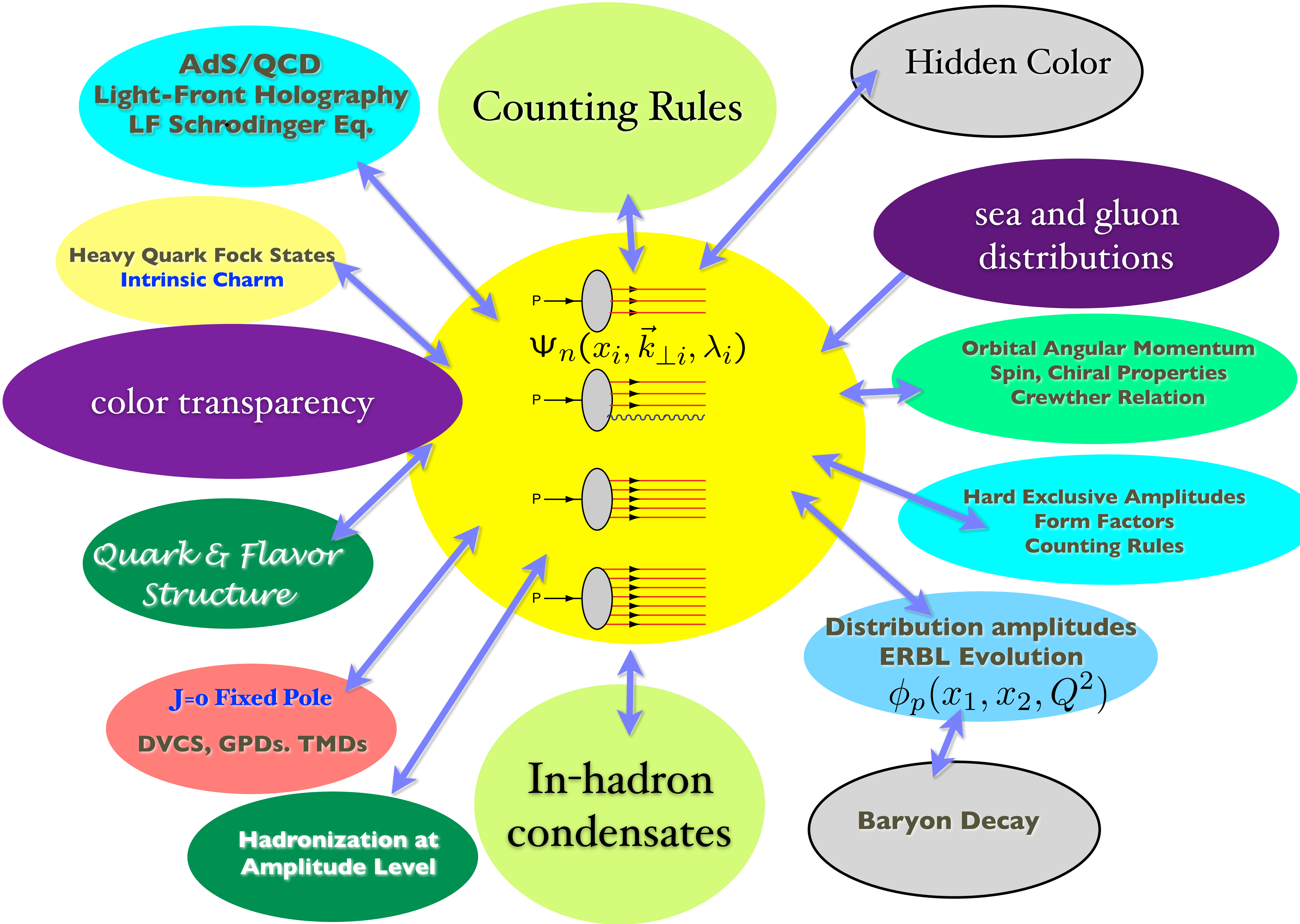
Lorce, Pasquini



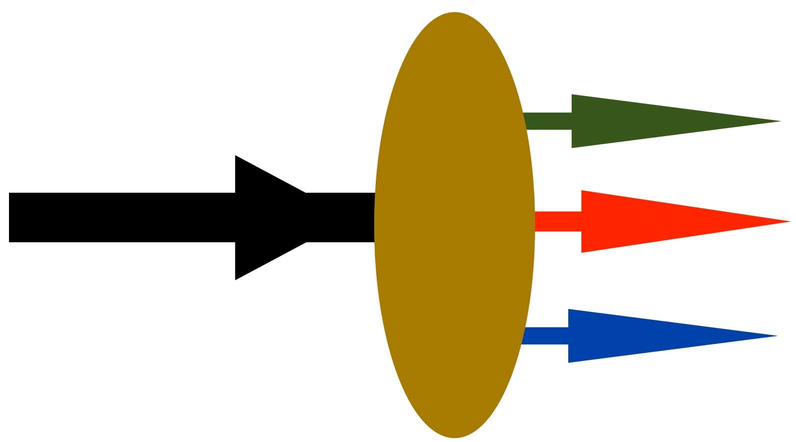


Sivers, T-odd from lensing

QCD and the LF Hadron Wavefunctions

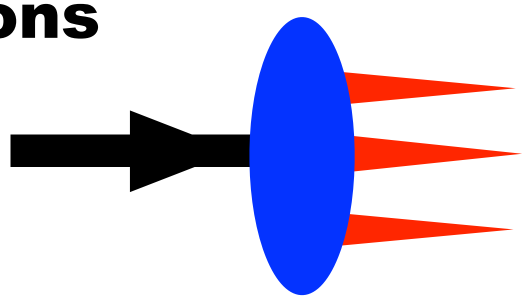


- *Hadron Physics without LFWFs is like Biology without DNA!*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LF wavefunctions play the role of Schrödinger wavefunctions in Atomic Physics**



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

- **LFWFs=Hadron Eigensolutions: Direct Connection to QCD Lagrangian**
- **Relativistic, frame-independent: no boosts, no disc contraction, Melosh built into LF spinors**
- **Hadronic observables computed from LFWFs: Form factors, Structure Functions, Distribution Amplitudes, GPDs, TMDs, Weak Decays, ... modulo `lensing' from ISIs, FSIs**
- **Cannot compute current matrix elements using instant form from eigensolutions alone -- need to include vacuum currents!**
- **Hadron Physics without LFWFs is like Biology without DNA!**

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Front Form

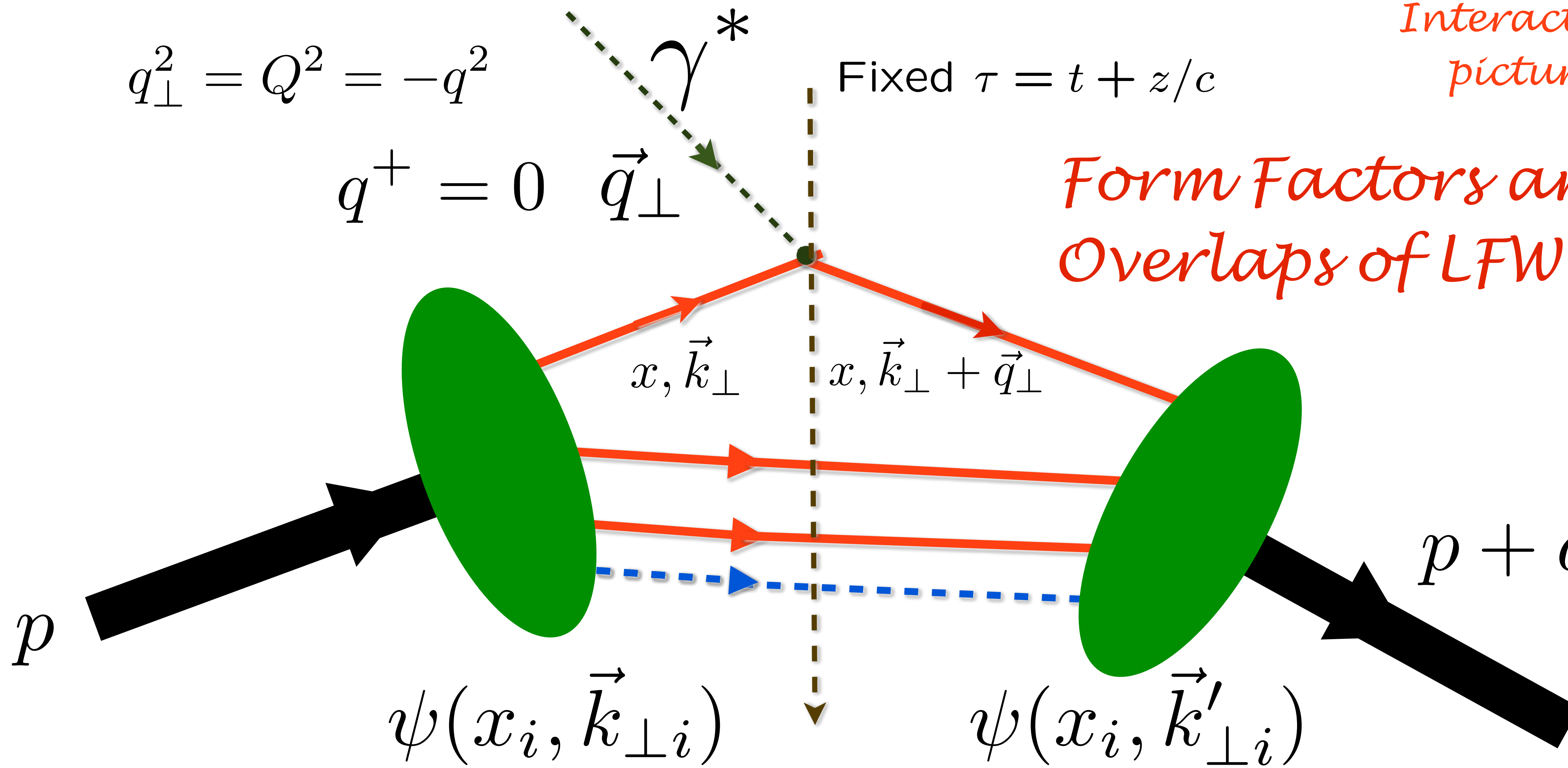
Interaction picture

$$q_\perp^2 = Q^2 = -q^2$$

$$q^+ = 0 \quad \vec{q}_\perp$$

Fixed $\tau = t + z/c$

Form Factors are Overlaps of LFWFs



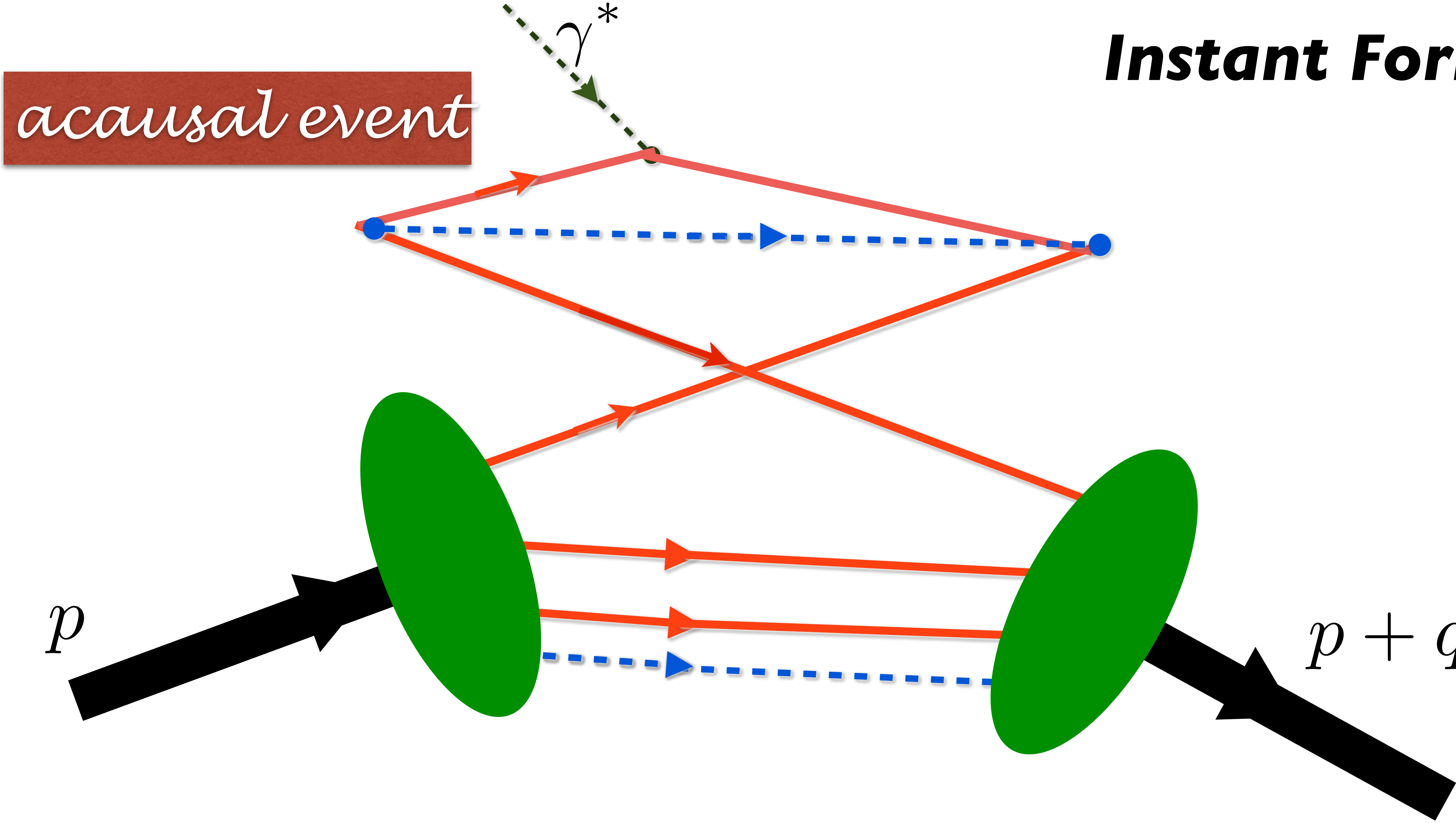
struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Drell, sjb

Instant Form



Must include vacuum-induced currents to compute form factors and other current matrix elements in instant form

Boosts are dynamical in instant form

Exact LF Formula for Pauli Form Factor

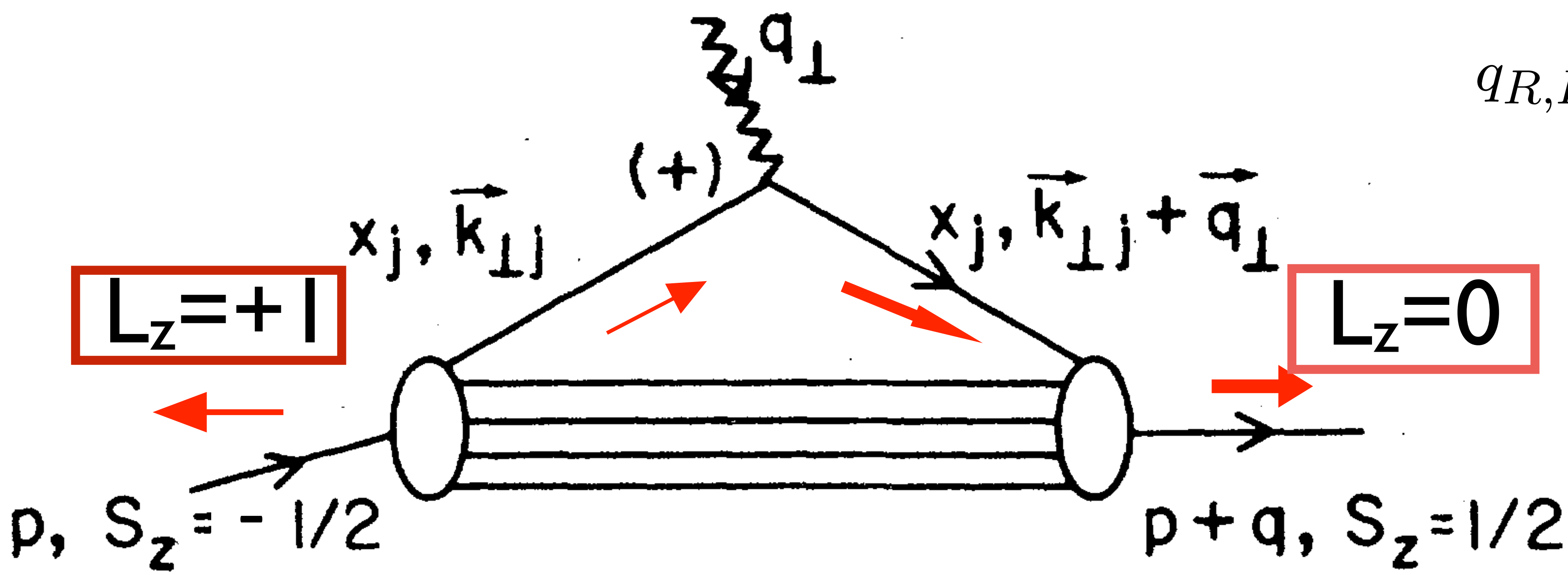
$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

Drell, sjb

$$\Big[-\frac{1}{q^L} \psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R} \psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i) \, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \Big]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

Light Front J^3 Conservation for Composite Systems

K. Chiu
sjb

single particle

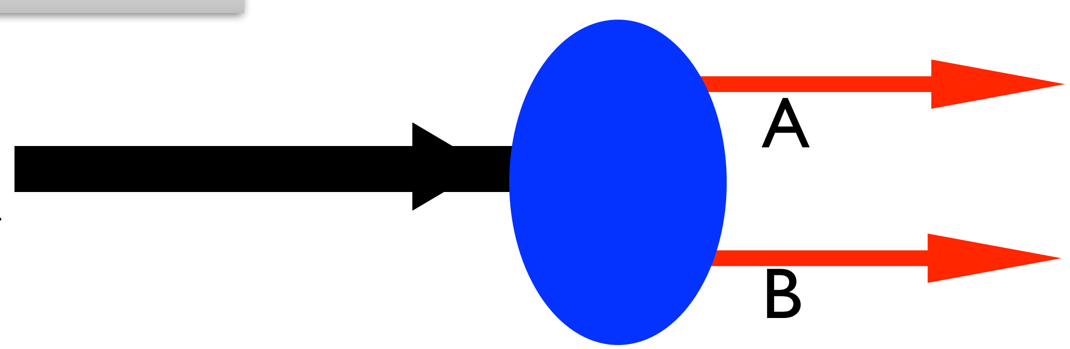
$$J^3\,|\vec{p},\lambda\rangle = \lambda\,|\vec{p},\lambda\rangle + i(p^2\frac{\partial}{\partial p_1} - p^1\frac{\partial}{\partial p_2})\,|\vec{p},\lambda\rangle$$

spin label

orbital angular momentum wrt fixed center

$$xp^+, x\vec{p}_\perp + \vec{k}_\perp$$

$$p^+, \vec{p}_\perp$$



$$(1-x)p^+, (1-x)\vec{p}_\perp - \vec{k}_\perp$$

$$J^3\,|p_A,\lambda_A\rangle \otimes |p_B,\lambda_B\rangle = (\lambda_A + \lambda_B + L^3(p_A) + L^3(p_B)\,|p_A,\lambda_A\rangle \otimes |p_B,\lambda_B\rangle\,,$$

constituent spins **constituent orbital**

$$\begin{aligned}&[L^3(p_A) + L^3(p_B))\\ &= [i(p^2\frac{\partial}{\partial p_1} - p^1\frac{\partial}{\partial p_2}) + i(k^2\frac{\partial}{\partial k_1} - k^1\frac{\partial}{\partial k_2})]\\ &= (L^3_o + L^3_i)\end{aligned}$$

L^3_o : Orbital Angular Momentum of the Bound State relative to a Fixed Center

L^3_i : Internal Orbital Angular Momentum of Constituent i

Composite System

$$(\lambda + L^3_o)\,|\vec{p},\lambda\rangle = (\lambda_A + \lambda_B + L^3_i + L^3_o)\,|p_A,\lambda_A\rangle \otimes |p_B,\lambda_B\rangle$$

$$\lambda = \sum_{i=1}^{n}\lambda_i + \sum_{i=1}^{n-1} l_{zi},$$

λ= sum of n internal spins plus n-1 internal orbital angular momenta

$$\boxed{H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>}$$

$$|p,S_z> = \sum_{n=3} \Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)|n;\vec{k}_{\perp_i},\lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i,\vec{k}_{\perp i},\lambda_i)$$

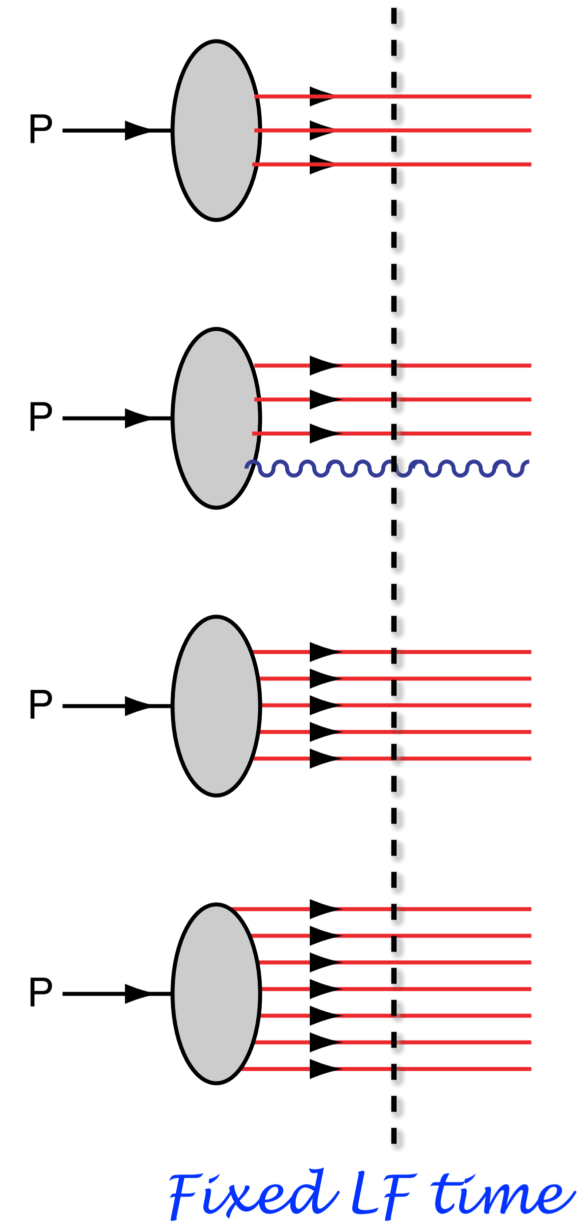
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fractions

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+,\ \sum_i^n x_i = 1,\ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



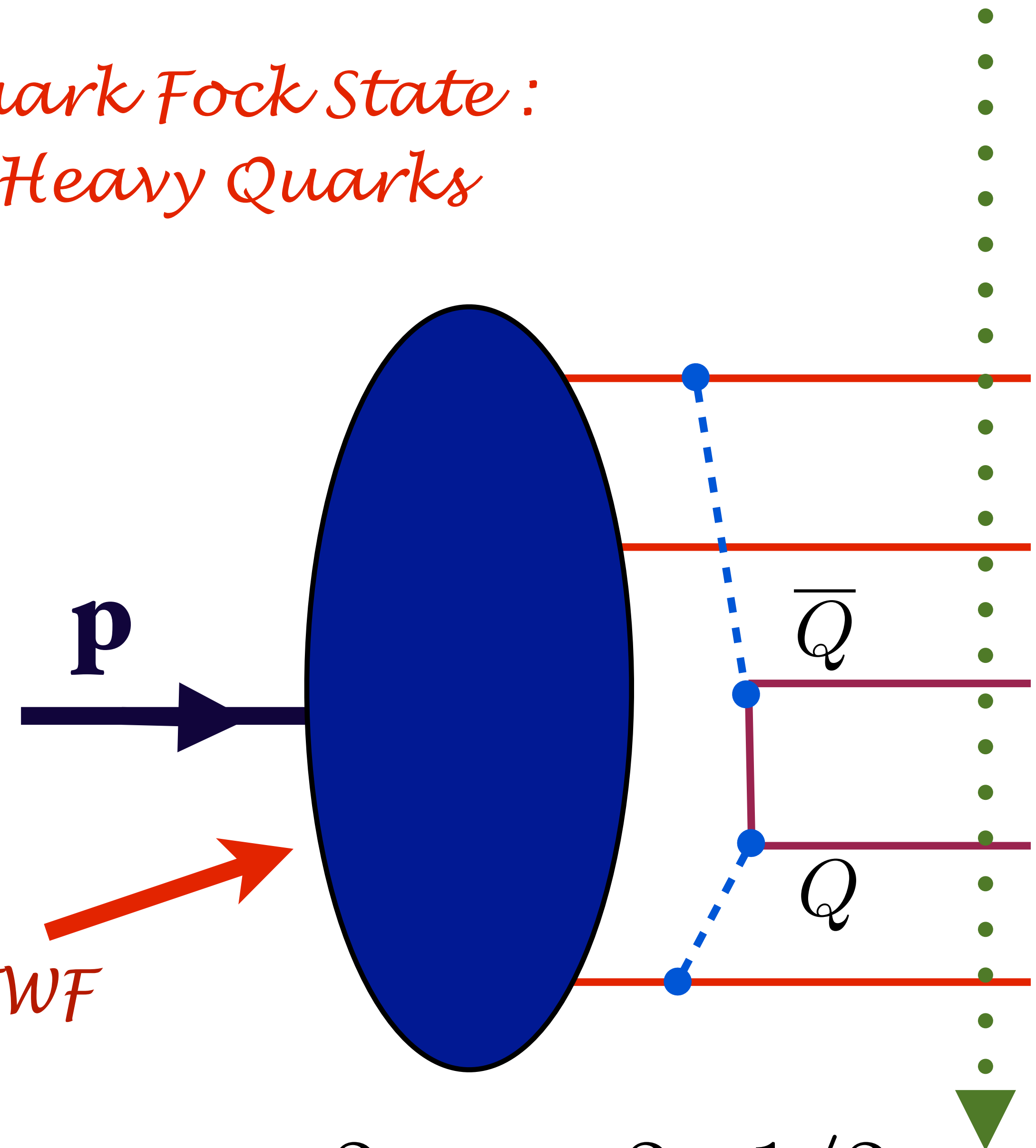
Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$$\bar{s}(x) \neq s(x)$$
$$\bar{u}(x) \neq \bar{d}(x)$$

Hidden Color

Fixed LF time

Proton 5-quark Fock State :
Intrinsic Heavy Quarks



Use AdS/QCD LFWF

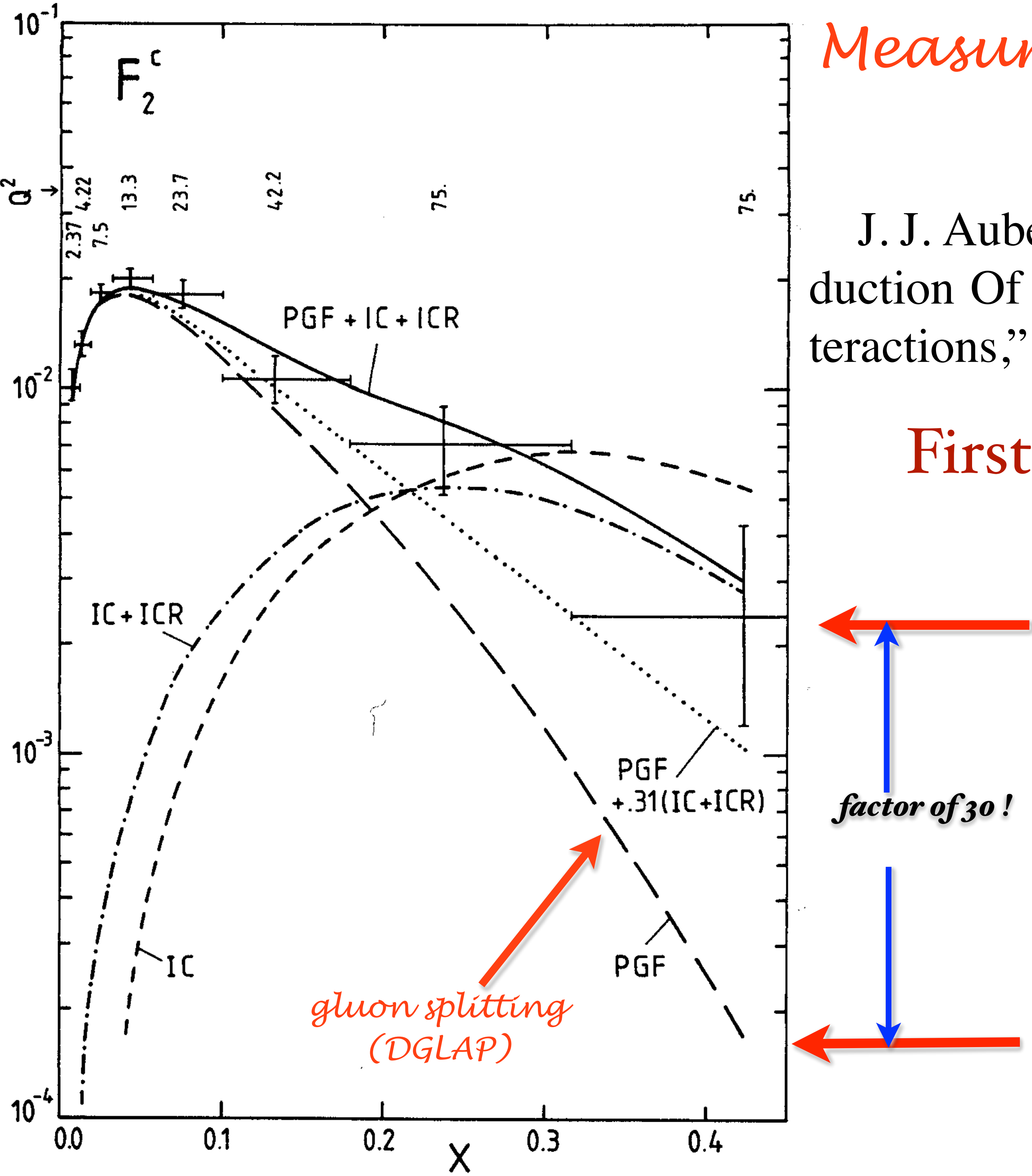
QCD predicts Intrinsic Heavy Quarks at high x

Minimal off-shellness

$$x_Q \propto (m_Q^2 + k_\perp^2)^{1/2}$$

$\text{Probability (QED)} \propto \frac{1}{M_\ell^4}$ $\text{Probability (QCD)} \propto \frac{1}{M_Q^2}$

Collins, Ellis, Gunion, Mueller, sjb
Polyakov, et al. Hoyer, Vogt, et al

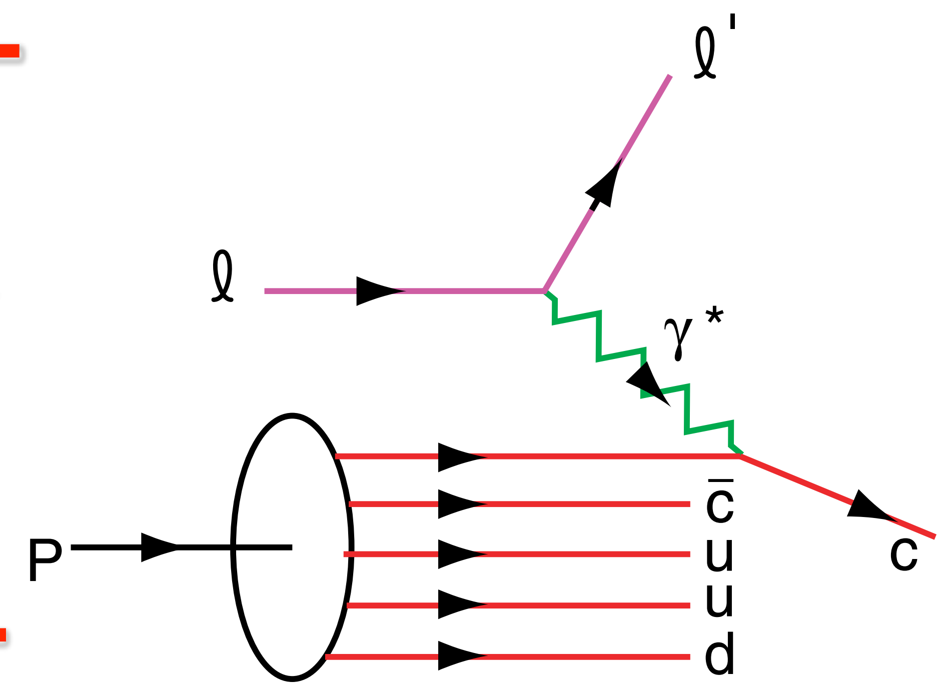


Measurement of Charm Structure Function!

