Continuum QCD

Craig Roberts

Selected Science Challenges for the coming decade

- Search for exotic hadrons – impossible in quark model
  - *Discovery would force dramatic reassessment of the distinction between the notions of matter fields and force fields*
- Exploit opportunities provided by new data on hadron elastic and transition form factors
  - *Chart infrared evolution of QCD's coupling and dressed-masses*
  - *Reveal correlations that are key to nucleon structure*
  - *Expose the facts or fallacies in contemporray descriptions of hadron structure*

Selected Science Challenges for the coming decade

- Precision experimental study of valence-quark region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*
- Explore and exploit opportunities to use precision-QCD as a probe for physics beyond the Standard Model

Overarching Science Challenge for the coming decade

Discover, theoretically

& experimentally:

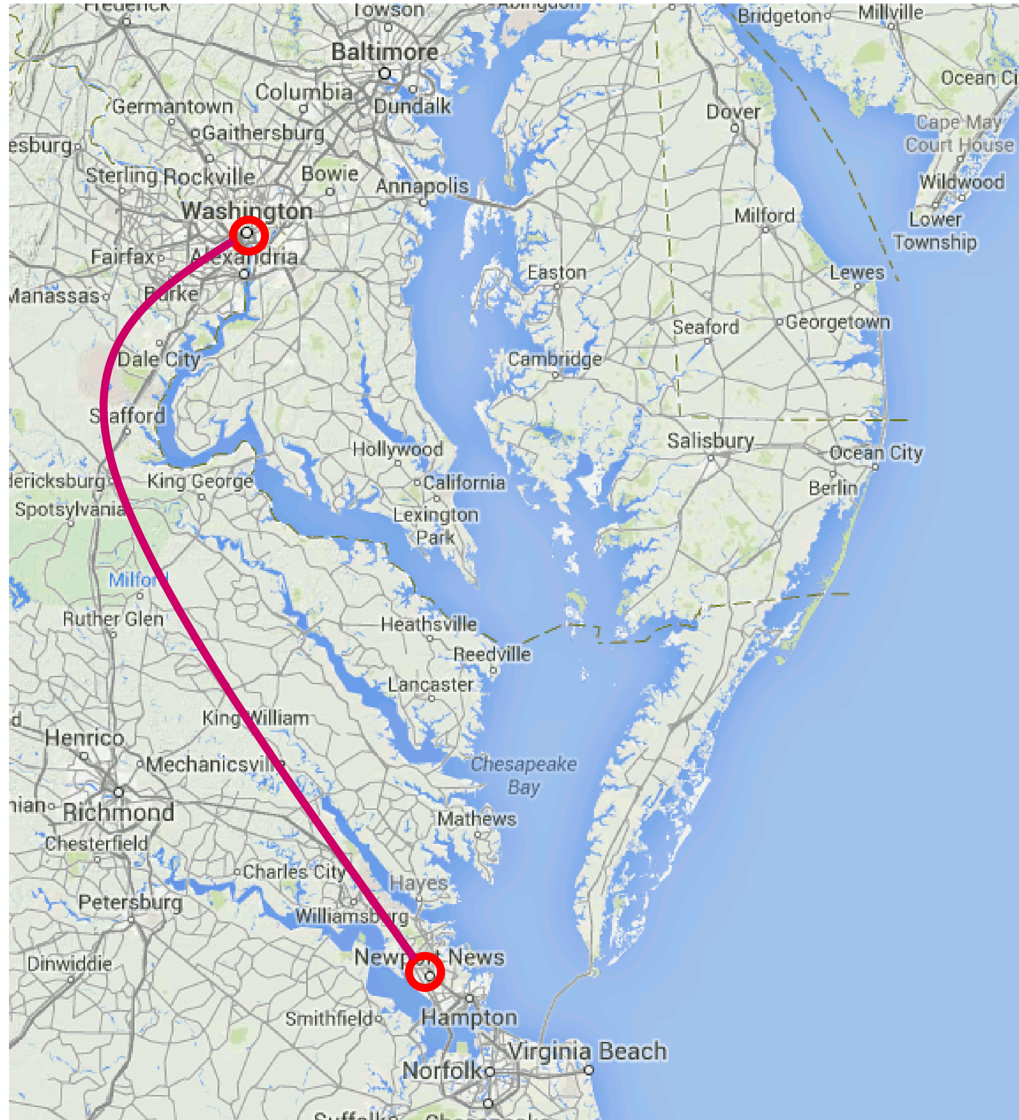
- *Realisation of confinement,*
- *its relationship to DCSB*

