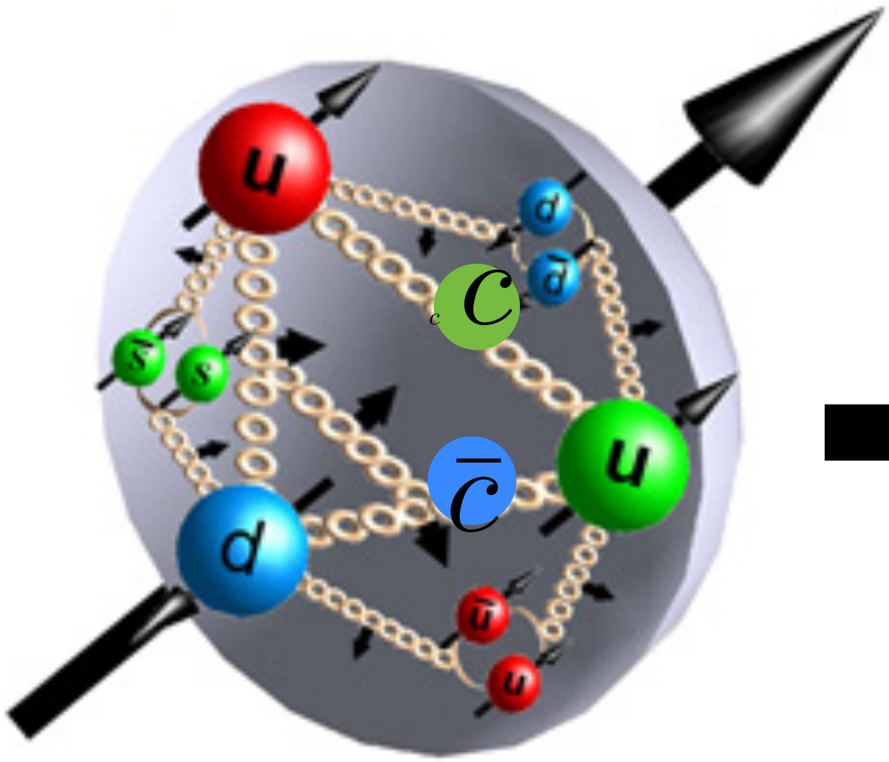
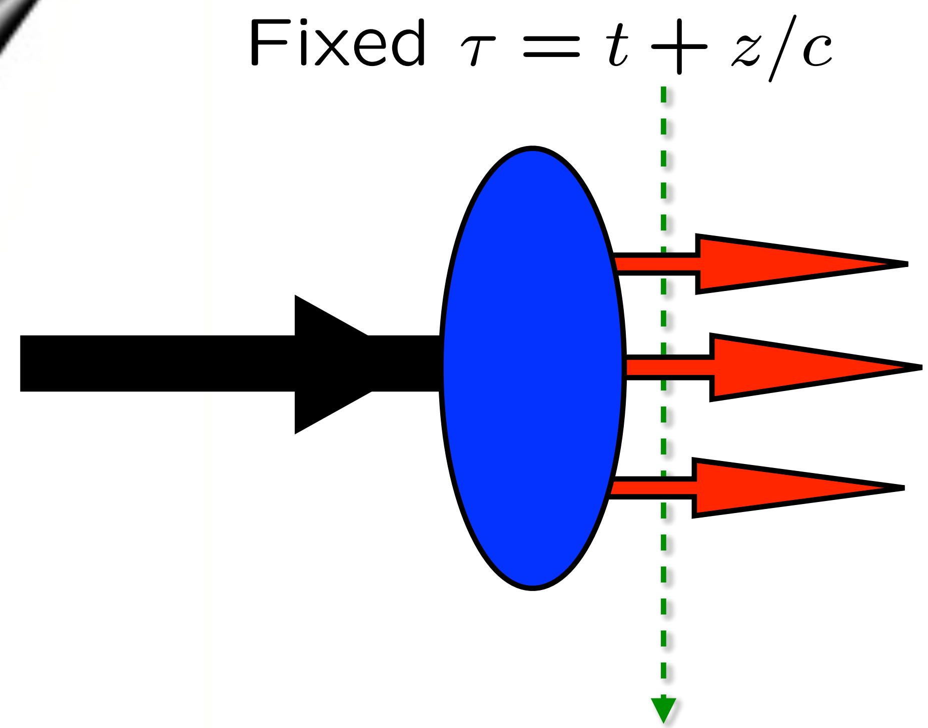
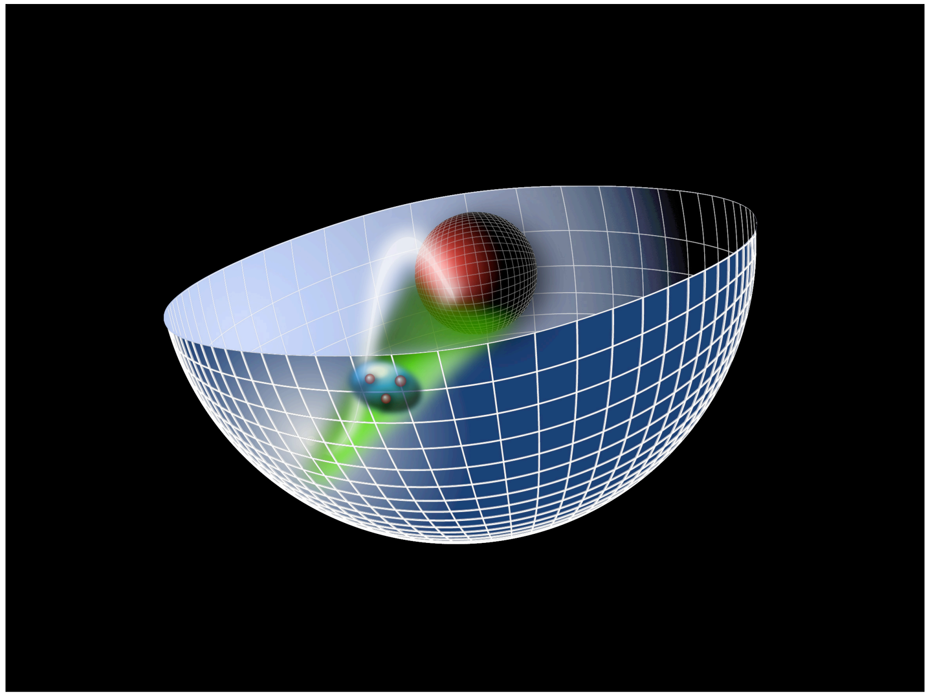
Lecture I

Light-Front Quantization and New Perspectives for Hadron Physics







Stan Brodsky

SLAC NATIONAL ACCELERATOR LABORATORY

LELAND STANFORD JUNIOR UNIVERSITY 1891

with Guy de Tèramond, Hans Günter Dosch, and Alexandre Deur

GGI School | Frontiers in Nuclear and Hadronic Physics | February 25 - March 8, 2019

Remarkable Fundamental Features of Hadrons, Nuclei

- Color confinement: Quarks and Gluons permanently confined in hadrons!
- Origin of the hadron mass scale: what determines the proton mass?
- Pion is a quark-antiquark bound state, but it is massless if the quark mass is zero!
- The QCD coupling at all scales, beyond asymptotic freedom
- How does one set the renormalization scale? QCD —> QED if Nc —> 0
- Poincare invariance: Physics independent of observer motion — no Lorentz contraction!
- Causality: No correlations exceeding the speed of light
- Light Front Theory: Relativistic Lorentz-invariant Bound State Dynamics
- Mesons and Baryons display supersymmetry!
- Exotic Phenomena: Color Transparency, Intrinsic Charm, Hidden Color, Exotic Hadrons
- Cosmological Constant

Light-Front Dynamics

Light-Front Quantization and New Perspectives for Hadron Physics

1. Introduction to QCD on the Light Front and Applications to Hadron Physics

2. Solving Quantum Field Theory Using Light-Front Hamiltonian Methods

3. Light-Front Holography and Super-Conformal Algebra: Applications to Hadron Spectroscopy and Dynamics

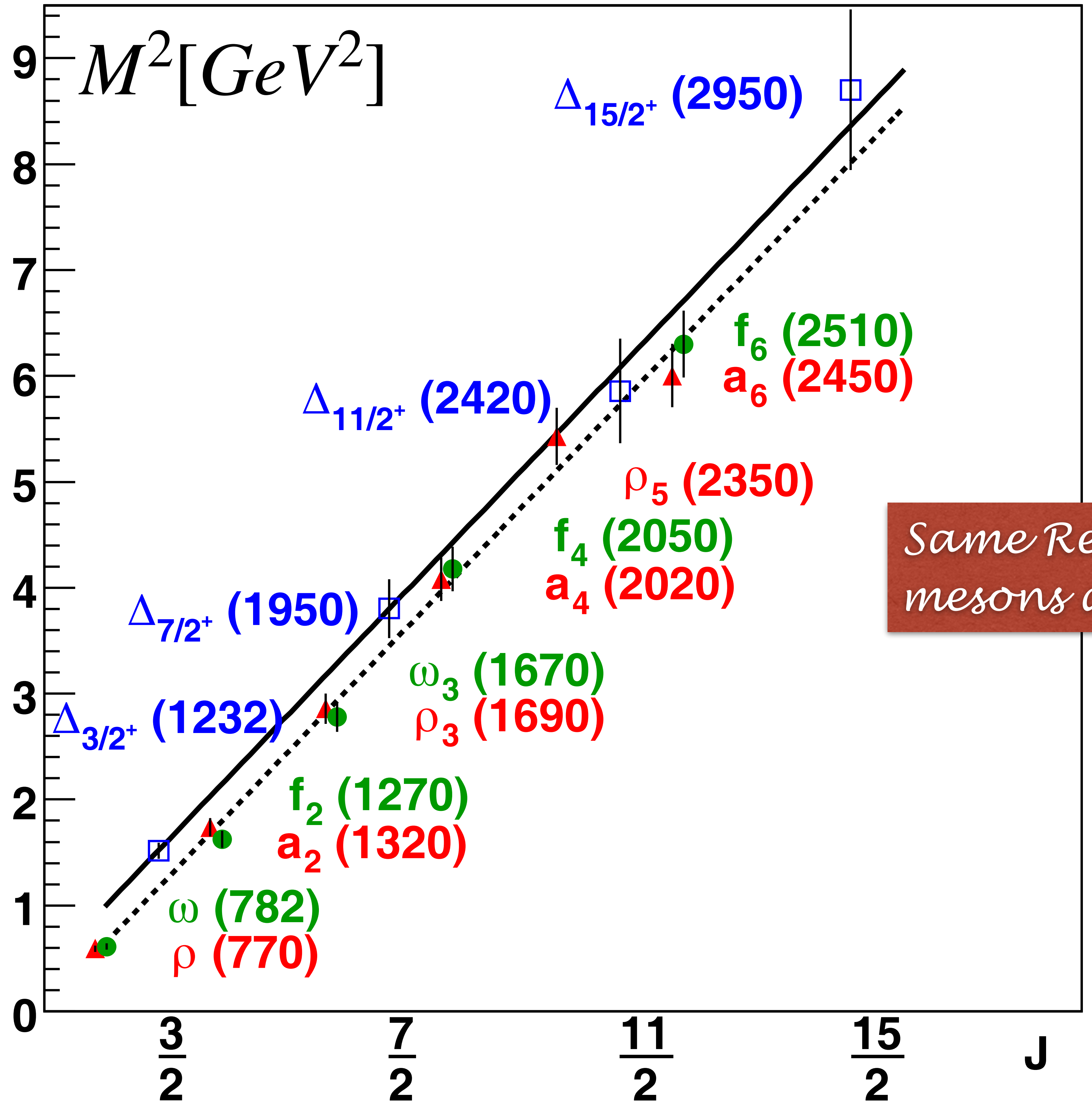
4. The Running QCD Coupling at All Scales and the QCD Light-Front Vacuum

5. Novel Features of QCD Phenomenology

6. Challenging Conventional Wisdom: Corrections to QCD Factorization Theorems and the Breakdown of Sum Rules

7. The Elimination of Renormalization and Factorization Scale Ambiguities.

E. Klempt and B. Ch. Metsch



Same Regge slope for mesons and baryons!

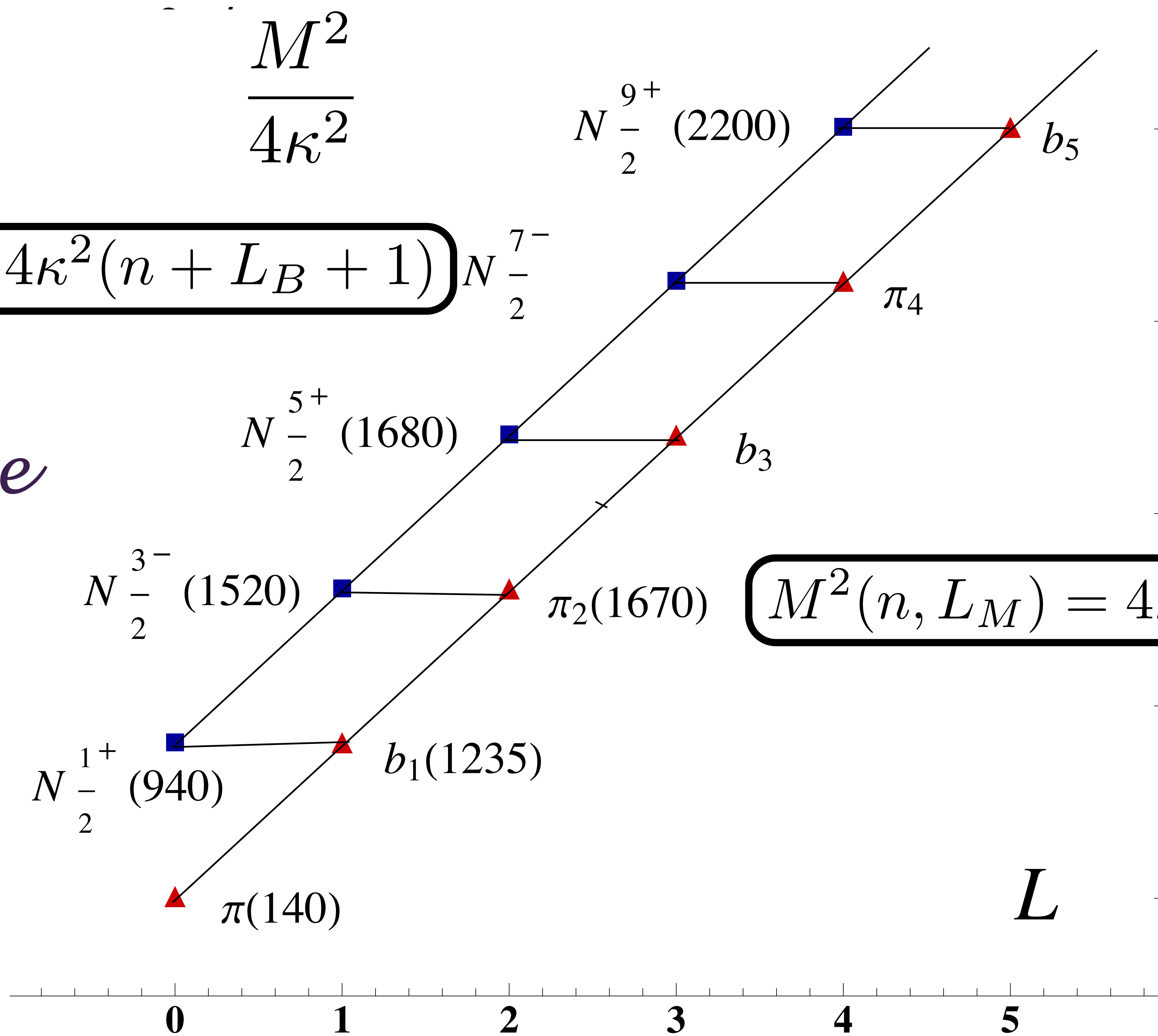
The leading Regge trajectory: Δ resonances with maximal J in a given mass range.
Also shown is the Regge trajectory for mesons with J = L+S.

Superconformal Quantum Mechanics

de Tèramond, Dosch, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

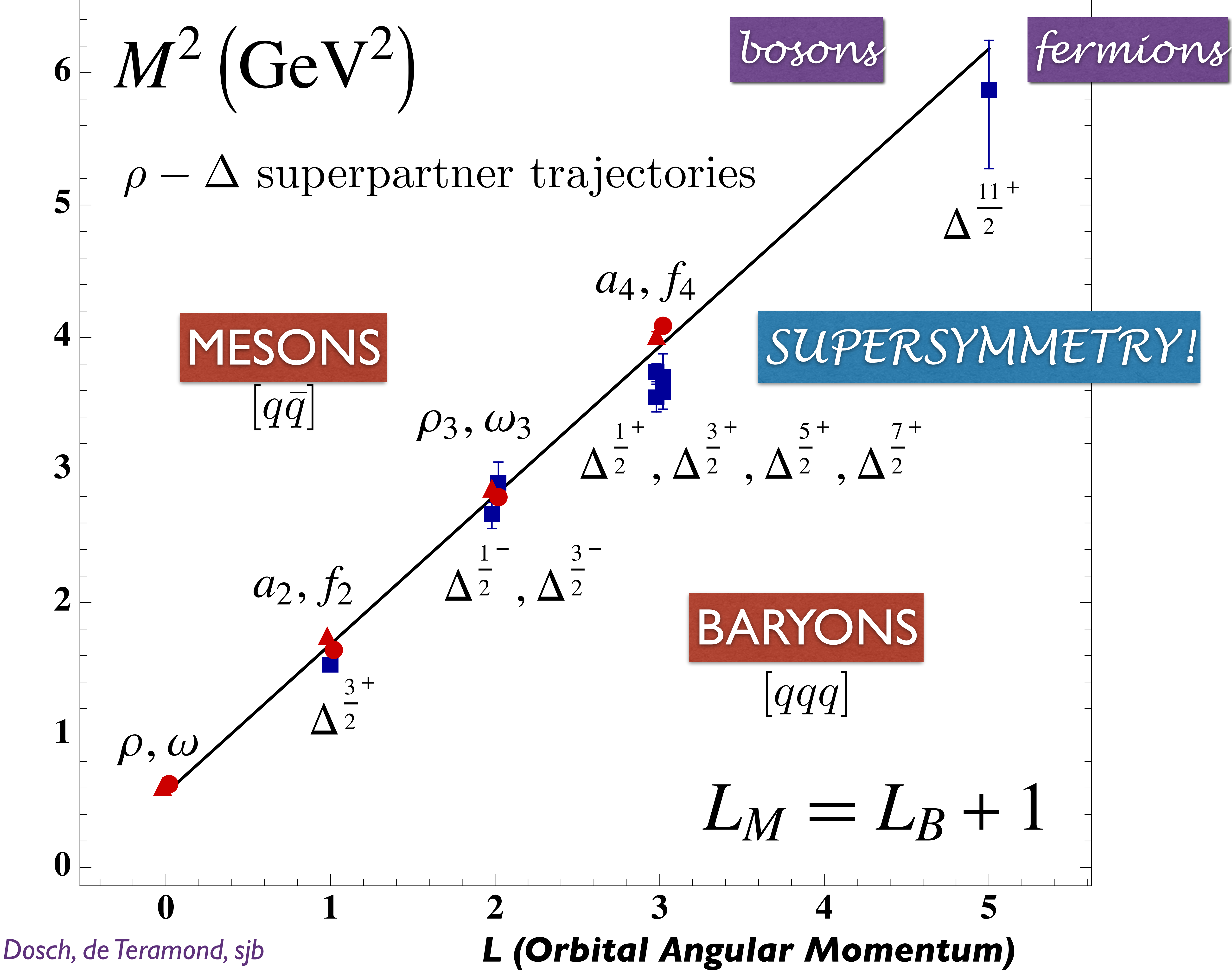
Same slope



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

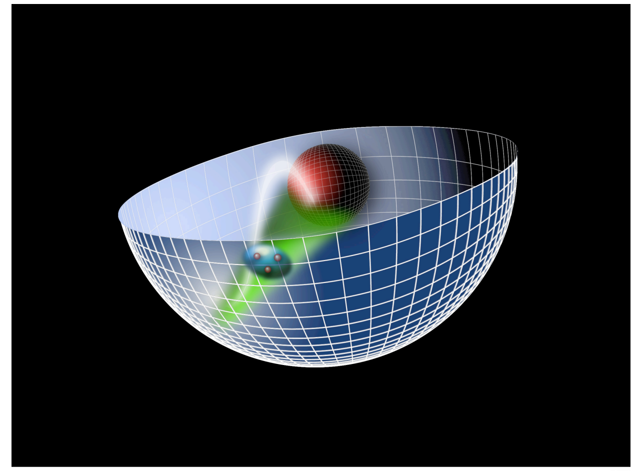
$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

Meson-Baryon Mass Degeneracy for $L_M = L_B + 1$



$M^2\ (\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
$\Delta_{\frac{11}{2}^+}$
a_4, f_4
MESONS
$[q\bar{q}]$
SUPERSYMMETRY!
ρ_3, ω_3
$\Delta_{\frac{1}{2}^+}, \Delta_{\frac{3}{2}^+}, \Delta_{\frac{5}{2}^+}, \Delta_{\frac{7}{2}^+}$
$\Delta_{\frac{1}{2}^-}, \Delta_{\frac{3}{2}^-}$
a_2, f_2
BARYONS
$[qqq]$
$\Delta_{\frac{3}{2}^+}$
ρ, ω
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

de Tèramond, Dosch, sjb



AdS/QCD Soft-Wall Model

Light-Front Holography

$$\zeta^2 = x(1-x)\mathbf{b}_\perp^2 .$$

$$\Big[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\Big]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Light-Front Schrödinger Equation

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Unique Confinement Potential!

Conformal Symmetry of the action

Confinement scale:

$$\kappa \simeq 0.6\ GeV$$

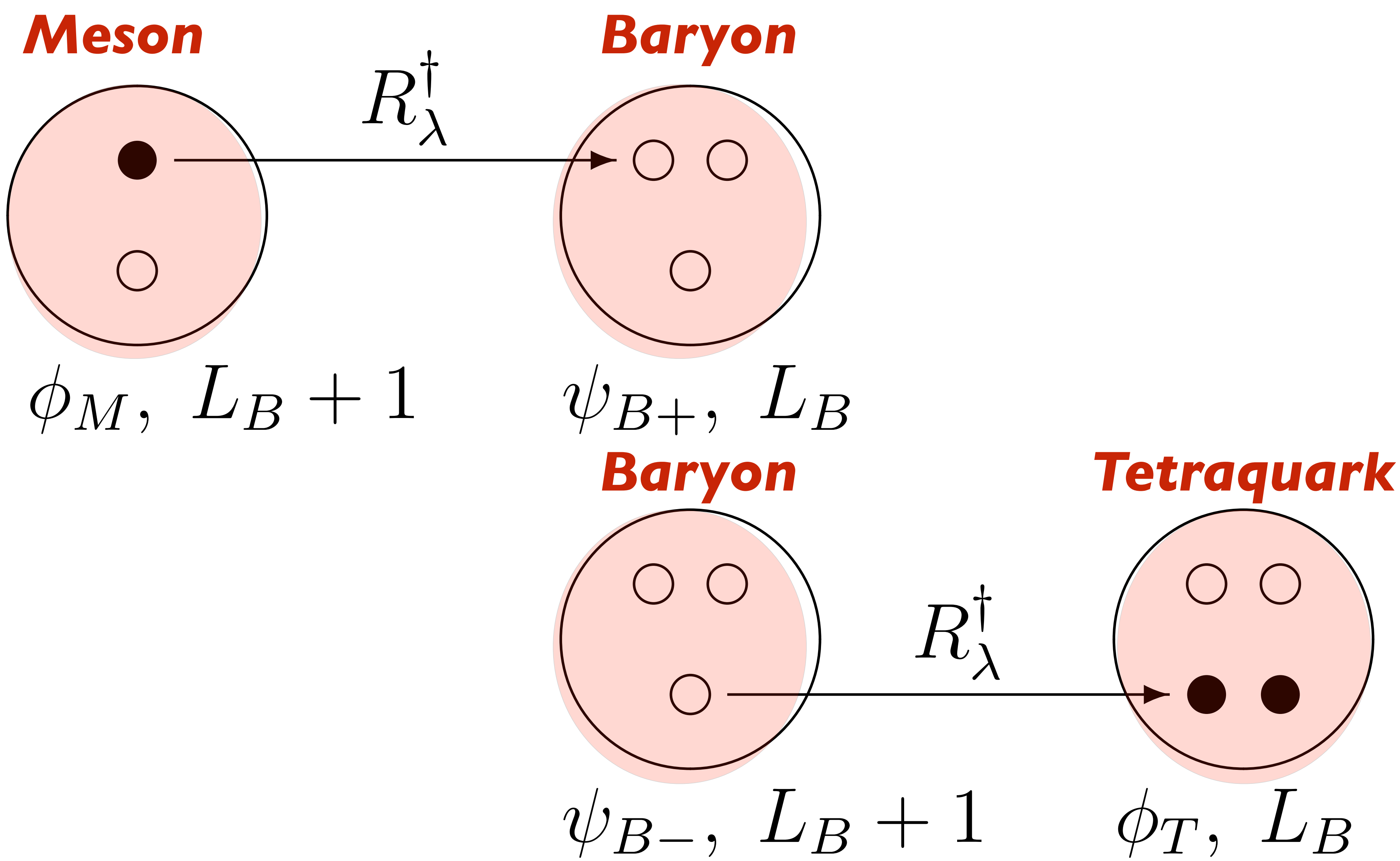
$$1/\kappa \simeq 1/3\ fm$$

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

Superconformal Algebra

2X2 Hadronic Multiplets

Bosons, Fermions with Equal Mass!



Proton: quark + scalar diquark $|q(qq)>$
(Equal weight: $L = 0, L = 1$)

LF Holography

Baryon Equation

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

S=1/2, P=+

both chiralities

Meson Equation

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

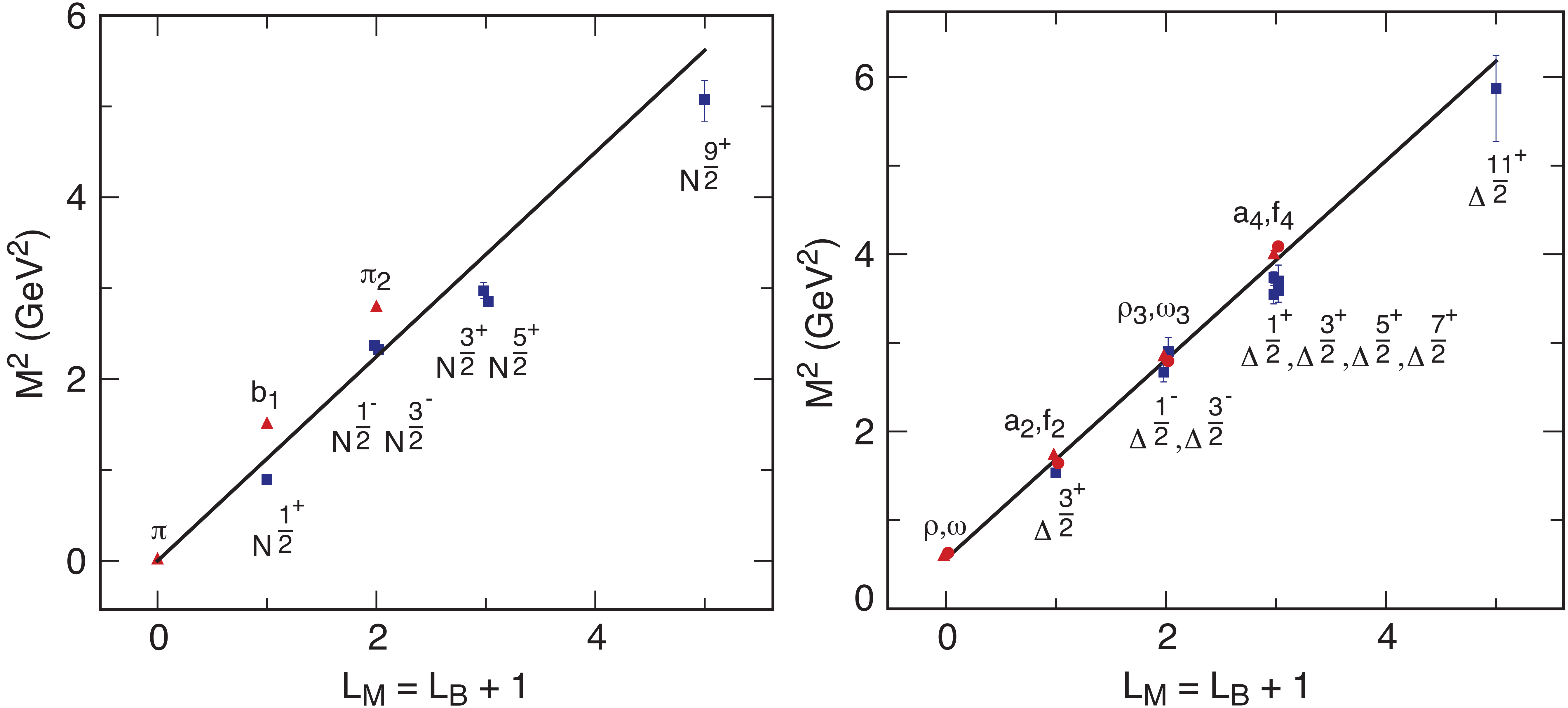
$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

Same κ!

S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon

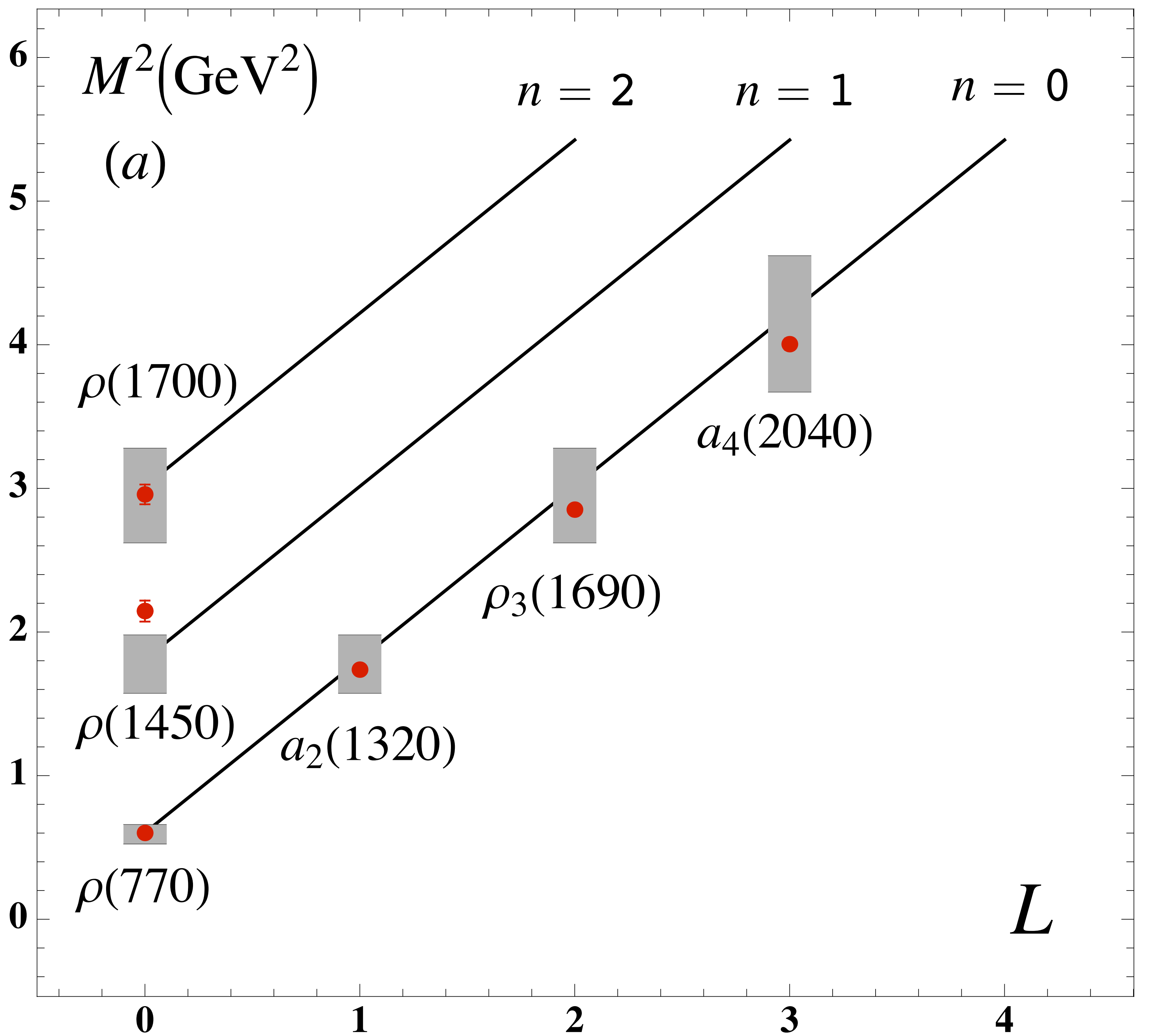
Meson-Baryon Degeneracy for $L_M = L_B + 1$

Solid line: $\kappa = 0.53$ GeV



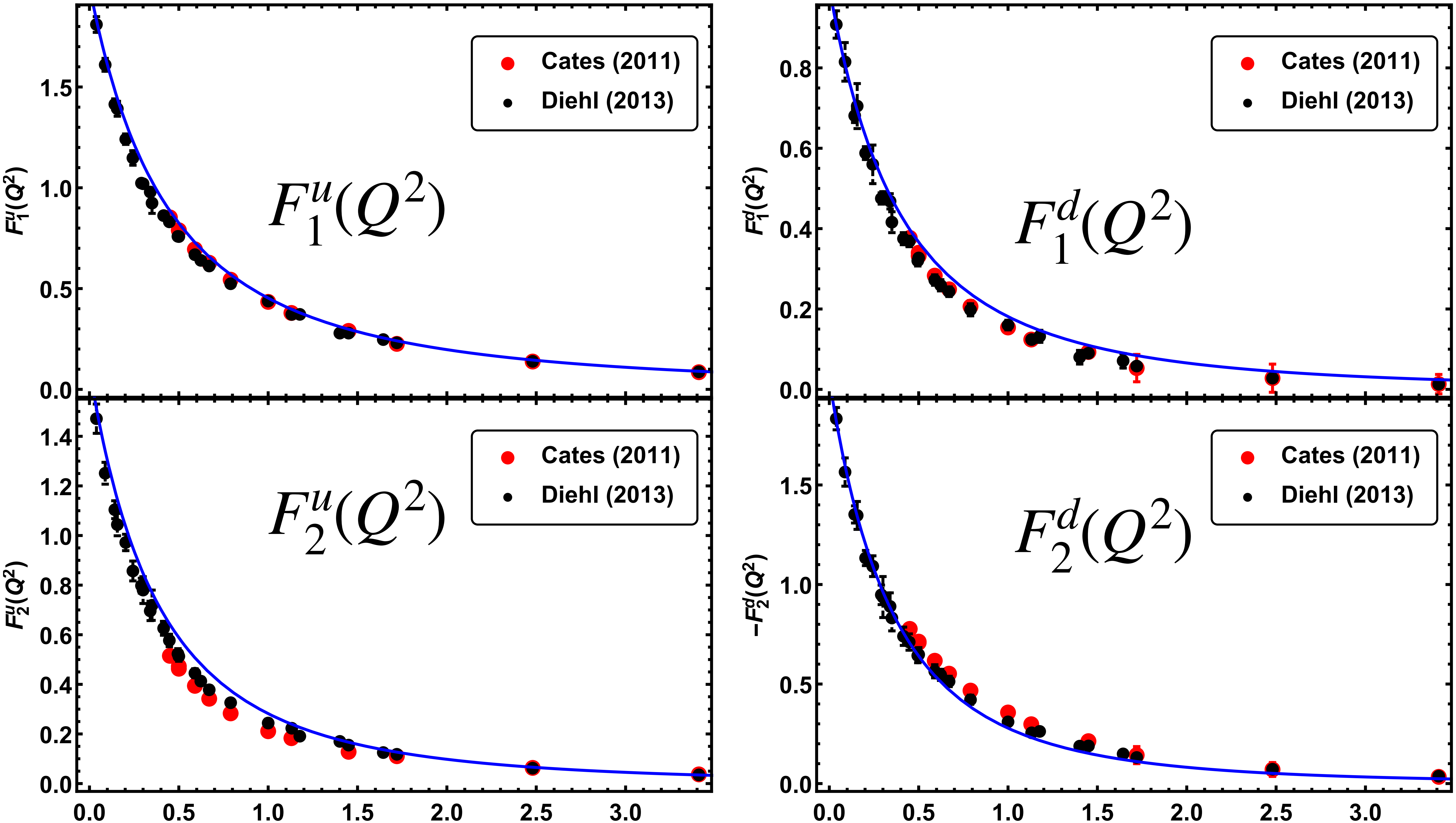
Superconformal meson-nucleon partners

de Tèramond, Dosch, sjb



$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

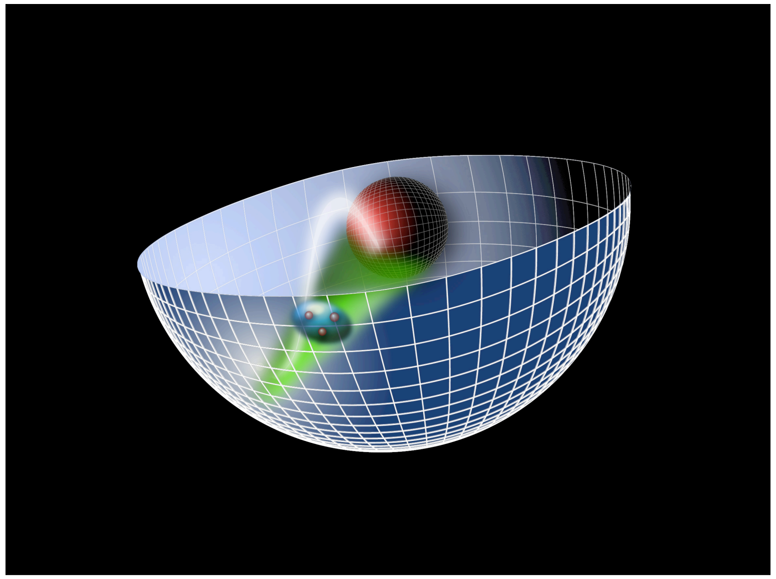
LFHQCD predictions for Nucleon Form Factors