J. J. Aubert et al. [European Muon Collaboration], "Production Of Charmed Particles In 250-Gev Mu+ - Iron Interactions," Nucl. Phys. B 213, 31 (1983).

First Evidence for Intrinsic Charm

Hoyer, Peterson, Sakai, sjb



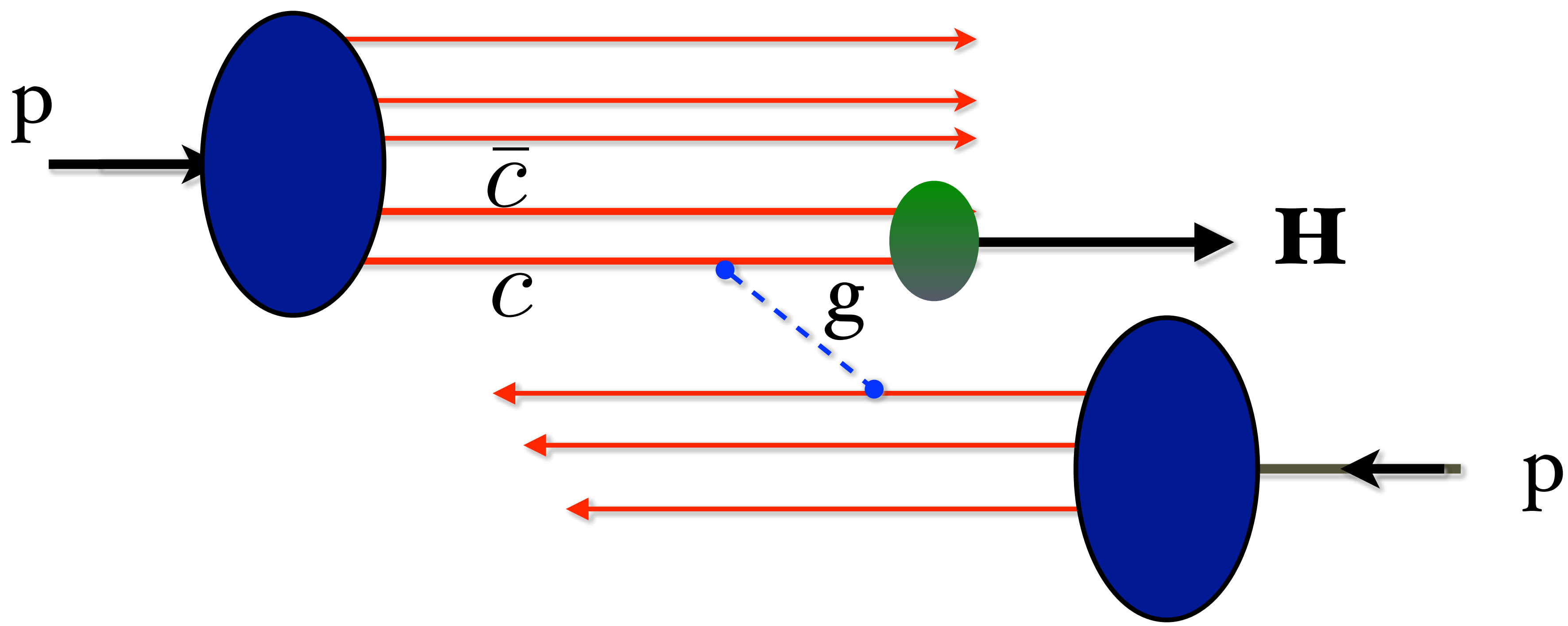
DGLAP / Photon-Gluon Fusion: factor of 30 too small

Two Components (separate evolution):

$$c(x,Q^2) = c(x,Q^2)_{\text{extrinsic}} + c(x,Q^2)_{\text{intrinsic}}$$

Goldhaber, Kopeliovich, Schmidt, Soffer sjb

Intrinsic Charm Mechanism for Inclusive High-X_F Higgs Production



$$pp \to H\,X$$

Also: intrinsic strangeness, bottom, top

Higgs can have > 80% of Proton Momentum!

New production mechanism for Higgs at the LHC

Properties of Hard Exclusive Amplitudes

- ***Form Factors (Elastic and Transition) are overlaps of Light-Front Wavefunctions***
- ***Key Input Hard Exclusive Processes: Distribution amplitudes***
- ***Factorization Theorems***

$$\phi_M(x,Q) = \int^Q d^2k_\perp \psi_{q\bar{q}}(x,\vec{k}_\perp)$$

- ***Hard Scattering Exclusive Hadron Amplitudes => Distribution amplitudes convoluted with hard subprocesses***
- ***ERBL Evolution of Distribution Amplitudes***
- ***Counting rules reflect leading twist LFWFS***
- ***Hadron-Helicity Conservation (Chiral Theory)***
- ***Quark Interchange Dominance***
- ***Color Transparency***
- ***Hidden Color***

Single-spin asymmetries

Leading Twist Sivers Effect

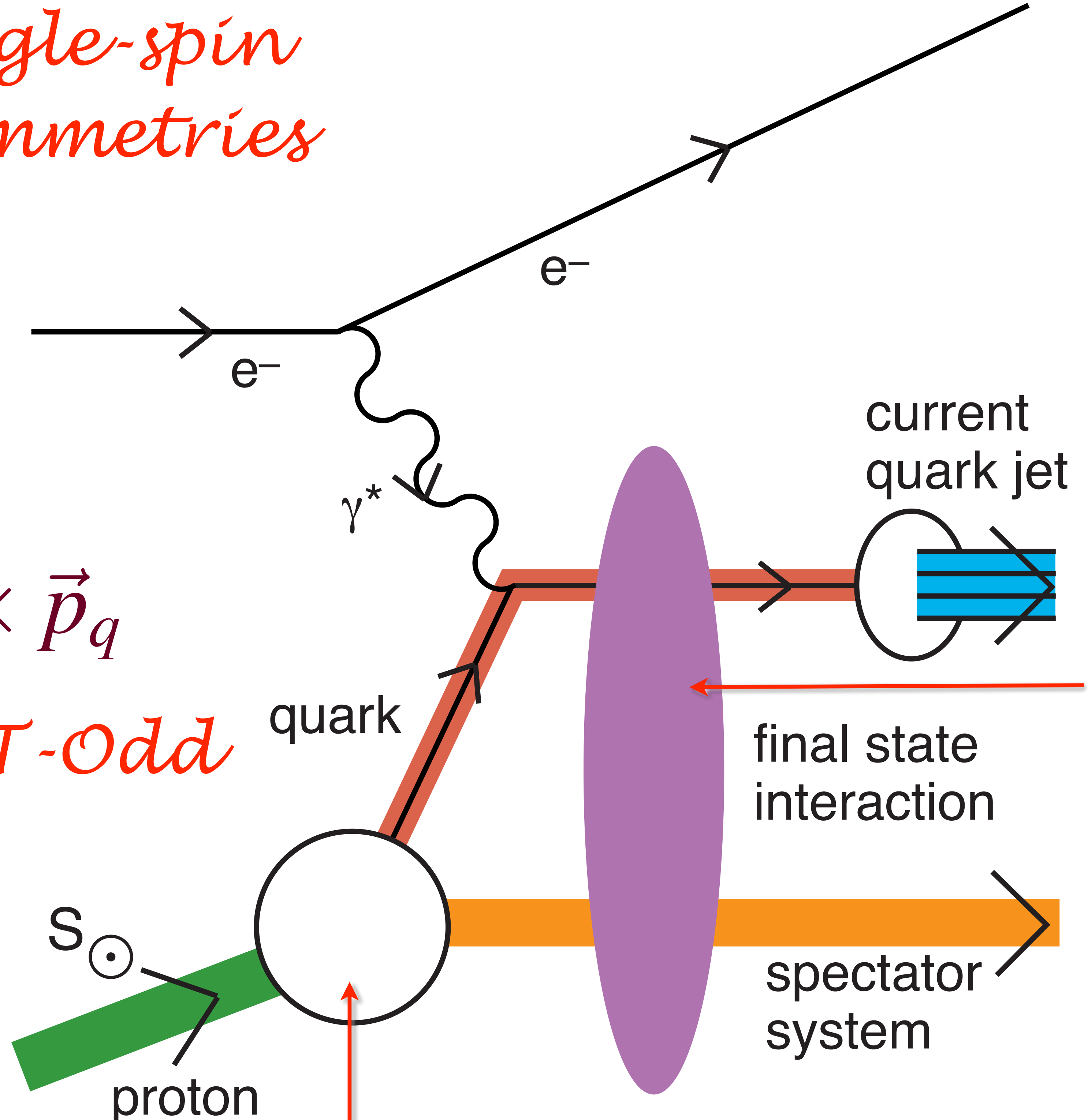
Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...

$$i\ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd

"Lensing" involves soft scales



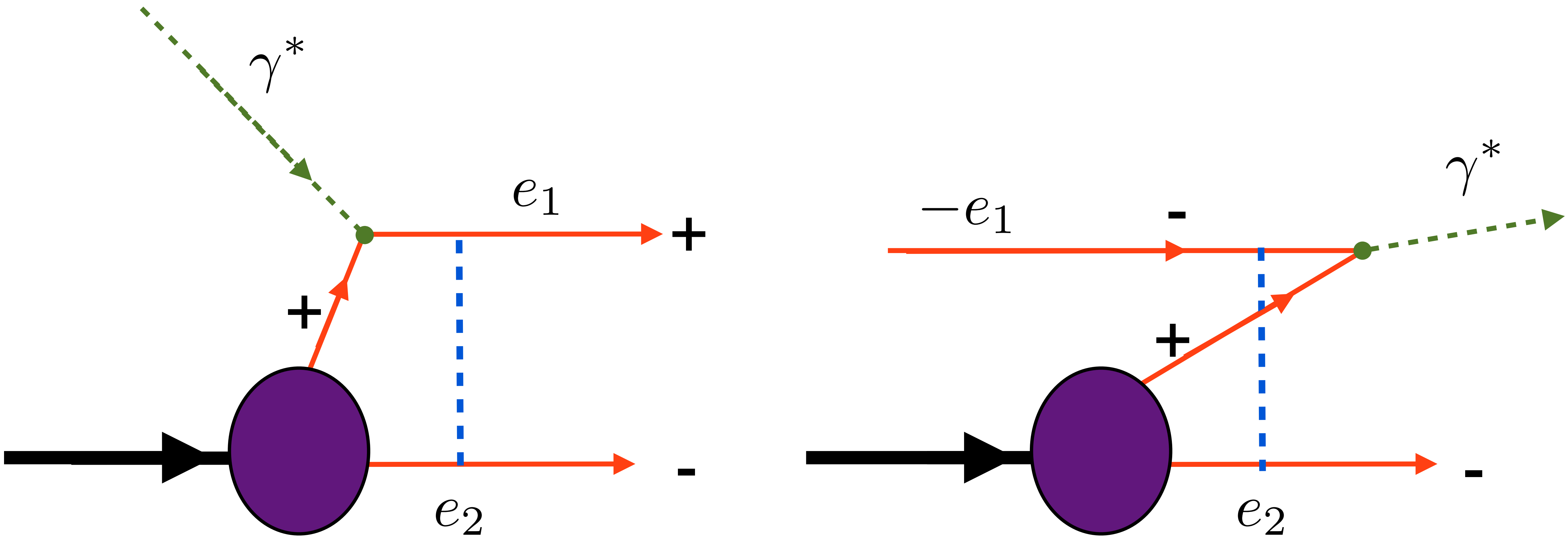
Light-Front Wavefunction S and P- Waves!

QCD S- and P- Coulomb Phases --Wilson Line

"Lensing Effect"

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!



DIS

Attractive, opposite-sign rescattering potential

DY

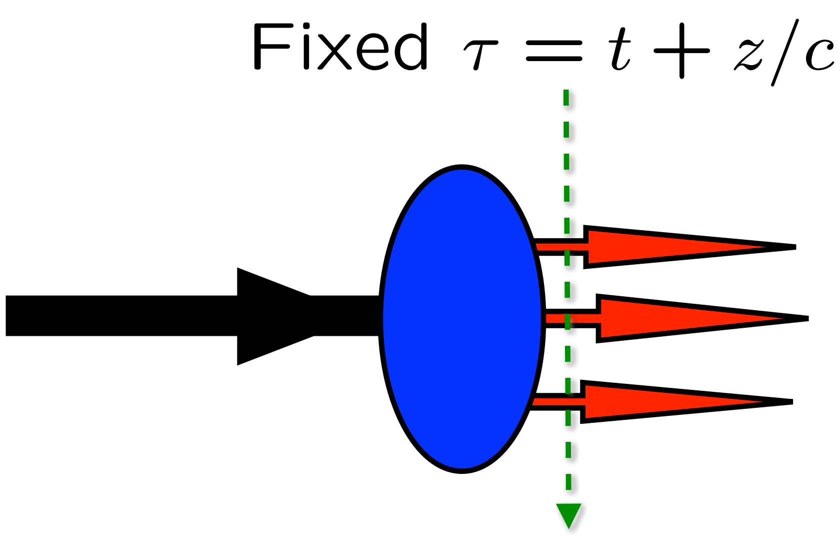
Repulsive, same-sign scattering potential

Dae Sung Hwang, Yuri V. Kovchegov, Ivan Schmidt, Matthew D. Sievert, sjb

Bound States in Relativistic Quantum Field Theory:

Light-Front Wavefunctions

Dirac's Front Form: Fixed $\tau = t + z/c$



$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Invariant under boosts. Independent of P^μ

$$\boxed{H^{QCD}_{LF} |\psi> = M^2 |\psi>}$$

Direct connection to QCD Lagrangian

Off-shell in invariant mass

Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space

QCD Lagrangian

$$\boxed{\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \cancel{\sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f}}$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classical Chiral Lagrangian is Conformally Invariant

Where does the QCD Mass Scale come from?

QCD does not know what MeV units mean!
Only Ratios of Masses Determined

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Unique confinement potential!

Goal: An analytic first approximation to nonperturbative QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What is the analytic form of the confining interaction?
- What sets the QCD mass scale?
- QCD Running Coupling at all scales
- Hadron Spectroscopy-Regge Trajectories
- Light-Front Wavefunctions
- Nonperturbative Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into QCD Condensates
- Chiral Symmetry
- Systematically improvable

$$H_{QED}$$

QED atoms: positronium and muonium

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

Coupled Fock states

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S},\vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

Effective two-particle equation

Includes Lamb Shift, quantum corrections

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r,S,\ell)]\ \psi(r) = E\ \psi(r)$$

Spherical Basis r, θ, ϕ

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

Coulomb potential

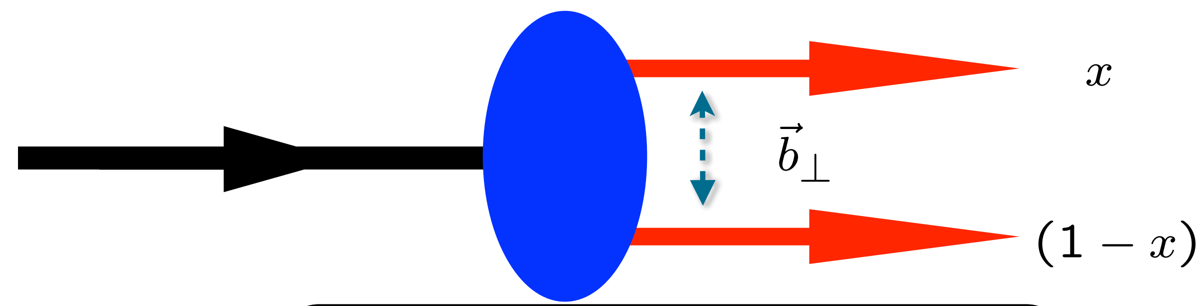
Bohr Spectrum

Semiclassical first approximation to QED

Schrödinger Eq.

Light-Front QCD

Fixed $\tau = t + z/c$



$$\zeta^2 = x(1-x)b^2_\perp$$

$$\mathcal{L}_{QCD} \rightarrow \boxed{H^{LF}_{QCD}}$$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states
and retarded interactions

$$[\frac{\vec{k}^2_\perp + m^2}{x(1-x)} + V^{LF}_{\text{eff}}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

Azimuthal Basis ζ, ϕ

$m_q = 0$

Single variable ζ

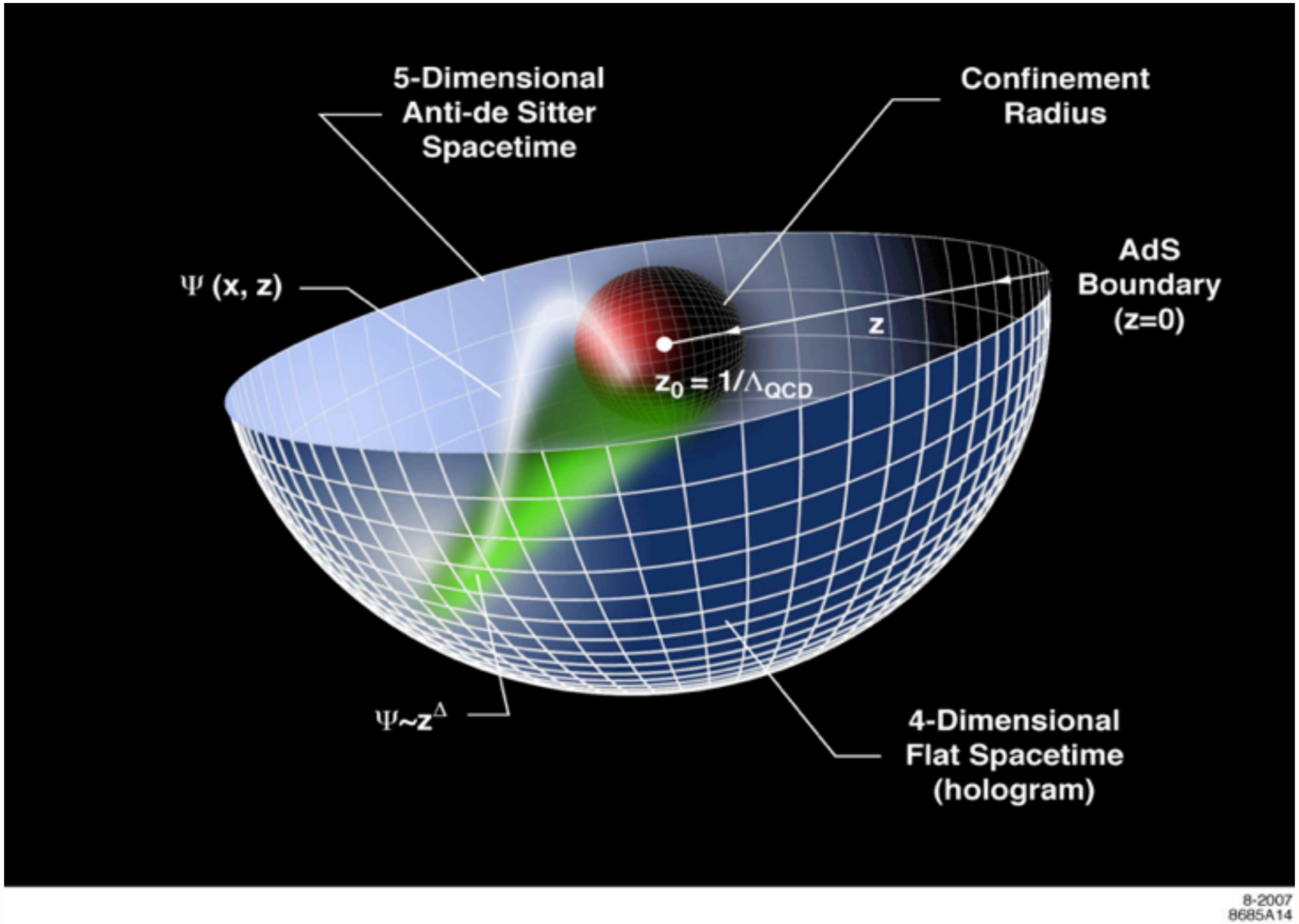
AdS/QCD:

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Confining AdS/QCD
potential!

Semiclassical first approximation to QCD

Sums an infinite # diagrams



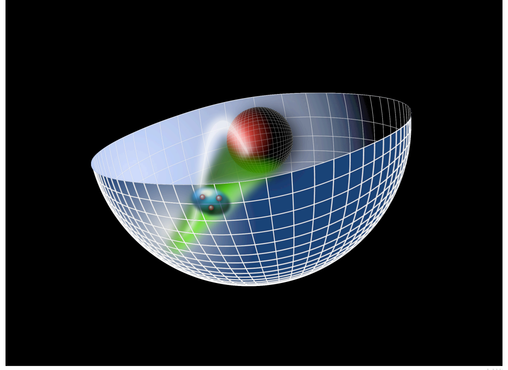
Changes in physical length scale mapped to evolution in the 5th dimension z

AdS_5

- Truncated AdS/CFT (Hard-Wall) model: cut-off at $z_0 = 1/\Lambda_{\rm QCD}$ breaks conformal invariance and allows the introduction of the QCD scale (Hard-Wall Model) Polchinski and Strassler (2001).

- Smooth cutoff: introduction of a background dilaton field $\varphi(z)$ – usual linear Regge dependence can be obtained (Soft-Wall Model) Karch, Katz, Son and Stephanov (2006).

AdS_5



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \; z \to \lambda z$, maps scale transformations into the holographic coordinate z.

- AdS mode in z is the extension of the hadron wf into the fifth dimension.

- Different values of z correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

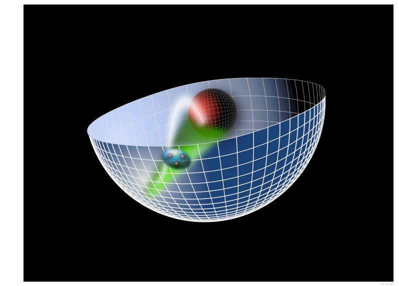
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

AdS/CFT

Dilaton-Modified AdS/QCD

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2}(\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- **Soft-wall dilaton profile breaks conformal invariance** $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- **Color Confinement in z**
- **Introduces confinement scale κ**
- **Uses AdS_5 as template for conformal theory**

Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

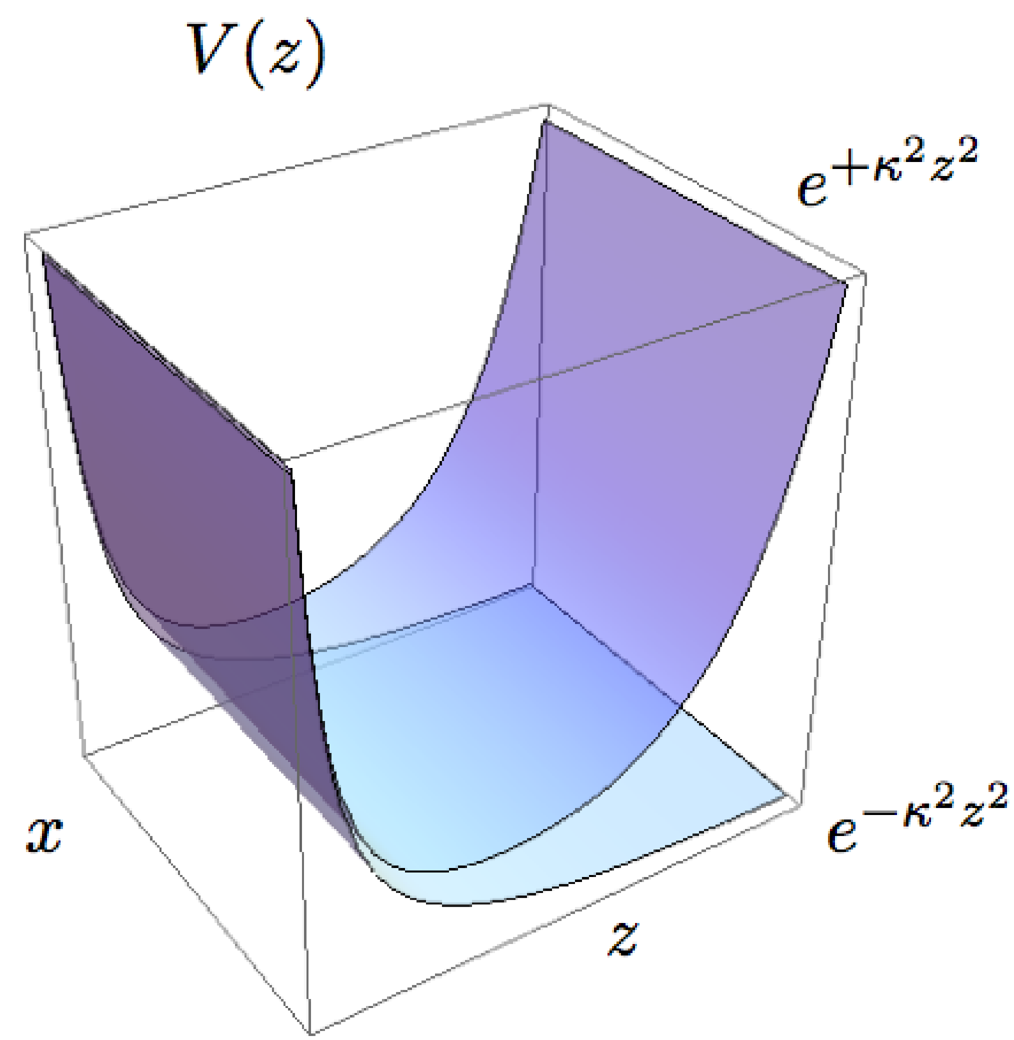
$$ds^2 = \frac{R^2}{z^2}\boxed{e^{\varphi(z)}}\left(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small z for geometries which are asymptotically AdS_5

- Gravitational potential energy for object of mass m

$$V = mc^2\sqrt{g_{00}} = mc^2 R\,\frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm\kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$



Klebanov and Maldacena

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- de Teramond, sjb

$$\boxed{e^{\varphi(z)} = e^{+\kappa^2 z^2}}$$

Positive-sign dilaton

- Dosch, de Teramond, sjb

AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

Derived from variation of Action for Dilaton-Modified AdS_5

Identical to Single-Variable Light-Front Bound State Equation in ζ!