  – the Origin of Visible Mass
- *and the link between them*

Jefferson Lab

Thomas Jefferson National Accelerator Facility (JLab)

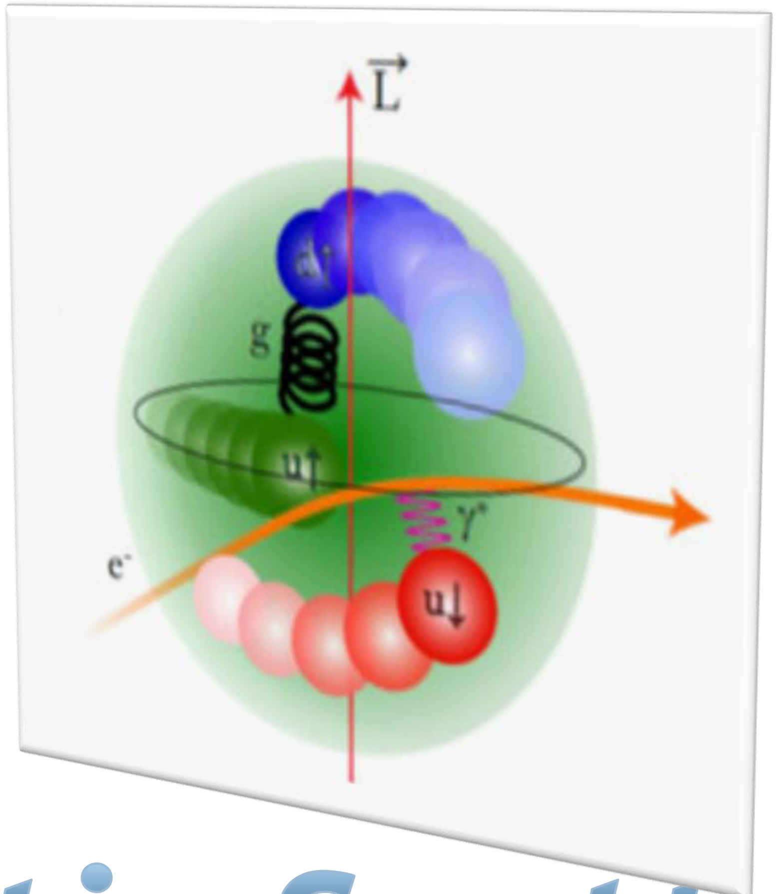
- Driving distance:
  Washington DC to JLab
  ≈ 270km



Thomas Jefferson National Accelerator Facility (JLab)

- 1984 … DoE provided initial funding for research, development and design
- 1987 … Construction begins on Continuous Electron Beam Accelerator Facility (CEBAF) - February 13
- 1994 … Accelerator reaches design energy of 4 GeV
- Construction cost in \$FY14 ≈ \$1-Billion
- *Goal … Write the book about the strongest force in nature - the force that holds nuclei together - and determine how that force can be explained in terms of gluons & quarks (quantum chromodynamics - QCD).*

$e(p) + H(q) \to e(p') + H(q')$



Elastic Scattering

Thomas Jefferson National Accelerator Facility (JLab)

e.g. S. J. Brodsky and G. R. Farrar, Phys. Rev. Lett. 31, 1153 (1973)

- One of the primary reasons for building CEBAF/JLab
  Prediction: at energy-scales greater than some *a priori* unknown minimum value, $\boldsymbol{Q_0}$, cross-sections and form factors behave as