From Neetika Sharma

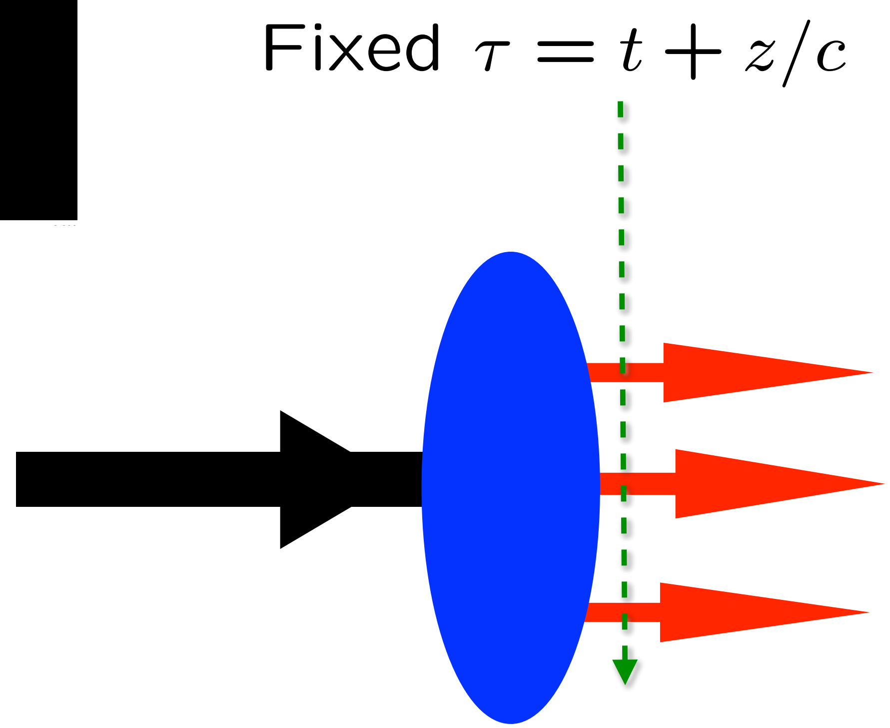
$\phi(z)$

AdS5: Conformal Template for QCD

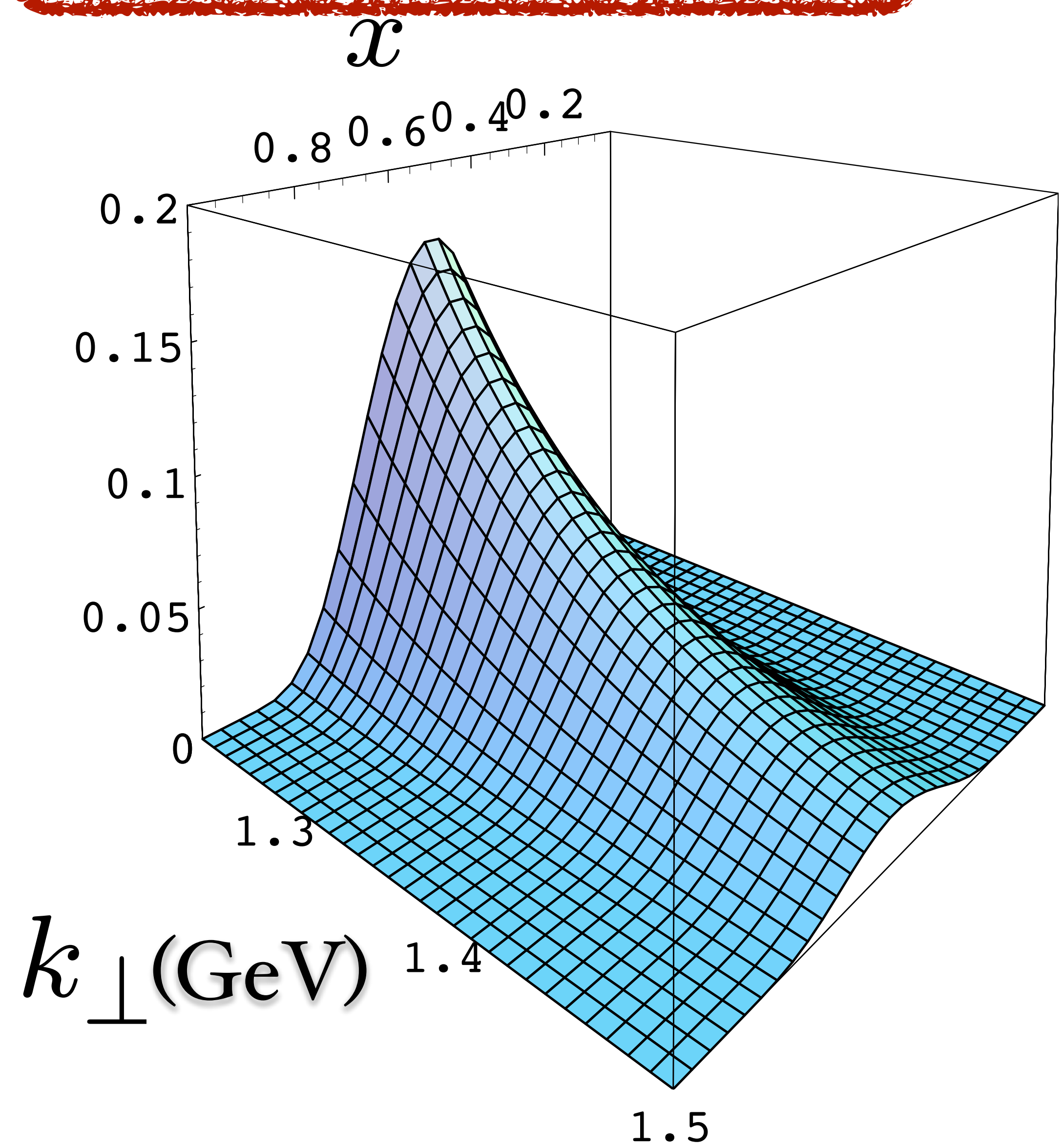


- *Light-Front Holography*

Duality of AdS$_5$ with LF Hamiltonian Theory



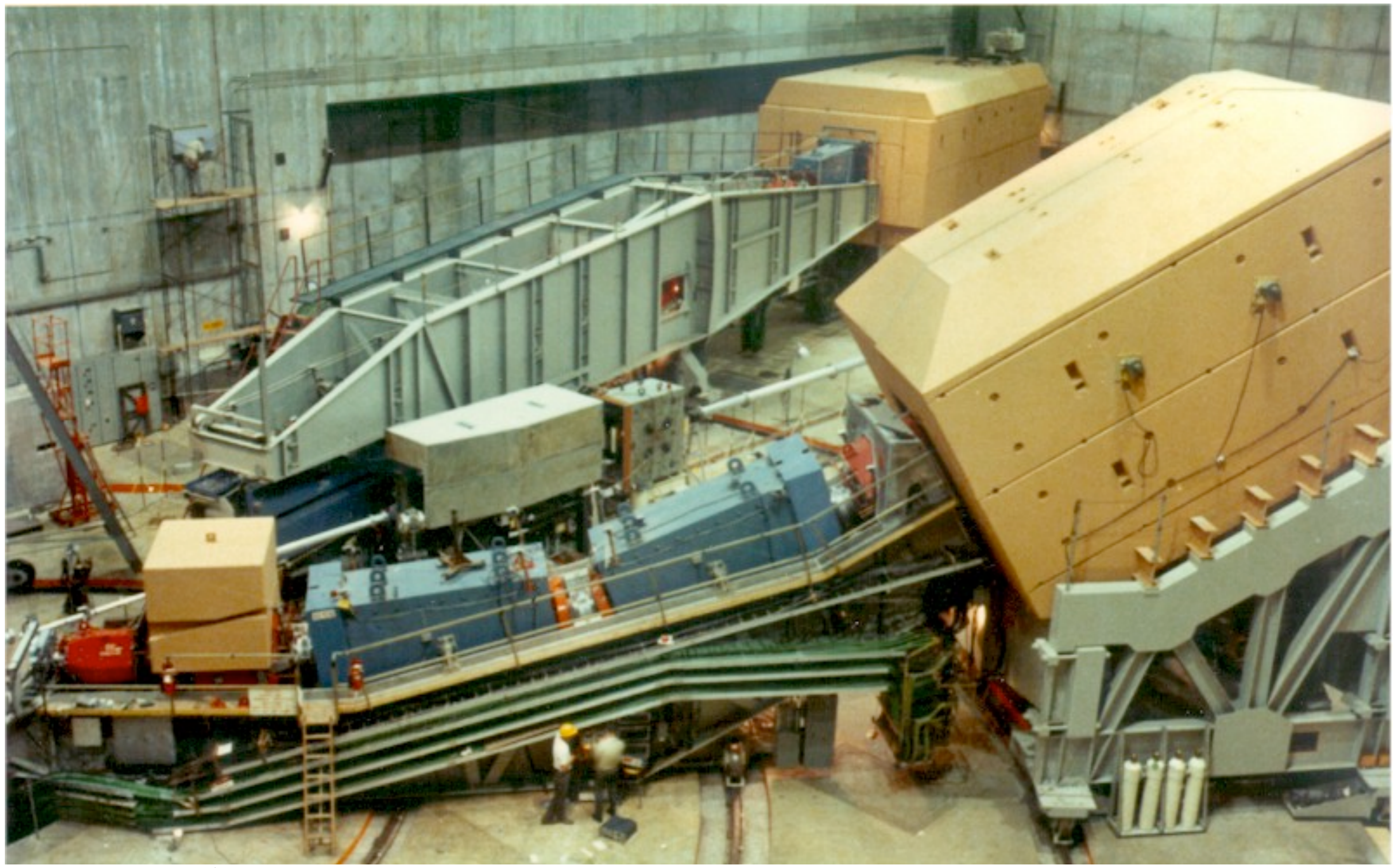
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$



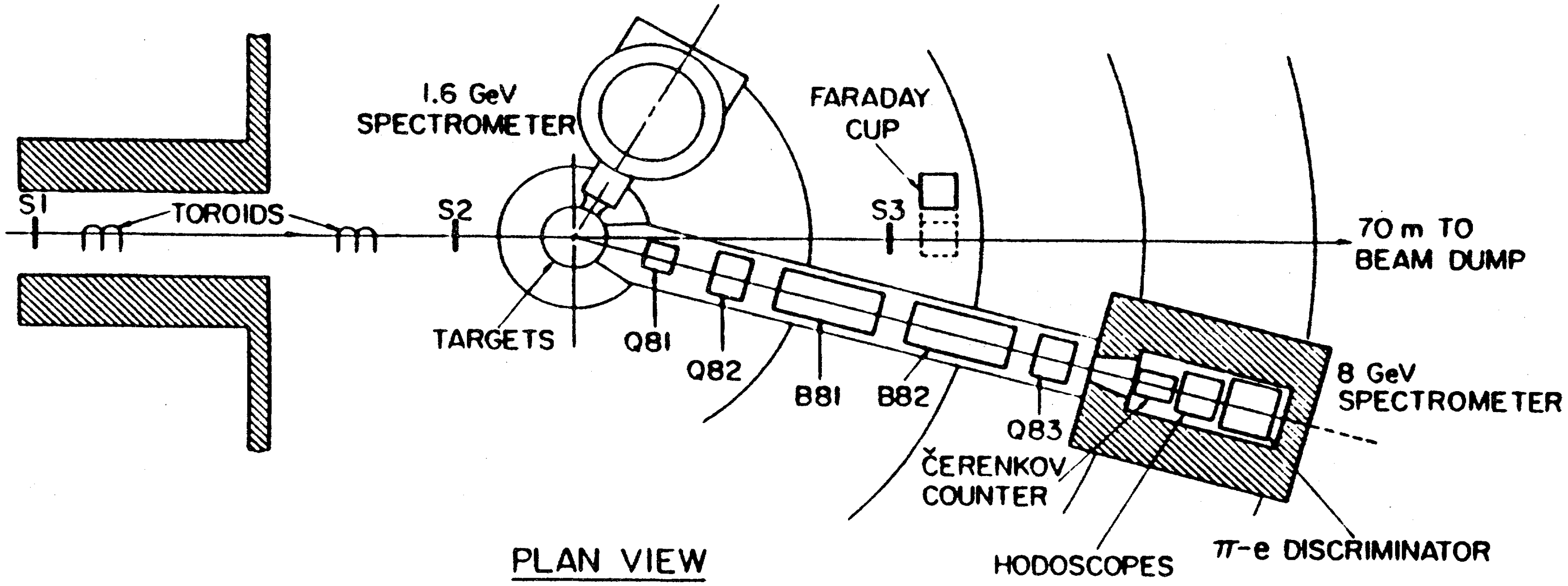
- *Light Front Wavefunctions:*

Light-Front Schrödinger Equation
Spectroscopy and Dynamics

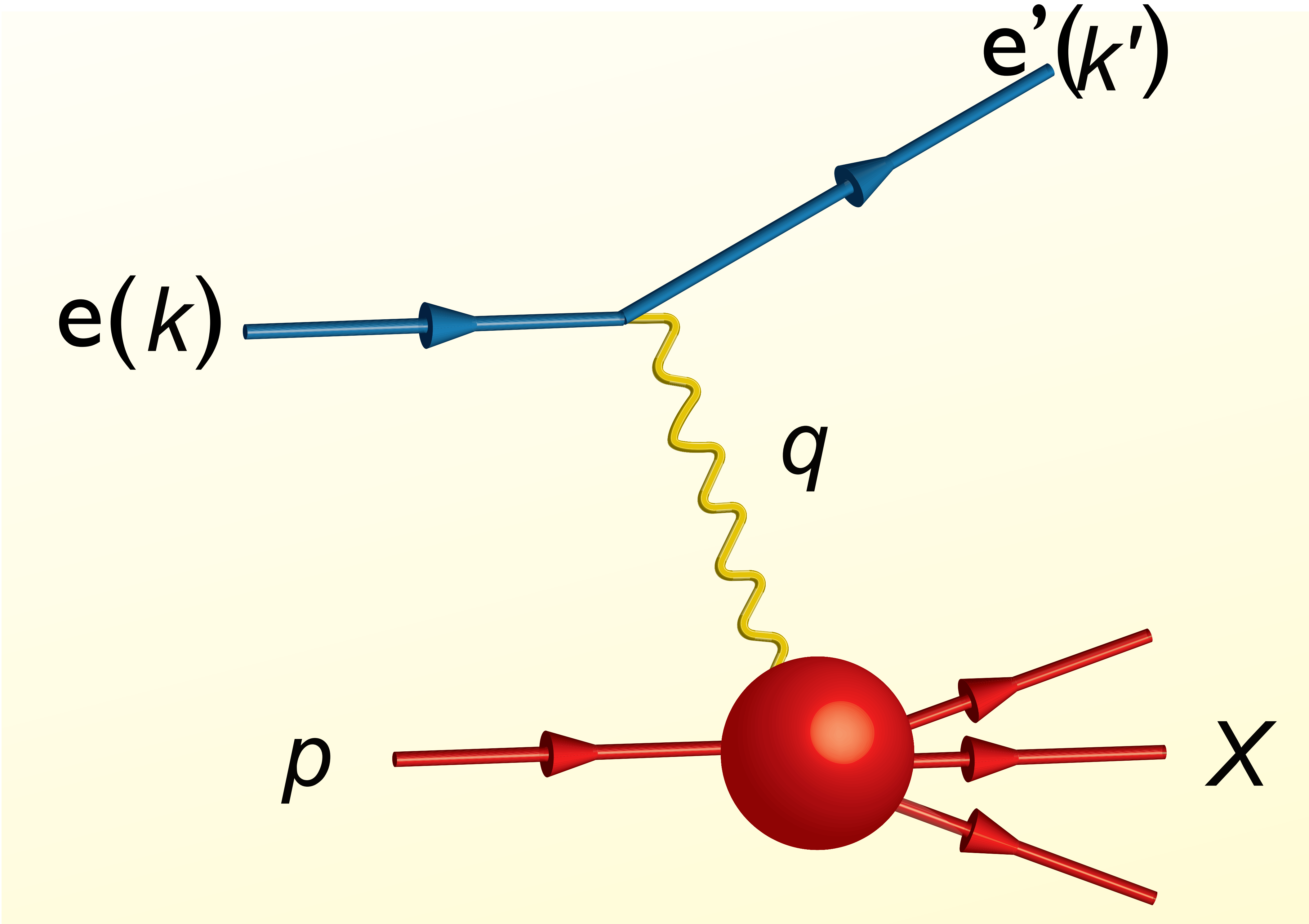
SLAC Two-Mile Linear Accelerator



Pief

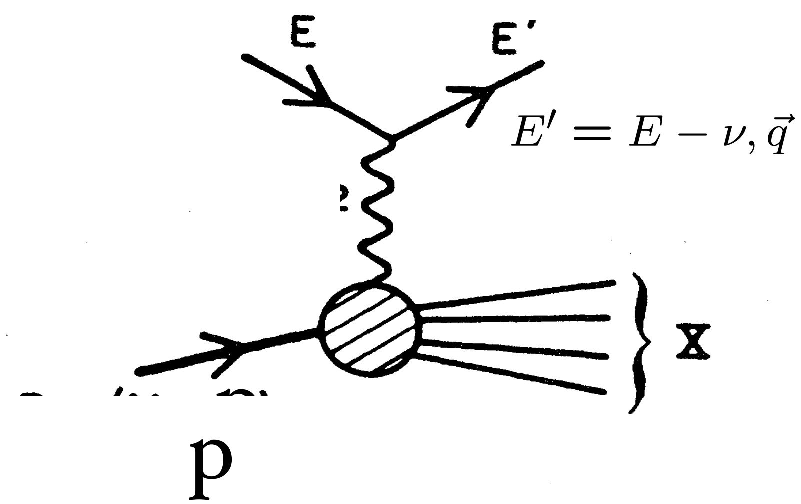


Causality: Information, correlations constrained by speed of light

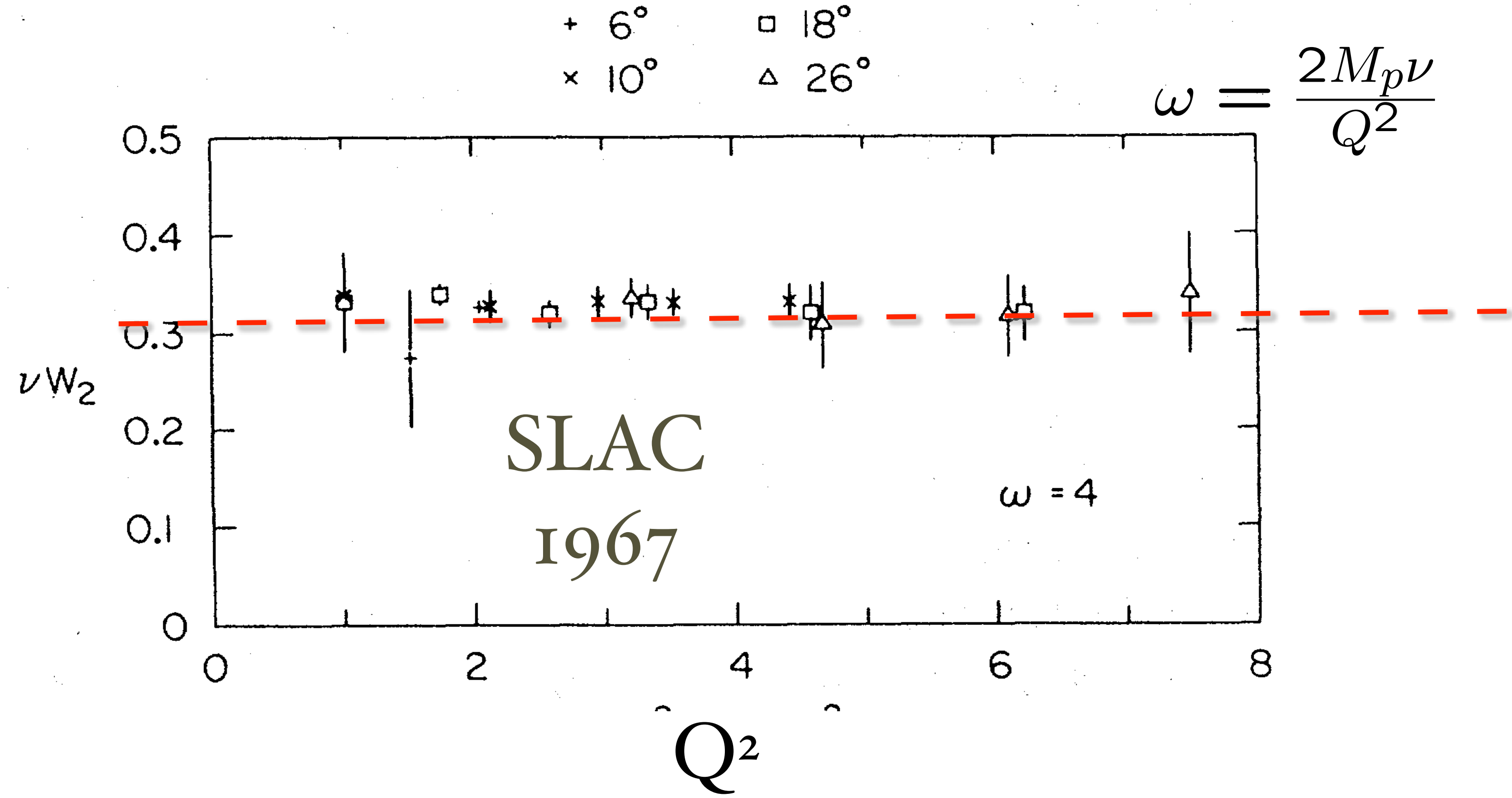


The scattered electron measures the proton's structure at the speed of light like a flash photograph

$$ep \to e' X$$



$$Q^2 = \vec{q}^2 - \nu^2$$



No intrinsic length scale !

Measure rate as a function of energy loss ν and momentum transfer Q

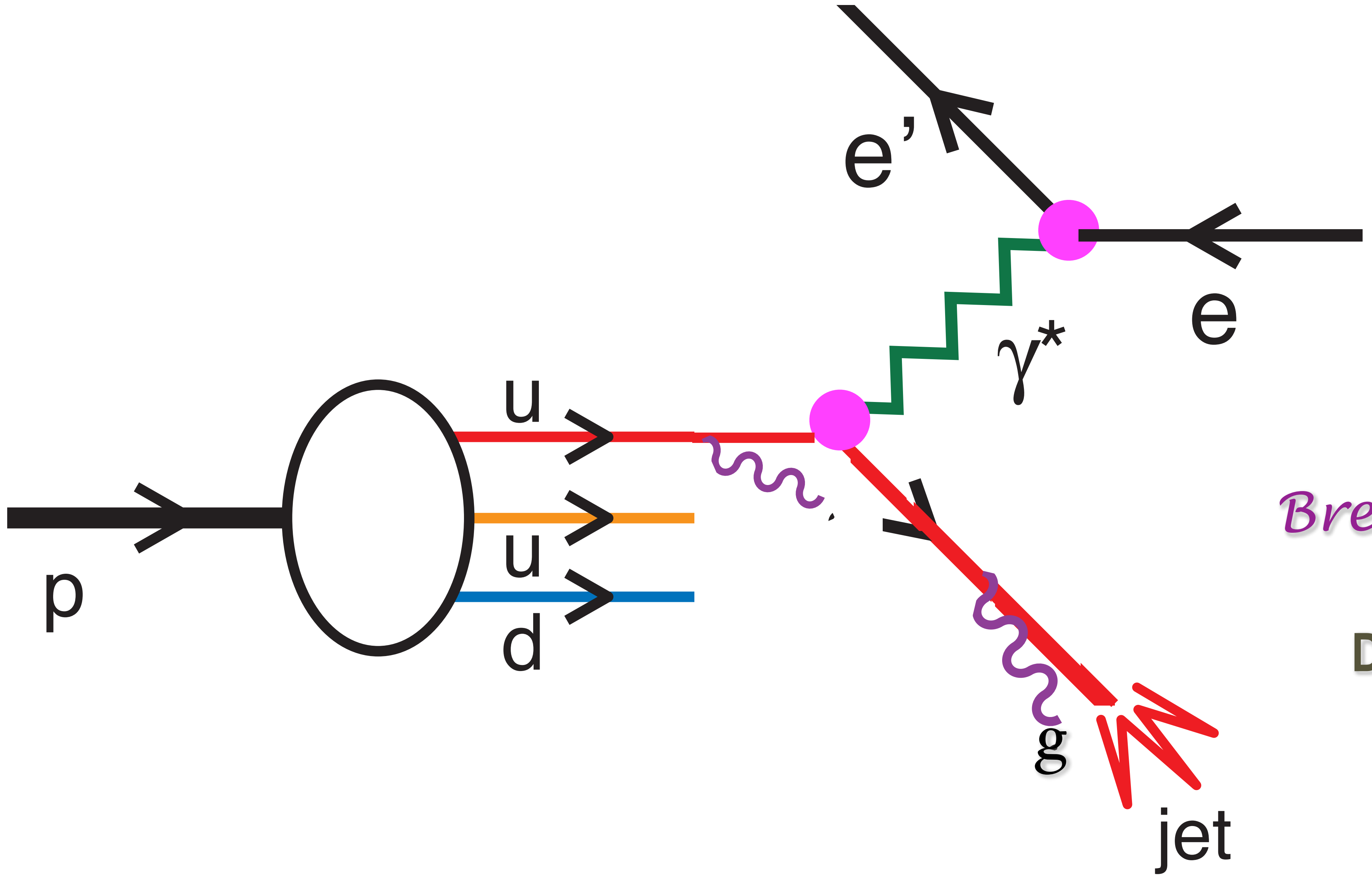
Scaling at fixed $x_{Bjorken} = \frac{Q^2}{2M_p\nu} = \frac{1}{\omega}$

$\omega = 4 \to x_{bj} = 0.25$ (quark momentum fraction)

Discovery of Bjorken Scaling:
Electron scatters on point-like quarks!

$Q^4 \times \frac{d\sigma}{dQ^2} = F(x_{Bj})$ independent of Q^2 *Scale-free*

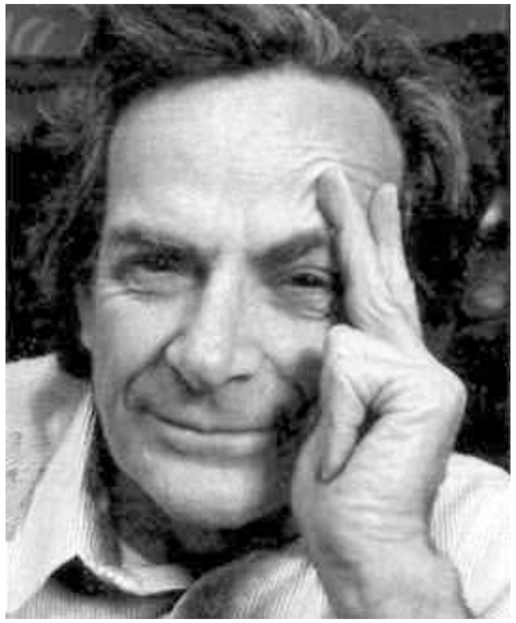
Deep Inelastic Electron-Proton Scattering



First Evidence for Quark Structure of Matter

But why do hadrons - not quarks - appear in the final state ?
Why are quarks confined within hadrons?

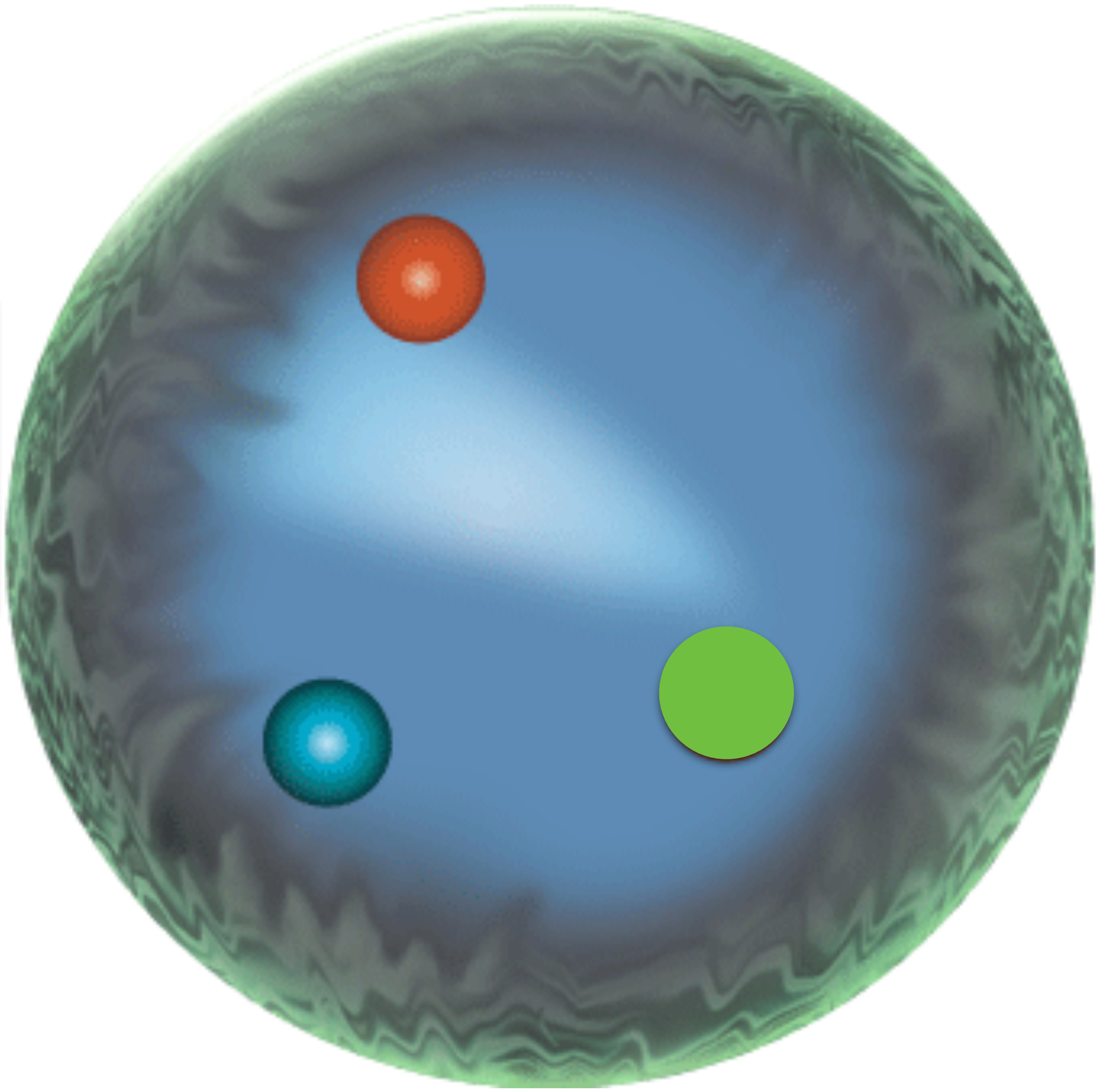
Quarks in the Proton



Feynman & Bjorken: “Parton” model

Bjorken: Scaling

p = (u u d)



$1\,fm$

$10^{-15}m = 10^{-13}cm$

Zweig: “Aces, Deuces, Treys”

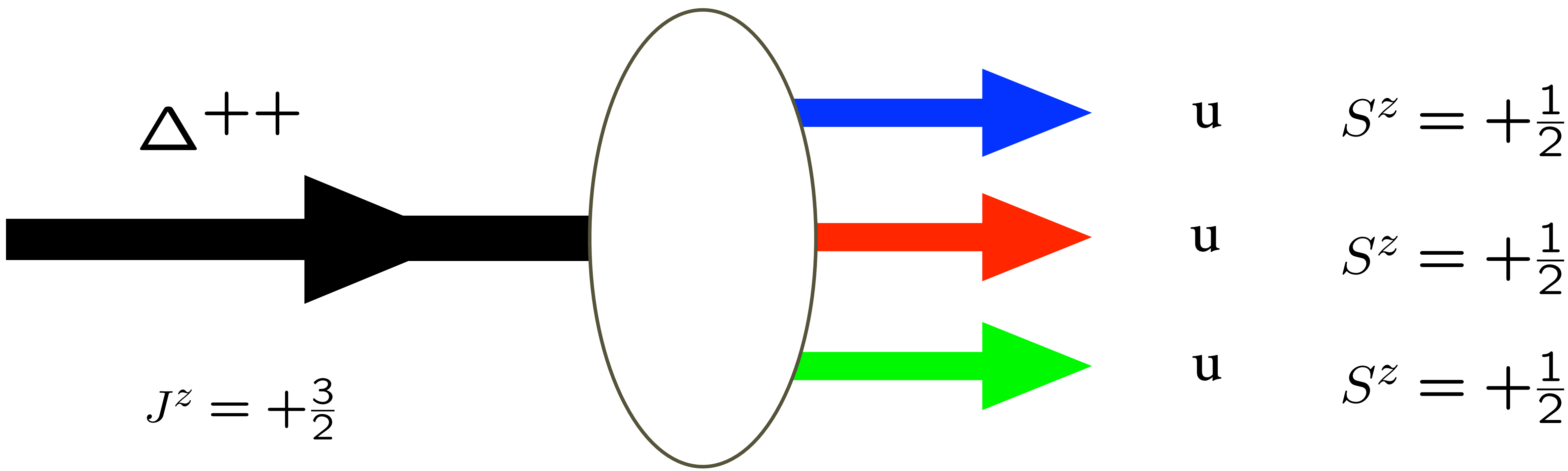
Gell Mann: “Three Quarks for Mr. Mark”

Why are there three colors of quarks?

Greenberg

Pauli Exclusion Principle!

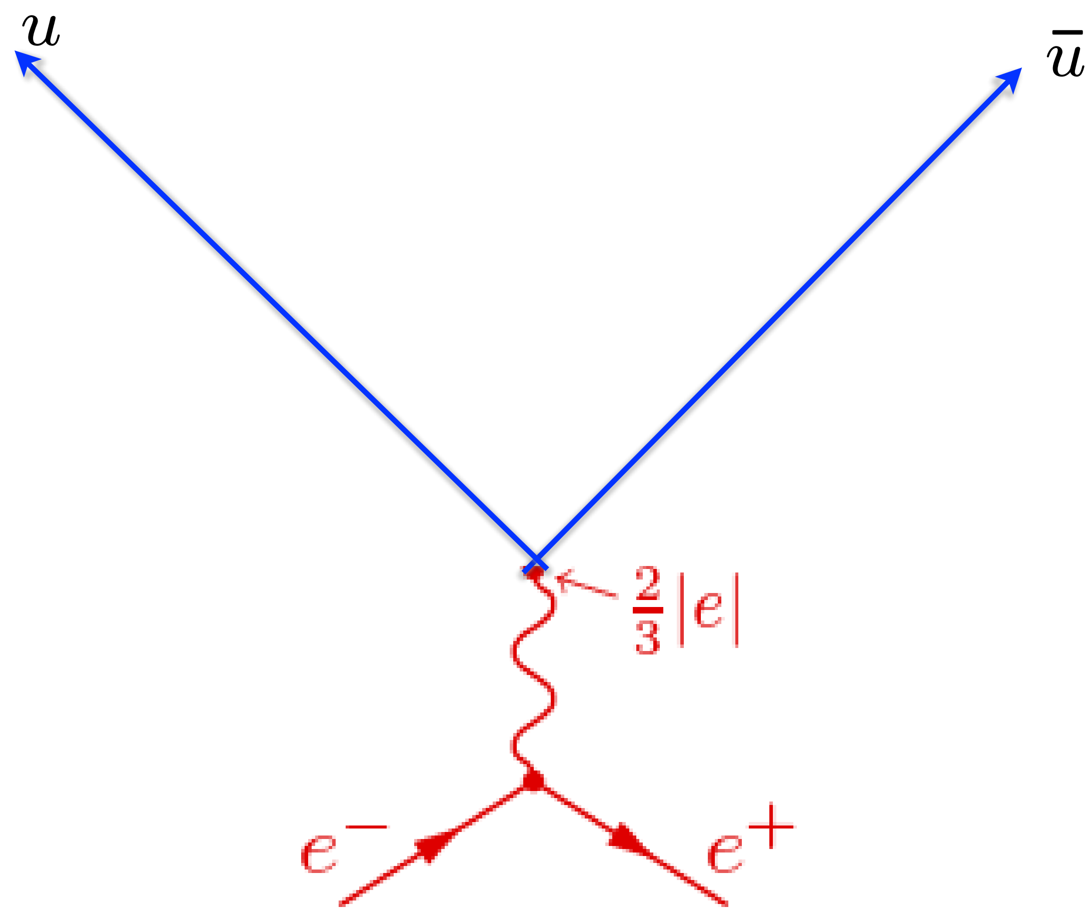
spin-half quarks cannot be in same quantum state !