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^{\,2}}$$

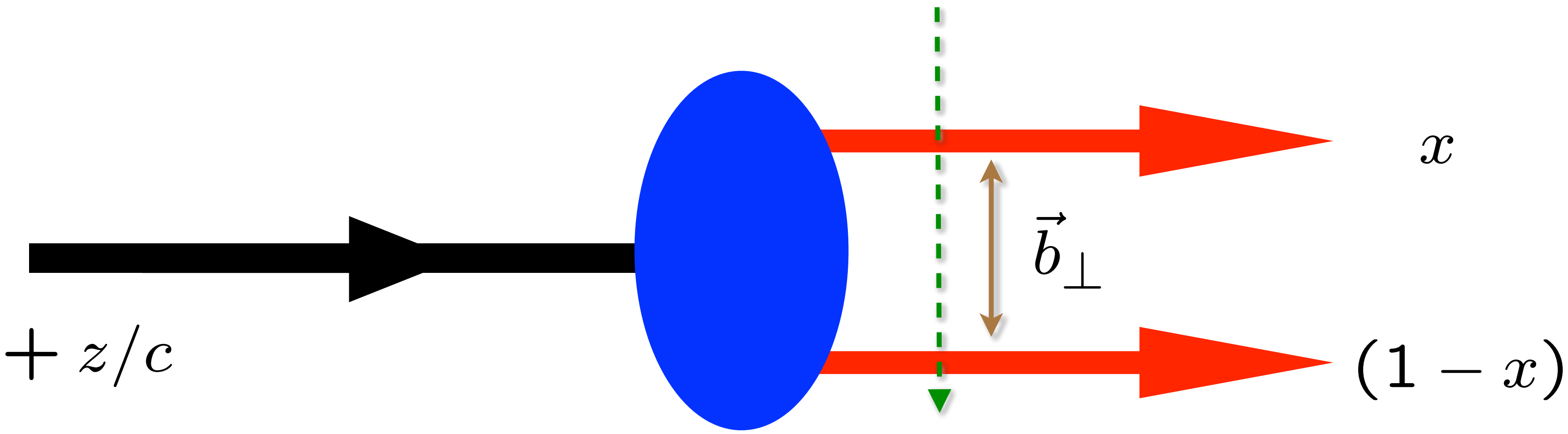
de Teramond, sjb

$LF(3+1) \longleftrightarrow AdS_5$

Light-Front Holographic Dictionary

$$\psi(x, \vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$



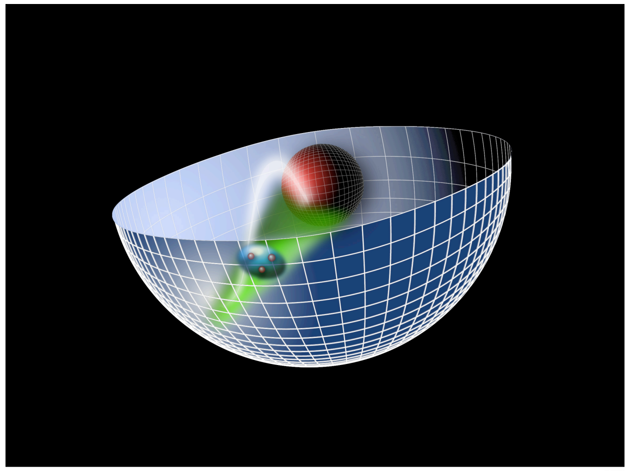
Fixed $\tau = t + z/c$

$$\psi(x, \zeta) = \sqrt{x(1-x)}\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

Light-Front Holography: *Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

de Tèramond, Dosch, Lorcè, sjb



AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathrm{b}^2_\perp .$$

Light-Front Holography

$$\boxed{[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)]\psi = M^2\psi}$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry
of the action

Single variable ζ

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

A very Practical Guide to Light Front Holographic QCD

Liping Zou and H.G. Dosch [1]

Institute of Modern Physics, Chinese Academy of Sciences
Lanzhou

The aim of these lectures is to convey a working knowledge of Light Front Holographic QCD and Supersymmetric Light Front Holographic QCD. We first give an overview of holographic QCD in general and then concentrate on the application of the holographic methods on QCD quantized in the light front form. We show how the implementation of the supersymmetric algebra fixes the interaction and how one can obtain hadron mass spectra with the minimal number of parameters. We also treat propagators and compare the holographic approach with other non-perturbative methods. In the last chapter we describe the application of Light Front Holographic QCD to electromagnetic form factors.

Constraints from conformal quantum mechanics

[V. de Alfaro, S. Fubini and G. Furlan, Nuovo Cim. A **34**, 569 (1976)]

[G. deTéramond, G. Dosch, sjb PLB **729**, 3 (2014)]

- Conformal Hamiltonian:

$$H = \tfrac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2}\right)$$

g dimensionless

- QM evolution

$$H|\psi(t)\rangle = i\frac{d}{dt}|\psi(t)\rangle$$

- H is one of the generators of the conformal group *Conf*(R^1). The two additional conserved generators, the dilatation generator:

$$D = \frac{i}{4}\left(x\frac{d}{dx} + \frac{d}{dx}x\right)$$

and the generator of special conformal transformations:

$$K = \tfrac{1}{2}x^2$$

close the conformal algebra

$$[H, D] = iH, \quad [H, K] = 2iD, \quad [K, D] = -iK$$

- **de Alfaro, Fubini, Furlan**

$$G|\psi(\tau) >= i\frac{\partial}{\partial \tau}|\psi(\tau) >$$

$$G = uH + vD + wK$$

New term

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

Retains conformal invariance of action despite mass scale!

$$4uw - v^2 = \kappa^4 = [M]^4$$

Identical to LF Hamiltonian with unique potential and dilaton!

- **Dosch, de Teramond, sjb**

$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)\right]\psi = M^2\psi$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

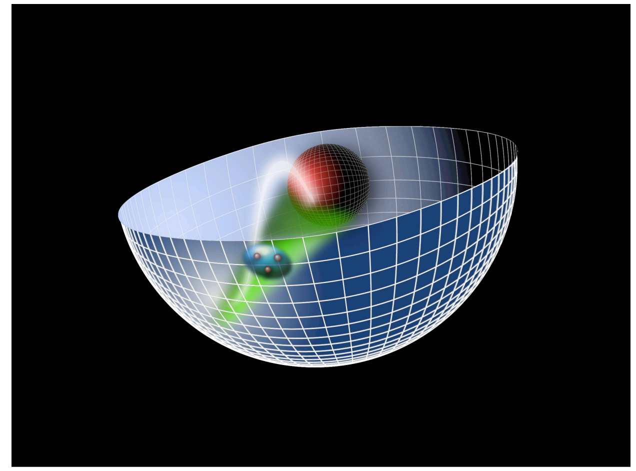
- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

Retains conformal invariance of action despite mass scale!

Fundamental Question: Origin of the QCD Mass Scale

- Pion massless for m_q=0
- What sets the mass of the proton when m_q=0 ?
- QCD: No knowledge of MeV units:
  Only ratios of masses can be predicted
- Novel proposal by de Alfaro, Fubini, and Furlan (DAFF):
  Mass scale κ can appear in Hamiltonian leaving the action conformal!
- Unique Color-Confinement Potential $\kappa^4\zeta^2$
- Eigenstates of Light-Front Hamiltonian determine hadronic mass spectrum and LF wavefunctions $\psi_H(x_i, \vec{k}_{\perp i}, \lambda_i)$
- Superconformal algebra: Degenerate meson and baryon mass spectrum
- Running QCD Coupling at all scales: $\text{Predict } \frac{\Lambda_{\overline{MS}}}{m_p}$

de Tèramond, Dosch, Lorcè, sjb



AdS/QCD Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

$$\zeta^2 = x(1-x)\mathbf{b}^2_\perp.$$

Light-Front Holography

$$[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)]\psi = M^2\psi$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Single variable ζ

Confinement scale: $\kappa \simeq 0.5\ GeV$

- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!

Meson Spectrum in Soft Wall Model

Massless pion!

$$\boxed{m_\pi = 0 \text{ if } m_q = 0}$$

Pion: Negative term for J=0 cancels positive terms from LFKE and potential

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2 \phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L} e^{-\kappa^2\zeta^2/2} L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

$$\boxed{\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)}$$

G. de Teramond, H. G. Dosch, sjb

Quark separation increases with L

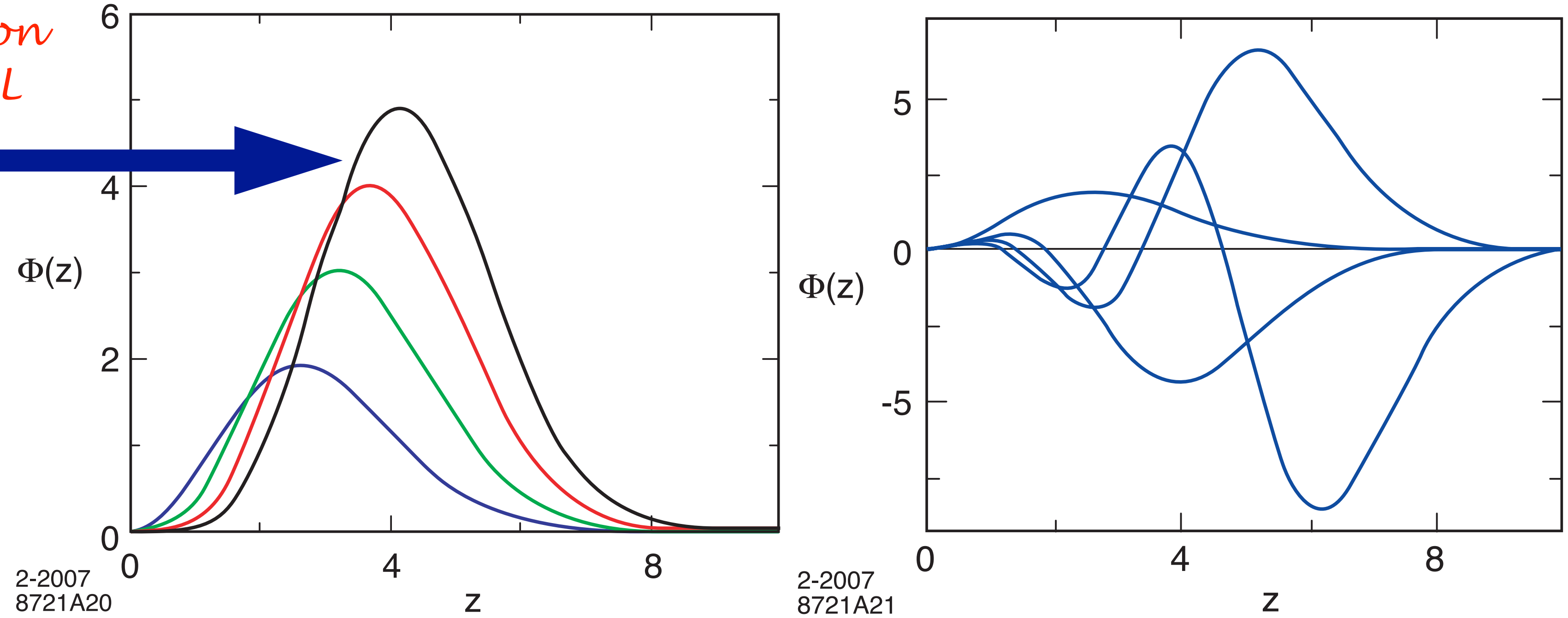
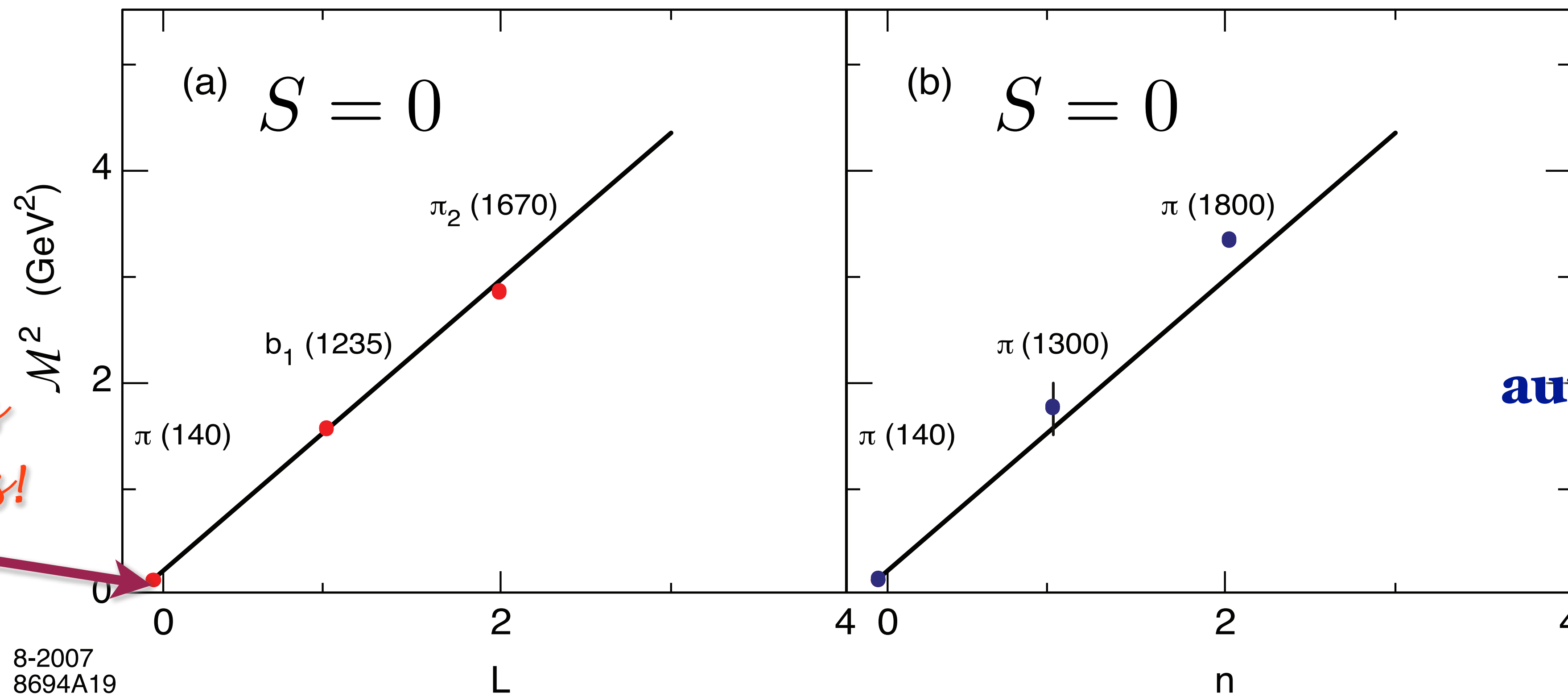


Fig: Orbital and radial AdS modes in the soft wall model for κ = 0.6 GeV .

Soft Wall Model

Same slope in n and L!



Pion has zero mass!

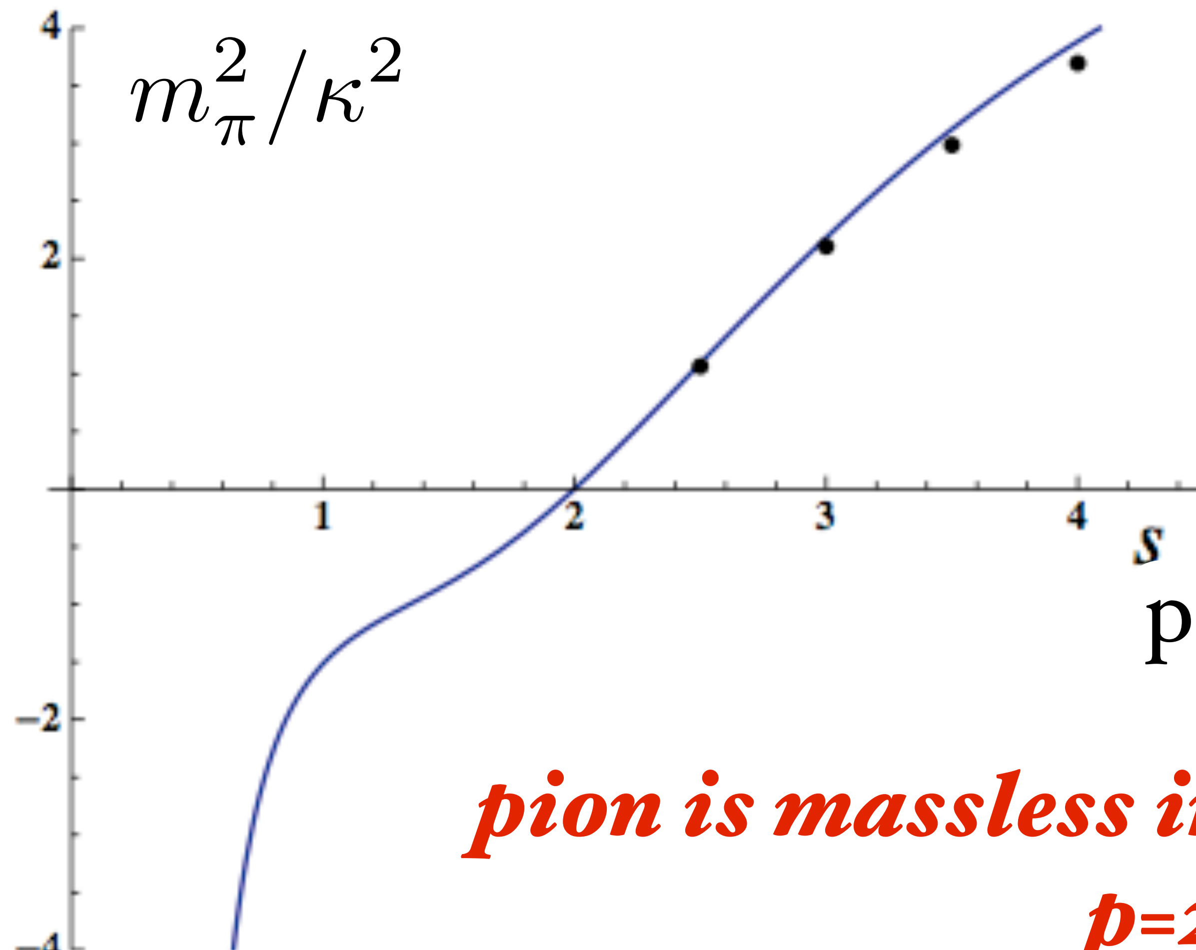
Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

Pion mass automatically zero!

$$m_q = 0$$

Uniqueness of Dilaton

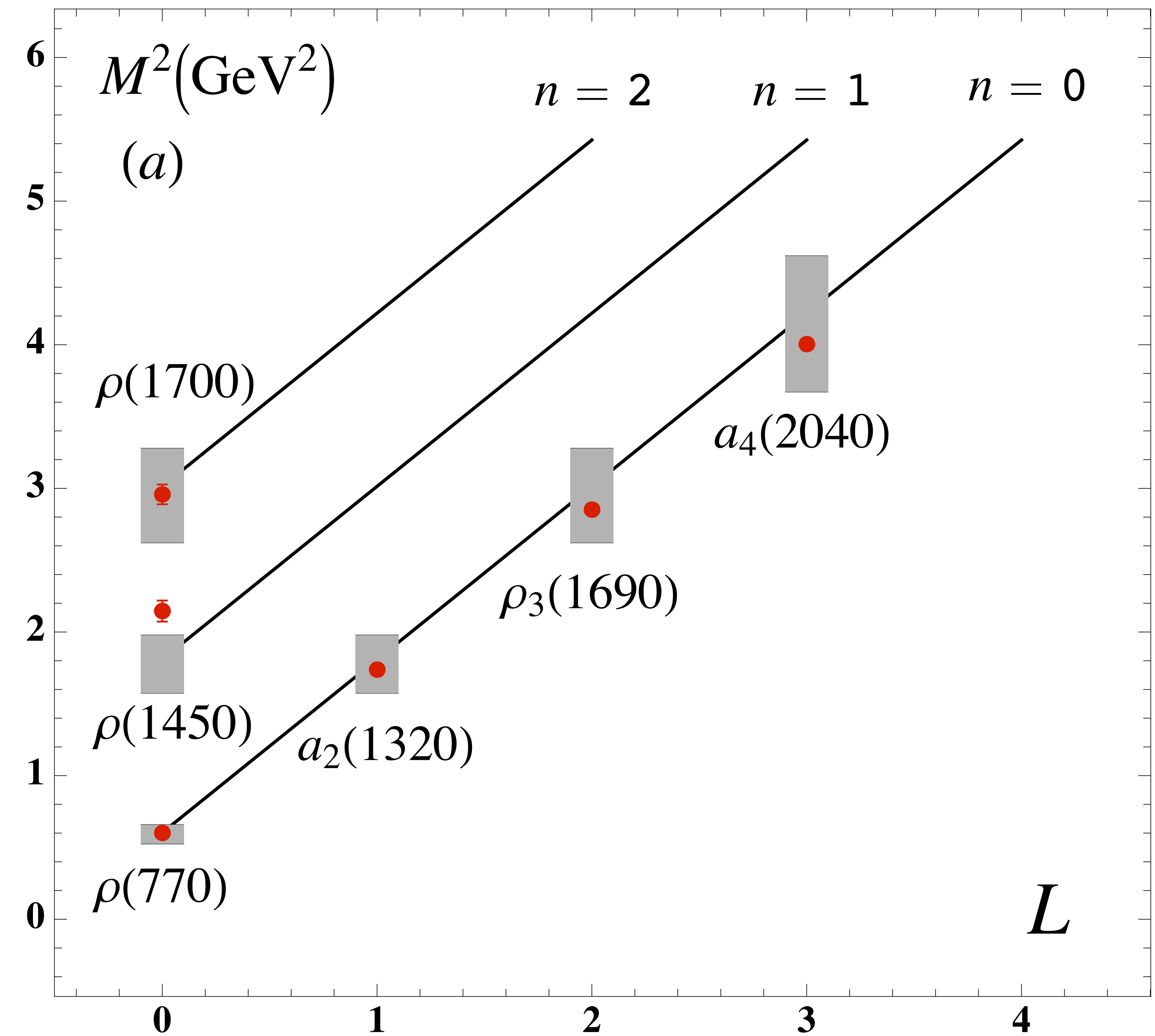
$$\varphi_p(z) = \kappa^p z^p$$



pion is massless in chiral limit iff p=2!

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

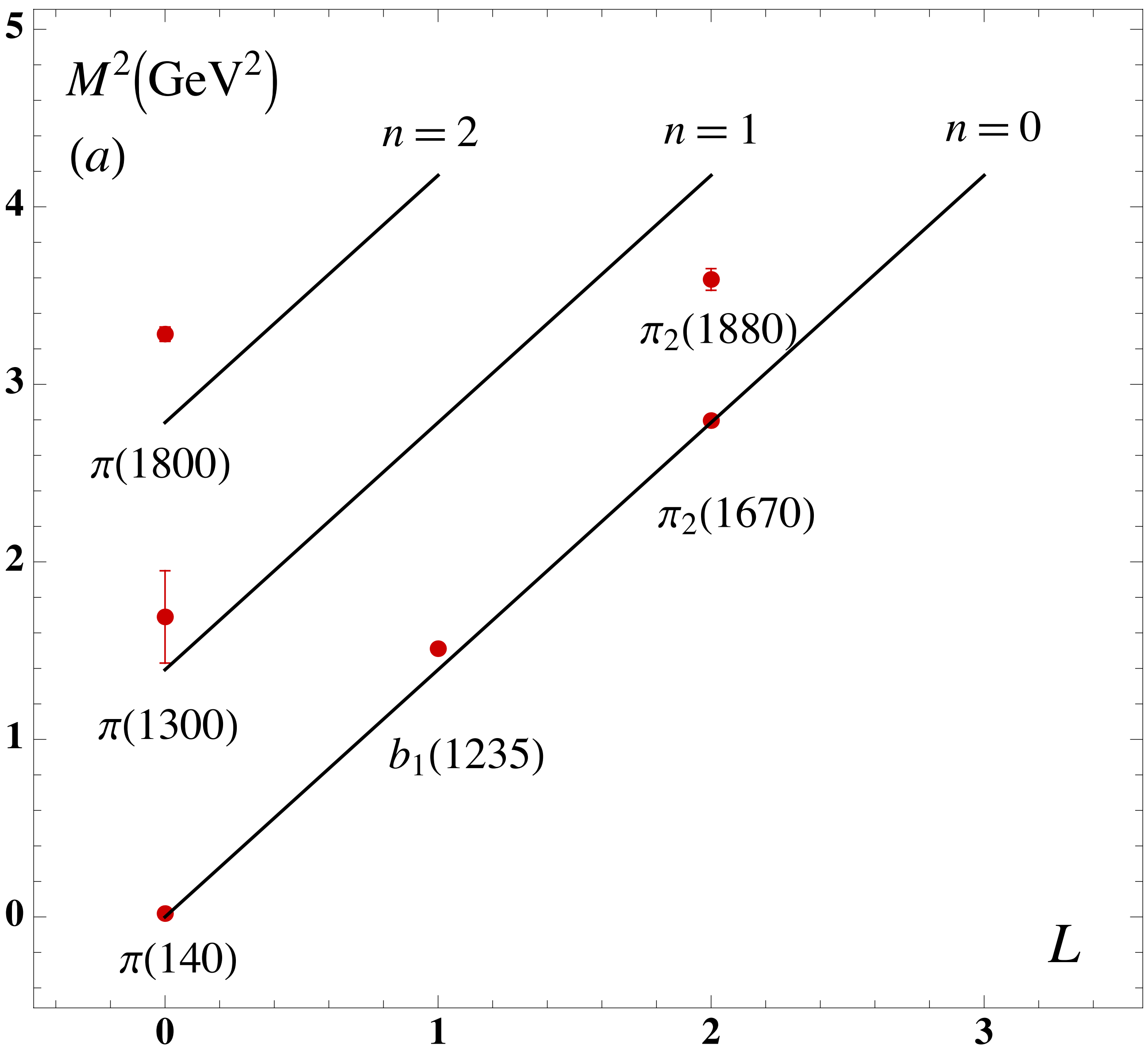
- **Dosch, de Tèramond, sjb**

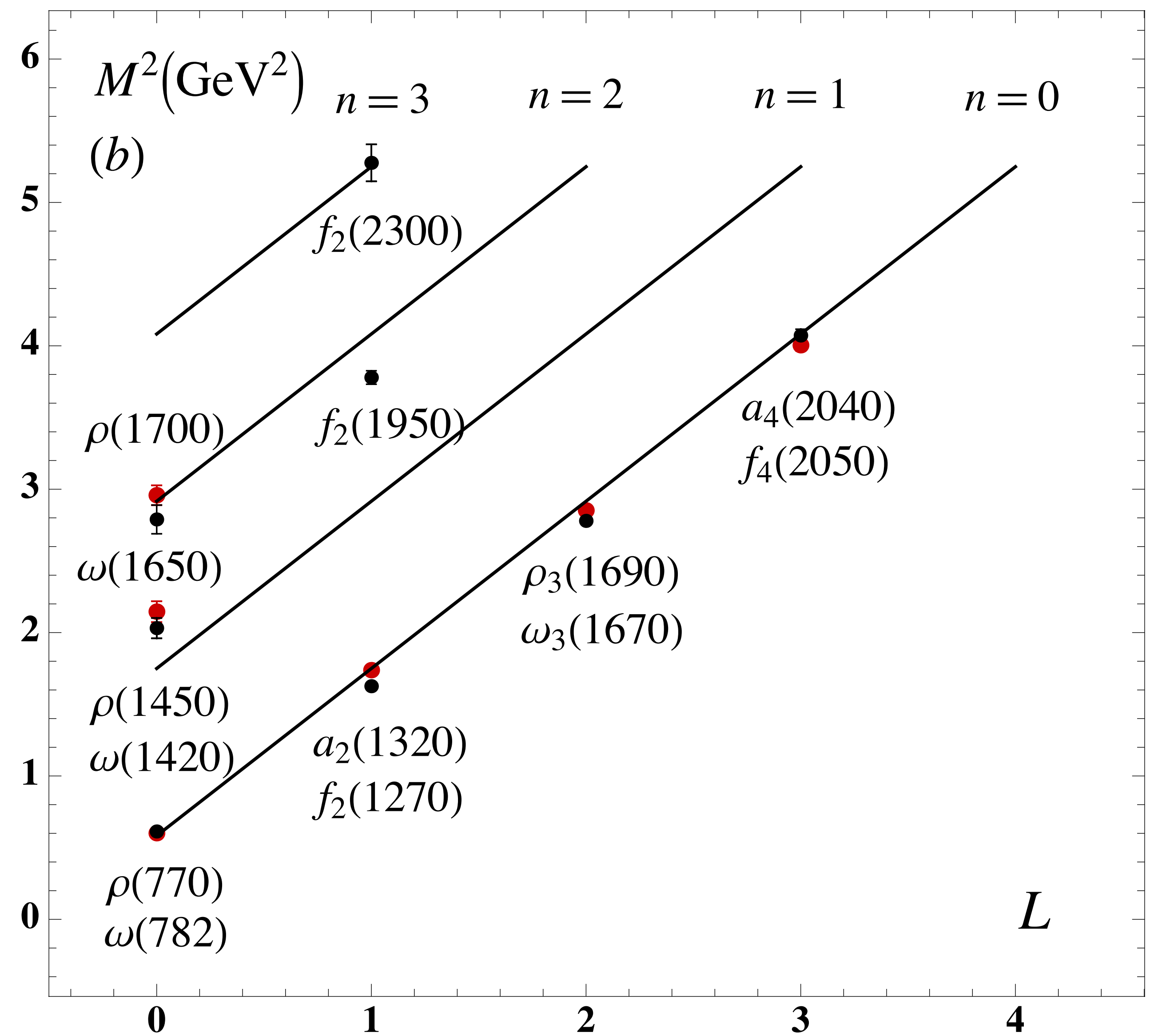


$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

$$m_u = m_d = 0$$

de Tèramond, Dosch, sjb





$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

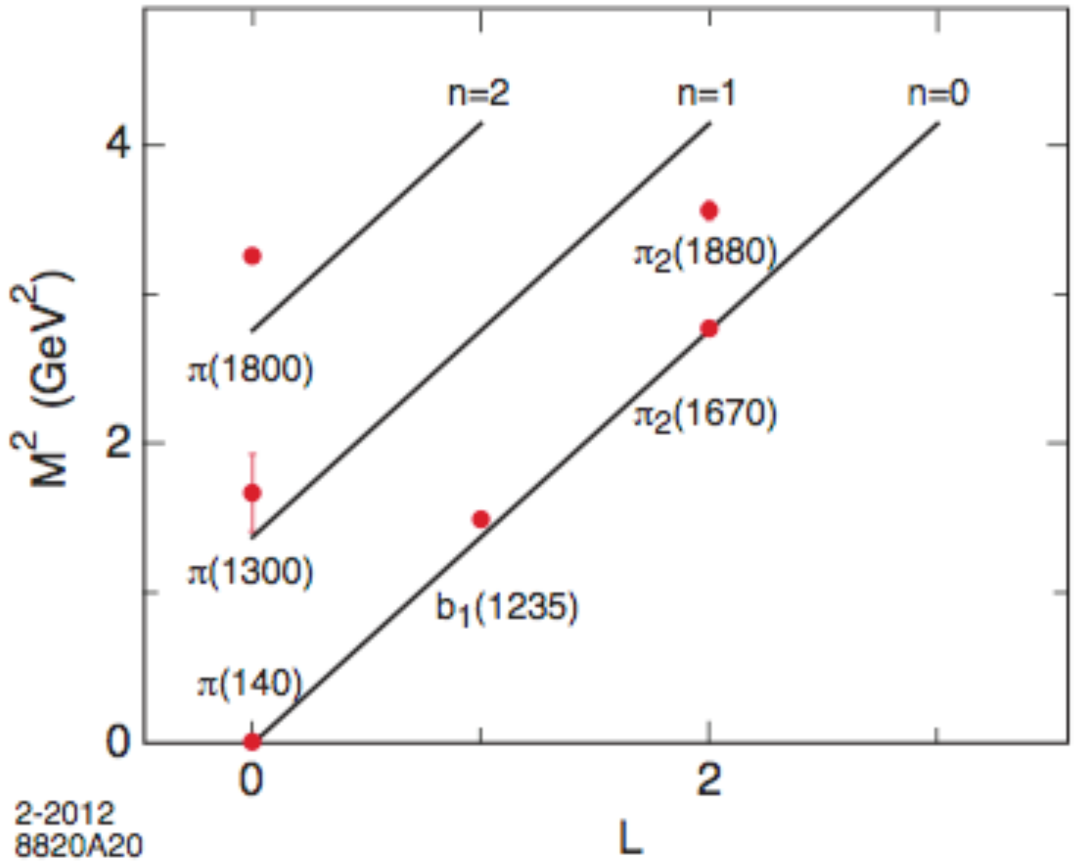
- $J = L + S, I = 1$ meson families $\boxed{\mathcal{M}^2_{n,L,S} = 4\kappa^2 \left(n + L + S/2\right)}$

