Topics to be discussed

Preliminaries : a bit more on QCD and a bit on complex calculus

Causality and Analyticity.

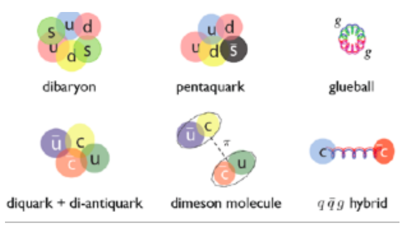
Scattering in non-relativistic quantum mechanics.

Kinematics of relativistic scattering and decays.

Relativistic partial wave analysis, unitarity, analyticity and resonances.

General parameterizations: N/D, K-matrix, connection with lattice QCD.

Physics of high energy collisions.



Regge limit and the Veneziano amplitude.

New hadrons observed in particle decays?

H.M.Nussenzveig, Causality and Dispersion relations
V.Gribov, Strong Interactions of Hadrons at High Energies
V.Gribov, Theory of Complex Angular Momentum
M.Perl, High Energy Hadron Physics

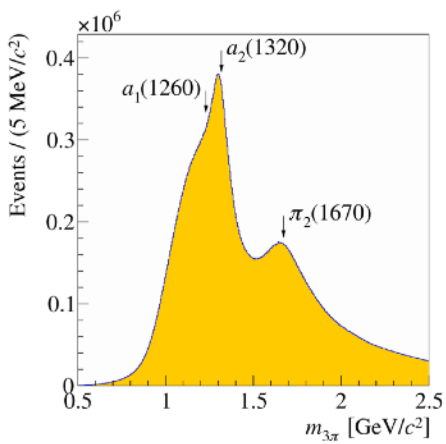
IU Online Course, P665, Fall 2019
E-mail Adam Szczepaniak
if you are interested

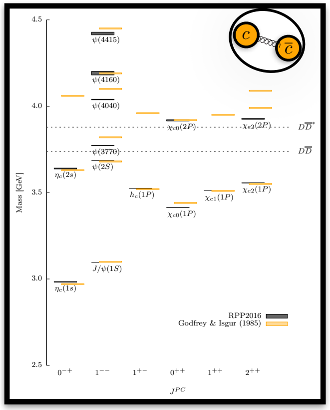
- Observables are smooth, analytical functions of variables. Physics law, constraints are manifested in singularities (poles, branch points)
- Cauchy theorem is a powerful tool to connect observables at different values of variables
- Physics is on 1st sheet but interesting phenomena happen on other sheets connected by analytical continuation, eg. Breit-Wigner formula

Identifying resonances

Data

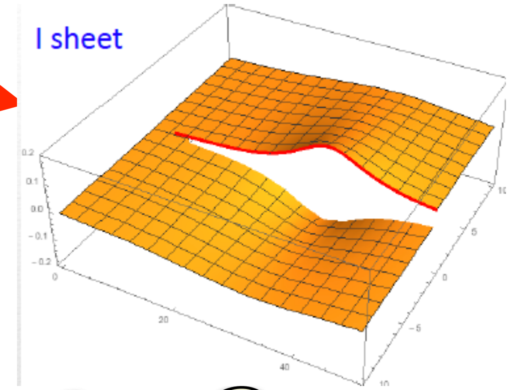
- Experimental or lattice signatures
cross section bumps and dips, energy levels





Reaction amplitudes

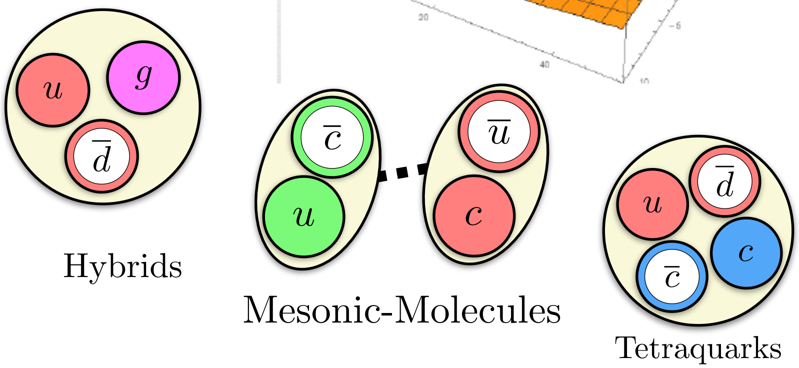
- Theoretical signatures (complex plane singularities: poles, cusps)



QCD Structure

- What is the interpretation (constituent quarks, molecules, …) ?

Theoretical uncertainty



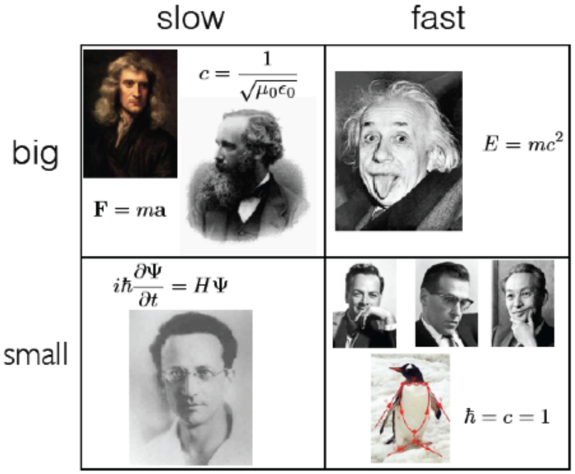
Stranger Things (of the Nuclear World)

What are the constituents of hadrons, (quarks and gluons) ?

small world (10^{-15}m)

of fast (v~c) particles

exerting ~1T forces !!!



$$\hbar = c = 1$$

[length] = [time] = $[\text{energy}]^{-1}$ = $[\text{momentum}]^{-1}$

Unit energy = 1GeV

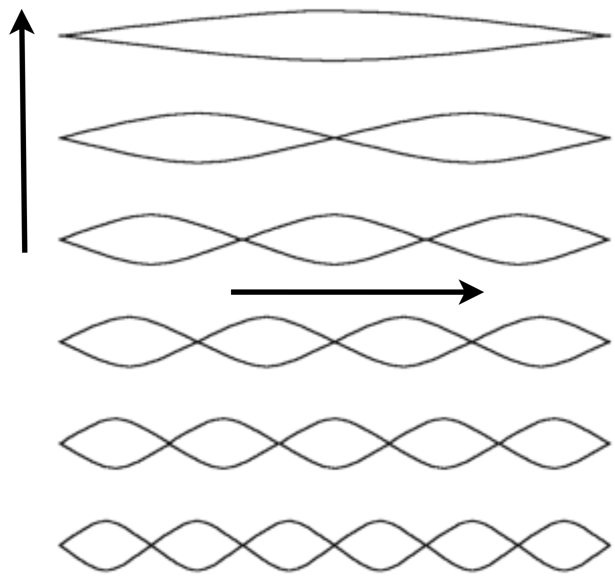
Unit length = 1GeV^{-1} = 0.197 fm

Particles vs Fields

In relativistic quantum mechanics (QFT) particles are emergent phenomena

(i.e. fields are not physically measurable but their “consequences” are, cf. potential vs electric field density)

“excitation of the aether” → field



collective motion → particle

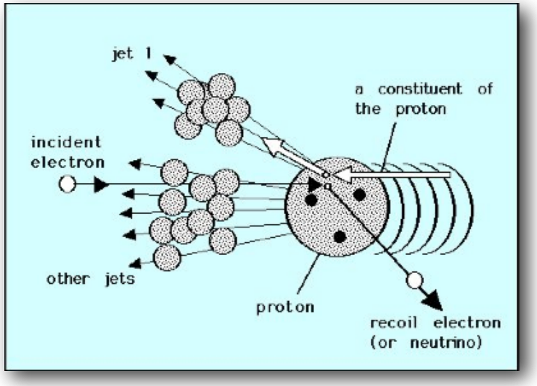
$H = H_{h.o} =$ harmonic oscillators

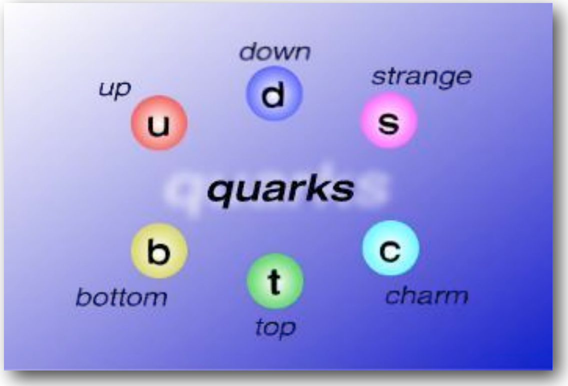
“bare” particles : eigenstates of $H_{h.o.}$

Bare particles are eigenstates of free Hamiltonian

"Bare (free)" particles of QCD: quarks and gluons

e.g. because of asymptotic freedom
measured in high energy collisions





- Gluon ~ 8 copies of a photon
- Photons do not cary electric charge : they only interact the matter (e.g.) electrons that do carry charge
- Gluons carry charge, i.e. interact with each other and with quarks.

Particles vs Fields: Hamiltonian vs Lagrangian

Legendre transformation

$$H(p_i, q_i) \longleftrightarrow L(q_i, \dot{q}_i)$$

Quantization

$$q_i = \vec{A}_i = \vec{A}(\vec{x}_i)$$

$$H\Psi[A_i] = E\Psi[A_i]$$

Ground state of H_o (pure gauge) is Gaussian

$$\langle q_i' | e^{-\beta H} | q_j' \rangle \to \int_{q'=q(\beta,q=q(0)} Dq e^{-S[q]}$$

Quantization

Easy to interpret
Hard to calculate
(particles, states, operators, etc.)

$\longleftrightarrow$

“Easy” to calculate
Hard to interpret
path integral, classical solution (solitons), etc.

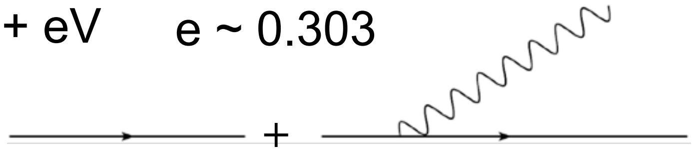
QED vs QCD

QED

- Bare particles are eigenstates of free Hamiltonian. If interactions are weak (e.g. QED) the "bare particle" ~ observed particle = (interacting particles)

$$H_{QED} = H_{c.h.o.} + eV \qquad e \sim 0.303$$

|electron> =

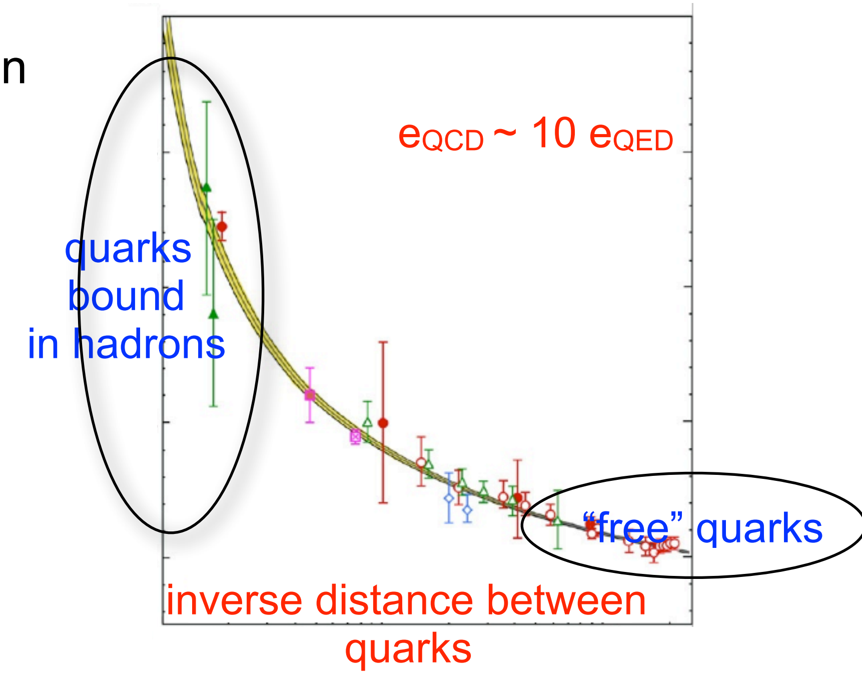


$+ \; O(e^2)$

|bare electron> eV|bare electron>

- Bound states, aka positronium require all orders, but can nevertheless be understood in terms of "bare" particles (choosing the "right gauge" = degrees of freedom is crucial)

QCD



- Quarks in hadrons have the effective color charge e > 3-4. Therefore there is in principle no reason for them to retain their identify in presence of strong interactions …
…but it seems they do

Quark Model : exploring flavor

AN SU_3 MODEL FOR STRONG INTERACTION SYMMETRY AND ITS BREAKING

II *)

G. Zweig **)

CERN—Geneva

ABSTRACT

Both mesons and baryons are constructed from a set of three fundamental particles called aces. The aces break up into an isospin doublet and singlet. Each ace carries baryon number 1/3 and is fractionally charged. SU_3 (but not the Eightfold Way) is adopted as a higher symmetry for the strong interactions. The breaking of this symmetry is assumed to be universal, being due to mass differences among the aces. Extensive space-time and group theoretic structure is then predicted for both mesons and baryons, in agreement with existing experimental information. Quantitative speculations are presented concerning resonances that have not as yet been definitively classified into representations of SU_3. A weak interaction theory based on right and left handed aces is used to predict rates for $|\Delta S| = 1$ baryon leptonic decays. An experimental search for the aces is suggested.

*) Version I is CERN preprint 8182/TH.401, Jan. 17, 1964.

**) This work was supported by the U.S. Air Force Office of Scientific Research and the National Academy of Sciences – National Research Council.

Volume 8, number 3 PHYSICS LETTERS 1 February 1964

A SCHEMATIC MODEL OF BARYONS AND MESONS *

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California Institute of Technology, Pasadena, California

Received 4 January 1964

If we assume that the strong interactions of baryons and mesons are correctly described in terms of the broken "eightfold way" [1-3], we are tempted to look for some fundamental explanation of the situation. A highly promised approach is the purely dynamical "bootstrap" model for all the strongly interacting particles within which one may try to derive isotopic spin and strangeness conservation and broken eightfold symmetry from self-consistency alone [4]. Of course, with only strong interactions, the orientation of the asymmetry in the unitary space cannot be specified; one hopes that in some way the selection of specific components of the F-spin by electromagnetism and the weak interactions determines the choice of isotopic spin and hypercharge directions.

Even if we consider the scattering amplitudes of strongly interacting particles on the mass shell only and treat the matrix elements of the weak, electromagnetic, and gravitational interactions by means of dispersion theory, there are still meaningful and important questions regarding the algebraic properties of these interactions that have so far been discussed only by abstracting the properties from a formal field theory model based on fundamental entities [3] from which the baryons and mesons are built up.

If these entities were octets, we might expect the underlying symmetry group to be SU(8) instead of SU(3); it is therefore tempting to try to use unitary triplets as fundamental objects. A unitary triplet t consists of an isotopic singlet s of electric charge z (in units of e) and an isotopic doublet (u, d) with charges z+1 and z respectively. The anti-triplet $\bar{t}$ has, of course, the opposite signs of the charges. Complete symmetry among the members of the triplet gives the exact eightfold way, while a mass difference, for example, between the isotopic doublet and singlet gives the first-order violation.

For any value of z and of triplet spin, we can construct baryon octets from a basic neutral baryon singlet b by taking combinations $(b t \bar{t})$, $(b t t \bar{t} \bar{t})$, etc. **. From $(b t \bar{t})$, we get the representations **1** and **8**, while from $(b t t \bar{t} \bar{t})$ we get **1**, **8**, **10**, $\overline{\mathbf{10}}$, and **27**. In a similar way, meson singlets and octets can be made out of $(t \bar{t})$, $(t t \bar{t} \bar{t})$, etc. The quantum number $n_t - n_{\bar{t}}$ would be zero for all known baryons and mesons. The most interesting example of such a model is one in which the triplet has spin $\frac{1}{2}$ and $z = -1$, so that the four particles d^-, s^-, u^0 and b^0 exhibit a parallel with the leptons.

A simpler and more elegant scheme can be constructed if we allow non-integral values for the charges. We can dispense entirely with the basic baryon b if we assign to the triplet t the following properties: spin $\frac{1}{2}$, $z = -\frac{1}{3}$, and baryon number $\frac{1}{3}$. We then refer to the members $u^{\frac{2}{3}}$, $d^{-\frac{1}{3}}$, and $s^{-\frac{1}{3}}$ of the triplet as "quarks" [6] q and the members of the anti-triplet as anti-quarks $\bar{q}$. Baryons can now be constructed from quarks by using the combinations $(q q q)$, $(q q q q \bar{q})$, etc., while mesons are made out of $(q \bar{q})$, $(q q \bar{q} \bar{q})$, etc. It is assuming that the lowest baryon configuration $(q q q)$ gives just the representations **1**, **8**, and **10** that have been observed, while the lowest meson configuration $(q \bar{q})$ similarly gives just **1** and **8**.

A formal mathematical model based on field theory can be built up for the quarks exactly as for p, n, Λ in the old Sakata model, for example [3] with all strong interactions ascribed to a neutral vector meson field interacting symmetrically with the three particles. Within such a framework, the electromagnetic current (in units of e) is just

$$i\{\tfrac{2}{3}\bar{u}\gamma_\alpha u - \tfrac{1}{3}\bar{d}\gamma_\alpha d - \tfrac{1}{3}\bar{s}\gamma_\alpha s\}$$

or $\mathscr{F}_{3\alpha} + \mathscr{F}_{8\alpha}/\sqrt{3}$ in the notation of ref. [3]. For the weak current, we can take over from the Sakata model the form suggested by Gell-Mann and Lévy [7], namely $i\,\bar{p}\gamma_\alpha(1+\gamma_5)(n\cos\theta + \Lambda\sin\theta)$, which gives in the quark scheme the expression ***

$$i\,\bar{u}\gamma_\alpha(1+\gamma_5)(d\cos\theta + s\sin\theta)$$

* Work supported in part by the U.S. Atomic Energy Commission.

** This is similar to the treatment in ref. [1]. See also ref. [5].

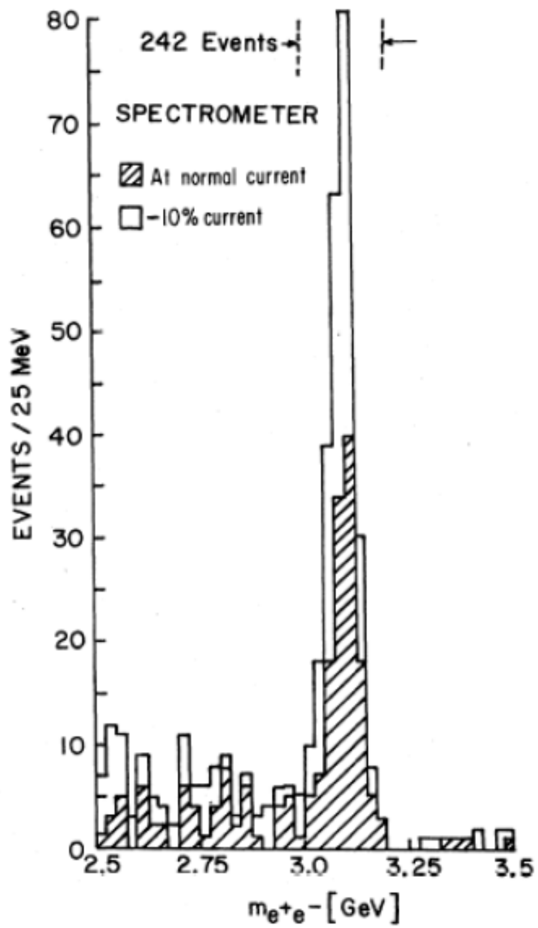
*** The parallel with $i\,\bar{\nu}_e\gamma_\alpha(1+\gamma_5)e$ and $i\,\bar{\nu}_\mu\gamma_\alpha(1+\gamma_5)\mu$ is obvious. Likewise, in the model with d^-, s^-, u^0, and b^0 discussed above, we would take the weak current to be $i(\bar{b}^0\cos\theta + \bar{u}^0\sin\theta)\gamma_\alpha(1+\gamma_5)s^- + i(\bar{u}^0\cos\theta - \bar{b}^0\sin\theta)\gamma_\alpha(1+\gamma_5)d^-$. The part with $\Delta(n_t - n_{\bar{t}}) = 0$ is just $i\,\bar{u}^0\gamma_\alpha(1+\gamma_5)(d^-\cos\theta + s^-\sin\theta)$.

Discovery of quarks e.g. the J/ψ

A narrow resonance was discovered in the 1974 November revolution of particle physics" in two reactions:

Proton + Be => e^+ e^- + anything at the BNL J. J. Aubert et al., "Experimental observation of a heavy particle J," Phys. Rev. Lett. 33, 1404 (1974).

$$J/\psi = c\bar{c}$$



e^+e^- annihilation to hadrons in the SPEAR storage ring at Stanford J. E. Augustin et al., "Discovery of a narrow resonance in e^+e^- annihilation," Phys. Rev. Lett. 33, 1406 (1974).

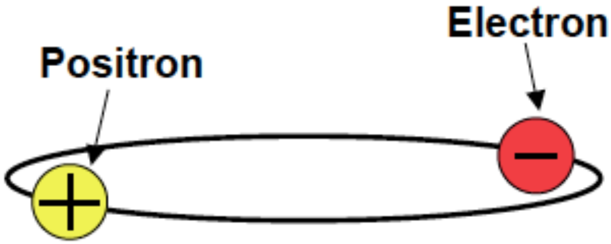
$$\text{mass} \;= 3096.87 \text{ MeV}$$

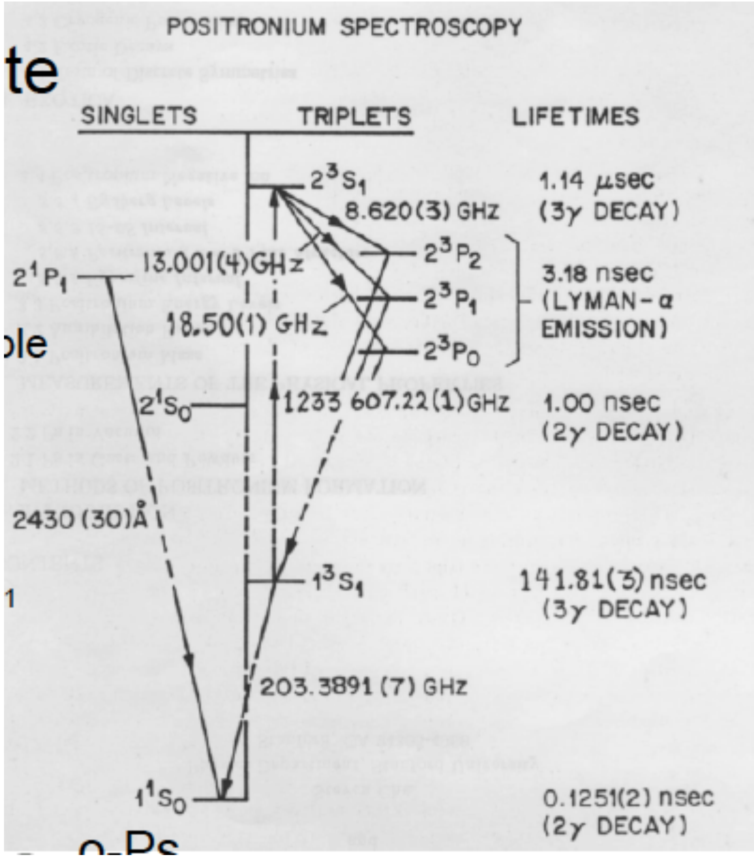
$$\Gamma = 87 \text{ keV}$$

10^3 longer lifetime !

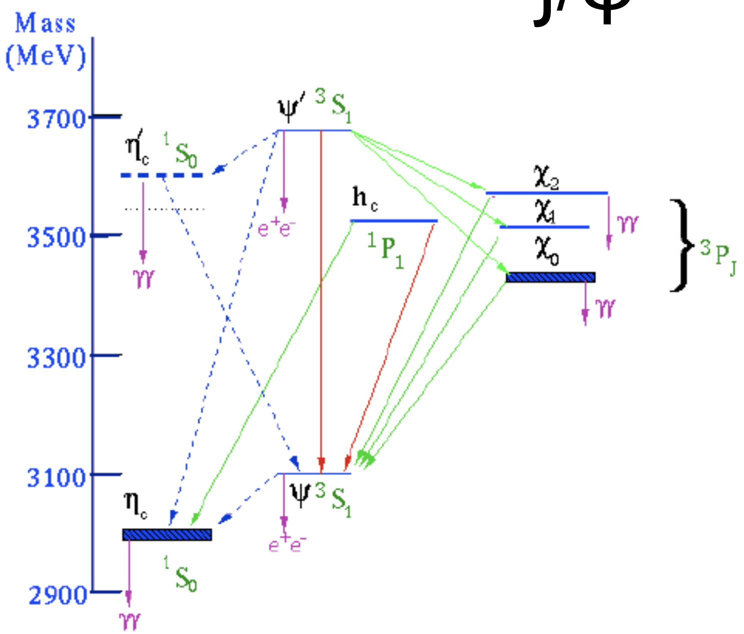
$$\text{typical hadronic width} \;= O(100 \text{ MeV})$$

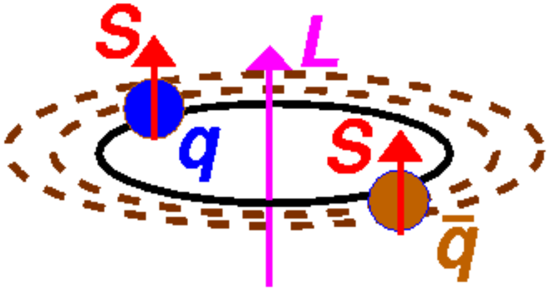
Charmonium spectrum





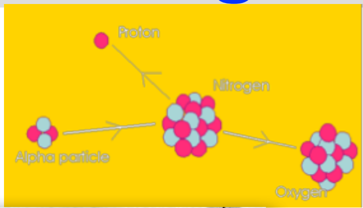
J/ψ





J/ψ is a bound state of $\bar{c}$ c

Hunting for Resonances

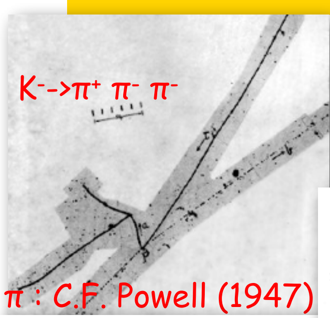


1909/1911 Rutherford/Geiger/Marsden discover the nucleus

1919 Rutherford discovers the proton

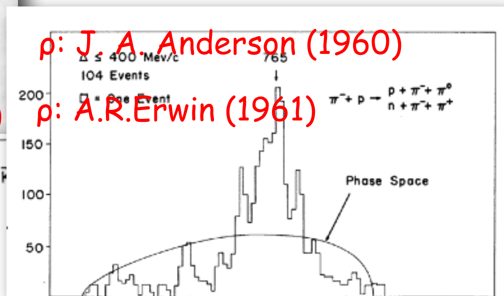
1932 Chadwick discovers the neutron

1940-now hundreds of resonances discovered



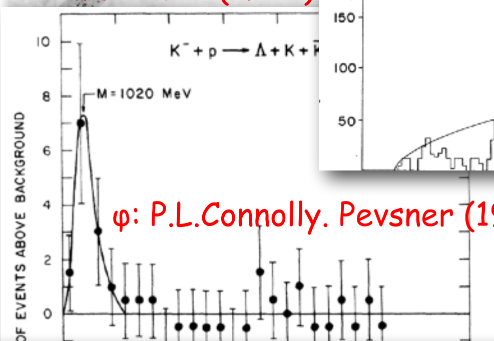
$K^- \to \pi^+ \pi^- \pi^-$

π : C.F. Powell (1947)

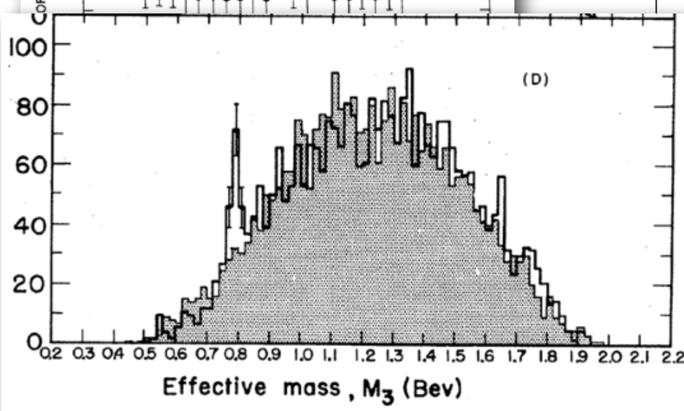


ρ: J. A. Anderson (1960)

ρ: A.R.Erwin (1961)



φ: P.L.Connolly. Pevsner (1962)



lifetime ~ 10^{-24} s

width ~ lifetime^{-1} = 150 MeV

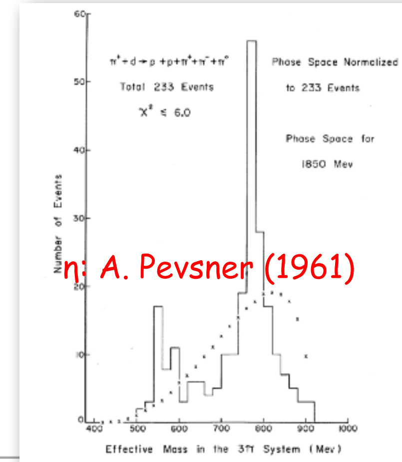
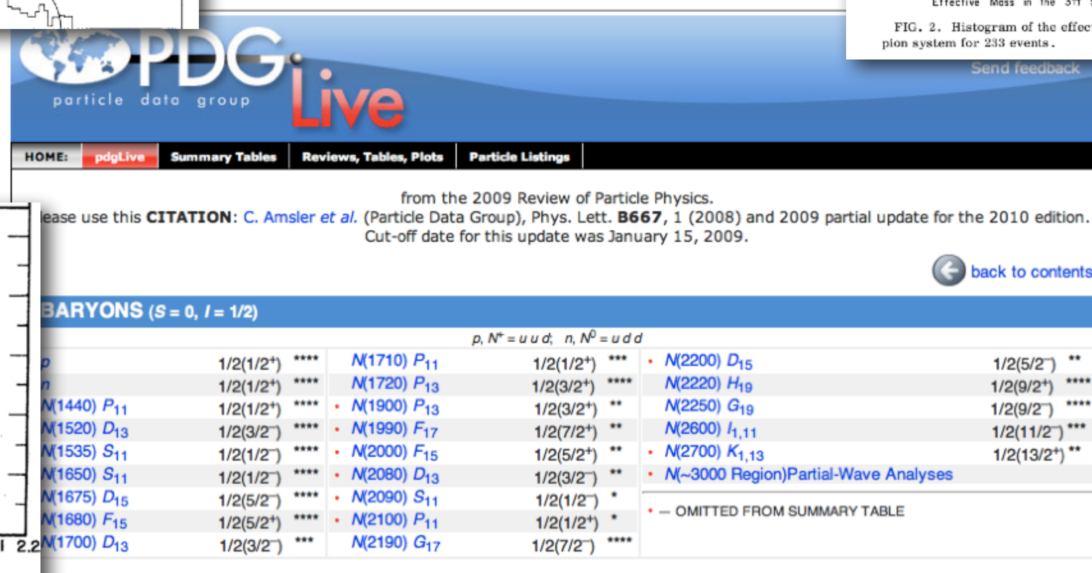


FIG. 2. Histogram of the effective mass of the three-pion system for 233 events.