Three Colors (Parastatistics) Solves Paradox

3 Colors Combine : WHITE $SU(N_C), N_C = 3$

How to Count Quarks



Color-triplet quark representation

For $10\ \text{GeV} < E_{\text{cm}} < 40\ \text{GeV}$,

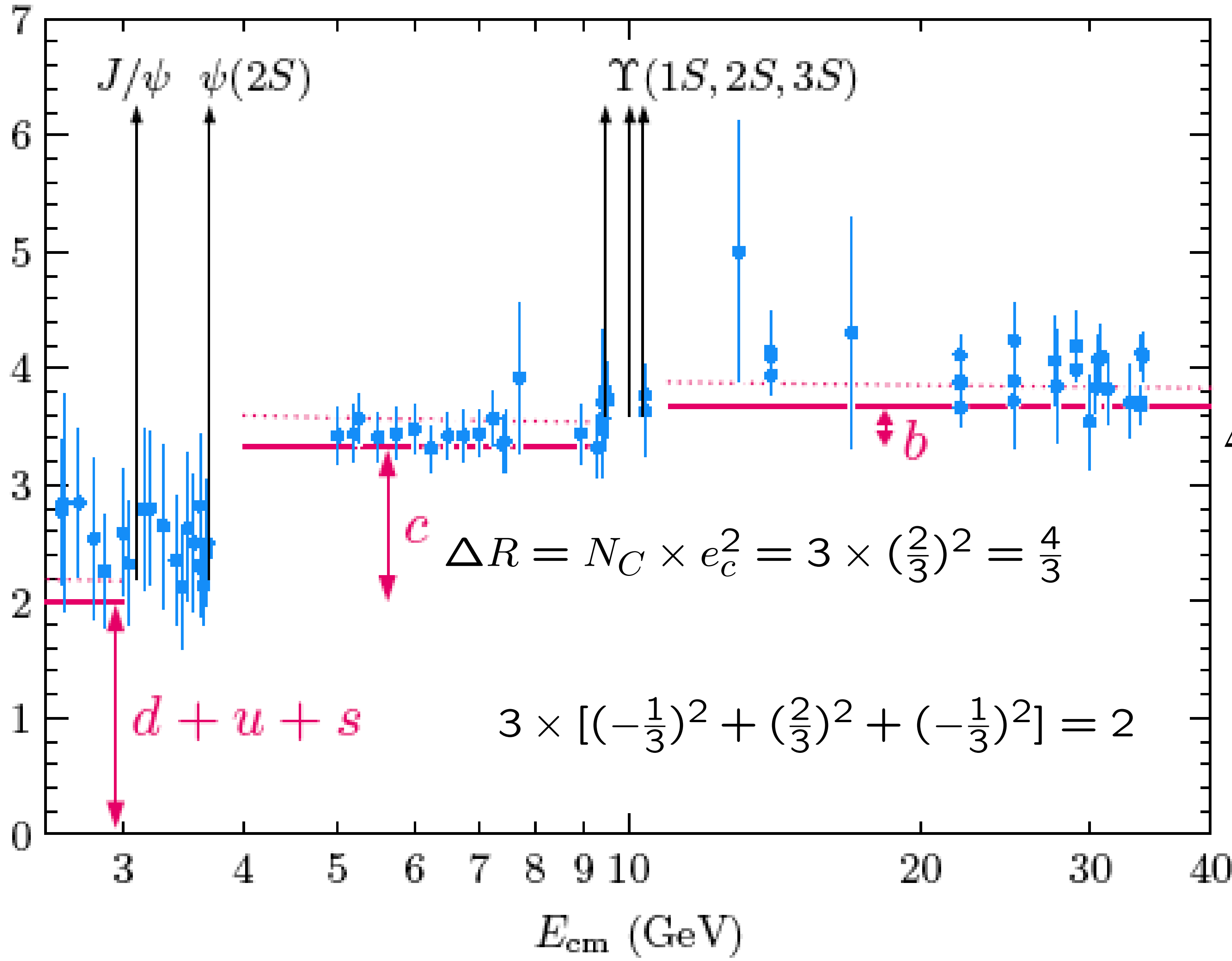
$$\frac{e^+e^- \to \text{hadrons}}{e^+e^- \to \mu^+\mu^-} = \underset{\substack{\uparrow \\ \text{colors}}}{3} \times \left[\underset{\substack{\uparrow \\ d}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ u}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ s}}{\left(\frac{1}{3}\right)^2} + \underset{\substack{\uparrow \\ c}}{\left(\frac{2}{3}\right)^2} + \underset{\substack{\uparrow \\ b}}{\left(\frac{1}{3}\right)^2} \right]$$

$J/\psi = (c\bar{c})_{1S}$

How to Count Quarks

$\Upsilon = (b\bar{b})_{1S}$

$$R = \sigma(\text{hadrons})/\sigma(\mu^+\mu^-)$$



$\Delta R_b = N_C \times e_b^2 = 3 \times (-\frac{1}{3})^2 = \frac{1}{3}$

$$R_{e^+e^-}(E_{cm}) = N_{colors} \times \sum_q e_q^2$$

$N_C = 3$

Evidence for Quarks

- Scale-Invariant Electron-Proton Inelastic Scattering: $ep \to e'X$
- Electron scatters on point-like constituents with fractional charge; final-state jets
- Electron-Positron Annihilation: $e^+e^- \to X$
  Production of point-like pairs with fractional charges
- 3 colors; quark, antiquark, gluon jets $\frac{dn_H}{dy}_g / \frac{dn_H}{dy}_q = \frac{C_A}{C_F} = 9/4$
- Exclusive hard scattering reactions: $pp \to pp,\ \gamma p \to \pi^+ n,\ ep \to ep$
- Probability that a hadron stays intact counts number of its point-like constituents:

Quark Counting Rules $F_H(t) \propto [\frac{1}{Q^2}]^{n-1}$ $\frac{d\sigma}{dt} = \frac{F(\theta_{CM})}{s^{n-2}}$

Quark interchange describes angular distributions

Farrar and sjb; Matveev et al; Lepage, sjb; Blankenbecler, Gunion, sjb; Sterman

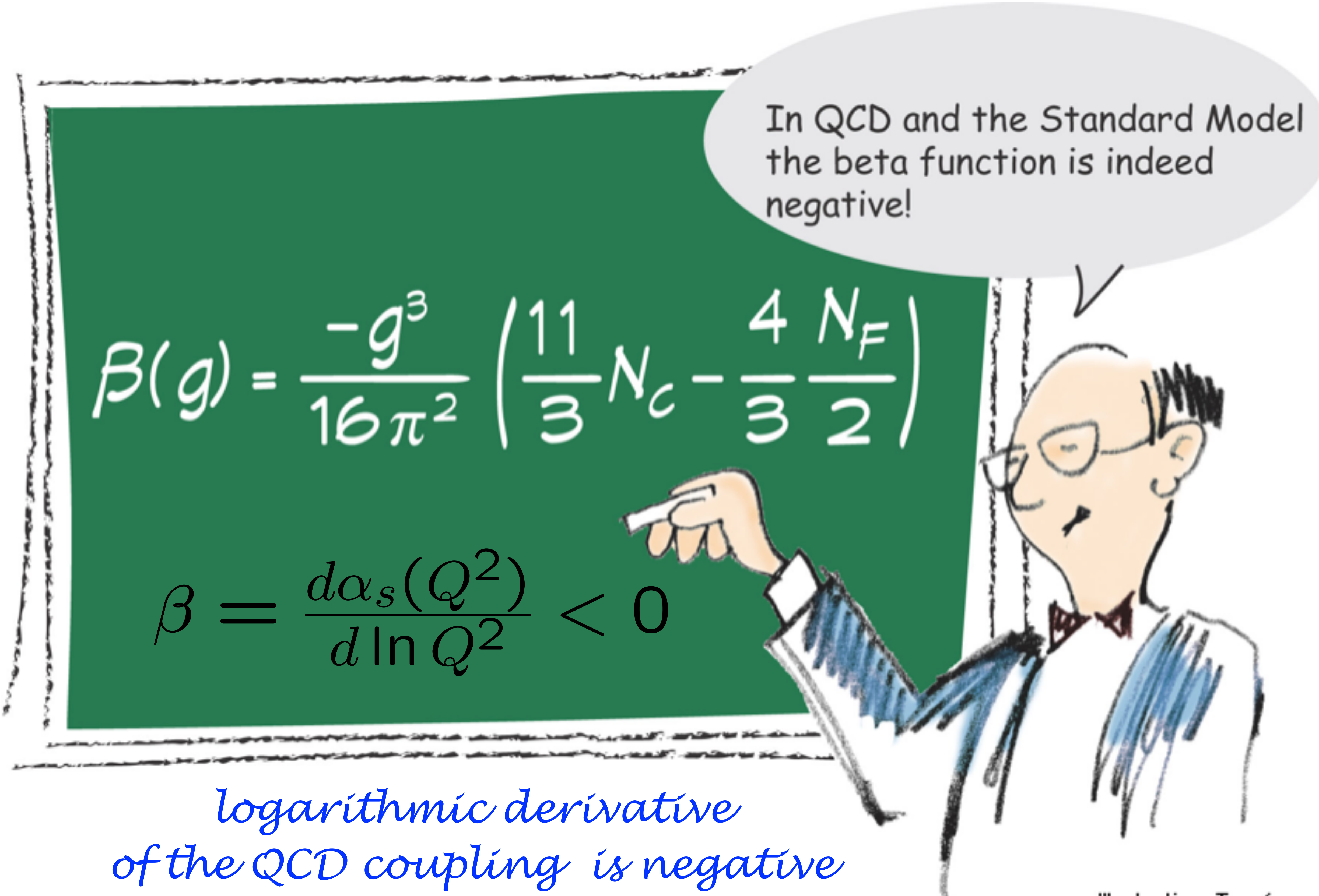


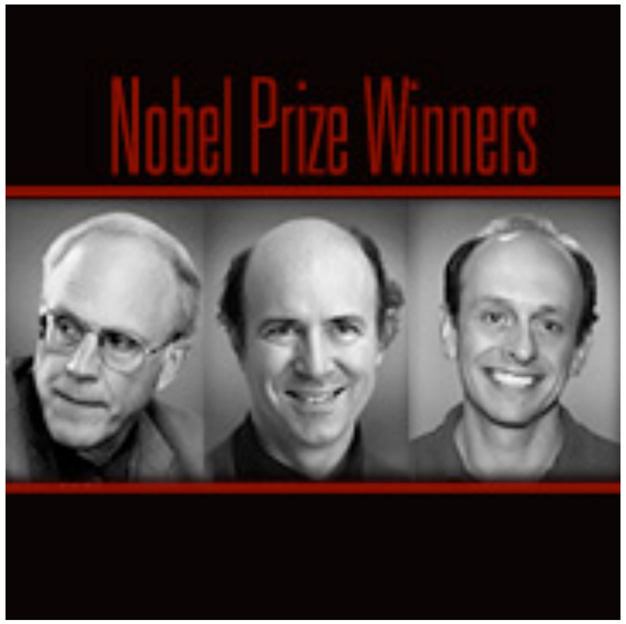
Illustration: Typoform

logarithmic derivative
of the QCD coupling is negative

Coupling becomes weaker at short distances = high momentum transfer

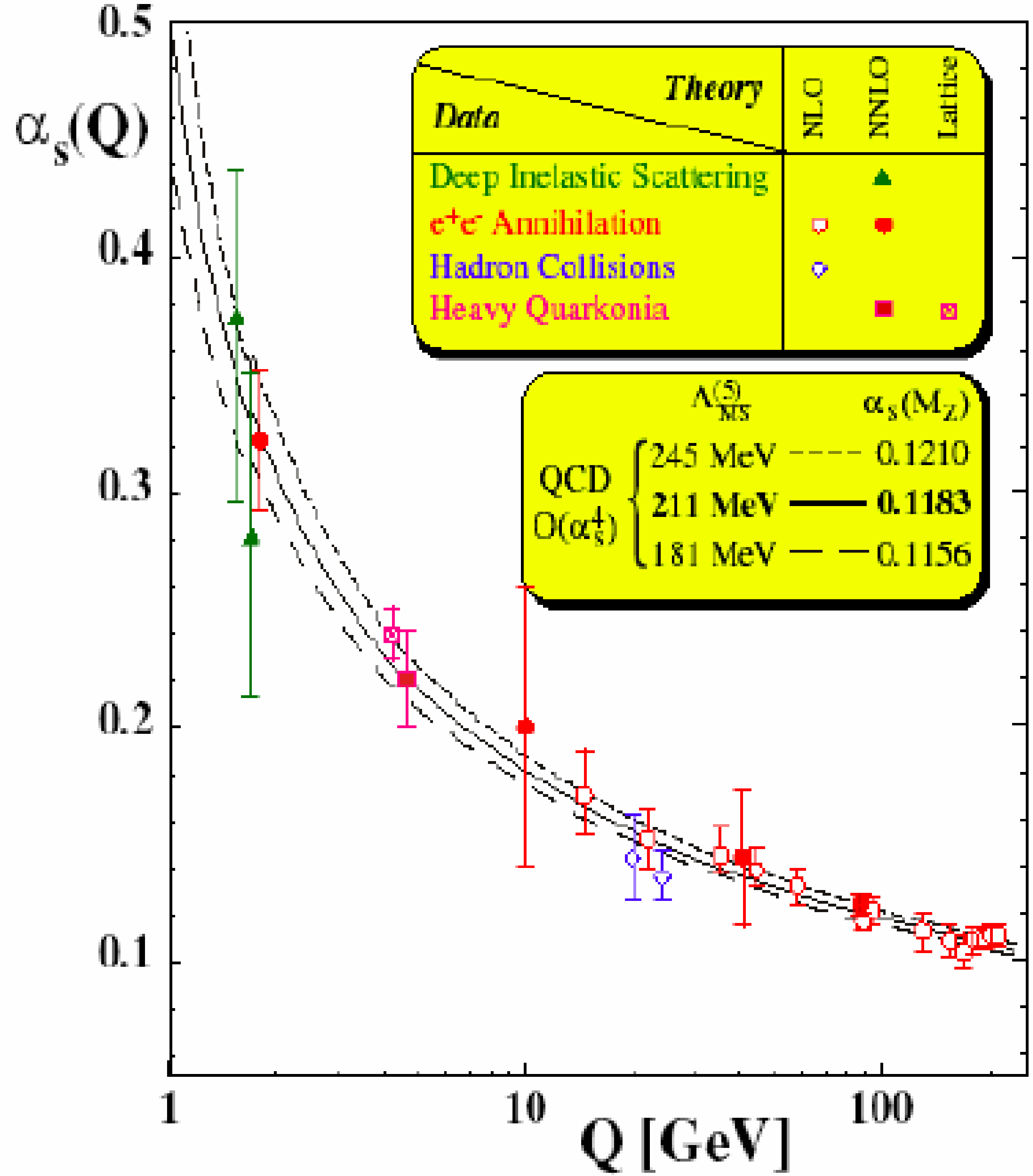
Verification of Asymptotic Freedom

$$\alpha_s(Q) \propto \frac{1}{\ln Q}$$



$$\frac{\sigma(e^+e^- \to \text{three jets})}{\sigma(e^+e^- \to \text{two jets})}$$

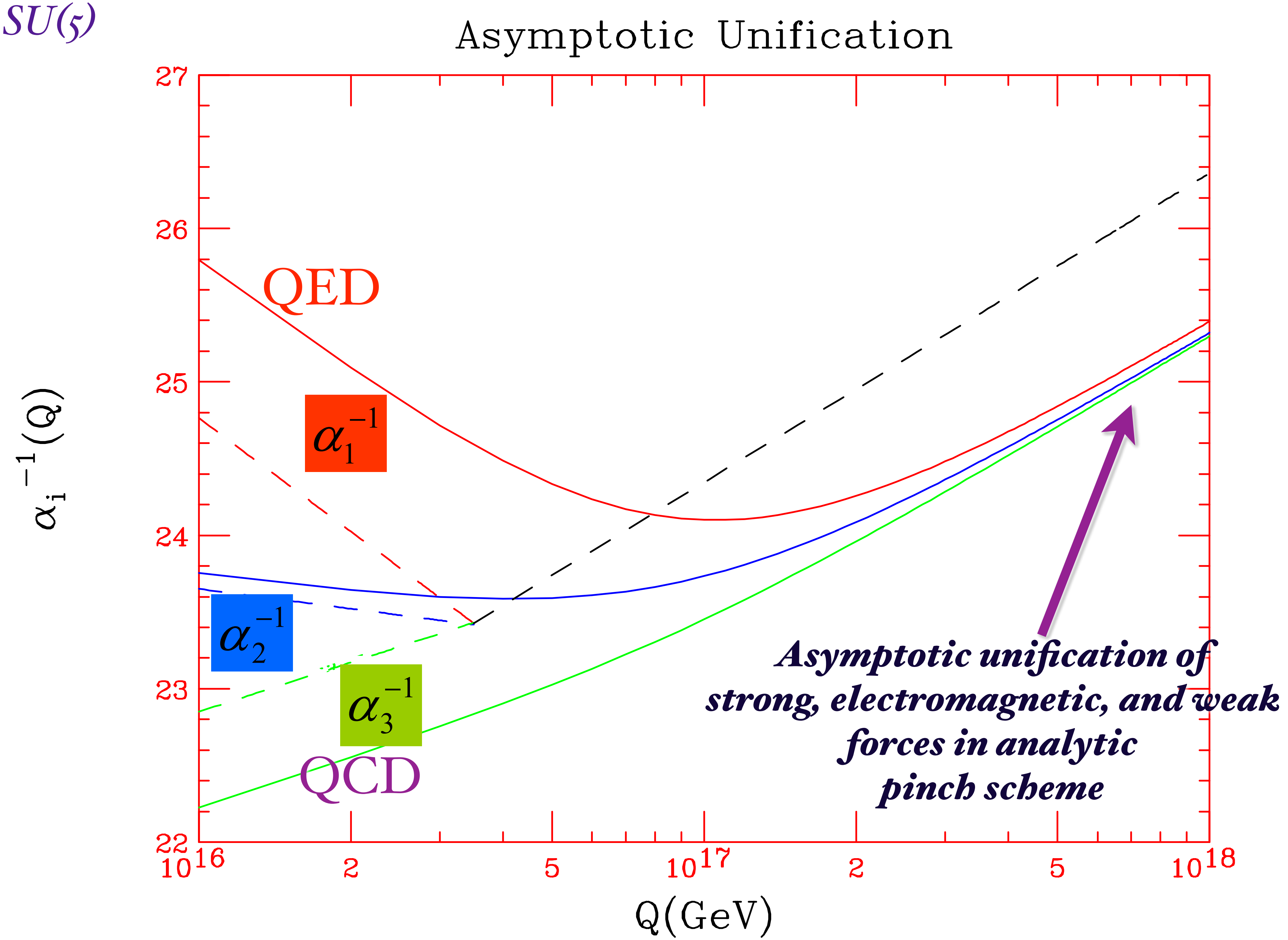
proportional to $\alpha_s(Q)$



Ratio of rate for $e^+e^- \to q\bar{q}g$ to $e^+e^- \to q\bar{q}$ at $Q = E_{CM} = E_{e^-} + E_{e^+}$

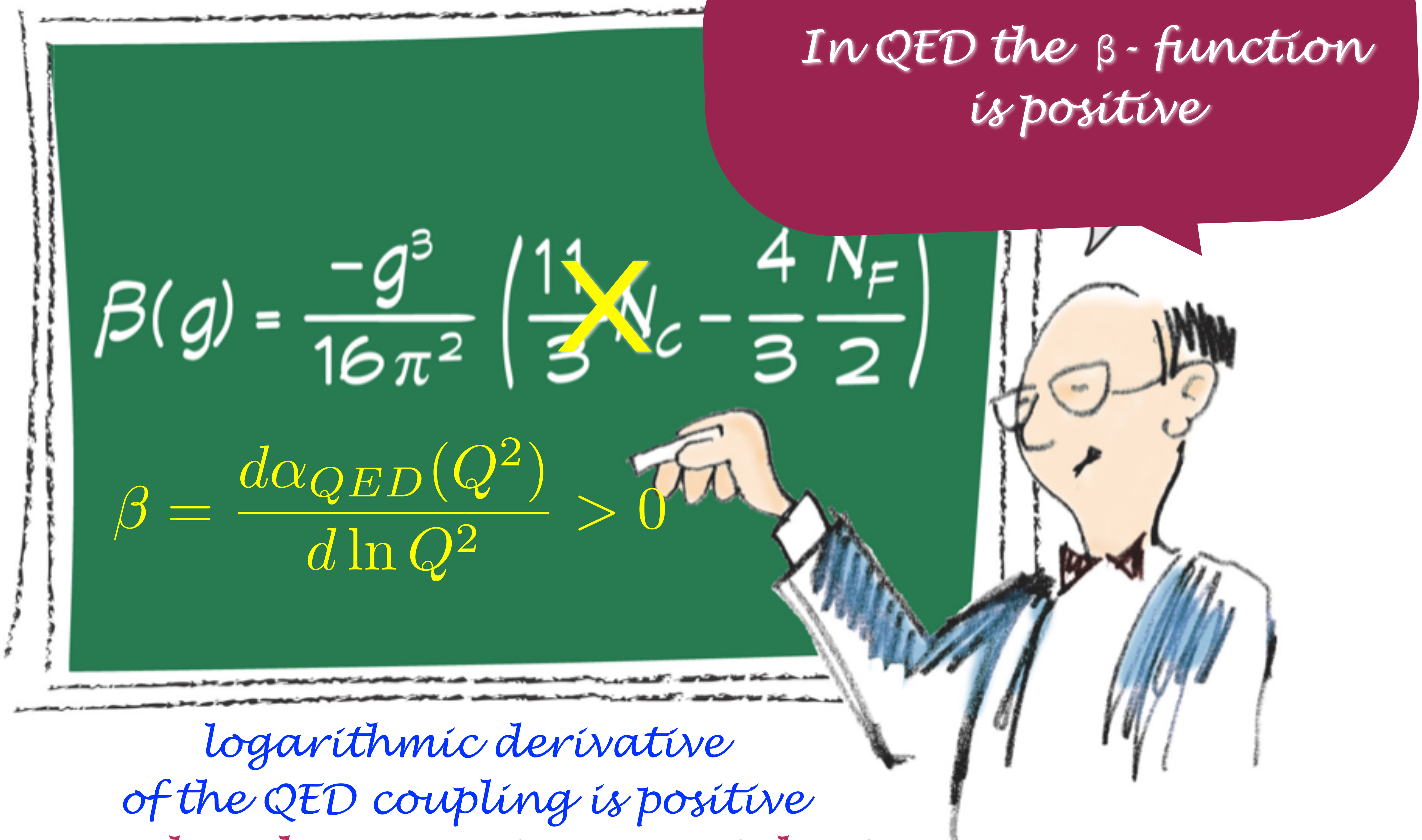
Supersymmetric SU(5)

Binger, sjb



Must Use Same Scale Setting Procedure! BLM/PMC

$QCD[(SU(N_C)] \rightarrow QED \; when \; N_C \rightarrow 0$



logarithmic derivative
of the QED coupling is positive
Coupling becomes stronger at short
distances = high momentum transfer

Landau Pole!

$$C_F = \frac{N_C^2 - 1}{2N_C}$$

P. Huet, sjb

$$\lim N_C \to 0 \text{ at fixed } \alpha = C_F \alpha_s, n_\ell = n_F / C_F$$

QCD $\to$ Abelian Gauge Theory

Analytic Feature of SU(Nc) Gauge Theory

All analyses for Quantum Chromodynamics must be applicable to Quantum Electrodynamics

Must Use Same Scale Setting Procedure! BLM/PMC

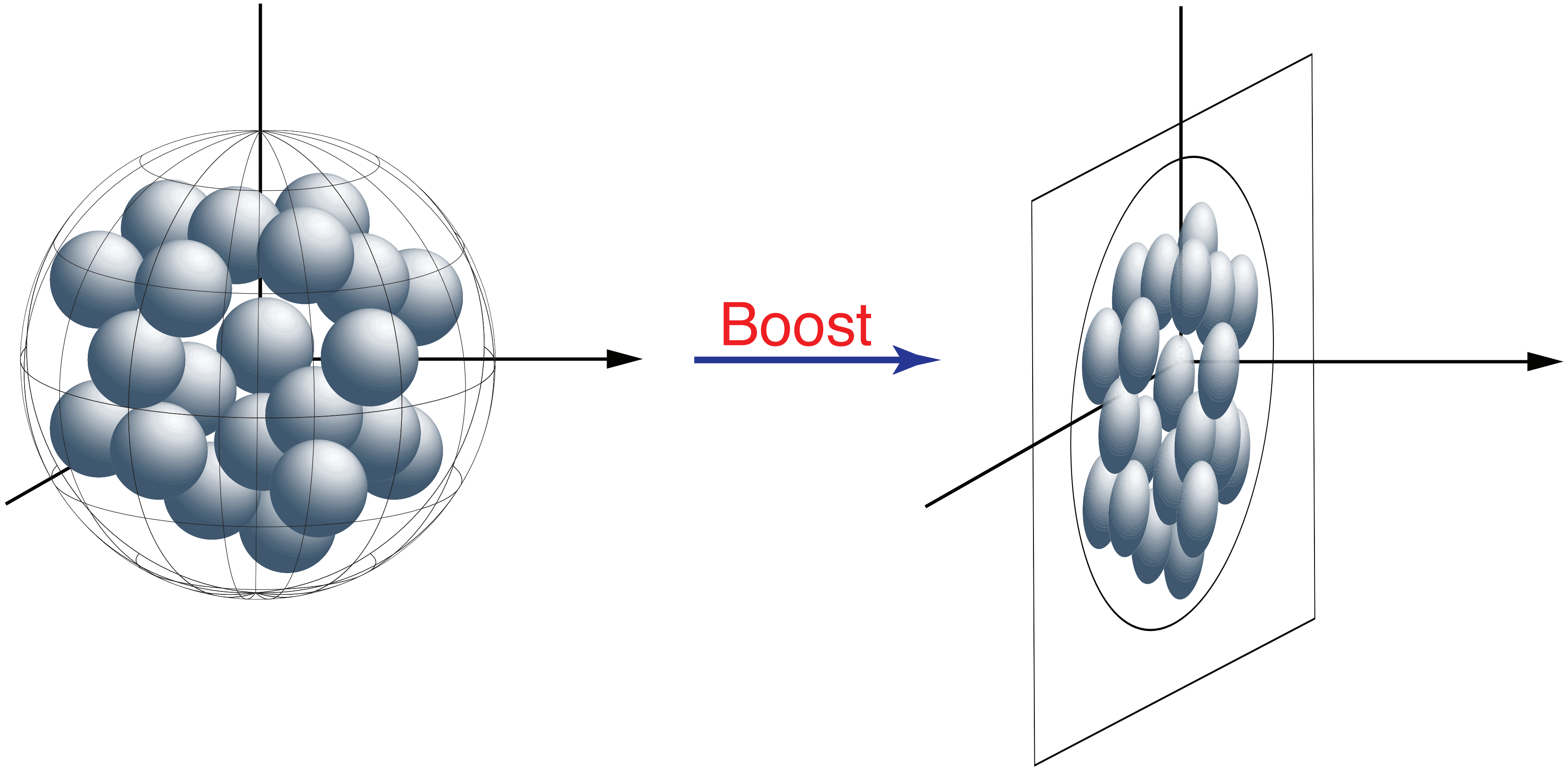
Advantages of the Dirac's Front Form for Hadron Physics
Lorentz Invariant

Physics Independent of Observer's Motion

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs

Terrell, Penrose

- LFWFs are frame-independent: no boosts, no pancakes!
- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum simple -- no vacuum condensates!
- Profound implications for Cosmological Constant



A large nucleus before and after an ultra-relativistic boost.

Is this really true? Will an electron-proton collider see different results than a fixed target experiment such as SLAC because the nucleus is squashed to a 'pancake'?

Violates Lorentz invariance

No length contraction — no pancakes!

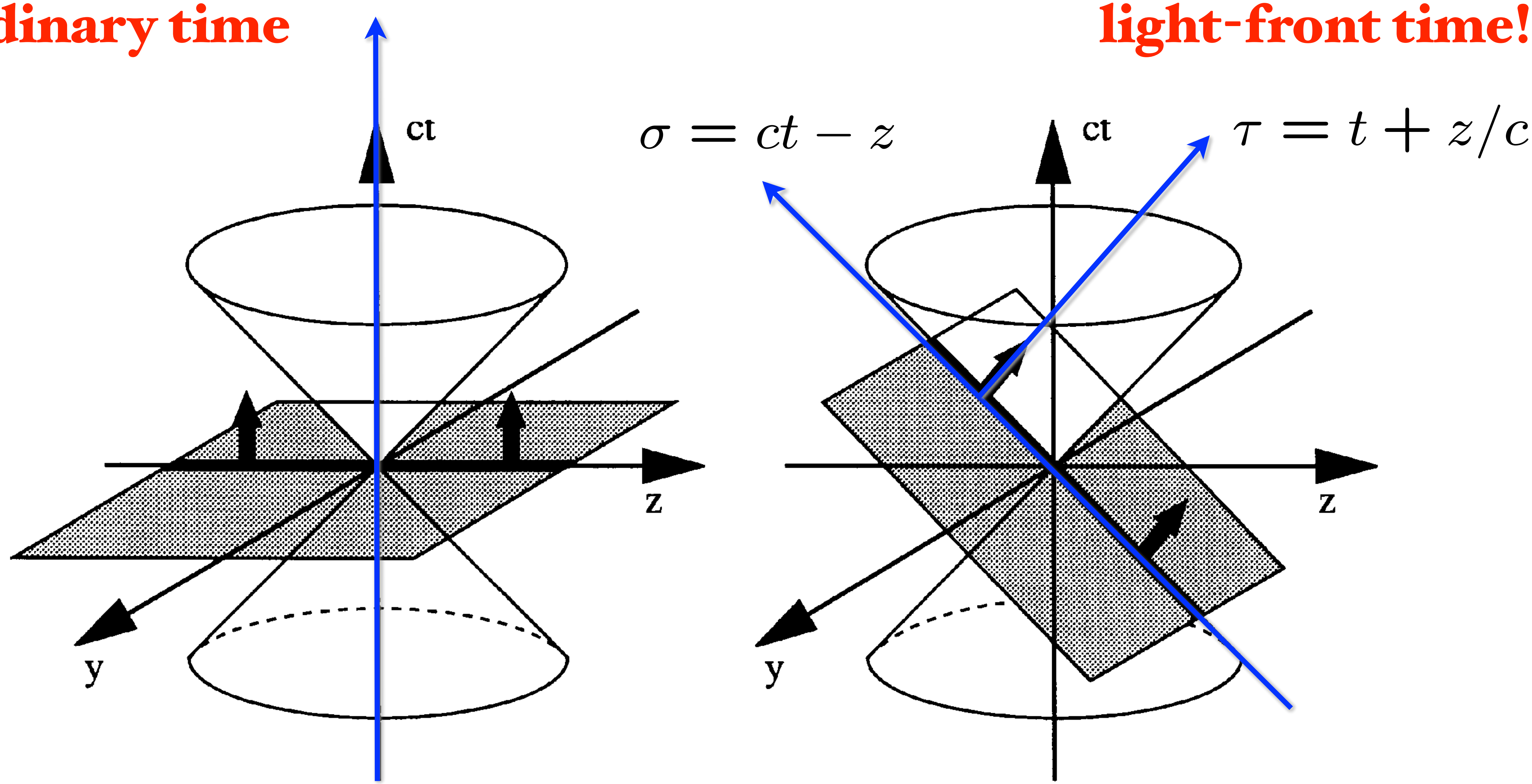
Penrose
Terrell
Weiskopf

We do not observe the nucleus at one time t!

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)

Evolve in ordinary time

Evolve in light-front time!



Instant Form **Front Form**

Light-Front Time

Each element of flash photograph illuminated at same LF time

$$\tau = t + z/c$$

Causal, frame-independent

$$P^{\pm} = P^0 \pm P^z$$

Evolve in LF time

$$P^- = i\frac{d}{d\tau}$$

Eigenstate -- independent of τ

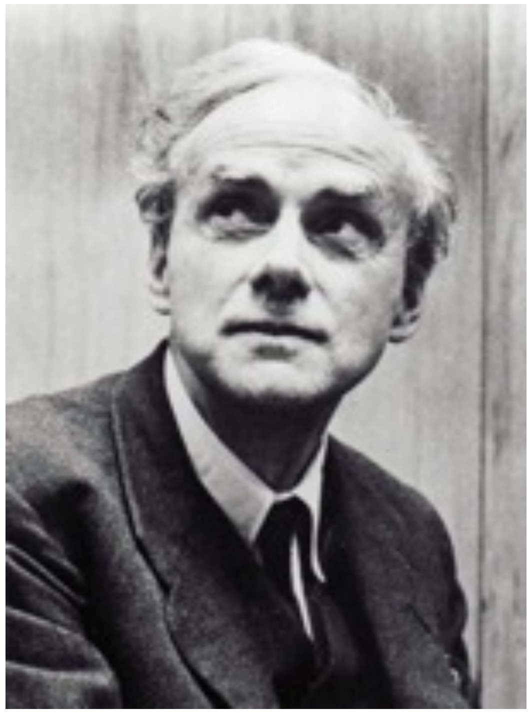
$$\text{Eigenvalue } P^- = \frac{\mathcal{M}^2 + \vec{P}_\perp^2}{P^+}$$

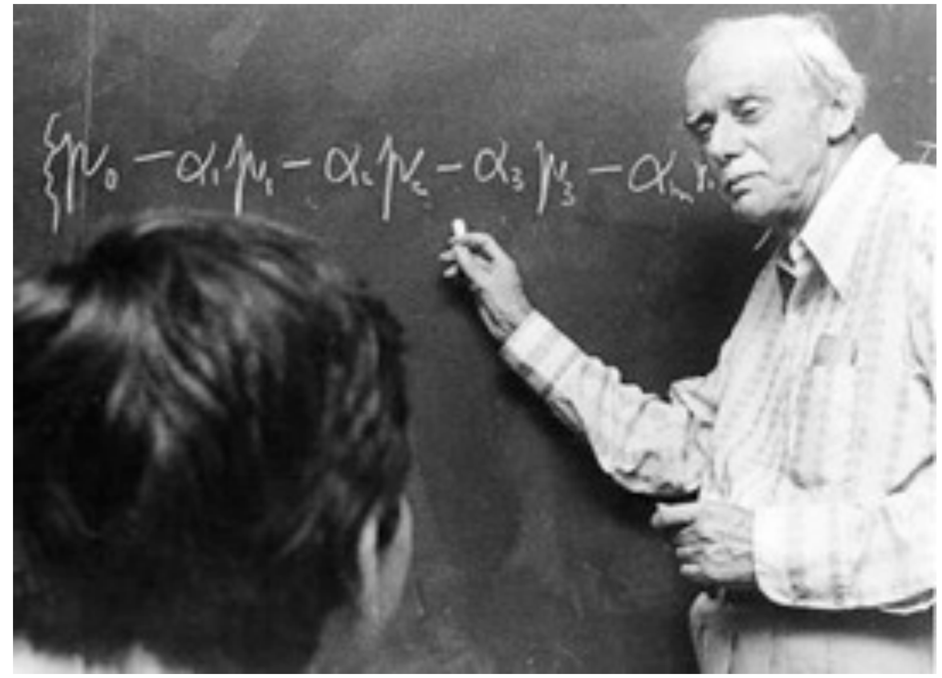
$$H_{LF} = P^+P^- - \vec{P}_\perp^2$$

$$H_{LF}^{QCD}|\Psi_h> = \mathcal{M}_h^2|\Psi_h>$$

HELEN BRADLEY - PHOTOGRAPHY

P.A.M Dirac, Rev. Mod. Phys. 21, 392 (1949)





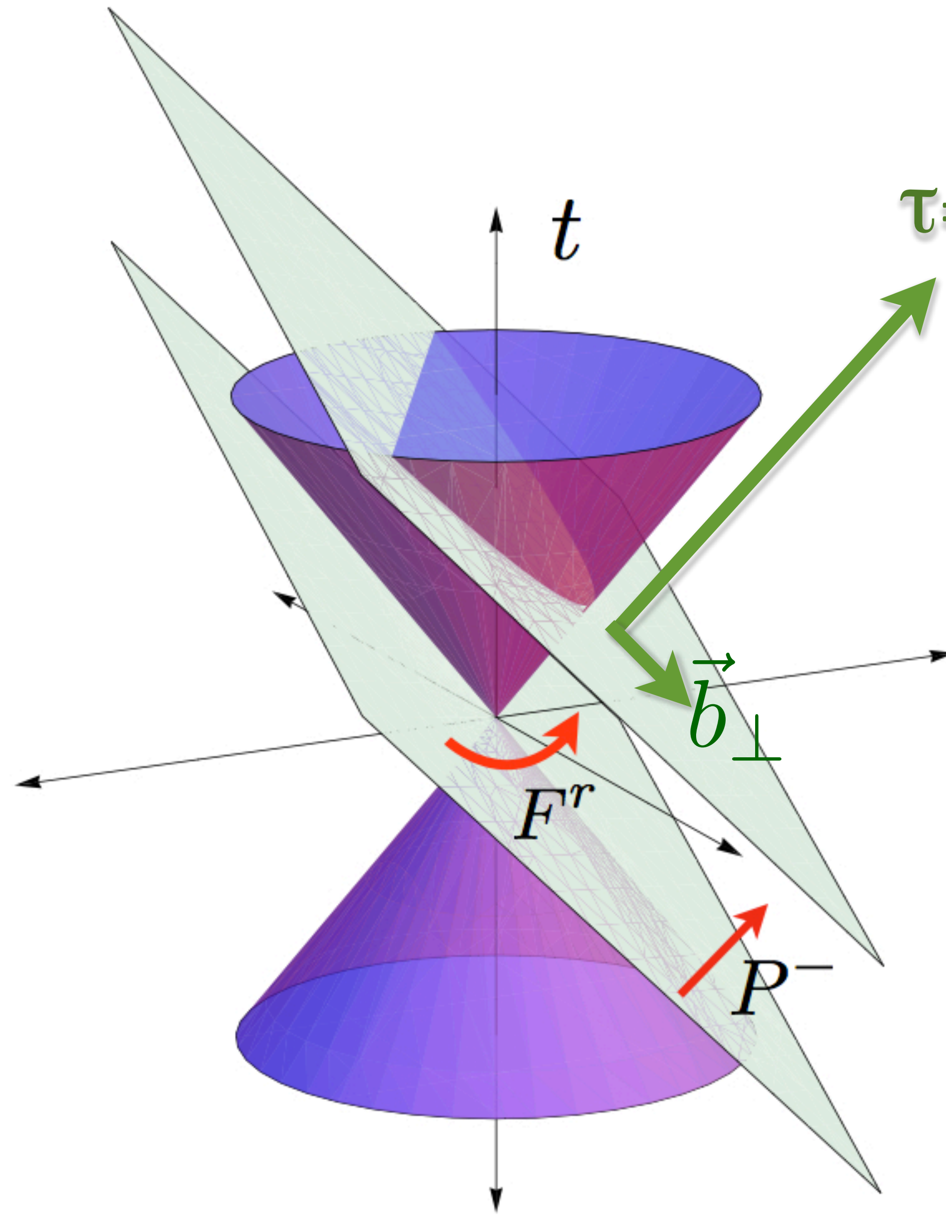
"Working with a front is a process that is unfamiliar to physicists.

But still I feel that the mathematical simplification that it introduces is all-important.

I consider the method to be promising and have recently been making an extensive study of it.

It offers new opportunities, while the familiar instant form seems to be played out " - P.A.M. Dirac (1977)

Silas R. Beane



$$\zeta_\perp^2 = b_\perp^2 x(1-x)$$

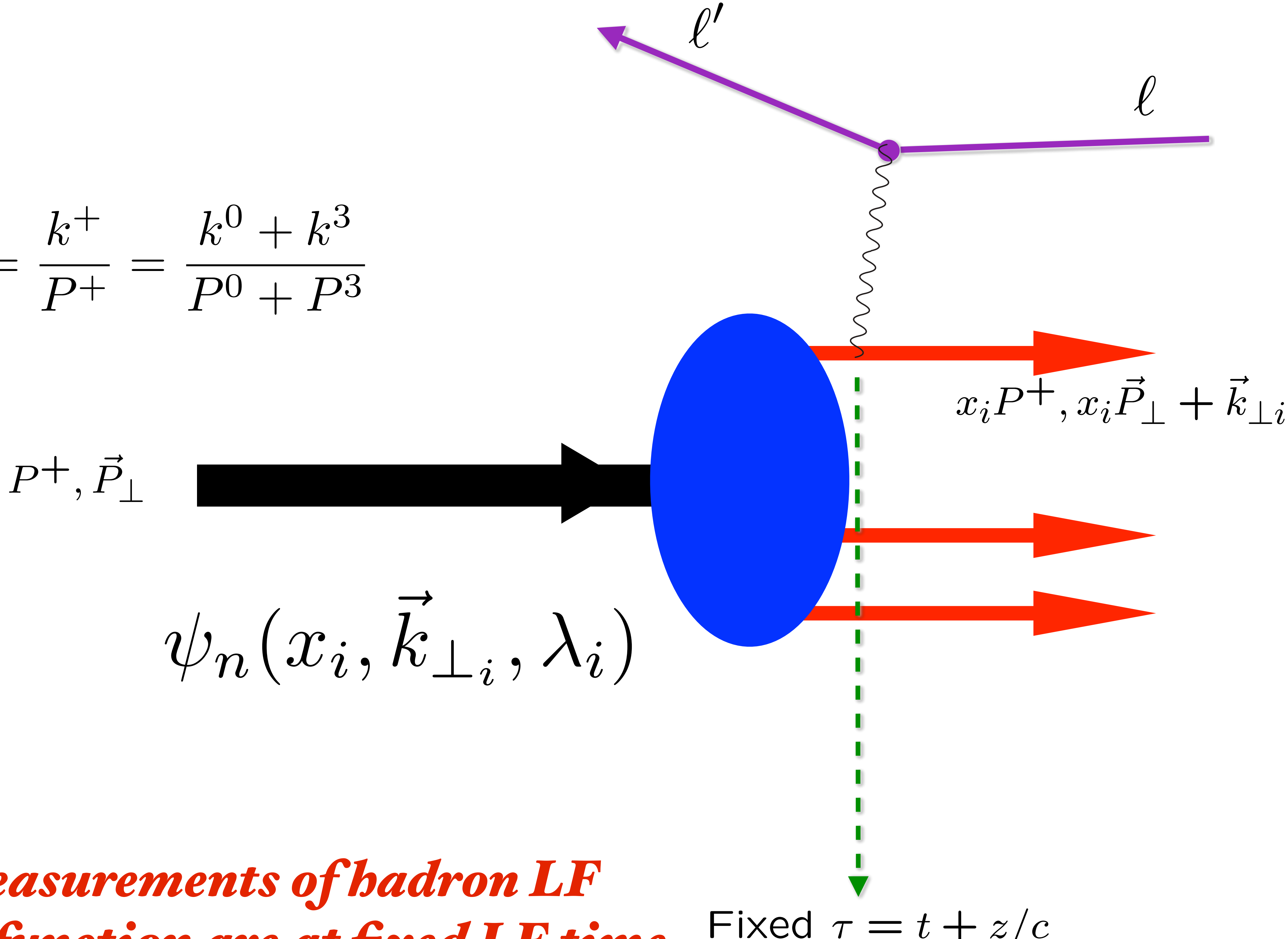
$$-\frac{d^2}{d\zeta_\perp^2} = \frac{k_\perp^2}{x(1-x)}$$

Null plane: a surface tangent to the light cone.

The null-plane Hamiltonians map the initial light-like surface onto some other surface, and therefore describe the dynamical evolution of the system.

The energy P^- translates the system in the null-plane time coordinate x^+, whereas the spin Hamiltonians F_r rotate the initial surface about the surface of the light cone.