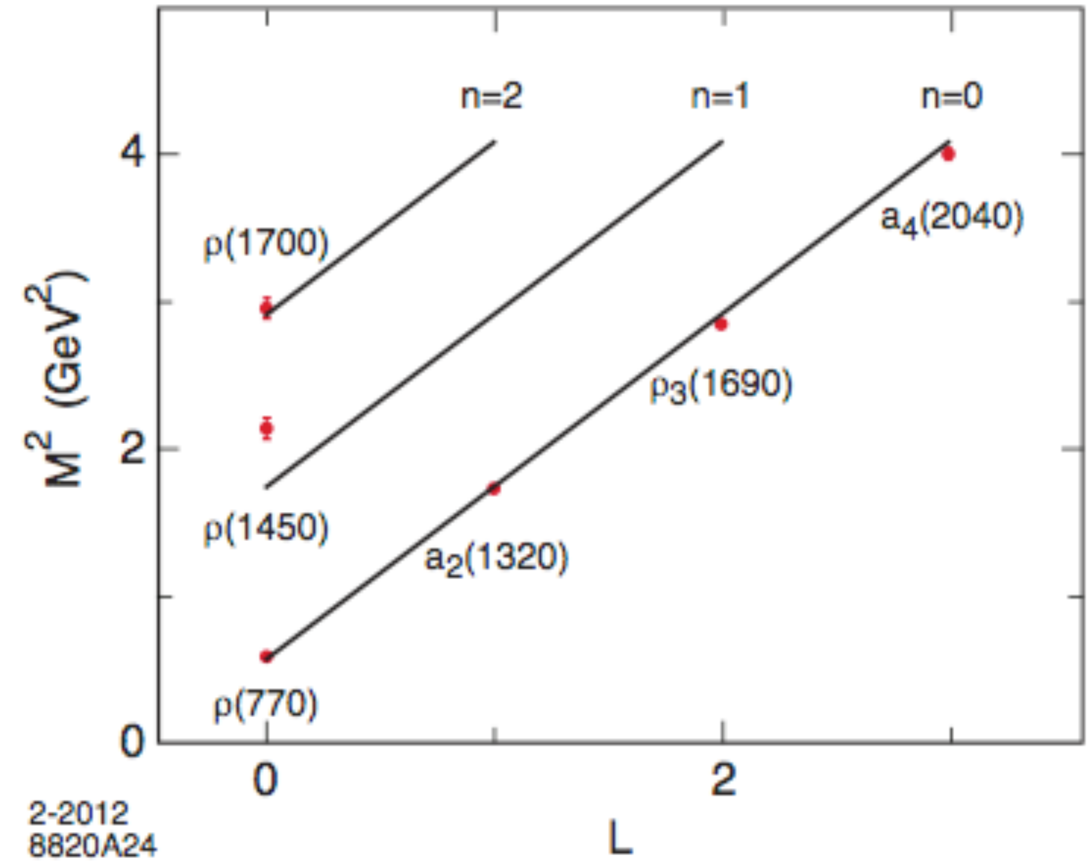
$4\kappa^2$ for $\Delta n = 1$
$4\kappa^2$ for $\Delta L = 1$
$2\kappa^2$ for $\Delta S = 1$

$\boxed{m_q = 0}$

Massless pion in Chiral Limit!

Same slope in n and L!





I=1 orbital and radial excitations for the π ($\kappa = 0.59$ GeV) and the ρ-meson families ($\kappa = 0.54$ GeV)

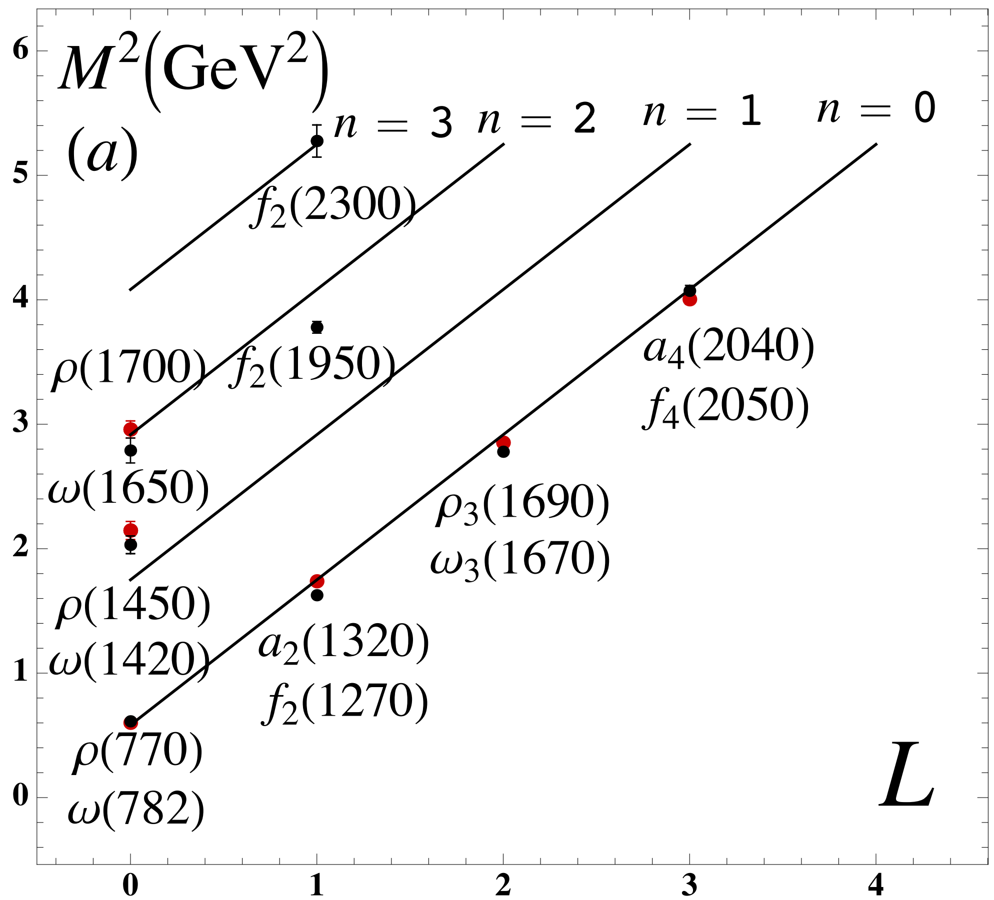
- Triplet splitting for the $I = 1, L = 1, J = 0, 1, 2$, vector meson a-states

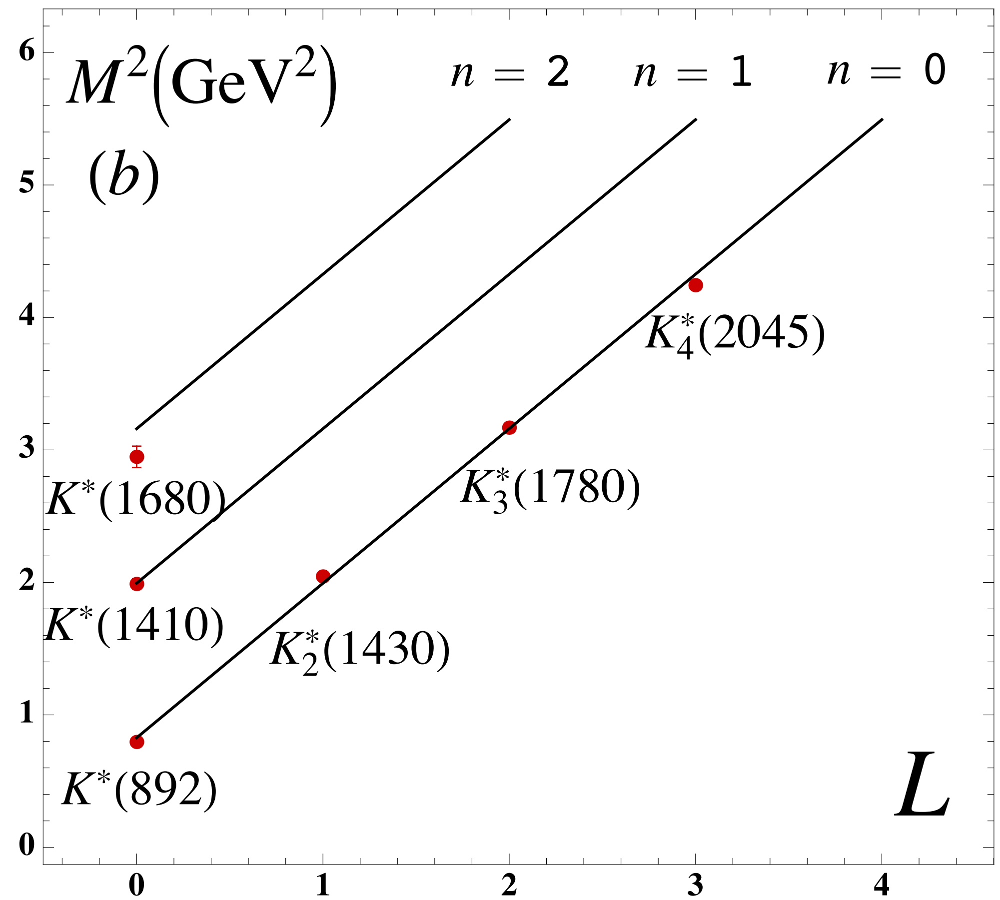
$$\mathcal{M}_{a_2(1320)} > \mathcal{M}_{a_1(1260)} > \mathcal{M}_{a_0(980)}$$

Mass ratio of the ρ and the a_1 mesons: coincides with Weinberg sum rules

G. de Teramond, H. G. Dosch, sjb

$$\kappa = \sqrt{\lambda} = 0.54\ GeV$$





Orbital and radial excitations for vector mesons

- Linear Regge trajectories, a massless pion and relation between the ρ and a_1 mass $M_{a_1}/M_\rho =$ usually obtained from Weinberg sum rules described by LF harmonic confinement model

De Teramond, Dosch, sjb

- Results easily extended to light quark masses (Ex: K-mesons)

- First order perturbation in the quark masses

$$\Delta M^2 = \langle \psi | \sum_a m_a^2 / x_a | \psi \rangle$$

- Holographic LFWF with quark masses

$$\lambda \equiv \kappa^2$$

$$\psi(x, \zeta) \sim \sqrt{x(1-x)}\, e^{-\frac{1}{2\lambda}\left(\frac{m_q^2}{x} + \frac{m_q^2}{1-x}\right)} e^{-\frac{1}{2}\lambda \zeta^2}$$

- Ex: Description of diffractive vector meson production at HERA

  [J. R. Forshaw and R. Sandapen, PRL **109**, 081601 (2012)]

- For the K^*

$$M_{n,L,S}^2 = M_{K^\pm}^2 + 4\lambda \left(n + \frac{J+L}{2}\right)$$

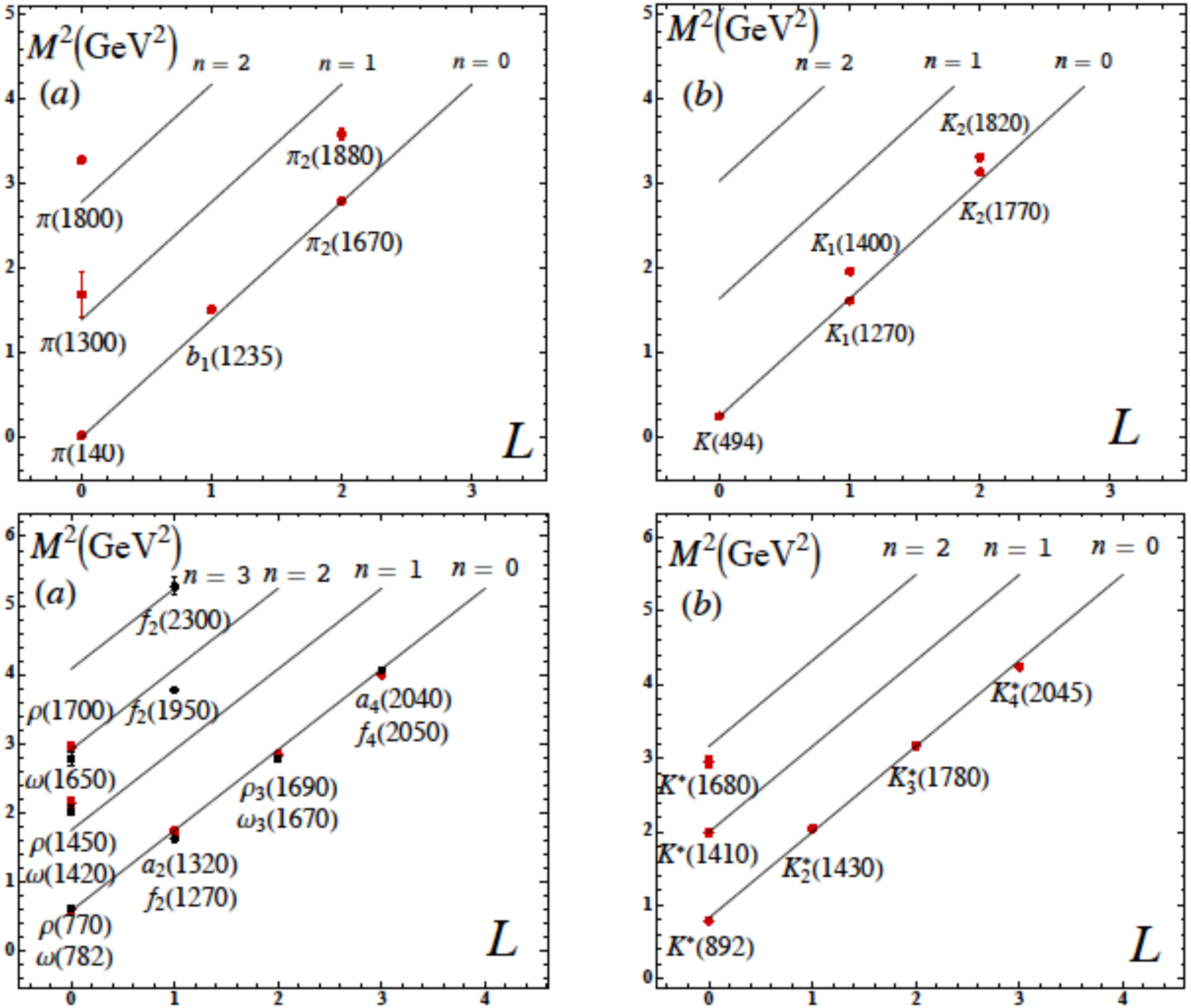
- Effective quark masses from reduction of higher Fock states as functionals of the valence state:

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

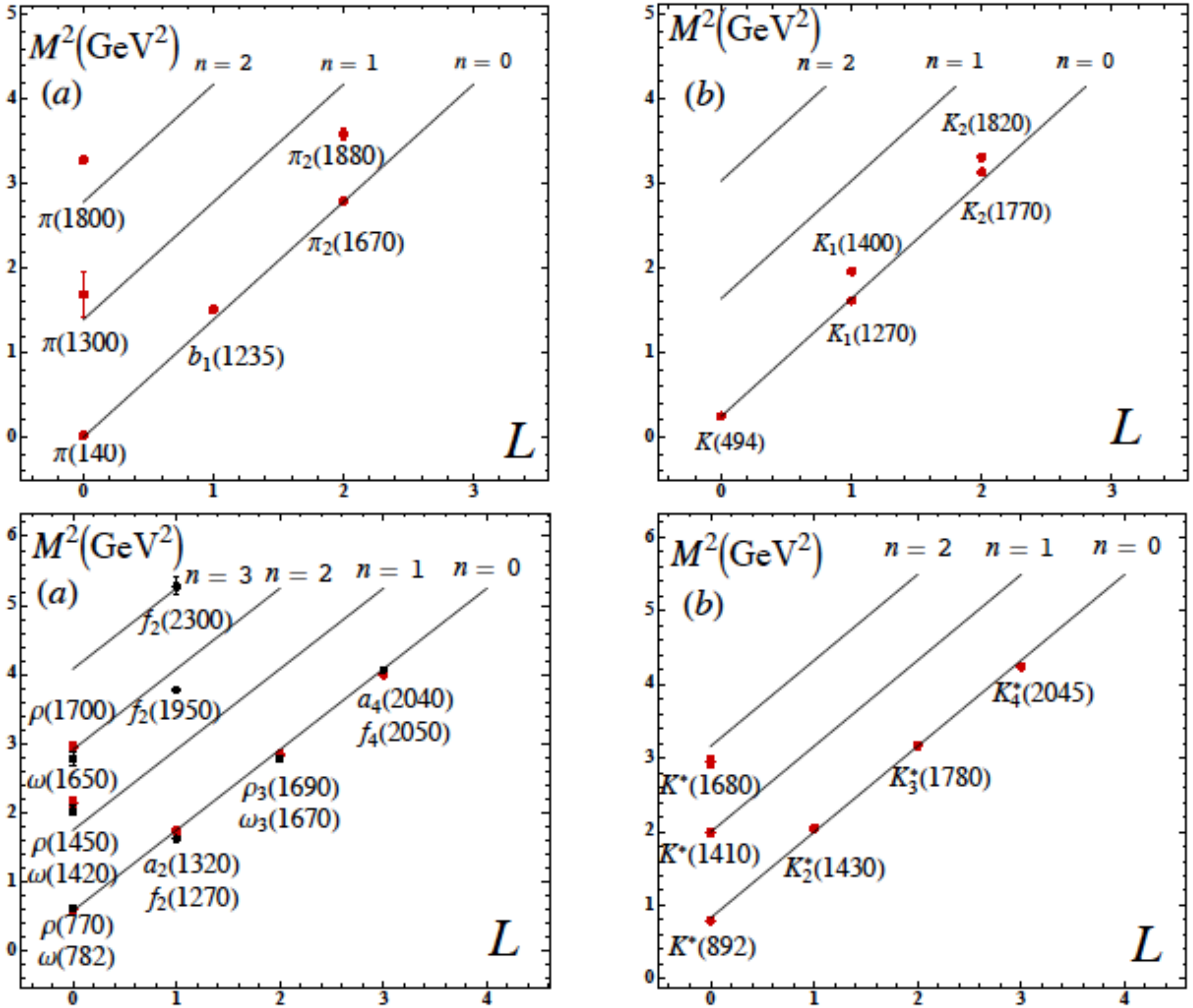
$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



De Teramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$



Remarkable, Fundamental Features of Hadrons, Nuclei

- *Color confinement: Quarks and Gluons permanently confined in hadrons!*
- *Origin of the hadron mass scale: what determines the proton mass?*
- *Pion is a quark-antiquark bound state, but it is massless if the quark mass is zero!*
- *The QCD coupling at all scales, beyond asymptotic freedom*
- *How does one set the renormalization scale? QCD —> QED if Nc —> 0*
- *Poincare invariance: Physics independent of observer motion — no Lorentz contraction!*
- *Causality: No correlations exceeding the speed of light*

Light-Front Dynamics

- *Relativity: Relativistic Bound State Dynamics*
- *Mesons and Baryons display supersymmetry!*
- *Exotic Phenomena: Color Transparency, Intrinsic Charm, Hidden Color, Exotic Hadrons*
- *Cosmological Constant is Zero!*

Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large Q^2***
- ***Computable at large Q^2 in any pQCD scheme***
- ***Universal β_0, β_1***

Running Coupling from Modified AdS/QCD

Deur, de Teramond, sjb

- Consider five-dim gauge fields propagating in AdS$_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$S = -\frac{1}{4}\int d^4x\, dz\, \sqrt{g}\, e^{\varphi(z)}\, \frac{1}{g_5^2}\, G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)} \frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$

- Coupling measured at momentum scale Q

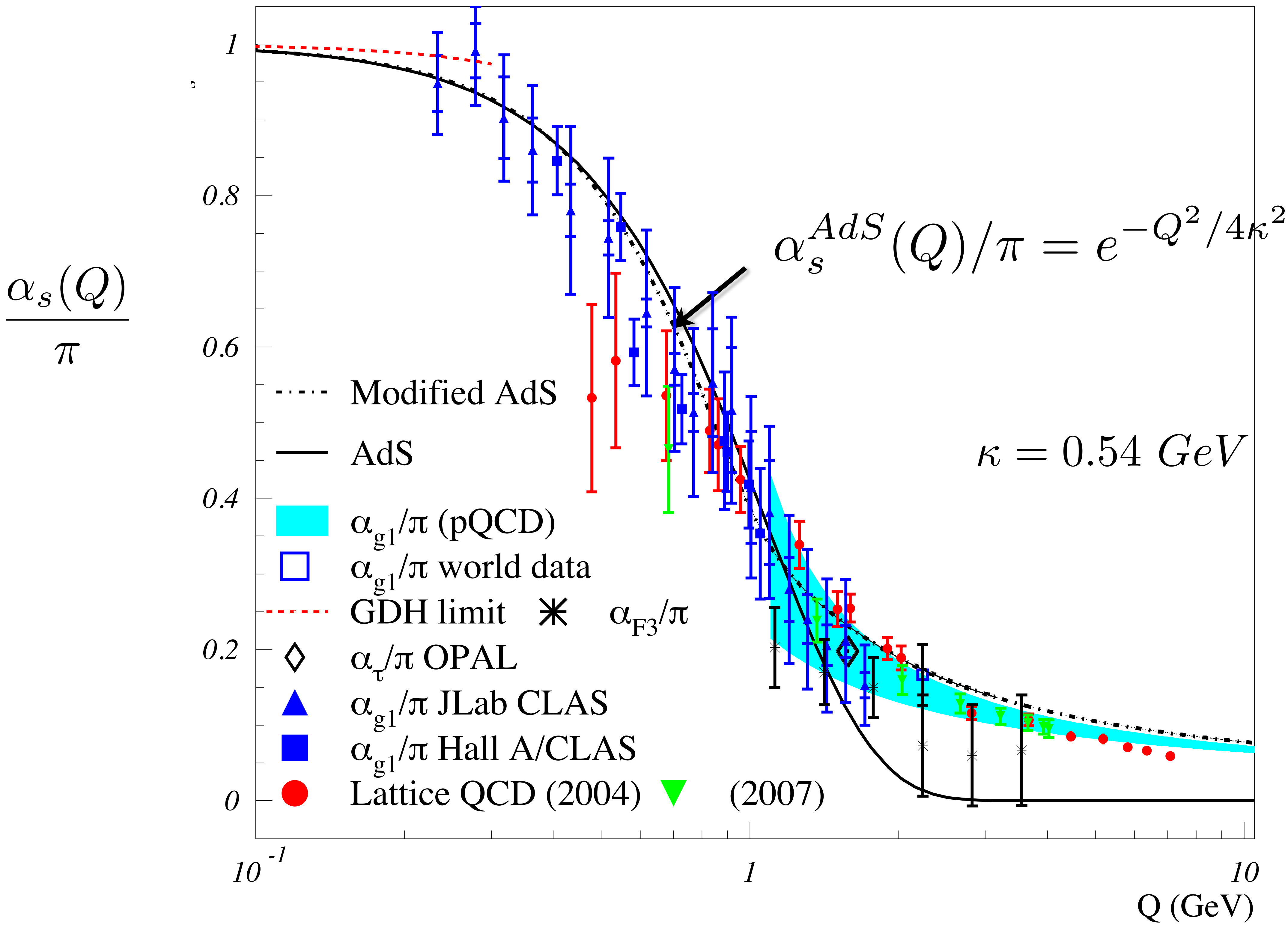
$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\, \alpha_s^{AdS}(\zeta)$$

- Solution

$$\boxed{\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\, e^{-Q^2/4\kappa^2}} \quad \text{from dilaton } e^{\kappa^2 z^2}$$

where the coupling α_s^{AdS} incorporates the non-conformal dynamics of confinement

Analytic, defined at all scales, IR Fixed Point



AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

Deur, de Teramond, sjb

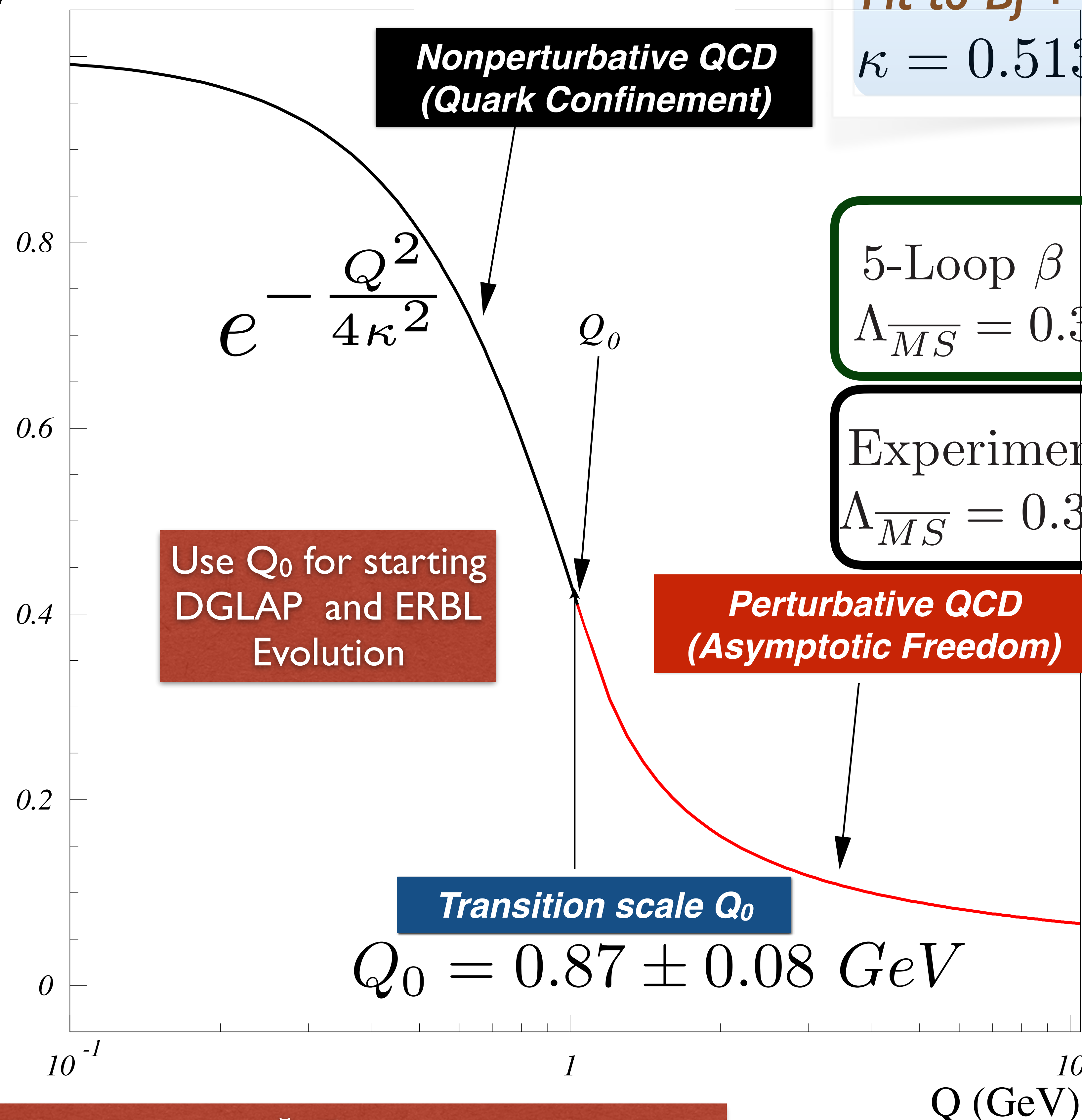
$$m_\rho = \sqrt{2}\kappa$$
$$m_p = 2\kappa$$

All-Scale QCD Coupling

Deur, de Tèramond, sjb

Fit to Bj + DHG Sum Rules:
$$\kappa = 0.513 \pm 0.007\ GeV$$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$



5-Loop β Prediction:
$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

Experiment:
$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ GeV$$

$$\lambda \equiv \kappa^2$$

Reverse Dimensional Transmutation!