η: A. Pevsner (1961)



p	1/2(1/2+) ****	N(1710) P_{11}	1/2(1/2+) ***	• N(2200) D_{15}	1/2(5/2−) **
n	1/2(1/2+) ****	N(1720) P_{13}	1/2(3/2+) ****	N(2220) H_{19}	1/2(9/2+) ****
N(1440) P_{11}	1/2(1/2+) ****	• N(1900) P_{13}	1/2(3/2+) **	N(2250) G_{19}	1/2(9/2−) ****
N(1520) D_{13}	1/2(3/2−) ****	• N(1990) F_{17}	1/2(7/2+) **	N(2600) $I_{1,11}$	1/2(11/2−) ***
N(1535) S_{11}	1/2(1/2−) ****	• N(2000) F_{15}	1/2(5/2+) **	• N(2700) $K_{1,13}$	1/2(13/2+) **
N(1650) S_{11}	1/2(1/2−) ****	• N(2080) D_{13}	1/2(3/2−) **	• N(~3000 Region)Partial-Wave Analyses	
N(1675) D_{15}	1/2(5/2−) ****	• N(2090) S_{11}	1/2(1/2−) *	• — OMITTED FROM SUMMARY TABLE	
N(1680) F_{15}	1/2(5/2+) ****	• N(2100) P_{11}	1/2(1/2+) *		
N(1700) D_{13}	1/2(3/2−) ***	N(2190) G_{17}	1/2(7/2−) ****		

Light mesons

Light (u,d,s) Mesons :

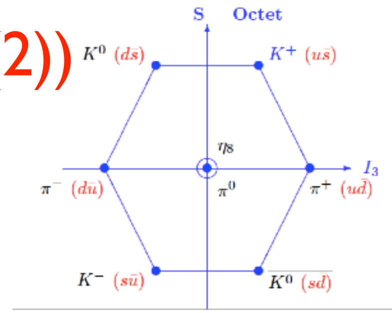
Flavor = 8 + 1 ($SU_f(3)$)

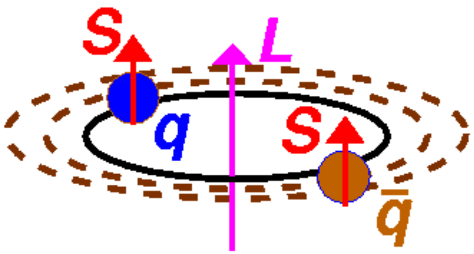
Spin S = 1/2 x 1/2 = 0 + 1 (SU(2))

Orbital L = 0,1,2... (O(3))

Radial n = 0,1,2... (various)

S,L => J,Parity (+,-), Charge conjugation)





$$P = (-1)^{L+1}$$

$$C = (-1)^{L+S}$$

$$P_Q = -P_{\bar{Q}}$$

$$C|Q\rangle = |\bar{Q}\rangle$$

8

J^{PC}	$I=1$	$I=0$
J^{-+}	π_J	η_J
J^{--}	ρ_J	ω_J
J^{+-}	b_J	h_J
J^{++}	a_J	f_J

$$\pi = 0^{-+} \to L = 0, S = 0, I = 1$$

$$\rho_2 = 2^{--} \to L = even = 2, S = 1, I = 1$$

quark model

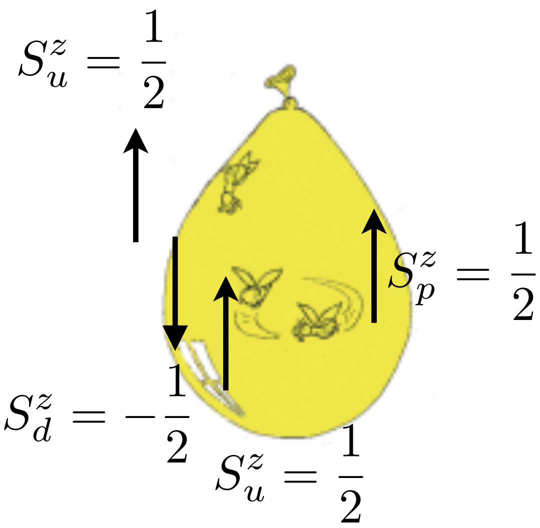
$$|B[8]\rangle = |Flavor\rangle_{8_{M_A}} \times |Spin\rangle_{8_{M_A}} + |Flavor\rangle_{8_{M_S}} \times |Spin\rangle_{8_{M_S}}$$

fully symmetric wave function (antisymmetric does not work!)
Color makes it into fully antisymmetric to respect Pauli principle

H. J. Lipkin

FERMILAB-Conf-84/125-T
November, 1984

34

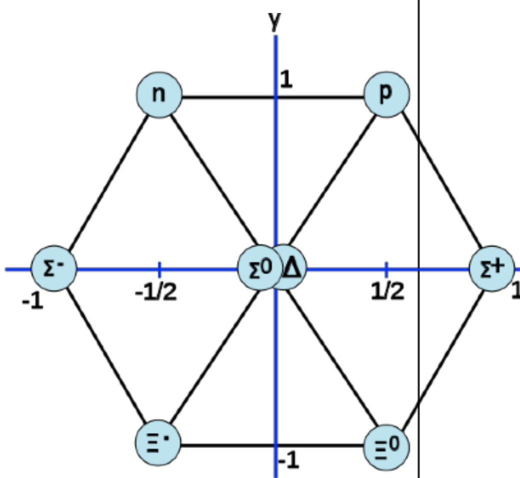


Baryon magnetic moments

Baryon Moment	1983 Data Ref[26]	From Naive Model[25]
μ(p)	2.793±0.000	2.79
μ(n)	-1.913±0.000	-1.86
μ(Λ)	-0.613±0.005	-0.58
μ(Σ+)	2.38±0.02	2.68
μ(Σ−)	-1.11±0.04[27]	-1.05
μ(Ξ°)	-1.25±0.014	-1.40
μ(Ξ−)	-0.60±0.04	-0.47

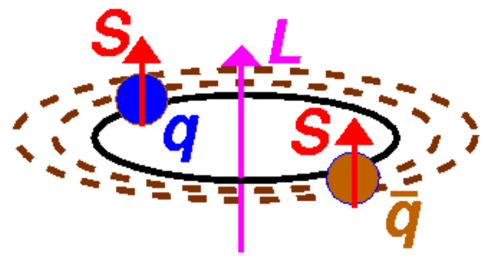
1995

$\mu_{\Omega^-} = (-2.019 \pm 0.054)\mu_N$ $\quad -1.84\mu_N$



better then 10% accuracy !!

quark model



$J=L+S$ $\qquad$ $^{(2S+1)}L_J$

$P=(-1)^{L+1}$

$C=(-1)^{L+S}$ $\qquad$ $^1S_0 = 0^{-+}$

$G=C\,(-1)^I$ $\qquad$ $^3S_1 = 1^{--}$

orbital excitations

radial excitations

L=0	ρ,ω,ϕ,K^*	1^{--}	L=1	a_2,f_2,f'_2,K_2	2^{++}
	π,η,η',K	0^{-+}		a_1,f_1,f'_1,K_1	1^{++}
				a_0,f_0,f'_0,K_0	0^{++}
				b_1,h_1,h'_1,K_1	1^{+-}

$$|J^{PC},n\rangle \to \Psi^{J^{PC},n}(\mathbf{r}_{q\bar{q}}, m_q m_{\bar{q}}, f_q f_{\bar{q}})\delta_{c_q c_{\bar{q}}}$$

$$S+L=J, \quad \Psi(\mathbf{r}_{q\bar{q}}) \to \Psi(|\mathbf{r}_{q\bar{q}}|)$$

$$H = \frac{\mathbf{p}_q^2}{2m_q} + \frac{\mathbf{p}_{\bar{q}}^2}{2m_{\bar{q}}} + V_C(r_{q\bar{q}}) + V_{SS} + V_{LS} + V_T + V_F$$

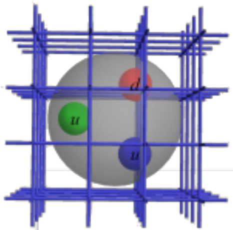
$m_u \sim m_d \sim 300\ \mathrm{GeV}$

constituent quarks

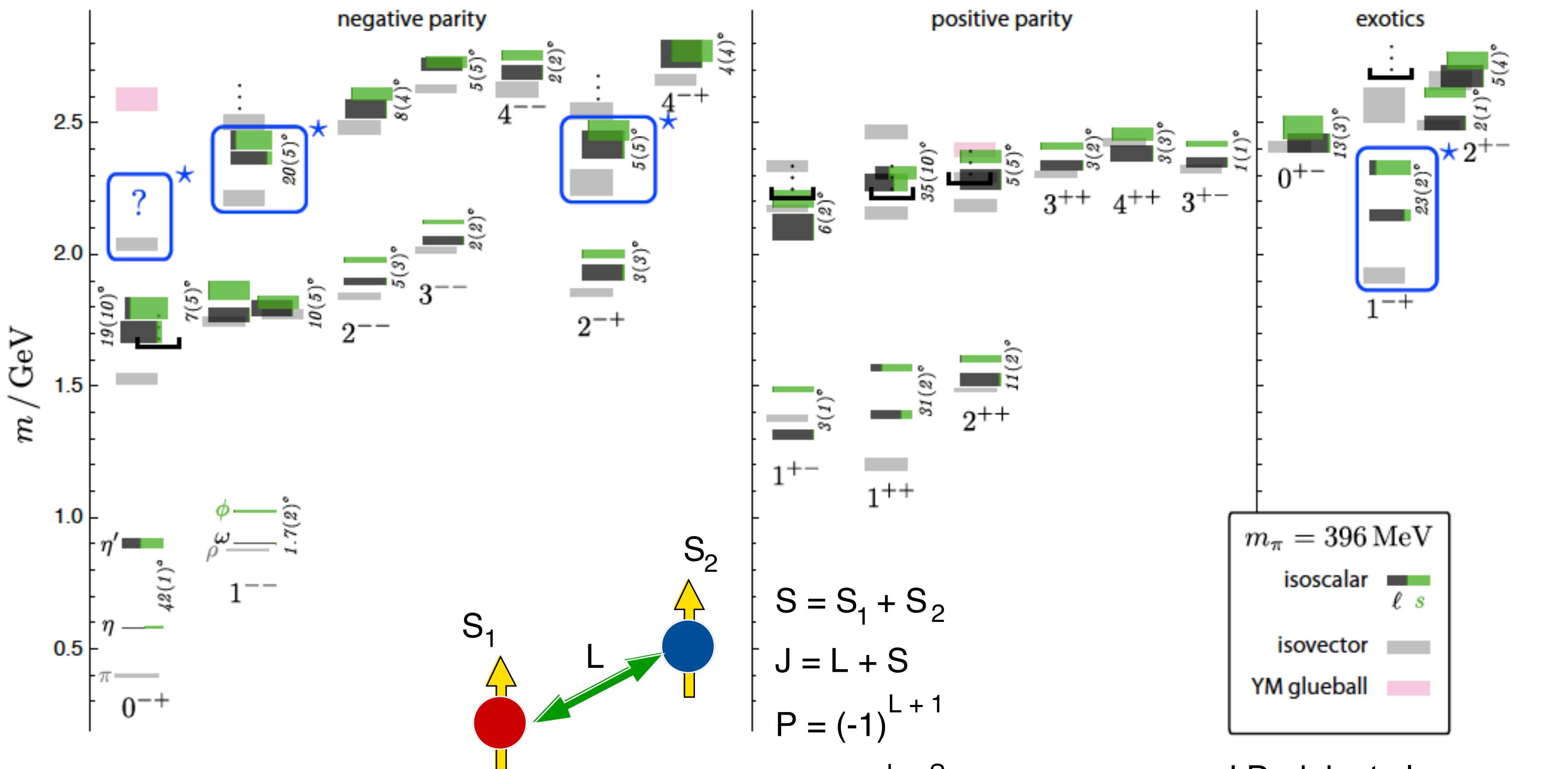
$m_u \sim m_d \sim\ \mathrm{few\ GeV}$

bare quarks

“Evidence” for Constituent Quarks:Light Quark Hadrons



Spectrum of mesons containing u,d,s quarks from numerical QCD simulations (lattice) resembles spectrum of quark models.



$S = S_1 + S_2$

$J = L + S$

$P = (-1)^{L+1}$

$C = (-1)^{L+S}$

J.Dudek et al.

QCD as a many body theory

QCD

$$L = -\frac{1}{4} F^a_{\mu\nu} F^a_{\mu\nu} - \bar{\psi}(\gamma_\mu D_\mu + m)\psi$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g f^{abc} A^b_\mu A^c_\nu$$

$$D_\mu = \partial_\mu + i g A^a_\mu T^a$$

$$[T^a, T^b]_{ij} = i f^{abc} T^c_{ij}$$

Variables:

$A^a_\mu(\mathbf{x};t)$ $\psi = \psi^i_\alpha(\mathbf{x};t)$

8 x 4 x 3N 3 x 4 x 3N

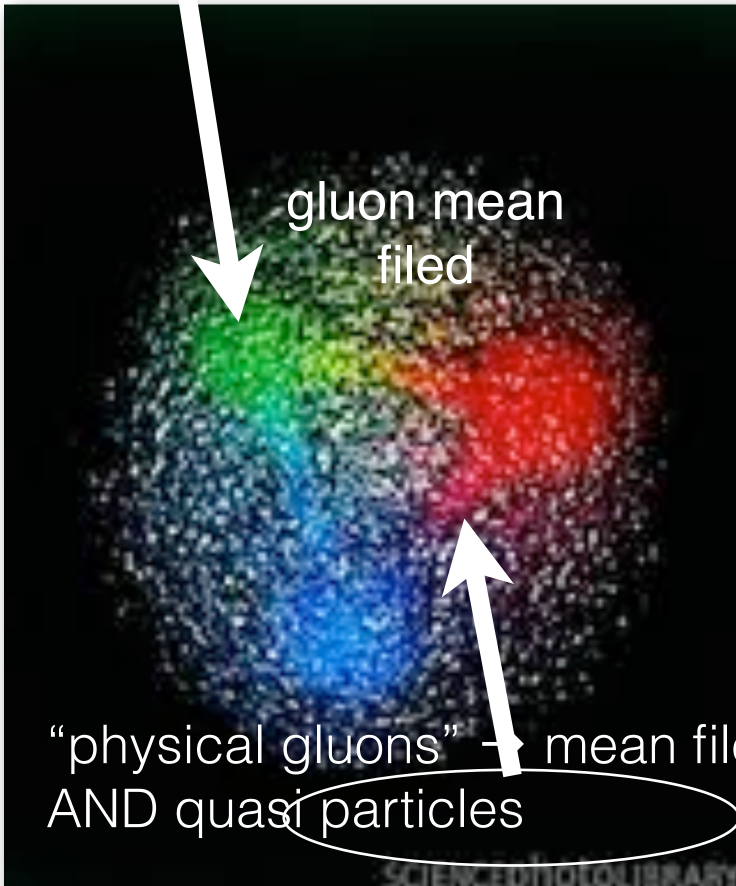
Parameters: g,m

Plausible scenario

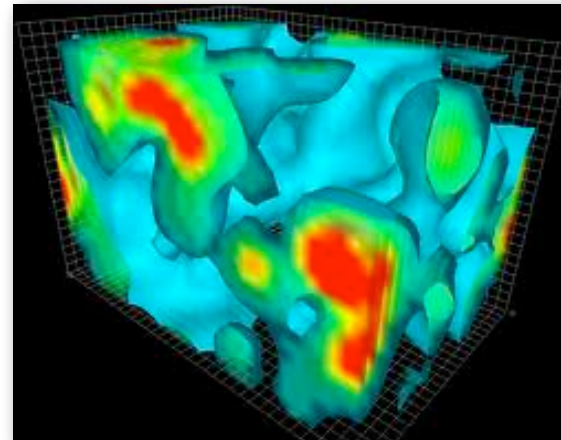
$$H_{QCD} = H_{c.h.o.} + \text{non-linear}$$

finite energy, localized solutions:
solitons (monopoles, vortices , ...)

“physical quarks” →
quasi particles in gluon mean filed



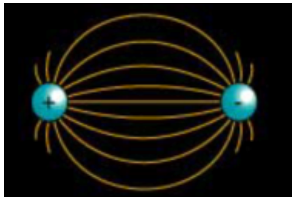
The QCD vacuum is not empty. Rather it contains quantum fluctuations in the gluon field at all scales. (Image: University of Adelaide)



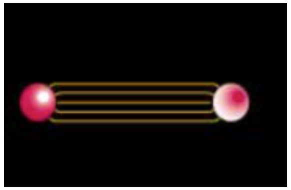
Monopoles have been long speculated as candidate gluon filed configurations responsible for confinement

Monopole confining scenario

in “empty vacuum”



in “magnetic condensate”



Type-II supper conductor

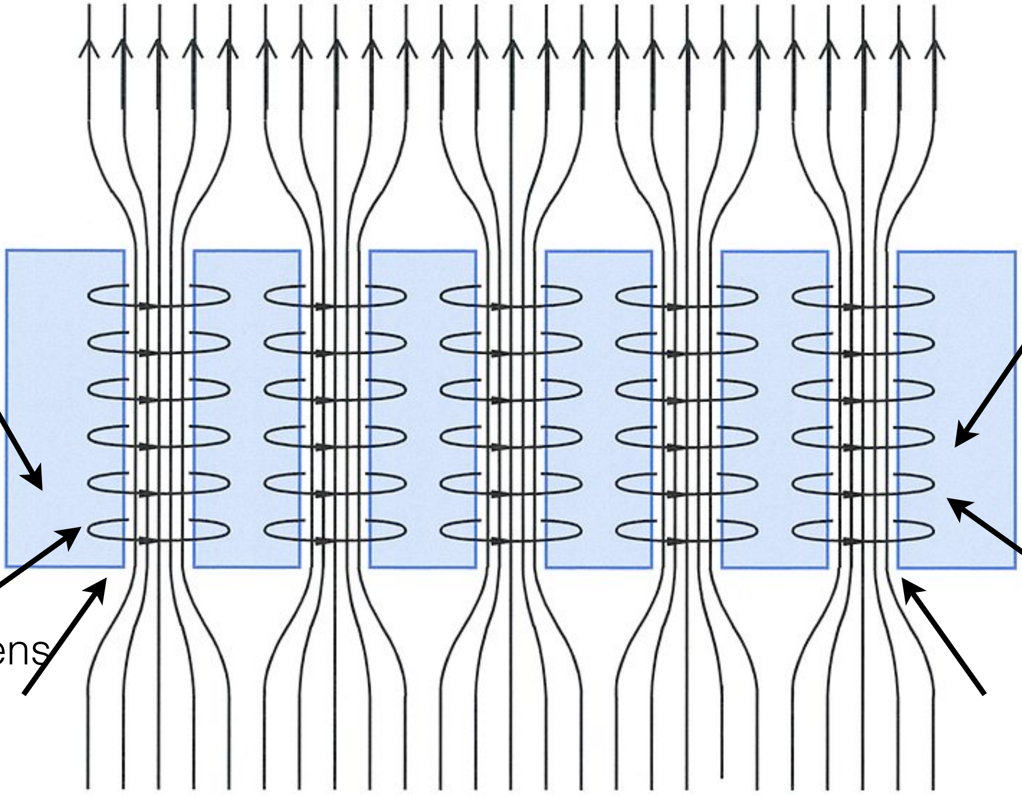
Dual Type-II supper conductor

QED

QCD

condensate of electric charges

condensate of magneti charges



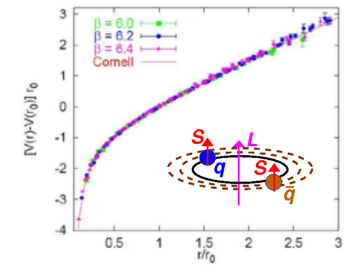
electric current screens magnetic lines

magnetic current screens electric lines

Emergence of constituent quarks

$$H = H_0 + V \qquad H_0 = \int d\mathbf{x} m_0 |\psi(\mathbf{x})|^2$$

Mean field approximation   Hartree + Fock (BCS theory)

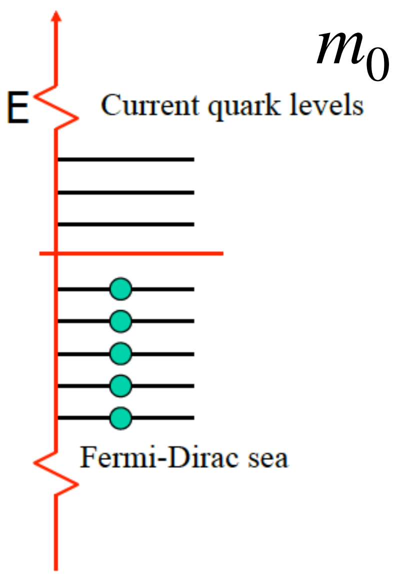


Instantaneous potential between (color) charges, e.g. Coulomb + Linear

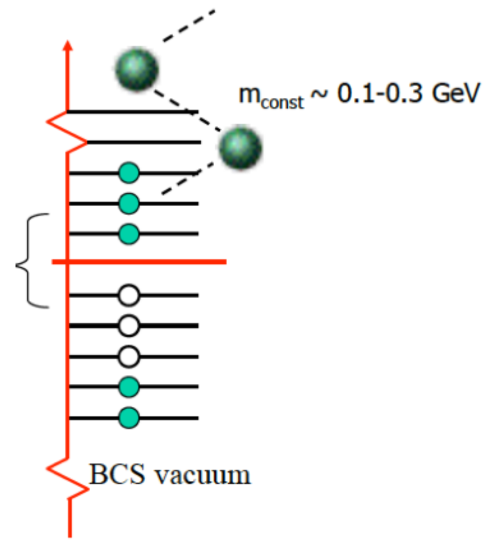
$$V = \int d\mathbf{x} d\mathbf{y} |\psi(\mathbf{x})|^2 V(\mathbf{x}-\mathbf{y}) |\psi(\mathbf{y})|^2$$

$$|\psi(\mathbf{y})|^2 \to \langle |\psi(\mathbf{y})|^2 \rangle = \text{condensate}$$

$$m_0 \to m_0 + V \times \text{ condensate}$$

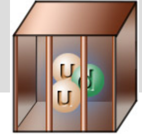


Interaction with the condensate increases energy of a quark added to the vacuum



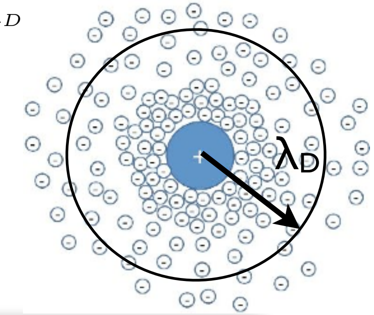
ground state contains a condensate of bare quarks

Confinement in QCD

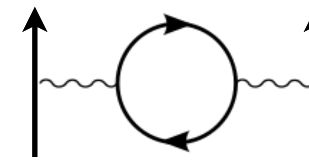


e.g. absence of isolated quarks applies to both screening and confinement

$$\int_{\partial V} \vec{E}\cdot d\vec{S} = Q \sim e^{-R/\lambda_D}$$



condensate (i.e. electrons in metal)

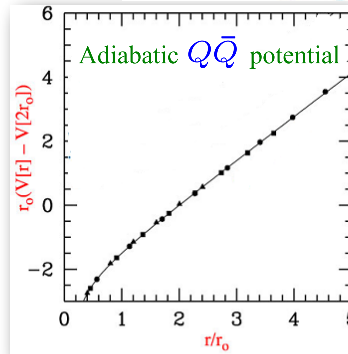


$$-\frac{e^2}{r} \to -\frac{e^2}{r}e^{-r/\lambda_D}$$

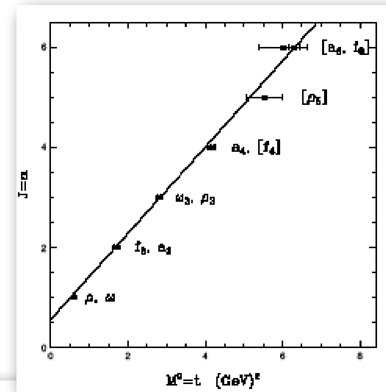
- absence of isolated quarks

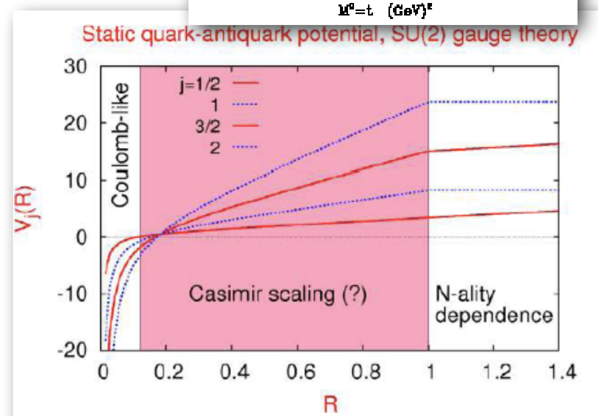
In absence of an order parameter we have to content with properties of confinement:

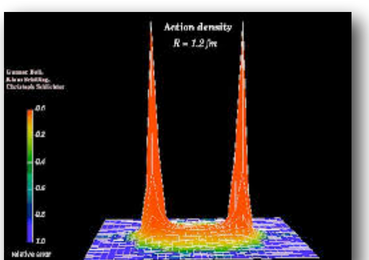
- linearly rising potential
- Regge trajectories
- Casimir and N-ality scaling
- string behavior

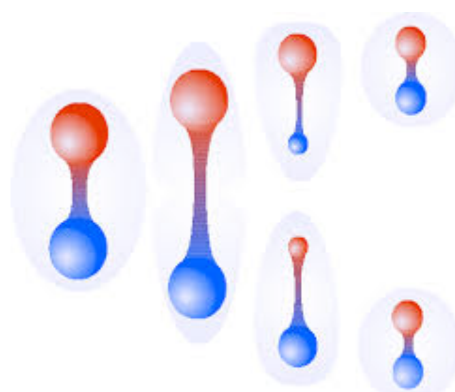


$r_0 = 0.5$ fm



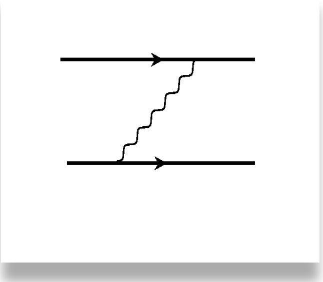






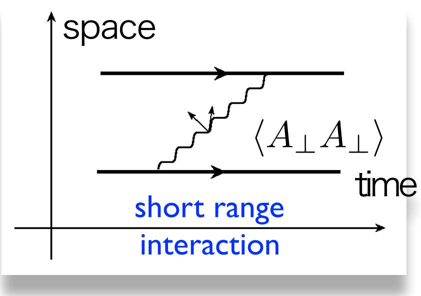
QCD vacuum and the role of gluons

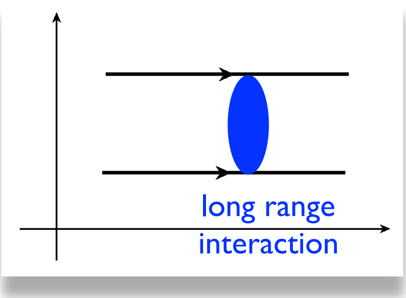
Gluons are responsible for confinement (aka effective potential between color charges) and are confined (aka contribute to the color charge)



Coulomb gauge

$$\nabla A^a(x) = 0$$





Coulomb gauge

Coulomb gauge Hamiltonian

$$B_i^a = \nabla_j A_k^a - \nabla_k A_j^a + g f^{abc} A_j^b A_k^c$$

$$H = \frac{1}{2}\int d\mathbf{x} \left[\mathcal{J}^{-1} \vec{\Pi}^a \mathcal{J} \vec{\Pi}^a + \vec{B}^a \vec{B}^a \right]$$

Jacobian (e.g. $r^{-1}\frac{d}{dr} r \frac{d}{dr}$)

$$\mathcal{J}(A) = Det \vec{\nabla}\mathcal{D}(\mathcal{A})$$

$$+ \int d\mathbf{x} \psi^\dagger \left[-i\vec{\alpha} \left(\vec{\nabla} - igA^aT^a \right) + \beta m \right] \psi$$

$$+ \frac{g^2}{2} \int d\mathbf{x} d\mathbf{y} \mathcal{J}^{-1} \rho^a(\mathbf{x}) K_{ab}[A](\mathbf{x},\mathbf{y}) \mathcal{J} \rho^b(\mathbf{y})$$

$$K = \frac{1}{\vec{\nabla}\mathcal{D}(A)} (-\vec{\nabla}^2) \frac{1}{\vec{\nabla}\mathcal{D}(A)} \qquad \rho^a = f^{abc} \vec{A}^b \vec{\Pi}^c + \psi^\dagger T^a \psi$$

$$H\left(\frac{\delta}{\delta A}, A\right) \Psi[A] = E\Psi[A], \;\; \int \mathcal{D}A \mathcal{J} |\Psi[A]|^2 = \langle | \rangle$$

$$\bar{H} = \mathcal{J}^{1/2} H \mathcal{J}^{-1/2}, \; \bar{\Psi} = \mathcal{J}^{1/2}\Psi \qquad \int \mathcal{D}A |\bar{\Psi}[A]|^2 = \langle | \rangle$$

Example of calculation Khriplovich, 1969

H_0 is a h.o.

$$|0\rangle \sim \exp(-\int dxdyA(x)\omega_0(x-y)A(y))$$

$$H = H_0 + gV$$

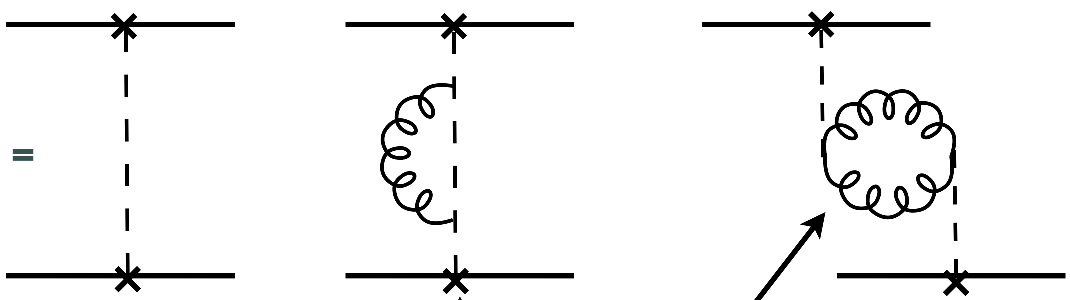
$$E = E_0 + gE_1 + g^2 E_2 + \cdots$$

calculate E for QQ in the perturbative QCD ground state

<H> enhanced in the IR from modes near horizon

real (quasi) particles propagating expected to be suppresses

E(R) =



$$E(R) = \frac{\alpha}{R}\left[1 + \frac{\alpha}{4\pi}[12 - 1]\log\left(\frac{1}{\Lambda R}\right)\right]$$

12 comes from the Coulomb potential

QCD

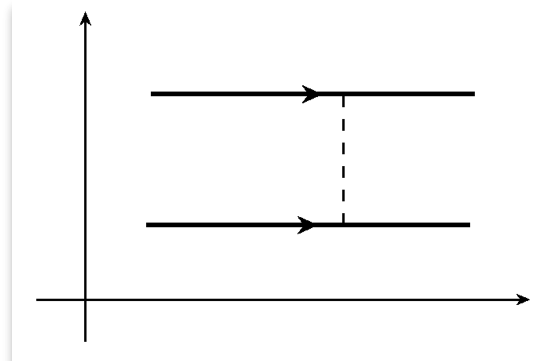
Debye screening

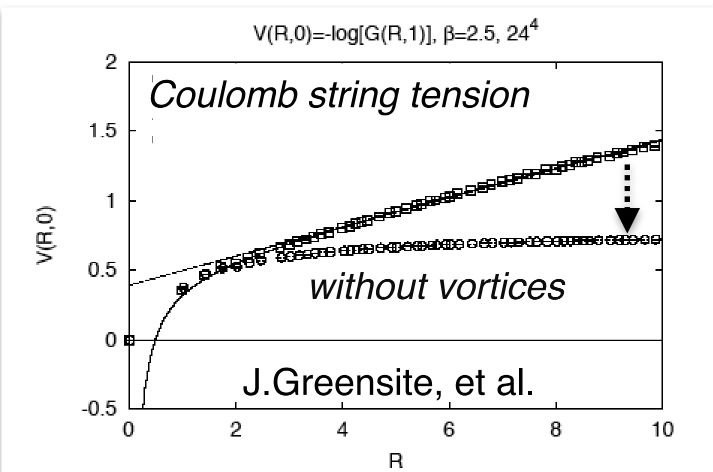
Confining Potential and the gluon condensate

$$H = H_{kin} + V \qquad H = H_{kin} + V$$

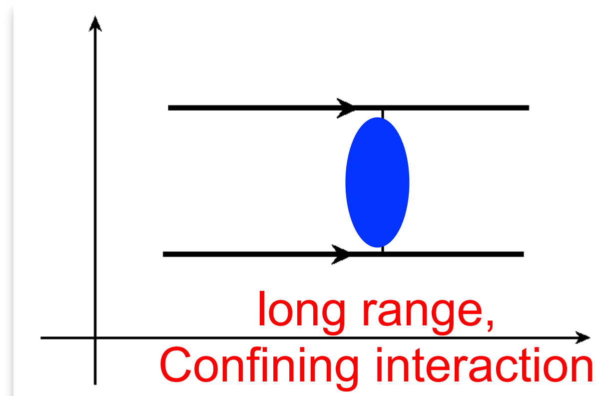
$$V = \int d\mathbf{x}d\mathbf{y}\,\rho(\mathbf{x})K[\mathbf{A},\mathbf{x},\mathbf{y}]\rho(\mathbf{y})$$

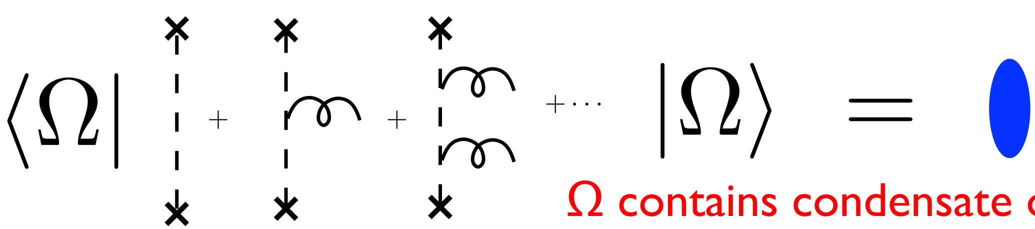
$$K \to -\frac{g^2}{\nabla^2} = \frac{\alpha}{|\mathbf{x}-\mathbf{y}|} = \quad V + \int d\mathbf{x}d\mathbf{y}\rho(\mathbf{x}V(\mathbf{x}-\mathbf{y})\rho(\mathbf{y})$$





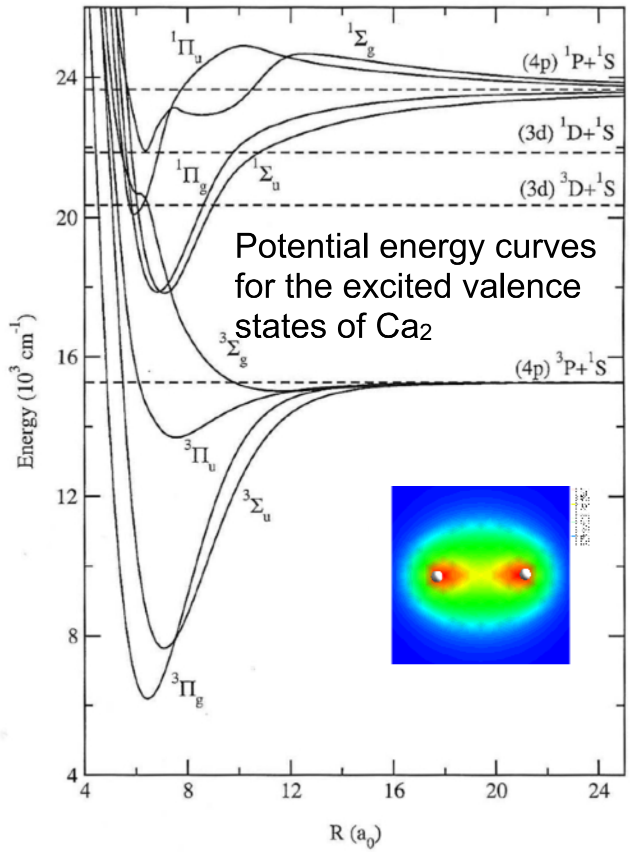
- Coulomb "Potential" between external (i.e. quark charges) depends on the distribution of gluons.
- In presence of a gluon condensate it produces a Confining force been external color charge



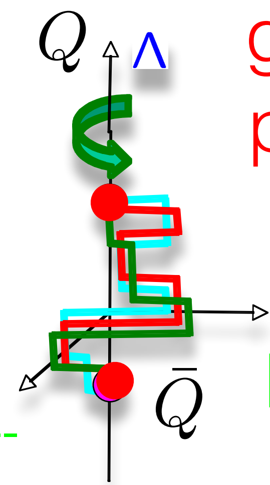


Ω contains condensate of monopoles, vortices, …

How to “measure the potential”



Potential energy curves for the excited valence states of Ca_2

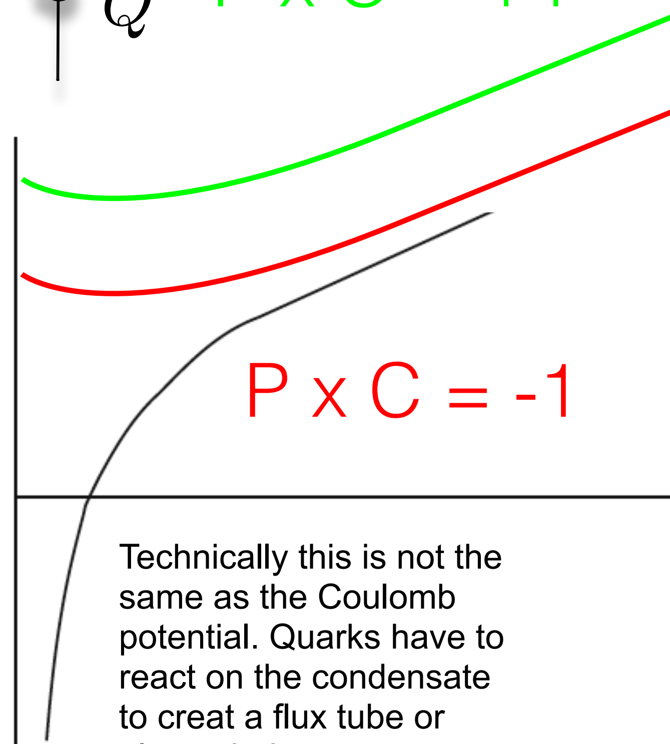


gluons behave as physical particles with $J^{PC} = 1^{+-}$

$J^{PC}=1^{--}$

P x C = +1

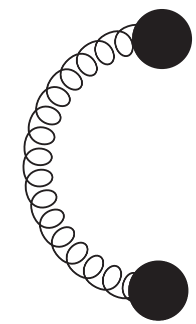
Energy of the gluon field



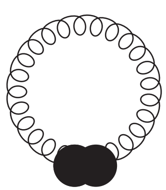
$J^{PC}=1^{+-}$

P x C = -1

$R \to 0$

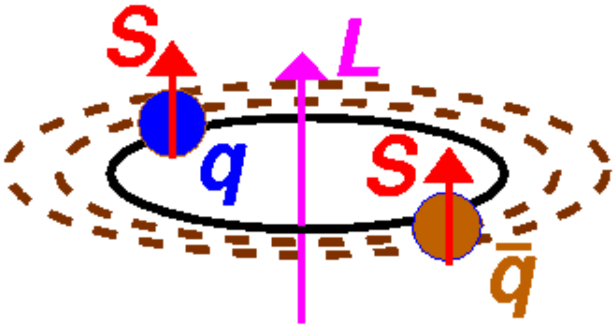


glue-lump



Technically this is not the same as the Coulomb potential. Quarks have to react on the condensate to creat a flux tube or gluon chain.