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



Measurements of hadron LF wavefunction are at fixed LF time

$$x_{bj} = x = \frac{k^+}{P^+}$$

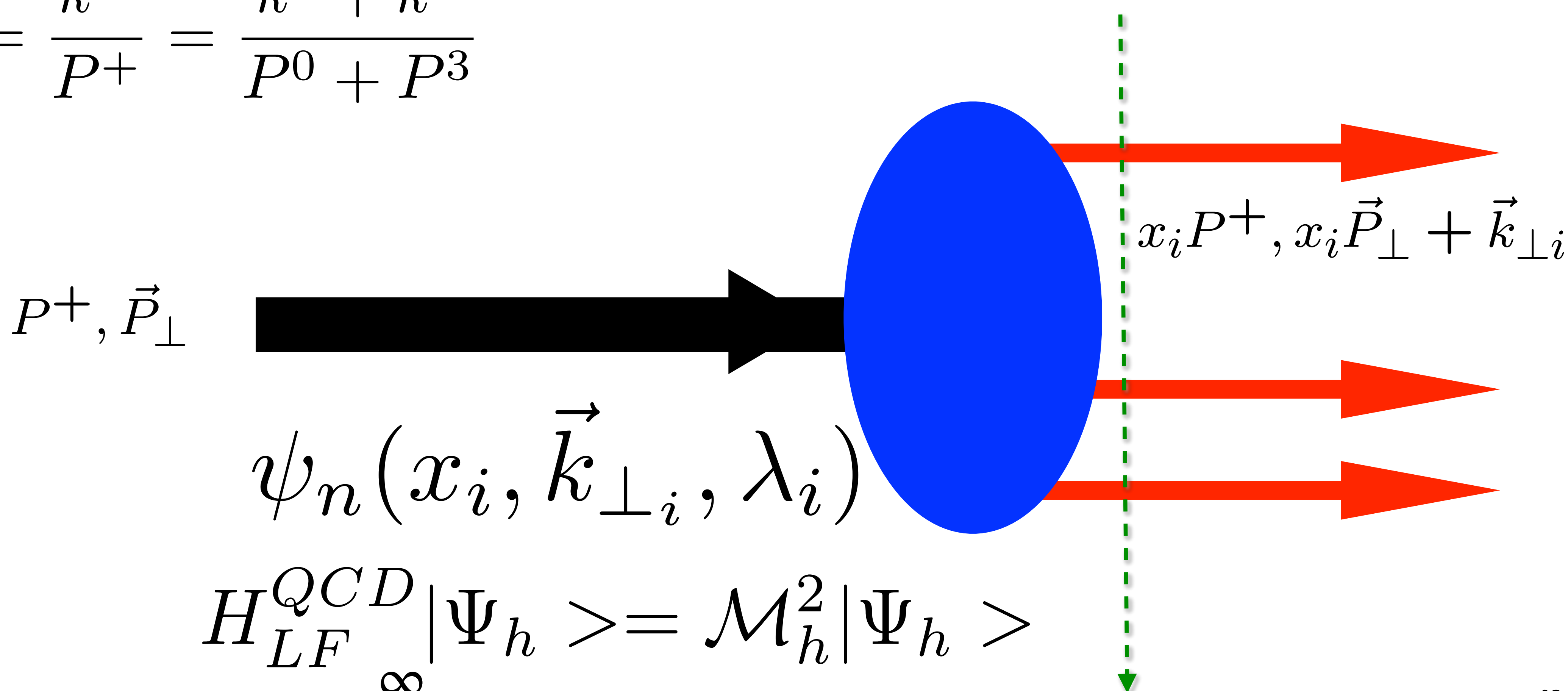
Like a flash photograph

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Eigenstate of LF Hamiltonian

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$



$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|\Psi_h> = \sum_{n=3}^{\infty} \psi_n(x_i, \overrightarrow{k}_{\perp i}, \lambda_i) |n; \overrightarrow{k}_{\perp i}, \lambda_i>$$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

- LF coordinates

$\tau = x^+ = x^0 + x^3$	light-front time	$P^+ = P^0 + P^3$	longitudinal momentum
$x^- = x^0 - x^3$	longitudinal space variable	$P^- = P^0 - P^3$	light-front Hamiltonian
$\mathbf{x}_\perp = (x^1, x^2)$	transverse space variable	$\mathbf{P}_\perp = (P^1, P^2)$	transverse momentum

- On shell relation $P_\mu P^\mu = P^- P^+ - \mathbf{P}_\perp^2 = \mathcal{M}^2$ leads to dispersion relation for LF Hamilnotian P^-

$$P^- = \frac{\mathbf{P}_\perp^2 + M^2}{P^+}, \quad P^+ > 0$$

- Hamiltonian equation for the relativistic bound state

$$i \frac{\partial}{\partial x^+} |\psi(P)\rangle = P^- |\psi(P)\rangle = \frac{M^2 + \mathbf{P}_\perp^2}{P^+} |\psi(P)\rangle$$

where P^- is derived from the QCD Lagrangian: kinetic energy of partons plus confining interaction

- Construct LF Lorentz invariant Hamiltonian $P^2 = P^- P^+ - \mathbf{P}_\perp^2$

$$P_\mu P^\mu |\psi(P)\rangle = M^2 |\psi(P)\rangle$$

- LF quantization is the ideal framework to describe hadronic structure in terms of constituents: simple vacuum structure allows unambiguous definition of partonic content of a hadron

Features of the LF Wavefunction

$$\psi_H(x_i, k_{\perp i}, \lambda_i)$$

$$P^+ = P^0 + P^z,\ k^+ = k^0 + k^z,\ x_i = \frac{k_i^+}{P^+}$$

- Independent of hadron momentum $P^+, P_\perp$
- Boost, Lorentz, and Poincare invariant
- Momentum conservation $\sum_1^n k^+ = P^+,\ \sum_1^n x_i = 1,\ \sum_1^n k_{\perp i} = 0$
- Angular Momentum conservation $J^z = \sum S_i^z + \sum L_i^z$
- General form: $\psi_{q\bar{q}}(x, k_\perp) = F(\frac{k_\perp^2}{x(1-x)})$
- Underlies all hadron observables!

Goal: an analytic first approximation to QCD

- As Simple as Schrödinger Theory in Atomic Physics
- Relativistic, Frame-Independent, Color-Confining
- Confinement in QCD -- What sets the QCD mass scale?
- QCD Coupling at all scales
- Hadron Spectroscopy
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- Constituent Counting Rules
- Hadronization at the Amplitude Level
- Insights into the QCD vacuum
- Chiral Symmetry
- Systematically improvable

Advantages of the Dirac's Front Form for Hadron Physics

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent -- no boosts!
- No dependence on observer's frame
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no condensates!
- Profound implications for Cosmological Constant

QCD Lagrangian

gluon dynamics — quark kinetic energy + quark-gluon dynamics — mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

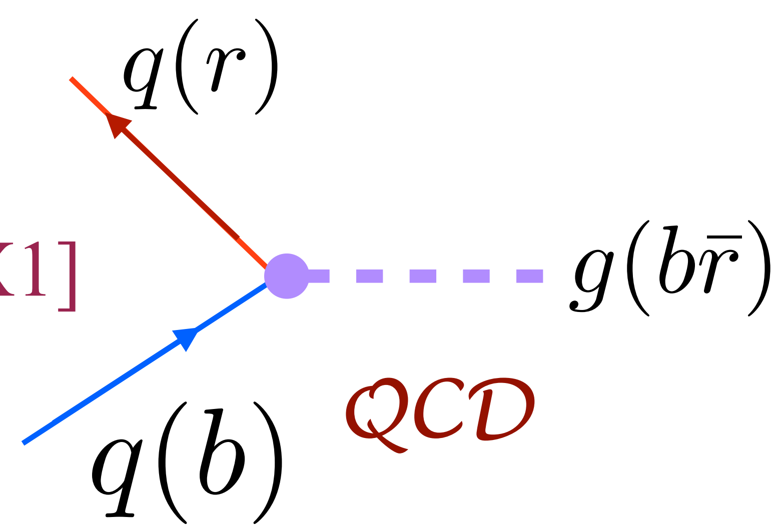
Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

Scale-Invariant Coupling
Renormalizable
Nearly-Conformal
Asymptotic Freedom
Color Confinement

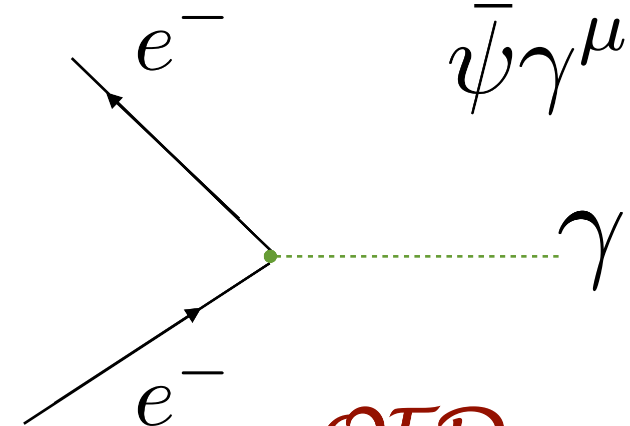
Fundamental Couplings of QCD and QED

$\bar{\psi}\gamma^{\mu} A^{\mu} \bar{\psi}$

[1X3] [3X3] [3X1]



QCD



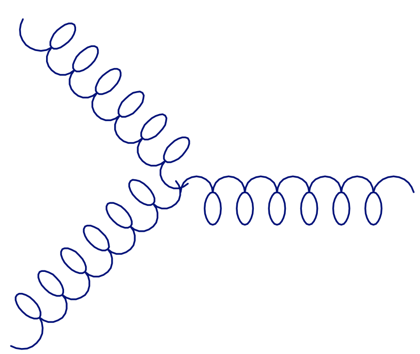
$\bar{\psi}\gamma^{\mu} A^{\mu} \bar{\psi}$

QED

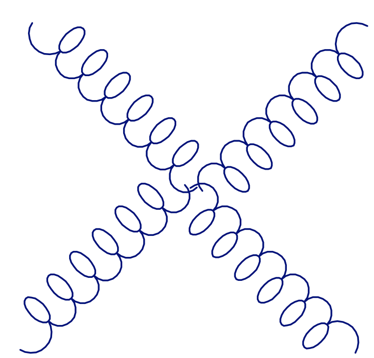
$$\mathcal{L}_{QCD} = -\frac{1}{4} Tr(G^{\mu\nu} G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_{\mu}\gamma^{\mu}\Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$G^{\mu\nu} = \partial^{\mu} A^{\mu} - \partial^{\nu} A^{\mu} - g[A^{\mu}, A^{\nu}]$$

Gluon vertices



QCD



$G^{\mu\nu} G_{\mu\nu}$

gluon self-couplings

QCD Lagrangian

Fundamental Theory of Hadron and Nuclear Physics

gluon dynamics

quark kinetic energy + quark-gluon dynamics

quark mass term

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \quad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

Classically Conformal if m_q=0

Yang Mills Gauge Principle: Color Rotation and Phase Invariance at Every Point of Space and Time

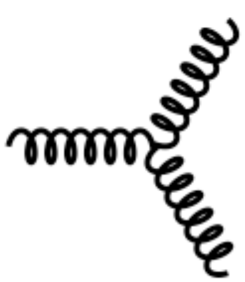
Scale-Invariant Coupling
Renormalizable
Asymptotic Freedom
Color Confinement

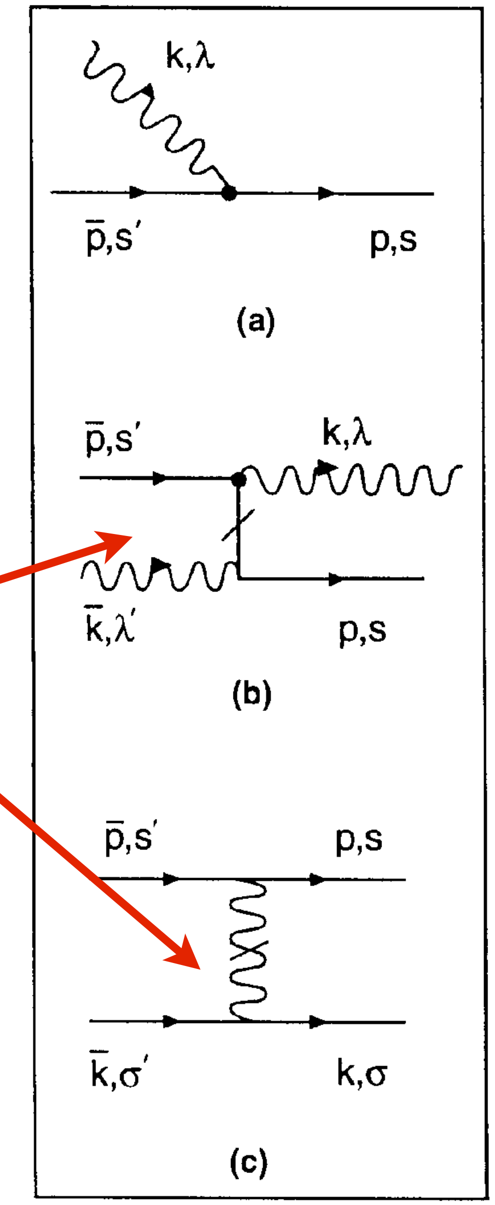
QCD Mass Scale from Confinement not Explicit

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

Physical gauge: $A^+ = 0$

$$\begin{aligned} H^{LF}_{QCD} = & \frac{1}{2}\int d^3x \bar{\tilde{\psi}}\gamma^+ \frac{(\mathrm{i}\partial^\perp)^2 + m^2}{\mathrm{i}\partial^+}\tilde{\psi} - A^i_a(\mathrm{i}\partial^\perp)^2 A_{ia} \\ & - \frac{1}{2}g^2 \int d^3x \mathbf{Tr}\left[\tilde{A}^\mu, \tilde{A}^\nu\right]\left[\tilde{A}_\mu, \tilde{A}_\nu\right] \\ & + \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \frac{1}{(\mathrm{i}\partial^+)^2} \bar{\tilde{\psi}}\gamma^+ T^a \tilde{\psi} \\ & - g^2 \int d^3x \bar{\tilde{\psi}}\gamma^+ \left(\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right)\tilde{\psi} \\ & + g^2 \int d^3x \mathbf{Tr}\left(\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\frac{1}{(\mathrm{i}\partial^+)^2}\left[\mathrm{i}\partial^+ \tilde{A}^\kappa, \tilde{A}_\kappa\right]\right) \\ & + \frac{1}{2}g^2 \int d^3x \bar{\tilde{\psi}} \tilde{\not{A}} \frac{\gamma^+}{\mathrm{i}\partial^+} \tilde{\not{A}} \tilde{\psi} \\ & + g \int d^3x \bar{\tilde{\psi}} \tilde{\not{A}} \tilde{\psi} \\ & + 2g \int d^3x \mathbf{Tr}\left(\mathrm{i}\partial^\mu \tilde{A}^\nu \left[\tilde{A}_\mu, \tilde{A}_\nu\right]\right) \end{aligned}$$





Light-Front QCD

Physical gauge: $A^+ = 0$

Exact frame-independent formulation of nonperturbative QCD!

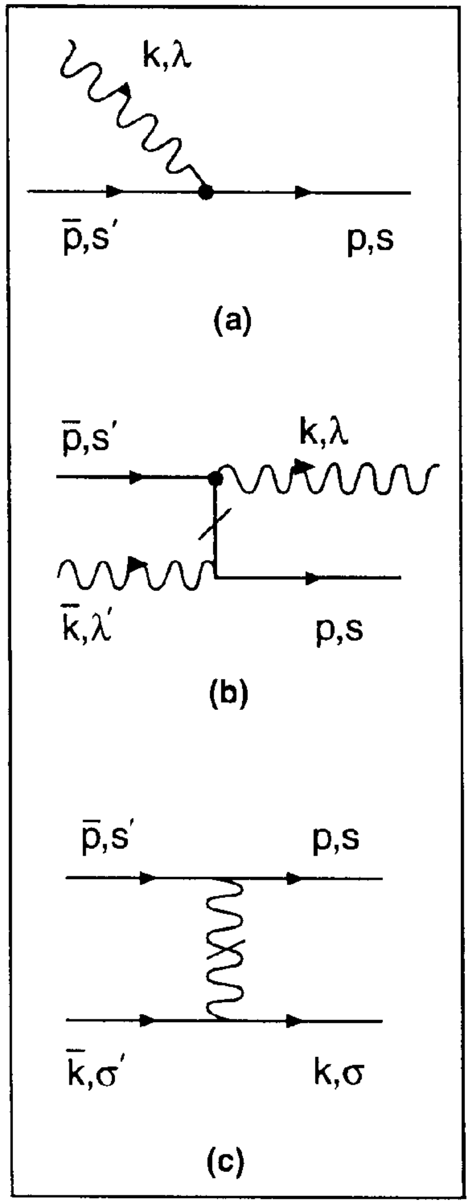
$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

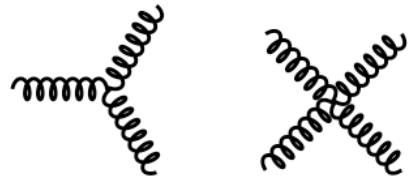
H^{int}_{LF}: Matrix in Fock Space

$$H^{QCD}_{LF}|\Psi_h> = \mathcal{M}^2_h|\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; x_i, \vec{k}_{\perp i}, \lambda_i>$$



Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions



H^{int}_{LF}

LFWFs: Off-shell in P- and invariant mass

Light-Front Hamiltonian Perturbation Theory

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{\rm initial} - \mathcal{M}^2_{\rm intermediate} + i\epsilon} H_I + \cdots$$

$$< i|T|j > = < i|H_I|j > + \sum_n < i|H_I|n > \frac{1}{\mathcal{M}^2_{\rm i} - \mathcal{M}^2_{\rm n} + i\epsilon} < n|H_I|j > + \cdots$$

All particles on their respective mass-shell

$k^2 = k^+ k^- - k^2_\perp = m^2$

All k^+ positive

Sum over all allowed intermediate states

$P^+, \vec{P}_\perp, J^z$ conserved

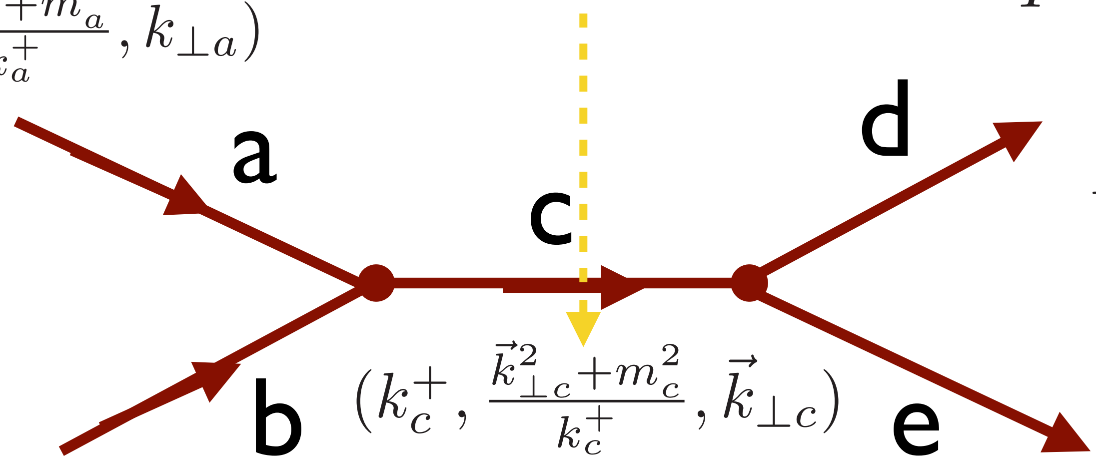
$$\Delta L^z = L^z_{initial} - L^z_{final} \leq number\ \ of\ \ vertices$$

K. Chiu
SJB

Light-Front Hamiltonian Perturbation Theory

$$< i|T|j > = < i|H_I|j > + \sum_n < i|H_I|n > \frac{1}{\mathcal{M}_{\rm i}^2 - \mathcal{M}_{\rm n}^2 + i\epsilon} < n|H_I|j > + \cdots$$

$(k_a^+, \frac{\vec{k}_{\perp a}^2 + m_a^2}{k_a^+}, \vec{k}_{\perp a})$



$(k_c^+, \frac{\vec{k}_{\perp c}^2 + m_c^2}{k_c^+}, \vec{k}_{\perp c})$

$(k_b^+, \frac{\vec{k}_{\perp b}^2 + m_b^2}{k_b^+}, \vec{k}_{\perp b})$

$$P^+ = k_a^+ + k_b^+ = k_c^+, \ \vec{P}_\perp = \vec{k}_{\perp a} + \vec{k}_{\perp b} = \vec{k}_{\perp c}$$

$$P^+ P^-_{\rm initial} = \mathcal{M}^2_{\rm initial} + \vec{P}_\perp^2 = s + \vec{P}_\perp^2$$

$$T = \frac{g^2}{s - m_c^2 + i\epsilon}$$

$$s = \mathcal{M}^2_{\rm initial} = (k_a + k_b)^2 = m_a^2 + m_b^2 + k_a^+ k_b^- + k_a^- k_b^+ - 2\vec{k}_{\perp a} \cdot \vec{k}_\perp$$

All particles on their respective mass-shell

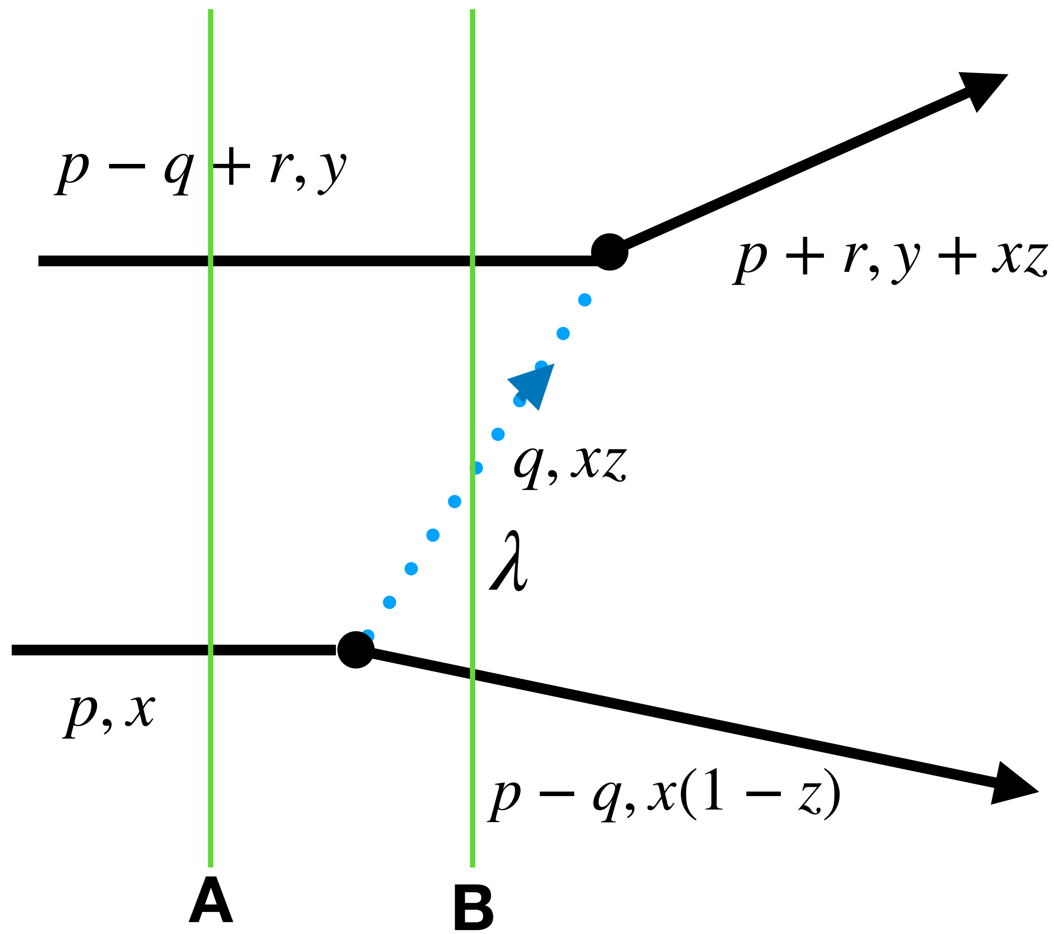
$P^+, \vec{P}_\perp, J^z$ conserved

$$k^2 = k^+ k^- - k_\perp^2 = m^2$$

Features of LF Perturbation Theory
Poincare' Invariant

- All intermediate states propagate on mass shell
- Wick theorem: sum of diagrams with positive k^+
- 3-dimensional Integrals: $\int d^2k_\perp \int_0^1 dx$
- Each amplitude is frame independent
- Unitarity is explicit
- *"History"*: Numerator is process independent!
- Jz Conservation at each vertex
- Spin projection along $\hat{z}$

Introduce virtuality t in LFPTH

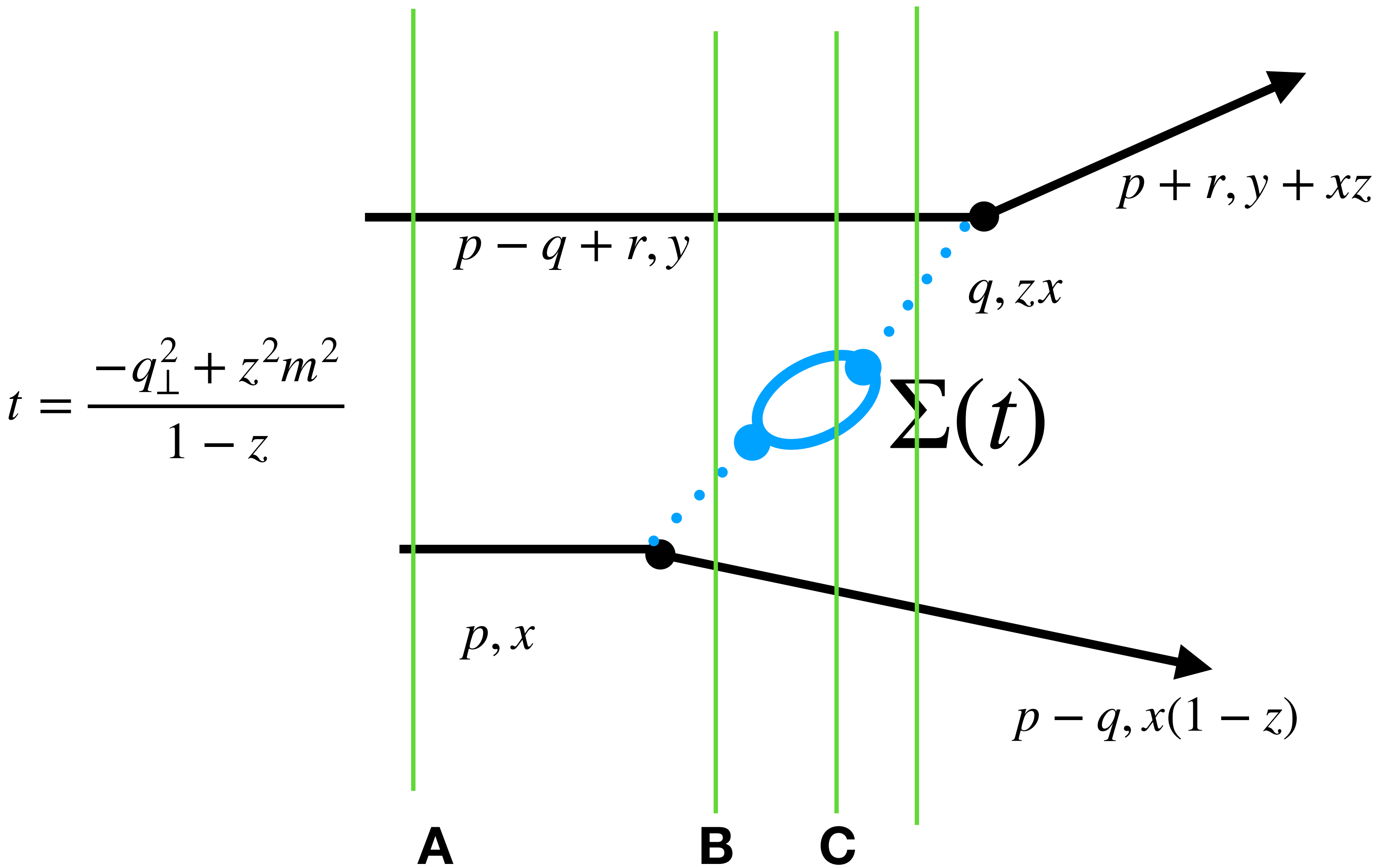


$$T = \frac{g^2}{xz[\frac{m^2}{x} - \frac{\lambda^2+q_\perp^2}{xz} - \frac{m^2+q_\perp^2}{x(1-z)}] + i\epsilon} \equiv \frac{g^2}{t-\lambda^2+i\epsilon}$$

$$Thus\ t = \frac{-q_\perp^2 - z^2m^2}{1-z}$$

This defines the Mandelstam variable t. It allows the concept of an off-shell propagator in LFPTH

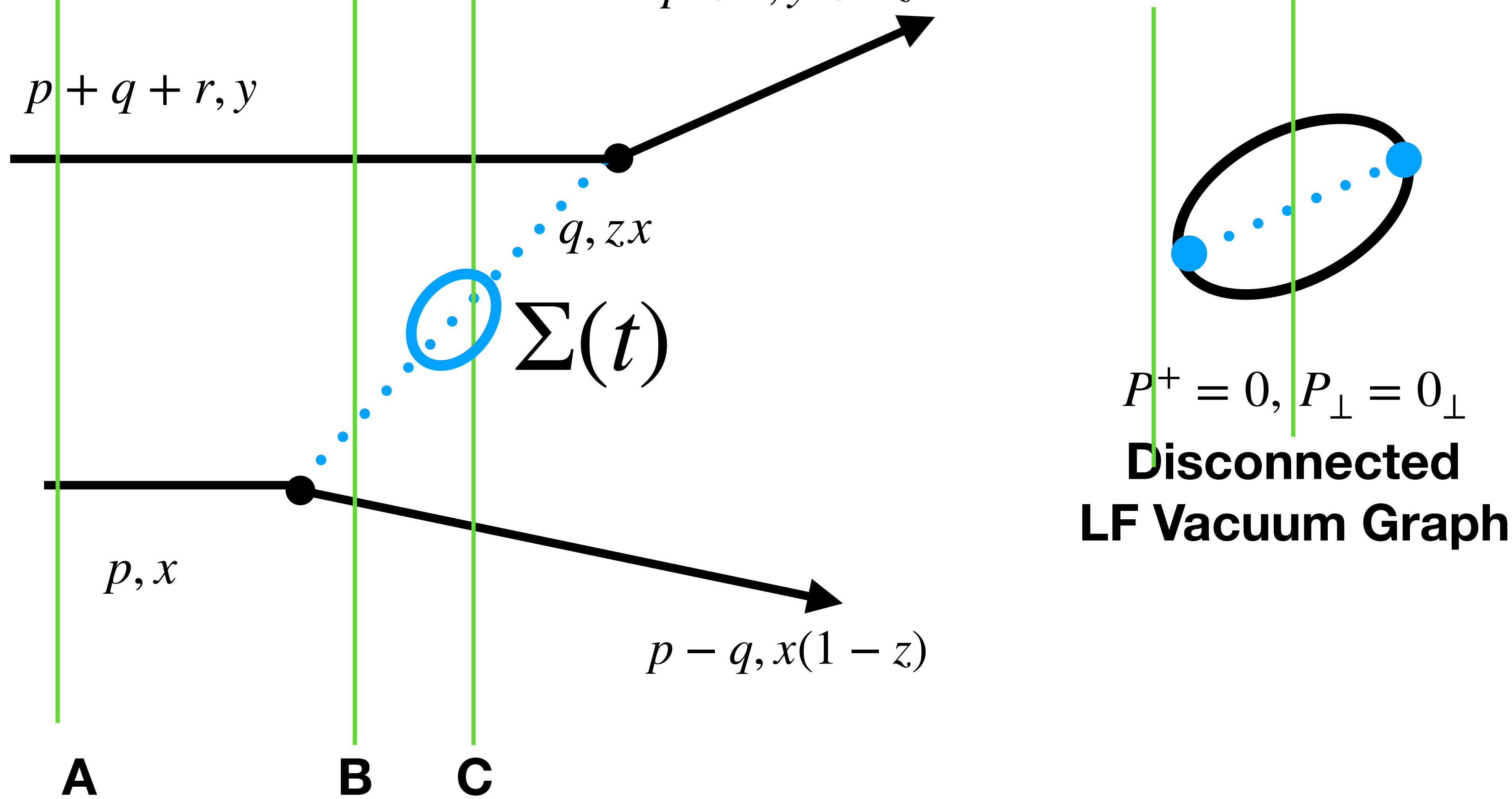
Note that one can choose the LF frame so that $t = -q_\perp^2$ if $z = 0$, as in the Drell Yan West analysis for form factors, or choose LF kinematics with $q_\perp = 0$ so that $t = \frac{-m^2z^2}{1-z}$.



Computing $\frac{1}{A-B}\frac{1}{A-C}$ in LFPTH is equivalent to computing the self-energy $\Sigma(t)$ from $\frac{1}{B-C}$ with an effective invariant mass squared $\mathcal{M}^2 = -t$.

One can thus compute $\Sigma(t)$ for $t \to 0$ by choosing $2 \to 2$ scattering kinematics, where $q_\perp = 0$, and taking the limit of small LF momentum fraction z.

$$q_\perp = 0, \;\lim\; z \to 0: \;\; t = \frac{-z^2m^2}{1-z} \to 0$$



Disconnected Vacuum graphs in LF theory have total $P^+ = 0, \vec{P}_\perp = 0_\perp$. They are invariant over all space $(x^-, \vec{x}_\perp)$ at fixed LF time $\tau = x^+ = t + z/c$.

The disconnected loop diagram is coupled to the vacuum eigenstate with $P^+ = 0, \vec{P}_\perp = 0$. It vanishes in LF theory since + momentum conservation cannot be satisfied. Unlike the connected self-energy insertion $\Sigma(p^2)$, it is not obtained from a limit process from a contribution with nonzero P^+.

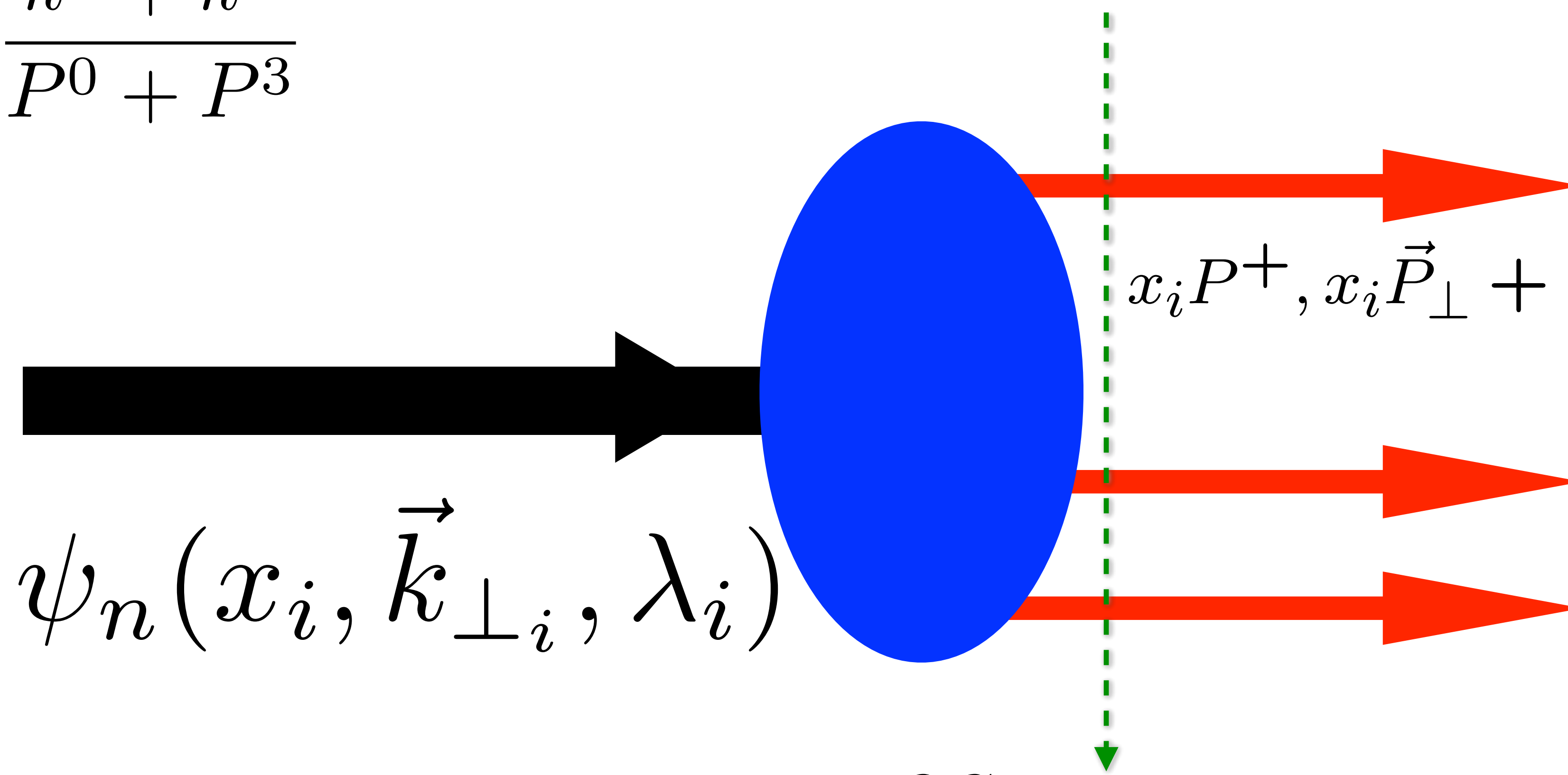
There is no limiting or rescaling process involved.

Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

Fixed $\tau = t + z/c$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

$P^+, \vec{P}_\perp$



$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

Eigenstate of LF Hamiltonian

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) | n; x_i, \vec{k}_{\perp i}, \lambda_i >$$

Invariant under boosts! Independent of P^μ

Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS

PHYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

Exclusive processes in perturbative quantum chromodynamics

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(Received 27 May 1980)

We present a systematic analysis in perturbative quantum chromodynamics (QCD) of large-momentum-transfer exclusive processes. Predictions are given for the scaling behavior, angular dependence, helicity structure, and normalization of elastic and inelastic form factors and large-angle exclusive scattering amplitudes for hadrons and photons. We prove that these reactions are dominated by quark and gluon subprocesses at short distances, and thus that the dimensional-counting rules for the power-law falloff of these amplitudes with momentum transfer are rigorous predictions of QCD, modulo calculable logarithmic corrections from the behavior of the hadronic wave functions at short distances. These anomalous-dimension corrections are determined by evolution equations for process-independent meson and baryon "distribution amplitudes" $\phi(x_i, Q)$ which control the valence-quark distributions in high-momentum-transfer exclusive reactions. The analysis can be carried out systematically in powers of $\alpha_s(Q^2)$, the QCD running coupling constant. Although the calculations are most conveniently carried out using light-cone perturbation theory and the light-cone gauge, we also present a gauge-independent analysis and relate the distribution amplitude to a gauge-invariant Bethe-Salpeter amplitude.

APPENDIX A: LIGHT-CONE PERTURBATION THEORY

One of the most convenient and physical formalisms for studying processes with large transverse momenta is light-cone quantization, or its equivalent, time-ordered perturbation theory in the infinite-momentum frame.[8] Defining $p^{\pm} \equiv p^0 \pm p^3$, we can parametrize a particle's momentum as

$$p^{\mu} = (p^+, p^-, \vec{p}_{\perp}) = \left(p^+, \frac{p_{\perp}^2 + m^2}{p^+}, \vec{p}_{\perp}\right),$$

where $p^2 = p^+p^- - p_{\perp}^2 = m^2$. [Note that in general $p \cdot k = \frac{1}{2}(p^+k^- + p^-k^+) - p_{\perp} \cdot k_{\perp}$.] These variables naturally distinguish between a particle's longitudinal and transverse degrees of freedom and when used in an appropriate frame lead to much simplification. This is particularly true in any analysis of collinear singularities as these appear as divergences only in integrations over transverse momenta, $k_{\perp}^2$.