$\overline{MS}$ *scheme*

Deur, de Tèramond, sjb

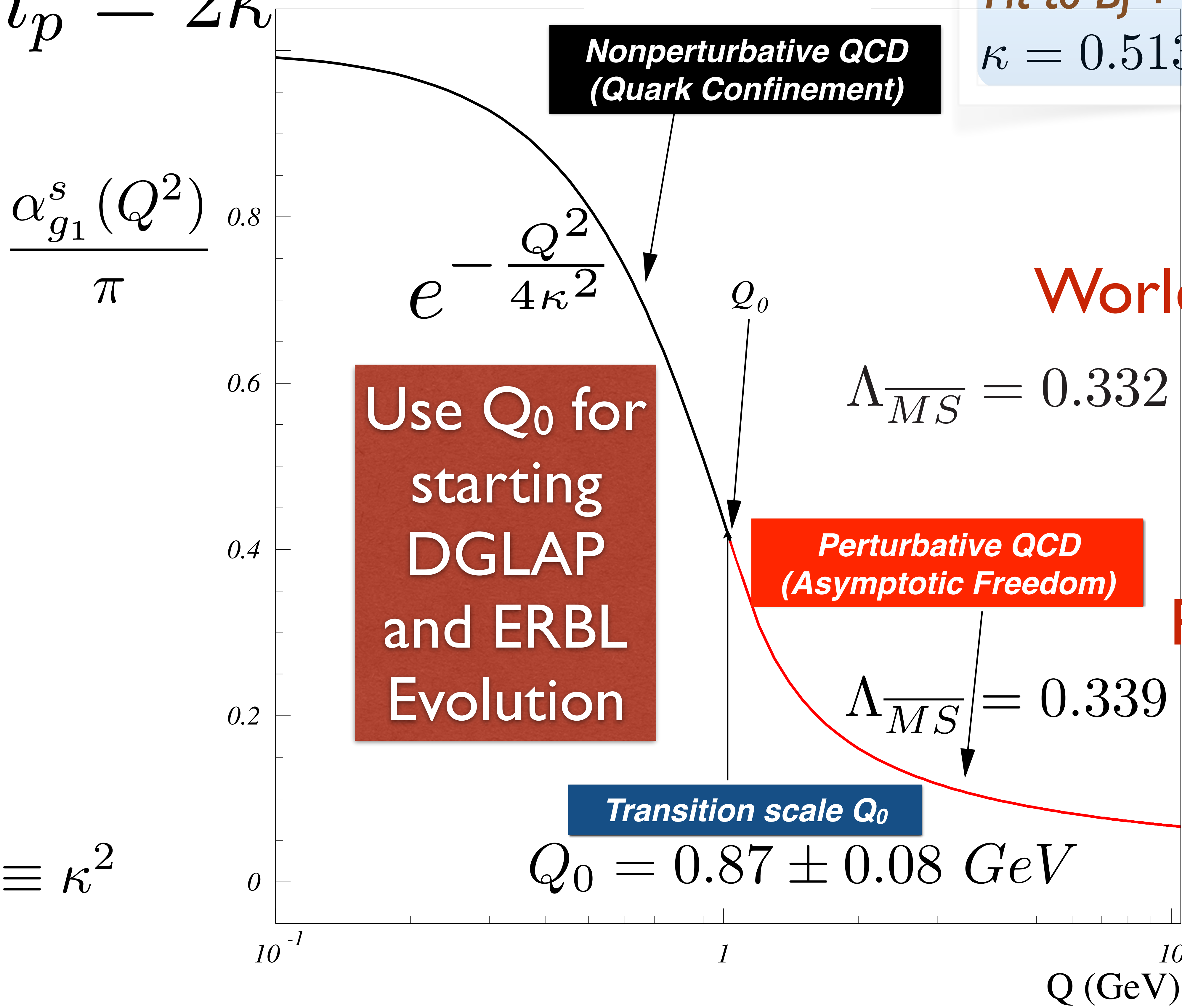
All-Scale QCD Coupling

$$m_\rho = \sqrt{2}\kappa$$

$$m_p = 2\kappa$$

Fit to Bj + DHG Sum Rules:

$$\kappa = 0.513 \pm 0.007\ GeV$$



World Data:

$$\Lambda_{\overline{MS}} = 0.332 \pm 0.017\ \text{GeV}$$

Prediction

$$\Lambda_{\overline{MS}} = 0.339 \pm 0.019\ GeV$$

$$Q_0 = 0.87 \pm 0.08\ GeV$$

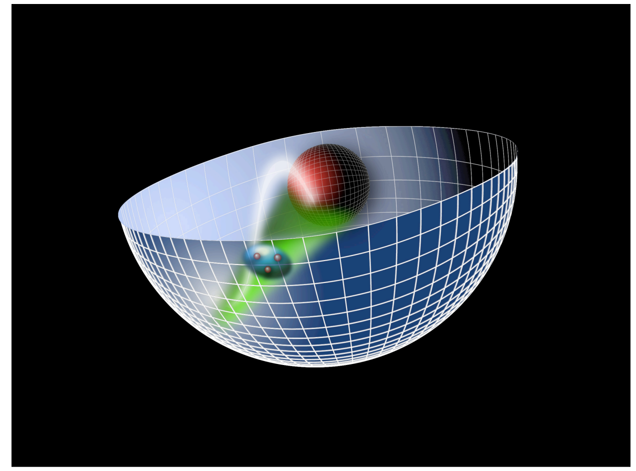
$\overline{MS}$ scheme

$$\lambda \equiv \kappa^2$$

de Tèramond, Dosch, Lorcè, sjb

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} + \frac{4L^2-1}{4\zeta^2} + U(\zeta^2)\right]\psi = M^2\psi$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique
Confinement Potential!

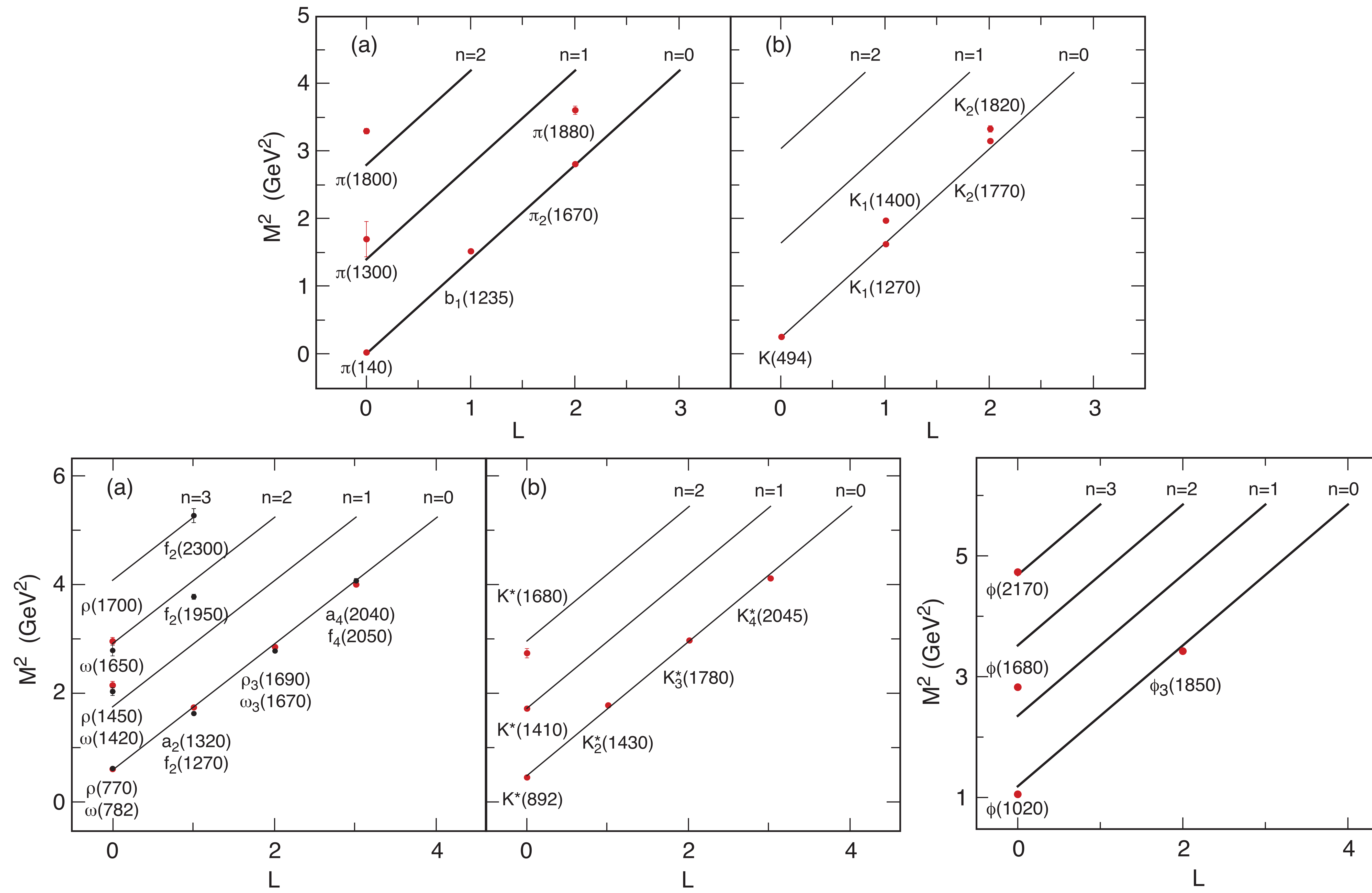
Conformal Symmetry
of the action

Single variable ζ

Confinement scale: $\kappa \simeq 0.5\ GeV$

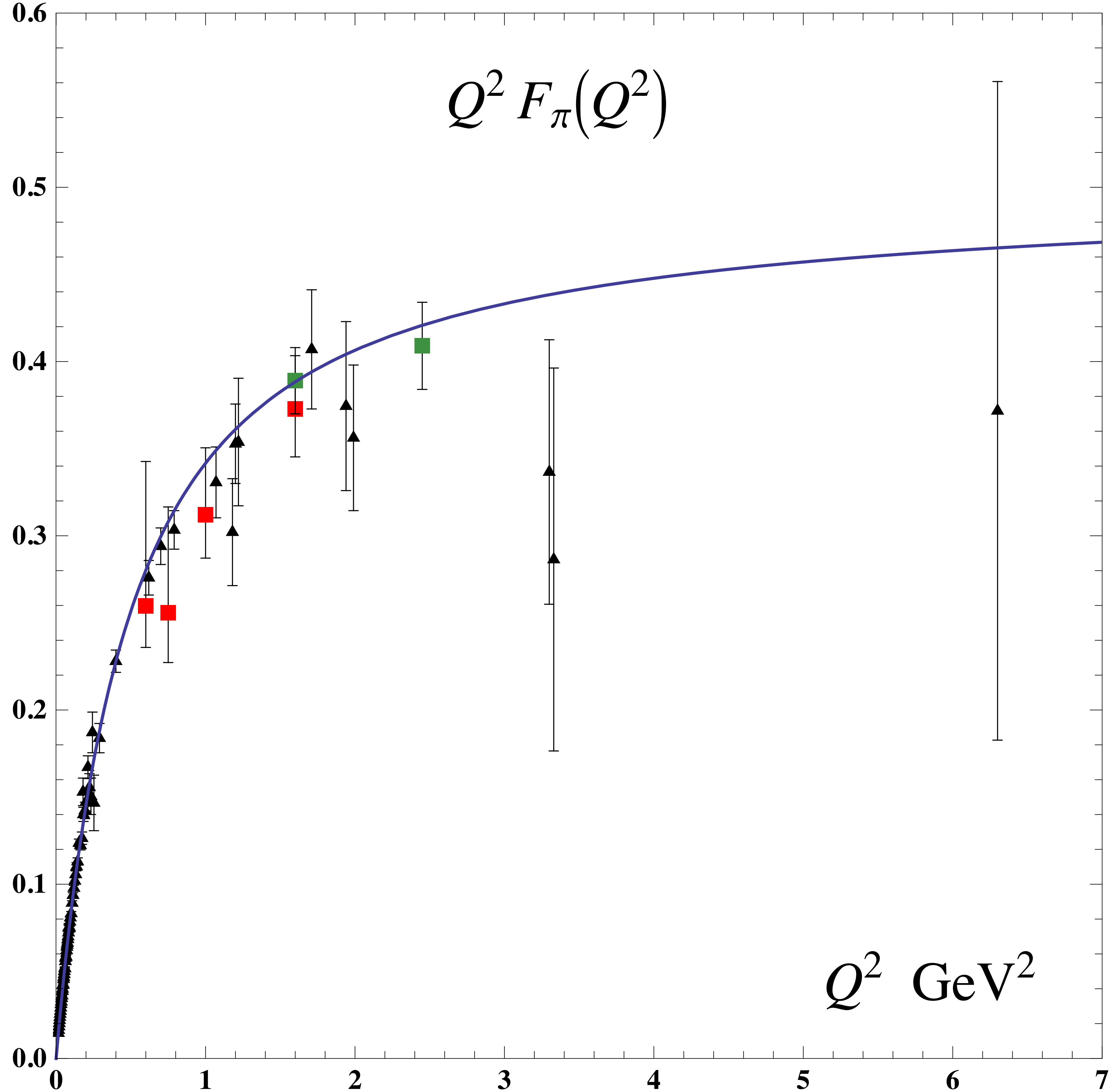
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici**

Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

Orbital and Radial Pseudoscalar and Vector Meson Excitations

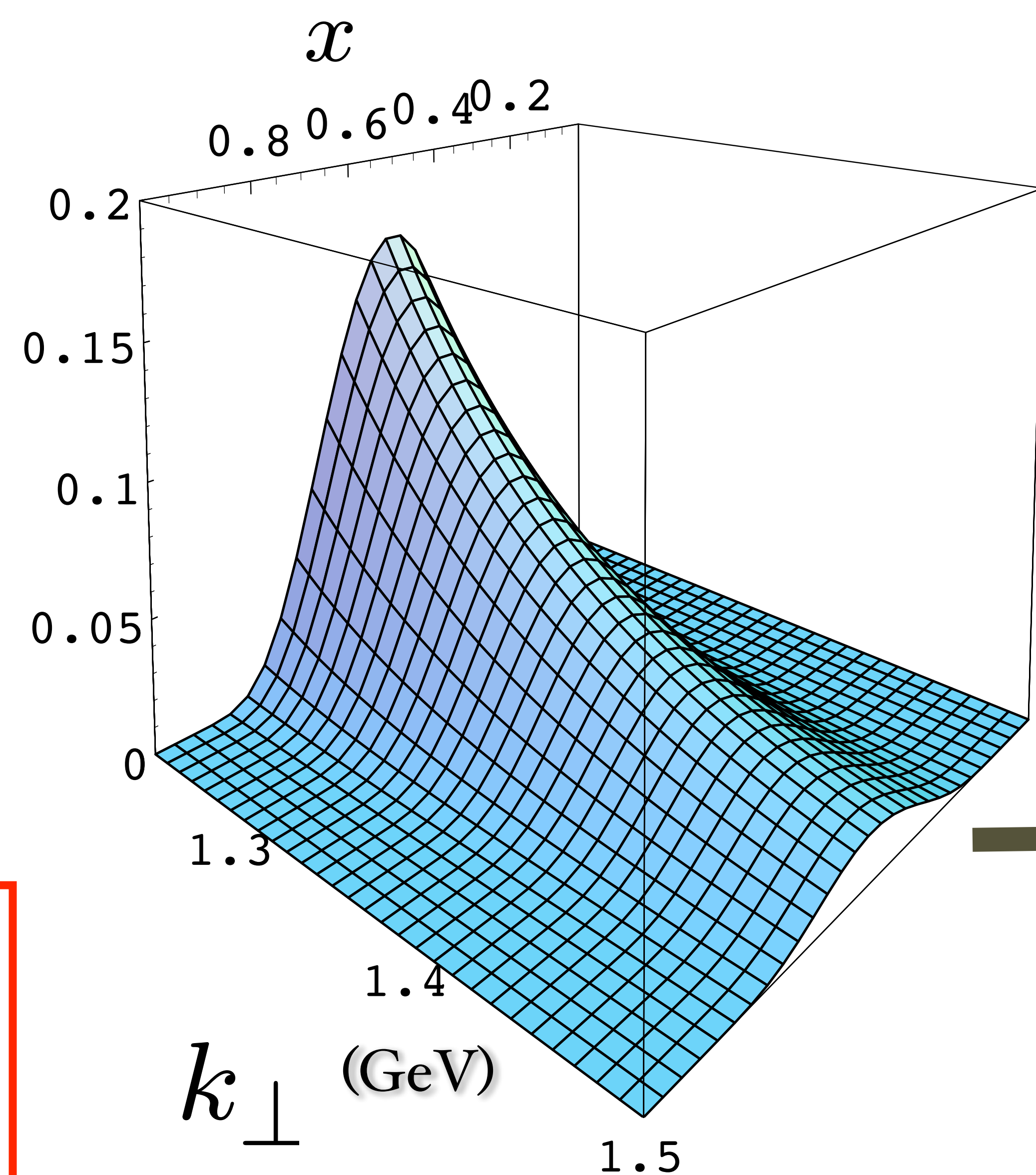


$Q^2 F_\pi(Q^2)$
Q^2 GeV^2
0.0
0.1
0.2
0.3
0.4
0.5
0.6
0
1
2
3
4
5
6
7

Prediction from AdS/QCD: Meson LFWF

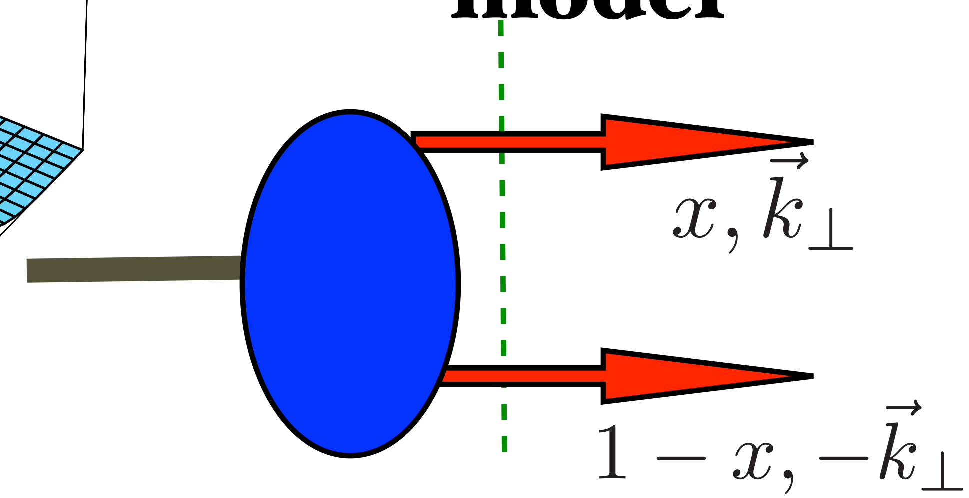
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

$$\psi_M(x, k_\perp^2)$$



de Teramond, Cao, sjb

"Soft Wall" model



massless quarks

Note coupling

$$k_\perp^2, x$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

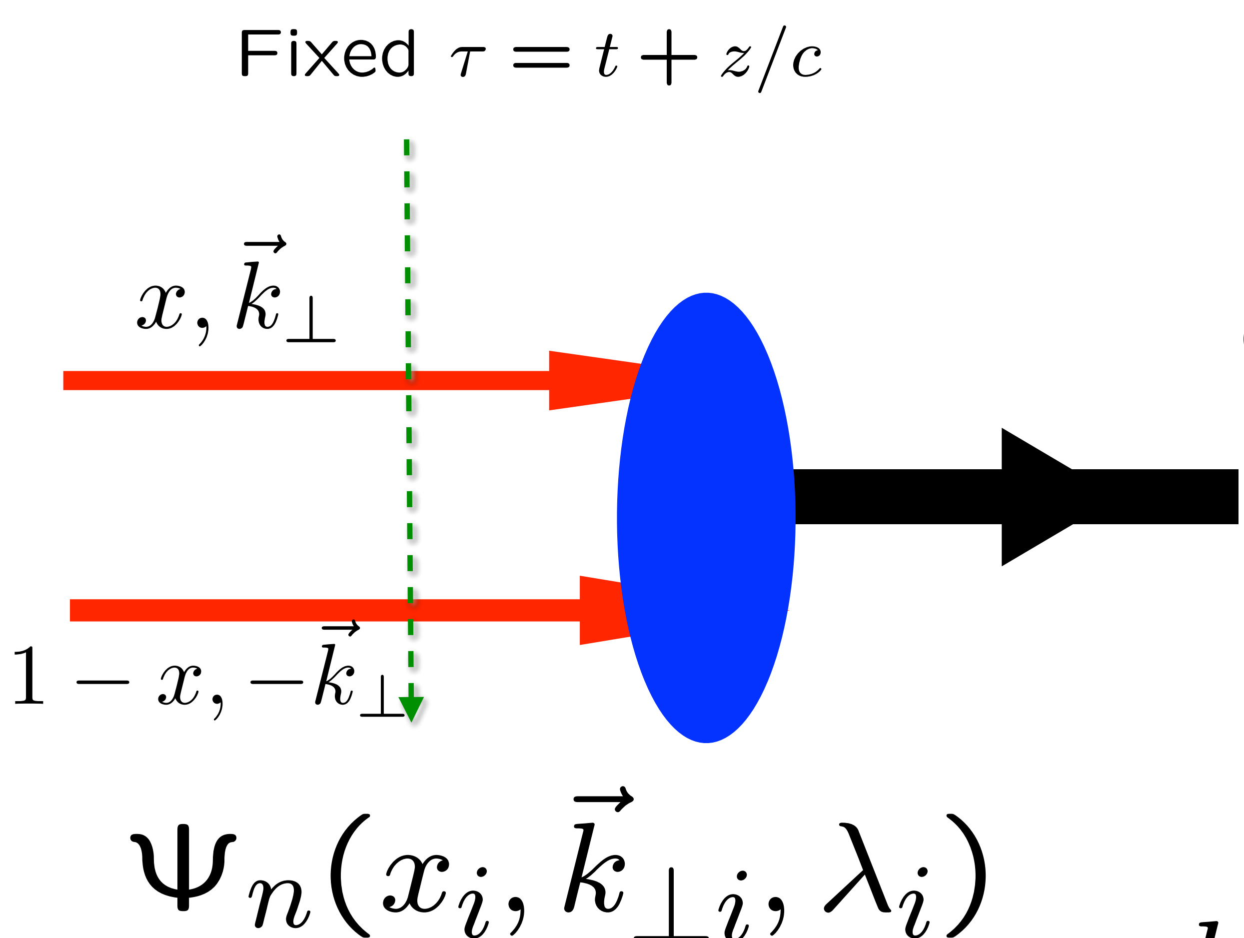
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

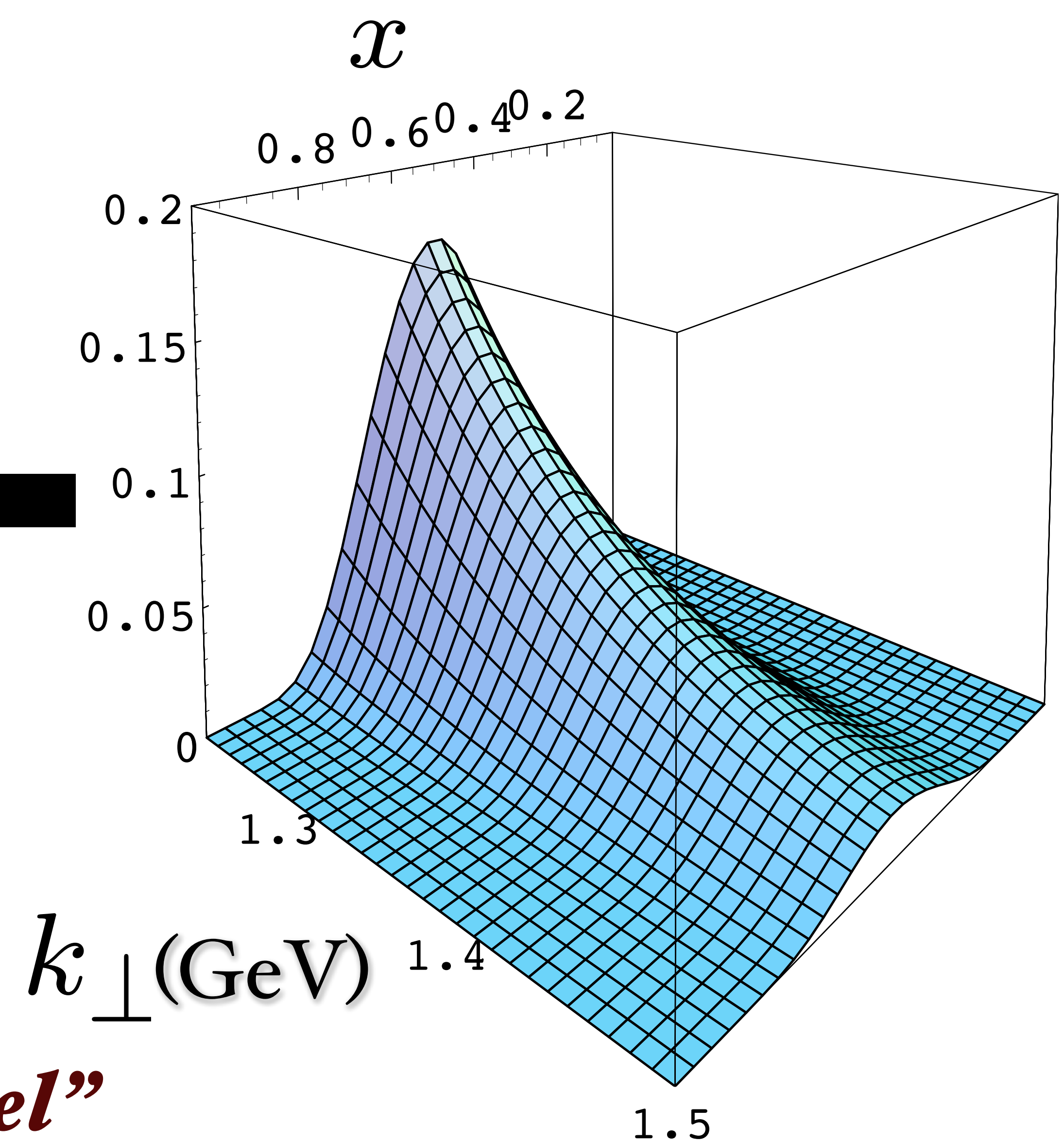
Same as DSE! **C. D. Roberts et al.**

Provides Connection of Confinement to Hadron Structure

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in P^- and invariant mass $\mathcal{M}^2_{q\bar{q}}$



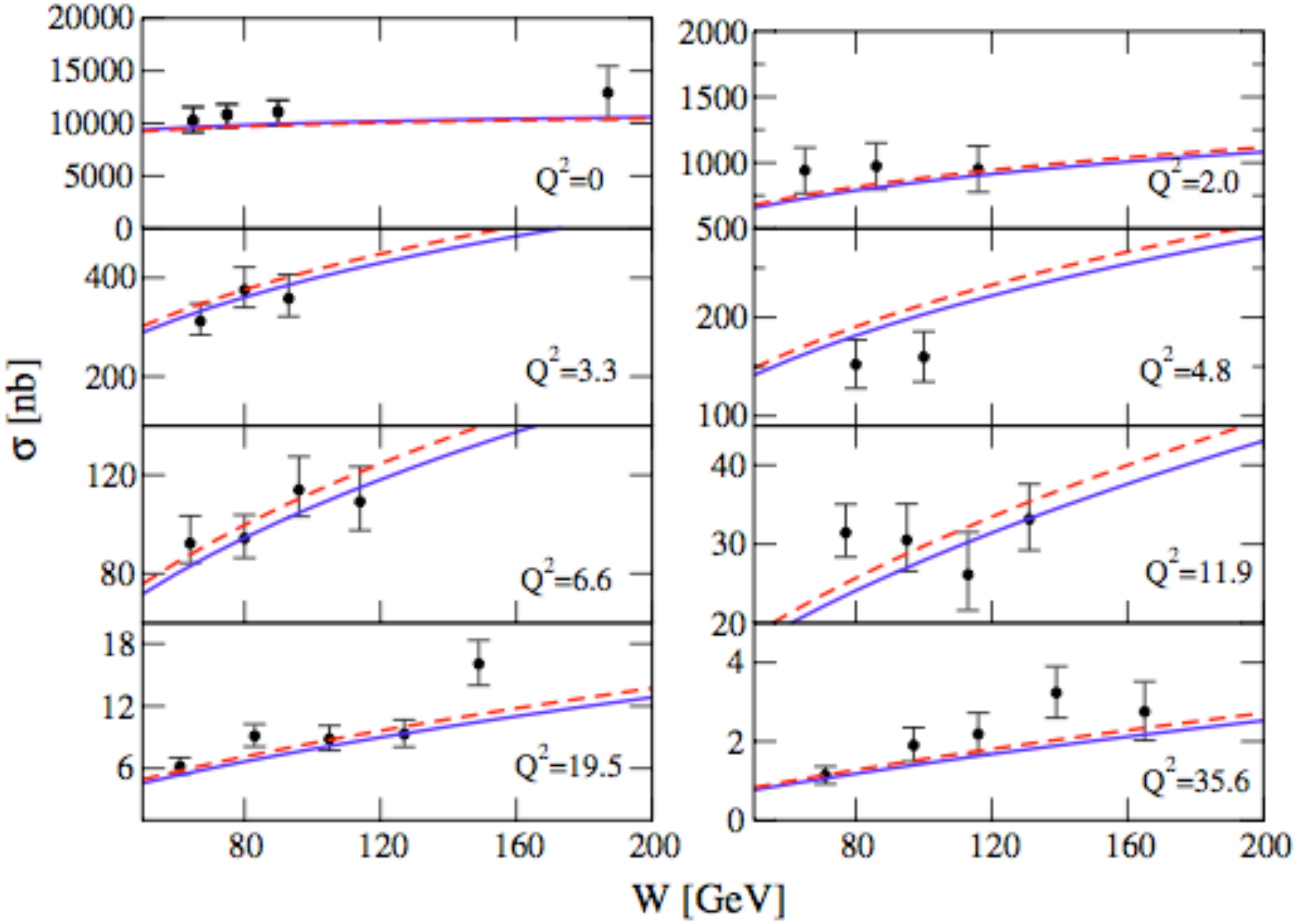


"Hadronization at the Amplitude Level"