Quark Model with Gluons : Hybrid States



$$P_{q\bar{q}} = (-1)^{L+1}$$

$$C_{q\bar{q}} = (-1)^{L+S}$$

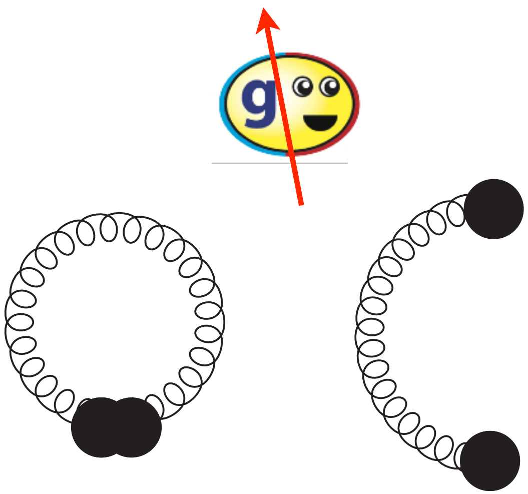
$$J_g^{PC} = 1^{+-}$$

J^{PC} glue

J^{PC} $Q\bar{Q}$

$$1^{+-} \times 0^{-+}_{S_{Q\bar{Q}}} = \boxed{1^{--}}$$

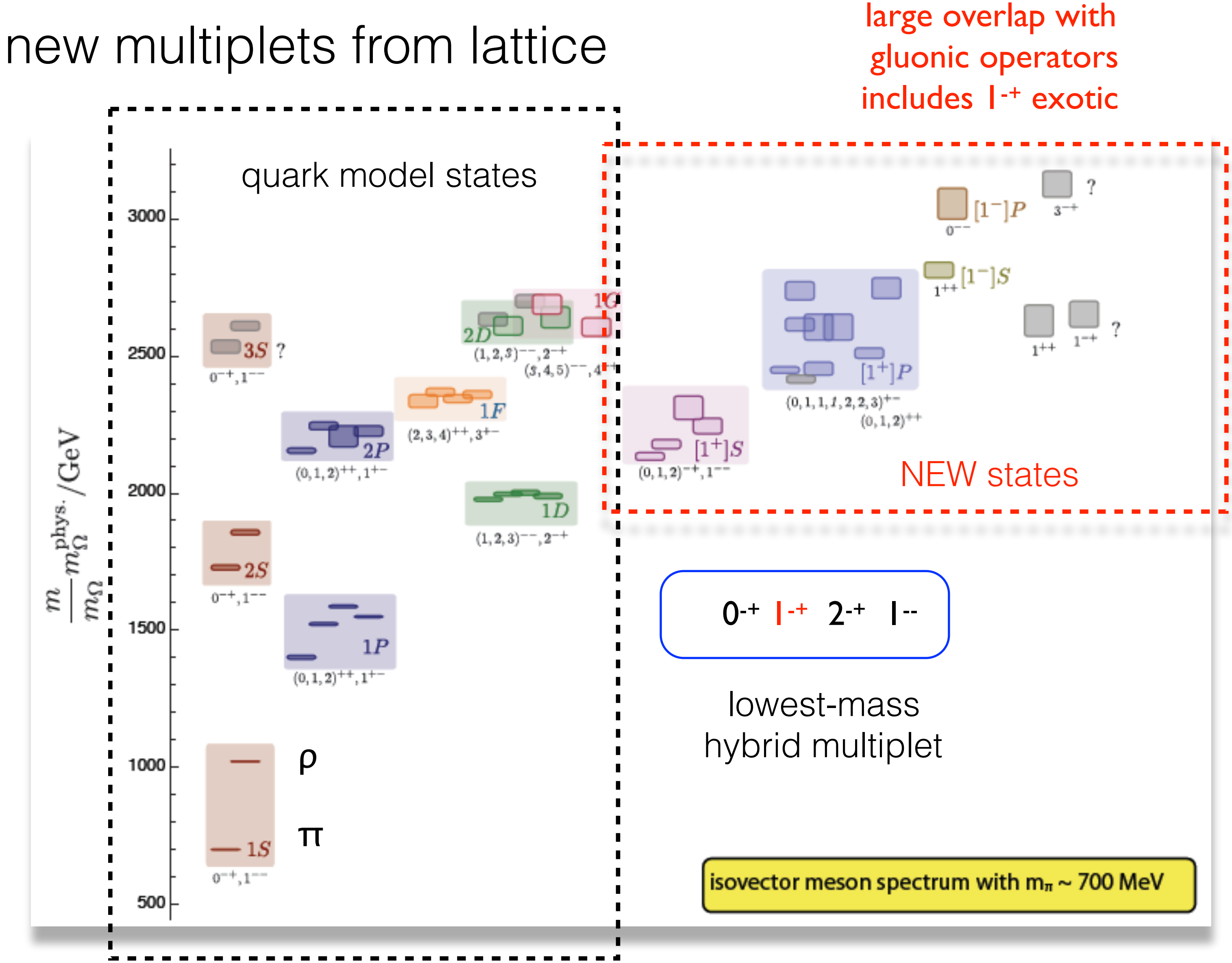
$$1^{+-} \times 1^{--}_{S_{Q\bar{Q}}=1} = \boxed{0^{-+}, 1^{-+}, 2^{-+}}$$



$J^{PC} = 1^{-+}$ is not a $q\bar{q}$ state

exotic quantum numbers

Meson Spectrum on the Lattice



J.Dudek et al. JLab

Light quark exotic candidate

$\pi^- p \to \eta\pi^- p$

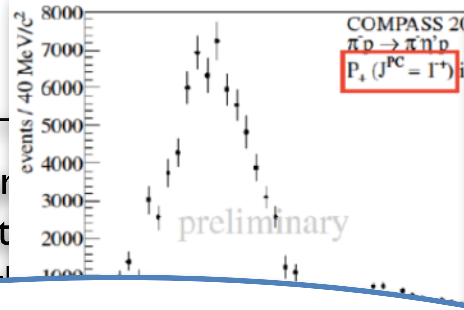
$M = 1370 \pm 16^{+50}_{-30}$ MeV / c^2

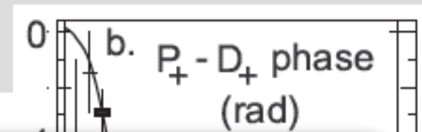
$\Gamma = 385 \pm 40^{+65}_{-105}$ MeV / c^2

$\pi^- p \to \eta\pi^0 n$

No consistent B-W interpr
possible but a weak ηπ int
exists and can reproduce t

search for

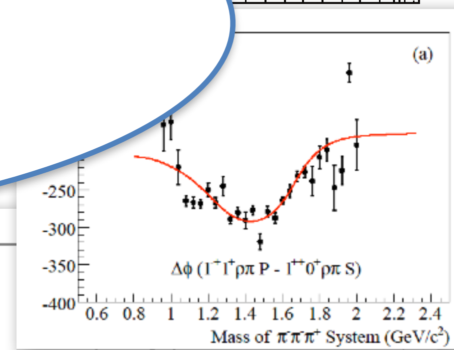




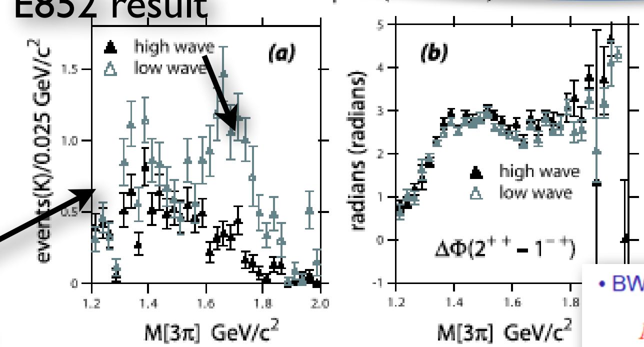
$\pi^- p \to \eta'\pi^-$

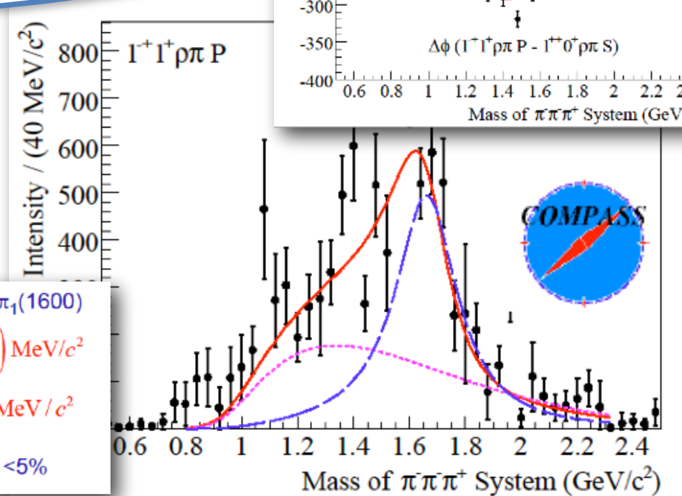
$\pi^- p \to \rho^0\pi^- p$

Need to be confirmed



E852 result





$\pi^- p \to \pi_2^-(1600)p$

$\pi_2^- \to \rho^0\pi^-$

$\rho^0 \to \pi^+\pi^-$

FIG. 25: (a) The $1^{-+}1^+$ P-wave $\rho\pi$ partial wave charged mode ($\pi^-\pi^-\pi^+$) for the high-wave set PWA a low-wave set PWA and (b) the phase difference $\Delta\Phi$ b the 2^{++} and 1^{-+} for the two wave sets.

- BW parameters for $\pi_1(1600)$

$M = \left(1660 \pm 10^{+0}_{-64}\right)$ MeV/c^2

$\Gamma = \left(269 \pm 21^{+42}_{-64}\right)$ MeV/c^2

- Leakage negligible: <5%

Beyond the quark model

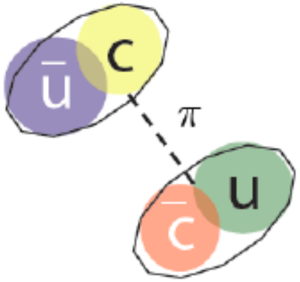
QCD: There are many other possible color singlets.

dibaryon

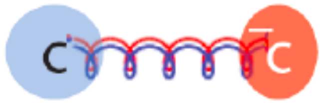
pentaquark

glueball

diquark + di-antiquark



dimeson molecule



$q\,\bar{q}\,g$ hybrid

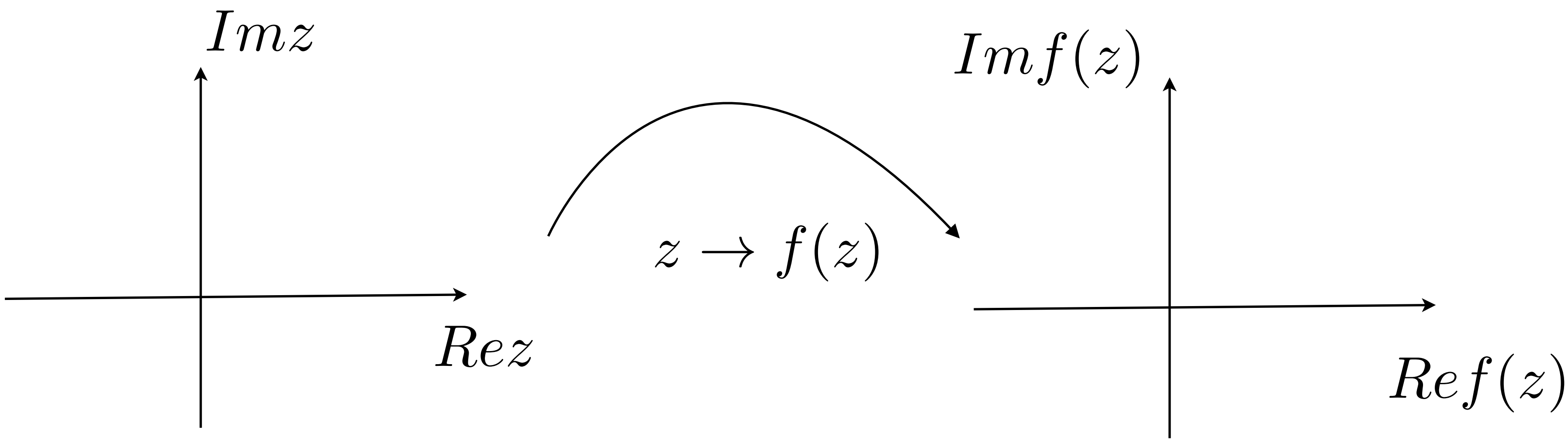
Quick Summary

- QCD vacuum has gluon condensate in the form color monopolies, vortices,…
- The condensate leads to an effective, confining potential between color charges
- Light quarks propagating through this medium acquire effective mass
- Static color charges (i.e. “very heavy” quarks) inserted into the vacuum polarize the condensate and change the background gluon distribution
- For large separation between the charges this leads to formation of a chromo electric flux tube (aka dual superconductor)
- For small separation between charges, the effect of vacuum polarization can be described as quasi-particles.
- Once the have quarks are allowed to move the polarized gluon filed (the quasi-particle of the flux tube) can result in a new type of hadrons -> hybrid mesons or baryons.

Intermezzo : Complex Calculus

$$z = a + bi \to f(z) = Ref(z) + iImf(z)$$

Elementary functions: you can also think of them as maps of one complex plane (z) to another (f(z)): z→ f(z)



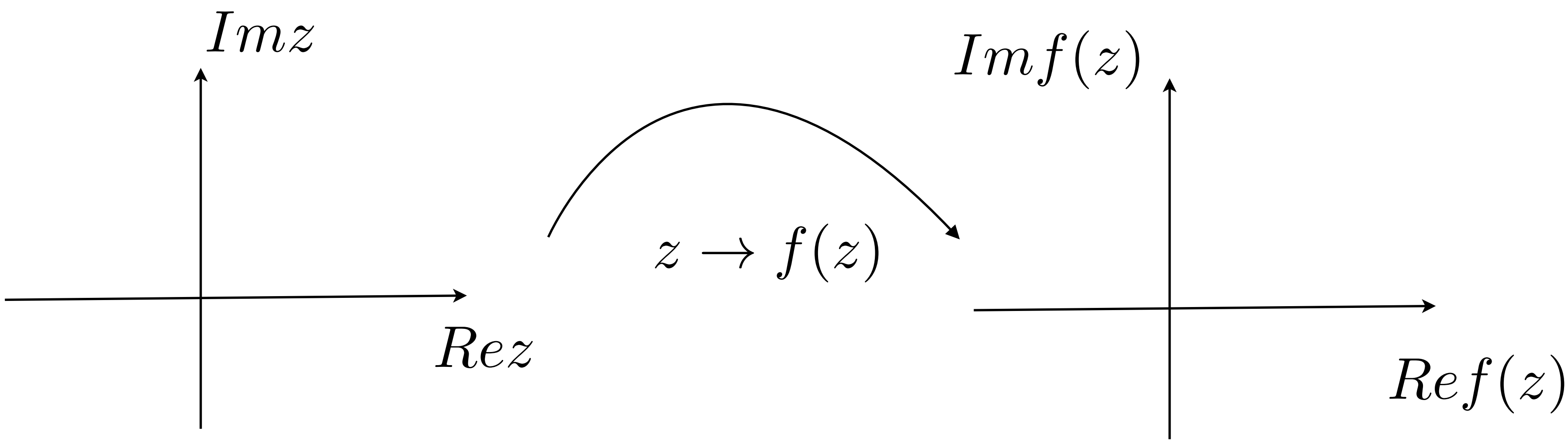
To define a function we can use the algebraic relations e.g

$f(z) = \sqrt{z}$ is such that $z = f(z) \times f(z)$

Complex functions

- Continuity imposes very strong conditions of functions (much stronger than in the case of real variables)
- "Smooth" (analytic) functions are "boring" all "action" is in the singularities (poles, cuts)
- Singularities determine functions "far away" from location of the singularity (e.g. local charge determines electric field)
- Physical observables are functions of real parameters, however physics law can be generalized to complex domains and become "smooth". Any "constraint" results in singularities.

Complex functions



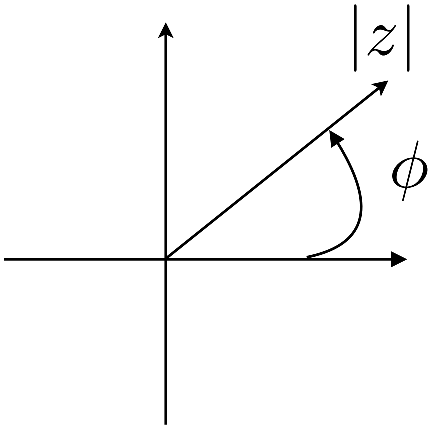
Often the mapping is not "one-to-one" and one needs to be careful in defining domains which give a unique value for the function, e.g. is

$$\sqrt{-25} = +5I \text{ or } -5I \ ?$$

$$z = |z| e^{i\phi}$$

$$\sqrt{z} \equiv \sqrt{|z|} e^{i\frac{\phi}{2}}$$

$$\sqrt{z}\sqrt{z} = \sqrt{|z|} e^{i\frac{\phi}{2}} \sqrt{|z|} e^{i\frac{\phi}{2}} = |z| e^{i\phi}$$



using $\phi = [-\pi, \pi)$

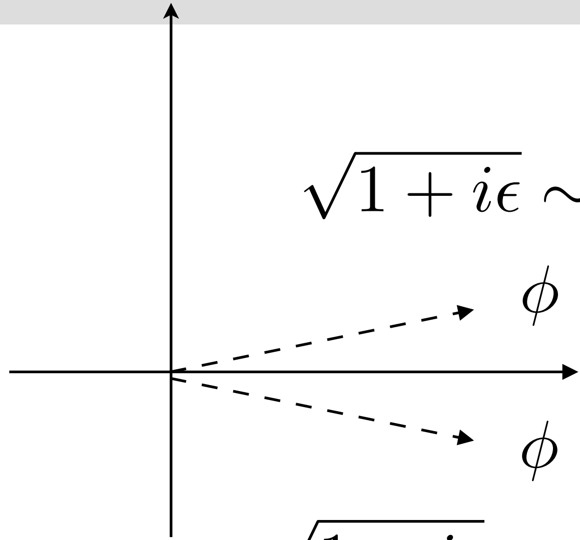
or $\phi = [0, 2\pi)$

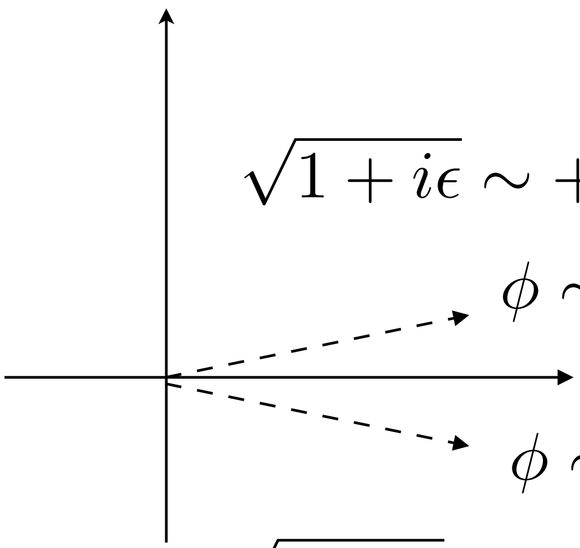
gives different results for $\sqrt{z}$

$$\sqrt{z} \equiv \sqrt{|z|} e^{i\frac{\phi}{2}}$$

- using $\phi = [-\pi, \pi)$

gives square root that is continuous near the positive real axis





- using $\phi = [0, 2\pi)$

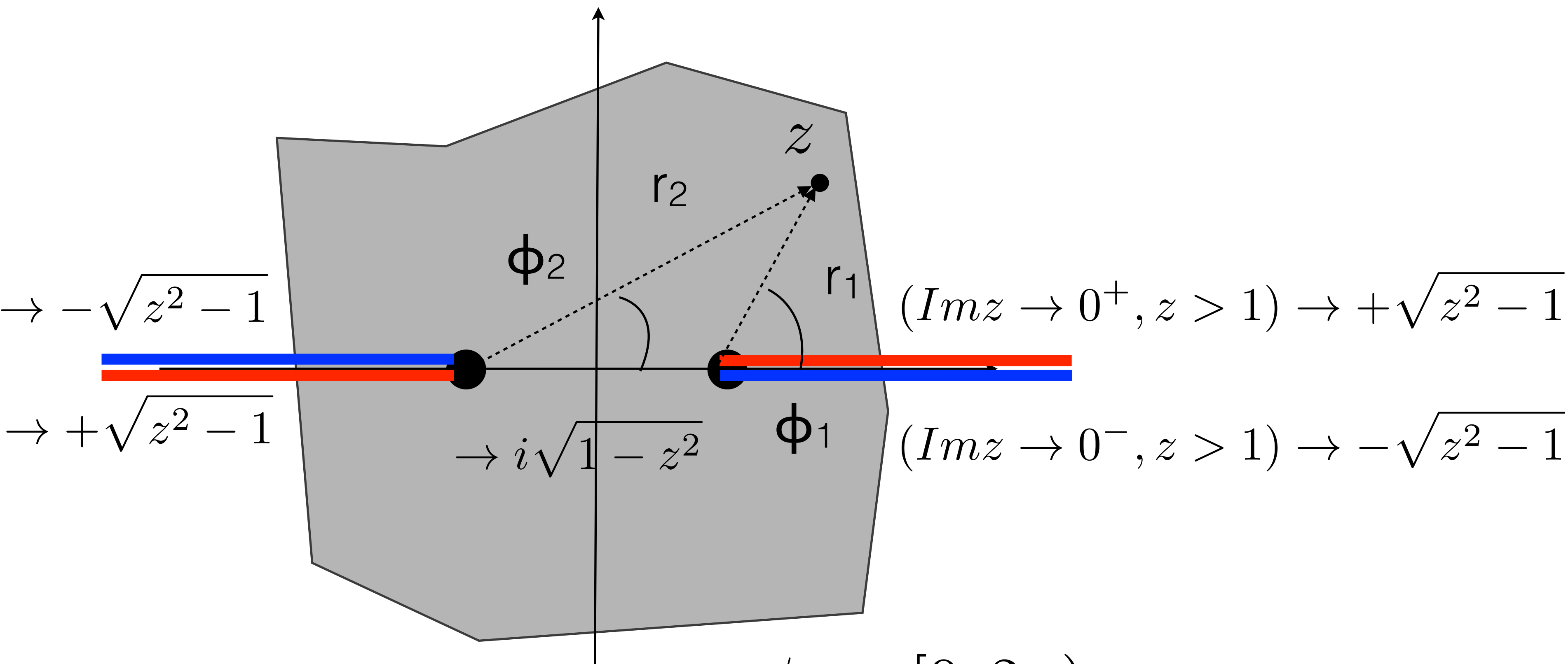
gives square root that is discontinuous near the positive real axis

In both case it has the same value when approaching the positive real axis rom above

More complicated functions

A. $z \to \sqrt{z^2 - 1}$

$$\sqrt{z^2 - 1} = \sqrt{r_1 r_2} e^{i\frac{\phi_1 + \phi_2}{2}}$$



$\phi_2 \in [-\pi, \pi)$ $\phi_1 \in [0, 2\pi)$

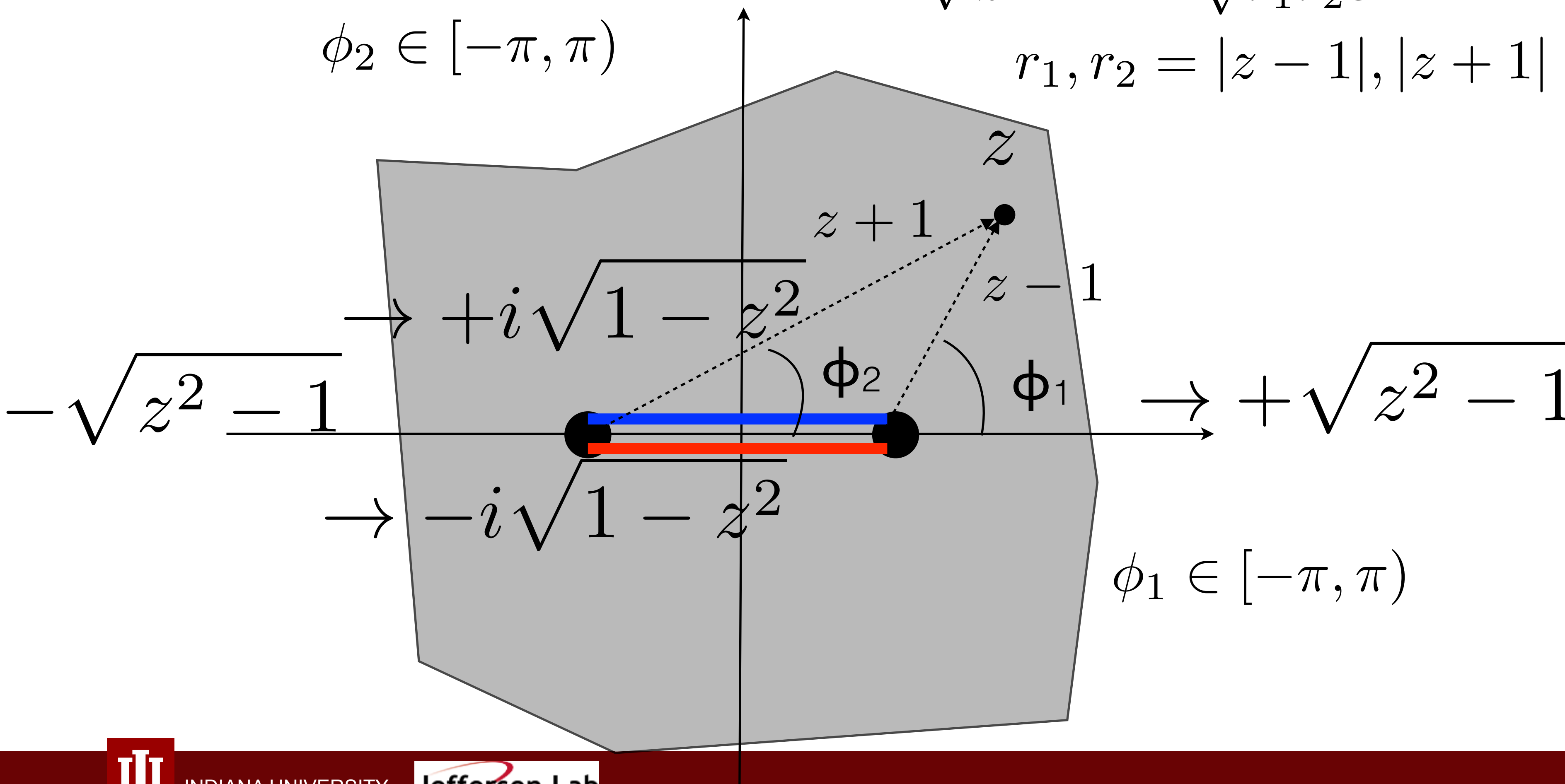
B. $z \to \sqrt{z^2 - 1}$

$$= \sqrt{z-1}\sqrt{z+1}$$

and use principal branches

$$\sqrt{z^2-1} = \sqrt{r_1 r_2} e^{i\frac{\phi_1+\phi_2}{2}}$$

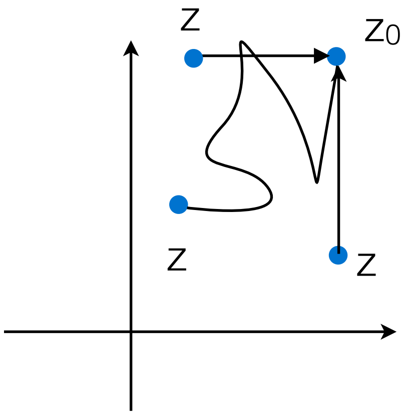
$$r_1, r_2 = |z-1|, |z+1|$$



Calculus: differentiation

f(z) is differentiable (holomorphic) if $\lim_{z \to z_0} \frac{f(z) - f(z_0)}{z - z_0} \equiv f'(z_0)$ exists

write $z = x + iy$ and f(z) as $f(z) = u(x,y) + i\, v(x,y)$. Since the procedure of taking the limit in definition of $f'(z_0)$ is independent of the path taken in $z \to z_0$, you can take two independent paths e.g. path 1: $x = x_0 + \varepsilon$, $y = y_0$ and path 2: $x = x_0$, $y = y + \varepsilon$: Cauchy relations:



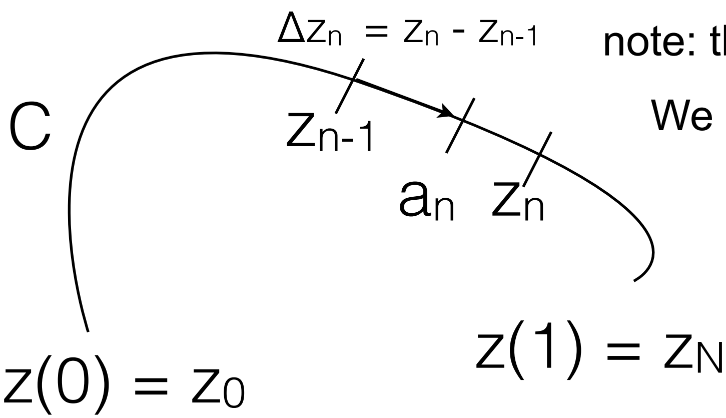
$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}, \quad \frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

This implies $\Delta u = \Delta v = 0$ where Δ is 2-dim Laplacian
u,v : harmonic functions

Calculus: integration

Line integrals: given a curve C in the complex plane parametrized by a real number $0 \le t \le 1$, $t \to z(t) = x(t) + iy(t)$ the integral of f over C is defined by

$$\int_C f(z)dz = \int_{t=0}^{1} f(z(t))\frac{dz}{dt}dt = \lim_{|\Delta z_n| \to 0, N \to \infty} \sum_{n=1}^{N} f(a_n)\Delta z_n$$



note: this is an ordered path

We can estimate the integral: if $|f(z)| \le M > 0$ along C then

$$\left| \int_C f(z)dz \right| \le Ms$$

where s it the length of the path

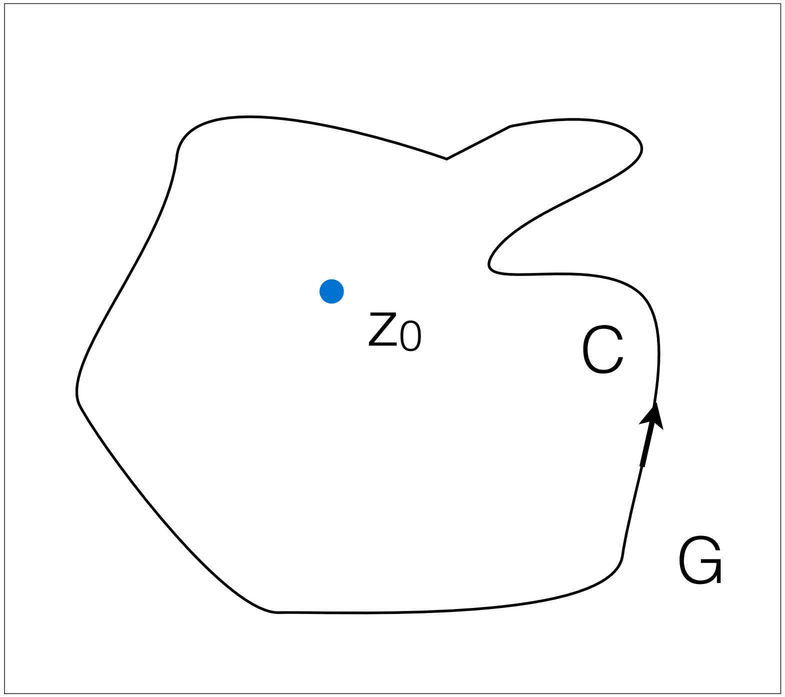
Cauchy-Goursat theorem: If f(z) is holomorphic in some region G and C is a closed contour (consisting of continuous or discontinuous cycles, double cycles, etc.) then

$$\oint f(z)dz = 0$$

(converse is also true)