For each time-ordered graph, the rules of light-cone perturbation theory are the following.

(R1) Assign a momentum k_{μ} to each line such that (a) k^+, $k_{\perp}$ are conserved at each vertex, and (b) $k^2 = m^2$; i.e., $k^- = (k_{\perp}^2 + m^2)/k^+$ and k_{μ} is on mass shell.

(R2) Include a factor $\theta(k^+)$ for each line—all quanta are forward moving ($k^3 > 0$) in the infinite-momentum frame.

(R3) For each gluon (or other vector-boson) line include a factor $d_{\mu\nu}^{(k)}/k^+$ where $d_{\mu\nu}$ is the (gauge-dependent) polarization sum. In Feynman gauge $d_{\mu\nu}$ equals $-g_{\mu\nu}$. In light-cone gauge $\eta \cdot A = A^+ = 0$,

$$d_{\mu\nu}^{(k)} = \sum_{\lambda=1,2} \epsilon_{\mu}(k, \lambda)\epsilon_{\nu}(k, \lambda)$$

$$= -g_{\mu\nu} + \frac{\eta_{\mu}k_{\nu} + \eta_{\nu}k_{\mu}}{\eta \cdot k},$$

where $k \cdot \epsilon = \eta \cdot \epsilon = 0$.[51] The singularity at $\eta \cdot k = 0$

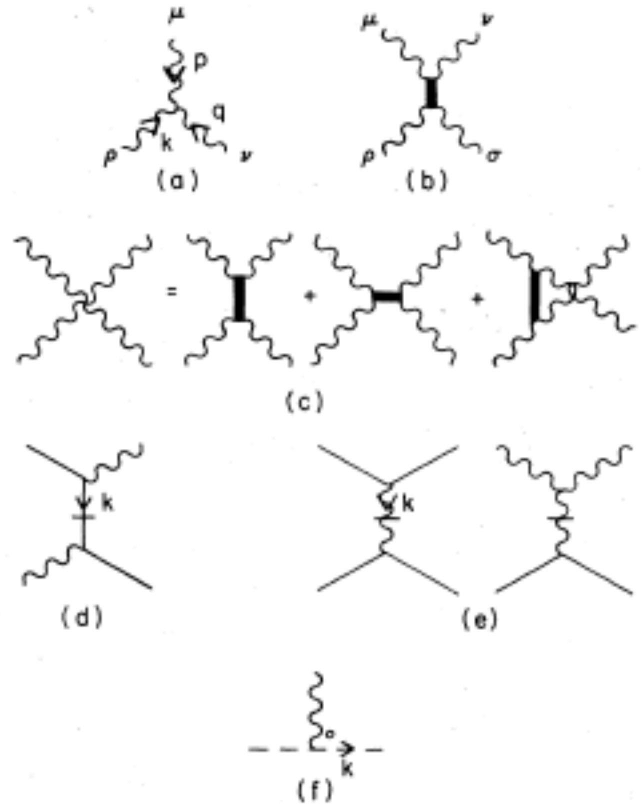

FIG. 30. Vertices appearing in QCD light-cone perturbation theory.

is regulated by replacing $1/\eta\cdot k \to \eta\cdot k/((\eta\cdot k)^2+\epsilon^2)$. Dependence on ϵ cancels in the total amplitude for a process.

(R4) The gluon-fermion vertices are

$$e_0\frac{\bar{u}(k)}{(k^+)^{1/2}}\gamma^\mu\frac{u(l)}{(l^+)^{1/2}}\,,\quad e_0\frac{\bar{u}(k)}{(k^+)^{1/2}}\gamma^\mu\frac{v(l)}{(l^+)^{1/2}}\,,$$

$$-e^0\frac{\bar{v}(k)}{(k^+)^{1/2}}\gamma^\mu\frac{u(l)}{(l^+)^{1/2}}\,,\quad -e_0\frac{\bar{v}(k)}{(k^+)^{1/2}}\gamma^\mu\frac{v(l)}{(l^+)^{1/2}}\,.$$

The factors $1/(k^+)^{1/2}$, $1/(l^+)^{1/2}$ are omitted for external fermions in a scattering amplitude.

(R5) The trigluon vertex is [Fig. 30(a)]

$$-e_0[(p-q)^\rho g^{\mu\nu}+(q-k)^\mu g^{\rho\nu}+(k-p)^\nu g^{\mu\rho}]$$

and the four-gluon vertex is [Fig. 30(b)]

$$e_0^2(g^{\mu\rho}g^{\nu\sigma}-g^{\mu\sigma}g^{\nu\rho})\,.$$

Generally there are three independent ways of inserting the four-gluon vertex [Fig. 30(c)]; all must be included.

(R6) For each intermediate state there is a factor

$$\frac{1}{\sum_{\text{inc}} k^- - \sum_{\text{interm}} k^- + i\epsilon}\,,$$

where the sums in the "energy denominator" are over the light-cone "energies," k^-, of the incident (inc) and intermediate (interm) particles.

(R7) In Feynman gauge, ghost loops occur. For each ghost line [with momentum as in (R1)] include a factor $-\theta(k^+)/k^+$. The gluon-ghost vertex is $e_0 k^\nu$ for Fig. 30(f). There are no ghosts in light-cone gauge.

(R8) The fermion propagator has an instantaneous part [$\gamma^+/2k^+$; Fig. 30(d)], as do the gluon propagator [$\eta^\mu\eta^\nu/k^{+2}$ in light-cone gauge; Fig. 30(e)] and the ghost propagator (in Feynman gauge). In each case, the instantaneous propagator can be absorbed into the regular propagator by replacing k, the momentum associated with the line, by

$$\tilde{k}=\Big(k^+,\sum_{\text{inc}}k^- - {\sum_{\text{interm}}}' k^-, k\Big)$$

in the numerator for those diagrams in which the fermion, gluon, or ghost propagates only over a single time interval (Fig. 31). Here $\sum_{\text{inc}}$ denotes summation over all initial particles in the diagram, while $\sum'_{\text{interm}}$ denotes summation over all particles in the intermediate state *other than* the particle of interest. Thus, in light-cone gauge, $\tilde{k}$ replaces k in the polarization sum $d_{\mu\nu}^{(k)}$, as well as in the trigluon coupling, for gluons appearing in a single intermediate state [Fig. 31(a)]. Similarly, $\sum_{\text{spins}} u(k)\bar{u}(k)$ is replaced by $\tilde{\not k}+m$, and $\sum_{\text{spins}} v(k)\times\bar{v}(k)$ by $\tilde{\not k}-m$, as in Fig. 31(b).

(R9) Integrate $\int_0^\infty dk^+\int d^2k_\perp/16\pi^3$ over each independent k and sum over internal spins and polarizations.

(R10) Color factors are computed as for covariant diagrams (see Ref. 52, for example).

In addition to these rules, there are several tricks which are useful in certain applications.

(T1) In amplitudes with an external line off shell (having momentum $q^\mu, q^2\neq m^2$), the energy denominators for intermediate states following the ver-

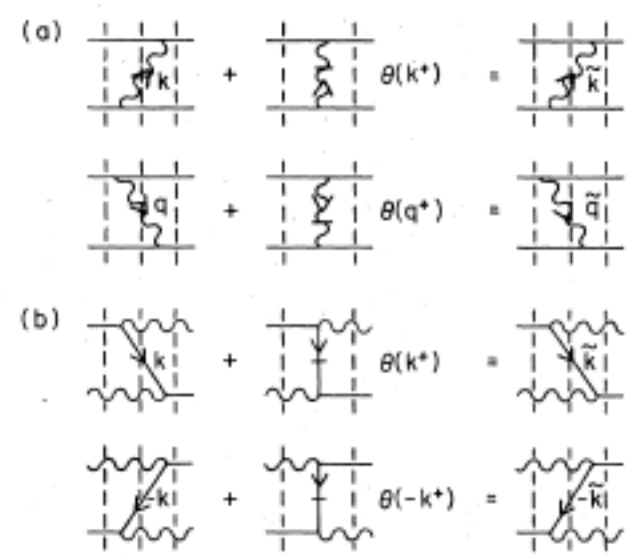

FIG. 31. Procedure for removing instantaneous propagators by redefining the noninstantaneous propagators.

PHYSICAL REVIEW D VOLUME 22, NUMBER 9 1 NOVEMBER 1980

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Exclusive processes in perturbative quantum chromodynamics

TABLE II. Dirac matrix elements for the helicity spinors of Appendix A.

Matrix element $\bar{u}_{\lambda'}\cdots u_{\lambda}$	Helicity ($\lambda \to \lambda'$) $\uparrow\to\uparrow$, $\downarrow\to\downarrow$	$\uparrow\to\downarrow$, $\downarrow\to\uparrow$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^+\frac{u(q)}{(q^+)^{1/2}}$	2	0
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\frac{2}{p^+q^+}(p_\perp\cdot q_\perp \pm ip_\perp\times q_\perp + m^2)$	$\mp\frac{2m}{p^+q^+}[(p^1\pm ip^2)-(q^1\pm iq^2)]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}+\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}$	$\mp m\left(\frac{p^+-q^+}{p^+q^+}\right)(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\frac{u(q)}{(q^+)^{1/2}}$	$m\left(\frac{p^++q^+}{p^+q^+}\right)$	$\mp\left(\frac{p^1\pm ip^2}{p^+}-\frac{q^1\pm iq^2}{q^+}\right)$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\frac{8}{p^+q^+}(p_\perp\cdot q_\perp \pm ip_\perp\times q_\perp + m^2)$	$\mp\frac{8m}{p^+q^+}[(p^1\pm ip^2)-(q^1\pm iq^2)]$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}\right)$	$\pm\frac{4m}{p^+}(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}\right)$	$\mp\frac{4m}{q^+}(\delta^{i1}\pm i\delta^{i2})$
$\frac{\bar{u}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma_\perp^j\frac{u(q)}{(q^+)^{1/2}}$	$2(\delta^{ij}\pm i\epsilon^{ij})$	0

$$\bar{v}_\mu(p)\gamma^\alpha v_\nu(q)=\bar{u}_\nu(q)\gamma^\alpha u_\mu(p) \qquad \bar{v}_\mu(p)v_\nu(q)=-\bar{u}_\nu(q)u_\mu(p)$$

$$\bar{v}_\mu(p)\gamma^\alpha\gamma^\beta\gamma^\delta v_\nu(q)=\bar{u}_\nu(q)\gamma^\delta\gamma^\beta\gamma^\alpha u_\mu(p)$$

$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma^+\frac{u(q)}{(q^+)^{1/2}}$	0	2
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\mp\frac{2m}{p^+q^+}[(p^1\pm ip^2)+(q^1\pm iq^2)]$	$\frac{2}{p^+q^+}(p_\perp\cdot q_\perp \pm ip_\perp\times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$\mp m\left(\frac{p^+ + q^+}{p^+q^+}\right)(\delta^{i1}\pm i\delta^{i2})$	$\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}+\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\frac{u(q)}{(q^+)^{1/2}}$	$\mp\left(\frac{p^1\pm ip^2}{p^+}-\frac{q^1\pm iq^2}{q^+}\right)$	$m\left(\frac{p^+ - q^+}{p^+q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\mp\frac{8m}{p^+q^+}[(p^1\pm ip^2)+(q^1\pm iq^2)]$	$\frac{8}{p^+q^+}(p_\perp\cdot q_\perp \pm ip_\perp\times q_\perp - m^2)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma^-\gamma^+\gamma_\perp^i\frac{u(q)}{(q^+)^{1/2}}$	$\mp\frac{4m}{p^+}(\delta^{i1}\pm i\delta^{i2})$	$4\left(\frac{p_\perp^i \mp i\epsilon^{ij}p_\perp^j}{p^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma^-\frac{u(q)}{(q^+)^{1/2}}$	$\mp\frac{4m}{q^+}(\delta^{i1}\pm i\delta^{i2})$	$4\left(\frac{q_\perp^i \pm i\epsilon^{ij}q_\perp^j}{q^+}\right)$
$\frac{\bar{v}(p)}{(p^+)^{1/2}}\gamma_\perp^i\gamma^+\gamma_\perp^j\frac{u(q)}{(q^+)^{1/2}}$	0	$2(\delta^{ij}\pm i\epsilon^{ij})$

antiparticles are

$$\left.\begin{matrix} u_\uparrow(p) \\ u_\downarrow(p)\end{matrix}\right\} = \frac{1}{(p^+)^{1/2}}(p^+ + \beta m + \alpha_\perp\cdot p_\perp)\times\begin{cases}\chi(\uparrow)\\ \chi(\downarrow)\end{cases},$$
$$\left.\begin{matrix} v_\uparrow(p) \\ v_\downarrow(p)\end{matrix}\right\} = \frac{1}{(p^+)^{1/2}}(p^+ - \beta m + \vec{\alpha}_\perp\cdot \vec{p}_\perp)\times\begin{cases}\chi(\downarrow)\\ \chi(\uparrow)\end{cases}. \qquad \text{(A3)}$$

Taking $p^+\to\infty$, we find that these are helicity eigenstates when viewed from the infinite-momentum frame. Notice also that the phases assigned the antiparticle spinors are conventional for spin-$\frac{1}{2}$ eigenstates. Thus a state $u_\uparrow\bar{v}_\downarrow - u_\downarrow\bar{v}_\uparrow$ has spin zero (in the infinite-momentum frame), for example.

Matrix elements involving these states are tabulated in Tables II and III.

In light-cone perturbation theory, a two-body bound state with total momentum $p^\mu = (p^+, (M^2 + p_\perp^2)/p^+, p_\perp)$ is described by a wave function

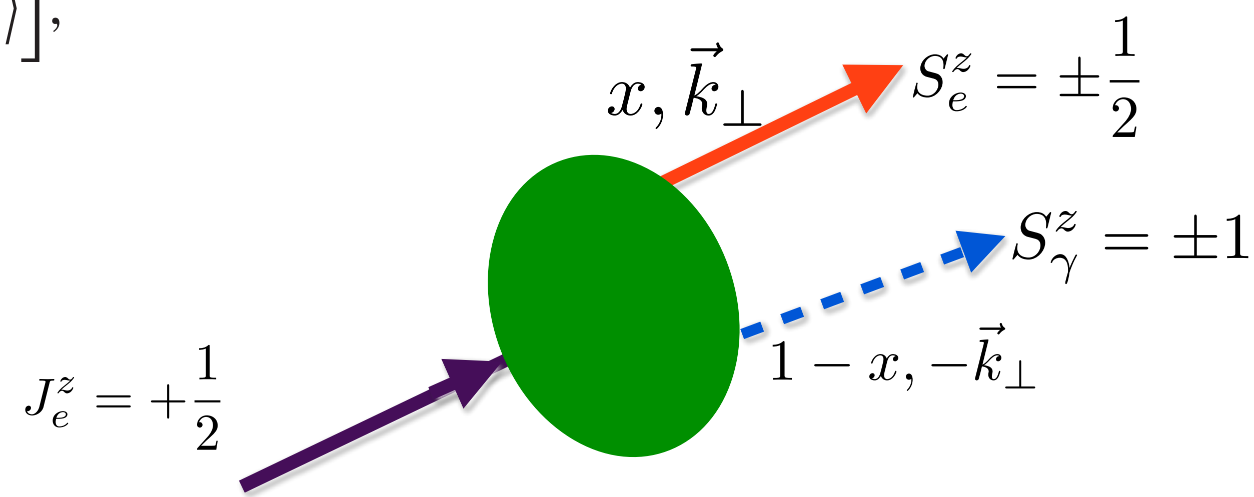
$$\Psi(x_i, k_\perp; p) = \frac{u_\lambda^{(1)}(x_1p^+, k_\perp + x_1p_\perp)}{\sqrt{x_1}}\frac{u_{\lambda'}^{(2)}(x_2p^+, -k_\perp + x_2p_\perp)}{\sqrt{x_2}}\psi_{\lambda\lambda'}(x_i, k_\perp), \qquad \text{(A4)}$$

where x_ip^+ is the longitudinal momentum carried by the ith constituent $(x_1 + x_2 = 1)$, and $\pm k_\perp$ is the constituents' transverse momentum relative to the bound states ($u^{(2)}$ is replaced by $\bar{v}$ for a bound state of a particle and an antiparticle). By Lorentz invariance [see (T2) above], $\psi(x_i, k_\perp)$ is independent of p^+ and $p_\perp$, and thus we can set $p^\mu = (1, M^2, 0_\perp)$ without loss of generality. This wave function is the positive-energy projection of the familiar Bethe-Salpeter wave function evaluated with the constituents at equal "time" $\tau = (z + t)$,

$$\int\frac{dk^-}{2\pi}\Psi_{BS}(k;p) = \frac{u^{(1)}(x_1, k_\perp)}{\sqrt{x_1}}\frac{u^{(2)}(x_2, -k_\perp)}{\sqrt{x_2}}\psi(x_i, k_\perp)$$
$$+ \text{ negative-energy components},$$

The two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations:

$$
\begin{aligned}
&\left|\Psi^{\uparrow}_{\text{two particle}}(P^+, \vec{P}_\perp = \vec{0}_\perp)\right\rangle \\
&= \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{\sqrt{x(1-x)}\, 16\pi^3} \Big[\psi^{\uparrow}_{+\frac{1}{2}\,+1}(x, \vec{k}_\perp) \left| +\tfrac{1}{2}\, +1;\, xP^+, \vec{k}_\perp \right\rangle \\
&\quad + \psi^{\uparrow}_{+\frac{1}{2}\,-1}(x, \vec{k}_\perp) \left| +\tfrac{1}{2} - 1;\, xP^+, \vec{k}_\perp \right\rangle + \psi^{\uparrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) \left| -\tfrac{1}{2} + 1;\, xP^+, \vec{k}_\perp \right\rangle \\
&\quad + \psi^{\uparrow}_{-\frac{1}{2}-1}(x, \vec{k}_\perp) \left| -\tfrac{1}{2} - 1;\, xP^+, \vec{k}_\perp \right\rangle \Big],
\end{aligned}
$$



$$
\begin{cases}
\psi^{\uparrow}_{+\frac{1}{2}+1}(x, \vec{k}_\perp) = -\sqrt{2}\, \dfrac{(-k^1 + \mathrm{i}k^2)}{x(1-x)}\, \varphi, \\
\psi^{\uparrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) = -\sqrt{2}\, \dfrac{(+k^1 + \mathrm{i}k^2)}{1-x}\, \varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) = -\sqrt{2} \left(M - \dfrac{m}{x} \right) \varphi, \\
\psi^{\uparrow}_{-\frac{1}{2}-1}(x, \vec{k}_\perp) = 0,
\end{cases}
$$

$$
\varphi = \varphi(x, \vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}.
$$

Hwang, Schmidt, Ma, sjb

$$\bar{K}_{\alpha\beta,\gamma\delta}=\frac{\bar{u}^{(1)}(x_1,k_\perp)}{\sqrt{x_1}}\frac{\bar{u}^{(2)}(x_2,-k_\perp)}{\sqrt{x_2}}\bar{K}(x_i,k_\perp;y_i,l_\perp;M^2)\frac{u^{(1)}(y_1,l_\perp)}{\sqrt{y_1}}\frac{u^{(2)}(y_2,-l_\perp)}{\sqrt{y_2}}\,, \tag{A6}$$

where in perturbation theory $\bar{K}$ is the sum of all truncated, two-particle irreducible[54] amplitudes as illustrated in Fig. 33(b). A scattering amplitude involving the bound state is given by

$$T=\int_0^1[dx]\int_0^\infty\frac{d^2k_\perp}{16\pi^3}\mathfrak{M}\Psi(x_i,k_\perp;p)=\int_0^1[dx]\int_0^\infty\frac{d^2k_\perp}{16\pi^3}\mathfrak{M}\frac{u^{(1)}}{\sqrt{x_1}}\frac{u^{(2)}}{\sqrt{x_2}}\psi(x_i,k_\perp)\,, \tag{A7}$$

where $\mathfrak{M}$ is the amplitude with the bound state replaced its constituents. Amplitude $\mathfrak{M}$ must be two-particle irreducible with respect to the constituent lines if double counting is to be avoided (Fig. 34). [Note that Eq. (A7) is consistent with rule (R4) which assigns the spinor factor $u/\sqrt{x}$ (or $v/\sqrt{x}$) to the interaction vertex of each internal fermion.] Equation (A7) has conventional (relativistic) normalization if the wave function is normalized so that

$$1=\int[dx]\frac{d^2k_\perp}{16\pi^3}|\psi(x_i,k_\perp)|^2-\int[dx]\frac{d^2k_\perp}{16\pi^3}\int[dy]\frac{d^2l_\perp}{16\pi^3}\psi^*(x_i,k_\perp)\frac{\partial}{\partial M^2}\bar{K}(x_i,k_\perp;y_i,l_\perp;M^2)\psi(y_i,l_\perp)\,. \tag{A8}$$

Notice that the second term in (A8) contributes only when the interaction potential is energy dependent (which is not the case in most nonrelativistic analyses).

For illustration, consider positronium. The kernel for one-photon exchange is

$$\bar{K}\simeq\frac{-16e^2m^2}{(k_\perp-l_\perp)^2+(x-y)^2m^2} \tag{A9}$$

in the nonrelatistic region $k_\perp, l_\perp\sim O(\alpha m)$ and $x\equiv x_1-x_2\sim O(\alpha)$, $y\equiv y_1-y_2\sim O(\alpha)$. Using this kernel and writting $M^2\simeq 4m^2+4m\epsilon$, Eq. (A5) is approximately

$$\left(\epsilon-\frac{k_\perp^2+x^2m^2}{m}\right)\psi(x_i,k_\perp)=(4x_1x_2)\int_{-1}^1 m\,dy\int_0^\infty\frac{d^2l_\perp}{(2\pi)^3}\frac{-e^2}{(k_\perp-l_\perp)^2+(x-y)^2m^2}\psi(y_i,l_\perp)\,.$$

This equation has ground-state energy $\epsilon\simeq-\alpha^2m/4$, as expected, and nonrelativistic wave functions

$$\Psi=\left(\frac{m\beta^3}{\pi}\right)^{1/2}\frac{64\pi\beta x_1x_2}{[k_\perp^2+(x_1-x_2)^2m^2+\beta^2]^2}\times\begin{cases}\dfrac{u_\uparrow\bar{v}_\downarrow-u_\downarrow\bar{v}_\uparrow}{(2x_1x_2)^{1/2}}, & \text{parapositronium},\\ \dfrac{u_\uparrow\bar{v}_\uparrow}{(x_1x_2)^{1/2}}, & \text{orthopositronium},\\ \cdots, & \end{cases}$$

where $\beta=\alpha m/2$.

For use in Secs. II and III, the free propagator in (A5) (i.e., S_0 in $S_0^{-1}\psi=\bar{K}\psi$) is replaced by the fully corrected propagator. Then $\bar{K}$ includes only those two-particle irreducible amplitudes in which the q-$\bar{q}$ lines are connected, to avoid double counting. Analyses for Fock states containing three or more particles are similar to that presented here for $q\bar{q}$ states. For example, the qqq Fock state in the nucleon is described by a wave function

$$\Psi(x_i,k_{\perp i};p)=\prod_{i=1}^3\frac{u_i^{(i)}(x_ip^+,k_{\perp i}+x_ip_\perp)}{\sqrt{x_i}}\psi_{\lambda_1\lambda_2\lambda_3}(x_i,k_{\perp i})\,,$$

where again ψ is independent of p^+ and $p_\perp$.

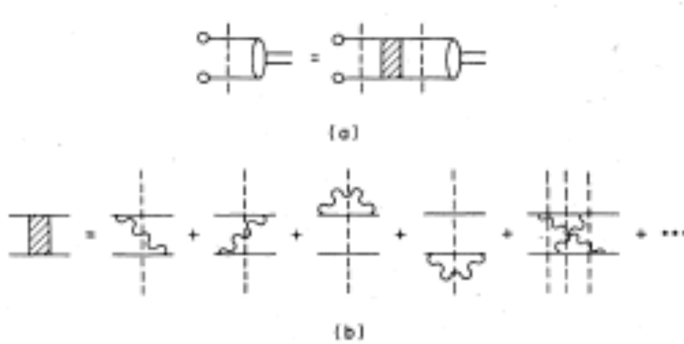


FIG. 33. The two-body bound-state equation in light-cone perturbation theory.

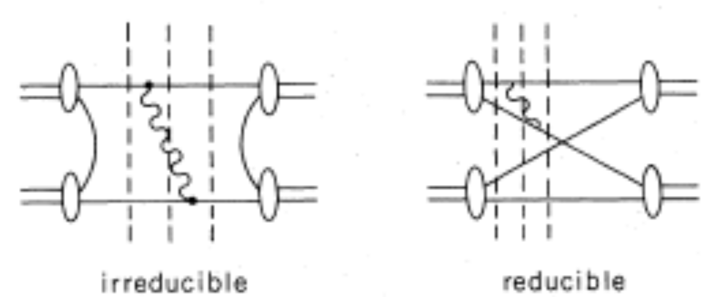


FIG. 34. Two-particle irreducible and reducible diagrams.

$$\left(M^2 - \frac{k_\perp^2 + m_1^2}{x_1} - \frac{k_\perp^2 + m_2^2}{x_2}\right)\psi(x_i, k_\perp) = \int_0^1 [dy] \int_0^\infty \frac{d^2 l_\perp}{16\pi^3} \tilde{K}(x_i, k_\perp; y_i, l_\perp; M^2)\psi(y_i, l_\perp)\,;$$

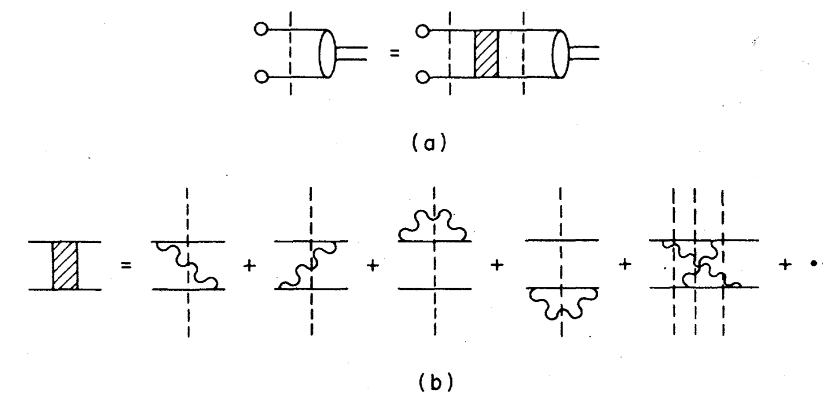


FIG. 33. The two-body bound-state equation in light-cone perturbation theory.

where $[dy] \equiv dy_1 dy_2 \delta(1 - y_1 - y_2)$. The interaction kernel $\tilde{K}$ is defined as

$$\tilde{K}_{\mu\nu,\rho\sigma} = \frac{\bar{u}^{(1)}(x_1, k_\perp)}{\sqrt{x_1}} \frac{\bar{u}^{(2)}(x_2, -k_\perp)}{\sqrt{x_2}} \bar{K}(x_i, k_\perp; y_i, l_\perp; M^2) \frac{u^{(1)}(y_1, l_\perp)}{\sqrt{y_1}} \frac{u^{(2)}(y_2, -l_\perp)}{\sqrt{y_2}}\,,$$

where in perturbation theory $\bar{K}$ is the sum of all truncated, two-particle irreducible[54] trated in Fig. 33(b). A scattering amplitude involving the bound state is given by

$$T = \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \mathfrak{M}\Psi(x_i, k_\perp; p) = \int_0^1 [dx] \int_0^\infty \frac{d^2 k_\perp}{16\pi^3} \mathfrak{M} \frac{u^{(1)}}{\sqrt{x_1}} \frac{u^{(2)}}{\sqrt{x_2}} \psi(x_i, k_\perp)\,, \tag{A7}$$

where $\mathfrak{M}$ is the amplitude with the bound state replaced its constituents. Amplitude $\mathfrak{M}$ must be two-particle irreducible with respect to the constituent lines if double counting is to be avoided (Fig. 34). [Note that Eq. (A7) is consistent with rule (R4) which assigns the spinor factor $u/\sqrt{x}$ (or $v/\sqrt{x}$) to the interaction vertex of each internal fermion.] Equation (A7) has conventional (relativistic) normalization if the wave function is normalized so that

$$1 = \int [dx] \frac{d^2 k_\perp}{16\pi^3} |\psi(x_i, k_\perp)|^2 - \int [dx] \frac{d^2 k_\perp}{16\pi^3} \int [dy] \frac{d^2 l_\perp}{16\pi^3} \psi^*(x_i, k_\perp) \frac{\partial}{\partial M^2} \tilde{K}(x_i, k_\perp; y_i, l_\perp; M^2)\psi(y_i, l_\perp)\,. \tag{A8}$$

Notice that the second term in (A8) contributes only when the interaction potential is energy dependent (which is not the case in most nonrelativistic analyses).

For illustration, consider positronium. The kernel for one-photon exchange is

$$\tilde{K} \simeq \frac{-16 e^2 m^2}{(k_\perp - l_\perp)^2 + (x - y)^2 m^2} \tag{A9}$$

in the nonrelatistic region $k_\perp, l_\perp \sim O(\alpha m)$ and $x \equiv x_1 - x_2 \sim O(\alpha)$, $y \equiv y_1 - y_2 \sim O(\alpha)$. Using this kernel and writting $M^2 \simeq 4m^2 + 4m\epsilon$, Eq. (A5) is approximately

$$\left(\epsilon - \frac{k_\perp^2 + x^2 m^2}{m}\right)\psi(x_i, k_\perp) = (4x_1 x_2) \int_{-1}^1 m\, dy \int_0^\infty \frac{d^2 l_\perp}{(2\pi)^3} \frac{-e^2}{(k_\perp - l_\perp)^2 + (x - y)^2 m^2} \psi(y_i, l_\perp)\,.$$

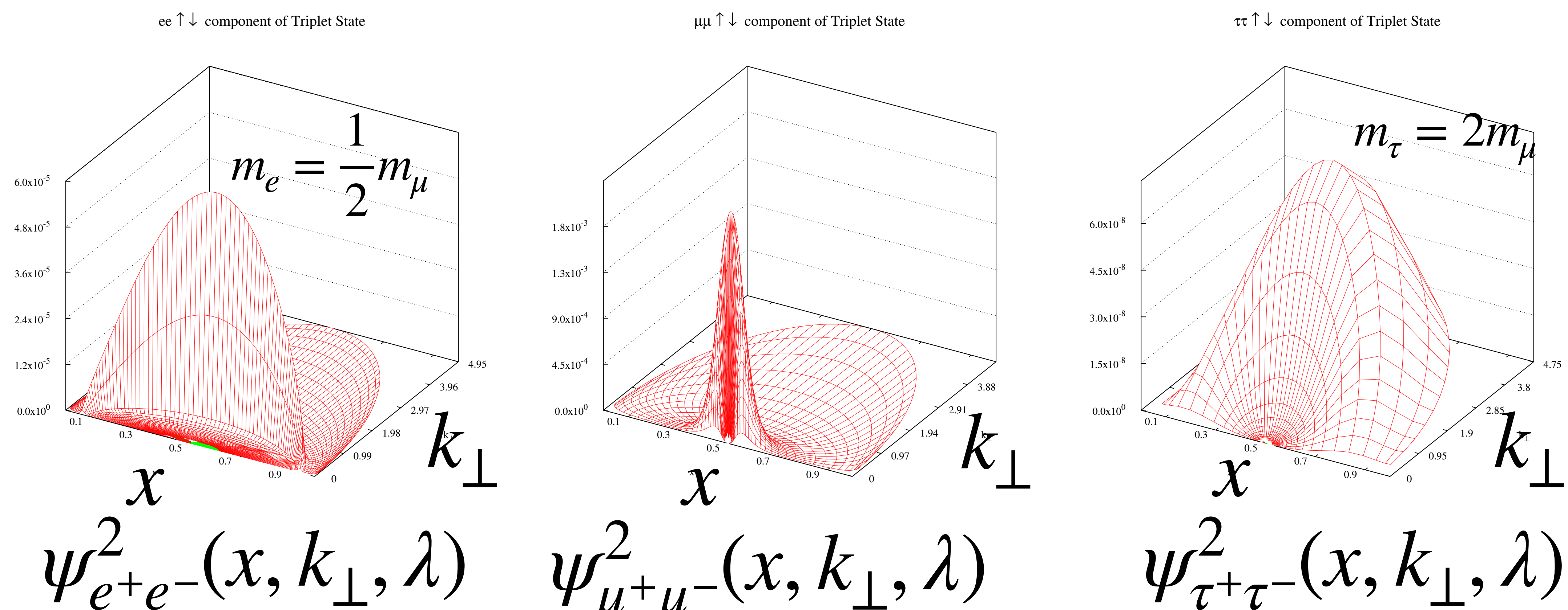
This equation has ground-state energy $\epsilon \simeq -\alpha^2 m/4$, as expected, and nonrelativistic wave functions

$$\Psi = \left(\frac{m\beta^3}{\pi}\right)^{1/2} \frac{64\pi\beta x_1 x_2}{[k_\perp^2 + (x_1 - x_2)^2 m^2 + \beta^2]^2} \times \begin{cases} \dfrac{u_\uparrow \bar{v}_\downarrow - u_\downarrow \bar{v}_\uparrow}{(2x_1 x_2)^{1/2}}, & \text{parapositronium}, \\[2ex] \dfrac{u_\uparrow \bar{v}_\uparrow}{(x_1 x_2)^{1/2}}, & \text{orthopositronium}, \end{cases} \qquad \text{where } \beta = \alpha m/2.$$

Henry Lamm · Richard F. Lebed

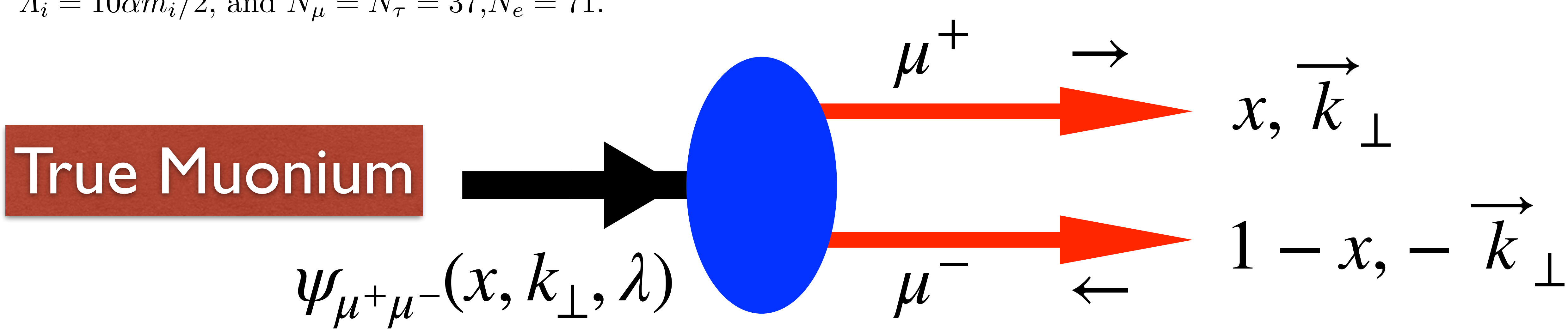
Nonperturbative True Muonium on the Light Front with TMSWIFT

DLCQ



The $1^3S_1^0$ probability density of (left) $\uparrow\downarrow$ $e\bar{e}$, (center) $\uparrow\downarrow$ $\mu\bar{\mu}$, and (right) $\uparrow\downarrow$ $\tau\bar{\tau}$ components of true muonium with $J_z = 0$, as functions of x and $k_\perp$, for $\alpha = 0.3$, $m_e = \frac{1}{2}m_\mu$, $m_\tau = 2m_\mu$ $\Lambda_i = 10\alpha m_i/2$, and $N_\mu = N_\tau = 37$,$N_e = 71$.