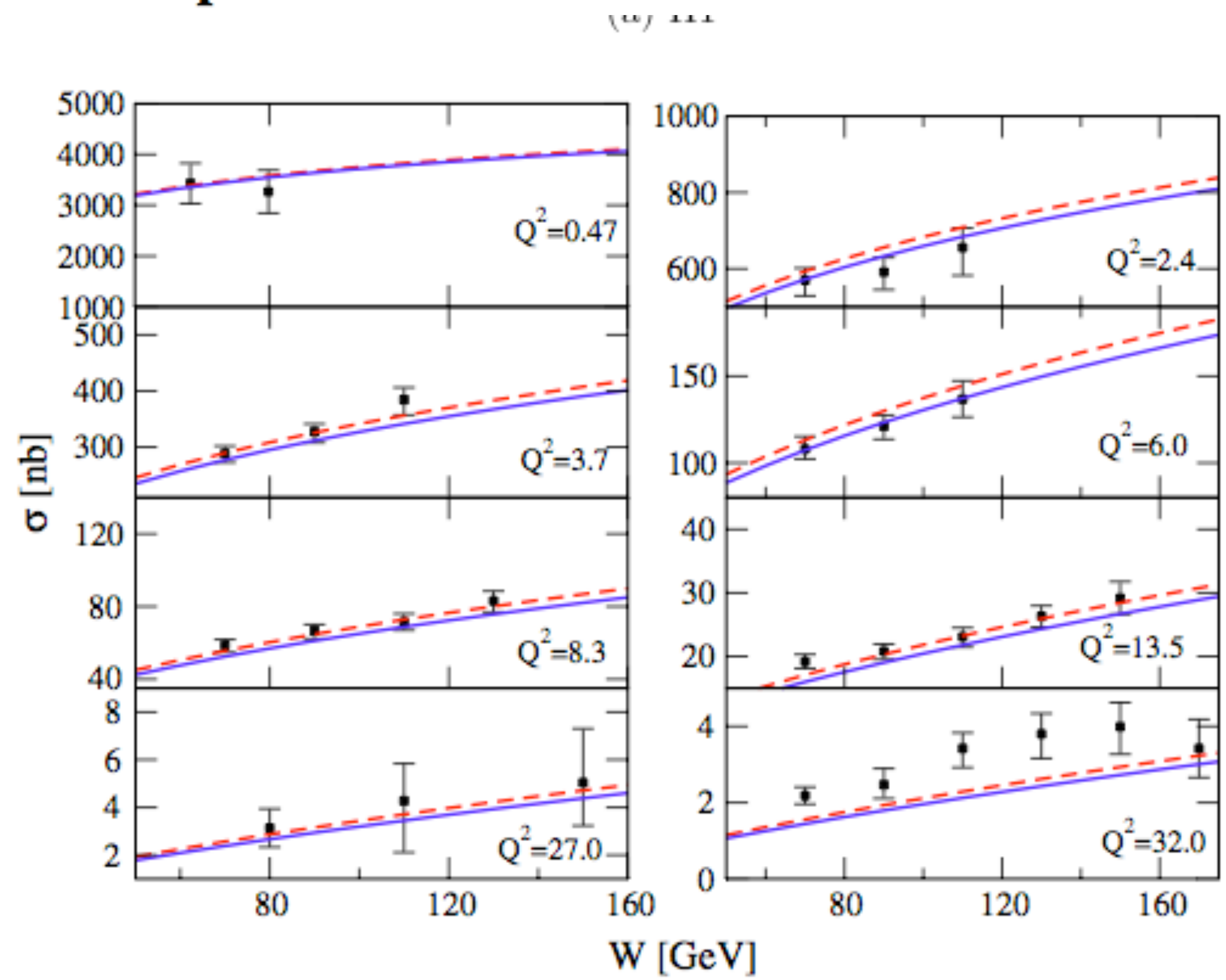
Boost-invariant LFWF connects confined quarks and gluons to hadrons

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

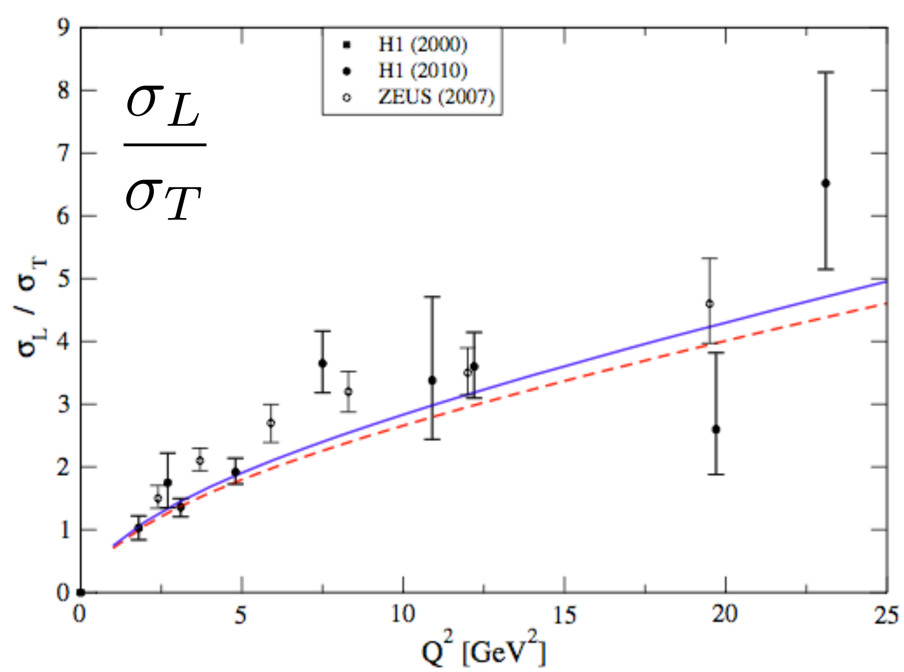
AdS/QCD Holographic Wave Function for the ρ Meson and Diffractive ρ Meson Electroproduction



(a) H1



(b) ZEUS



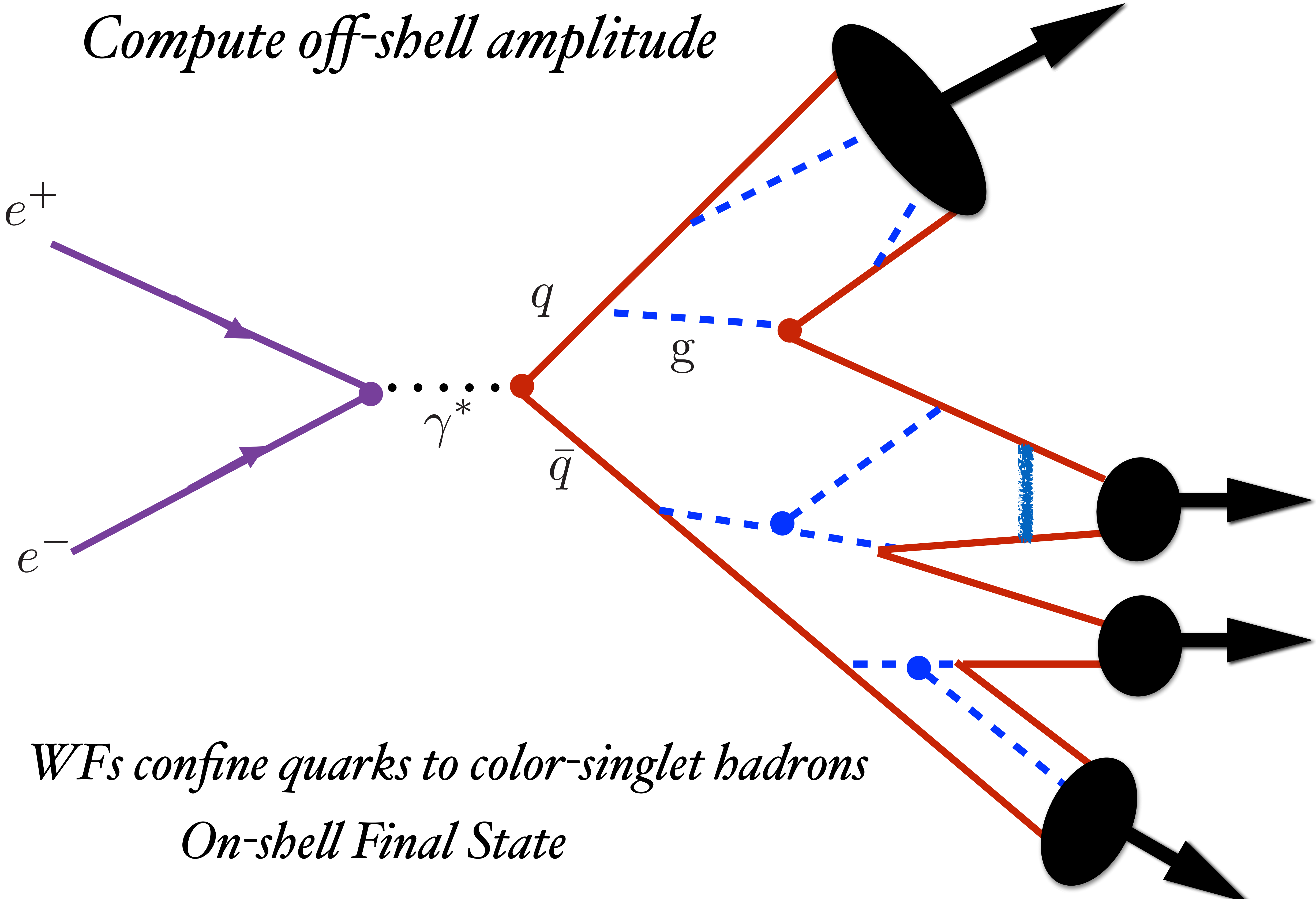
$$\frac{\sigma_L}{\sigma_T}$$

J. R. Forshaw, R. Sandapen

$$\gamma^* p \to \rho^0 p'$$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

Compute off-shell amplitude



WFs confine quarks to color-singlet hadrons

On-shell Final State

Quantum-Mechanical Entangled Amplitude

Remarkable Features of Light-Front Schrödinger Equation

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$\boxed{U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2 (L + S - 1)}$$

Interpretation of Mass Scale κ

- Does not affect conformal symmetry of QCD action
- Self-consistent regularization of IR divergences
- Determines all mass and length scales for zero quark mass
- Compute scheme-dependent $\Lambda_{\overline{MS}}$ determined in terms of κ
- Value of κ itself not determined -- place holder
- Need external constraint such as f_π

Haag, Lopuszanski, Sohnius (1974)

Fubini and Rabinovici

Superconformal Quantum Mechanics

de Teramond Dosch, Lorce, sjb

1+1

$$\{\psi, \psi^+\} = 1$$

two anti-commuting fermionic operators

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

Realization as Pauli Matrices

$$Q = \psi^+[-\partial_x + W(x)], \quad Q^+ = \psi[\partial_x + W(x)], \qquad W(x) = \frac{f}{x}$$

(Conformal)

$$S = \psi^+ x, \quad S^+ = \psi x$$

Introduce new spinor operators

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\boxed{Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}}$$

$$\{Q, Q\} = \{Q^+, Q^+\} = 0, \quad [Q, H] = [Q^+, H] = 0$$

(4) Superconformal quantum mechanics and light-front dynamics

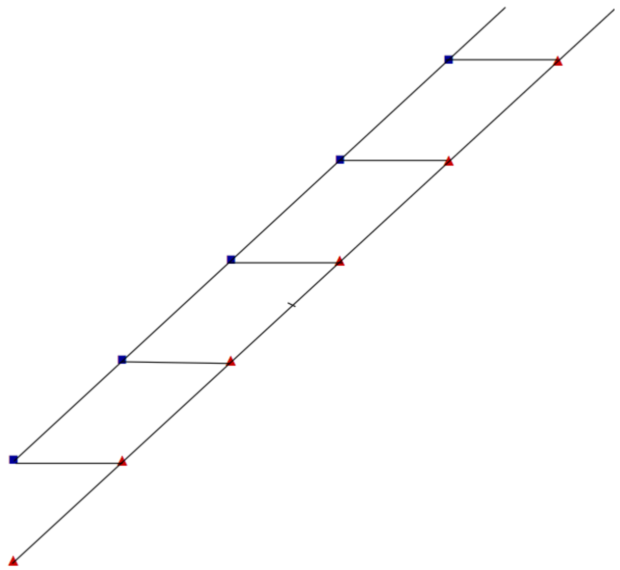
[GdT, H.G. Dosch and S. J. Brodsky, Phys. Rev. D **91**, 045040 (2015)]

- SUSY QM contains two fermionic generators Q and $Q^\dagger$, and a bosonic generator, the Hamiltonian H

  [E. Witten, NPB **188**, 513 (1981)]

- Closure under the graded algebra $sl(1/1)$:

$$\frac{1}{2}\{Q, Q^\dagger\} = H$$
$$\{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0$$
$$[Q, H] = [Q^\dagger, H] = 0$$



Note: Since $[Q^\dagger, H] = 0$ the states $|E\rangle$ and $Q^\dagger|E\rangle$ have identical eigenvalues E

- A simple realization is

$$Q = \chi\,(ip + W)\,, \qquad Q^\dagger = \chi^\dagger\,(-ip + W)$$

where χ and $\chi^\dagger$ are spinor operators with anticommutation relation

$$\{\chi, \chi^\dagger\} = 1$$

- In a 2×2 Pauli-spin matrix representation: $\chi = \frac{1}{2}\,(\sigma_1 + i\sigma_2)\,, \;\; \chi^\dagger = \frac{1}{2}\,(\sigma_1 - i\sigma_2)$

$$[\chi, \chi^\dagger] = \sigma_3$$

- Following Fubini and Rabinovici consider a 1-dim QFT invariant under conformal and supersymmetric transformations [S. Fubini and E. Rabinovici, NPB **245**, 17 (1984)]

- Conformal superpotential (f is dimensionless)

$$W(x) = \frac{f}{x}$$

- Thus 1-dim QFT representation of the operators

$$Q \;=\; \chi\left(\frac{d}{dx} + \frac{f}{x}\right), \qquad Q^\dagger = \chi^\dagger\left(-\frac{d}{dx} + \frac{f}{x}\right)$$

- Conformal Hamiltonian $H = \frac{1}{2}\{Q, Q^\dagger\}$ in matrix form

$$H = \frac{1}{2}\begin{pmatrix} -\frac{d^2}{dx^2} + \frac{f(f-1)}{x^2} & 0 \\ 0 & -\frac{d^2}{dx^2} + \frac{f(f+1)}{x^2} \end{pmatrix}$$

- Conformal graded-Lie algebra has in addition to Hamiltonian H and supercharges Q and $Q^\dagger$, a new operator S related to generator of conformal transformations K

$$S = \chi\, x, \qquad S^\dagger = \chi^\dagger x$$

- Find enlarged algebra (Superconformal algebra of Haag, Lopuszanski and Sohnius (1974))

$$\begin{aligned}
\tfrac{1}{2}\{Q,Q^\dagger\} &= H, \qquad \tfrac{1}{2}\{S,S^\dagger\} = K \\
\tfrac{1}{2}\{Q,S^\dagger\} &= \frac{f}{2} + \frac{\sigma_3}{4} + iD \\
\tfrac{1}{2}\{Q^\dagger,S\} &= \frac{f}{2} + \frac{\sigma_3}{4} - iD
\end{aligned}$$

where the operators

$$\begin{aligned}
H &= \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{f^2 - \sigma_3 f}{x^2}\right) \\
D &= \frac{i}{4}\left(\frac{d}{dx}x + x\frac{d}{dx}\right) \\
K &= \tfrac{1}{2}x^2
\end{aligned}$$

satisfy the conformal algebra

$$[H,D] = iH, \quad [H,K] = 2iD, \quad [K,D] = -iK$$

- Following F&R define a supercharge R, a linear combination of the generators Q and S

$$R = \sqrt{u}\, Q + \sqrt{w}\, S$$

and consider the new generator $G = \frac{1}{2}\{R, R^\dagger\}$ which also closes under the graded algebra

$$\begin{aligned} &\tfrac{1}{2}\{R, R^\dagger\} = G && \tfrac{1}{2}\{Q, Q^\dagger\} = H \\ &\{R, R\} = \{R^\dagger, R^\dagger\} = 0 && \{Q, Q\} = \{Q^\dagger, Q^\dagger\} = 0 \\ &[R, H] = [R^\dagger, H] = 0 && [Q, H] = [Q^\dagger, H] = 0 \end{aligned}$$

- New QM evolution operator

$$G = uH + wK + \tfrac{1}{2}\sqrt{uw}\,(2f + \sigma_3)$$

is compact for $uw > 0$: Emergence of a scale since Q and S have different units

- Light-front extension of superconformal results follows from

$$x \to \zeta, \quad f \to \nu + \tfrac{1}{2}, \quad \sigma_3 \to \gamma_5, \quad 2G \to H_{LF}$$

- Obtain:

$$H_{LF} = -\frac{d^2}{d\zeta^2} + \frac{\left(\nu + \frac{1}{2}\right)^2}{\zeta^2} - \frac{\nu + \frac{1}{2}}{\zeta^2}\gamma_5 + \lambda^2\zeta^2 + \lambda(2\nu + 1) + \lambda\gamma_5$$

where coefficients u and w are fixed to $u = 2$ and $w = 2\lambda^2$

- Take the 'square root' of the LF Hamiltonian $H_{LF} = \{R, R^\dagger\}$

$$H_{LF}\, \psi = D_{LF}^2 \psi = M^2 \psi$$

with the linear Dirac equation

$$\left(D_{LF} - M\right) \psi = 0$$

- In a 2×2 component representation $\psi_\pm$

$$\begin{aligned} - \frac{d}{d\zeta}\psi_- - \frac{\nu + \frac{1}{2}}{\zeta}\psi_- - \lambda\, \zeta \psi_- &= M\psi_+ \\ \frac{d}{d\zeta}\psi_+ - \frac{\nu + \frac{1}{2}}{\zeta}\psi_+ - \lambda\, \zeta \psi_+ &= M\psi_- \end{aligned}$$

where the chiral spinors are defined by $\psi_\pm = \frac{1}{2}\left(1 \pm \gamma_5\right) \psi$

- Note: In a 4×4 Dirac-matrix representation the spinor operators χ and $\chi^\dagger$ satisfy the relations

$$\{\chi, \chi^\dagger\} = 1 \quad \text{and} \quad [\chi, \chi^\dagger] = \gamma_5$$

Haag, Lopuszanski, Sohnius (1974)

Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

Superconformal Quantum Mechanics

Fubini and Rabinovici

Baryon Equation

$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

$\boxed{\text{Consider } R_w = Q + wS;}$

w: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

Fubini and Rabinovici

New Extended Hamiltonian G is diagonal:

$$G_{11} = \Big(- \partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f + \frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$G_{22} = \Big(- \partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f - \frac{1}{2})^2 - 1}{4x^2} \Big)$$

$$\boxed{\text{Identify } f - \tfrac{1}{2} = L_B \ , \quad w = \kappa^2}$$

Eigenvalue of G: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ!

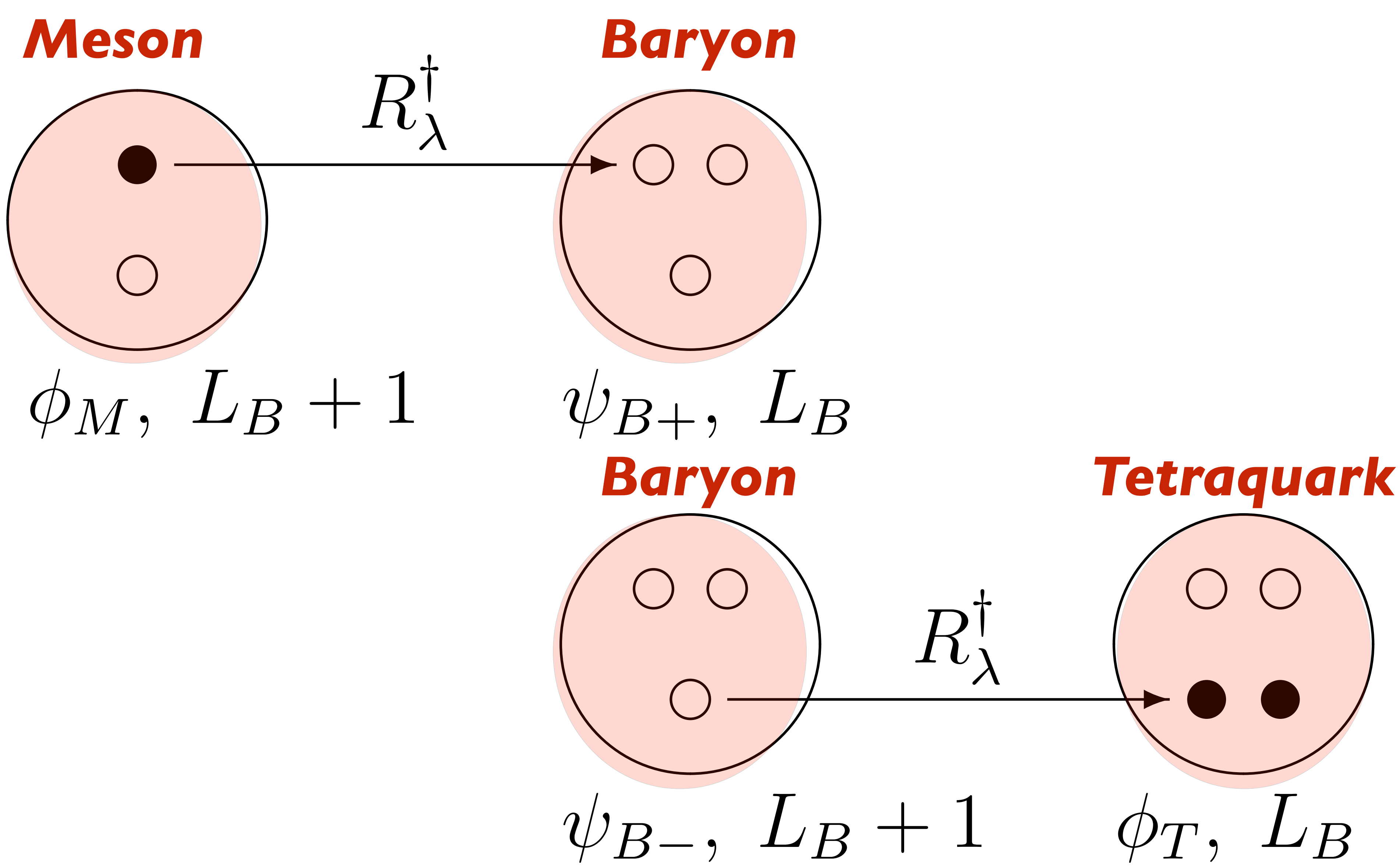
S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

Meson-Baryon Degeneracy for $L_M = L_B + 1$

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!

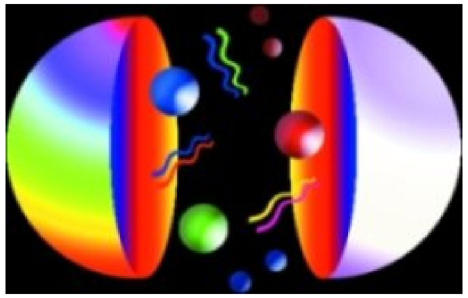


Proton: quark + scalar diquark $|q(qq)>$
(Equal weight: $L=0, L=1$)

Fermionic Modes and Baryon Spectrum

[Hard wall model: GdT and S. J. Brodsky, PRL **94**, 201601 (2005)]

[Soft wall model: GdT and S. J. Brodsky, (2005), arXiv:1001.5193]



From Nick Evans

- Nucleon LF modes

$$\psi_+(\zeta)_{n,L} = \kappa^{2+L}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{3/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+1}\left(\kappa^2\zeta^2\right)$$

$$\psi_-(\zeta)_{n,L} = \kappa^{3+L}\frac{1}{\sqrt{n+L+2}}\sqrt{\frac{2n!}{(n+L)!}}\zeta^{5/2+L}e^{-\kappa^2\zeta^2/2}L_n^{L+2}\left(\kappa^2\zeta^2\right)$$

- Normalization

$$\int d\zeta\,\psi_+^2(\zeta) = \int d\zeta\,\psi_-^2(\zeta) = 1$$

- Eigenvalues

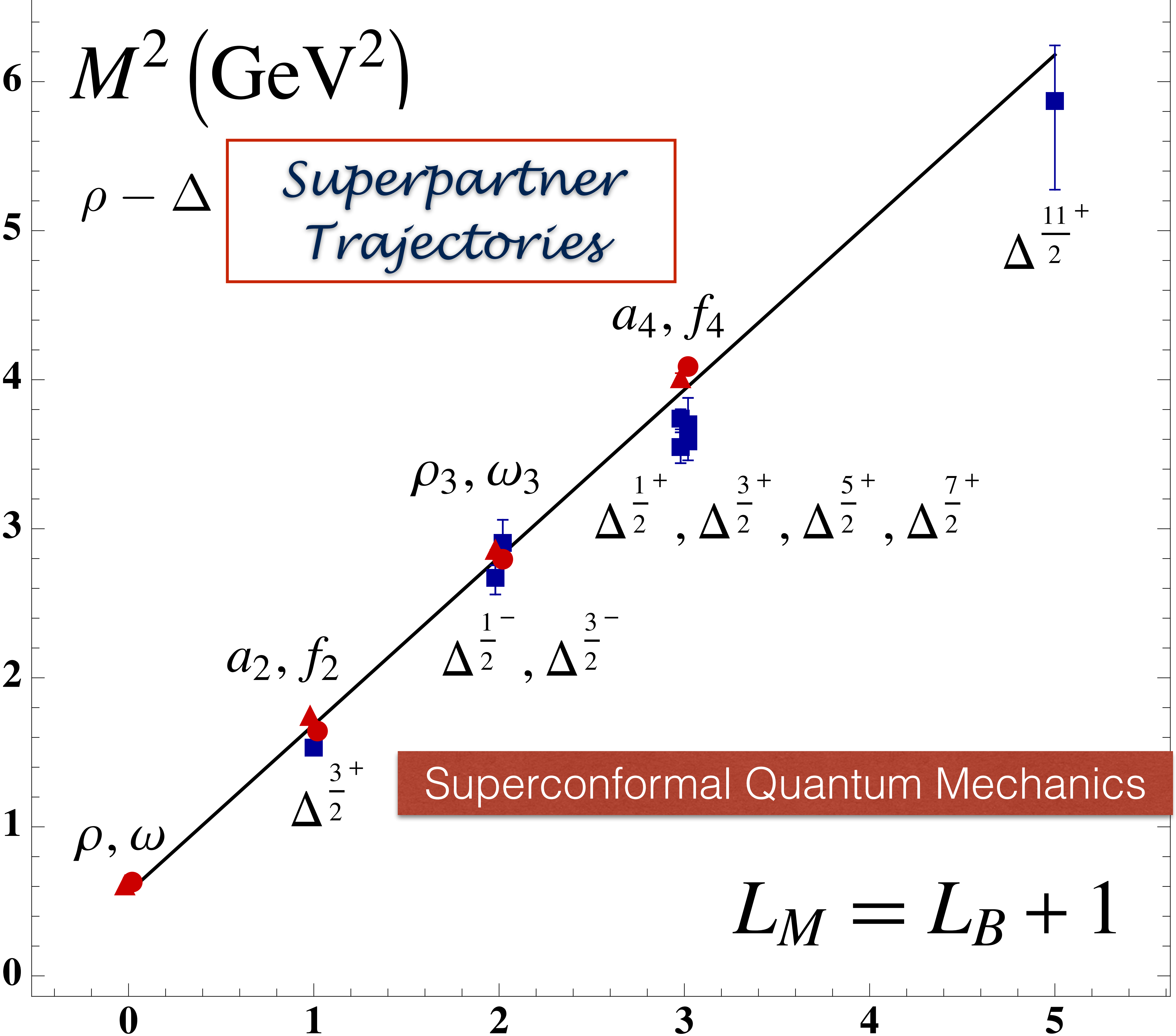
$$\mathcal{M}^2_{n,L,S=1/2} = 4\kappa^2\left(n+L+1\right)$$

- “Chiral partners”

$$\frac{\mathcal{M}_{N(1535)}}{\mathcal{M}_{N(940)}} = \sqrt{2}$$

Quark Chiral Symmetry of Eigenstate!