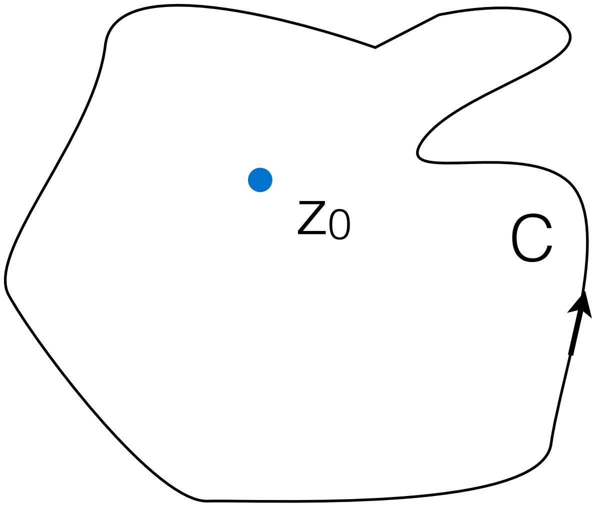
Cauchy formula

The Cauchy integral formula: if f(z) holomorphic in G, $z_0 \in G$, and C a closed curve (cycle), which goes around z_0 once in positive (counterclockwise) direction, then



$$f(z_0) = \frac{1}{2\pi i} \oint_C \frac{f(z)dz}{z - z_0}$$

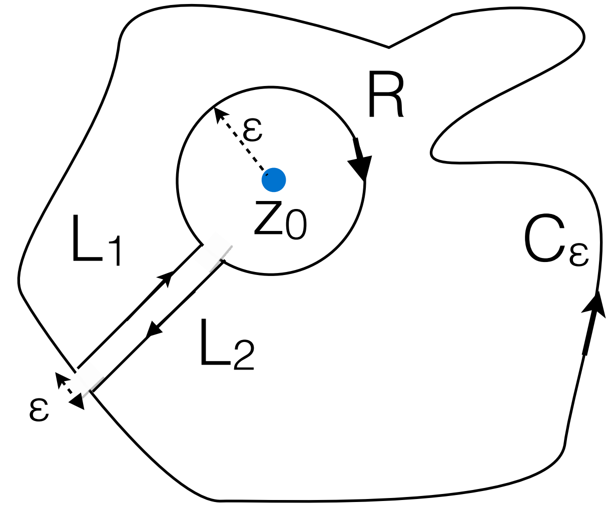
Proof



$\lim_{\varepsilon\to 0} C_\varepsilon = C$

$\lim_{\varepsilon\to 0} L_1 = -L_2$

$$\oint_{C'} \frac{f(z)dz}{z-z_0} = 0$$



$C' = C_\varepsilon + L_1 + L_2 + R$

$$0 = \oint_{C'} = \lim_{\epsilon\to 0}\left[\int_{L_1} + \int_{L_2} + \int_R + \int_{C_\epsilon}\right] = \lim_{\epsilon\to 0}\int_R + \int_{C_\epsilon}$$

$$\int_R \frac{f(z)dz}{z-z_0} = f(z_0)\int_R \frac{dz}{z-z_0} + \int_R \frac{f(z)-f(z_0)}{z-z_0}dz$$

$\varepsilon\to 0$: $\quad -2\pi i \qquad O(\epsilon)\to 0$

$z - z_0 = \epsilon e^{i\phi}$

$$-2\pi i f(z_0) + \int_C = 0$$

INDIANA UNIVERSITY Jefferson Lab

Exercise

Integrals:

$$\int_\gamma dz \qquad \int_\gamma z^n dz$$

$$\int_\gamma \frac{dz}{z} \qquad \int_{\gamma'} \frac{dz}{z}$$

$$\int_\gamma \frac{dz}{z^2}$$

γ = unit circle

γ’ = unit square

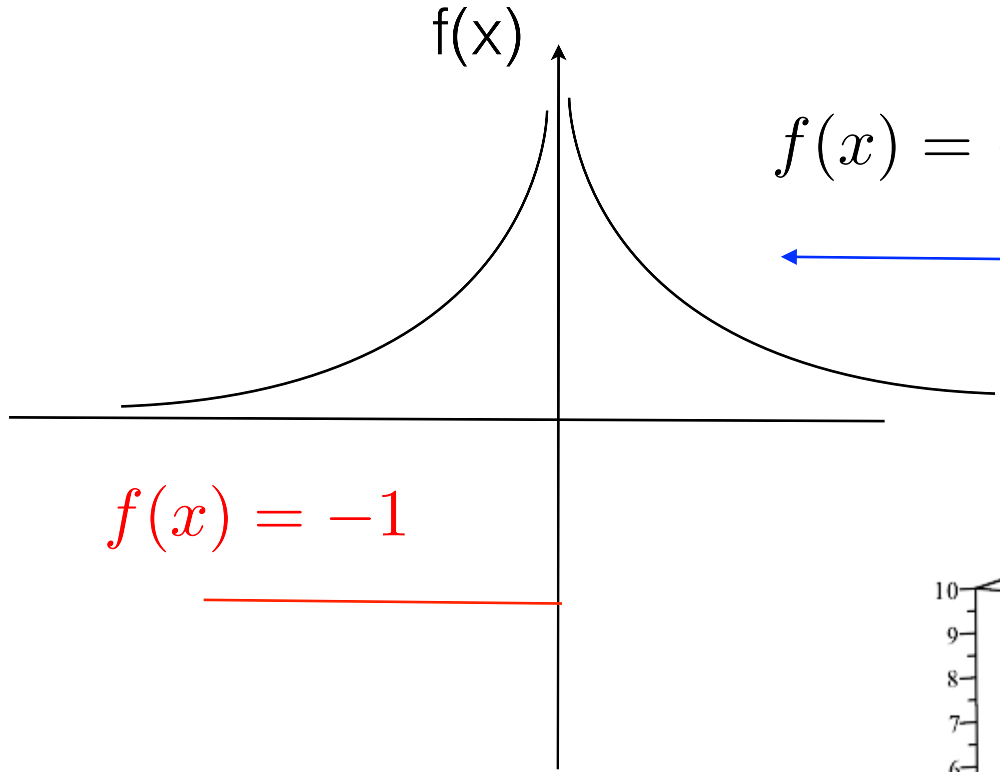
Contour deformations !!!!!

$$\int_{-1}^{1} dx \frac{1}{\sqrt{1-x^2}} = \pi$$

$$\int_{1}^{\infty} \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{2}$$

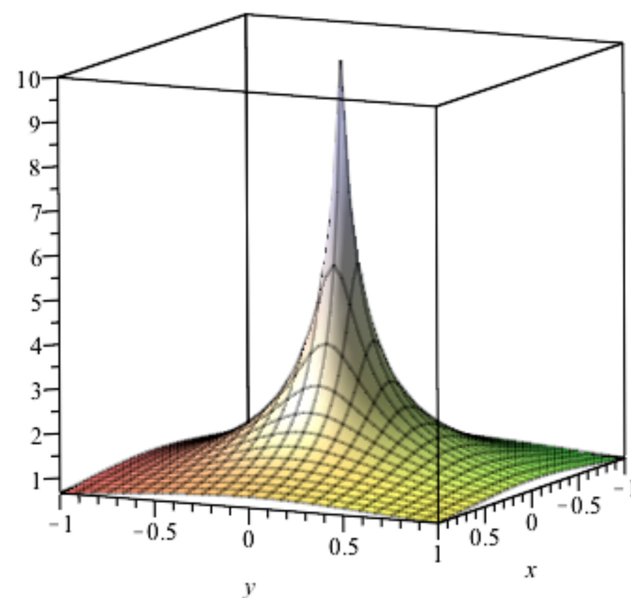
Analytical continuation

For real functions it does not work



$$f(x) = \frac{1}{x^2}$$

$$f(x) = -1$$



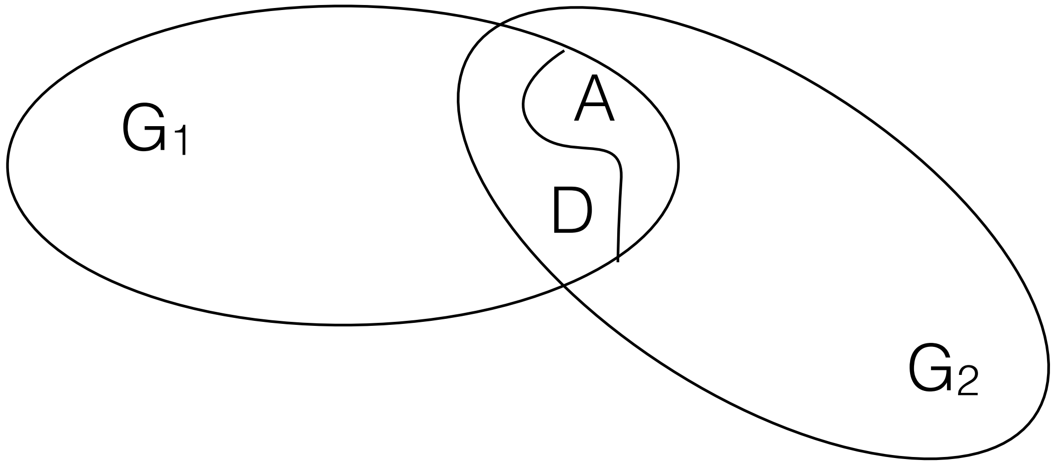
but for complex functions you can go continuously around the z=0 singularity and *analytically continue* from one region to another

Analytical continuation

Let $f_1(z)$ be holomorphic in G_1 and $f_2(z)$ in G_2, G_1 and G_2 intersect on an arch A (or domain D), and $f_1 = f_2$ on A (or D) then f_1 and f_2 are analytical continuation of each other and

$$f(z) = \begin{cases} f_1(z), z \in G_1 \\ f_2(z), z \in G_2 \end{cases}$$

is holomorphic in the union of G_1 and G_2



Examples

$1 + z + z^2 + \cdots$ is holomorphic in |z|<1

$\int_0^\infty e^{-(1-z)t} dt$ is holomorphic in Re z < 1

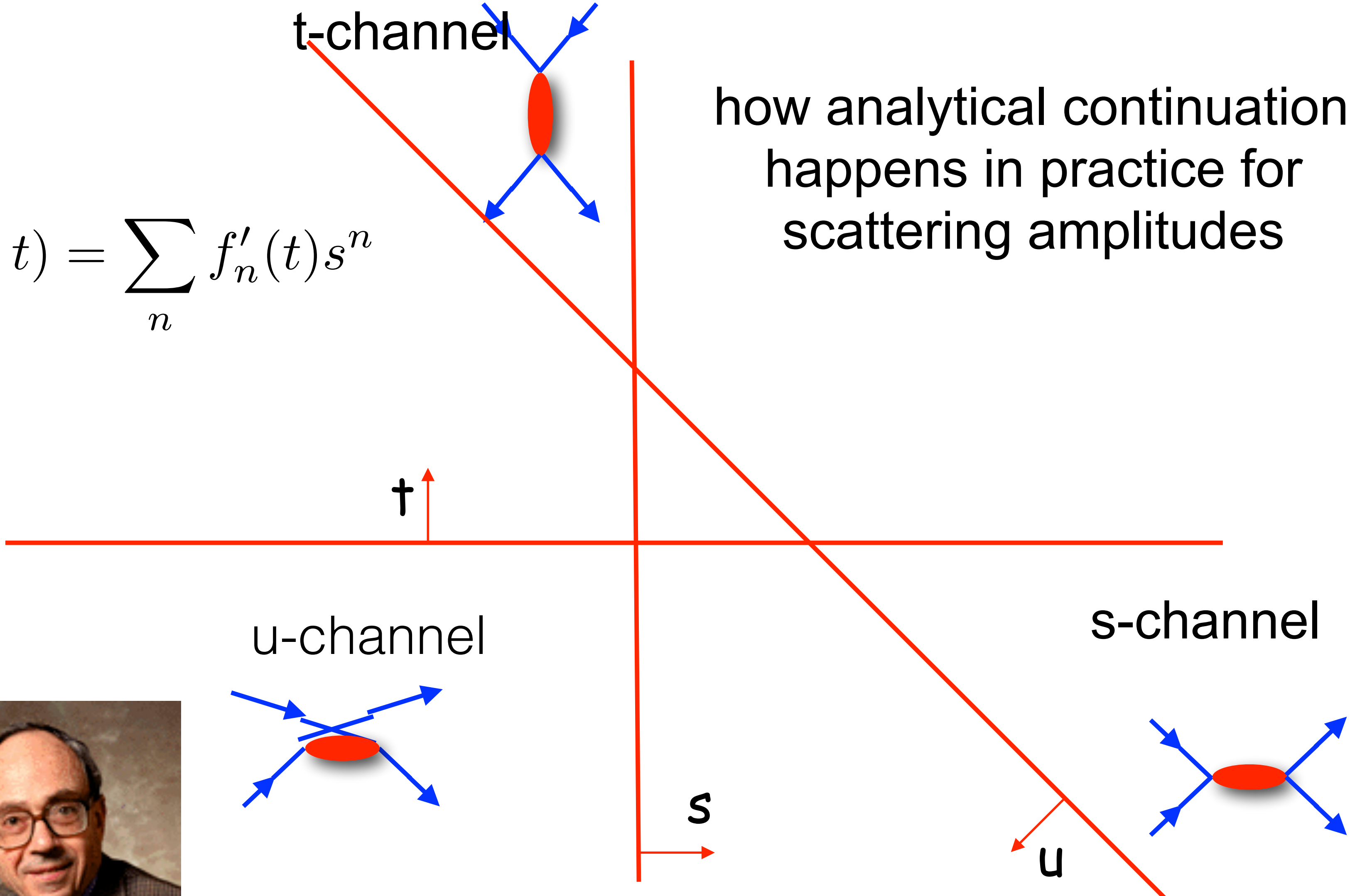
$-(1 + 1/z + 1/z^2 + \cdots)$ is holomorphic in |z|>1

all these functions represent f(z) = 1/(1-z) in different domains, which is holomorphic everywhere except at the point z=1

A(s,t) : how to continue from between s and t

how analytical continuation happens in practice for scattering amplitudes

$$f(s,t) = \sum_n f'_n(t) s^n$$



$$f(s,t) = \sum_n f_n(s) t^n$$

Continuation of integrals

what are the possibilities for g(s) to be singular?

$$g(s) = \int_C dz f(z,s) dz$$

g(s) can be singular at $s_0 \in G$ only if

1. $f(z,s_0)$ in z-plane has a singularity coinciding with the end points of the arc C (end-point singularity)

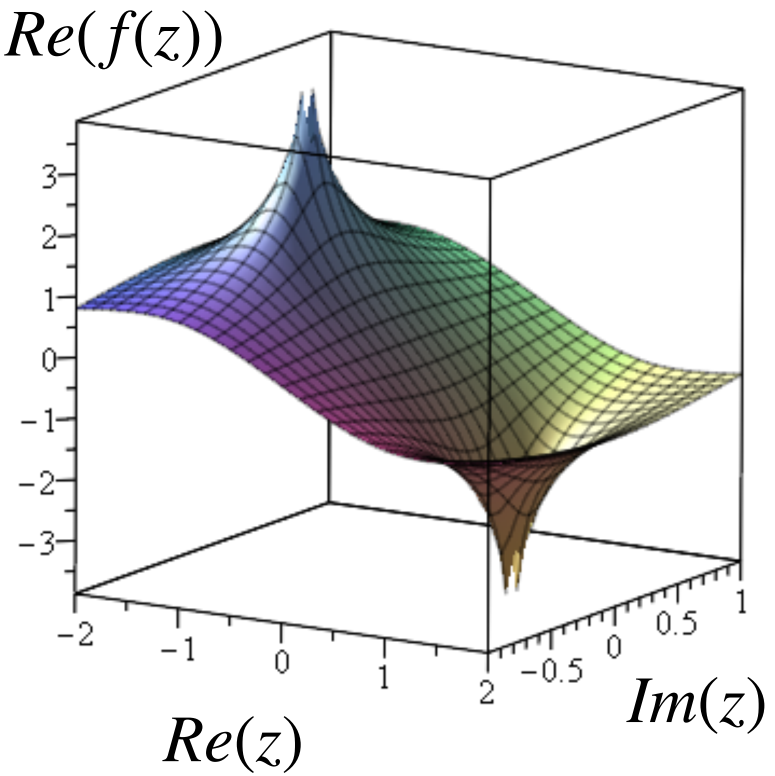
2. two singularities of f, $z_1(s)$ and $z_2(s)$, approach the arc C from opposite sides and pinch the arc precisely at $s=s_0$. (pinch singularity)

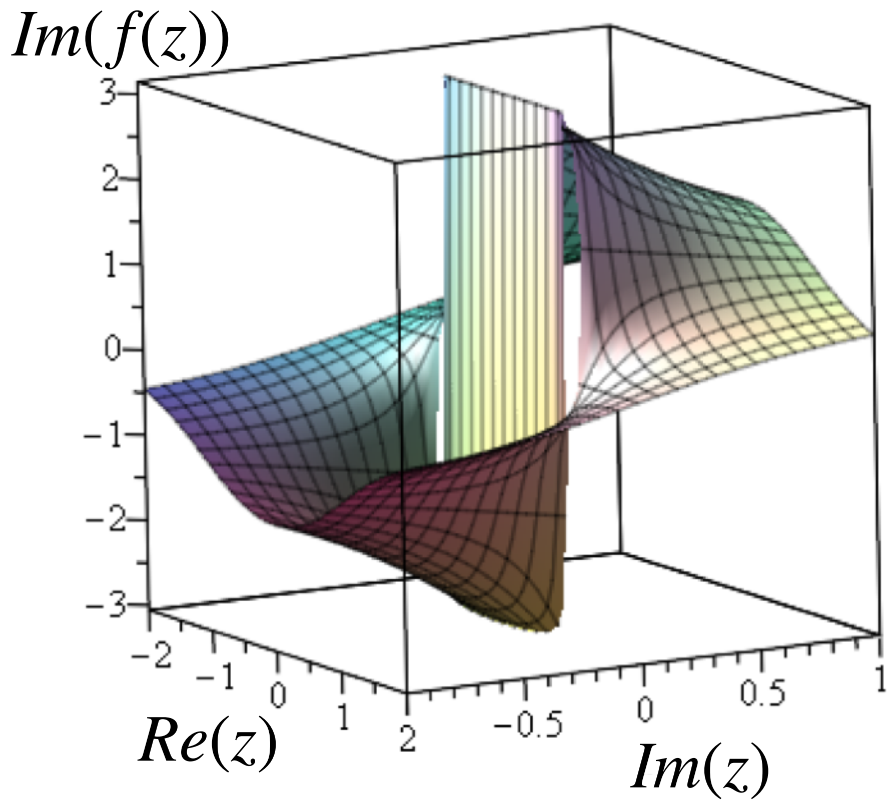
3. a singularity z(s) tents to infinity as $s \to s_0$ deforming the contour with itself to infinity; one has to change variables to bring the point ∞ to the finite plane to see what happens.

Example

$$f(z) = \int_{-1}^{1} \frac{dx}{x - z} = log(1 - z) - log(-1 - z)$$

where are the singularities ?

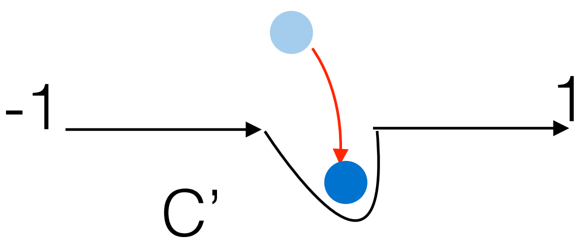




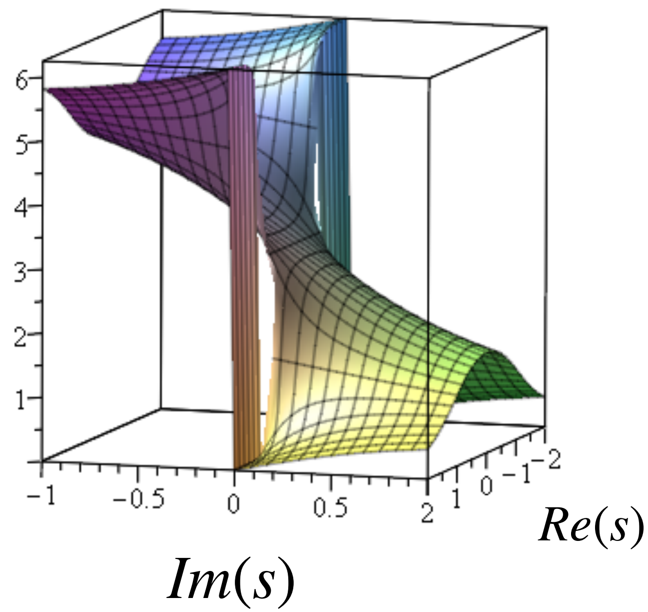
Before analytical continuation the result is on the 1st sheet !

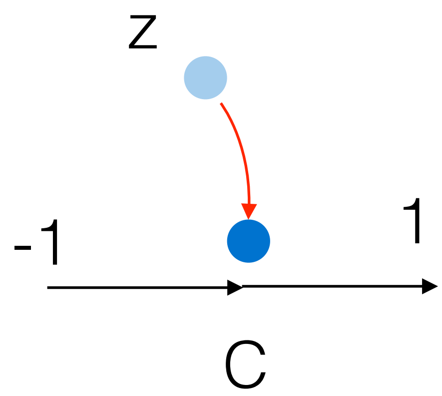
Analytical continuation takes us to "other" sheets

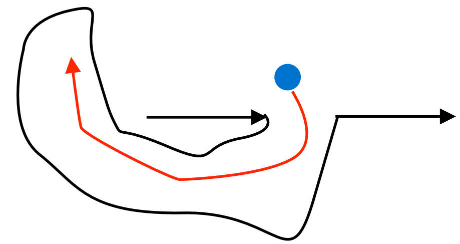
$$f(z) = \int_{-1}^{1} \frac{dx}{x - z} \quad C = [-1,1]$$



when z approaches x deforming C allows to define a function f(z) which changes continuously







Q: So on which sheet is physics

A: All. The 1st sheet is protected by analyticity. Other sheets have singularities which have physical interpretation

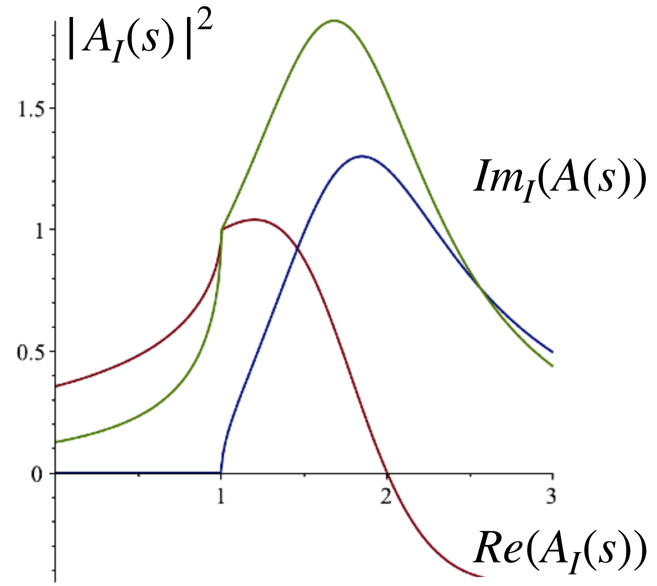
Breit-Wigner Formula (1st sheet)

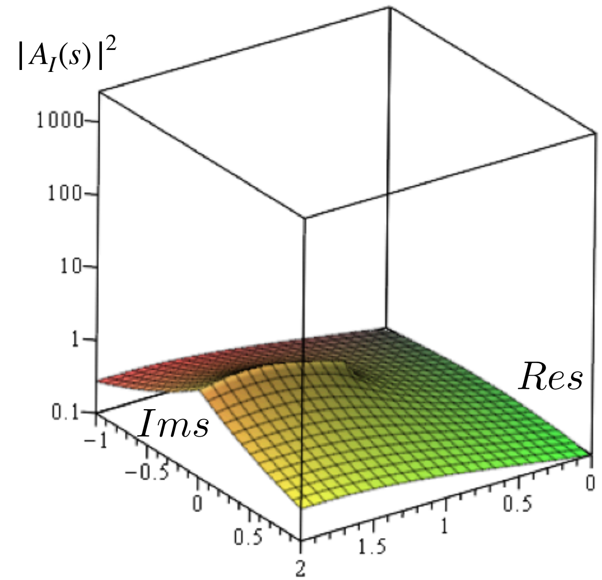
$$A(s) = \frac{1}{m^2 - s - im\Gamma(s)} \qquad \Gamma(s) = \sqrt{s - s_{st}} \times \text{rest(s)}$$

$$A(s) = \frac{1}{2 - s - 0.8i\sqrt{s-1}}$$

$$\downarrow$$

$$A_I(s) = \frac{1}{2 - s + 0.8\sqrt{1-s}}$$





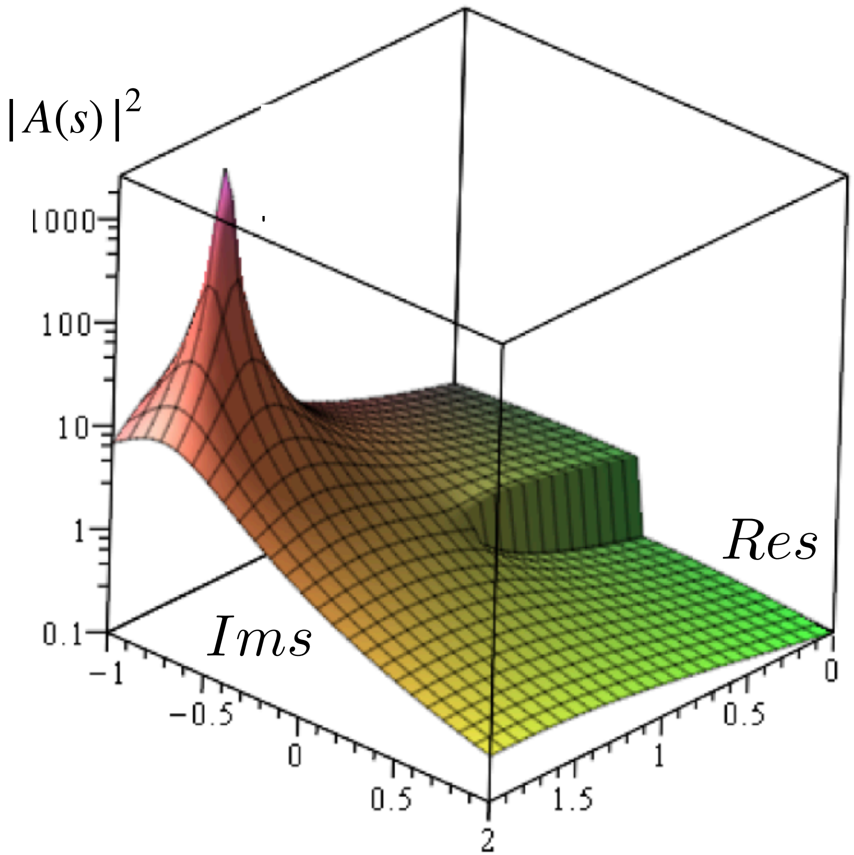
- Physics : s real i0 above the real axis
- Unitarity : cuts the real axis above thresholds.
- Analytical continuation to 2nd sheet displays resonances, etc.

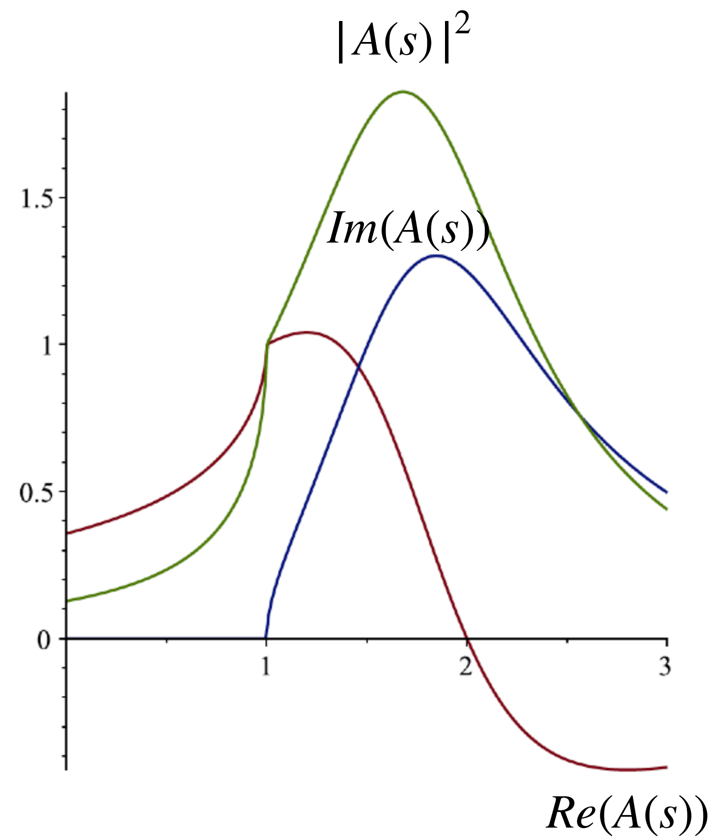
This formula is valid entirely on the 1st sheet

There is no peak !!!

Breit-Wigner Formula

$$A(s) = \frac{1}{2 - s - 0.8i\sqrt{s-1}}$$





This formula is valid on the 1st sheet for Im(s) > 0 and 2nd sheet for Im(s)<0

- Observables are smooth, analytical functions of variables. Physics law, constraints are manifested in singularities (poles, branch points)
- Cauchy theorem is a powerful tool to connect observables at different values of variables
- Physics is on 1st sheet but interesting phenomena happen on other sheets connected by analytical continuation, eg. Breit-Wigner formula

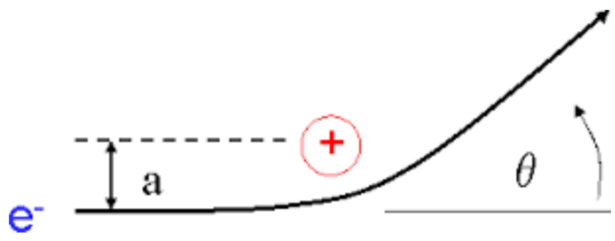
Probing QCD resonances (using physical states)

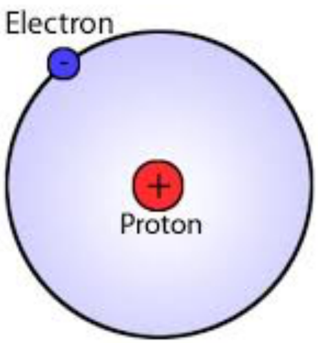
- When (color neutral) mesons and baryons a smashed, their quarks overlap, “stick together” and form resonances (quasi QCD eigenstates). They are short lived and decay to lowest energy, asymptotic states (pions, K’s, proton,…)
- Resonances are fundamental to our understanding of QCD dynamics because they are formed by all-order (aka beyond perturbation theory) interactions. Resonances challenge QFT practitioners to develop all orders calculations (still ways to go).
- (QCD) Resonance lead to extremely rich phenomenology, e.g. XYZ states, gluonic excitations, etc.
- In practice, one requires tools that relate asymptotic states before collision to asymptotic states after collision that include flexible parametrization of the microscopic dynamics. This is often referred to as amplitude analysis. The rest of these lectures will focus on this topic.

Bound states/Resonances/Asymptotic states

$$\left[\frac{p^2}{2m_e} - \frac{\alpha}{r}\right]\psi(r) = E\psi(r)$$

$$\alpha = \alpha_{QED} = \frac{1}{137}$$





$$\psi(r) = \frac{e^{-ikr}}{r} - S\frac{e^{+ikr}}{r} \qquad (1)$$

$$S = 1 + O(\alpha)$$

$$\psi(r) = e^{-\alpha m_e r} \qquad (2)$$

Born approximation : “weak” perturbation (lowest order) to free motion

Bound states: compact wave function contains interaction to all orders.

Resonances: particles interact to all orders (like bound states) but eventually decay (connect with asymptotically free states). Their effect appears in the S-matrix : Compare (1) and (2) ! $(k = i\alpha m_e)$

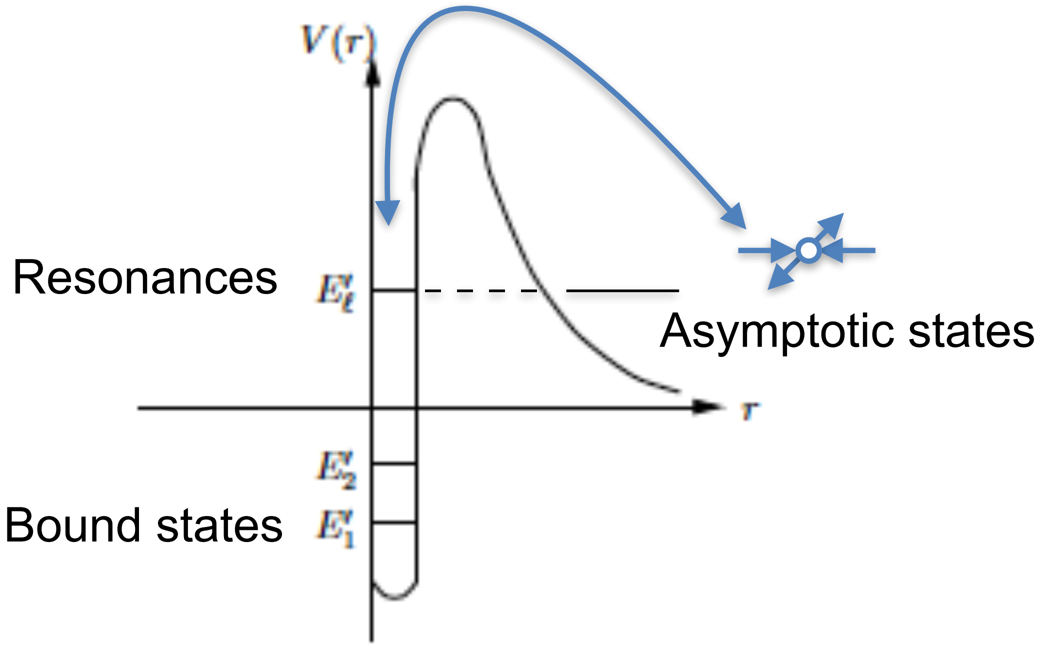
Causality implies Analyticity

Actual relation depends on the type of problem (mechanics, Q.M., QFT, ...)

You will see similarities, though, i.e. absence of singularities on the physical sheet.