$\alpha = 0.3$

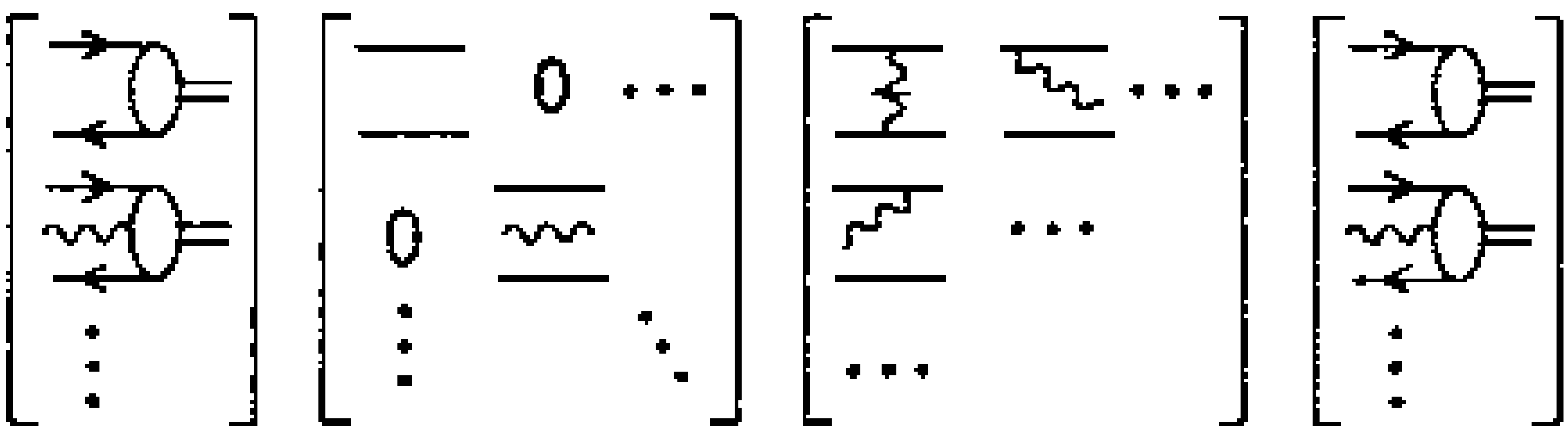


LIGHT-FRONT MATRIX EQUATION

Rigorous Method for Solving Non-Perturbative QCD!

$$\left(M_\pi^2 - \sum_i \frac{\vec{k}_{\perp i}^2 + m_i^2}{x_i}\right) \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix} = \begin{bmatrix} \langle q\bar{q}|\, V\, |q\bar{q}\rangle & \langle q\bar{q}|\, V\, |q\bar{q}g\rangle & \cdots \\ \langle q\bar{q}g|\, V\, |q\bar{q}\rangle & \langle q\bar{q}g|\, V\, |q\bar{q}g\rangle & \cdots \\ \vdots & \vdots & \ddots \end{bmatrix} \begin{bmatrix} \psi_{q\bar{q}/\pi} \\ \psi_{q\bar{q}g/\pi} \\ \vdots \end{bmatrix}$$

$A^+ = 0$



Minkowski space; frame-independent; no fermion doubling; no ghosts

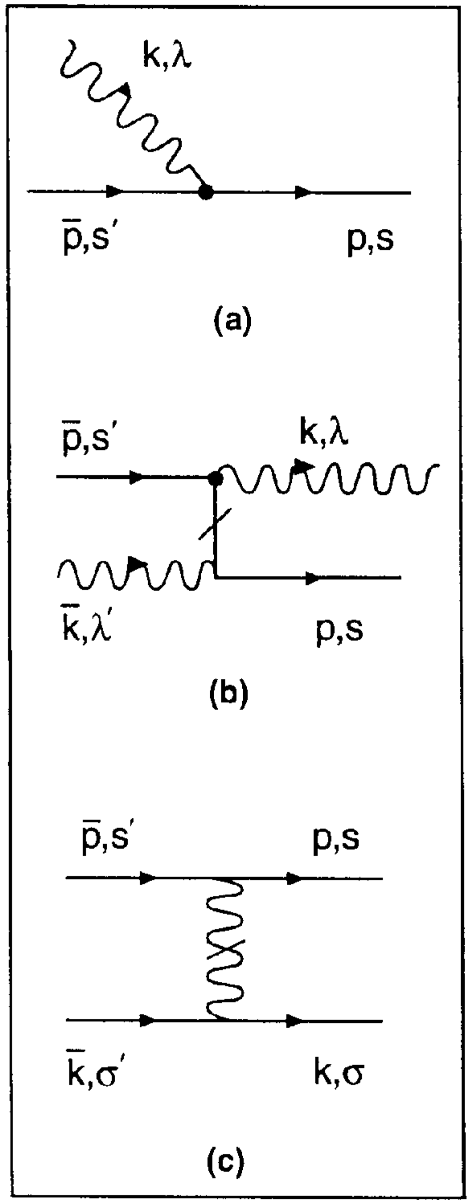
- *Light-Front Vacuum = vacuum of free Hamiltonian!*

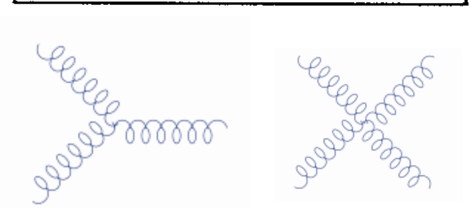
Light-Front QCD Heisenberg Equation

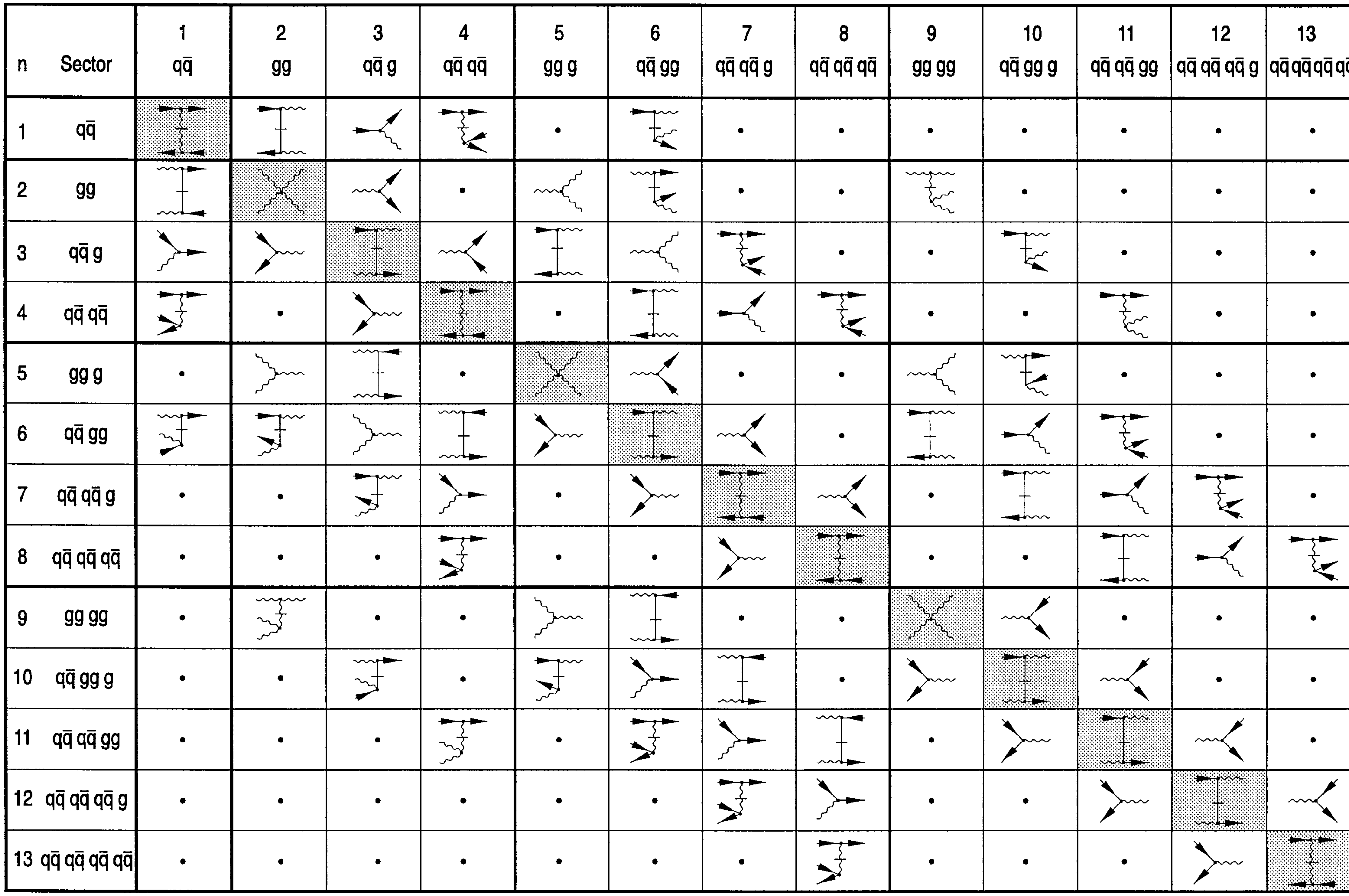
$$H_{LC}^{QCD}|\Psi_h\rangle = \mathcal{M}_h^2\,|\Psi_h\rangle$$

DLCQ: Solve QCD(1+1) for any quark mass and flavors

Hornbostel, Pauli, sjb

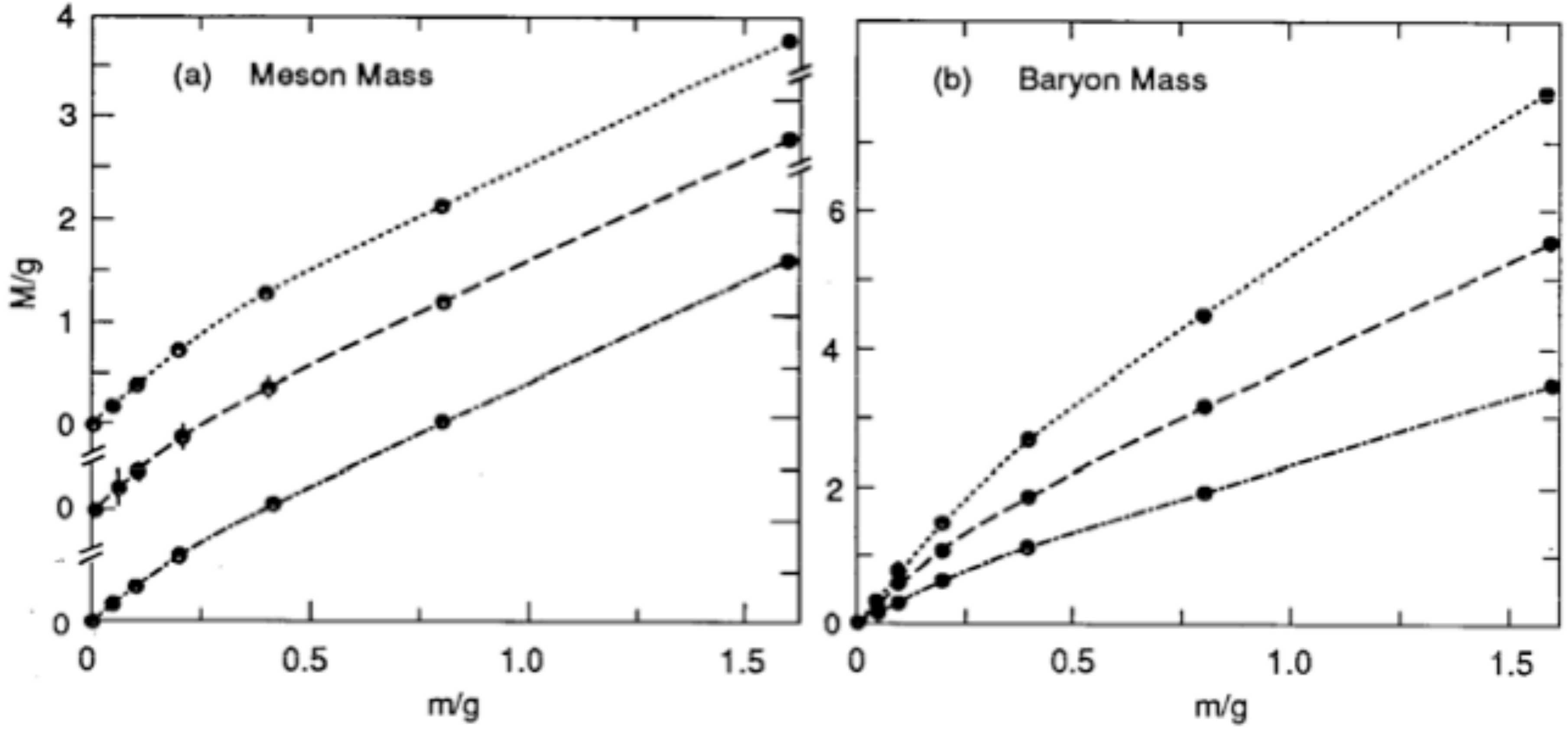




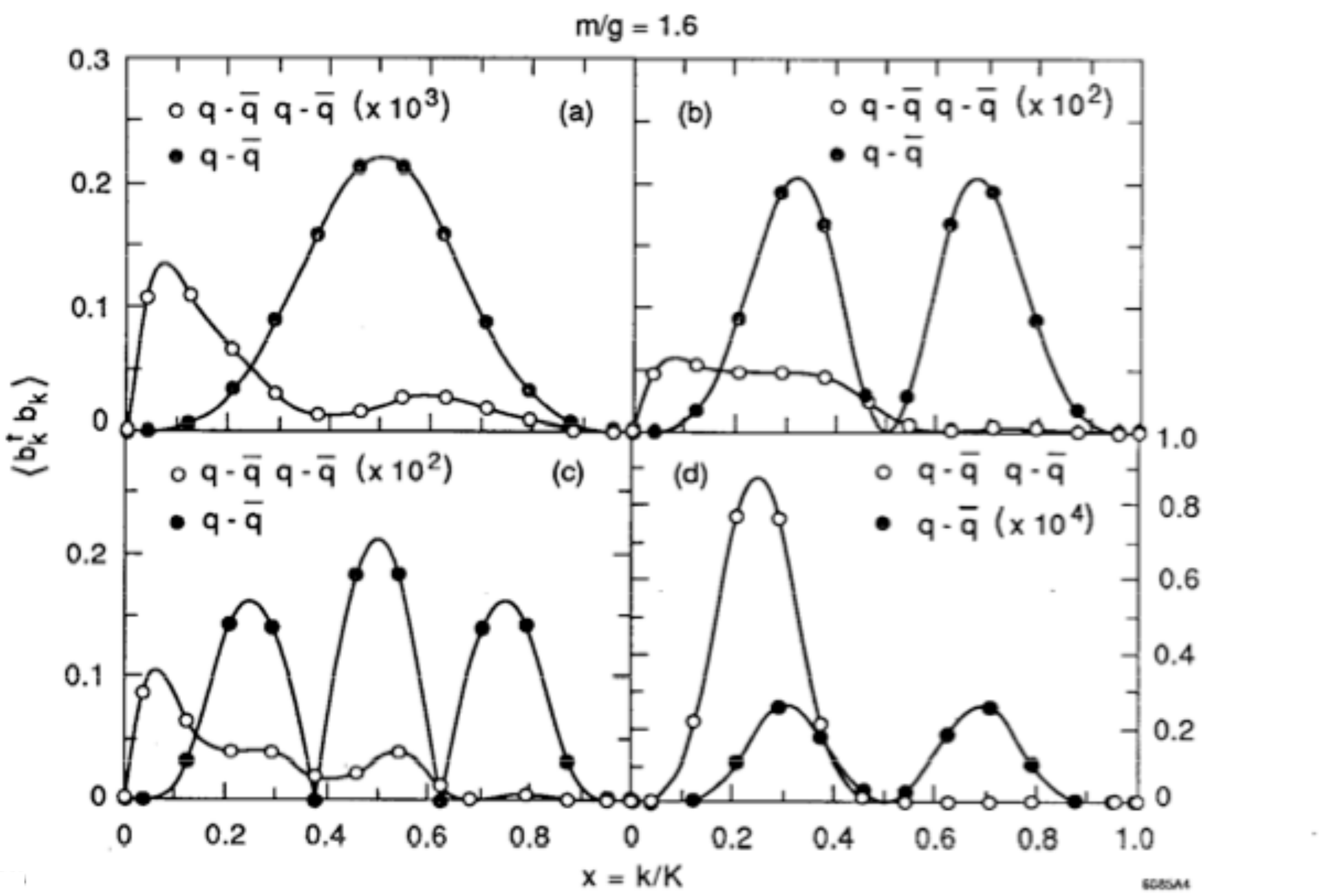


Minkowski space; frame-independent; no fermion doubling; no ghosts trivial vacuum

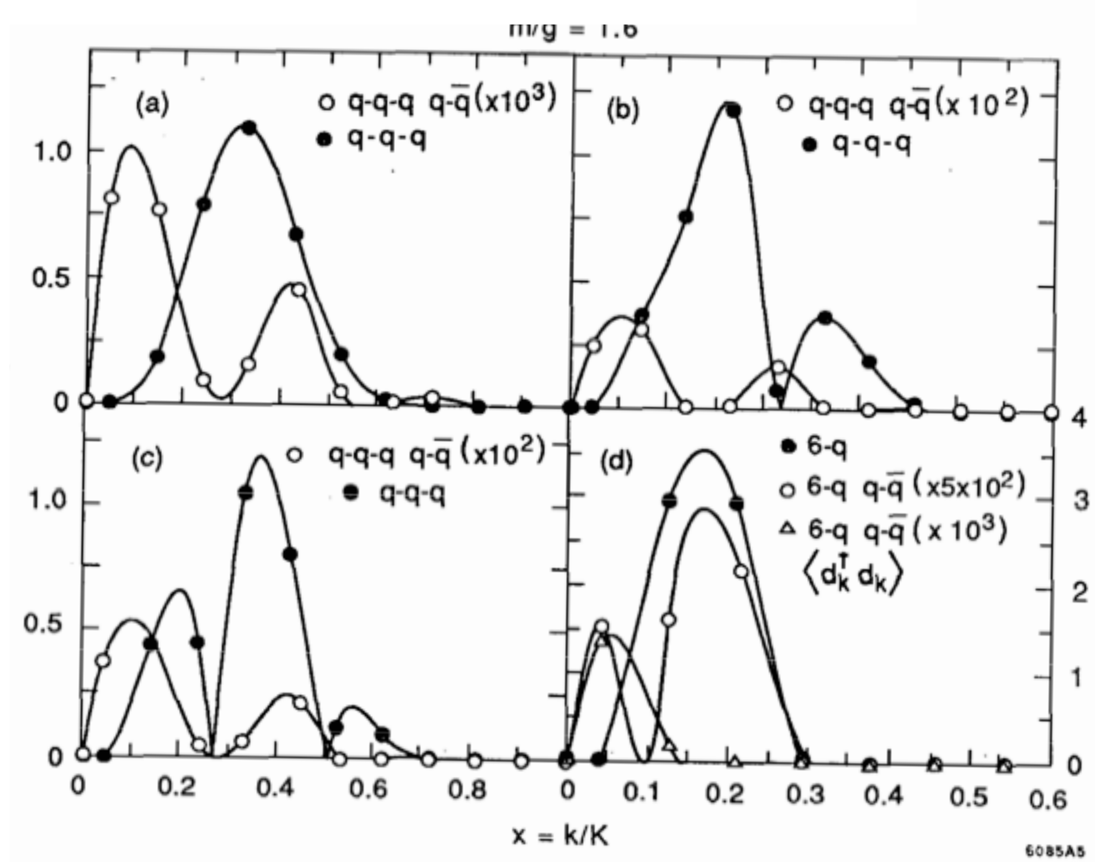
DLCQ: Solve QCD(1+1) for any quark mass and flavors



Extrapolated masses for $N = 2, 3$ and 4 meson and baryon.



a-c) First three states in $N = 3$ meson spectrum for $m/g = 1.6$, 2K=24. d) Eleventh state.



a-c) First three states in $N = 3$ baryon spectrum, 2K=21. d) First $B = 2$ state.

Hornbostel, Pauli, sjb

$$|p, S_z> = \sum_{n=3} \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)|n; \vec{k}_{\perp_i}, \lambda_i>$$

sum over states with n=3, 4, ...constituents

The Light Front Fock State Wavefunctions

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

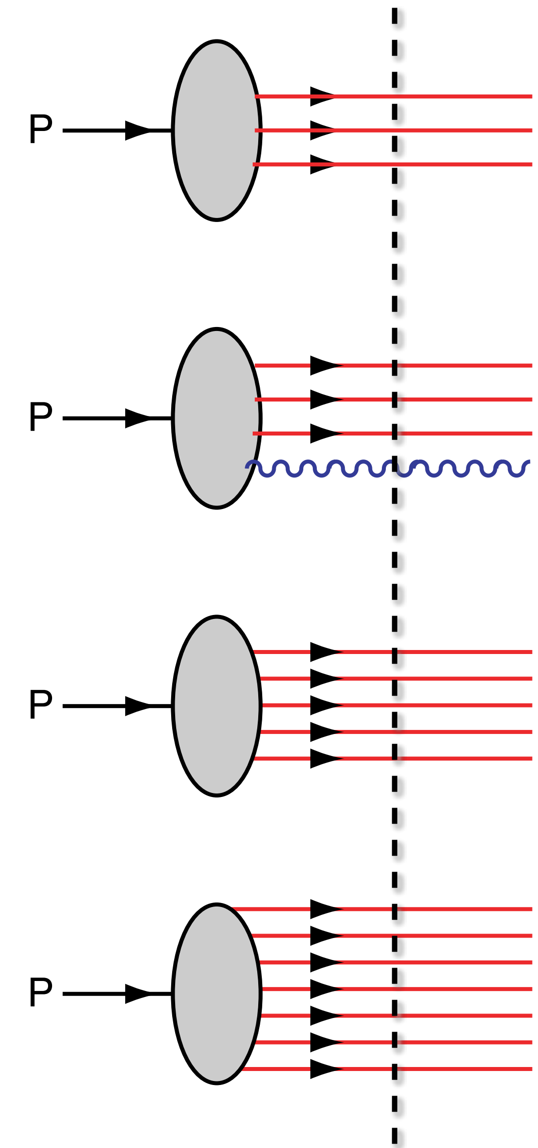
are boost invariant; they are independent of the hadron's energy and momentum P^μ.

The light-cone momentum fraction

$$x_i = \frac{k_i^+}{p^+} = \frac{k_i^0 + k_i^z}{P^0 + P^z}$$

are boost invariant.

$$\sum_i^n k_i^+ = P^+, \ \sum_i^n x_i = 1, \ \sum_i^n \vec{k}_i^\perp = \vec{0}^\perp.$$



Fixed LF time

Intrinsic heavy quarks
s(x), c(x), b(x) at high x !

$\bar{s}(x) \neq s(x)$
$\bar{u}(x) \neq \bar{d}(x)$

Hidden Color

Hidden Color in QCD

Lepage, Ji, sjb

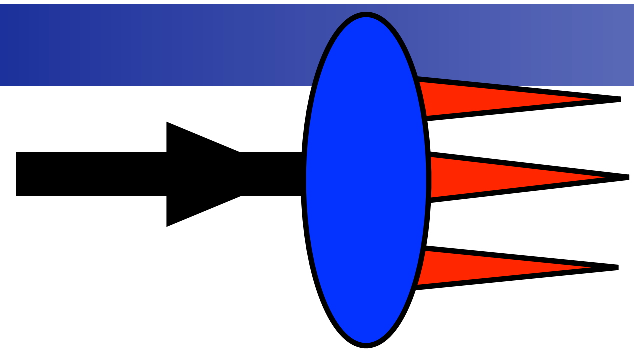
- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photo-disintegration at high momentum transfer
- Predict

$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

$$|\,[ud]_{\bar{3}C}[ud]_{\bar{3}C}(ud)_{\bar{3}C} >$$

hexaquark: 3 diquarks

- *Light Front Wavefunctions:*



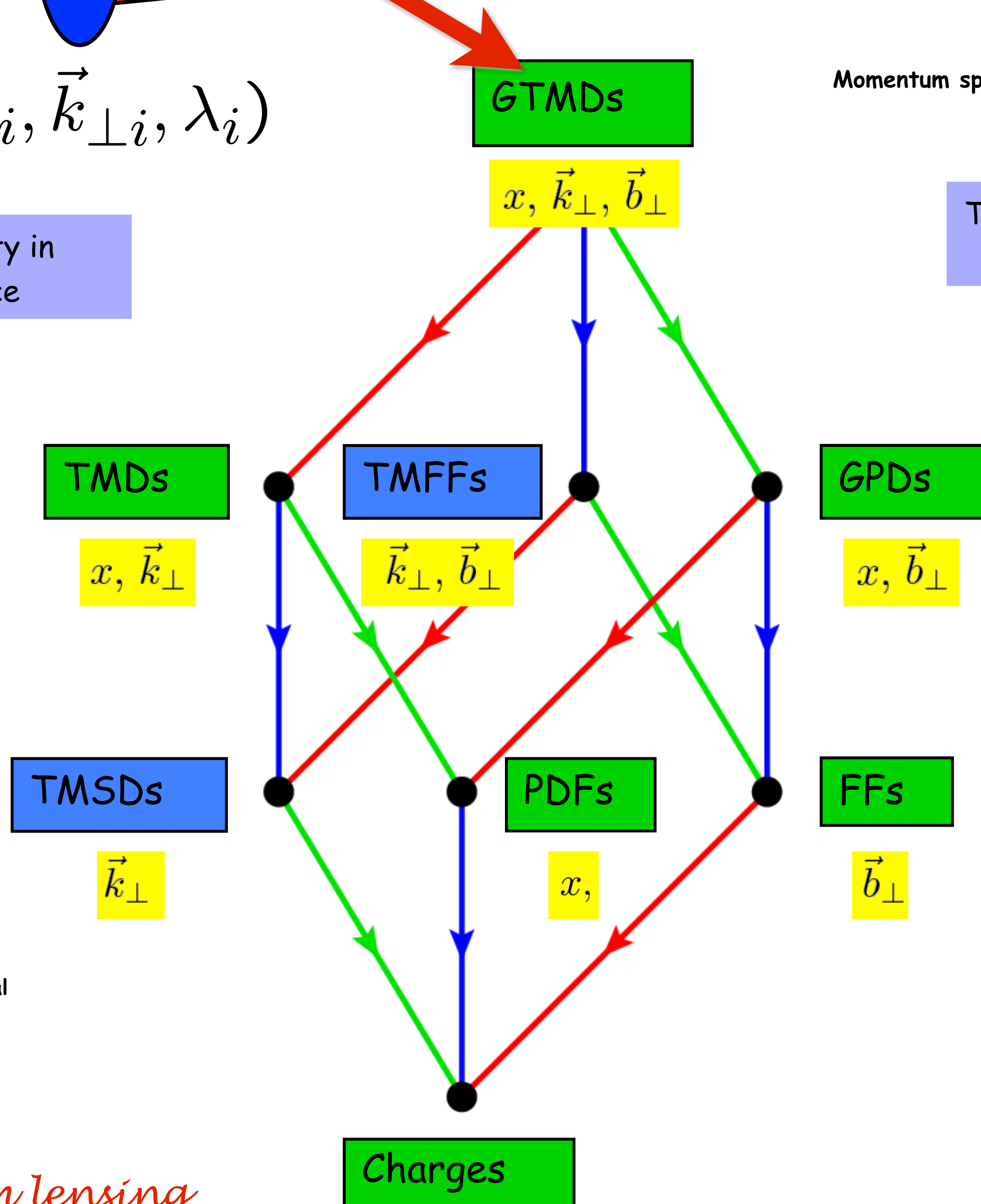
$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

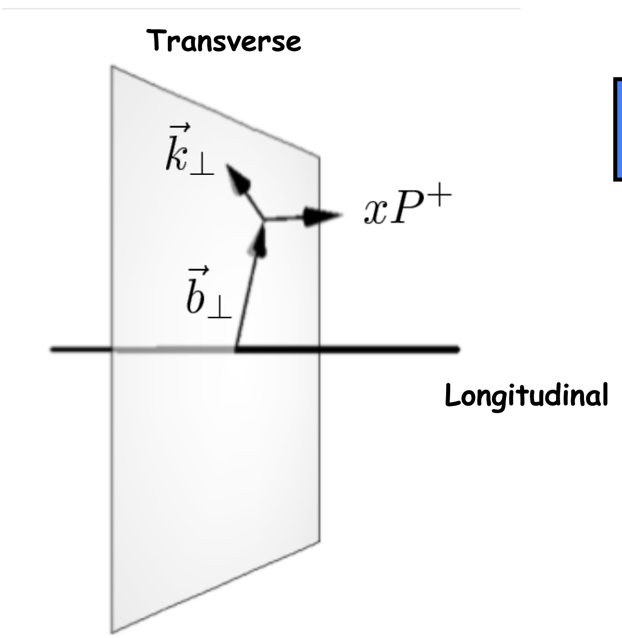
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

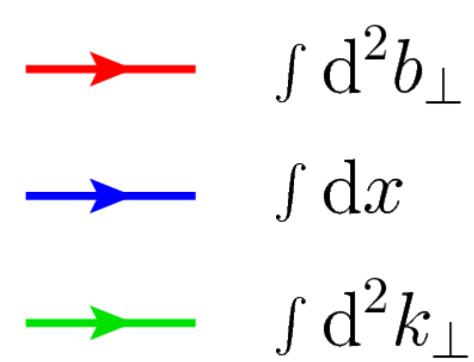
Transverse density in momentum space

Transverse density in position space



Lorce, Pasquini

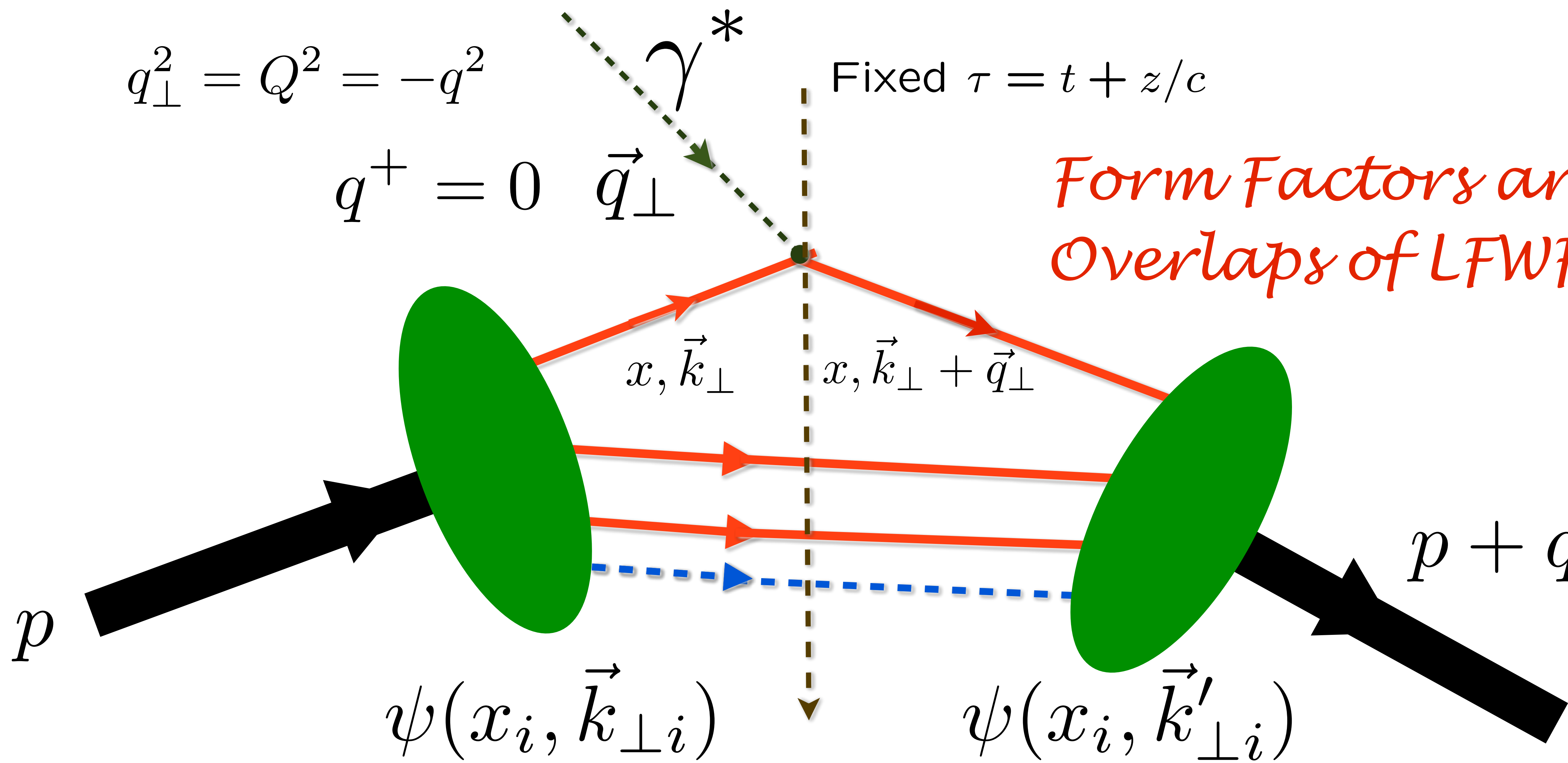




Sivers, T-odd from lensing

$$< p+q|j^+(0)|p > = 2p^+F(q^2)$$

Interaction picture



Form Factors are Overlaps of LFWFs

struck $\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp$

spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Exact LF Formula for Pauli Form Factor

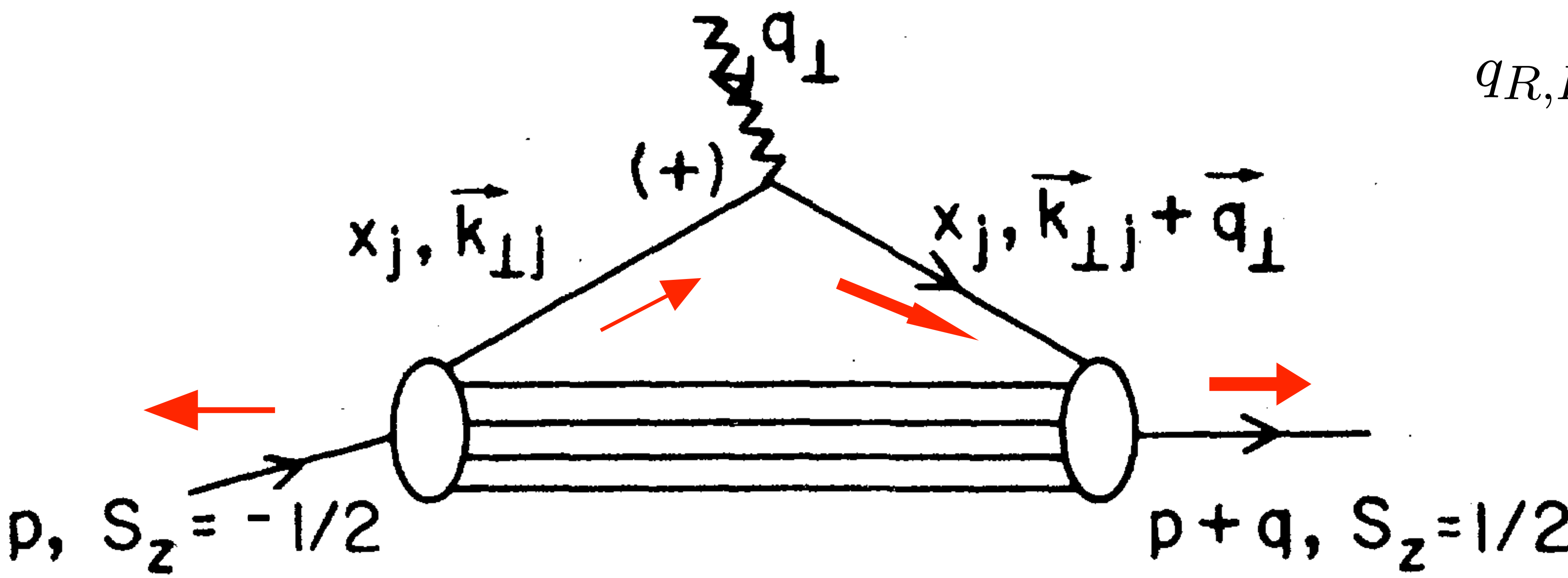
Drell, sjb

$$\frac{F_2(q^2)}{2M} = \sum_a \int [\mathrm{d}x][\mathrm{d}^2\mathbf{k}_\perp] \sum_j e_j \, \frac{1}{2} \times$$

$$\left[-\frac{1}{q^L}\psi_a^{\uparrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) + \frac{1}{q^R}\psi_a^{\downarrow *}(x_i, \mathbf{k}'_{\perp i}, \lambda_i)\, \psi_a^{\uparrow}(x_i, \mathbf{k}_{\perp i}, \lambda_i) \right]$$

$$\mathbf{k}'_{\perp i} = \mathbf{k}_{\perp i} - x_i \mathbf{q}_\perp \qquad \mathbf{k}'_{\perp j} = \mathbf{k}_{\perp j} + (1 - x_j)\mathbf{q}_\perp$$

$$q_{R,L} = q^x \pm i q^y$$



Must have $\Delta \ell_z = \pm 1$ to have nonzero $F_2(q^2)$

Nonzero Proton Anomalous Moment -->
Nonzero orbital quark angular momentum

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Nuclear Physics B 593 (2001) 311–335

NUCLEAR PHYSICS B

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Light-cone representation of the spin and orbital angular momentum of relativistic composite systems ☆

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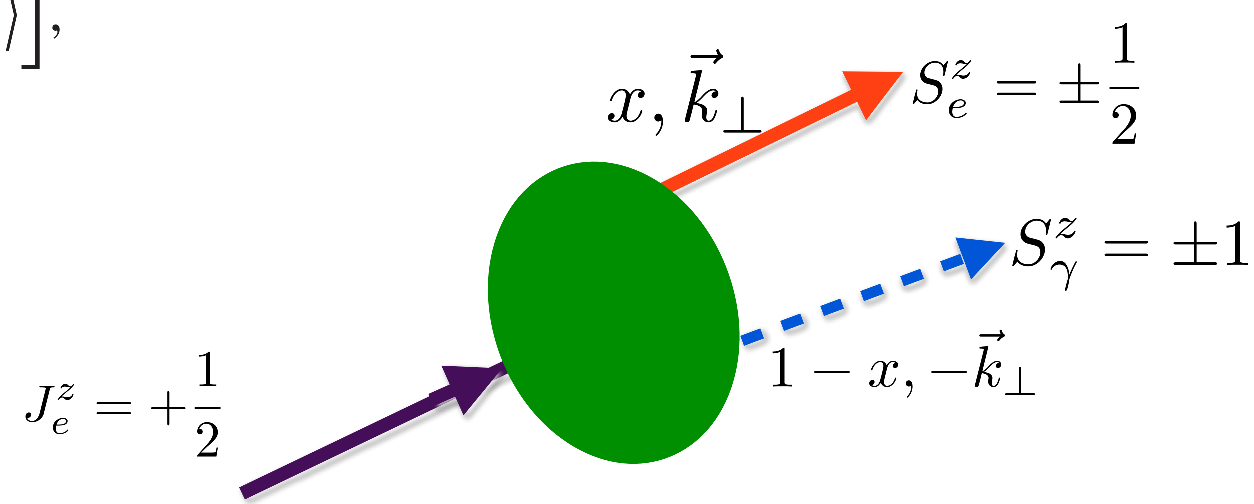
[b] Department of Physics, Sejong University, Seoul 143-747, South Korea
[c] CCAST (World Laboratory), P.O. Box 8730, Beijing 100080, China
[d] Department of Physics, Peking University, Beijing 100871, China
[e] Institute of High Energy Physics, Academia Sinica, P.O. Box 918(4), Beijing 100039, China
[f] Departamento de Física, Universidad Técnica Federico Santa María, Casilla 110-V, Valparaíso, Chile

The two-particle Fock state for an electron with $J^z = +\frac{1}{2}$ has four possible spin combinations:

$$\left|\Psi^{\uparrow}_{\text{two particle}}(P^+, \vec{P}_\perp = \vec{0}_\perp)\right\rangle$$
$$= \int \frac{d^2\vec{k}_\perp\, dx}{\sqrt{x(1-x)}\,16\pi^3}\Big[\psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}+1;\, xP^+,\, \vec{k}_\perp\right\rangle$$
$$+\,\psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp)\left|+\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle + \psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}+1;\, xP^+,\, \vec{k}_\perp\right\rangle$$
$$+\,\psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp)\left|-\tfrac{1}{2}-1;\, xP^+,\, \vec{k}_\perp\right\rangle\Big],$$



$$\begin{cases} \psi^{\uparrow}_{+\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(-k^1 + ik^2)}{x(1-x)}\,\varphi, \\ \psi^{\uparrow}_{+\frac{1}{2}-1}(x,\vec{k}_\perp) = -\sqrt{2}\,\dfrac{(+k^1 + ik^2)}{1-x}\,\varphi, \\ \psi^{\uparrow}_{-\frac{1}{2}+1}(x,\vec{k}_\perp) = -\sqrt{2}\left(M - \dfrac{m}{x}\right)\varphi, \\ \psi^{\uparrow}_{-\frac{1}{2}-1}(x,\vec{k}_\perp) = 0, \end{cases}$$

$$\varphi = \varphi(x,\vec{k}_\perp) = \frac{e/\sqrt{1-x}}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}.$$

Hwang, Schmidt, Ma, sjb

$$
\begin{aligned}
F_2(q^2) &= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \langle \Psi^{\uparrow}(P^+, \vec{P}_\perp = \vec{q}_\perp) | \Psi^{\downarrow}(P^+, \vec{P}_\perp = \vec{0}_\perp) \rangle \\
&= \frac{-2M}{(q^1 - \mathrm{i}q^2)} \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3} \Big[\psi^{\uparrow\,*}_{+\frac{1}{2}-1}(x, \vec{k}'_\perp)\, \psi^{\downarrow}_{+\frac{1}{2}-1}(x, \vec{k}_\perp) \\
&\qquad\qquad + \psi^{\uparrow\,*}_{-\frac{1}{2}+1}(x, \vec{k}'_\perp)\, \psi^{\downarrow}_{-\frac{1}{2}+1}(x, \vec{k}_\perp) \Big] \\
&= 4M \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3} \frac{(m - Mx)}{x} \varphi(x, \vec{k}'_\perp)^* \varphi(x, \vec{k}_\perp) \\
&= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3} \frac{(m - xM)}{x(1-x)} \\
&\quad \times \frac{1}{M^2 - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + m^2)/x - ((\vec{k}_\perp + (1-x)\vec{q}_\perp)^2 + \lambda^2)/(1-x)} \\
&\quad \times \frac{1}{M^2 - (\vec{k}^2_\perp + m^2)/x - (\vec{k}^2_\perp + \lambda^2)/(1-x)}. \qquad (30)
\end{aligned}
$$

$$
F_2(q^2) = \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}\alpha \int_0^1 \mathrm{d}x \, \frac{m - xM}{\alpha(1-\alpha)\frac{1-x}{x}\vec{q}^{\,2}_\perp - M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}}.
$$

$$\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\uparrow \right\rangle = F_1(q^2), \tag{5}$$

$$\left\langle P+q,\uparrow \left| \frac{J^+(0)}{2P^+} \right| P,\downarrow \right\rangle = -(q^1 - \mathrm{i}q^2)\frac{F_2(q^2)}{2M}. \tag{6}$$

The magnetic moment of a composite system is one of its most basic properties. The magnetic moment is defined at the $q^2 \to 0$ limit,

$$\mu = \frac{e}{2M}\left[F_1(0) + F_2(0)\right], \tag{7}$$

where e is the charge and M is the mass of the composite system. We use the standard light-cone frame ($q^\pm = q^0 \pm q^3$):

$$\begin{aligned} q &= (q^+, q^-, \vec{q}_\perp) = \left(0, \frac{-q^2}{P^+}, \vec{q}_\perp\right), \\ P &= (P^+, P^-, \vec{P}_\perp) = \left(P^+, \frac{M^2}{P^+}, \vec{0}_\perp\right), \end{aligned} \tag{8}$$

where $q^2 = -2P\cdot q = -\vec{q}_\perp^{\,2}$ is 4-momentum square transferred by the photon.