Parton model scaling

$$\mathcal{A}(\sigma^2) \overset{\sigma \gg Q_0}{\propto} \left[\frac{Q_0^2}{\sigma^2}\right]^N \ln\left[\frac{Q_0^2}{\sigma^2}\right]^{\gamma_{\mathcal{A}}}$$

QCD scaling violations

power = (number $_{\text{valence-quarks}}$ − 1 + $\Delta\lambda$)

$\Delta\lambda$=0,1, depending on whether helicity is conserved or flipped

… prediction of $1/k^2$ vector-boson exchange

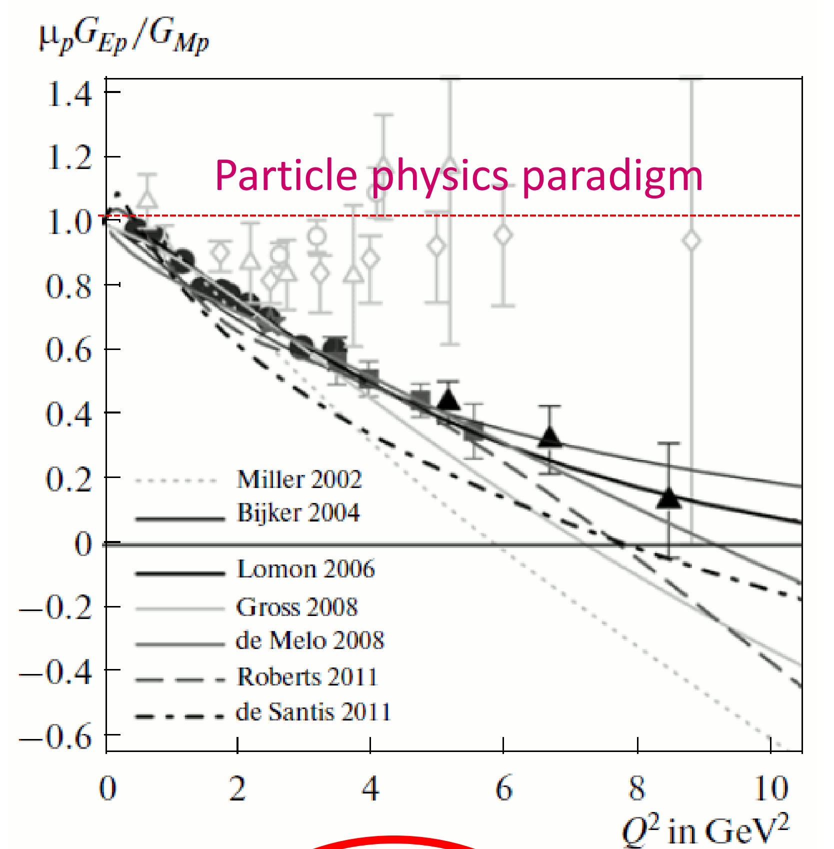
logarithm = distinctive feature & concrete prediction of QCD

- Initially imagined that $\boldsymbol{Q_0}$ = 1GeV!
- So, JLab was initially built to reach 4GeV.

Thomas Jefferson National Accelerator Facility (JLab)

- 1994 – 2004
  - An enormous number of fascinating experimental results
  - Including an empirical demonstration that the distribution of charge and magnetisation within the proton are completely different
  - Suggesting that quark-quark correlations play a crucial role in nucleon structure