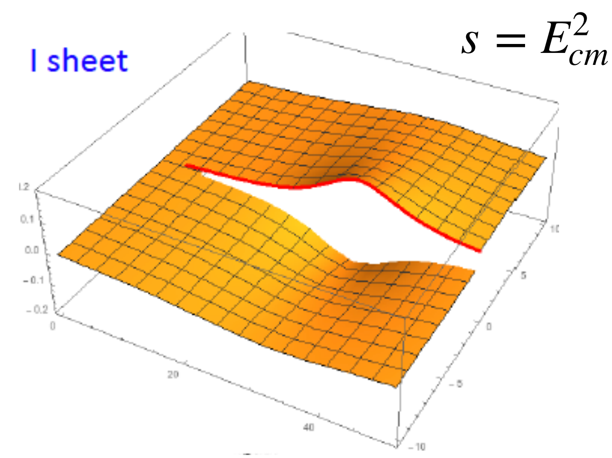
Conservation law i.e. probability deals with time dependent flow “messes with analyticity” and introduces singularities outside physical sheet.

Amplitude analyticity: it is much about complex functions



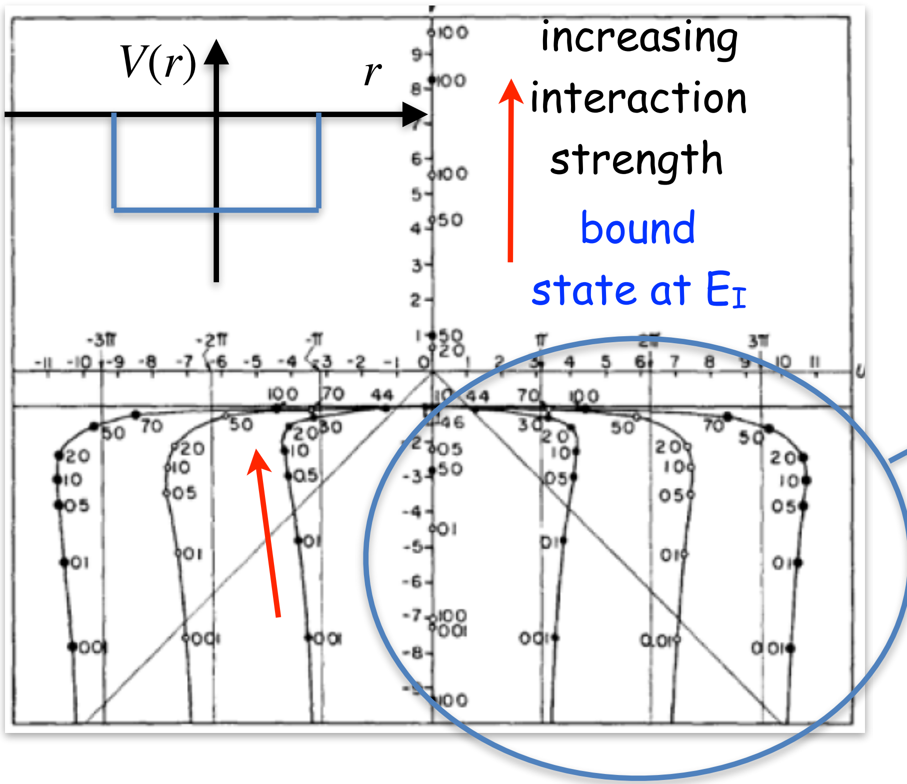
$$A_{physical} = A(s + i\epsilon) \rightarrow A(s = complex)$$

analytical continuation

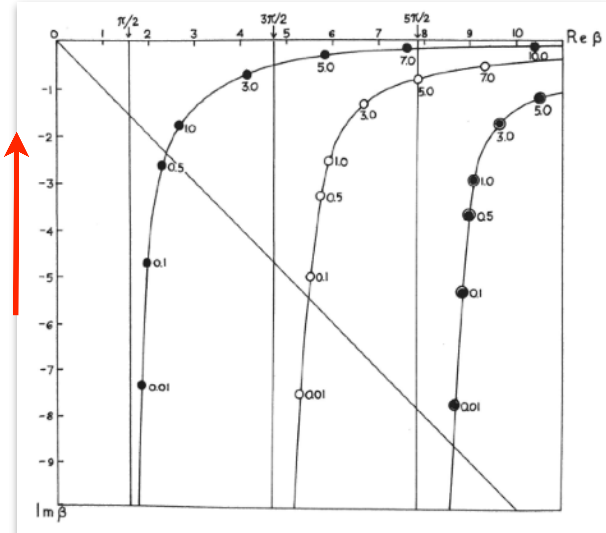


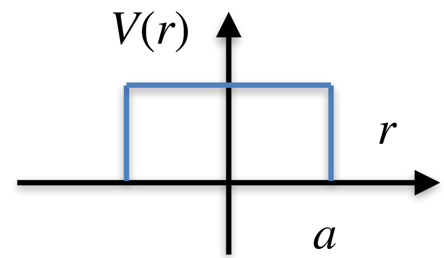
- Scattering amplitude describes evolution between asymptotic states. The information related to formation of resonances is “hidden” in unphysical domains (sheets) of the kinematical variables.
- The “bump” in the right figure is an indication of a “hidden” phenomenon. To uncover it one needs to analytically continue outside the physical sheet.

Potential Well vs Barrier : not very pole is a resonance



increasing
interaction
strength





- Resonances have minimum width before they become bound states

$$\Gamma \sim \frac{1}{\tau} \sim \frac{v}{a}$$

- Average velocity inside the Well is always finite

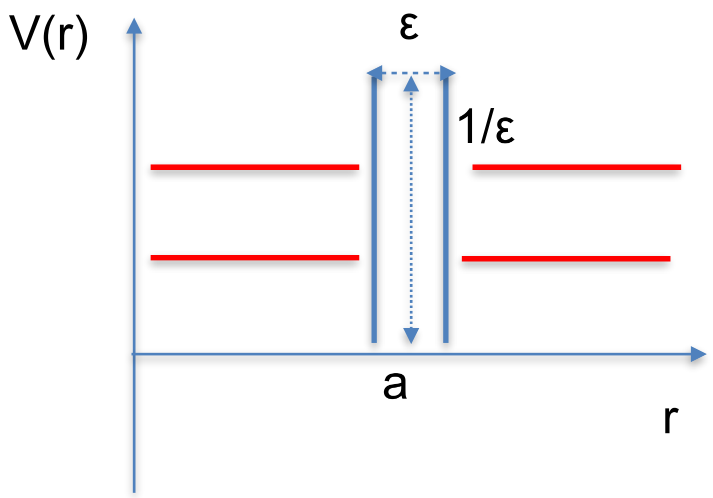
$$\sim \frac{k}{a} \sim \frac{\sqrt{E-V}}{a}$$

- Resonances move to + ∞ with wishing width
- Average velocity of the wave infinitesimal -> long time spend on top of the barrier

Every pole is a resonance (positive energy finite lifetime) but not all resonances (poles) are connected to bound states

Infinite, thin shell

$$H = \frac{p^2}{2\mu} + V \qquad V = \frac{\lambda}{2\mu a^2}\delta(r-a) \qquad \mathrm{dim}\lambda = -1$$



“Relation” to QCD

Inside the shell (0<r<a) particles are confined (like quarks in hadrons)
The shell is thin allowing for free asymptotic states (hadron decays)

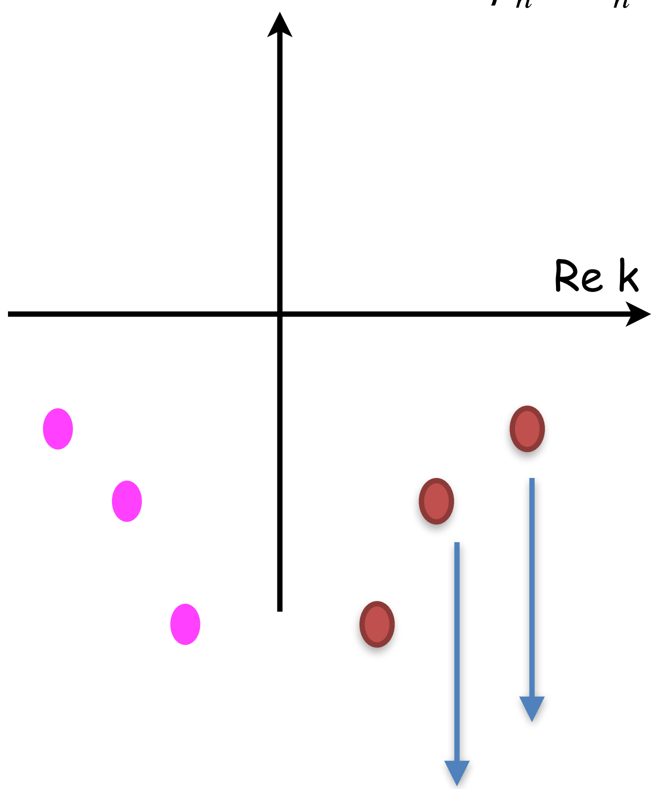
Method 1: In coordinate space (as before)

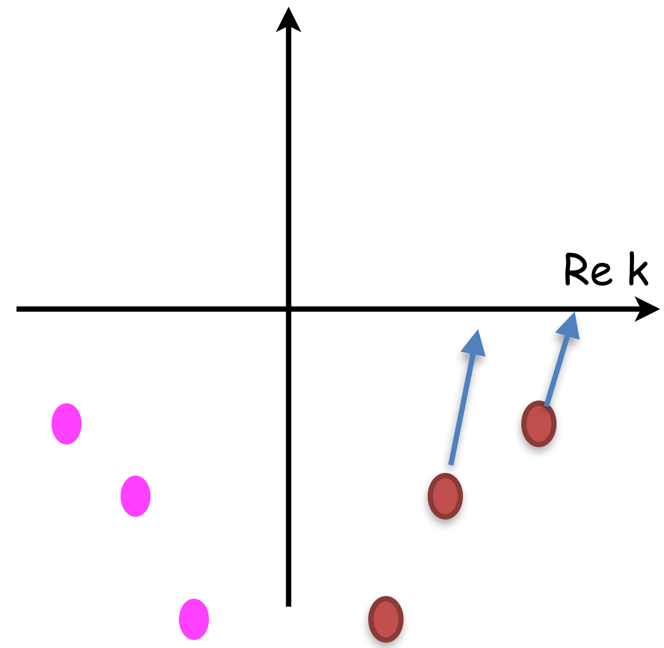
Method 2: Lippmann-Schwinger

$$T = V + VG_0V + \cdots$$

- For any strength of the potential there is an infinite number of resonances
- There is one pole in each strip $(n-1)\pi < \Re(\beta_n) < n\pi \quad (n = 1,2,\cdots)$

$$\beta_n = k_n a$$





- as potential strength decreases :

$$\beta_n \to (n - \frac{1}{2}) - i\infty$$

- as potential strength increases :

$$\beta_n \to n\pi\left(1 - \frac{1}{1+A}\right) - i\left(\frac{n\pi}{A}\right)^2$$

$$A = \lambda / a$$

S-matrix properties (in relativistic theory)

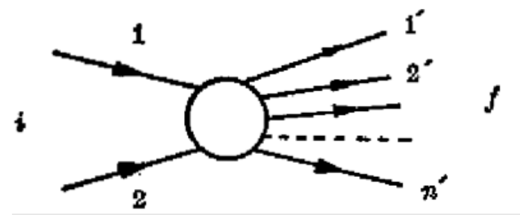
- There are no potentials
- Particles and antiparticles are related by crossing
- There are NO exact, non perturbative methods in QFT (major challenge for mathematicians)
- Physics lows are manifested as singularities of analytical functions (observables)

First order of business: understand properties of reactions enforced by these general principles.

S-matrix properties (in relativistic theory)

- Related to transition probability

$$P_{fi} = |\langle f|S|i\rangle|^2 = \langle i|S^\dagger|f\rangle\langle f|S|i\rangle$$



- Conservation of Probability = Unitarity

$$\sum_f P_{fi} = 1$$

$$S^\dagger S = I$$

$$2ImT_{ft} = \sum_n 2\pi\delta(E_i - E_n)T^*_{fn}T_{ni}$$

- Lorentz symmetry: T is a product of Lorentz scalars and covariant factors representing wave functions of external states, e.g for $\pi(k_1) + N(p_1,\lambda_1) \to \pi(k_2) + N(p_2,\lambda_2)$

$$\bar{u}(p_1,\lambda_1)[A(s,t) + (k_1+k_2)_\mu\gamma^\mu B(s,t)]u(p_2,\lambda_2)$$

- Crossing symmetry: the same scalar functions describe all process related by permutation of legs between initial and final states (only the wave function change)

$$\pi(k_1) + \pi(-k_2) \to \bar{N}(-p_1,\mu_1) + N(p_2,\mu_2)$$

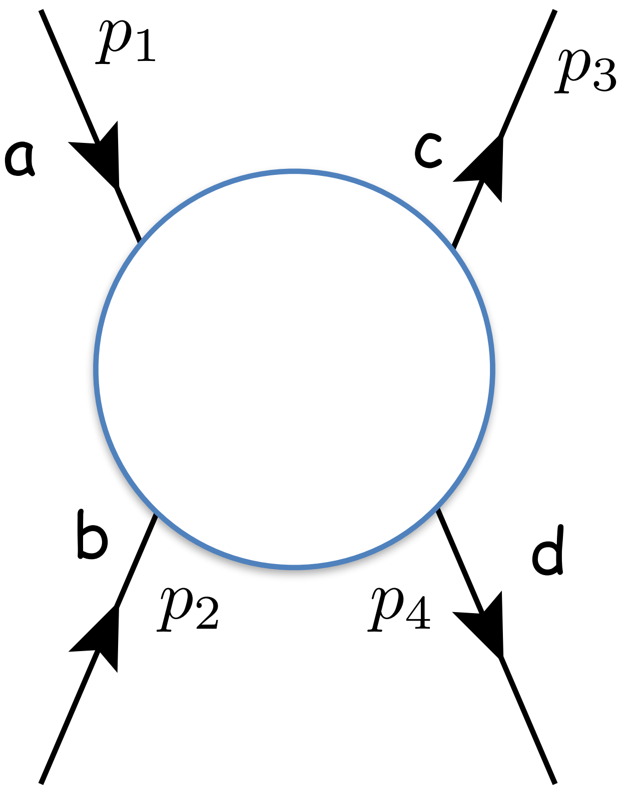
$$\bar{v}(p_1,\mu_1)[A(s,t) + (k_1+k_2)_\mu\gamma^\mu B(s,t)]u(p_2,\mu_2)$$

- Analyticity: The scalar functions are analytical functions of invariants

Lorentz symmetry

N-to-M scattering depends on 4(N+M)-(N+M)-10 = 3(N+M)-10 invariants

e.g for 2-to-2: 2 invariants related to the c.m. energy and scattering angle



$$s = (p_1 + p_2)^2 \; = (E_{1,cm} + E_{2,cm})^2$$

$$t = (p_1 - p_3)^2$$

$$t = m_1^2 + m_2^2 - 2E_{1,cm}E_{2,cm} + 2|p_{1,cm}||p_{2,cm}|z_s$$

$$u = (p_1 - p_4)^2 \qquad s + t + u = \sum_i m_i^2$$

$$u = m_1^2 + m_4^2 - 2E_{1,cm}E_{4,cm} - 2|p_{1,cm}||p_{4,cm}|z_s$$

$$2\pi\delta(E_f - E_i) iT = \langle c, d|(S-1)|a, b\rangle$$

Dimensions $\langle p', \beta | p, \alpha\rangle = 2E(\mathbf{p})\delta(\mathbf{p}_f - \mathbf{p}_i)\delta_{\alpha,\beta}$

$$T = (2\pi)^3\delta(\mathbf{p}_f - \mathbf{p}_i)A(s,t,u)$$

r.h.s has dim = -4

A(s,t,u) is a scalar function of mass dimension =0

Question

How many independent variables describe

- Decay proces $A \to a + b + c$
- Three particle production $A + B \to a + b + c$

Helicity amplitudes

We work in the c.m. frame $$\frac{\vec{S}\cdot\vec{p}}{|\vec{p}|}|p,\lambda\rangle = \lambda|p,\lambda\rangle$$

$$\langle p_3,\lambda_3;p_4,\lambda_4|A|p_1,\lambda_1;p_2,\lambda_2\rangle = A_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(s,t,u)$$

Helicity states vs canonical spin states: $$S_z|p,m\rangle_z = m|p,m\rangle_z$$

$$|p,m\rangle_z = \Lambda(\vec{p}\leftarrow 0)|0,m\rangle_z$$

$$|p,\lambda\rangle = R(\hat{p})\Lambda(|\vec{p}|\hat{z}\leftarrow 0)|0,m\rangle_z$$

Exercise show this: $$|p,\lambda\rangle_z = \sum_{m=-S}^{S}|p,m\rangle_z D^S_{m,\lambda}(\hat{p})$$

- Even though this looks non relativistic it is relativistic. Notion of LS amplitudes, LS vs. helicity relations are relativistic

η = naturally

Parity $$A_{\lambda_1,\lambda_2,\lambda_3,\lambda_4}(s,t,u) = \eta A_{-\lambda_1,-\lambda_2,-\lambda_3,-\lambda_4}(s,t,u)$$

Question

How many independent scalar functions describe

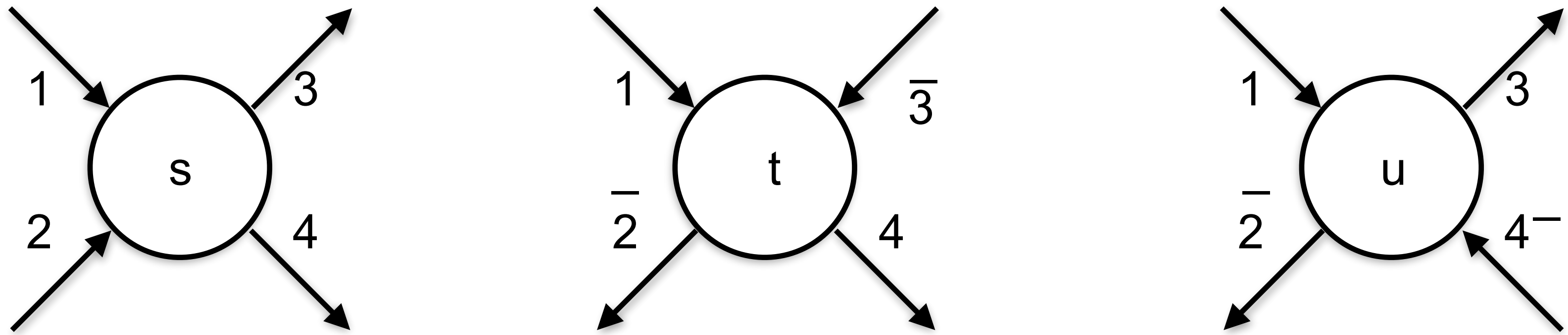
$J/\psi \rightarrow \pi^+ \pi^- \pi^0$

$\gamma\, p \rightarrow \pi^0\, p$

Crossing symmetry

$$\bar{p}_i = -p_i = (-\vec{p}_i, -E_i)$$

$a(p_1) + \bar{c}(\bar{p}_3) \to \bar{b}(\bar{p}_2) + d(p_4)$



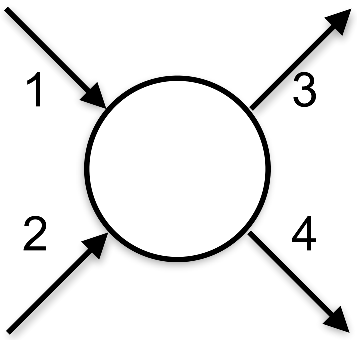
$a(p_1) + b(p_2) \to c(p_3) + d(p_4)$

$a(p_1) + \bar{d}(\bar{p}_4) \to c(p_3) + \bar{b}(\bar{p}_2)$

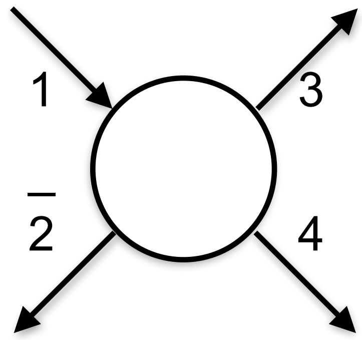
$E_{c.m}$	$s = (p_1 + p_2)^2$	$t = (p_1 + p_{\bar{3}})^2$	$u = (p_1 + p_{\bar{4}})^2$
$\cos(\theta)$	$t = (p_1 - p_3)^2$	$s = (p_1 - p_{\bar{2}})^2$	$t = (p_1 - p_3)^2$
$\cos(\theta)$	$u = (p_1 - p_4)^2$	$u = (p_1 - p_4)^2$	$s = (p_1 - p_{\bar{2}})^2$

$$A^{(s)}_{\lambda_1,\cdots}(s + i\epsilon, t, u) \to \sum_{\lambda'_1,\cdots} [D^{S_1}_{\lambda_1,\lambda'_1} \cdots] A^{(t)}_{\lambda'_1,\cdots}(s, t + i\epsilon, u) \to \cdots$$

- The $i\varepsilon$ is important. Function values at, e.g. $s + i\varepsilon$ vs $s - i\varepsilon$ are different !



$a(p_1) + b(p_2) \rightarrow c(p_3) + d(p_4)$



$a(p_1) \rightarrow \bar{b}(\bar{p}_2) + c(p_3) + d(p_4)$

$$A(s,t,u) \to A(M_1^2 + i\epsilon, s + i\epsilon, t + i\epsilon, u + i\epsilon)$$

- In decay kinematics, the decaying mass becomes a dynamical variable, (iε important)
- Crossing from one kinematical region (e.g. s-channel) to another (e.g. t-channel) requires taking the corresponding variables off the real axis and to the complex plane : analytical continuation.

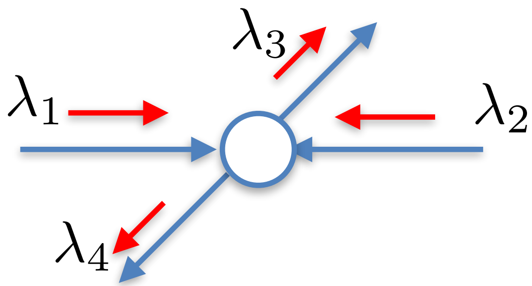
Kinematical vs Dynamical Singularities

For particles with spin

$$A_{\lambda_i}(s,t) = 16\pi \sum_{J=-M}^{M} (2J+1) f^J_{\lambda_i}(s) d^J_{\lambda,\lambda'}(\theta)$$

$$M = \max(|\lambda|, |\lambda'|)$$

$$\lambda' = \lambda_3 - \lambda_4 \quad \lambda = \lambda_1 - \lambda_2$$



$$f^J_{\lambda_i}(s) = \frac{1}{32\pi} \int_{-1}^{1} dz_s A_{\lambda_i}(s, t(s,\theta)) d^J_{\lambda,\lambda'}(\theta)$$

- Wigner d-functions lead to kinematical singularities
- Threshold (barrier factors) originate from kinematical factors in relation between t and cos(θ) (through dependence of A_λ on t)
- Unequal masses give lead to “daughter poles”
- Dynamical singularities : from dynamical (unitary cuts) in A(s,t).

Analyticity

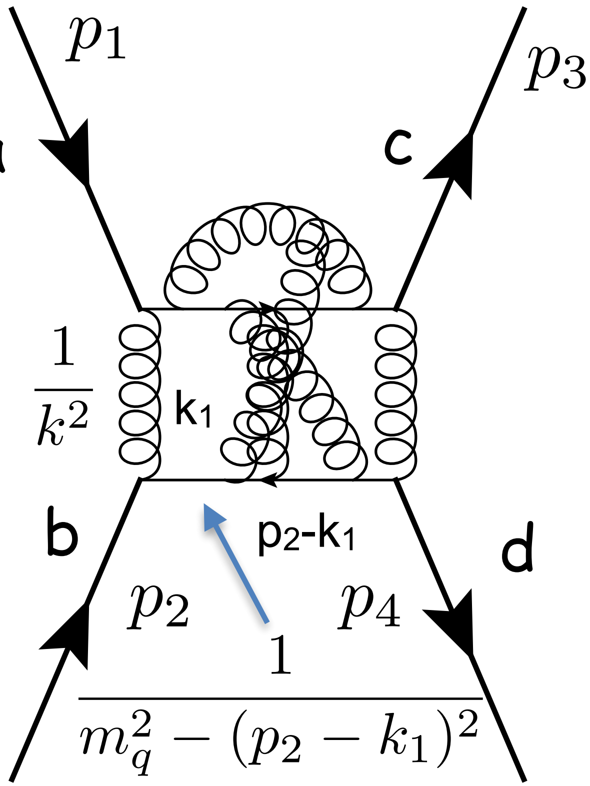
Feynman diagrams

$$A(p_1,\cdots)\propto\int[\Pi_j d^4k_j]\frac{\text{polynomial in k}_j}{(m_q^2-(p_i-k_j)^2-i\epsilon)((k_i-k_j)^2-i\epsilon)\cdots}$$

$$m^2-p^2=[m^2+\mathbf{p}^2]-(p^0)^2$$

$$m^2-p^2=0\to p^0=\pm(m^2+\mathbf{p}^2)^{1/2}$$

- Integrand becomes singular when intermediate states go on shell.
- Thresholds for producing physical intermediate are the only reason why amplitudes are singular.
- Production of intermediate states is related to unitarity. Thus we expect unitarity to determine singularities of the amplitudes.



On the role of iε $\quad \mathrm{Im}\left[\frac{1}{\sqrt{m^2+\mathbf{p}^2}\mp i\epsilon-p^0}\right]=\pm\pi\delta(p_0-\sqrt{m^2+\mathbf{p}^2})$

Relativistic S-matrix fundamentals

Causality: Determines domain of analyticity of reaction amplitudes as function of kinematical variables.

Unitarity: Determines singularities.

Crossing: Dynamical relation, aka reaction amplitudes in the exchange channel (forces) are analogous to amplitude in the direct channel (resonance)

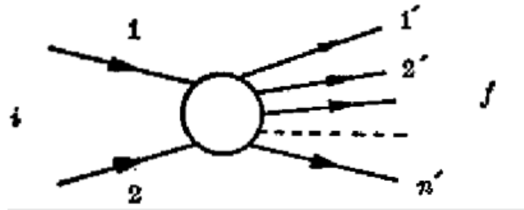
These principles constrain the amplitude on the physical sheet. But on the unphysical sheet, there poles and other singularities, i.e. triangle singularity brains points, that arise from the underlying dynamics. Thus in reality it is the unphysical sheet which is of interest.

Amplitude analysis = make hypothesis about these singularities and use analytical continuation to obtain the amplitude on the physical sheet where you fit to data.

S-matrix properties (in relativistic theory)

- Related to transition probability

$$P_{fi} = |\langle f|S|i\rangle|^2 = \langle i|S^\dagger|f\rangle\langle f|S|i\rangle$$



- Conservation of Probability = Unitarity

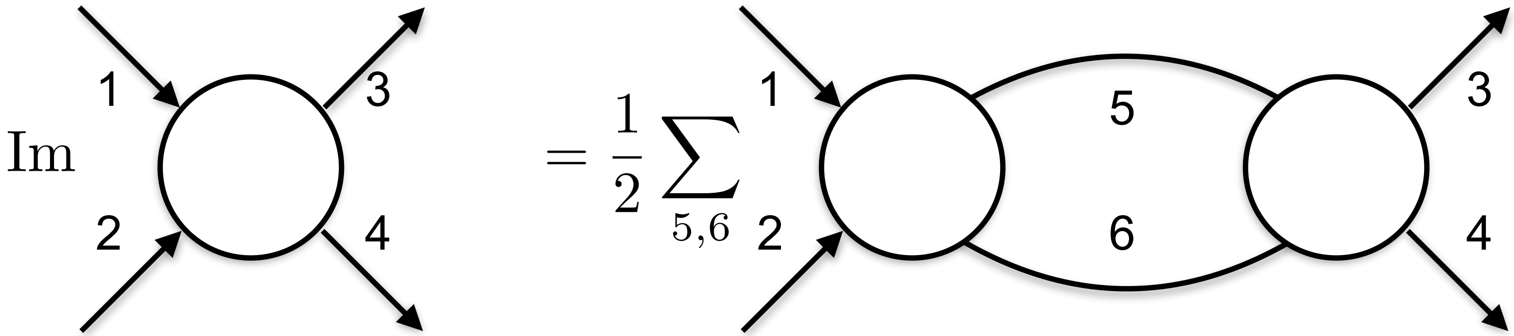
$$\sum_f P_{fi} = 1$$

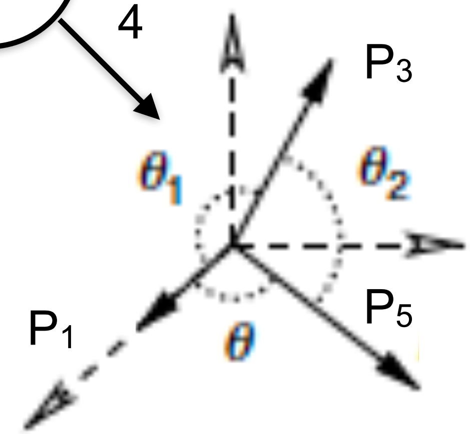
$$S^\dagger S = I$$

$$2ImT_{ft} = \sum_n 2\pi\delta(E_i - E_n)T^*_{fn}T_{ni}$$

How unitarity constrains singularities: simple example

$$2ImT_{ft} = \sum_n 2\pi\delta(E_i - E_n)T^*_{fn}T_{ni}$$





Consider elastic scattering of spineless particles

$$ImA(s,t) = \frac{\rho(s)}{16\pi}\int\frac{d\Omega}{4\pi}A(s,\cos\theta_1)A^*(s,\cos\theta_2)$$

$$\rho(s) = 2k_{cm}(s)/\sqrt{s}$$

At fixed s, this is a complicated, integral relation w.r.t momentum transfer, t
It is simplified (diagonalized) by expanding A(s,t) in partial waves

$$A(s,t) = 16\pi\sum_{l=0}^{\infty}(2l+1)f_l(s)P_l(\cos\theta) \qquad Imf_l(s) = \rho(s)|f_l(s)|^2$$

How unitarity constrains singularities

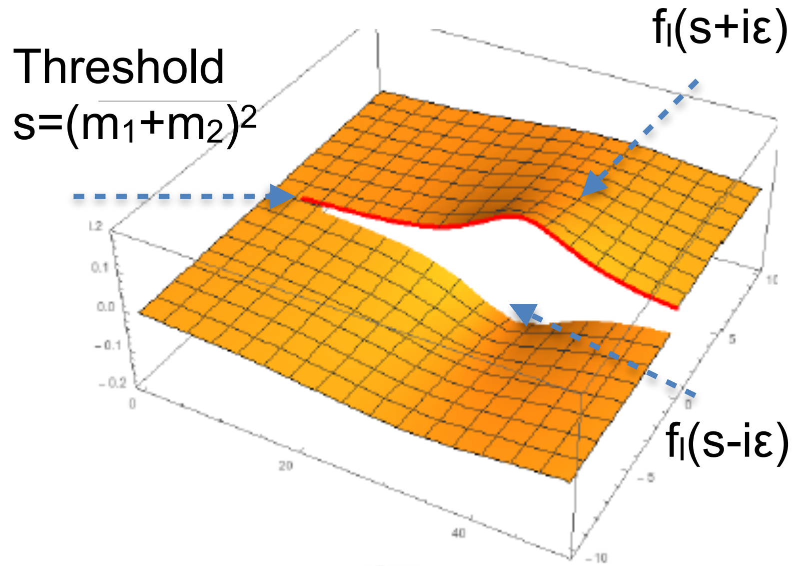
Properties of the partial wave, $f_l(s)$ (for fixed l as function of s):

- $f_l(s)$ is real for s below threshold
- Im $f_l(s)$ is finite above threshold.
- $f_l(s)$ is analytical (since A(s,t) is)

$$f_l(s) = \frac{1}{32\pi}\int_{-1}^{1} d\cos\theta P_l(\cos\theta) A(s, t(s,\cos\theta))$$

for simplicity ignore singularities in t

→ Reflection theorem (Calculus 101): $f_l(s^*) = f_l(s^*)$



$$\frac{1}{2i}[f_l(s+i\epsilon) - f_l(s-i\epsilon)] = \rho(s) f_l(s+i\epsilon) f_l(s-i\epsilon)$$

Lets look for a function, $f_{II}(s)$ that, for s-iε is equal to $f_l(s+i\varepsilon)$. Theorem of analytical continuation implies there is only one such function