Nucleon: Equal Probability for L=0,1



de Tèramond, Dosch, Lorce, sjb

Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

AdS/QCD + Light Front Holography: Proton is bound state of a quark + scalar diquark

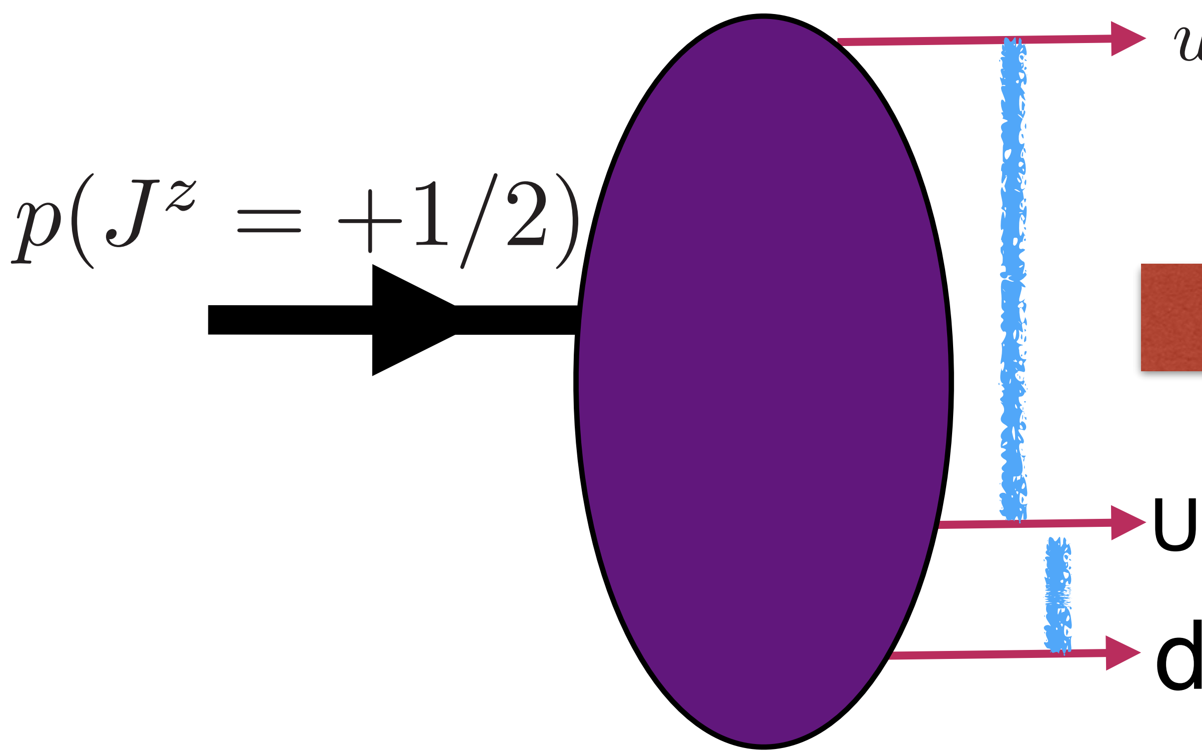
de Teramond, Dosch, Lorce, sjb

Skyrme model: Ellis, Karliner, sjb

LF J^z conservation: K. Chiu, sjb

$$3_C \times 3_C = \bar{3}_C + \cancel{6}_C$$

$$|p> = |u_{3C}[ud]_{\bar{3}C}>$$



$u(S^z = +1/2)$ $u(S^z = -1/2)$

$L^z = 0$ $L^z = -1$

Equal probability L=0, L=1

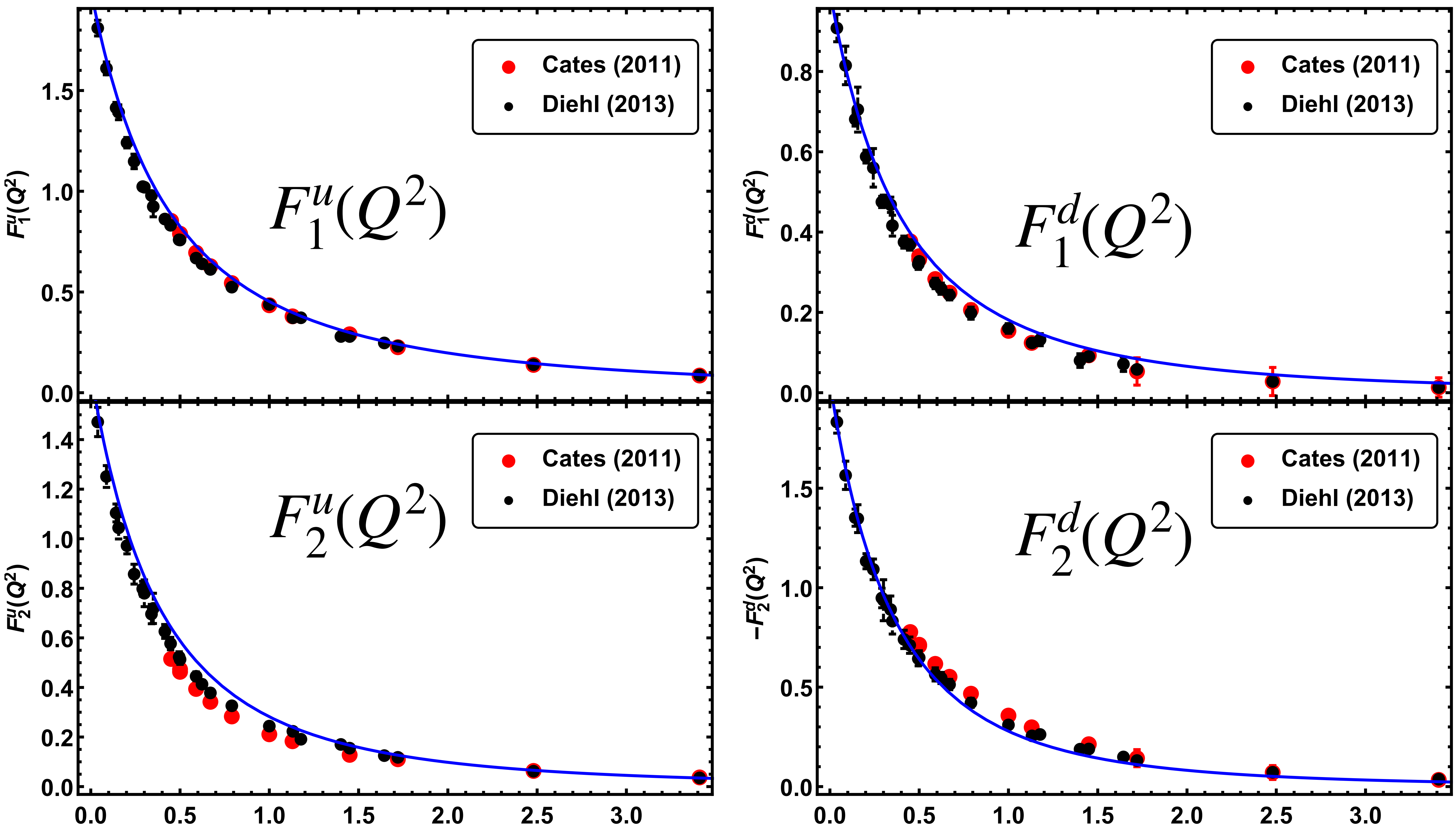
J=0

Quark chiral symmetry
Anomalous moment nonzero
Leading Twist Sivers Effect

Gluonic distribution reflects quark+diquark color structure of the proton

Color confinement potential —> high density gluon field: flux tube

LFHQCD predictions for Nucleon Form Factors



From Neetika Sharma

Collisions of flux tubes of protons

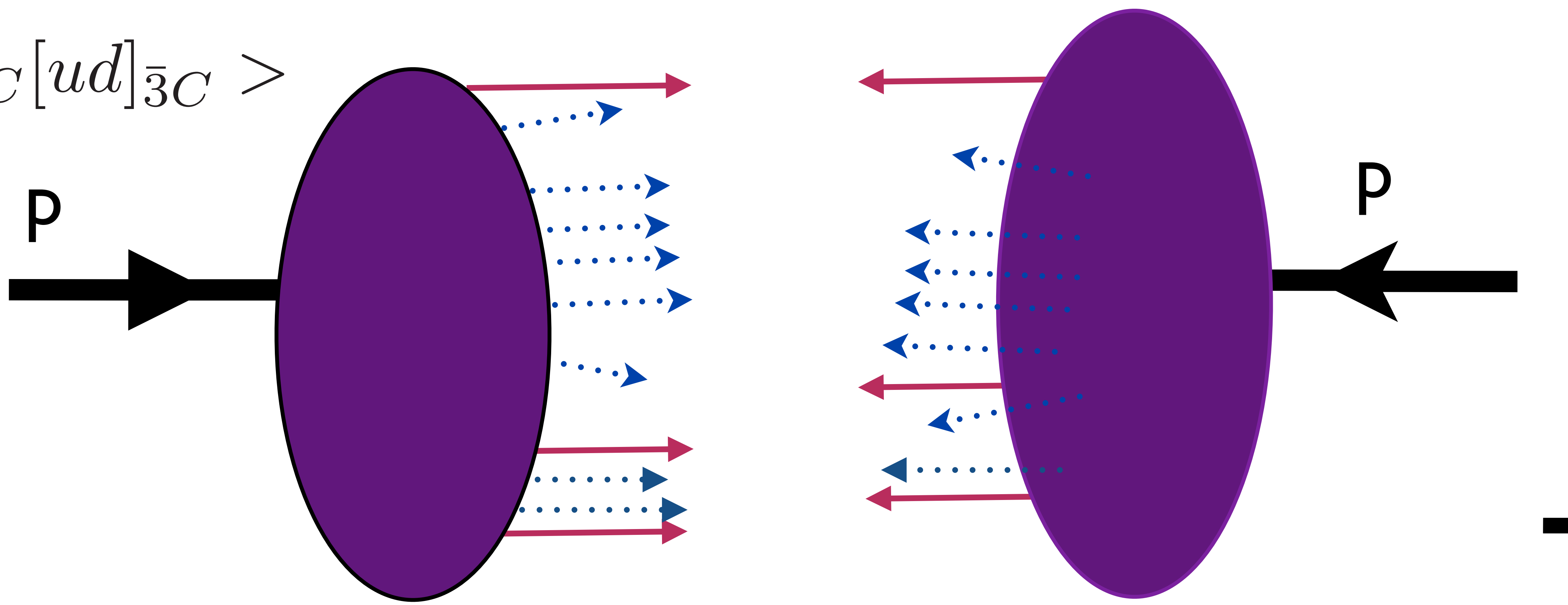
Color confinement potential —> high density gluon field: flux tube

Highest hadron multiplicity produced when the two flux tubes are aligned and overlap completely along their length.

Bjorken, Goldhaber, sjb

Hadrons produced from the collisions of flux tubes

$$|p>= |u_{3C}[ud]_{\bar{3}C}>$$



Gluonic distribution reflects quark+diquark color structure of the protons

v_2 (dominant) + v_3 (from `Y' quark + diquark configurations)

- Strangeness and charm enhancements

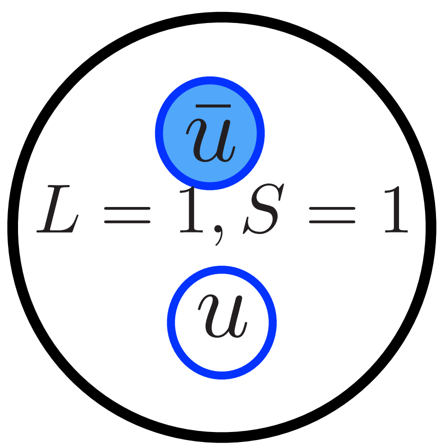
Superconformal Algebra 4-Plet

$R_\lambda^\dagger \quad \bar{q} \to (qq) \quad S=1$

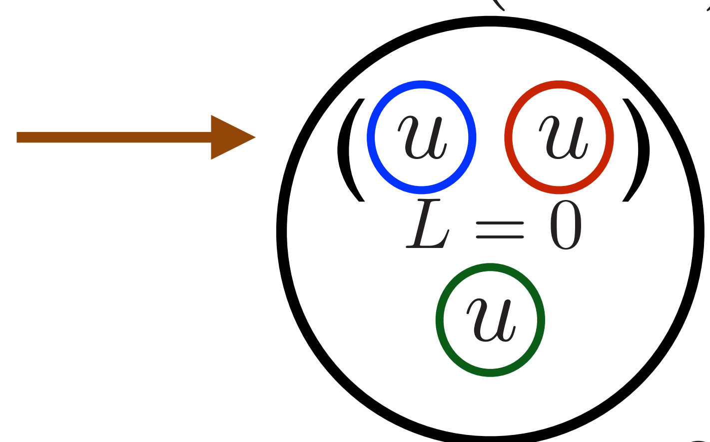
$\bar{3}_C \to \bar{3}_C$

Vector ()+ Scalar [] Diquarks

$f_2(1270)$



$\Delta^+(1232)$



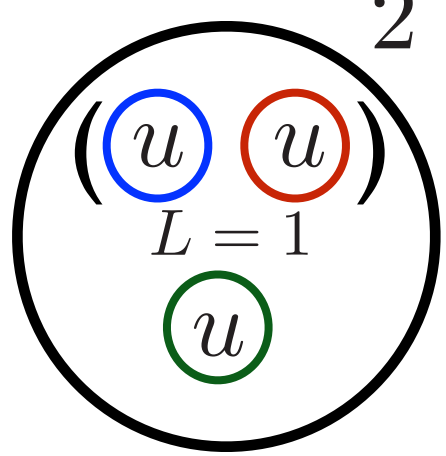
Tetraquark

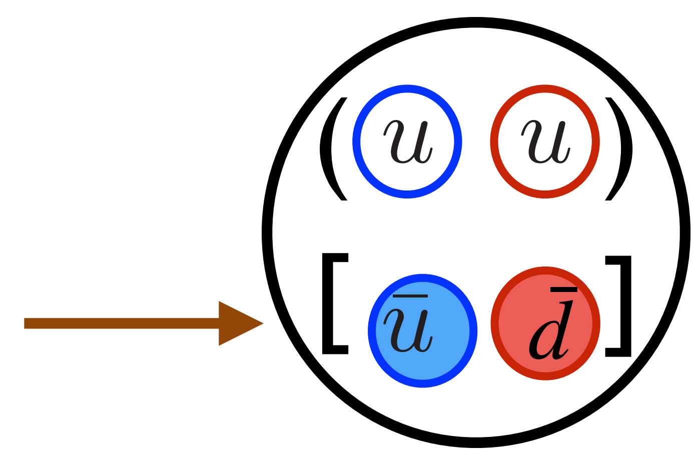
$J^{PC} = 1^{++}$

$a_1(1260)$

$J^P = \frac{3}{2}^+$

$J^{PC} = 2^{++}$





$S=0$

$L=0$

$R_\lambda^\dagger \quad q \to [\bar{q}\bar{q}]$

$3_C \to 3_C$

Meson **Baryon**

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}q$	0^{-+}	$\pi(140)$	—	—	—	—	—	—
$\bar{q}q$	1^{+-}	$h_1(1170)$	$[ud]q$	$(1/2)^+$	$N(940)$	$[ud][\bar{u}\bar{d}]$	0^{++}	$\sigma(500)$
$\bar{q}q$	2^{-+}	$\eta_2(1645)$	$[ud]q$	$(3/2)^-$	$N_{\frac{3}{2}^-}(1520)$	$[ud][\bar{u}\bar{d}]$	1^{-+}	—
$\bar{q}q$	1^{--}	$\rho(770),\ \omega(780)$	—	—	—	—	—	—
$\bar{q}q$	2^{++}	$a_2(1320),\ f_2(1270)$	$(qq)q$	$(3/2)^+$	$\Delta(1232)$	$(qq)[\bar{u}\bar{d}]$	1^{++}	$a_1(1260)$
$\bar{q}q$	3^{--}	$\rho_3(1690),\ \omega_3(1670)$	$(qq)q$	$(3/2)^-$	$\Delta_{\frac{3}{2}^-}(1700)$	$(qq)[\bar{u}\bar{d}]$	1^{-+}	$\pi_1(1600)$
$\bar{q}q$	4^{++}	$a_4(2040),\ f_4(2050)$	$(qq)q$	$(7/2)^+$	$\Delta_{\frac{7}{2}^+}(1950)$	$(qq)[\bar{u}\bar{d}]$	—	—
$\bar{q}s$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{q}s$	1^+	$\bar{K}_1(1270)$	$[ud]s$	$(1/2)^+$	$\Lambda(1115)$	$[ud][\bar{s}\bar{q}]$	0^+	$K_0^*(1430)$
$\bar{q}s$	2^-	$K_2(1770)$	$[ud]s$	$(3/2)^-$	$\Lambda(1520)$	$[ud][\bar{s}\bar{q}]$	1^-	—
$\bar{s}q$	0^-	$K(495)$	—	—	—	—	—	—
$\bar{s}q$	1^+	$K_1(1270)$	$[sq]q$	$(1/2)^+$	$\Sigma(1190)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$a_0(980)$ $f_0(980)$
$\bar{s}q$	1^-	$K^*(890)$	—	—	—	—	—	—
$\bar{s}q$	2^+	$K_2^*(1430)$	$(sq)q$	$(3/2)^+$	$\Sigma(1385)$	$(sq)[\bar{u}\bar{d}]$	1^+	$K_1(1400)$
$\bar{s}q$	3^-	$K_3^*(1780)$	$(sq)q$	$(3/2)^-$	$\Sigma(1670)$	$(sq)[\bar{u}\bar{d}]$	2^-	$K_2(1820)$
$\bar{s}q$	4^+	$K_4^*(2045)$	$(sq)q$	$(7/2)^+$	$\Sigma(2030)$	$(sq)[\bar{u}\bar{d}]$	—	—
$\bar{s}s$	0^{-+}	$\eta'(958)$	—	—	—	—	—	—
$\bar{s}s$	1^{+-}	$h_1(1380)$	$[sq]s$	$(1/2)^+$	$\Xi(1320)$	$[sq][\bar{s}\bar{q}]$	0^{++}	$f_0(1370)$ $a_0(1450)$
$\bar{s}s$	2^{-+}	$\eta_2(1870)$	$[sq]s$	$(3/2)^-$	$\Xi(1620)$	$[sq][\bar{s}\bar{q}]$	1^{-+}	—
$\bar{s}s$	1^{--}	$\Phi(1020)$	—	—	—	—	—	—
$\bar{s}s$	2^{++}	$f_2'(1525)$	$(sq)s$	$(3/2)^+$	$\Xi^*(1530)$	$(sq)[\bar{s}\bar{q}]$	1^{++}	$f_1(1420)$ $a_1(1420)$
$\bar{s}s$	3^{--}	$\Phi_3(1850)$	$(sq)s$	$(3/2)^-$	$\Xi(1820)$	$(sq)[\bar{s}\bar{q}]$	—	—
$\bar{s}s$	2^{++}	$f_2(1640)$	$(ss)s$	$(3/2)^+$	$\Omega(1672)$	$(ss)[\bar{s}\bar{q}]$	1^+	$K_1(1650)$

Meson Baryon Tetraquark

New Organization of the Hadron Spectrum

M. Nielsen, sjb

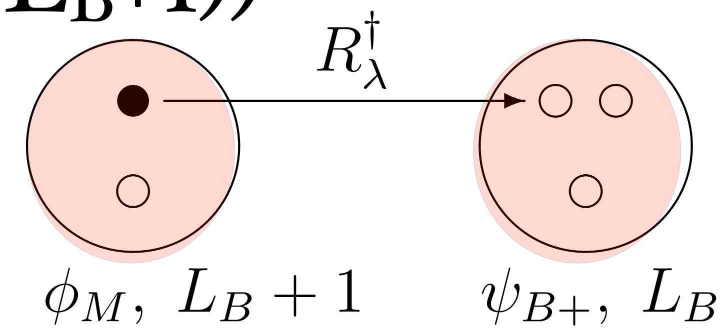
de Tèramond, Dosch, Lorce, sjb

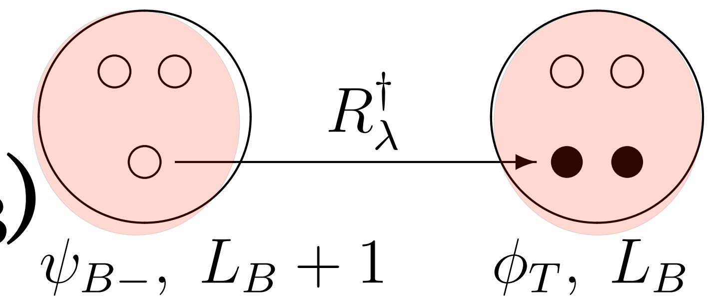
Superconformal Algebra

2X2 Hadronic Multiplets

$$\begin{pmatrix} \phi_M(L_M = L_B + 1) & \psi_{B-}(L_B + 1) \\ \psi_{B+}(L_B) & \phi_T(L_T = L_B) \end{pmatrix}$$

- quark-antiquark meson ($L_M = L_B$+1))
- quark-diquark baryon (L_B)
- quark-diquark baryon (L_B+1)
- diquark-antidiquark tetraquark ($L_T = L_B$)
- Universal Regge slopes $\lambda = \kappa^2$





$$M^2_H/\lambda = \overbrace{\underbrace{(2n + L_H + 1)}_{kinetic} + \underbrace{(2n + L_H + 1)}_{potential}}^{contribution\ from\ 2\text{-}dim\ light\text{-}front\ harmonic\ oscillator} + \overbrace{2(L_H + s) + 2\chi}^{contribution\ from\ AdS\ and\ superconformal\ algebra} + < \sum_i \frac{m_i^2}{x_i} >$$

$\chi(mesons) = -1 \qquad \chi(baryons, tetraquarks) = +1$

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

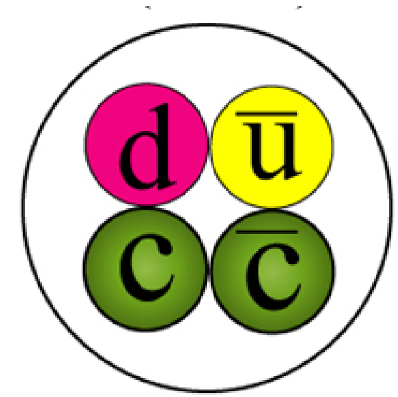
Bound!

- Diquark Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states
- Confinement Force Similar to quark-antiquark mesons $\bar{3}_C \times 3_C = 1_C$
- Isospin $I = 0, \pm 1, \pm 2$ Charge $Q = 0, \pm 1, \pm 2$

Complete Regge spectrum in n, L

$uu\bar{d}\bar{d}$ $\quad$ $uu\bar{s}\bar{d}$ $\quad$ $uu\bar{s}\bar{s}$

Q= +2



Q= - 1

de Tèramond, Dosch, Lorce, sjb

New World of Tetraquarks

$$3_C \times 3_C = \bar{3}_C + 6_C$$

Bound!

- Diquark: Color-Confined Constituents: Color $\bar{3}_C$
- Diquark-Antidiquark bound states $\bar{3}_C \times 3_C = 1_C$
- $$\sigma(TN) \simeq 2\sigma(pN) - \sigma(\pi N)$$

$$2\left[\sigma([\{qq\}N) + \sigma(qN)\right] - \left[\sigma(qN) + \sigma(\bar{q}N)\right] = \left[\sigma(\{qq\}N) + \sigma(\{qq\}N)\right]$$

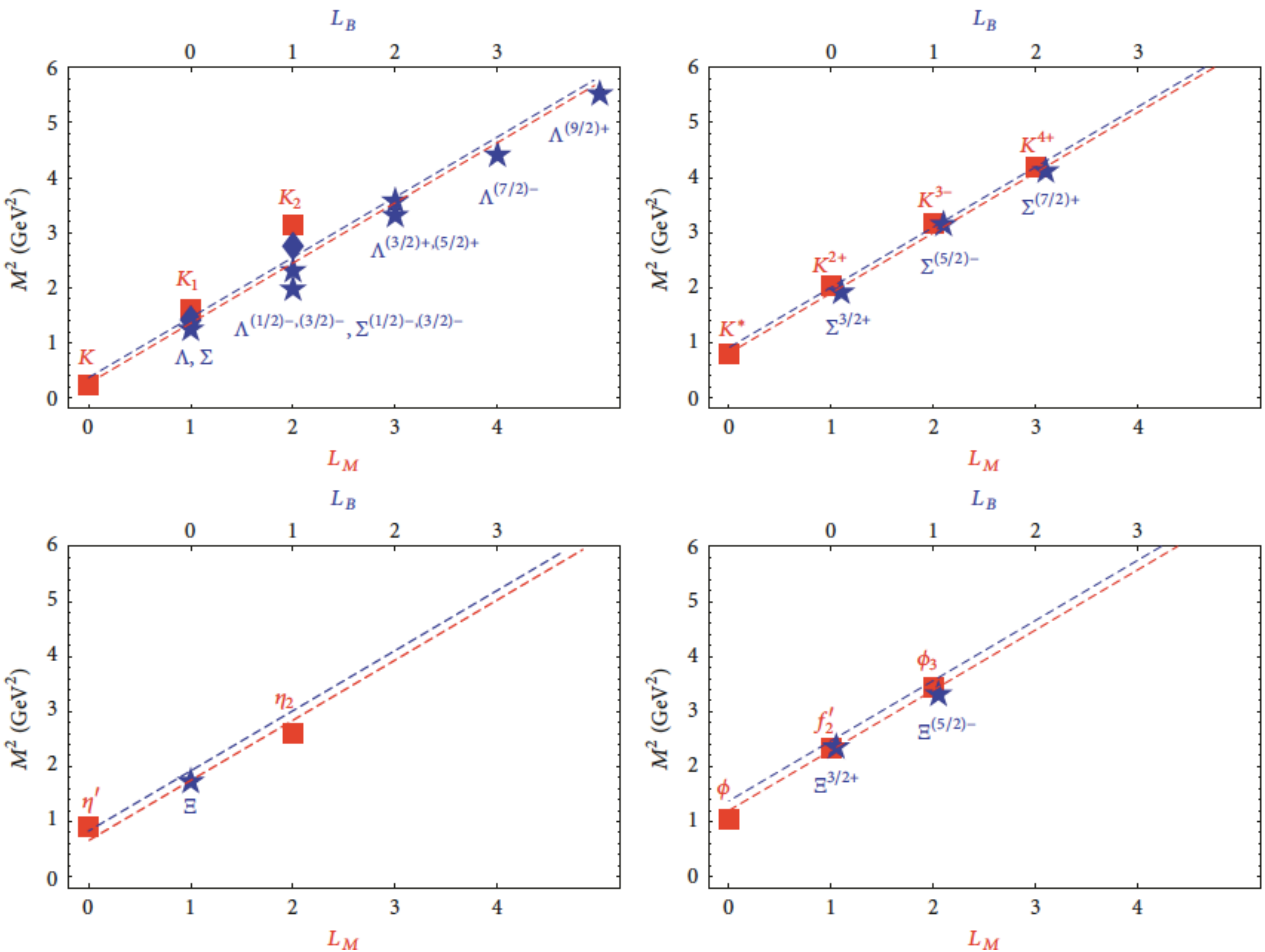
$$\text{Candidates } f_0(980) I = 0, J^P = 0^+, \text{ partner of proton}$$

$$a_1(1260) I = 0, J^P = 1^+, \text{ partner of } \Delta(1233)$$

Test twist=4, power-law fall-off of form factors

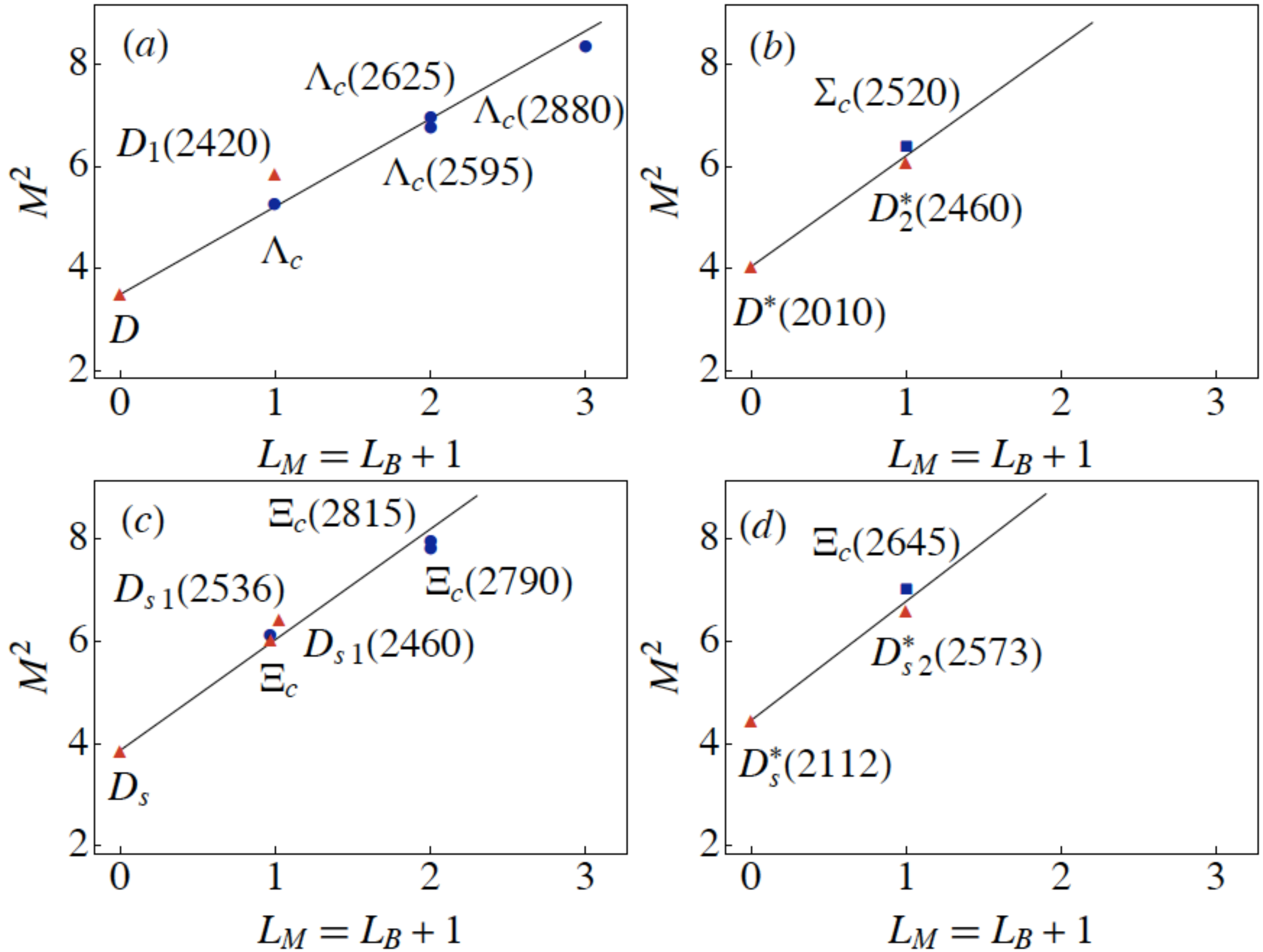
Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Superpartners for states with one c quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}c$	0^-	$D(1870)$	—	—	—	—	—	—
$\bar{q}c$	1^+	$D_1(2420)$	$[ud]c$	$(1/2)^+$	$\Lambda_c(2290)$	$[ud][\bar{c}\bar{q}]$	0^+	$\bar{D}_0^*(2400)$
$\bar{q}c$	2^-	$D_J(2600)$	$[ud]c$	$(3/2)^-$	$\Lambda_c(2625)$	$[ud][\bar{c}\bar{q}]$	1^-	—
$\bar{c}q$	0^-	$\bar{D}(1870)$	—	—	—	—	—	—
$\bar{c}q$	1^+	$\bar{D}_1(2420)$	$[cq]q$	$(1/2)^+$	$\Sigma_c(2455)$	$[cq][\bar{u}\bar{d}]$	0^+	$D_0^*(2400)$
$\bar{q}c$	1^-	$D^*(2010)$	—	—	—	—	—	—
$\bar{q}c$	2^+	$D_2^*(2460)$	$(qq)c$	$(3/2)^+$	$\Sigma_c^*(2520)$	$(qq)[\bar{c}\bar{q}]$	1^+	$D(2550)$
$\bar{q}c$	3^-	$D_3^*(2750)$	$(qq)c$	$(3/2)^-$	$\Sigma_c(2800)$	$(qq)[\bar{c}\bar{q}]$	—	—
$\bar{s}c$	0^-	$D_s(1968)$	—	—	—	—	—	—
$\bar{s}c$	1^+	$D_{s1}(2460)$	$[qs]c$	$(1/2)^+$	$\Xi_c(2470)$	$[qs][\bar{c}\bar{q}]$	0^+	$\bar{D}_{s0}^*(2317)$
$\bar{s}c$	2^-	$D_{s2}(\sim 2860)?$	$[qs]c$	$(3/2)^-$	$\Xi_c(2815)$	$[sq][\bar{c}\bar{q}]$	1^-	—
$\bar{s}c$	1^-	$D_s^*(2110)$	—	—	—	—	—	—
$\bar{s}c$	2^+	$D_{s2}^*(2573)$	$(sq)c$	$(3/2)^+$	$\Xi_c^*(2645)$	$(sq)[\bar{c}\bar{q}]$	1^+	$D_{s1}(2536)$
$\bar{c}s$	1^+	$D_{s1}(\sim 2700)?$	$[cs]s$	$(1/2)^+$	$\Omega_c(2695)$	$[cs][\bar{s}\bar{q}]$	0^+	??
$\bar{s}c$	2^+	$D_{s2}^*(\sim 2750)?$	$(ss)c$	$(3/2)^+$	$\Omega_c(2770)$	$(ss)[\bar{c}\bar{s}]$	1^+	??