But no sign of parton model scaling and certainly not of scaling violations

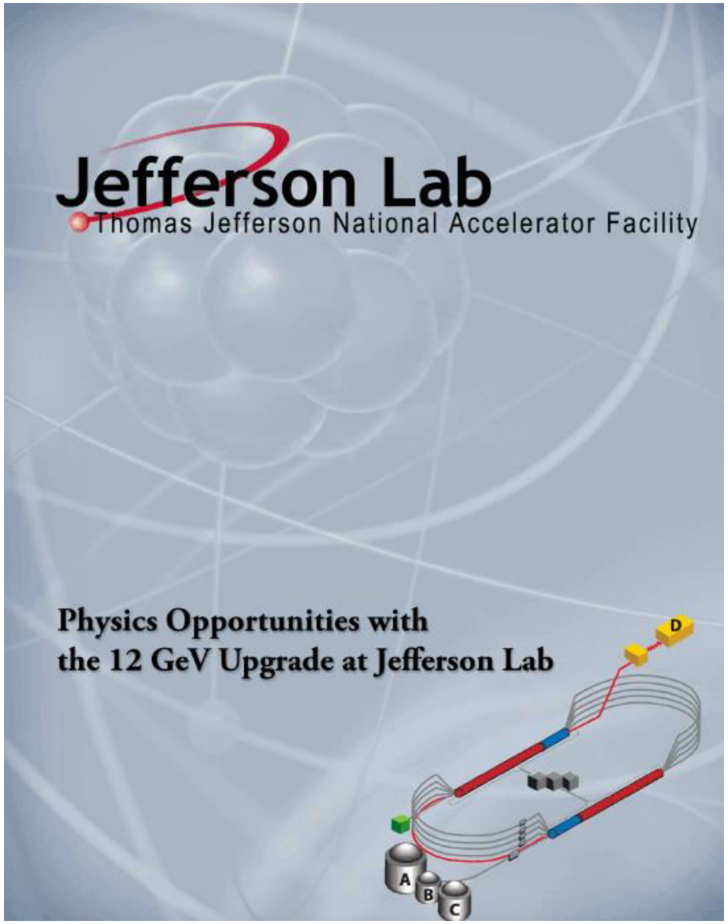


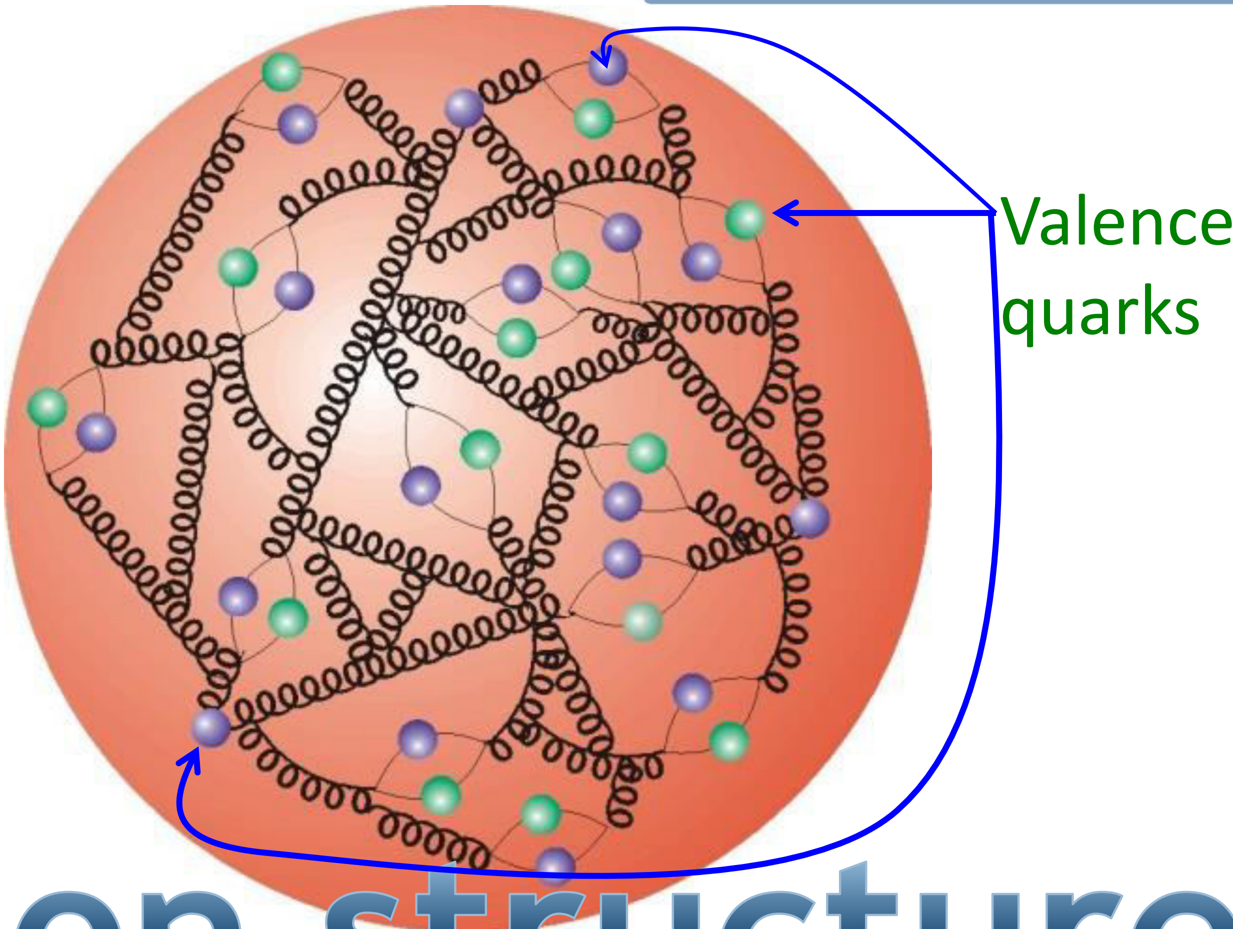
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)} = 1$$