Second sheet

$$f(s+i\epsilon) = \frac{f(s-i\epsilon)}{1-2i\rho(s)f(s-i\epsilon)}$$

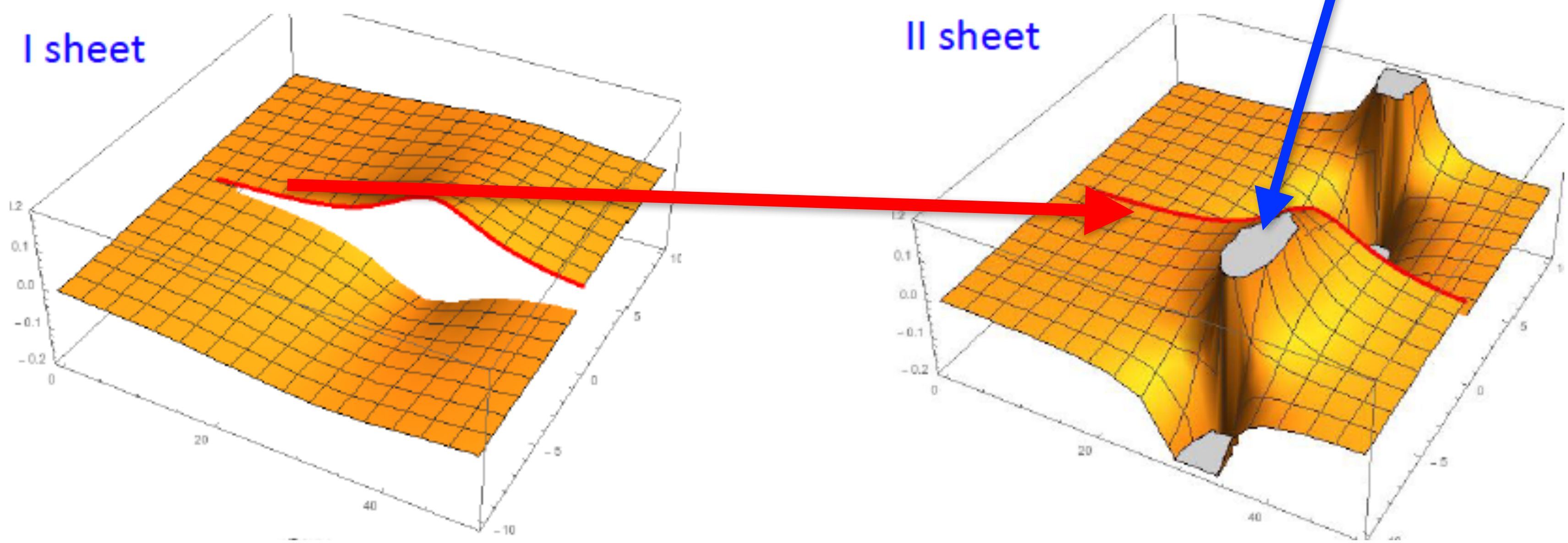
Singularity = Resonance at complex s when

$$f(s) = \frac{1}{2i\rho(s)}$$

Define for Im s < 0 $$f_{II}(s) = \frac{f(s)}{1-2i\rho(s)f(s)}$$

$$f_{II}(s-i\epsilon) = f(s+i\epsilon)$$

This is analytical continuation of f(s) below the real axis



Breit-Wigner

$$f(s) = \frac{g^2\sqrt{s_{tr}-s}}{m^2 - s + g^2\sqrt{s_{tr}-s}}$$

when Im s < 0

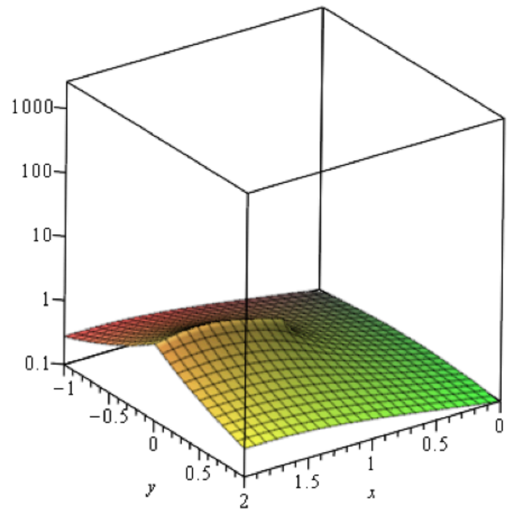
$$m^2 - s + ig^2\sqrt{s-s_{tr}}$$

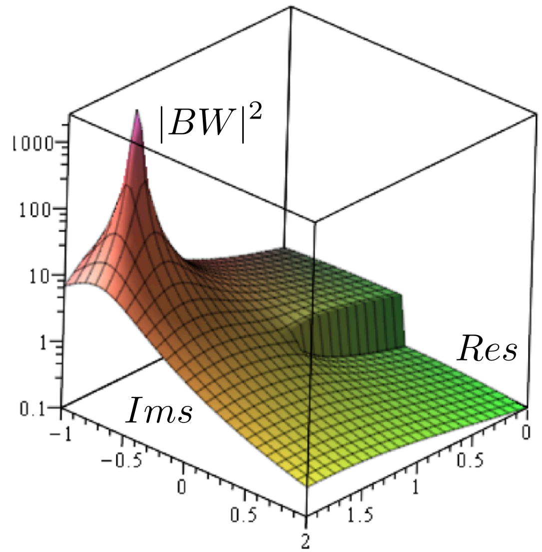
$$\rho(s) = \sqrt{s - s_{tr}}$$

$$f_{II}(s) = \frac{f(s)}{1-2i\rho(s)f(s)} = \frac{g^2\sqrt{s_{tr}-s}}{m^2 - s + g^2\sqrt{s_{tr}-s} - 2ig^2\sqrt{s-s_{tr}}}$$

when Im s < 0

$$= \frac{g^2\sqrt{s_{tr}-s}}{m^2 - s - ig^2\sqrt{s-s_{tr}}}$$





Phenomenology of hadron interaction

- Evidence for resonance scattering : connection to QCD bound states.
- Kinematical range for resonance scattering.
- Features of high energy scattering : physics of cross channels
- Space-time interpretation of high and low energy scattering
- Dual models

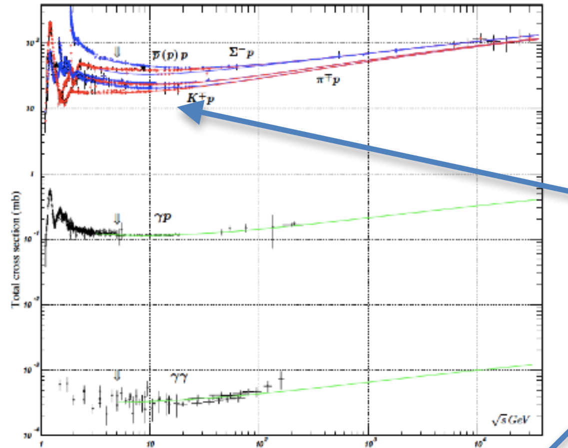
$$\sigma_{a+b\to a+b} \propto \int \frac{dt}{s^2} |A(s,t)|^2$$

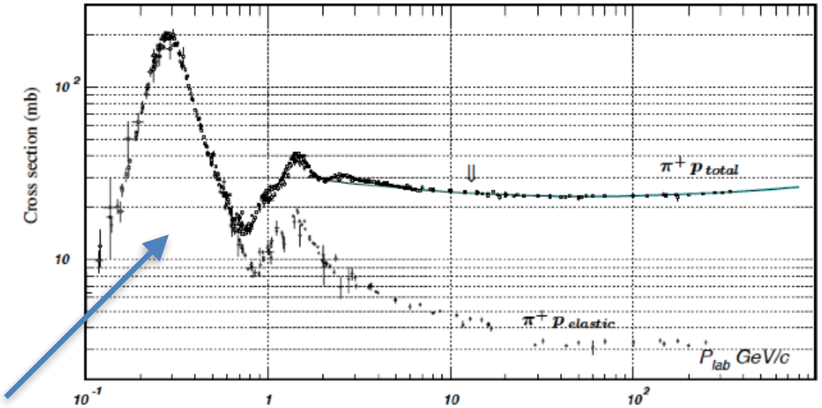
$$\sigma_{a+b\to X} \propto \frac{ImA(s,0)}{s}$$ from unitarity

Phenomenology of hadron interaction

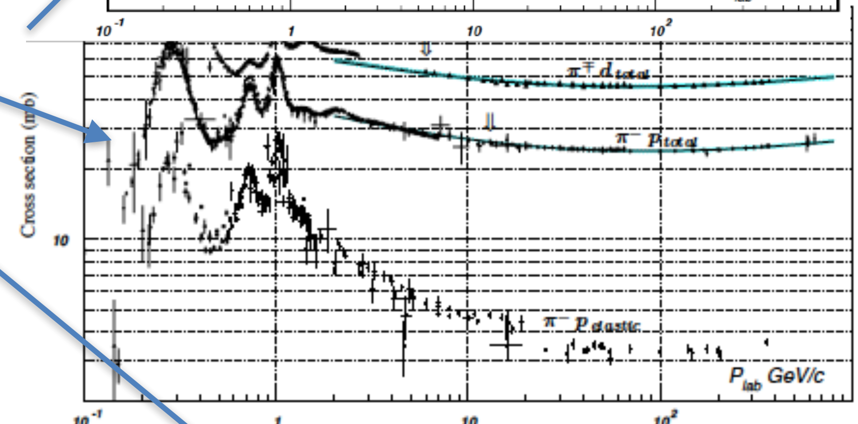
$$\sigma_{a+b\to a+b} \propto \int \frac{dt}{s^2} |A(s,t)|^2$$

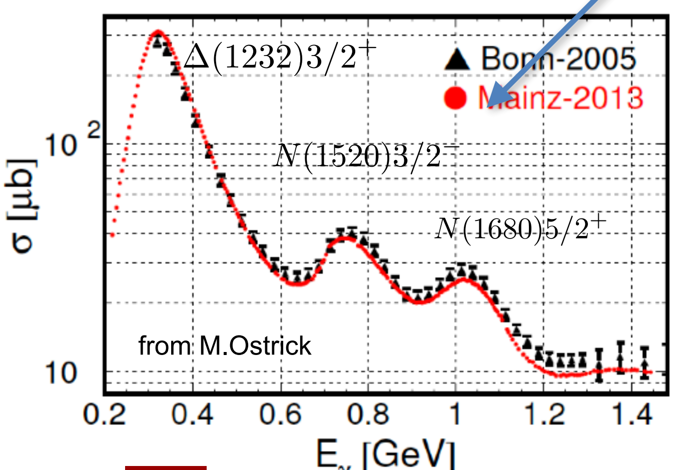
$$\sigma_{a+b\to X} \propto \frac{Im A(s,0)}{s}$$ from unitarity



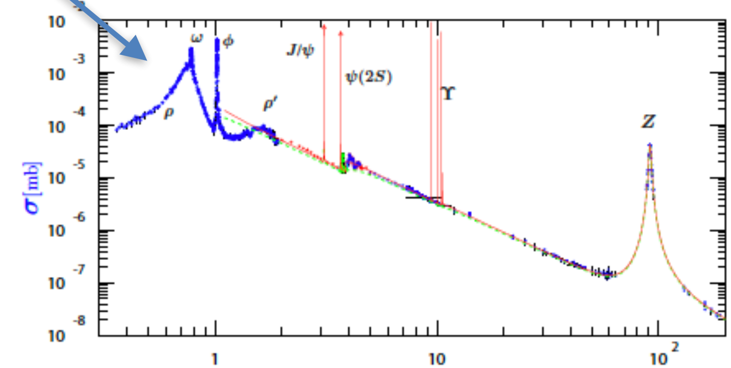


Resonance scattering





$$A(s,t) \sim \frac{1}{m_r^2 - s}$$

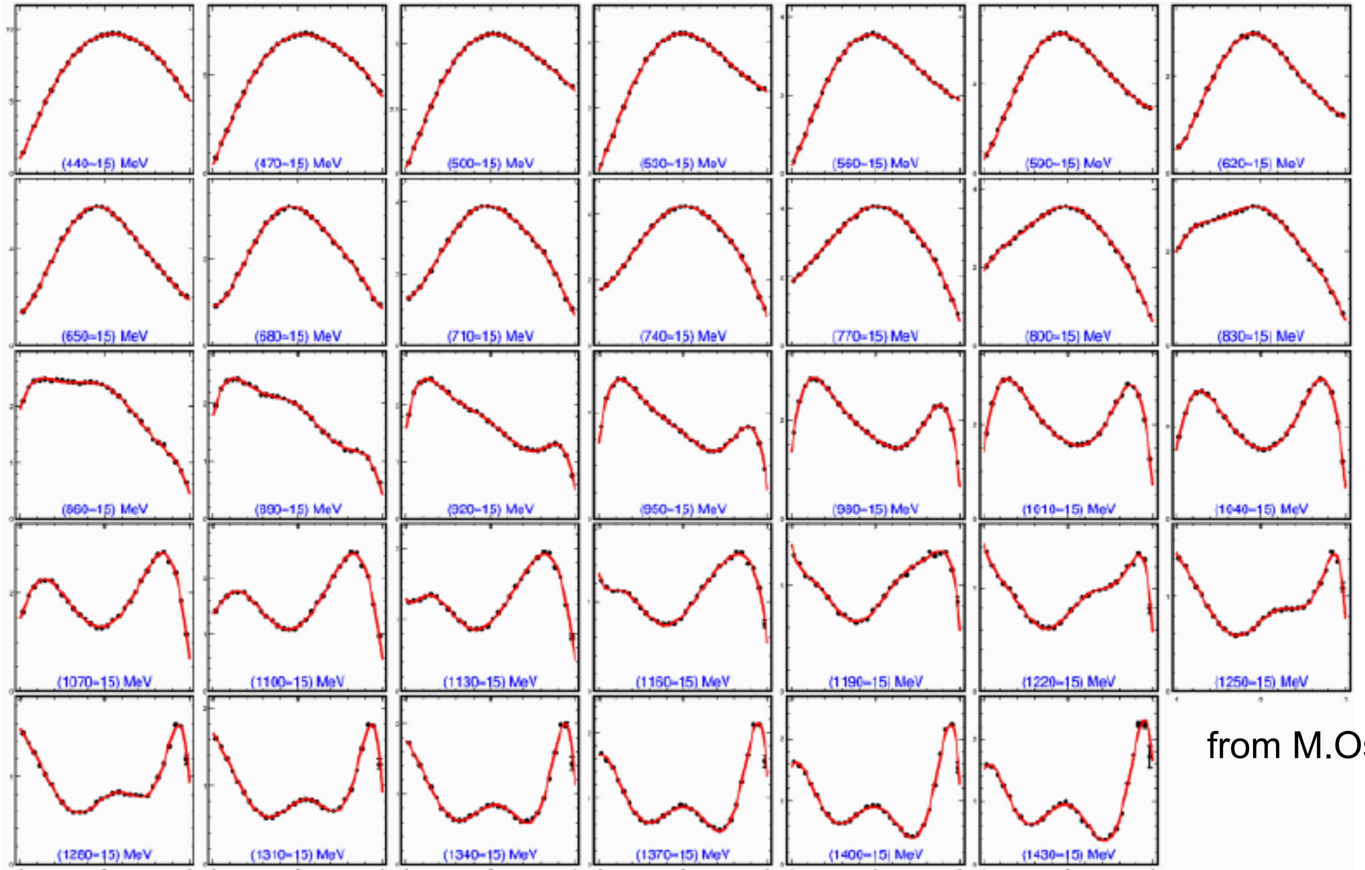


Resonance Scattering : look at angular distribution

$$\frac{d\sigma}{dt} \propto \frac{|A(s,t)|^2}{s^2}$$

Angular distribution: a few "wiggles"

$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(z_s(t))$$



from M.Ostrick

more pronounced forward/backward peaks as energy increases

Resonance scattering

- Due to confinement, we expect an infinite number of resonances (poles at positive energy — recall the potential shell example) of arbitrary large mass and spin.

- String/flux tube breaking leads to screening of color charge and these poles decay. As mass increases they coach to multi-particle final states. The poles are still there, but dive deeper into to complex plane and are more difficult to identify. However, when making a model it makes more sense to parametrize amplitude with BW resonances as compared to some arbitrary background functions.

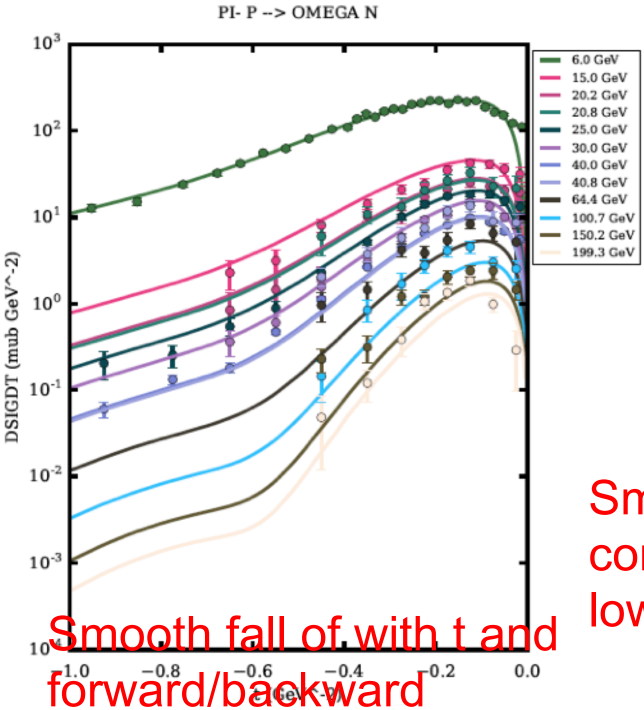
$$p = l/r$$

- For l_{max} ~ 5 and interaction range r_0 ~0.5fm this gives p_{lab} <~ 10/fm ~ 2GeV, [or W ~ (2 P_{lab} m_p $)^{1/2}$ ~ 2GeV]

- For resonance scattering

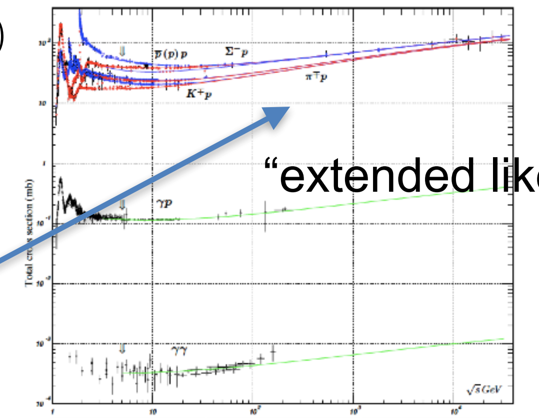
$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(z_s(t)) \quad \longrightarrow \quad A(s,t) \sim \frac{P_{l_R}(z_s(t))}{s - s_R}$$

Scattering at High energies



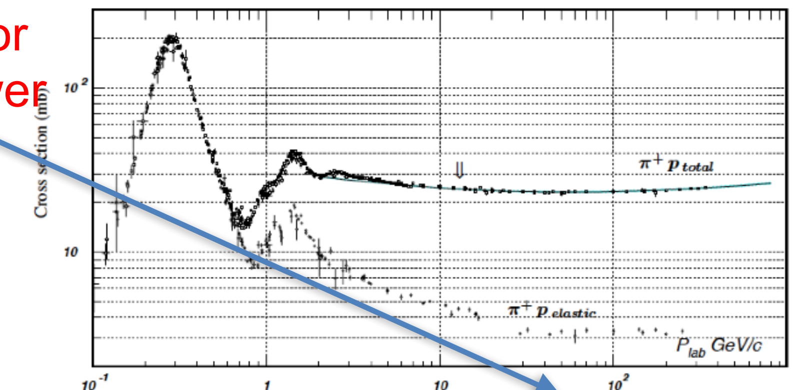
$$\sigma_{a+b\to X} = \frac{1}{s} Im A_{ab\to ab}(s,0)$$

$$\frac{d\sigma}{dt}(s) = \frac{1}{s^2}|A(s,t)|^2$$

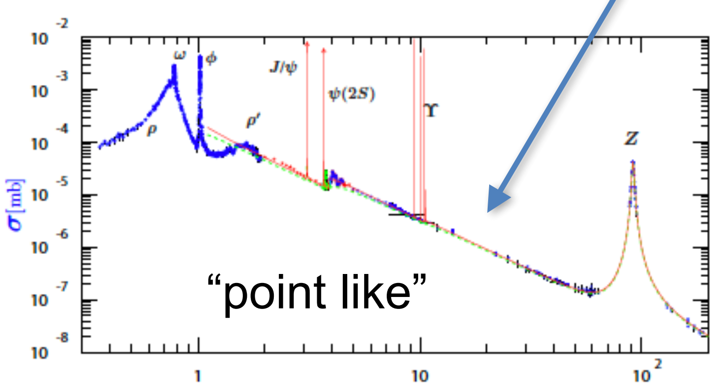


"extended like"

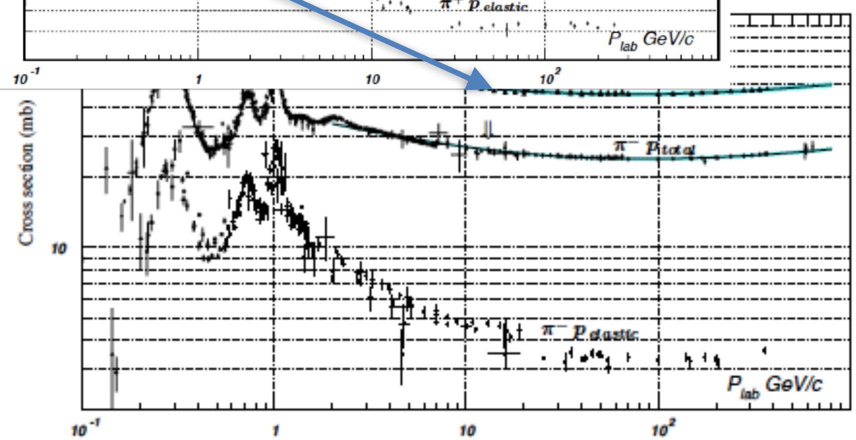
Smooth behavior constant or power low fall off



Smooth fall of with t and forward/backward peking

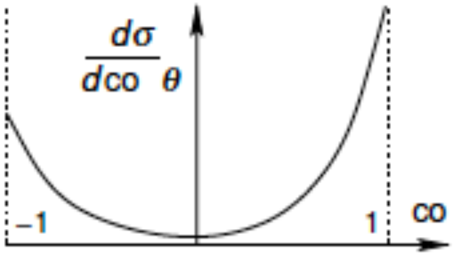


"point like"



Scattering at high energies

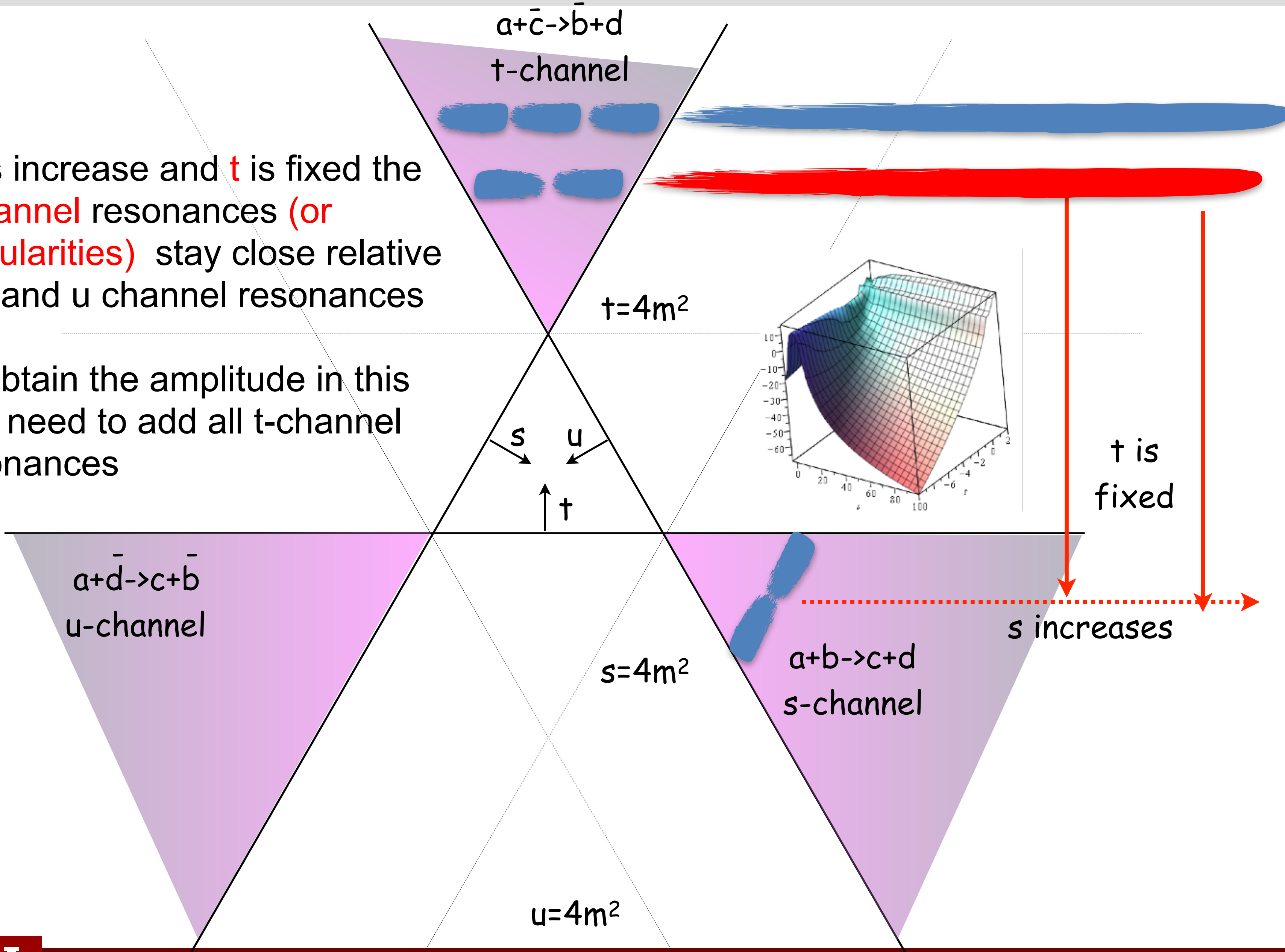
- s-dependence:
  - many intermediate particles can be produced, unitarity becomes complicated and less useful.
- t-dependence:
  - high partial waves become important, several Legendre functions are needed.
- There is universality in both s and t-dependencies: smooth (constant or falling s-dependence), and forward/(backward) peaking in t. The universality hints into importance of t/(u) channel singularities.



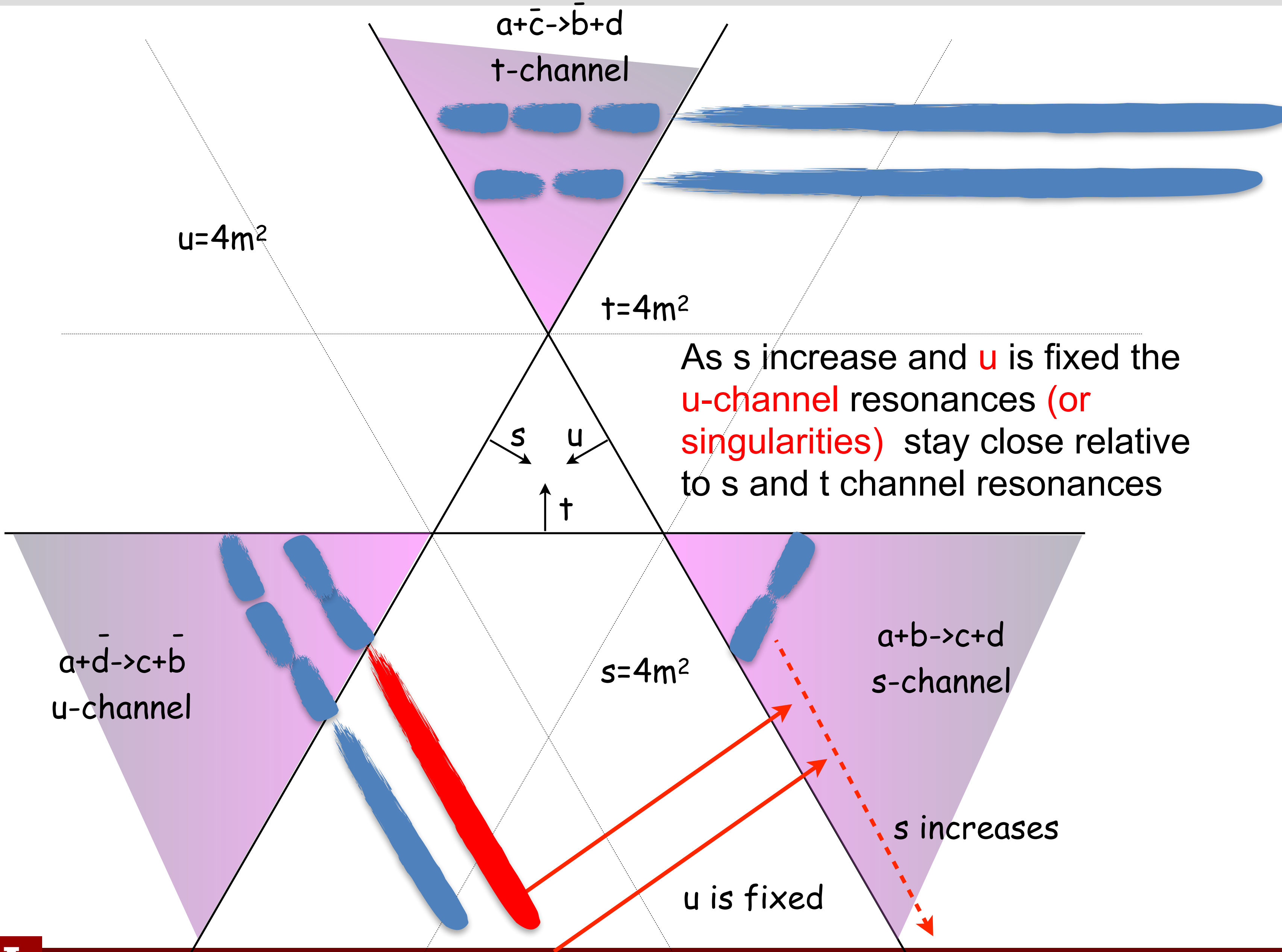
From t-channel to s-channel (high energy forward scattering)

As s increase and t is fixed the t-channel resonances (or singularities) stay close relative to s and u channel resonances

To obtain the amplitude in this limit need to add all t-channel resonances



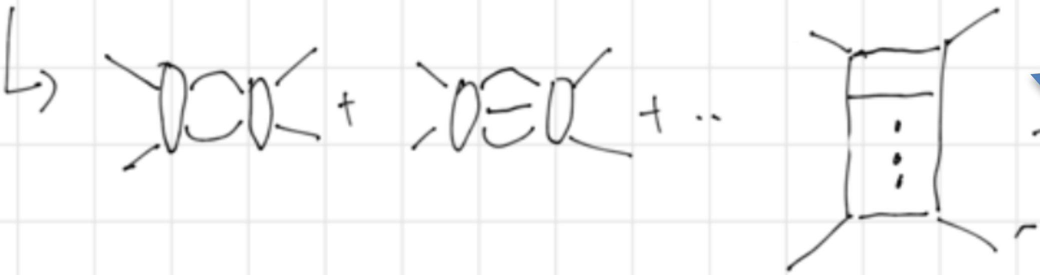
From u-channel to s-channel (high energy backward scattering)



As s increase and u is fixed the u-channel resonances (or singularities) stay close relative to s and t channel resonances

analytical continuation from s to t

$A(s,t) = \sum_{\ell} (2\ell+1) f_\ell(s) P_\ell(z_s)$ is s-channel



Recall Schrödinger equation: $T = V + VGV + \dots$

To form bound state need ∞ number of terms

bound state or resonance

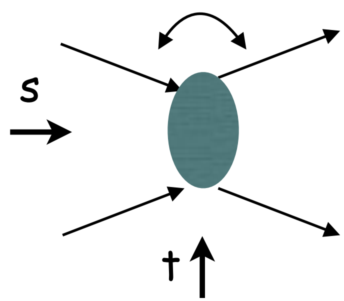
Sum of a large number of particle productions at high-s looks like an exchange of various resonances in the t-channel.

Use t-channel partial waves and analytically continue to large-s

$$A(s,t) = \sum_l (2l+1) f_l(s) P_l(\cos\theta)$$

converges if |cosθ|<1 : (e.g. 1+x+x²+... = finite for |x|<1)

$z = \cos\theta$



$$z = 1 + \frac{2t}{s - 4m^2}$$

$$t = -\frac{(1-z)}{2}(s - 4m^2) < 0 \text{ for } |z| < 1 \text{ and } s > 4m^2$$

"s-channel"

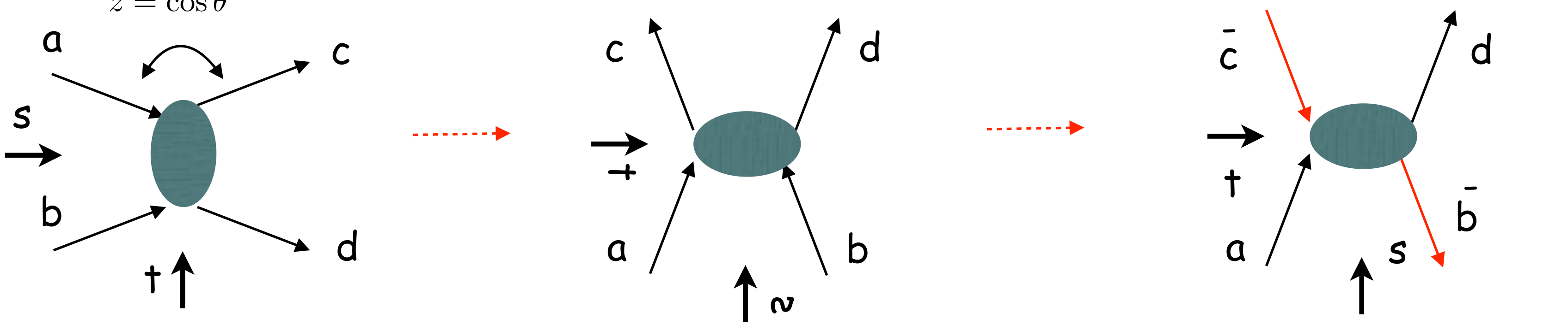
$$s = -\frac{t - 4m^2}{2}(1 - z_t)$$

$$A(s,t) = \sum_l (2l+1) f_l(t) P_l(z_t)$$

"t-channel"

$$t = -\frac{(1-z)}{2}(s - 4m^2) > 4m^2 \text{ for } |z| > 1 \text{ and } s < 0$$

$z = \cos\theta$



a+b -> c+d

(e.g. what is the value of 1+x+x²+... when x>1 ?)

Example of analytical continuation

$$A(s,t) = \sum_l (2l+1) f_l(t) P_l(z_t) \qquad s = -\frac{t-4m^2}{2}(1-z_t)$$

The series converges for $|z_t|<1$ (cosine of scattering angle in the t-channel), i.e. in the t-channel physical region. We want to know A(s,t) for in the s-channel physical region, in particular for large s, with corresponds to $|z_t| >> 1$.

For example, assume $f_l(t) = \frac{1}{l-\alpha(t)}$ i.e. it has a pole (resonance) where α(t)=l

$$A(s,t) \sim J(z_t) = \sum_l \frac{z_t^l}{l-\alpha(t)}$$ for α<0 and $|z_t| < 1$ use $$\frac{1}{l-\alpha} = \int_0^\infty dx e^{-x(l-\alpha)}$$

to obtain $$J(z) = \int_0^\infty dx \left[\frac{e^{x\alpha}}{1+ze^{-x}}\right] = z^\alpha \int_0^z \frac{dy}{y^{\alpha+1}(1+y)} \qquad y = ze^{-x}$$

provides analytical continuation for α>0 for large z =z(s) ~ s

$$J(z) = -\frac{z^\alpha \pi}{\sin \pi\alpha} + z^\alpha \int_z^\infty \frac{dy}{y^{\alpha+1}(1+y)} \to -\frac{z^\alpha \pi}{\sin \pi\alpha} \qquad z \to \infty$$

this is analog of

$$f(x) = 1 + x + x^2 + \cdots$$

$$f(x) = \frac{1}{1-x}$$

Reggeon

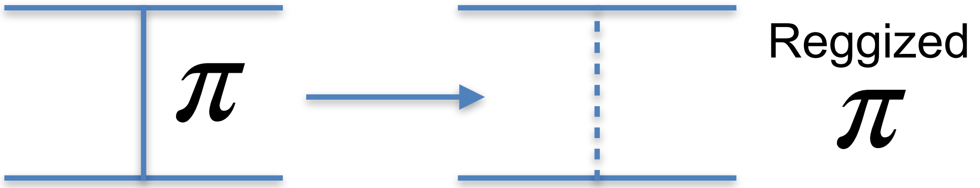
s-channel partial wave expansion $$A(s,t) = \sum_l (2l+1) f_l^{(s)}(s) P_l(\cos\theta_s)$$

t-channel partial wave expansion $$A(s,t) = \sum_l (2l+1) f_l^{(t)}(t) P_l(\cos\theta_t)$$

The amplitude at large-s (in the s-channel physical region) is dominated by a selected, infinite set of t-channel partial waves (t-channel resonances).