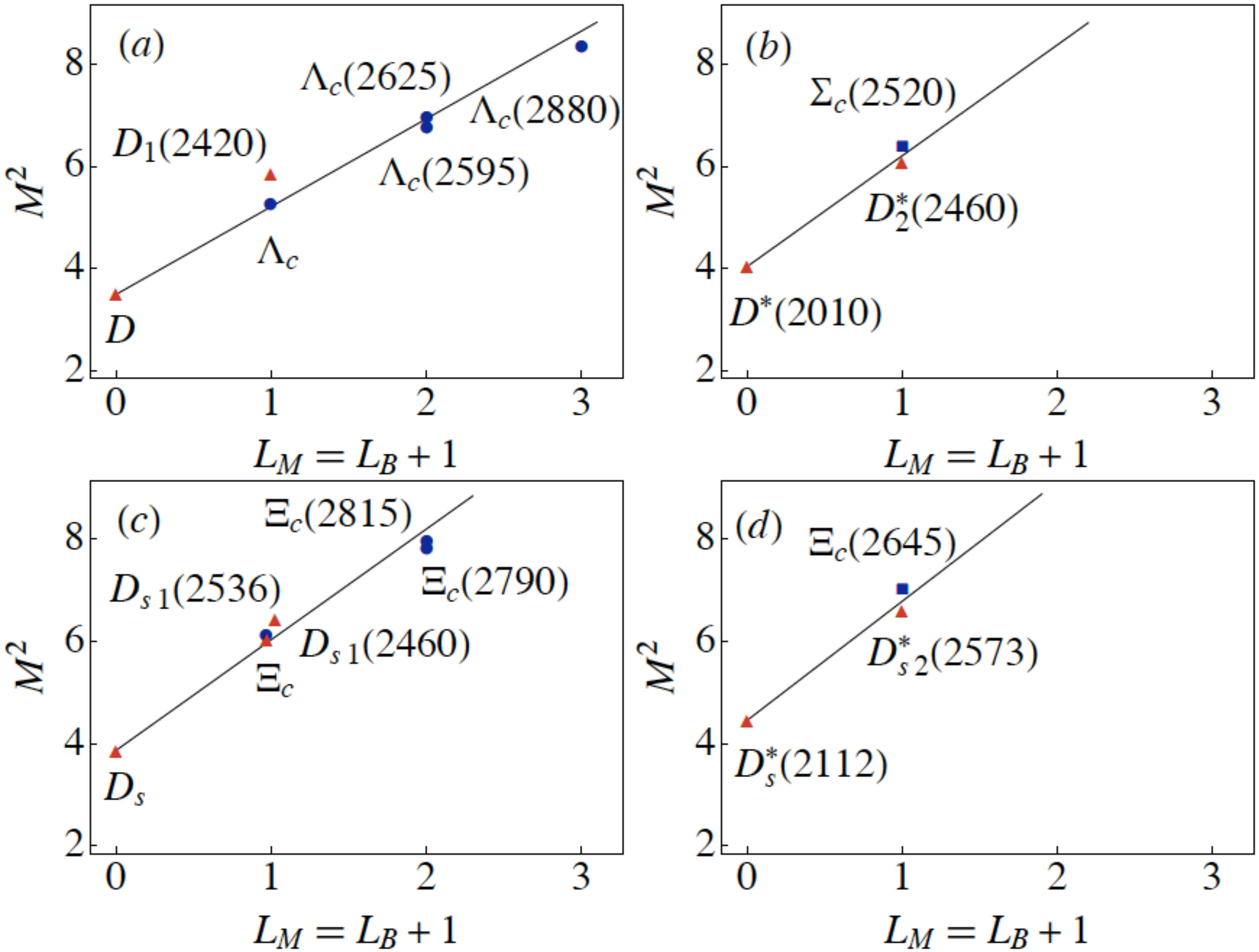
M. Nielsen, sjb

predictions

beautiful agreement!

Dosch, de Teramond, sjb

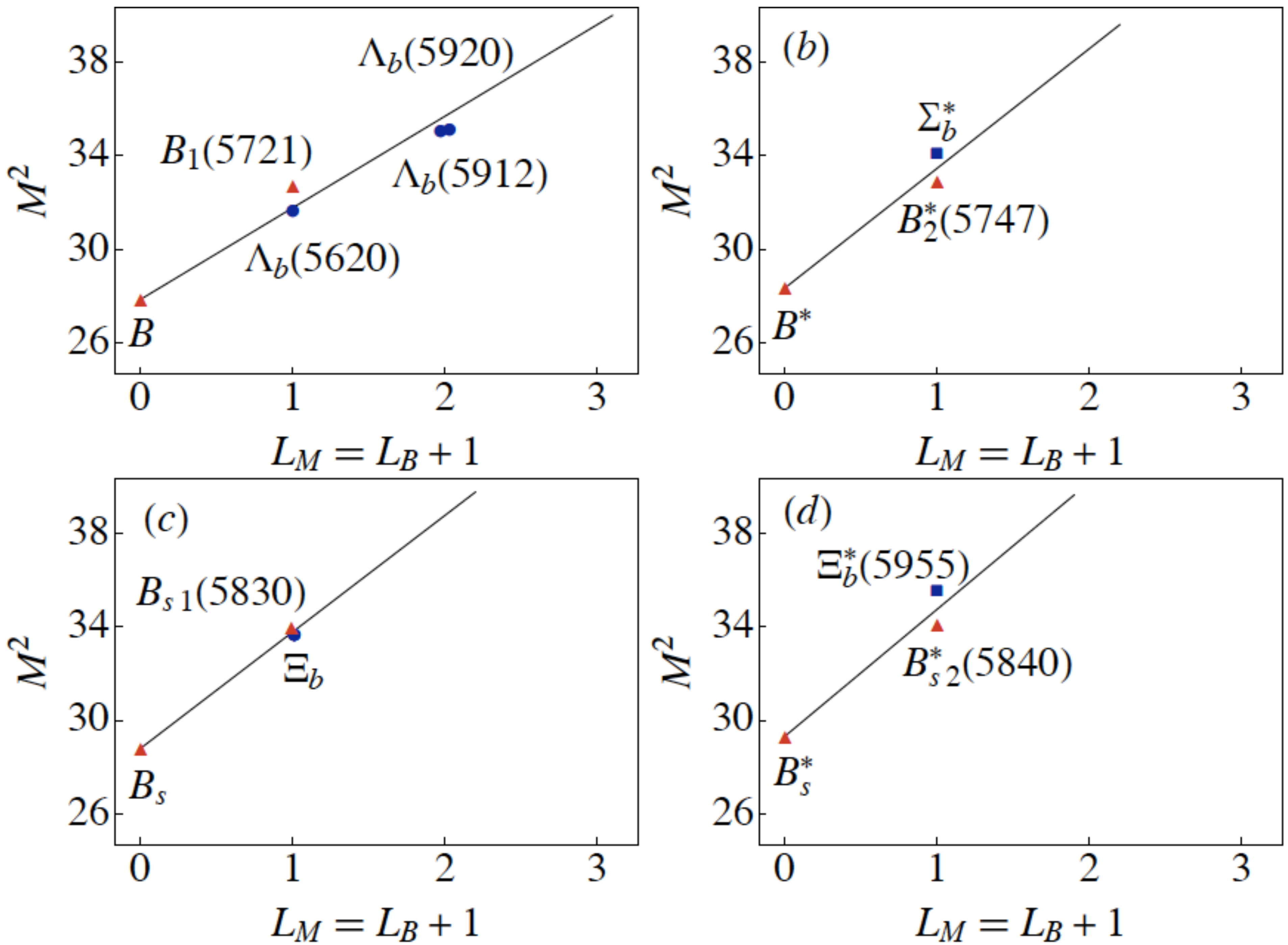
Supersymmetry across the light and heavy-light spectrum



Heavy charm quark mass does not break supersymmetry

Dosch, de Teramond, sjb

Supersymmetry across the light and heavy-light spectrum

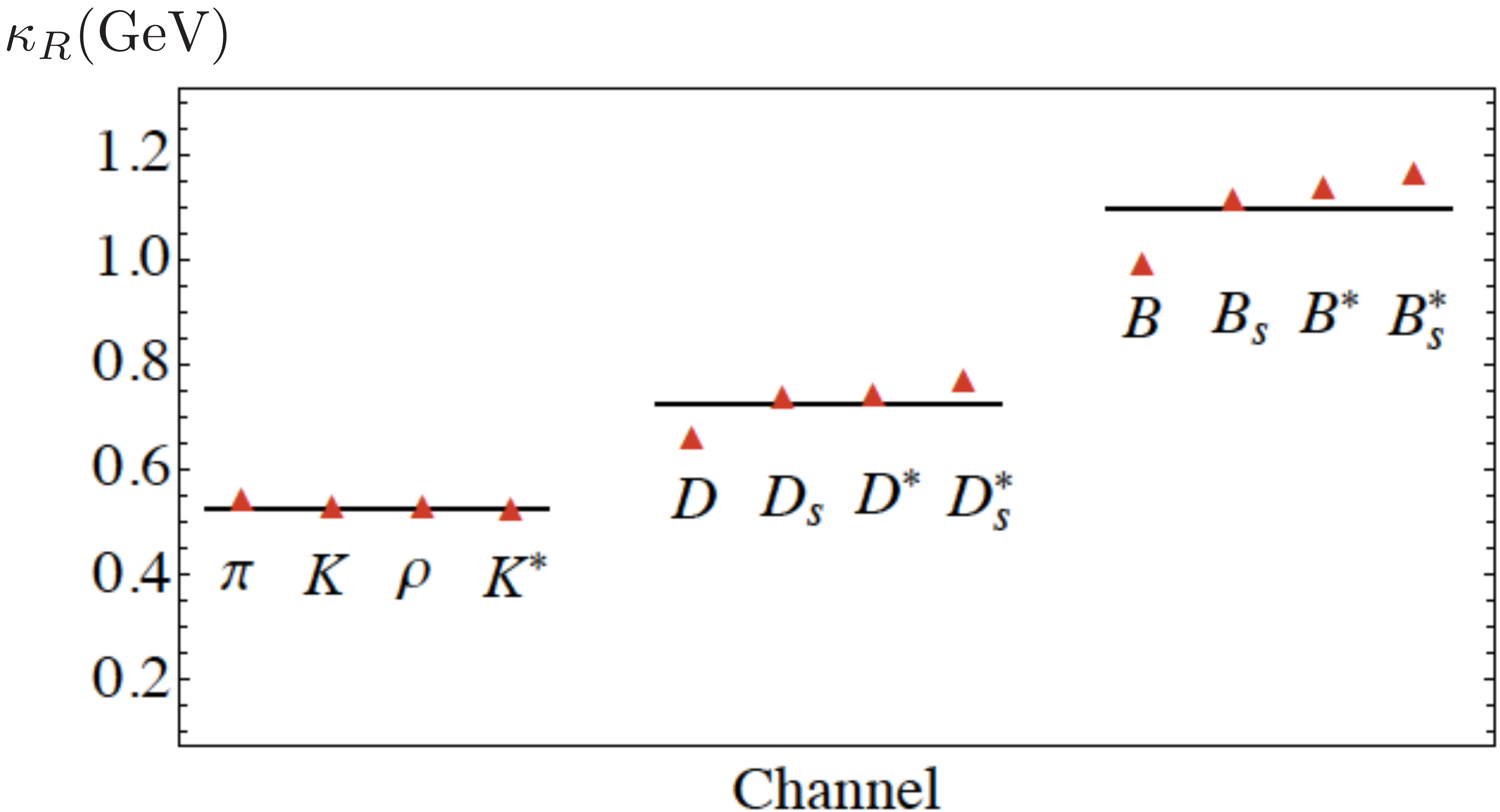


Heavy bottom quark mass does not break supersymmetry

Superpartners for states with one b quark

Meson			Baryon			Tetraquark		
q-cont	$J^{P(C)}$	Name	q-cont	J^P	Name	q-cont	$J^{P(C)}$	Name
$\bar{q}b$	0^-	$\bar{B}(5280)$	—	—	—	—	—	—
$\bar{q}b$	1^+	$\bar{B}_1(5720)$	$[ud]b$	$(1/2)^+$	$\Lambda_b(5620)$	$[ud][\bar{b}\bar{q}]$	0^+	$B_J(5732)$
$\bar{q}b$	2^-	$\bar{B}_J(5970)$	$[ud]b$	$(3/2)^-$	$\Lambda_b(5920)$	$[ud][\bar{b}\bar{q}]$	1^-	—
$\bar{b}q$	0^-	$B(5280)$	—	—	—	—	—	—
$\bar{b}q$	1^+	$B_1(5720)$	$[bq]q$	$(1/2)^+$	$\Sigma_b(5815)$	$[bq][\bar{u}\bar{d}]$	0^+	$\bar{B}_J(5732)$
$\bar{q}b$	1^-	$B^*(5325)$	—	—	—	—	—	—
$\bar{q}b$	2^+	$B_2^*(5747)$	$(qq)b$	$(3/2)^+$	$\Sigma_b^*(5835)$	$(qq)[\bar{b}\bar{q}]$	1^+	$B_J(5840)$
$\bar{s}b$	0^-	$B_s(5365)$	—	—	—	—	—	—
$\bar{s}b$	1^+	$B_{s1}(5830)$	$[qs]b$	$(1/2)^+$	$\Xi_b(5790)$	$[qs][\bar{b}\bar{q}]$	0^+	$\bar{B}_{s0}^*(\sim 5800)$?
$\bar{s}b$	1^-	$B_s^*(5415)$	—	—	—	—	—	—
$\bar{s}b$	2^+	$B_{s2}^*(5840)$	$(sq)b$	$(3/2)^+$	$\Xi_b^*(5950)$	$(sq)[\bar{b}\bar{q}]$	1^+	$B_{s1}(\sim 5900)$?
$\bar{b}s$	1^+	$B_{s1}(\sim 6000)$?	$[bs]s$	$(1/2)^+$	$\Omega_b(6045)$	$[bs][\bar{s}\bar{q}]$	0^+	??

predictions

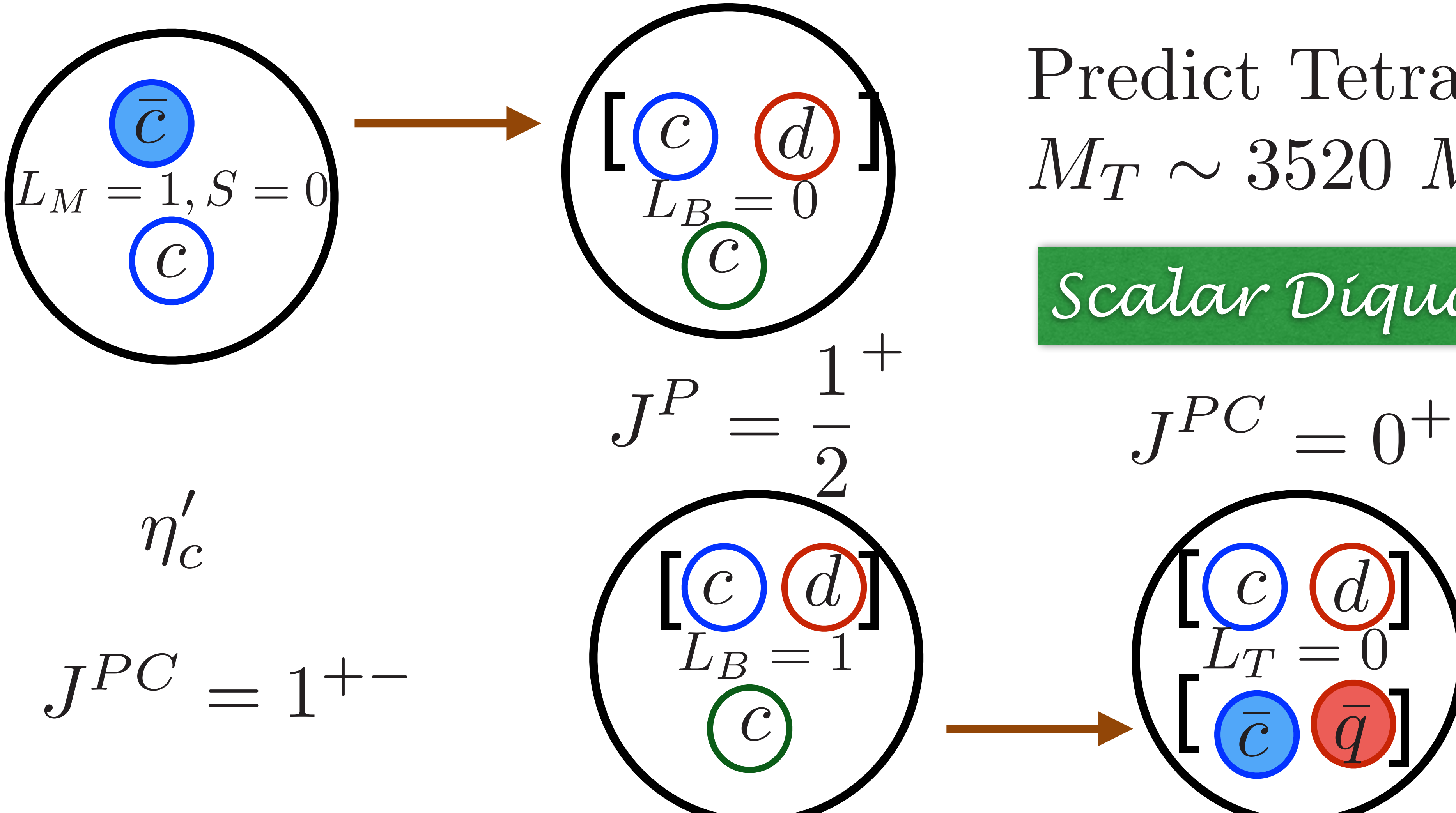


Regge slope for heavy-light mesons, baryons: increases with heavy quark mass

Double-Charm Baryon (SELEX)

$$S = 0$$
$$R_\lambda^\dagger \ \bar{q} \to [qq]$$
$$\bar{3}_C \to \bar{3}_C$$

$h_c(3525)$ $\quad$ $\Xi_{CC}^+(3520)$



Predict Tetraquark $T_{c\bar{c}q\bar{q}}$

$M_T \sim 3520 \ MeV$

Scalar Diquarks

$$R_\lambda^\dagger \ q \to [\bar{q}\bar{q}] \qquad S = 0$$
$$3_C \to 3_C$$

Features of Supersymmetric Equations

- J =L+S baryon simultaneously satisfies both equations of G with L , L+1 with same mass eigenvalue

- $J^z = L^z + 1/2 = (L^z + 1) - 1/2$ $\qquad S^z = \pm 1/2$

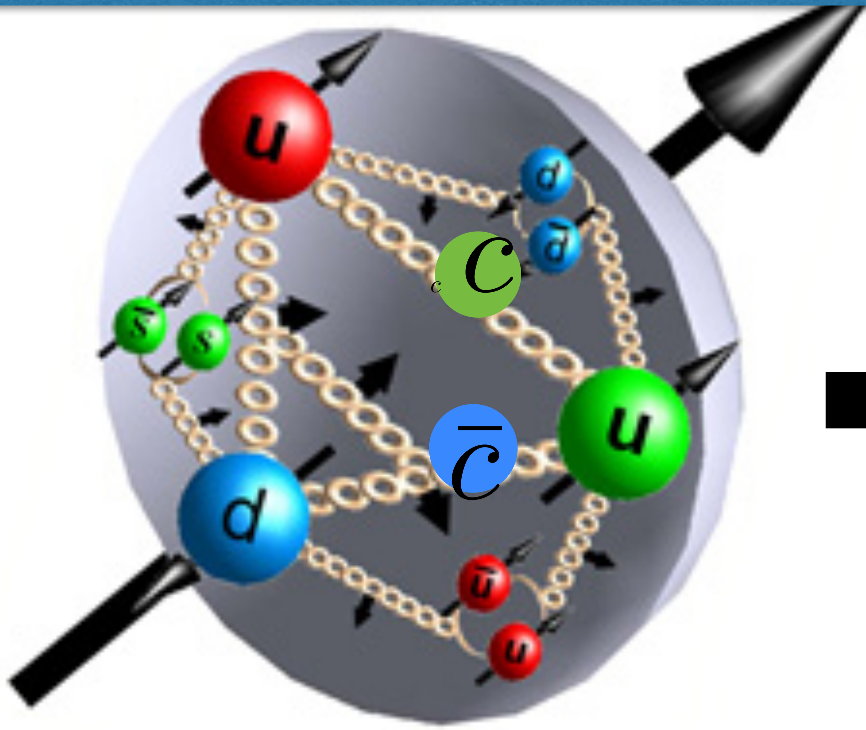
- Proton spin carried by quark L^z

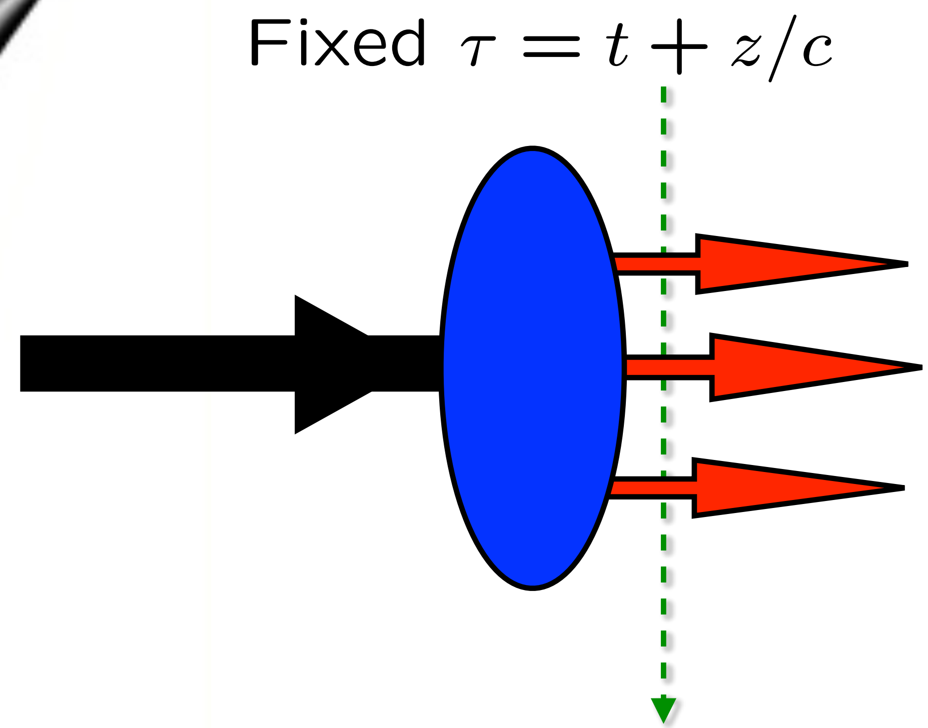
$$\boxed{< J^z >= \frac{1}{2}(S_q^z = \frac{1}{2}, L^z = 0) + \frac{1}{2}(S_q^z = -\frac{1}{2}, L^z = 1) =< L^z >= \frac{1}{2}}$$

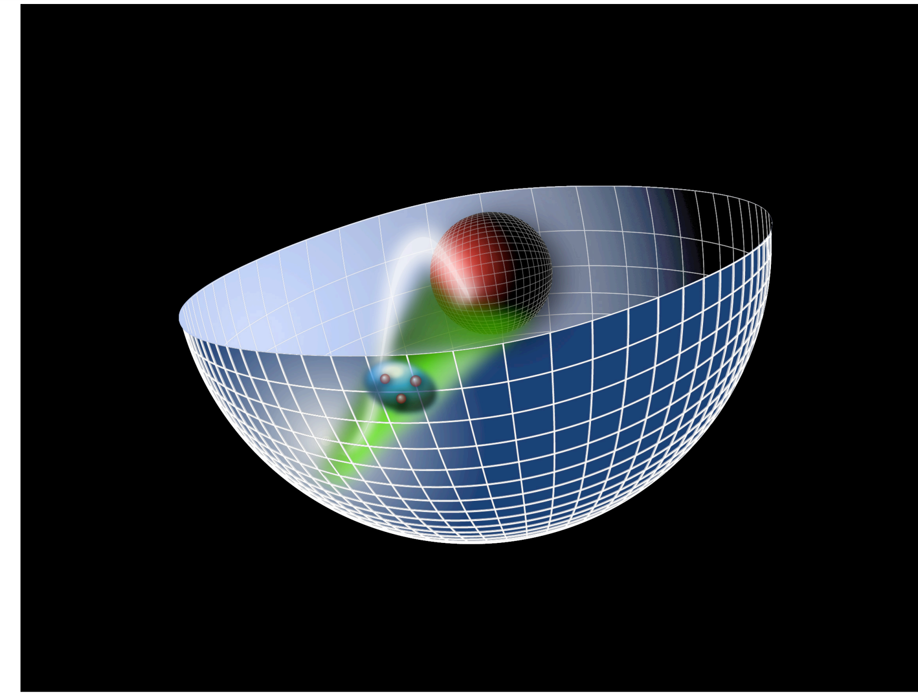
- Mass-degenerate meson "superpartner" with $L_M = L_B + 1$. *"Shifted meson-baryon Duality"*

- Mesons and baryons have same κ !

Supersymmetric Properties of Hadron Spectroscopy and Predictions for Hadron Dynamics from Light-Front Holography and Superconformal Algebra







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY

with Guy de Tèramond, Hans Günter Dosch, and Alexandre Deur

GGI School | Frontiers in Nuclear and Hadronic Physics | February 25 - March 8, 2019