The Pauli form factor and the anomalous magnetic moment $\kappa = \frac{e}{2M}F_2(0)$ can then be calculated from the expression

$$-(q^1 - \mathrm{i}q^2)\frac{F_2(q^2)}{2M} = \sum_a \int \frac{\mathrm{d}^2\vec{k}_\perp\, \mathrm{d}x}{16\pi^3} \sum_j e_j\, \psi_a^{\uparrow *}(x_i, \vec{k}'_{\perp i}, \lambda_i)\, \psi_a^{\downarrow}(x_i, \vec{k}_{\perp i}, \lambda_i), \tag{9}$$

where the summation is over all contributing Fock states a and struck constituent charges e_j. The arguments of the final-state light-cone wavefunction are [1,2]

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} + (1 - x_i)\vec{q}_\perp \tag{10}$$

for the struck constituent and

$$\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp \tag{11}$$

The anomalous moment is obtained in the limit of zero momentum transfer:

$$
\begin{aligned}
F_2(0) &= 4Me^2 \int \frac{\mathrm{d}^2\vec{k}_\perp\,\mathrm{d}x}{16\pi^3}\,\frac{(m - xM)}{x(1-x)}\,\frac{1}{[M^2 - (\vec{k}_\perp^{\,2} + m^2)/x - (\vec{k}_\perp^{\,2} + \lambda^2)/(1-x)]^2} \\
&= \frac{Me^2}{4\pi^2} \int_0^1 \mathrm{d}x\, \frac{m - xM}{-M^2 + \frac{m^2}{x} + \frac{\lambda^2}{1-x}},
\end{aligned}
\tag{32}
$$

which is the result of Ref. [8]. For zero photon mass and $M = m$, it gives the correct order α Schwinger value $a_e = F_2(0) = \alpha/2\pi$ for the electron anomalous magnetic moment for QED.

$$\left|\psi_p(P^+,\vec{P}_\perp)\right\rangle = \sum_n \prod_{i=1}^{n} \frac{\mathrm{d}x_i\, \mathrm{d}^2\vec{k}_{\perp i}}{\sqrt{x_i}\,16\pi^3} 16\pi^3 \delta\left(1-\sum_{i=1}^{n} x_i\right) \delta^{(2)}\left(\sum_{i=1}^{n} \vec{k}_{\perp i}\right)$$

$$\times\, \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \left| n;\, x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}, \lambda_i \right\rangle.$$

$$q_{\lambda_q/\Lambda_p}(x, \Lambda) = \sum_{n, q_a} \int \prod_{j=1}^{n} \mathrm{d}x_j\, \mathrm{d}^2\vec{k}_{\perp j} \sum_{\lambda_i} \left|\psi^{(\Lambda)}_{n/H}(x_i, \vec{k}_{\perp i}, \lambda_i)\right|^2$$

$$\times\, \delta\left(1-\sum_{i}^{n} x_i\right) \delta^{(2)}\left(\sum_{i}^{n} \vec{k}_{\perp i}\right) \delta(x - x_q) \delta_{\lambda_a \lambda_q} \Theta(\Lambda^2 - \mathcal{M}_n^2),$$

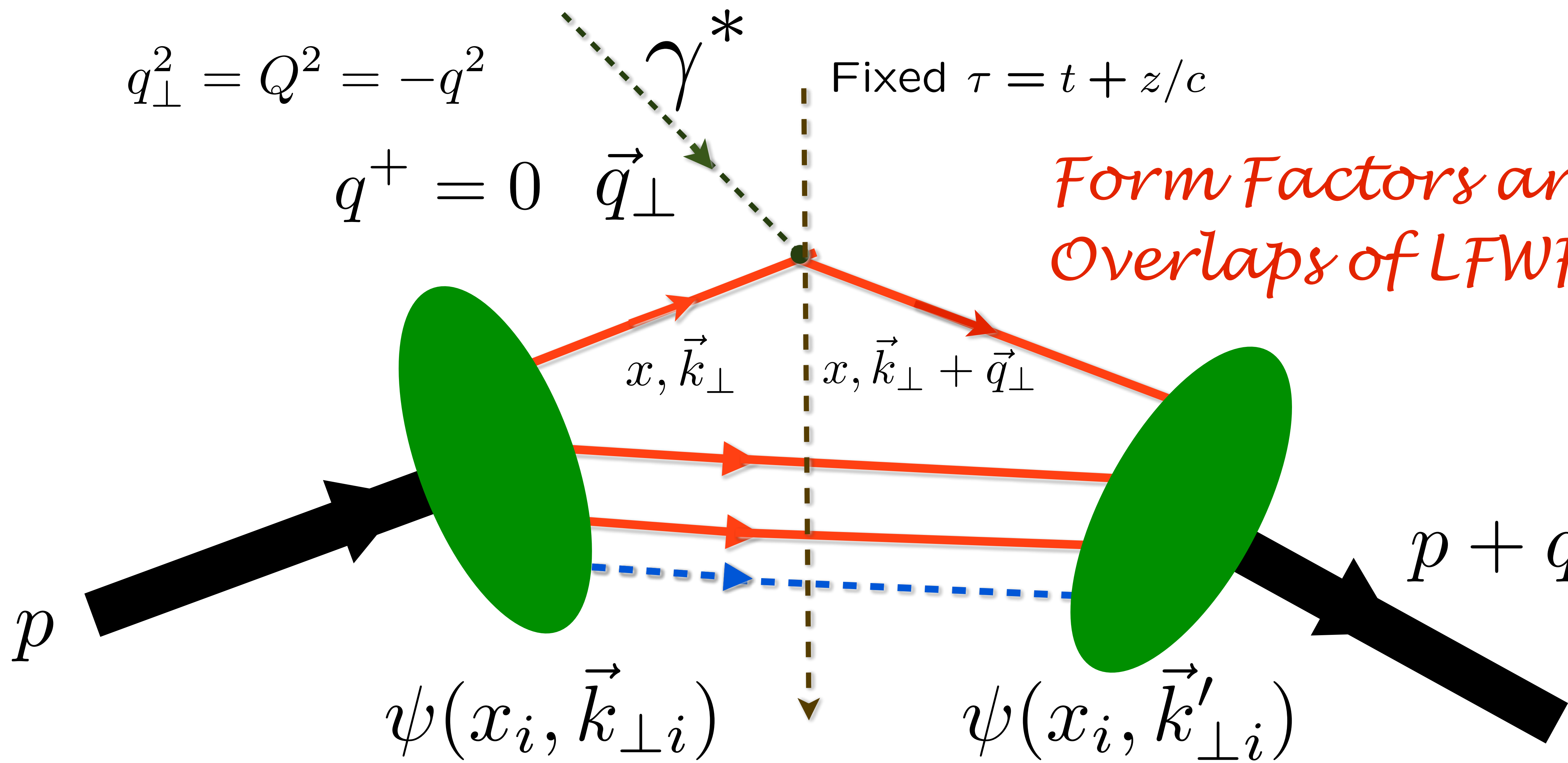
Obeys DGLAP Evolution *Defines quark distributions*

Connection to Bethe-Salpeter:

$$\int dk^- \Psi_{BS}(k, P) \to \psi_{LF}(x, \vec{k}_\perp) \qquad \Psi_{BS}(x, P)|_{x^+=0}$$

$$< p+q|j^+(0)|p >= 2p^+F(q^2)$$

Interaction picture



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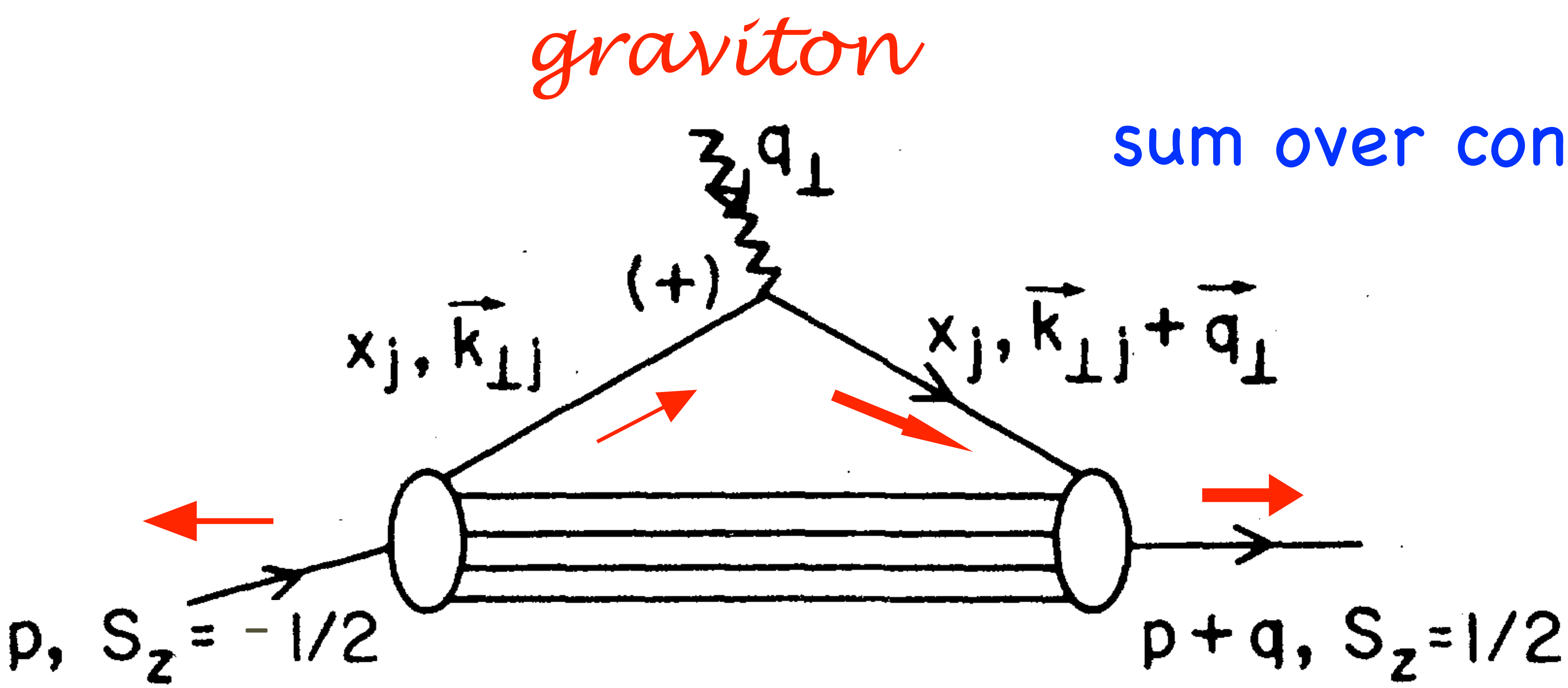
spectators $\vec{k}'_{\perp i} = \vec{k}_{\perp i} - x_i\vec{q}_\perp$

Drell &Yan, West
Exact LF formula!

Vanishing Anomalous gravitomagnetic moment B(0)

Terayev, Okun, et al: *B(0) Must vanish because of Equivalence Theorem*



sum over constituents

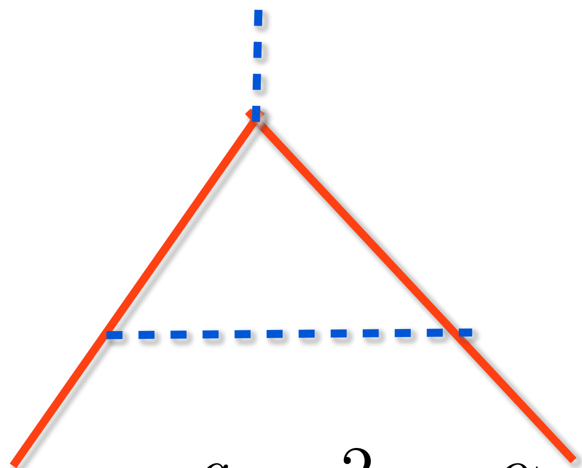
Hwang, Schmidt, sjb;
Holstein et al
Chiu, Lowdon, sub

B(0) = 0

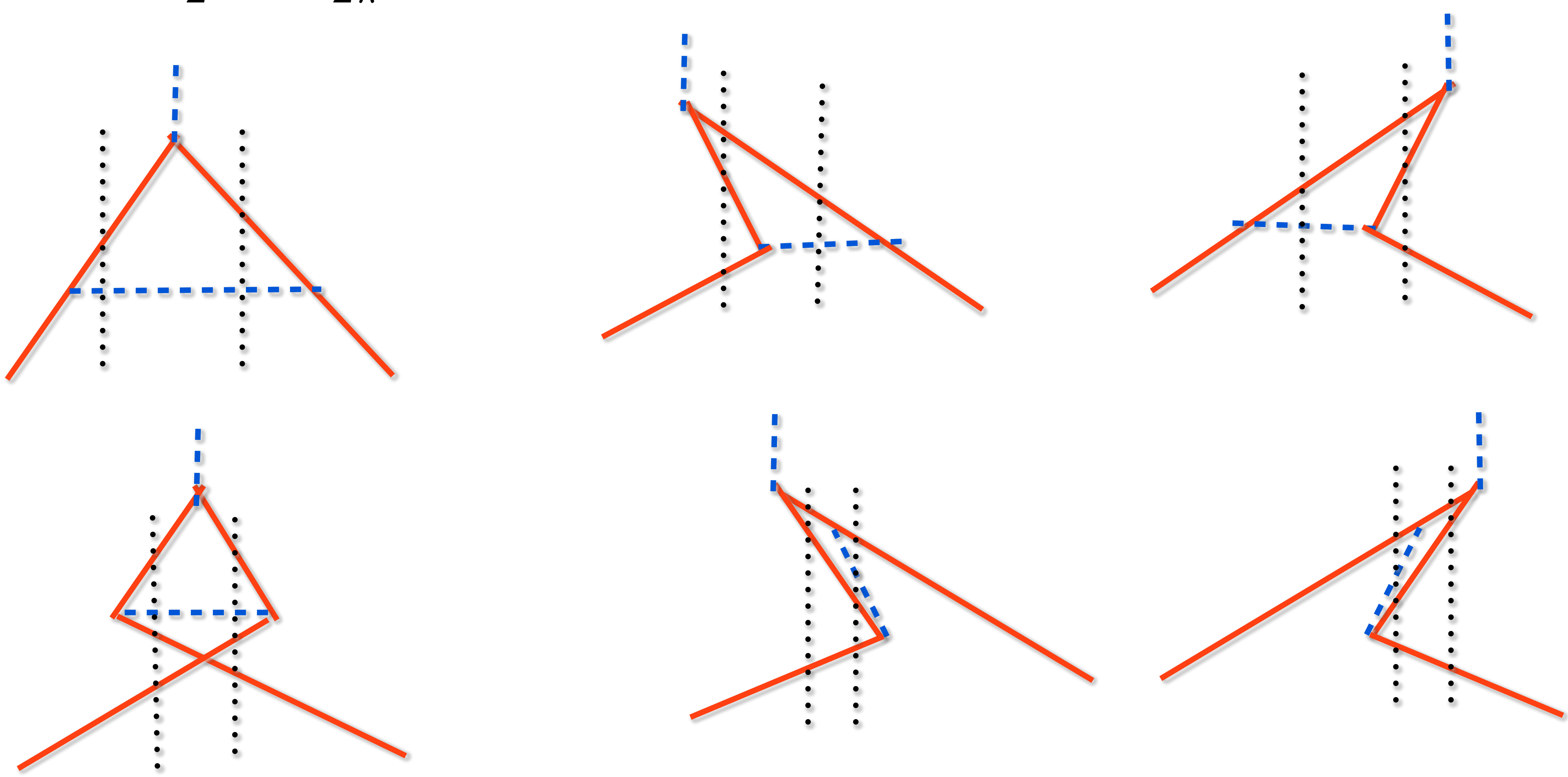
Each Fock State

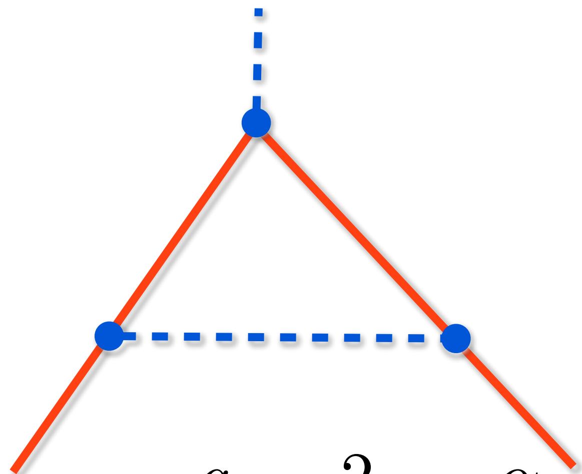
Wick Theorem

Feynman diagram = sum n!
instant-form time-ordered diagrams

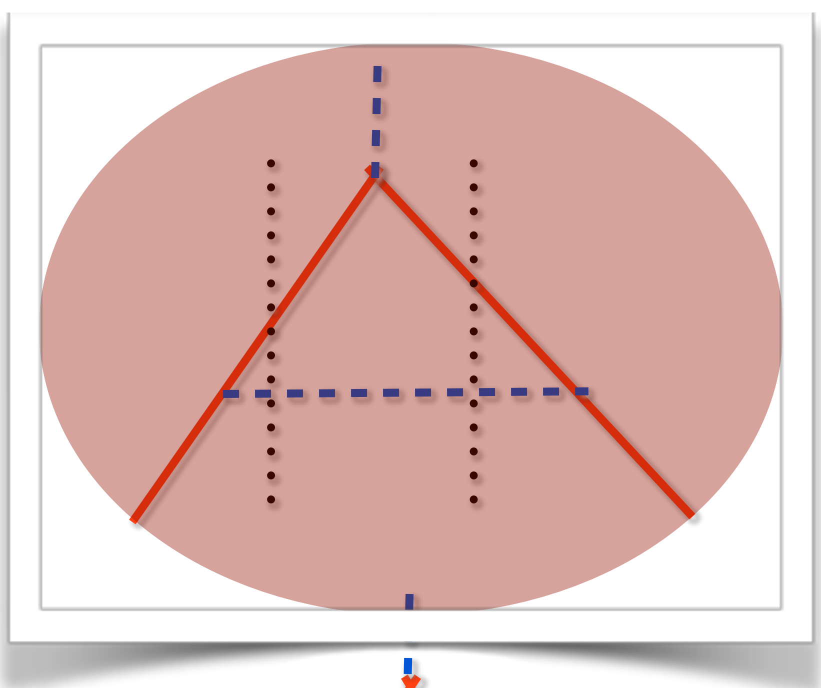


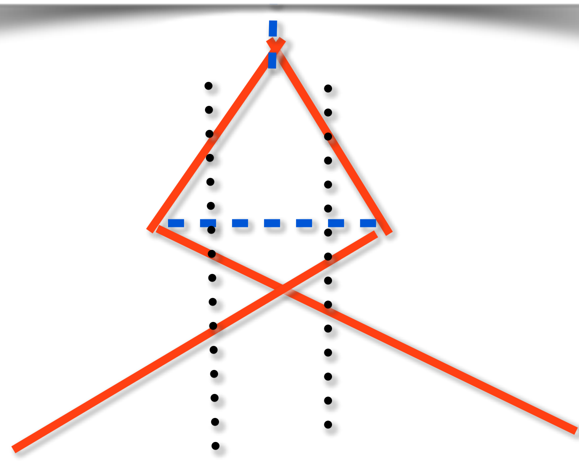
$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$





$$a_e = \frac{g_e - 2}{2} = \frac{\alpha}{2\pi}$$

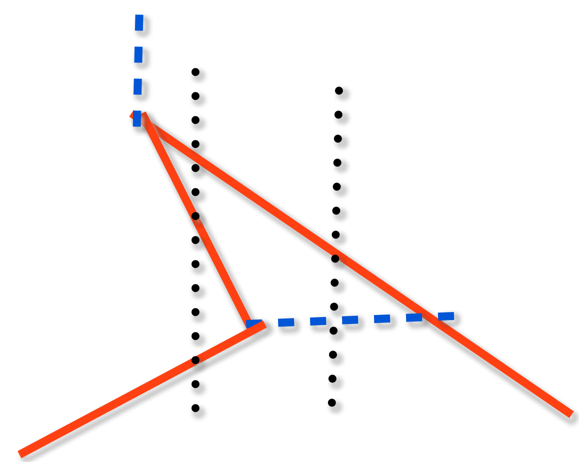


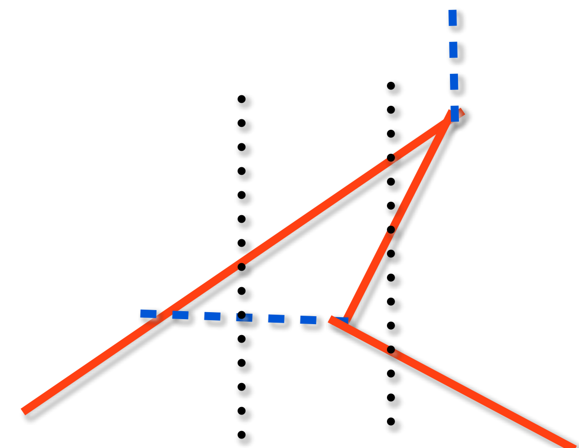


Wick Theorem

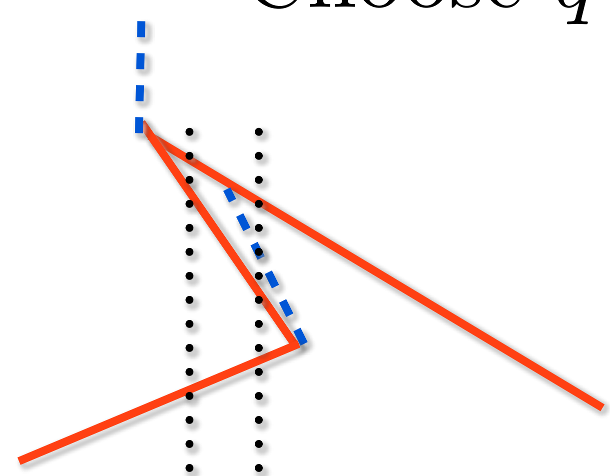
Feynman diagram =
single front-form time-ordered diagram!

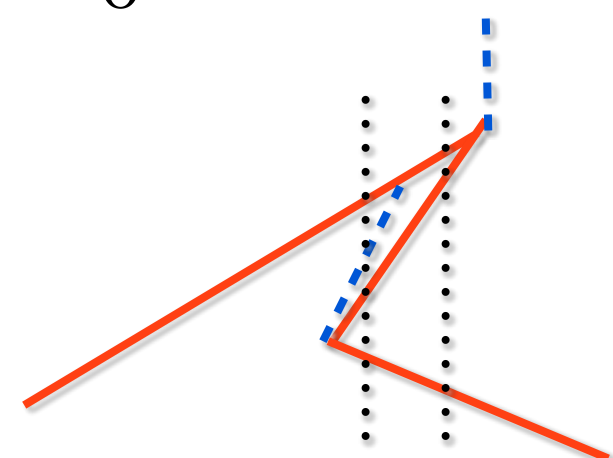
Also $P \to \infty$ observer frame (Weinberg)





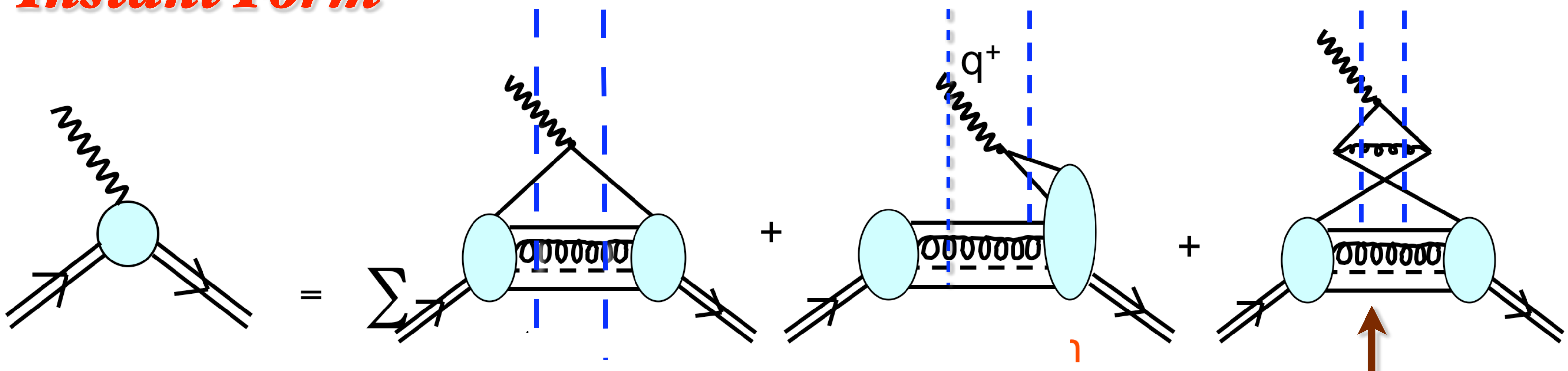
Choose $q^+ = 0$





Calculation of Form Factors in Equal-Time Theory

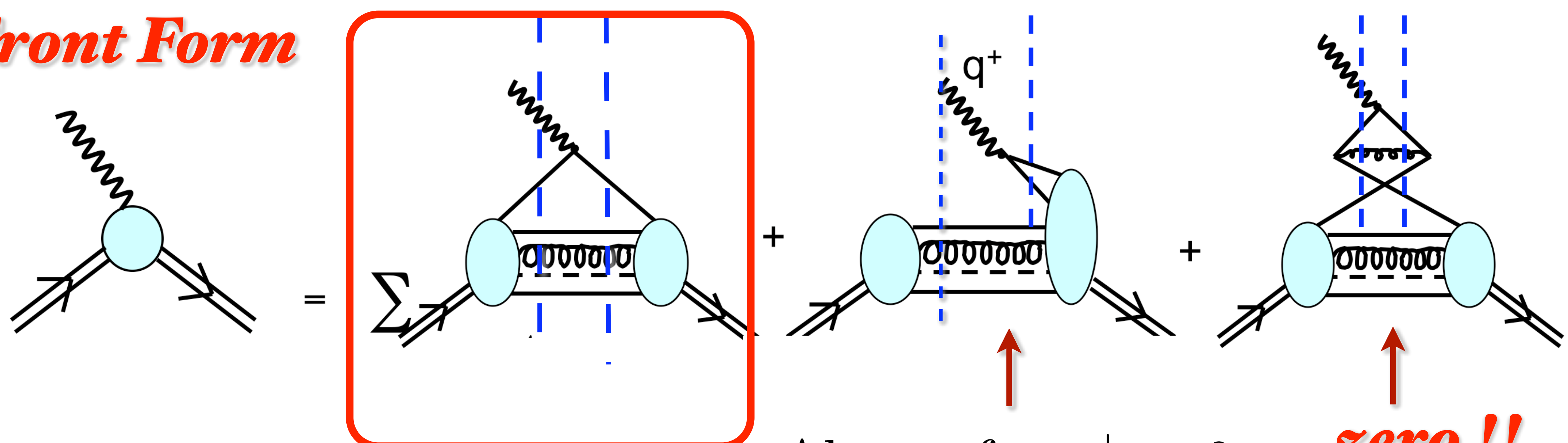
Instant Form



Need vacuum-induced currents

Calculation of Form Factors in Light-Front Theory

Front Form



Exact Answer!

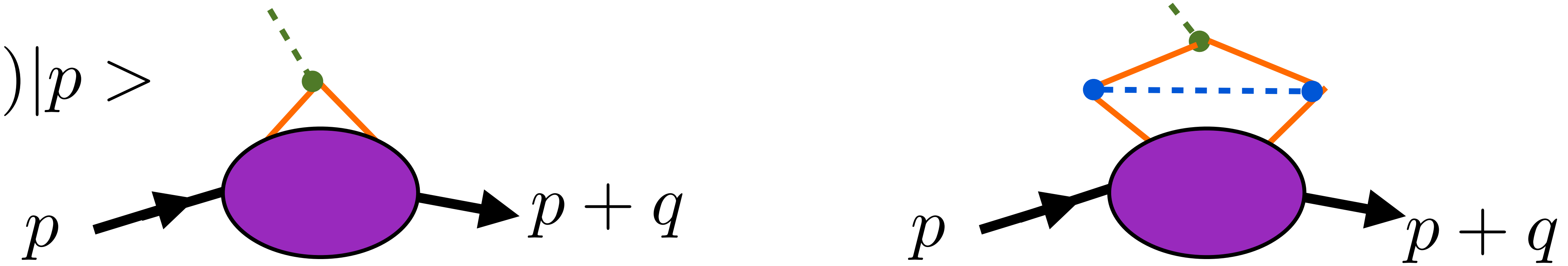
Absent for $q^+ = 0$

zero !!

No vacuum graphs

Calculation of proton form factor in Instant Form

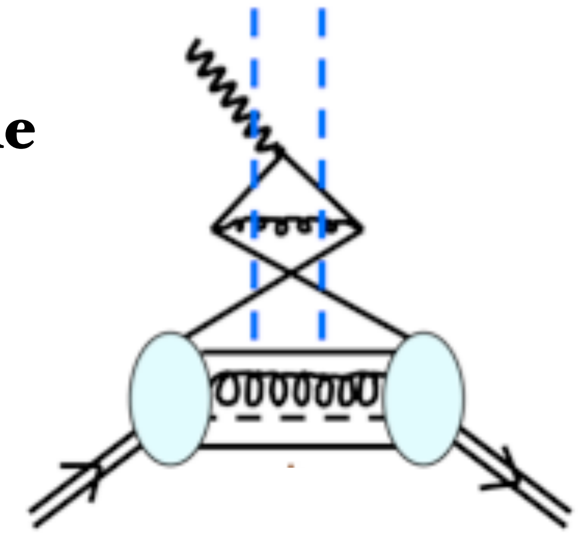
$< p+q|J^{\mu}(0)|p >$



- **Need to boost proton wavefunction from p to p+q: Extremely complicated dynamical problem; even the particle number changes**
- **Need to couple to all currents arising from vacuum!! Remains even after normal-ordering**
- **Each time-ordered contribution is frame-dependent**
- **Divide by disconnected vacuum diagrams**
- **Instant form: acausal boundary conditions**

Disadvantages of the Instant Form

- Boosts are dynamical, change particle number: not Melosh!
- Famous wrong proof showing violation of LET and DHG sum rule
- States defined at one instant of time over all space - acausal!
- Current matrix elements involve connected vacuum currents -- eigensolutions insufficient!
- N! time-ordered graphs, each frame-dependent
- Vacuum is complex: apparently gives huge vacuum energy density
- Normal-ordering required to compute observables
- Cluster decomposition theorem fails in relativistic systems
- Virtually no valid calculations of dynamics of relativistic composite systems use the instant form
- Why Feynman invented Feynman diagrams!



Drell Hearn Gerasimov Sum Rule

$$\int_{\omega_{\rm th}}^{\infty} \frac{\sigma_P(\omega) - \sigma_A(\omega)}{\omega} d\omega = 8\pi^2 (\mu - \frac{Z_T e}{2\mathcal{M}})^2$$

anomalous magnetic moment squared

Proof

Optical Theorem from Unitarity

Forward spin-flip amplitude given by LET $M^{\uparrow\to\downarrow}(\theta = 0)$

Un-subtracted dispersion relation

$$M_{fi} = \frac{1}{2\omega}(2\pi)^3\, \delta^3(P_f - P_i)\left[\frac{{Z_T}^2 e^2}{\mathcal{M}}\hat{\mathbf{e}}' \cdot \hat{\mathbf{e}}\delta_{fi} + 2i\omega\left(\mu - \frac{Z_T e}{2\mathcal{M}}\right)^2 \boldsymbol{\sigma}_{fi} \cdot \hat{\mathbf{e}}' \times \hat{\mathbf{e}} + O(\omega^2)\right]$$

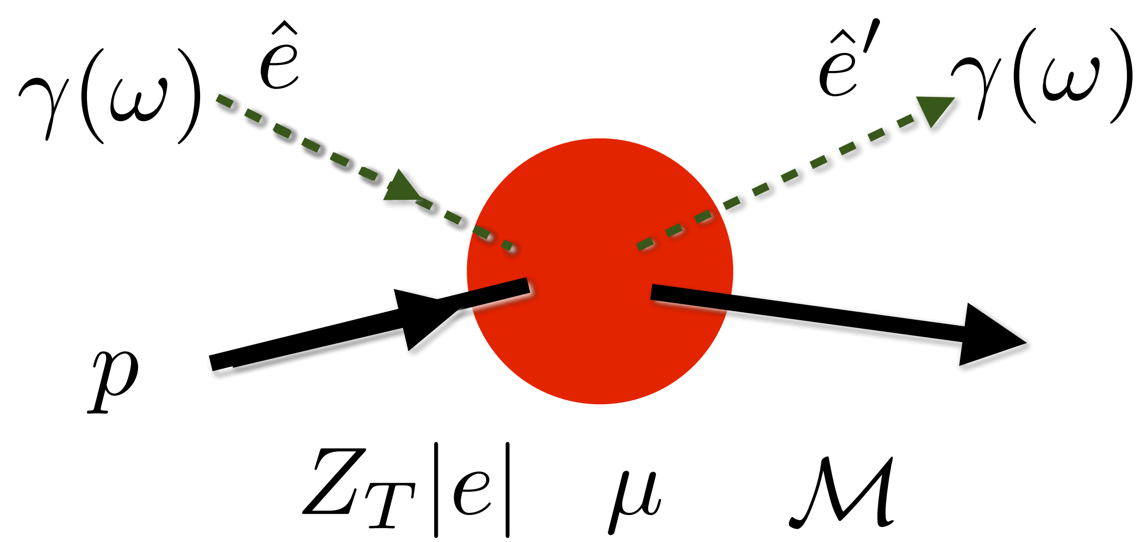
Erroneous claim (Barton &Dombey): LET and DHG Wrong!

Low Energy Forward Compton Scattering

Low energy theorem: Spin-1/2 Target

$$S_{fi} = -2\pi i \delta(E_f - E_i)\, M_{fi}$$

$$M_{fi} = \frac{1}{2\omega}(2\pi)^3\, \delta^3(P_f - P_i)\left[\frac{{Z_T}^2 e^2}{\mathcal{M}}\,\hat{\mathbf{e}}' \cdot \hat{\mathbf{e}}\,\delta_{fi} + 2i\omega\left(\mu - \frac{Z_T e}{2\mathcal{M}}\right)^2 \boldsymbol{\sigma}_{fi} \cdot \hat{\mathbf{e}}' \times \hat{\mathbf{e}} + O(\omega^2)\right]$$



Amplitude determined by static properties of target

$k \cdot p = \omega \mathcal{M}$ Photon lab energy $\omega \to 0, \theta \to 0$

Erroneous claim (Barton &Dombey): LET Wrong!

PHYSICAL REVIEW VOLUME 174, NUMBER 5 25 OCTOBER 1968

Electromagnetic Interactions of Loosely-Bound Composite Systems*

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Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305
(Received 13 June 1968)

Contrary to popular assumption, the interaction of a composite system with an external electromagnetic field is not equal to the sum of the individual Foldy-Wouthyusen interactions of the constituents if the constituents have spin. We give the correct interaction, and note that it is consistent with the Drell-Hearn-Gerasimov sum rule and the low-energy theorem for Compton scattering. We also discuss the validity of additivity of the individual Dirac interactions, and the corrections to this approximation, with particular reference to the atomic Zeeman effect, which is of importance in the fine-structure and Lamb-shift measurements.

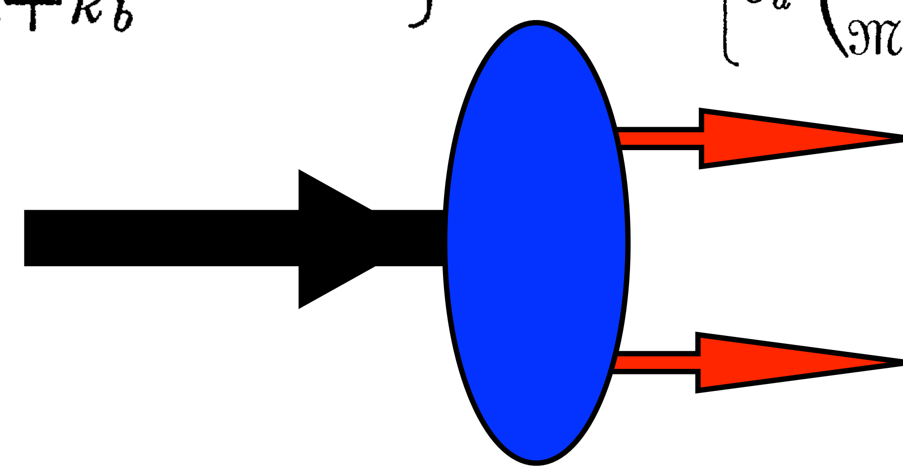
Dynamical boost contribution

$$\begin{Bmatrix} 1 \\ \dfrac{1}{2m_a+k_a}\boldsymbol{\sigma}_a\cdot\mathbf{p} \end{Bmatrix} \otimes \begin{Bmatrix} 1 \\ \dfrac{1}{2m_b+k_b}\boldsymbol{\sigma}_b\cdot(-\mathbf{p}) \end{Bmatrix}$$

$$\vec{P} \neq 0$$

$$\begin{Bmatrix} 1+\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{Bmatrix} \otimes \begin{Bmatrix} 1-\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathfrak{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathfrak{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{Bmatrix}$$

Instant Form WF



Also: Hugh Osborne

Single particle wave-packet

Primack, sjb

$$\phi(x) = \int \frac{d^3p}{(2\pi)^{3/2}} \sqrt{\frac{m}{p^0}}\, u(p)\, \phi(p)\, e^{-ip.x}$$

$$u(p) = \sqrt{\frac{p^0+m}{2m}} \begin{pmatrix} 1 \\ \frac{\boldsymbol{\sigma}\cdot\mathbf{p}}{p^0+m} \end{pmatrix} \chi.$$

Instant Form Wavefunction of moving bound state:

Not product of independent boosts!

$$\varphi_{EP}(\mathbf{x}_a \quad \mathbf{x}_b\,,\, X^0)_{SM}$$

$$= \frac{E+\mathscr{M}}{2\mathscr{M}} \int \frac{d^3p}{(2\pi)^{3/2}} \left(\frac{p_a{}^0+m_a}{2p_a{}^0}\,\frac{p_b{}^0+m_b}{2p_b{}^0}\right)^{1/2}$$

$$\times \begin{pmatrix} 1+\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{P}}{\mathscr{M}+E}\,\dfrac{\boldsymbol{\sigma}_a\cdot\mathbf{p}}{2m_a+k_a} \\ \boldsymbol{\sigma}_a\cdot\left(\dfrac{\mathbf{P}}{\mathscr{M}+E}+\dfrac{\mathbf{p}}{2m_a+k_a}\right) \end{pmatrix} \otimes \begin{pmatrix} 1-\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{P}}{\mathscr{M}+E}\,\dfrac{\boldsymbol{\sigma}_b\cdot\mathbf{p}}{2m_b+k_b} \\ \boldsymbol{\sigma}_b\cdot\left(\dfrac{\mathbf{P}}{\mathscr{M}+E}-\dfrac{\mathbf{p}}{2m_b+k_b}\right) \end{pmatrix}$$

$$\times\ \phi_{\mathscr{M}}(\mathbf{p})\, \chi_{SM} \exp[i\mathbf{p}\cdot\tilde{\mathbf{x}} + i\mathbf{P}\cdot\mathbf{X}] \exp[-iEX^0].$$

$$\tilde{\mathbf{x}} = \mathbf{x} + (\gamma-1)\,\hat{\mathbf{V}}\hat{\mathbf{V}}\cdot\mathbf{x} \;;\quad p^0_{a,b} = \sqrt{\mathbf{p}^2+m^2_{a,b}}\,,\quad k_{a,b} \equiv -\tau_{b,a}(U+W).$$

Correct Boosted Wavefunction needed for LET, DGH!