Particle physics paradigm

Thomas Jefferson National Accelerator Facility (JLab)

- 2004 … Mission Need Agreed on upgrade of CEBAF (JLab's accelerator) to 12GeV
- 2014 … 12GeV commissioning beams now being delivered to the experimental halls
- Final cost of upgrade is approximately $370-Million
- Physics of JLab at 12GeV arXiv:1208.1244 [hep-ex]
- *12 GeV Era has begun*





Parton structure of hadrons

Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it’s how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC
- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*

Lepage, G. Peter and Brodsky, Stanley J: *Exclusive Processes in Perturbative Quantum Chromodynamics,* Phys. Rev. D 22 (1980) pp. 2157-2198

Hard Exclusive Processes & PDAs

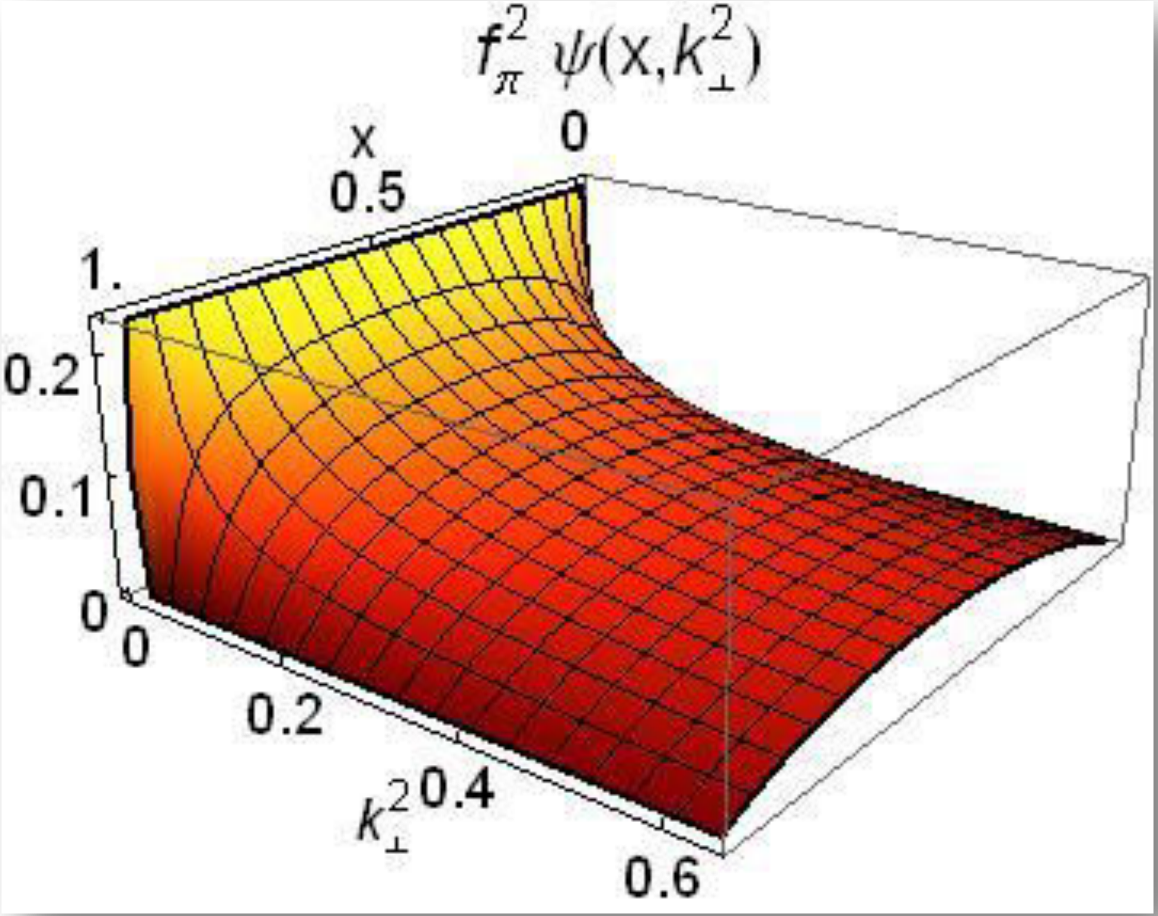
- In the theory of strong interactions, the cross-sections for many *hard* exclusive hadronic reactions can be expressed in terms of the PDAs of the hadrons involved
- Example: pseudoscalar-meson elastic electromagnetic form factor