This sum is referred to as a Reggeon or a Regge exchange.

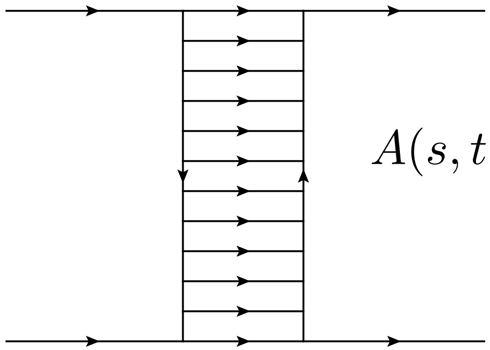
Since Reggeon is a collection of partial waves and partial waves have quantum numbers of resonances, so do Reggeon. They are like special kind of virtual particles. For example in perturbation theory pion we can talk about virtual, single pion exchange. A collection of all pion like exchange becomes a Reggion with pion quantum numbers. “Reggized pion”



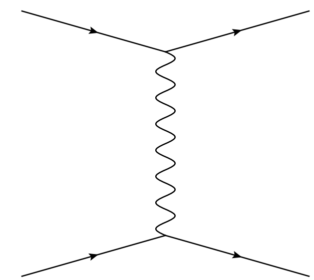
Pomeron vs Reggeons

s-channel: multi-particle production

t-channel: collection of resonances: “Regge” exchanges



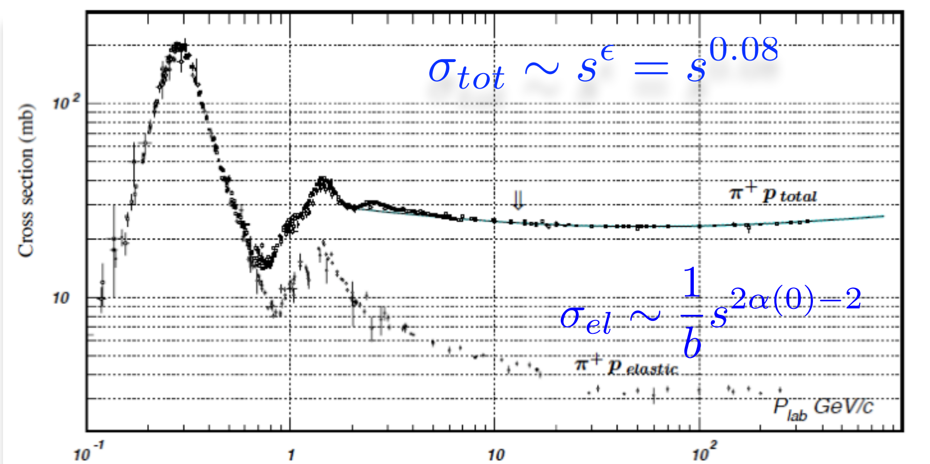
$$A(s,t) \propto r(t) s^{\alpha(t)} \quad \texttt{<--} \quad f_l(t) = \frac{r(t)}{l - \alpha(t)}$$

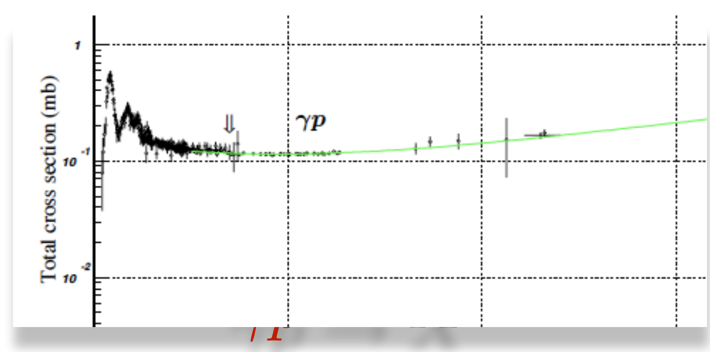


$$A(s, t \sim 0) \sim i s^{\alpha(0)} \sim s\sigma_{tot}$$

* Exchange of t-channel partial wave with quantum numbers of the vacuum is called the Pomeron

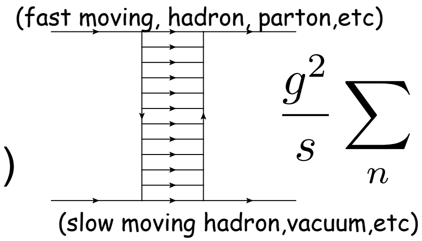
(exchange of non-vacuum q.n. falls with energy)





Growing Radius, partons, saturation,…

* Where does to parton model come from



adding correlated partons is beneficial (expansion not in g^2 but in $g^2 \log s$)

$$\frac{g^2}{s}\sum_n \frac{\beta^{n-1}(t)}{(n-1)!}\log^{n-1} s \to s^{\alpha(-k_\perp^2)}$$

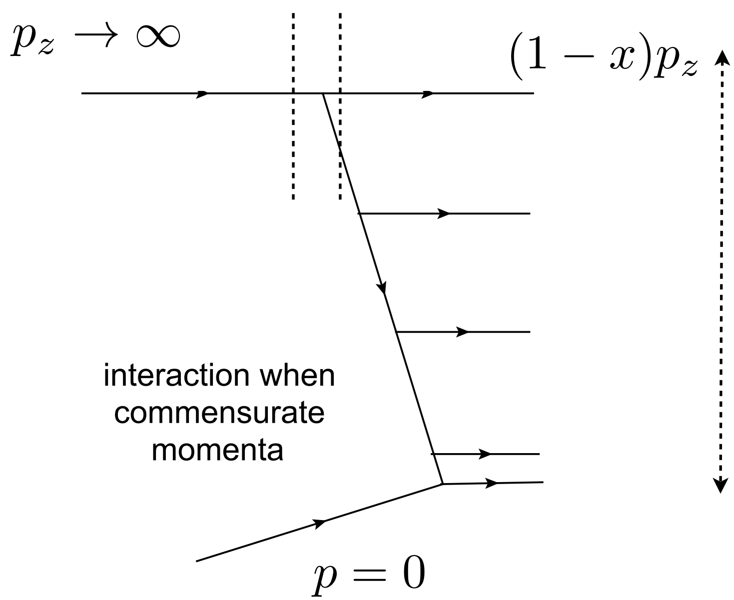
$$\alpha(t) = -1 + \beta(t)$$

... and in space-time assuming Pomeron $\alpha(0)=1$

$$A(s, r_\perp) \sim \int d^2k_\perp e^{ik_\perp r_\perp} e^{\alpha(-k_\perp^2)\log s} \sim \frac{1}{\log(s)} e^{-r_\perp^2/\log(s)}$$

hadron swells

$$\Delta E \sim \frac{\mu_\perp^2}{x(1-x)p_z}$$



* long lived fluctuations finite <x>

$$\langle x\rangle^{\langle n\rangle} = \frac{p_z}{\mu} \qquad \langle n\rangle \sim \log(s)$$

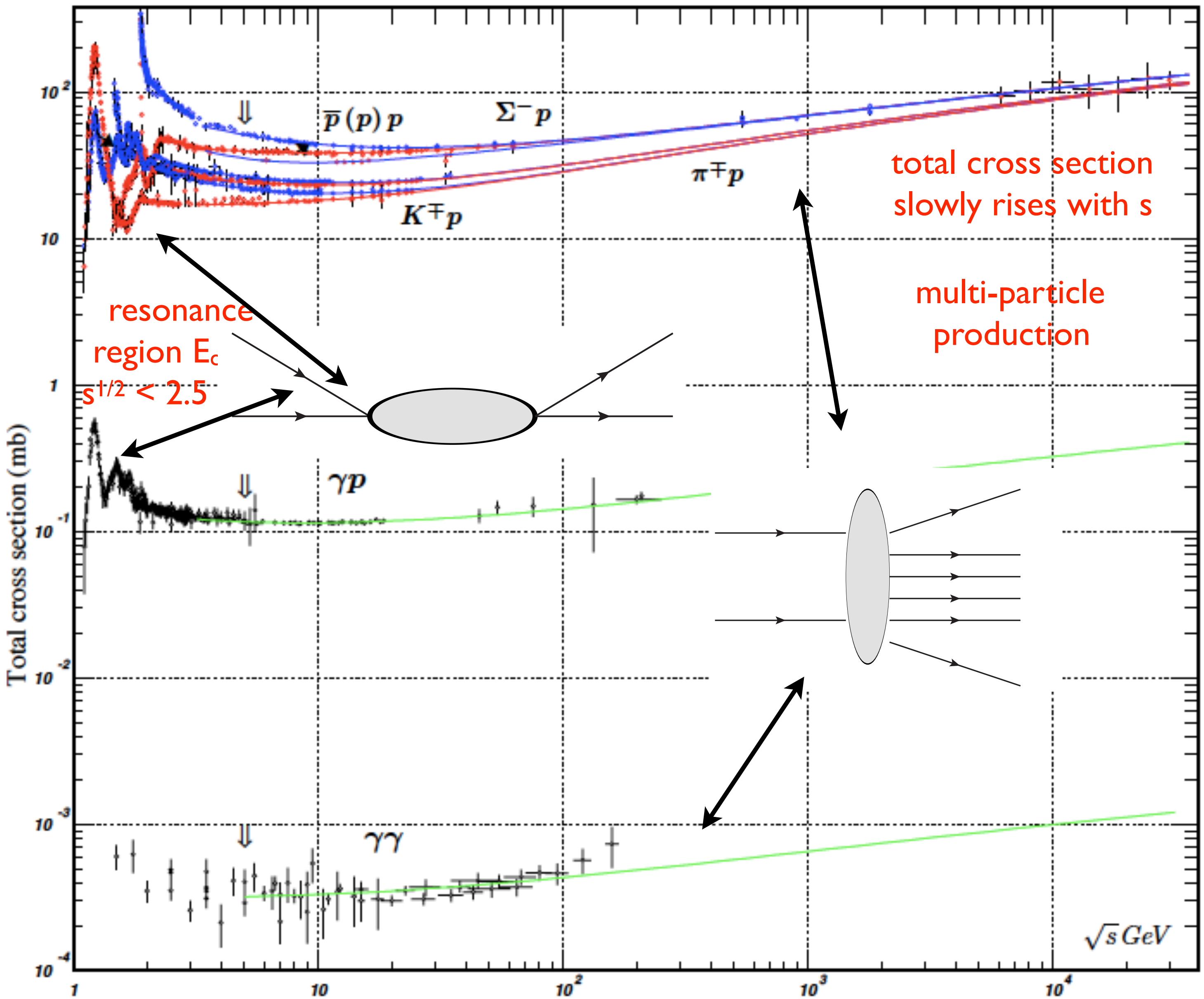
random walk in transverse space

* $$\langle r_\perp\rangle \sim \sqrt{\langle n\rangle \frac{1}{\mu_\perp}} \sim \log^{1/2}(s)$$

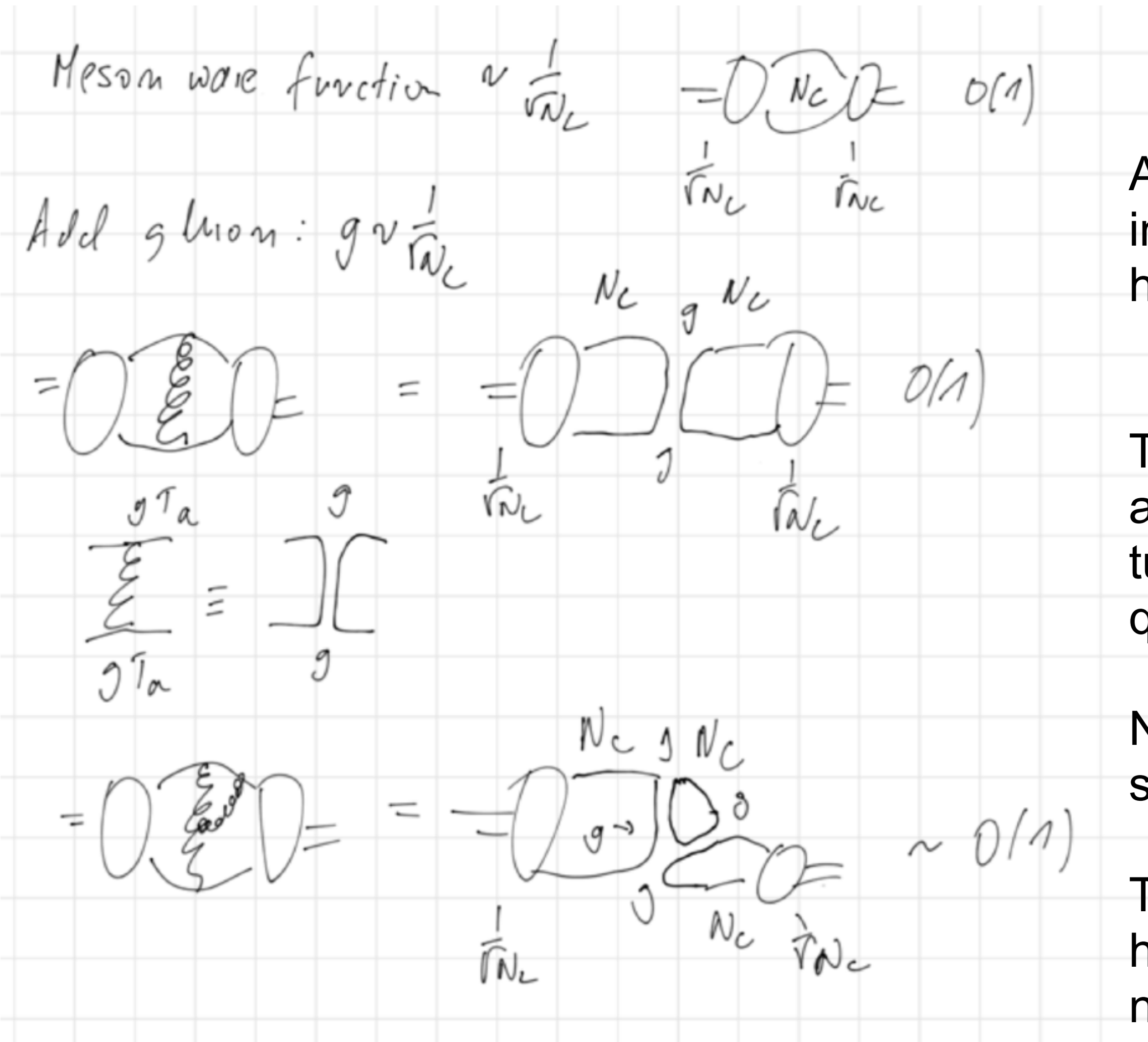
* large-s behavior universal (Pomeron = vacuum pole, universal mid-rapidity)

* it takes "a long time" to develop a low-x parton out of a fast one

Comparing with Experiment



Large N_c



$$N_c \to \infty$$

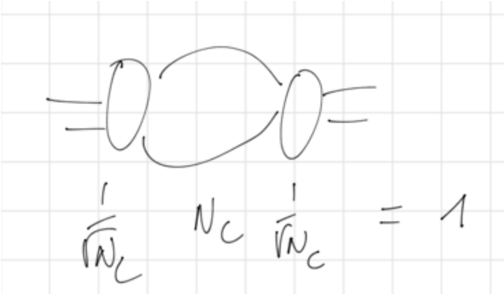
$$g^2 N_C = const\,.$$

An empty digram represents infinite number of process that happen in a plane !

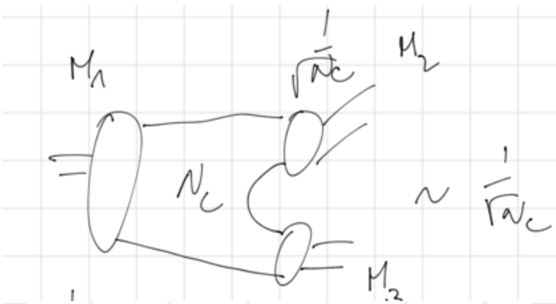
The plane can be intercepted as a world sheet of a string/flux tube connecting the valance quarks

Non planar diagrams are suppressed by 1/Nc

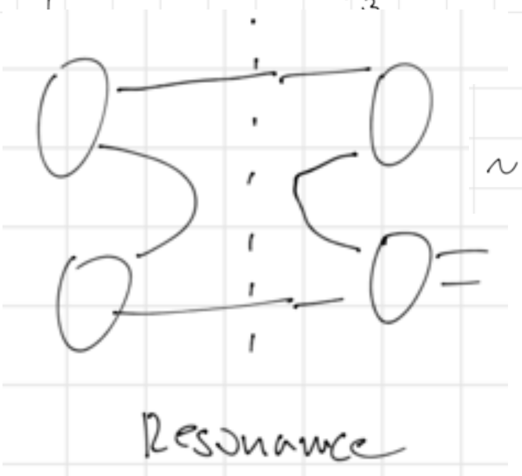
To leading order in 1/Nc hadrons do not decay, that to not scatter.



$$\langle M | H | M \rangle = O(1)$$



$$\langle M_1 | H | M_2 M_3 \rangle = g = O(1/\sqrt{N_C})$$



$$\sim \left(\frac{1}{\sqrt{N_c}}\right)^4 \times N_c = \frac{1}{N_c}$$

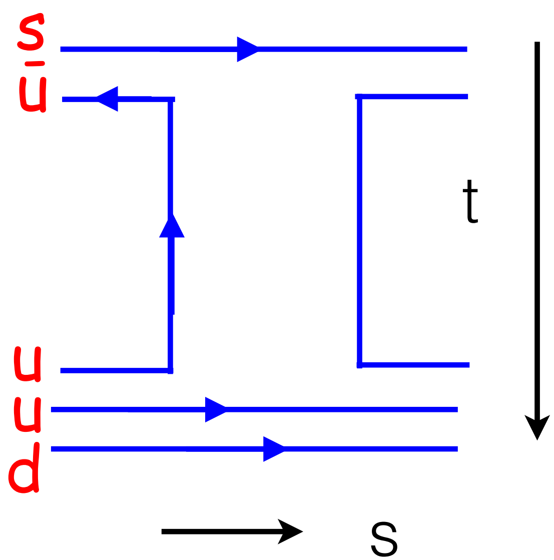
$$\sim \frac{\Gamma}{m^2 - s - i\Gamma} \qquad \Gamma = O(1/N_c) = g^2$$

Dualities

planar diagrams may be considered as either
s-channel or t-channel

K-p

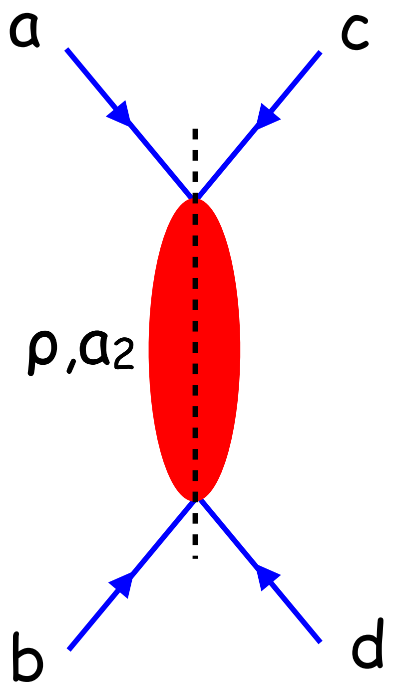
K-p has “normal mesons” in the t-channel and “normal baryons” in the s channel



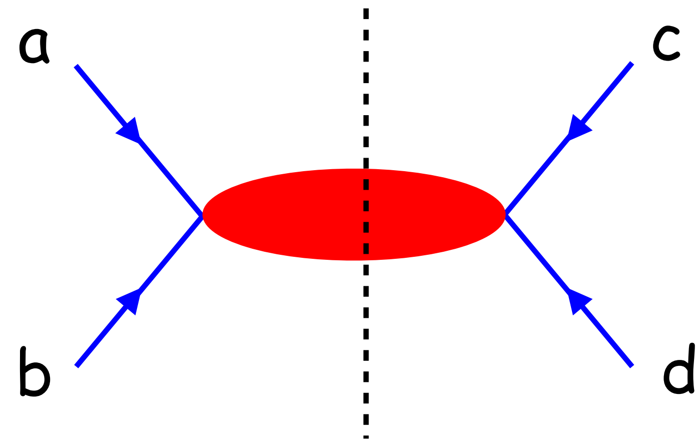
Interpretation of what happens in s-channel is dual to what happens in the t-channel : Mesons require baryons and vice versa

Regge phenomena :
sum of t-channel resonances determines large-s behavior of the s0channel and vice versa.

Does it work ?

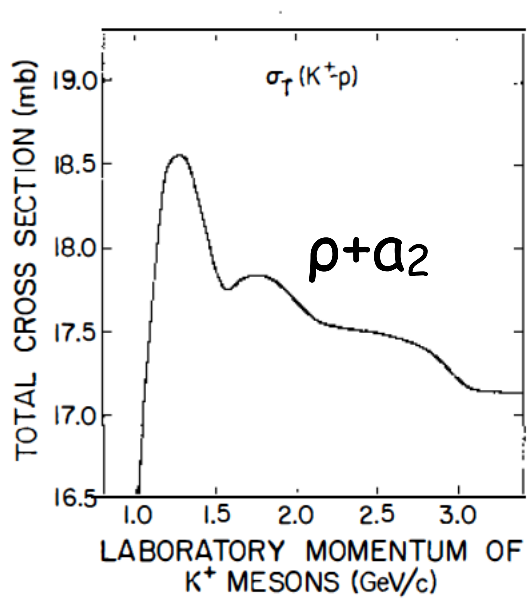


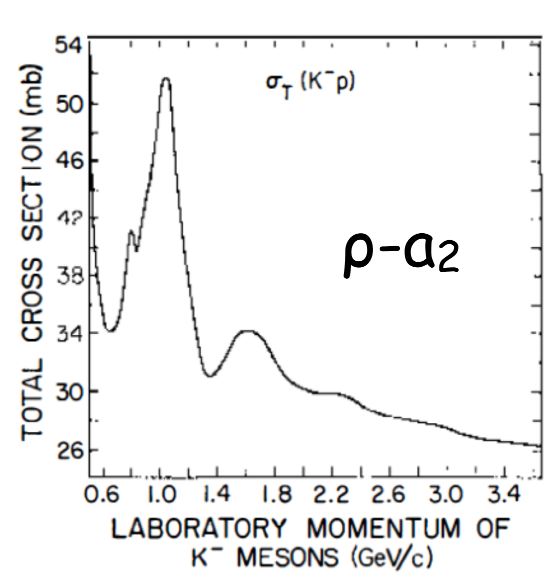
In K-p scattering
imaginary parts of a2
and rho add up
In K+p they cancel !



$a_2 \sim 1 + \exp(i \pi \alpha(t))$

$\rho \sim 1 - \exp(i \pi \alpha(t))$





Dolen Horn Schmit duality

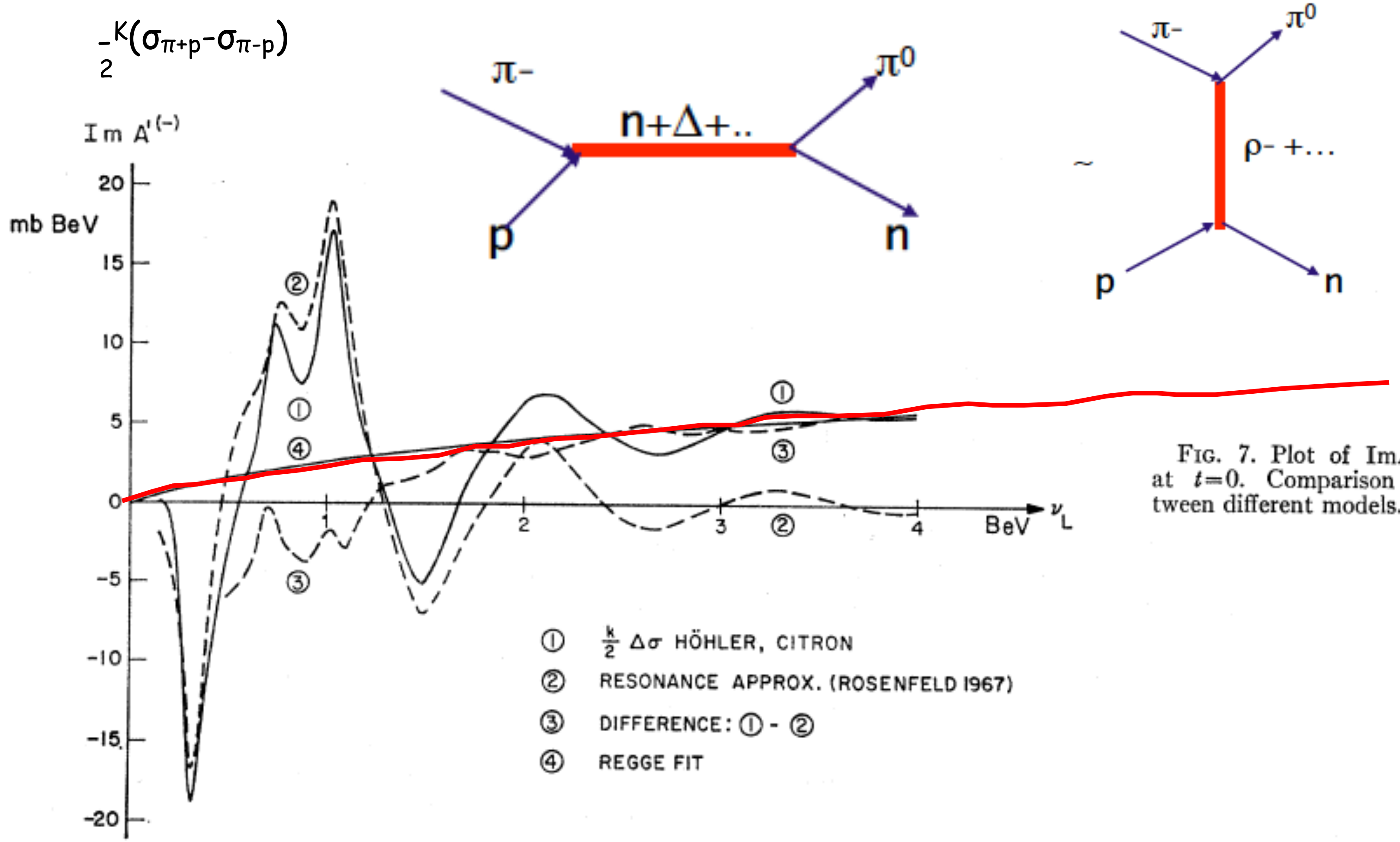
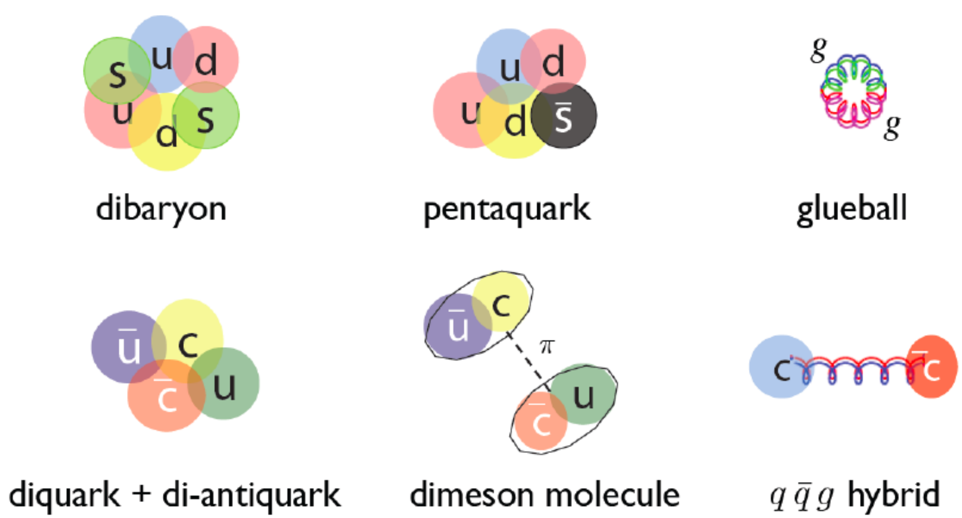


FIG. 7. Plot of Im$A'^{(-)}$ at $t=0$. Comparison between different models.

$$\bar{u}(p_1,\lambda_1)[A(s,t)+(k_1+k_2)_\mu\gamma^\mu B(s,t)]u(p_2,\lambda_2)$$

What about “exotic” hadrons

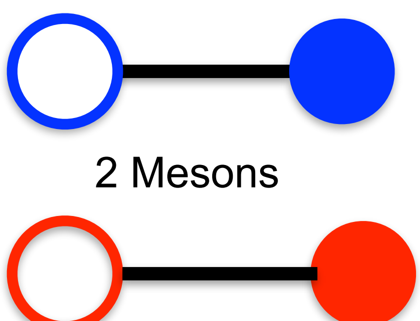


Standard argument for non-existence of multi quark sates: they can fall apart to ordinary mesons and baryons

For example 2 quarks and 2 anti quarks can rearrange into 2 quark-antiquark pairs

But confinement requires quarks are connected by flux tubes and it is possible that certain multi quark configurations are more favorable than “fall apart configurations”

$$3 \times \bar{3} = 8 + 1$$

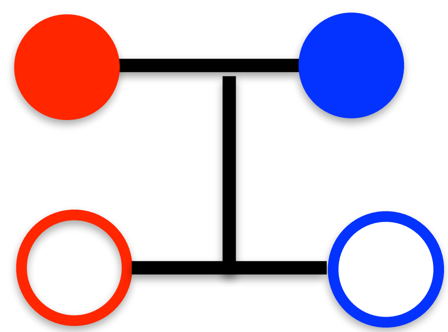


2 Mesons

$$3 \times \bar{3} = 8 + 1$$

vs

$$3 \times 3 = 6 + \bar{3}$$



$$\bar{3} \times \bar{3} = 6 + 3$$

2 di quarks = teraquark

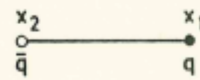
Need to introduce strings

Talk by G.Rossi

Hadronic states → irreducible gauge invariant operators in QCD

Table IIa

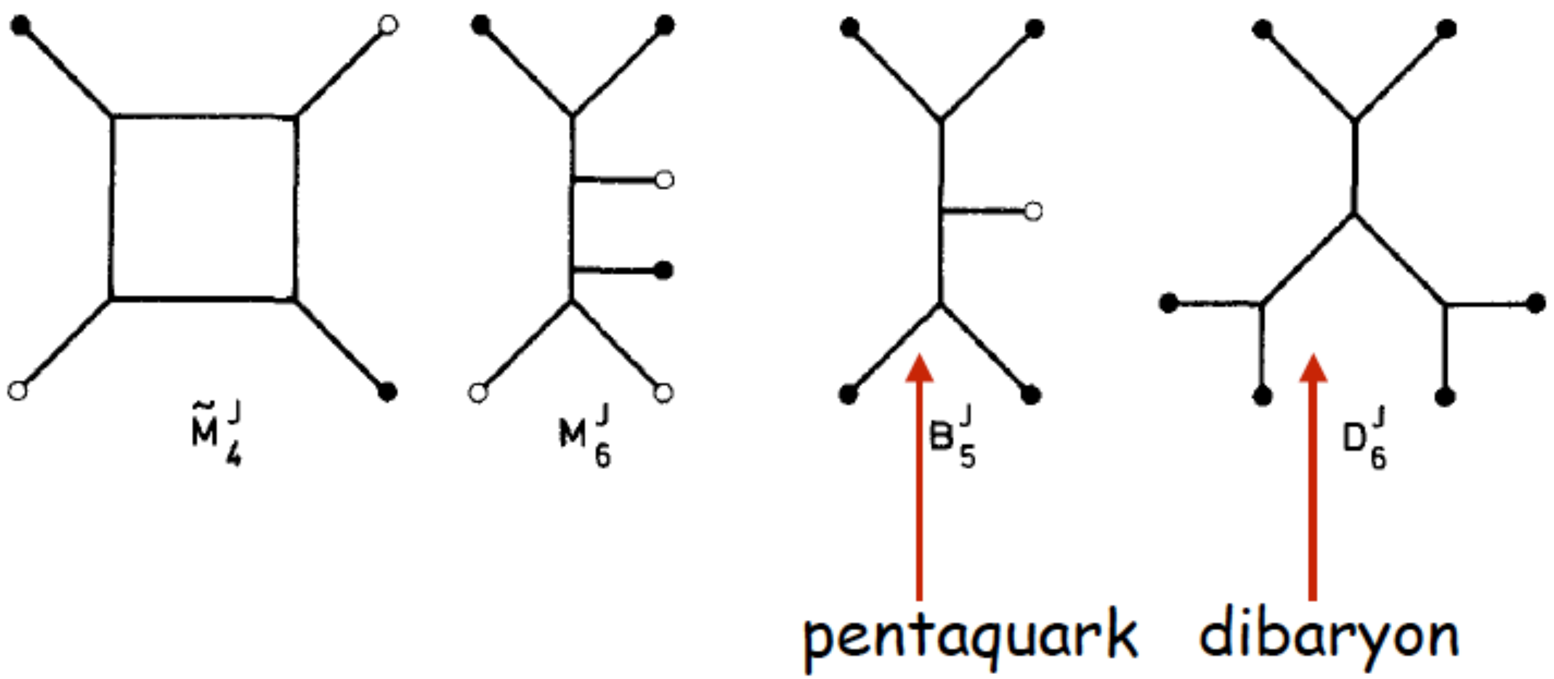
Simplest mesons and baryons : colour structure and string picture

HADRON	GAUGE INVARIANT OPERATOR	STRING PICTURE
$M_2 = q\bar{q}$ meson	$\bar{q}^{j_2}(x_2)\left[P\exp\left(ig\int_{x_1}^{x_2} A_\mu\, dx^\mu\right)\right]_{j_2}^{j_1} q_{j_1}(x_1)$	
M_o = quarkless meson	$\mathrm{Tr}\left[P\exp\left(ig\oint A_\mu\, dx^\mu\right)\right]$	
$B_3 = qqq$ baryon	$\varepsilon^{j_1j_2j_3}\left[P\exp\left(ig\int_{x_1}^{x} A_\mu\, dx^\mu\right) q(x_1)\right]_{j_1}$ $\left[P\exp\left(ig\int_{x_2}^{x} A_\mu\, dx^\mu\right) q(x_2)\right]_{j_2}\left[P\exp\left(ig\int_{x_3}^{x} A_\mu\, dx^\mu\right) q(x_3)\right]_{j_3}$	