Remarkable Advantages of the Front Form

- Light-Front Time-Ordered Perturbation Theory: Elegant, Physical
- Frame-Independent
- Few LF Time-Ordered Diagrams (not n!) -- all k^+ must be positive
- $J^z = L^z + S^z$ conserved at each vertex
- Automatically normal-ordered; LF Vacuum trivial up to zero modes
- Renormalization: Alternate Denominator Subtractions: Tested to three loops in QED
- Reproduces Parke-Taylor Rules and Amplitudes (Stasto)
- Hadronization at the Amplitude Level with Confinement

Angular Momentum on the Light-Front

LC gauge A⁺=0

$$J^z = \sum_{i=1}^{n} s_i^z + \sum_{j=1}^{n-1} l_j^z .$$

Conserved
LF Fock state by Fock State

Gluon orbital angular momentum defined in physical lc gauge

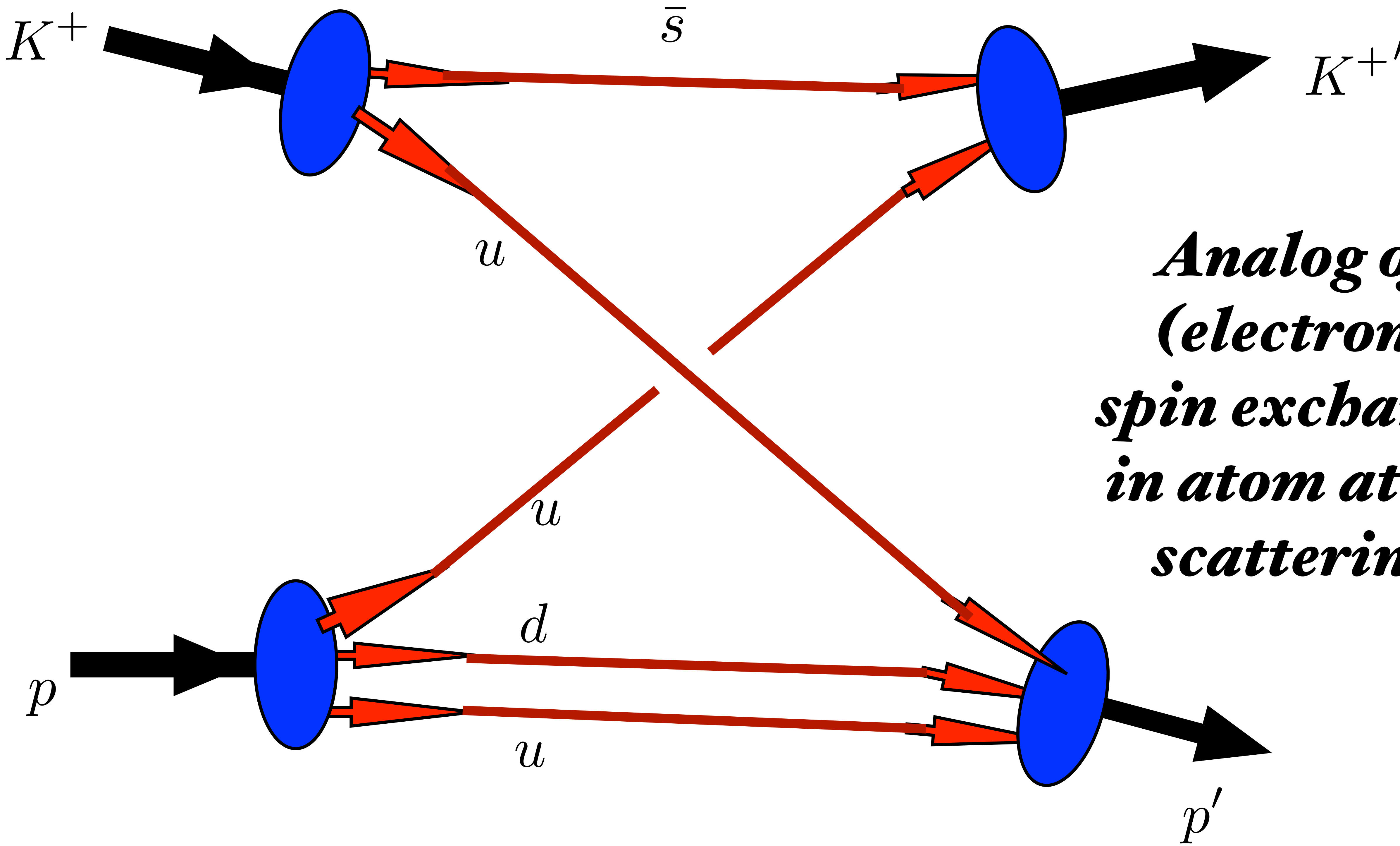
$$l_j^z = -\mathrm{i}\left(k_j^1 \frac{\partial}{\partial k_j^2} - k_j^2 \frac{\partial}{\partial k_j^1}\right)$$

n-1 orbital angular momenta

Orbital Angular Momentum is a property of LFWFS

Nonzero Anomalous Moment -->
Nonzero quark orbital angular momentum!

$$K^+ p \to K^+ p$$

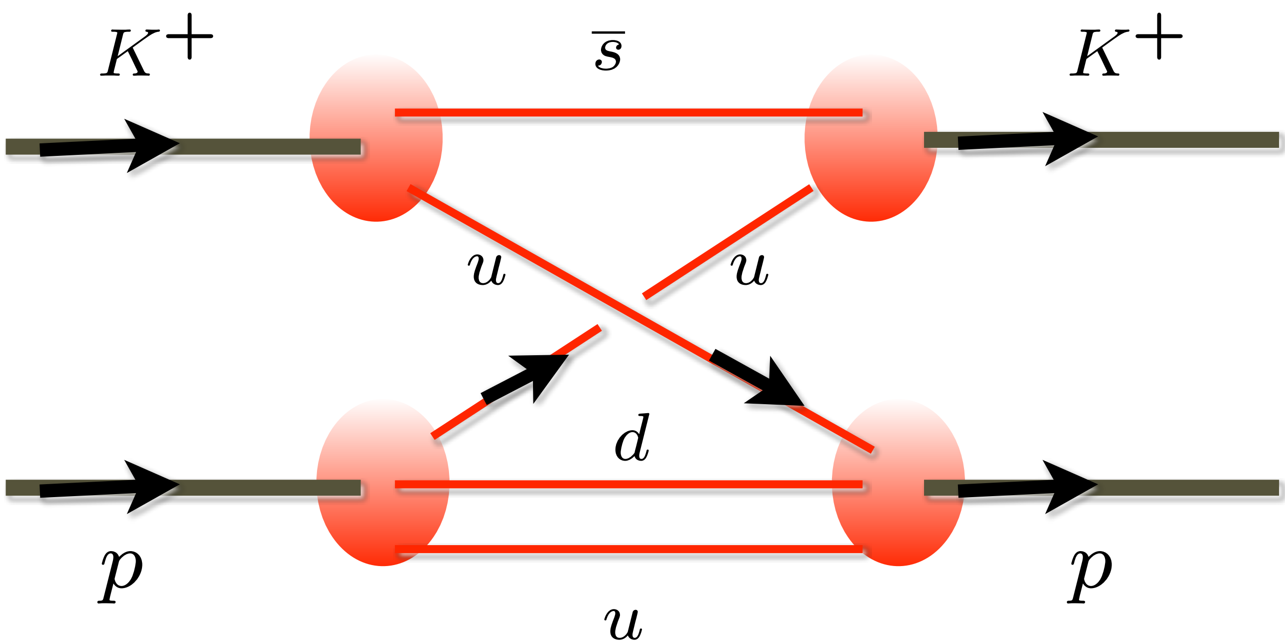


Analog of (electron) spin exchange in atom atom scattering

Constituent Interchange

Blankenbecler, Gunion, sjb

Blankenbecler, Gunion, sjb



$$\frac{d\sigma}{dt} = \frac{|M(s,t)|^2}{s^2}$$

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

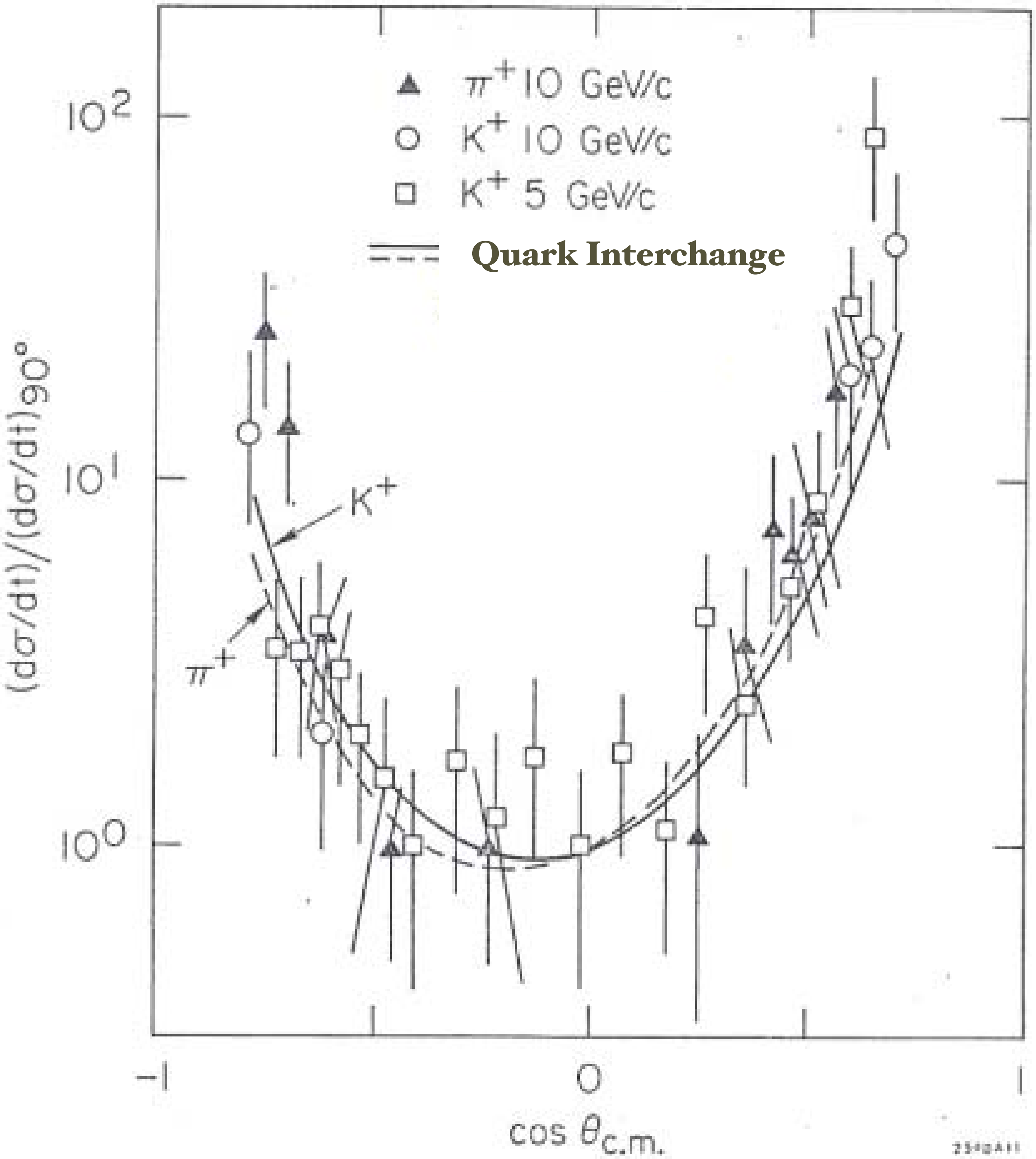
$$M(s,t)_{A+B\to C+D}$$

$$= \frac{1}{2(2\pi)^3}\int d^2k \int_0^1 \frac{dx}{x^2(1-x)^2}\,\Delta\psi_C(\vec{k}_\perp - x\vec{r}_\perp, x)\psi_D(\vec{k}_\perp + (1-x)\vec{q}_\perp, x)\psi_A(\vec{k}_\perp - x\vec{r}_\perp + (1-x)\vec{q}_\perp, x)\psi_B(\vec{k}_\perp, x)$$

$$\Delta = s - \sum_i \frac{k_{\perp i}^2 + m_i^2}{x_i}$$

Product of four light-front wavefunctions

Agrees with electron exchange in atom-atom scattering in nonrelativistic limit



AdS/CFT explains why quark interchange is dominant interaction at high momentum transfer in exclusive reactions

$$M(t,u)_{\text{interchange}} \propto \frac{1}{ut^2}$$

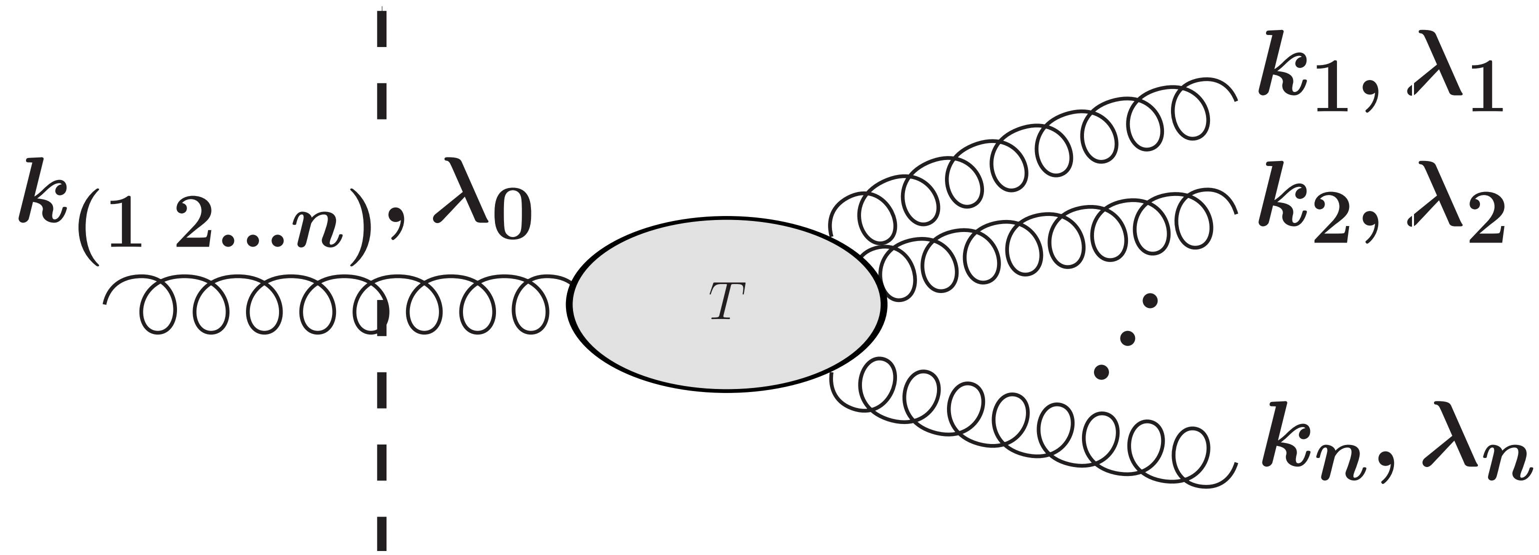
Non-linear Regge behavior:

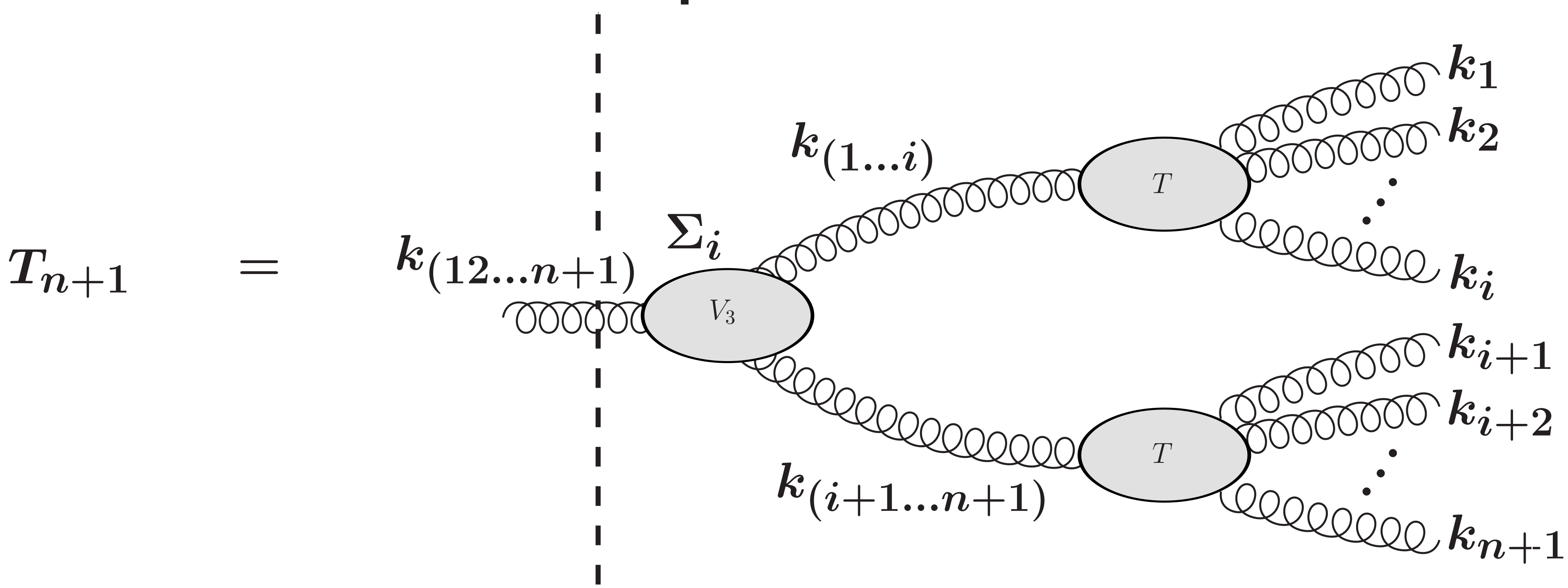
$$\alpha_R(t) \to -1$$

Recursion Relations and Scattering Amplitudes in the Light-Front Formalism

Cruz-Santiago & Stasto

Cluster Decomposition Theorem for relativistic systems: **C. Ji & sjb**





Parke-Taylor amplitudes reflect LF angular momentum conservation

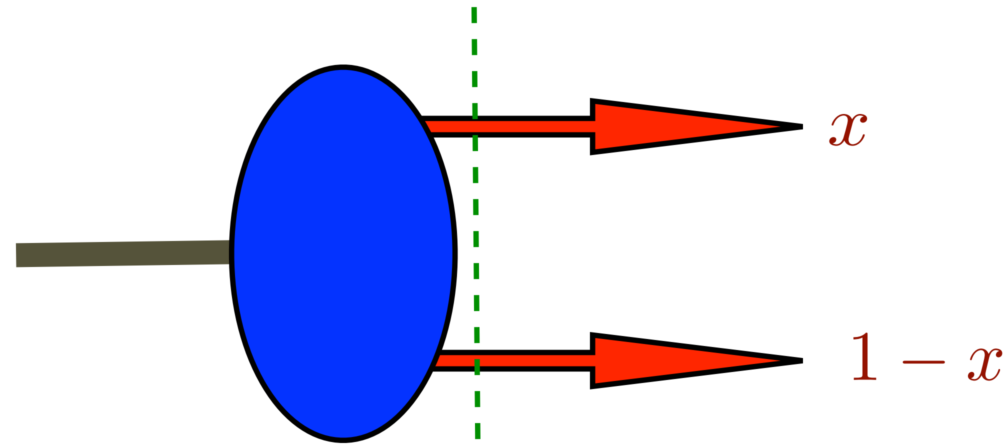
$$\langle i j \rangle = \sqrt{z_i z_j}\, \underline{\epsilon}^{(-)} \cdot \left(\frac{\underline{k}_i}{z_i} - \frac{\underline{k}_j}{z_j} \right) =$$

***K. Chiu* & sjb**

Hadron Distribution Amplitudes

$A^+ = 0$

$$\phi_M(x, Q) = \int^Q d^2\vec{k}\ \psi_{q\bar{q}}(x, \vec{k}_\perp)$$



$k_\perp^2 < Q^2$

$$\sum_i x_i = 1$$

Fixed $\tau = t + z/c$

- Fundamental gauge invariant non-perturbative input to hard exclusive processes, heavy hadron decays. Defined for Mesons, Baryons

Lepage, sjb

Efremov, Radyushkin

- Evolution Equations from PQCD, OPE

Sachrajda, Frishman Lepage, sjb

- Conformal Expansions

Braun, Gardi

- Compute from valence light-front wavefunction in light-cone gauge

Gravitational Form Factors

$$\langle P'|T^{\mu\nu}(0)|P\rangle = \overline{u}(P')\Big[A(q^2)\gamma^{(\mu}\overline{P}^{\nu)} + B(q^2)\frac{i}{2M}\overline{P}^{(\mu}\sigma^{\nu)\alpha}q_\alpha + C(q^2)\frac{1}{M}(q^\mu q^\nu - g^{\mu\nu}q^2)\Big]u(P) ,$$

where $q^\mu = (P'-P)^\mu$, $\overline{P}^\mu = \frac{1}{2}(P'+P)^\mu$, $a^{(\mu}b^{\nu)} = \frac{1}{2}(a^\mu b^\nu + a^\nu b^\mu)$

$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\uparrow\right\rangle = A(q^2) ,$$

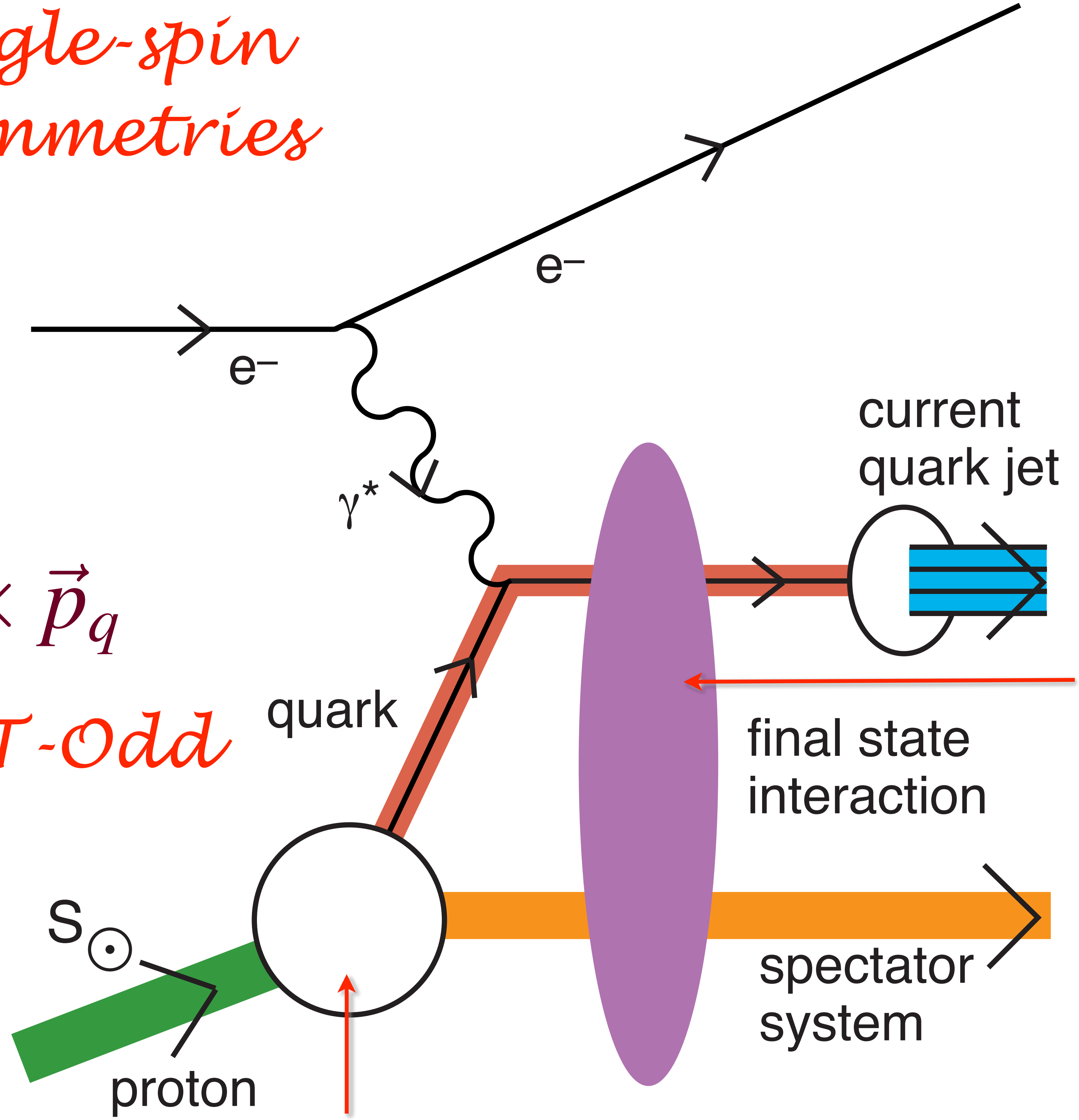
$$\left\langle P+q,\uparrow\left|\frac{T^{++}(0)}{2(P^+)^2}\right|P,\downarrow\right\rangle = -(q^1 - \mathrm{i}q^2)\frac{B(q^2)}{2M} .$$

Single-spin asymmetries

Leading Twist Sivers Effect

Hwang, Schmidt, sjb

Collins, Burkardt, Ji, Yuan. Pasquini, ...



$$i \ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

Pseudo- T-Odd

QCD S- and P-Coulomb Phases --Wilson Line

“Lensing Effect”

QED: Lensing involves soft scales

Light-Front Wavefunction S and P- Waves!

Leading-Twist Rescattering Violates pQCD Factorization!

Sign reversal in DY!

Static	Dynamic
Square of Target LFWFs	Modified by Rescattering: ISI & FSI
No Wilson Line	Contains Wilson Line, Phases
Probability Distributions	No Probabilistic Interpretation
Process-Independent	Process-Dependent - From Collision
T-even Observables	T-Odd (Sivers, Boer-Mulders, etc.)
No Shadowing, Anti-Shadowing	Shadowing, Anti-Shadowing, Saturation
Sum Rules: Momentum and J^z	Sum Rules Not Proven
DGLAP Evolution; mod. at large x	DGLAP Evolution
No Diffractive DIS	Hard Pomeron and Odderon Diffractive DIS

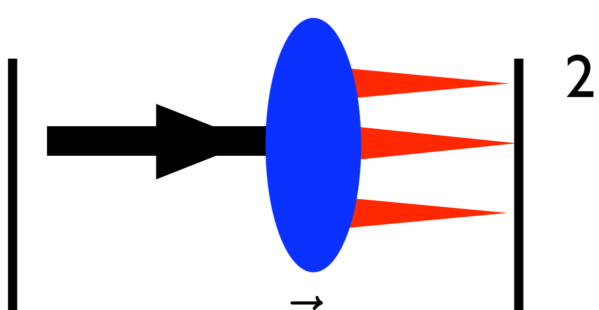
Hwang, Schmidt, sjb,

Mulders, Boer

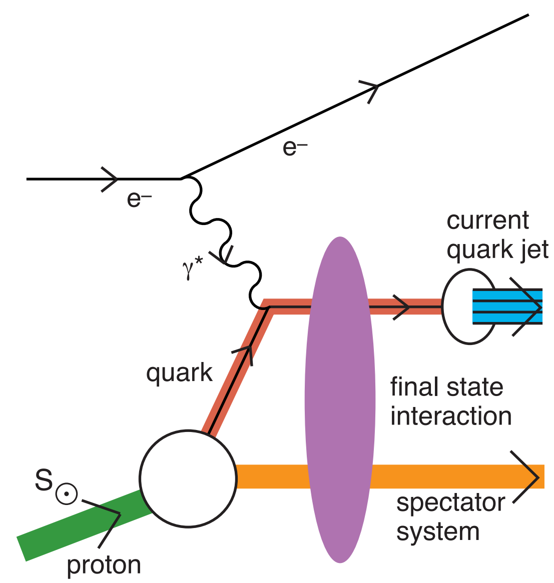
Qiu, Sterman

Collins, Qiu

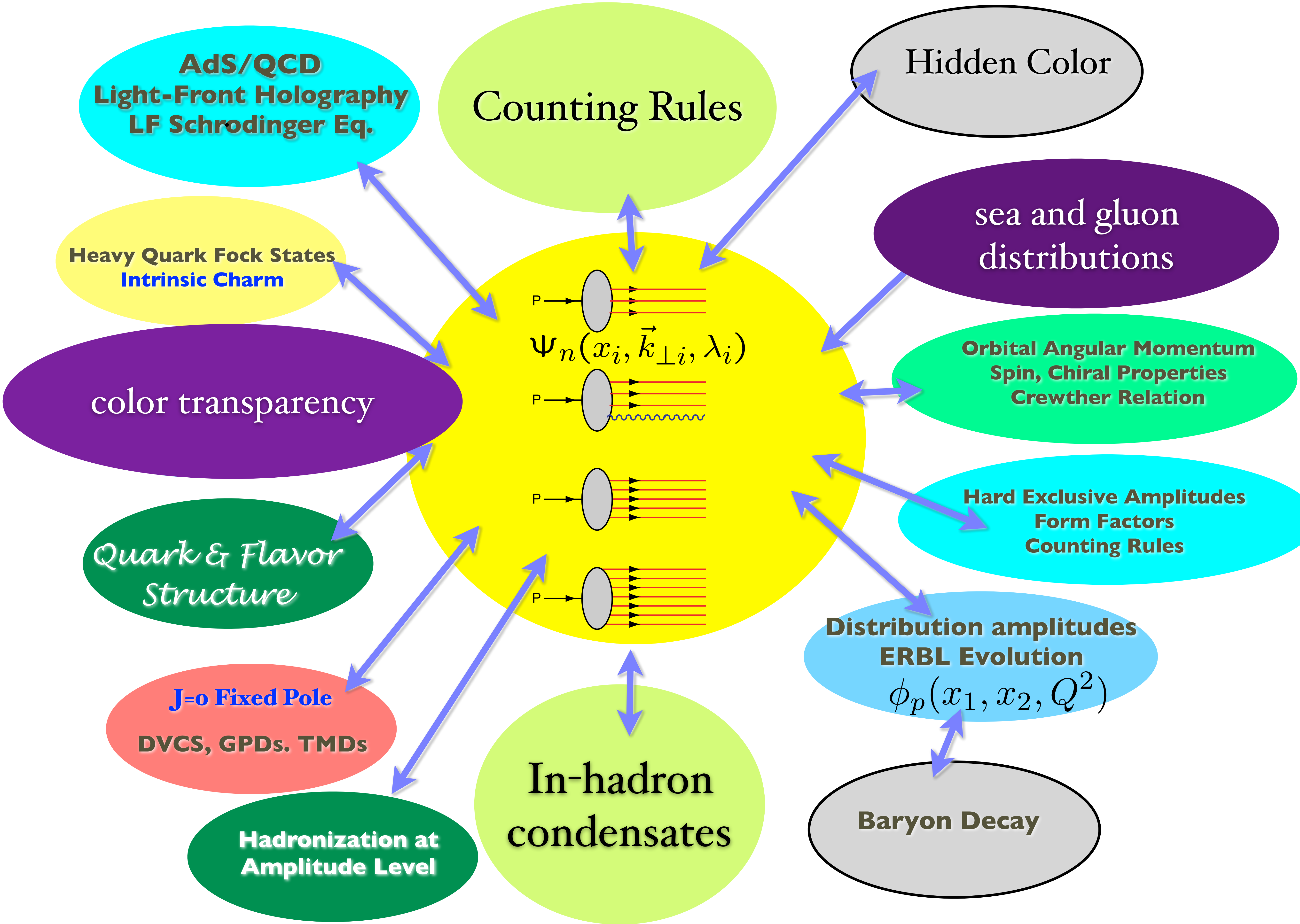
Pasquini, Xiao, Yuan, sjb



$$\left| \Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) \right|^2$$



QCD and the LF Hadron Wavefunctions



Hidden Color in QCD

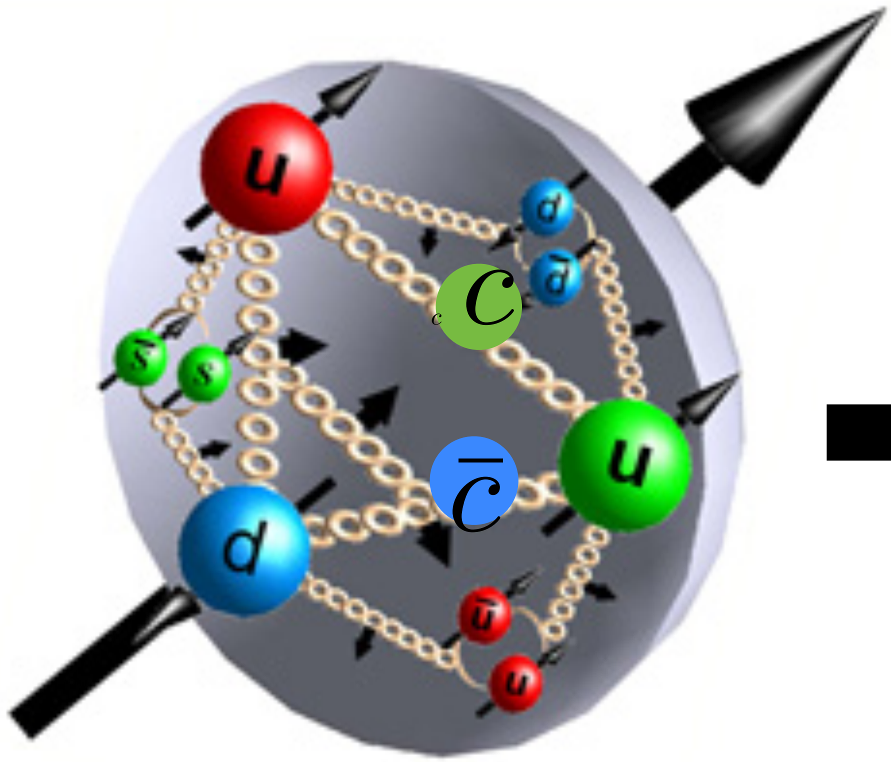
Lepage, Ji, sjb

- Deuteron six quark wavefunction:
- 5 color-singlet combinations of 6 color-triplets -- one state is |n p>
- Components evolve towards equality at short distances
- Hidden color states dominate deuteron form factor and photodisintegration at high momentum transfer
- Predict

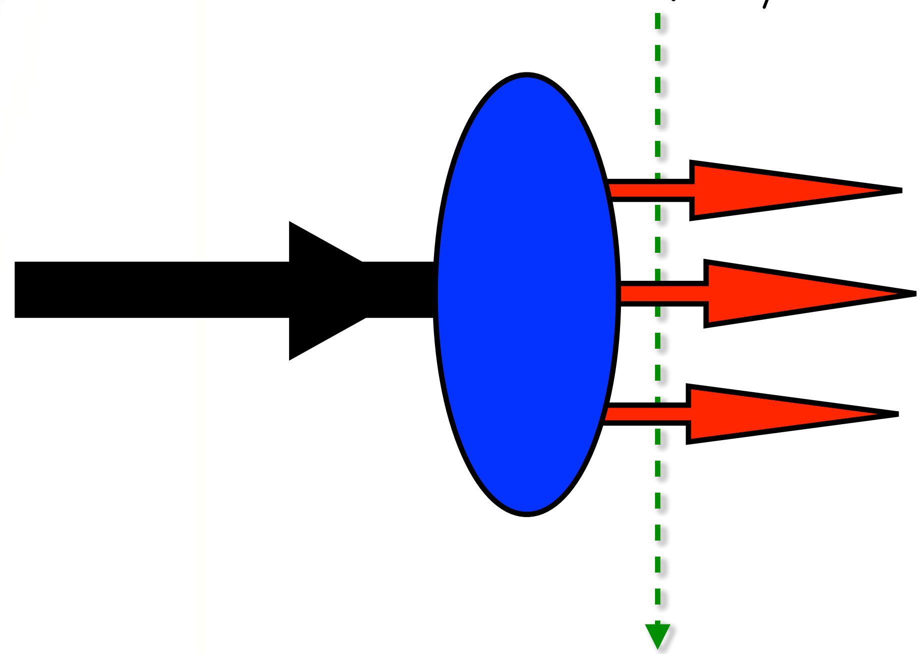
$$\frac{d\sigma}{dt}(\gamma d \to \Delta^{++}\Delta^{-}) \simeq \frac{d\sigma}{dt}(\gamma d \to pn) \text{ at high } Q^2$$

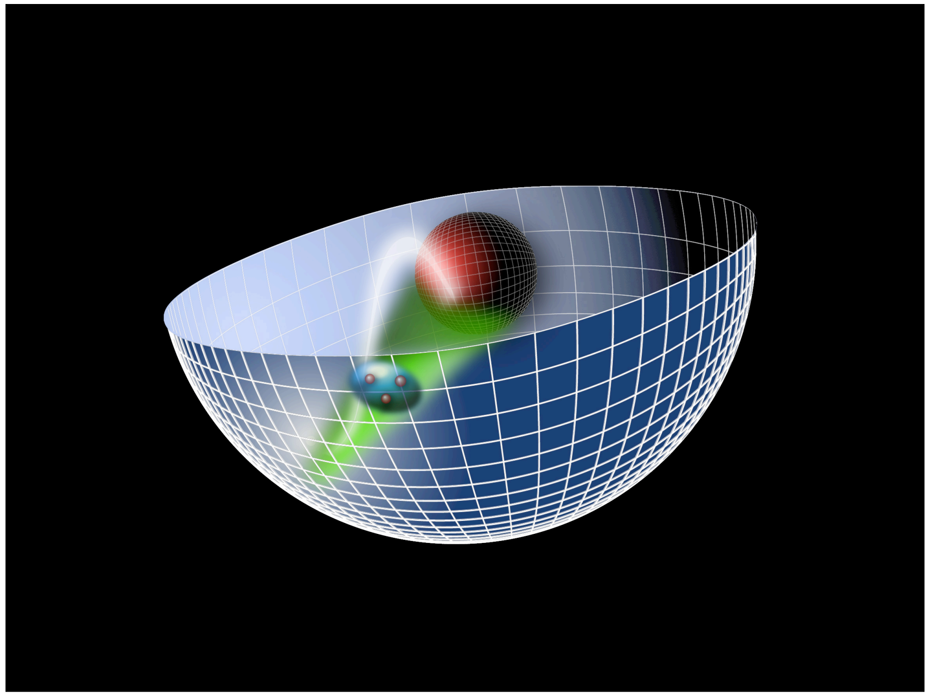
Lecture I

Light-Front Quantization and New Perspectives for Hadron Physics



Fixed $\tau = t + z/c$





Stan Brodsky

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