$$\exists Q_0 > \Lambda_{\rm QCD} \,|\, Q^2 F_P(Q^2) \overset{Q^2>Q_0^2}{\approx} 16\pi\alpha_s(Q^2) f_P^2 w_\varphi^2,$$

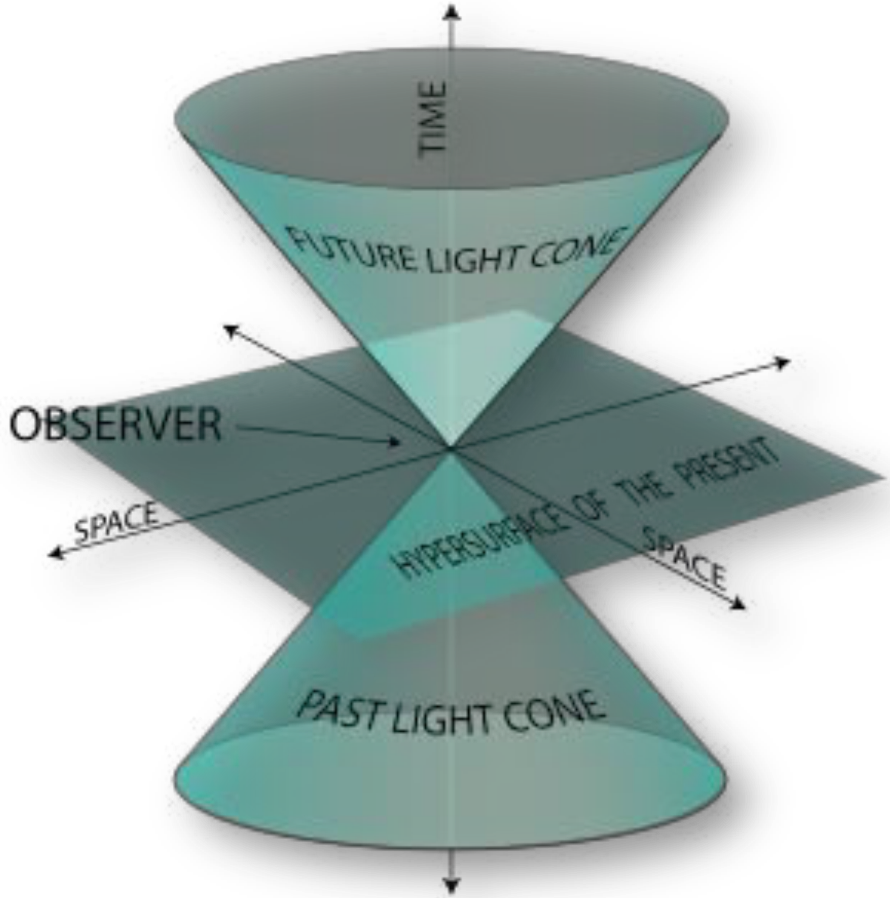
$$w_\varphi = \frac{1}{3}\int_0^1 dx\, \frac{1}{x}\varphi^P(x),$$