Other states

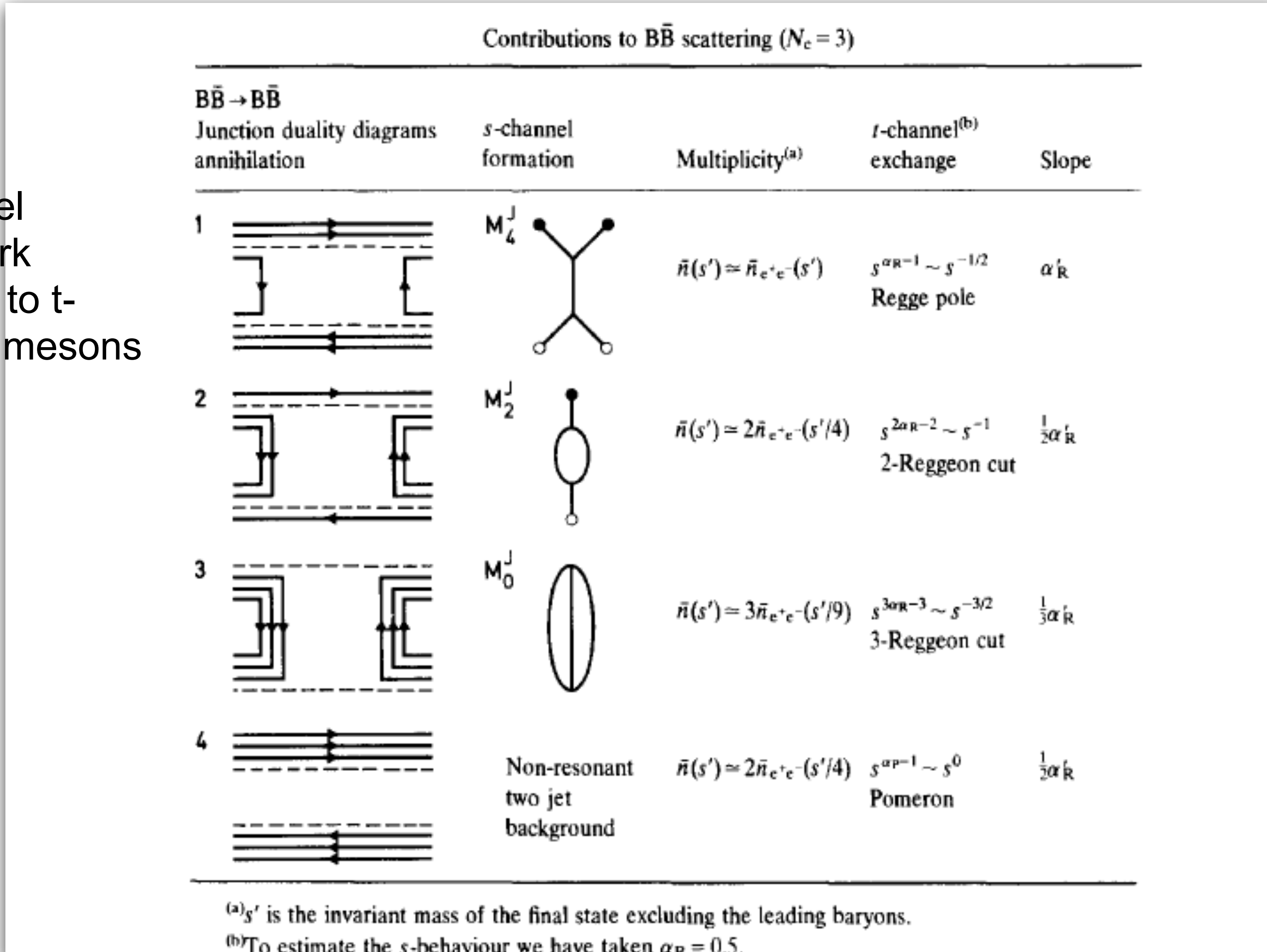
Other multiquark states

(from G. C. Rossi & GV, Phys. Rep. 1982)



Muti-quark states can be related to ordinary states by duality

s-channel tetraquark are dual to t-channel mesons



Contributions to $B\bar{B}$ scattering ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation	s-channel formation	Multiplicity[a]	t-channel[b] exchange	Slope
1	M_4^J	$\bar{n}(s') \simeq \bar{n}_{e^+e^-}(s')$	$s^{\alpha_R - 1} \sim s^{-1/2}$ Regge pole	α'_R
2	M_2^J	$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{2\alpha_R - 2} \sim s^{-1}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$
3	M_0^J	$\bar{n}(s') \simeq 3\bar{n}_{e^+e^-}(s'/9)$	$s^{3\alpha_R - 3} \sim s^{-3/2}$ 3-Reggeon cut	$\frac{1}{3}\alpha'_R$
4	Non-resonant two jet background	$\bar{n}(s') \simeq 2\bar{n}_{e^+e^-}(s'/4)$	$s^{\alpha_P - 1} \sim s^0$ Pomeron	$\frac{1}{2}\alpha'_R$

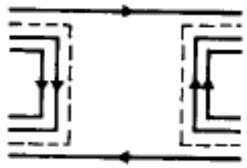
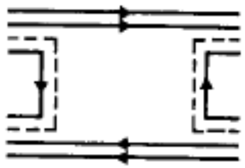
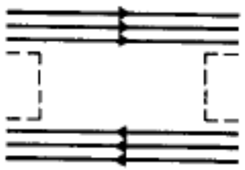
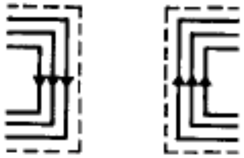
[a] s' is the invariant mass of the final state excluding the leading baryons.
[b] To estimate the s-behaviour we have taken $\alpha_R = 0.5$.

Muti-quark states can be related to ordinary states by duality

s-channel mesons are dual to t-channel tetra quarks

tetra quarks should form Regge trajectories just like mesons

Contribution to $B\bar{B}$ annihilation ($N_c = 3$)

$B\bar{B} \to B\bar{B}$ Junction duality diagrams annihilation	s-channel formation	Multiplicity	t-channel[(a)] exchange	Slope
1 	$1q\bar{q}$ – jet	$\bar{n}(s) \simeq \bar{n}_{e^+e^-}(s)$	$s^{\alpha(M_4^J)-1} \sim s^{-3/2}$ Regge pole	$\alpha'(M_4^J) \sim \alpha'_R$
2 	$2q\bar{q}$ – jets	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{\alpha(M_2^J)-1} \sim s^{-1}$ Regge pole	$\alpha'(M_2^J) \sim \frac{1}{2}\alpha'_R$
3 	$3q\bar{q}$ – jets	$\bar{n}(s) \simeq 3\bar{n}_{e^+e^-}(s/9)$	$s^{\alpha(M_0^J)-1} \sim s^{-1/2}$ Regge pole	$\alpha'(M_0^J) \sim \frac{1}{3}\alpha'_R$
4 	M_0 	$\bar{n}(s) \simeq 2\bar{n}_{e^+e^-}(s/4)$	$s^{2\alpha_B-2} \sim s^{-2}$ 2-Reggeon cut	$\frac{1}{2}\alpha'_R$

[(a)] To estimate the s-behaviour we have taken $\alpha_B \simeq 0$.

Veneziano Model

$$f(s) = \frac{1}{K^{-1}(s) - i\Gamma(s)}$$

∞ number of poles : confinement

Quadratically spaced radial trajectories

$$K(s) = \sum_{r=1}^{\infty} \frac{g_r^2}{m_r^2 - s} \to \sum_r \frac{1}{r^2 - s} \sim \frac{\cos(\pi\sqrt{s})}{\sin(\pi\sqrt{s})}$$

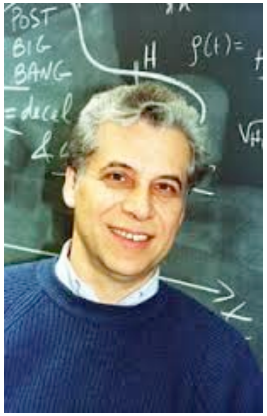
Linearly spaced radial trajectories (Veneziano)

$$K(s) \sim \frac{\Gamma(a-s)}{\Gamma(b-s)}$$

Veneziano amplitude : crossing symmetric:

$$A(s,t) = \frac{\Gamma(1-\alpha(s))\Gamma(1-\alpha(t))}{\Gamma(2-\alpha(s)-\alpha(t))}$$

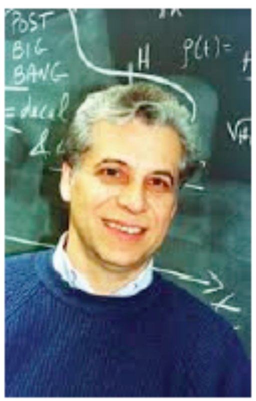
$$\alpha(s) = a + bs$$

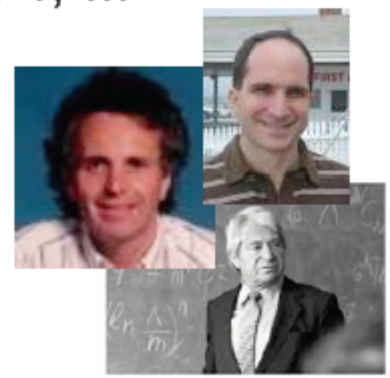


relativistic h.o.

string of relativistic oscillators

QCD, loop representation, large-N_c, AdS/CFT, ...





$\omega \rightarrow 3\pi$

$$\boxed{A(s,t) = \frac{\Gamma(-J(s))\Gamma(-J(t))}{\Gamma(-J(s)-J(t))}}$$

string revolution

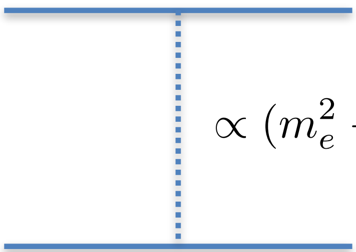
Other effects of partial wave analyticity

Scalar particle scattering 1+2 -> 3 + 4

$$A_l(s) = \int dz_s A(s, t(s, z_s), u(s, z_s)) P_l(\cos\theta)$$

Partial waves have “right hand” singularity (from s) and “left hand” (from t and u)

For example assume equal masses



$$\propto (m_e^2 - t(s, z_s))^{-1}$$

$$t = -\frac{(s - 4m^2)}{2}(1 - z_s)$$

$$A_0(s) \sim \int_{-1}^{1} dz_s \frac{1}{m_e^2 + \frac{(s-4m^2)}{2}(1 - z_s)}$$

For $s>4m^2$ integral is finite

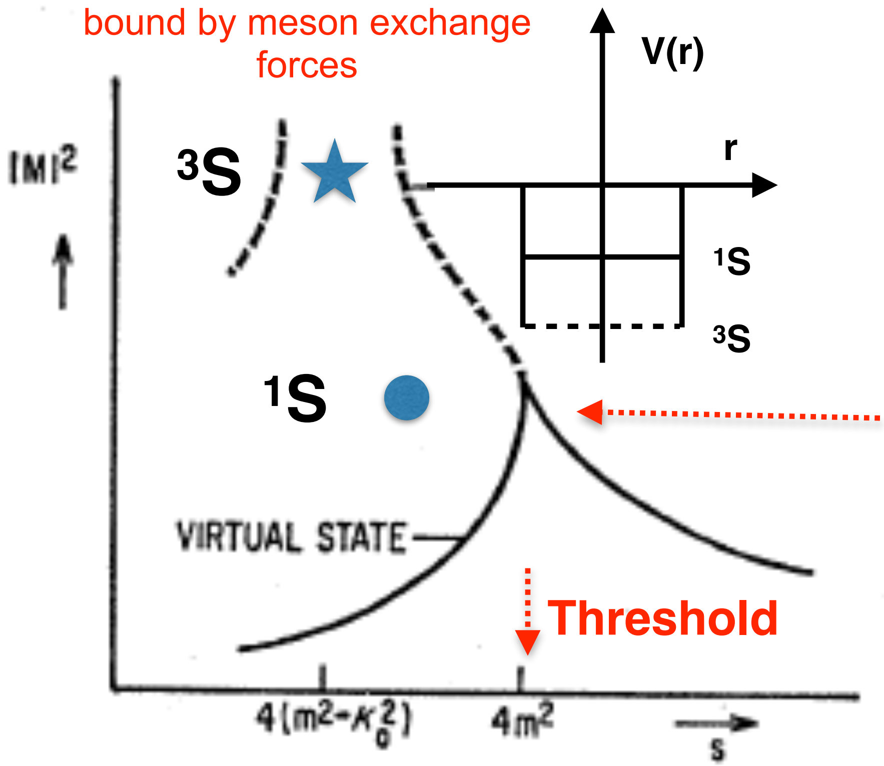
For $s<4m^2 - m_e^2$ the detonator crosses 0 within integration limi, implying $A_0(s)$ has a cut for negative s

Scalar amplitudes have simple singularity structure, but partial waves a much more complicated. They also have kinematical singularities when spin and/or unequal masse are involved

Bound states and Virtual States

- f0(980),
- a0(980),
- a1(1420),
- Lambda(1405),
- XYZ,
- …

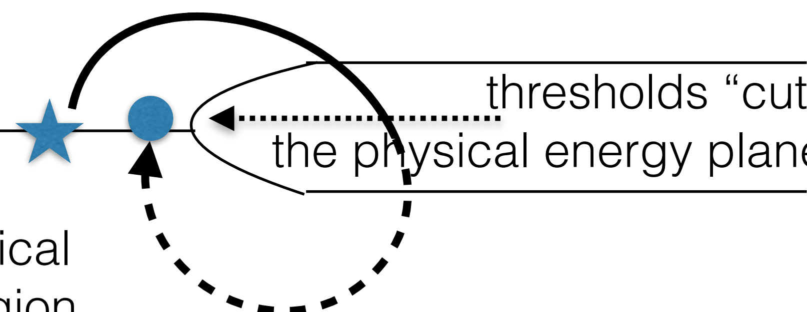
Deuteron the np molecule bound by meson exchange forces



- **Thresholds are "windows" to singularities (particles, visual states, forces") located on the nearby unphysical sheet.**

- **They appear as cusps (if below threshold) or bumps (is above)**

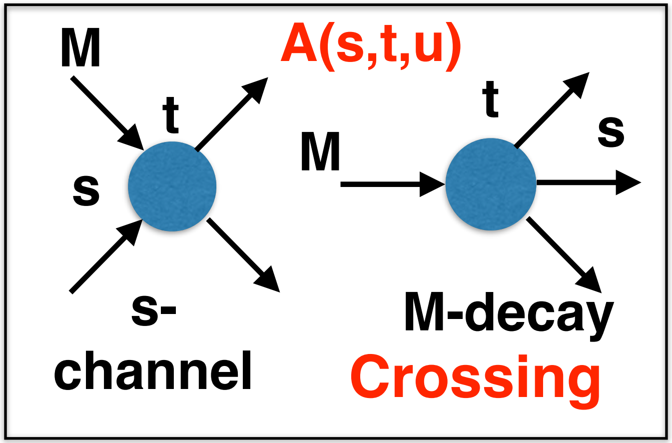
bound state : pole on the physical energy plane



thresholds "cut" the physical energy plane

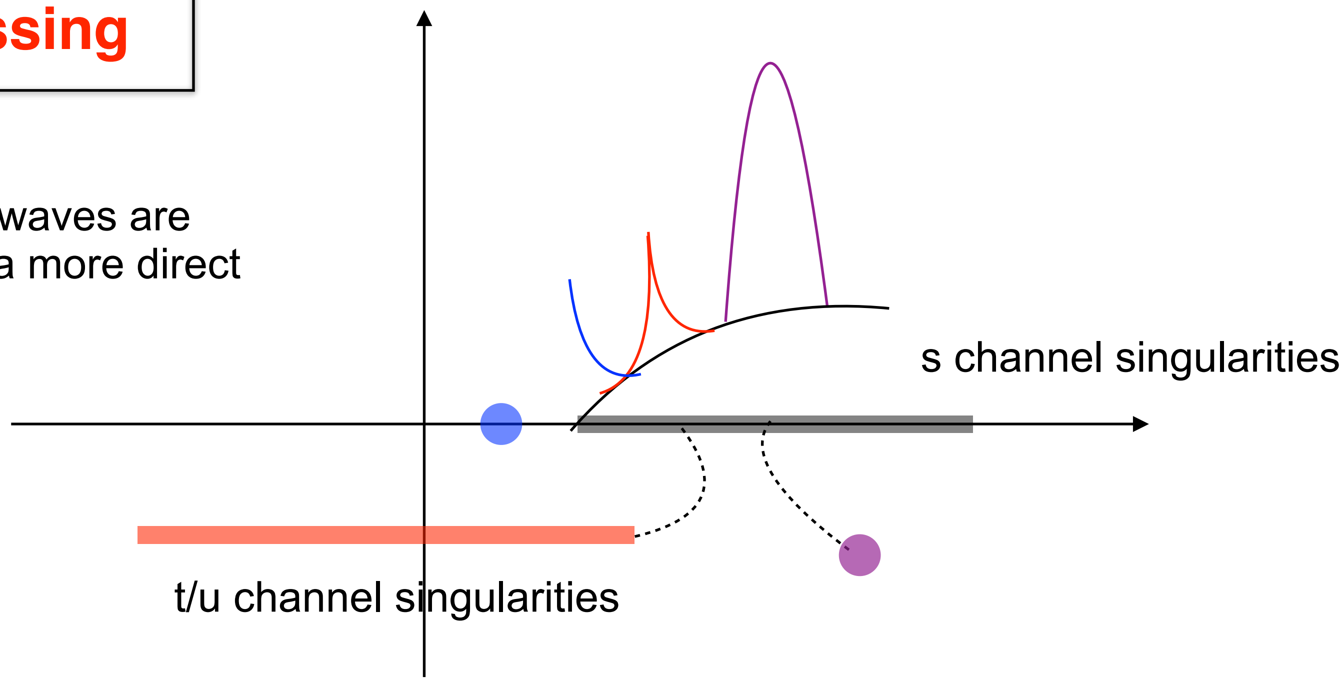
virtual state : pole on "unphysical sheet" closest the physical region

Amplitude singularities



- A(s,t,u) has simple singularity structure. Its connection to particles arises through (complicated) partial waves

$$A_l(s) = \frac{1}{2}\int_{-1}^{1} dz_s A(s, t(s,z), u(s,z))$$



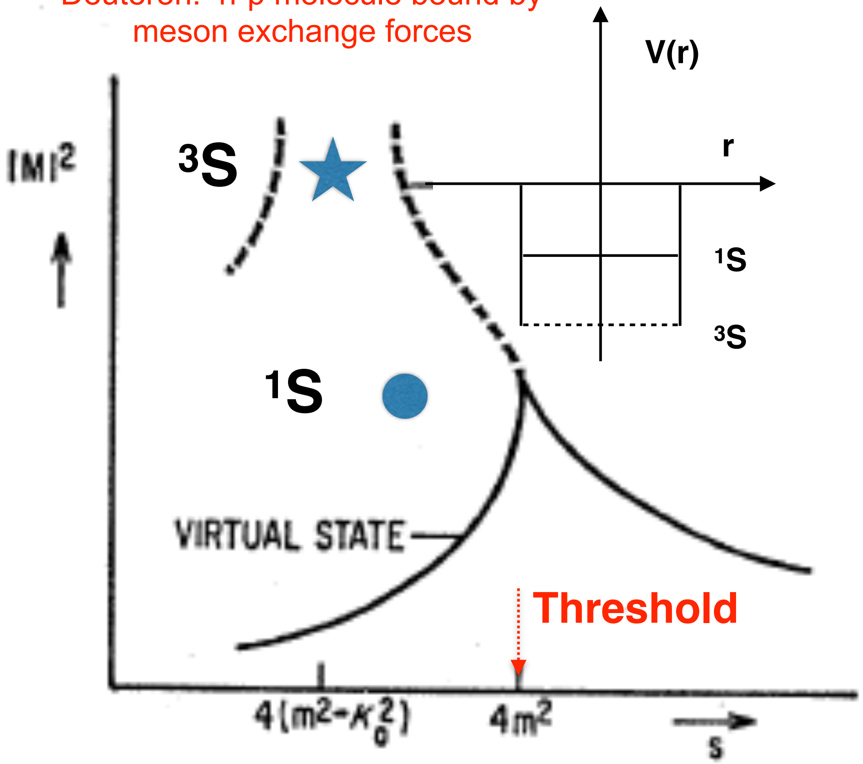
- Singularities of partial waves are complicated but have a more direct physical interpretation

- However, X-sections are given by A(s,t,u) and not by partial waves. In general “bumps” in partial waves are “washed out” and require partial wave analysis.

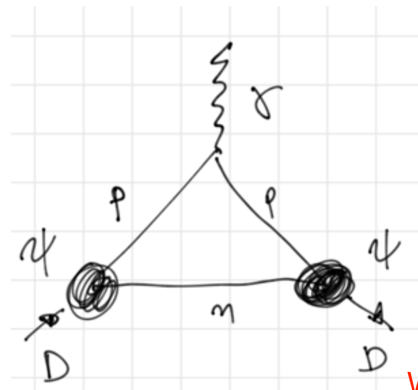
s inherit singularities of cross-

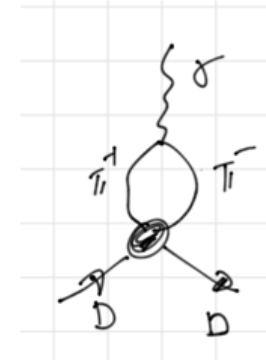
Well known examples of cusps

Deuteron: n-p molecule bound by meson exchange forces

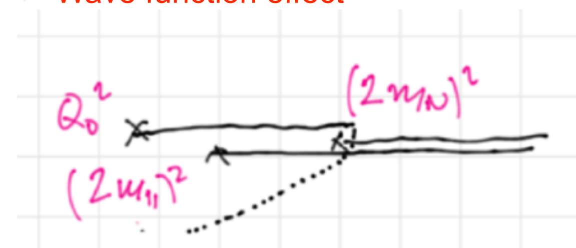


$$Q_0 \sim 100\ \mathrm{MeV} < 2m_\pi << 2m_N$$



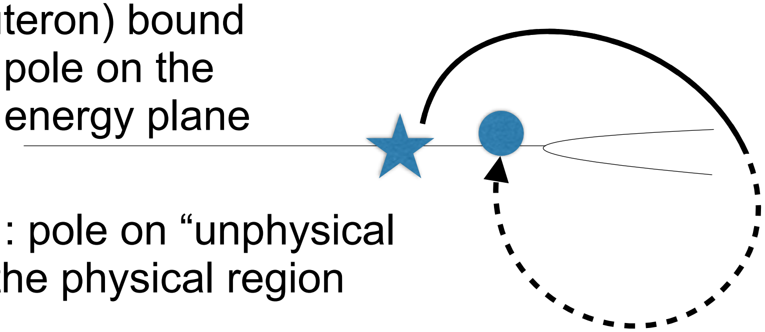


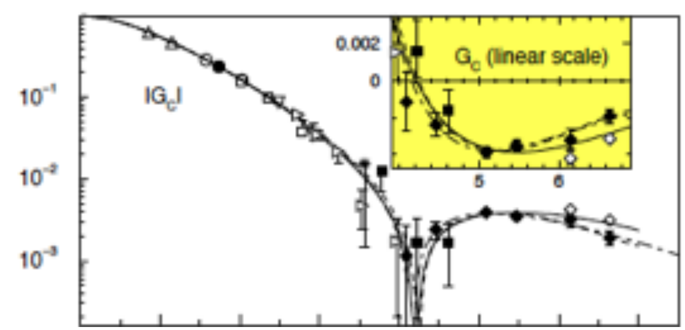
Wave function effect

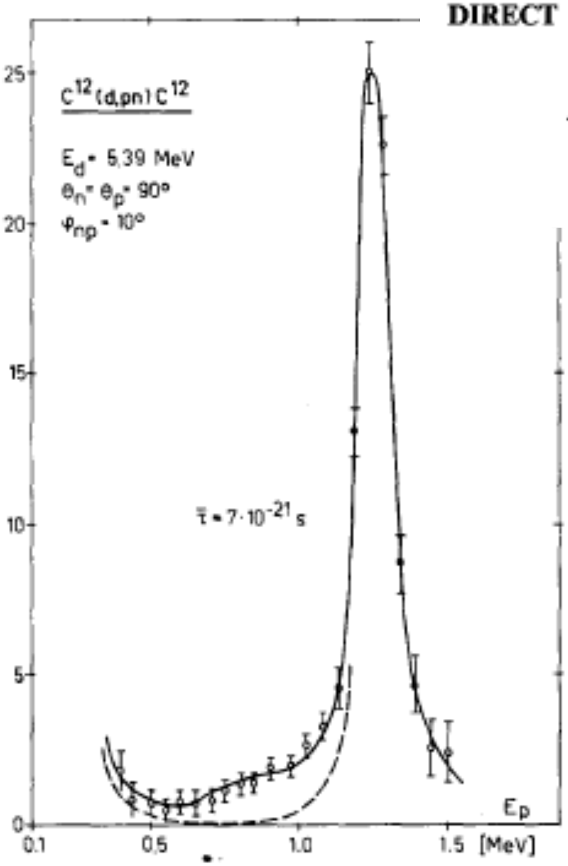


$^3S_1$ (deuteron) bound state : pole on the physical energy plane

$^1S_1$ virtual state : pole on "unphysical sheet" close the physical region





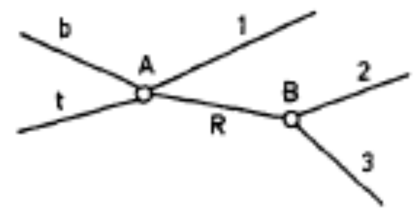


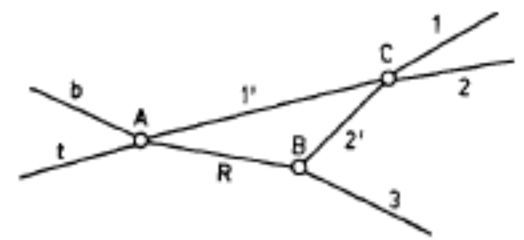
DIRECT DETERMINATION OF A SHORT NUCLEAR LIFETIME ($\approx 10^{-20}$ s) BY THE PROXIMITY SCATTERING METHOD

J. LANG, R. MÜLLER, W. WÖLFLI, R. BÖSCH and P. MARMIER
Laboratorium für Kernphysik, Eidg. Techn. Hochschule, Zürich †

Received 4 February 1966

$$b+t \rightarrow 1+2+3 \qquad [d+{}^{12}C \rightarrow n+p+{}^{12}C],$$



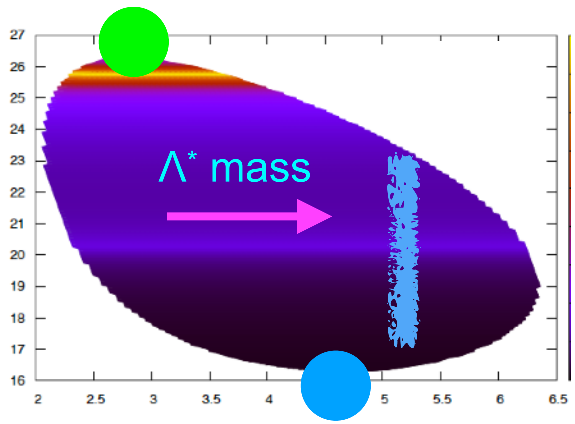


$$b+t \rightarrow 1+R, \qquad \text{with } Q\text{-value } Q_1 \qquad [d+{}^{12}C \rightarrow n+{}^{13}N^*, \qquad Q_1 = -3.82 \text{ MeV}],$$

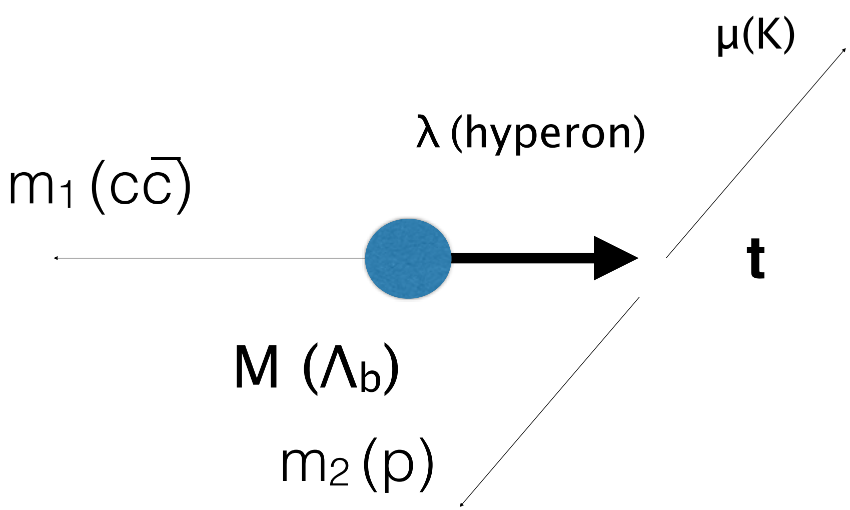
$$R \rightarrow 2+3 \qquad \text{with } Q\text{-value } Q_2 \qquad [{}^{13}N^* \rightarrow p+{}^{12}C, \qquad Q_2 = 1.59 \text{ MeV}],$$

Classical picture

Coleman-Norton



t-channel resonance can produce s-channel “band” if:



all particles on-shell

m_2 and m_1 collinear

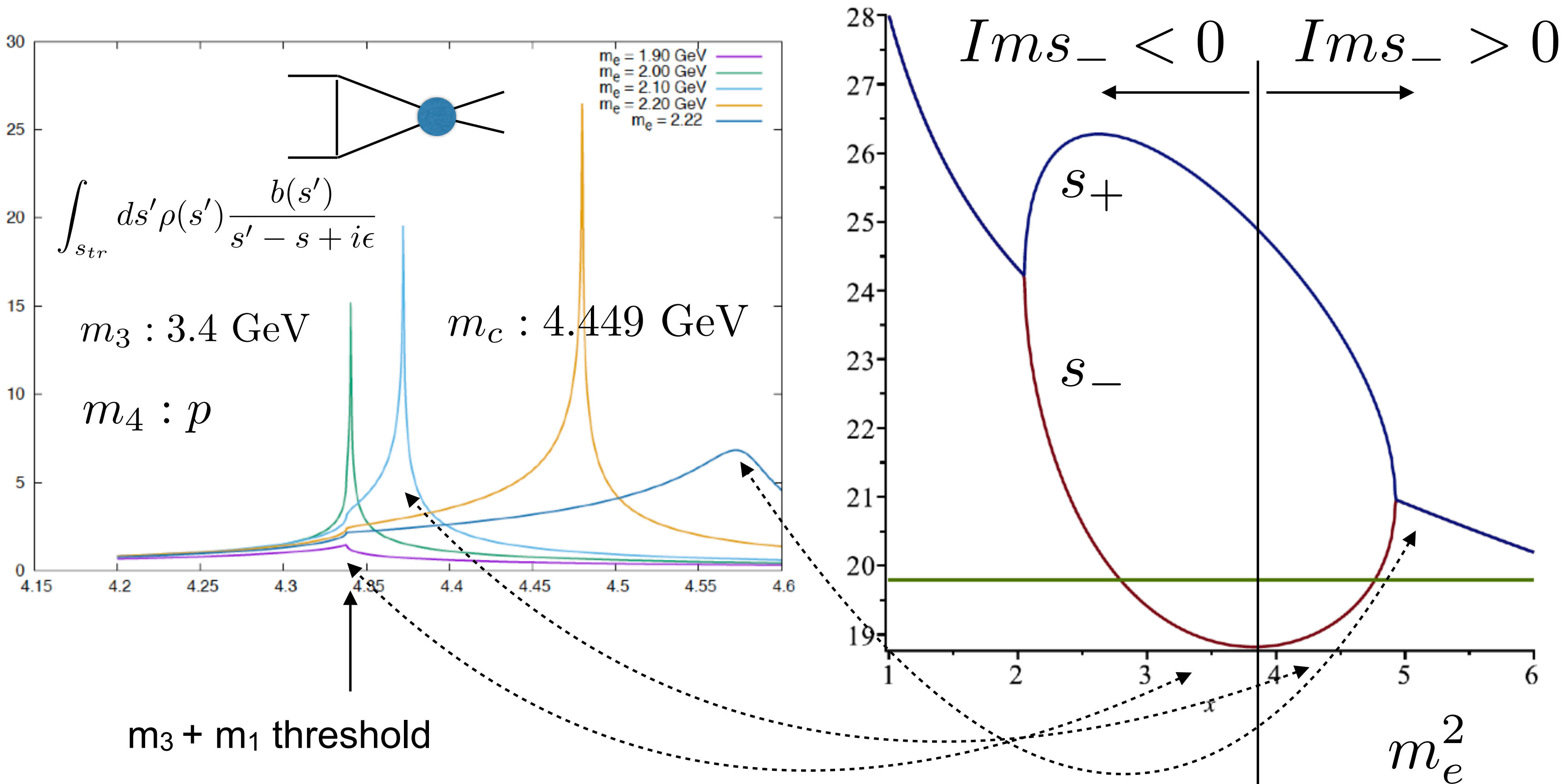
$v(m_2) > v(m_1)$

Example : Pc Kinematics

m_1	m_3
m_2	m_4

$$s_{\pm} = -m_e^2 + p_2^2 + p_3^2 + \frac{(m_e^2 + p_1^2 - p_3^2)(m_e^2 + p_4^2 - p_2^2)}{2m_e^2} \pm \frac{\lambda^{1/2}(m_e^2, p_1^2, p_3^2)\lambda^{1//2}(m_e^2, p_2^2, p_4^2)}{2m_e^2}$$

$m_1 : \Lambda_b \quad m_2 : K$



- Singularities of b(s) are at s=s±

$$b_l(s) = \frac{1}{2}\int_{-1}^{1} dz_s \frac{P_l(z_s)}{m_\Lambda^2 - t(s,z)}$$

Summary

- In QCD light quark resonances appear clearly up to ~2GeV But one expected there to be an infinite number of them.

- At higher masses they are harder to find. To help discriminating between various hypotheses one should “consult” with expectations from quark model and duality arguments.

- Duality arguments are consistent with existence of multi quark hadrons.

- Veneziano model and generalizations could be used to implement these ideas in data analysis.

- Unlike non-relativistic theory, besides resonance poles one should work about “left-hand cuts’ (cusps), however, so far there is no unambiguous evidence for them in the data.

Thank you for your attention !