  - $\alpha_S(Q^2)$ is the strong running coupling,
  - $\varphi_\pi(u)$ is the meson's twist-two valence-quark PDA
  - f_P is the meson's leptonic decay constant

It was promised that JLab would verify this fundamental prediction



Pion's Wave Function

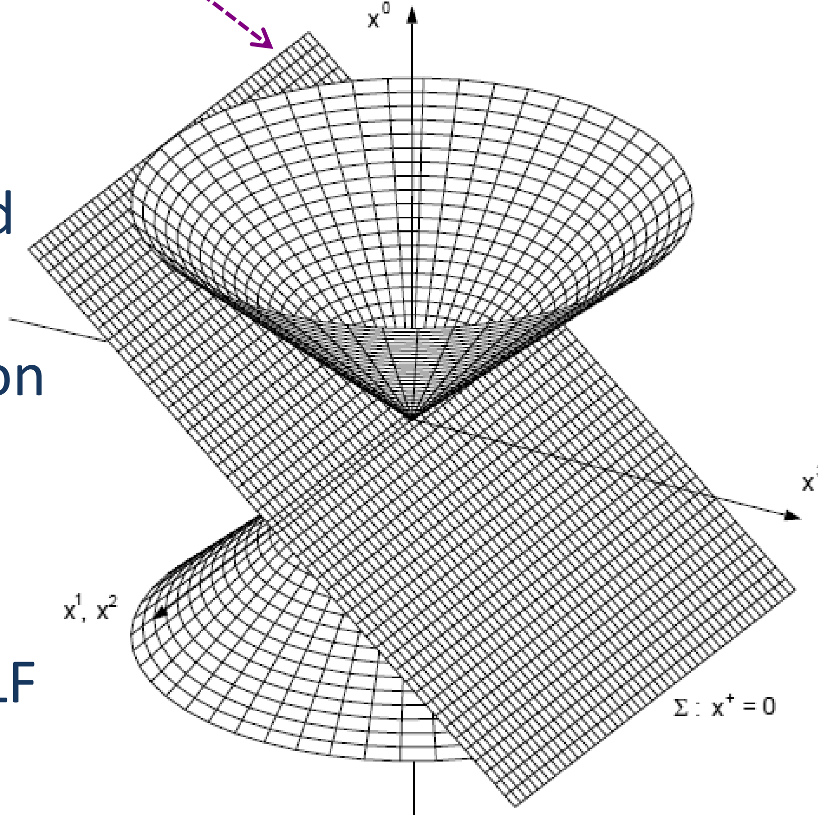


Why light-front?

Light-front Quantisation

- Hamiltonian formulation of quantum field theory.
  - Fields are specified on a particular initial surface:

    Light front $x^+ = x^0 + x^3 = 0$
- Using LF quantisation:
  - ✓ quantum mechanics-like wave functions can be defined;
  - ✓ quantum-mechanics-like expectation values can be defined and evaluated
  - ✓ Parton distributions are correlation functions at equal LF-time x^+ ; namely, within the initial surface $x^+ = 0$, and can thus be expressed directly in terms of ground state LF wavefunctions



Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n\cdot P)\, \gamma_5\gamma\cdot n\, S(k)\Gamma_\pi(k;P)S(k-P)$$

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

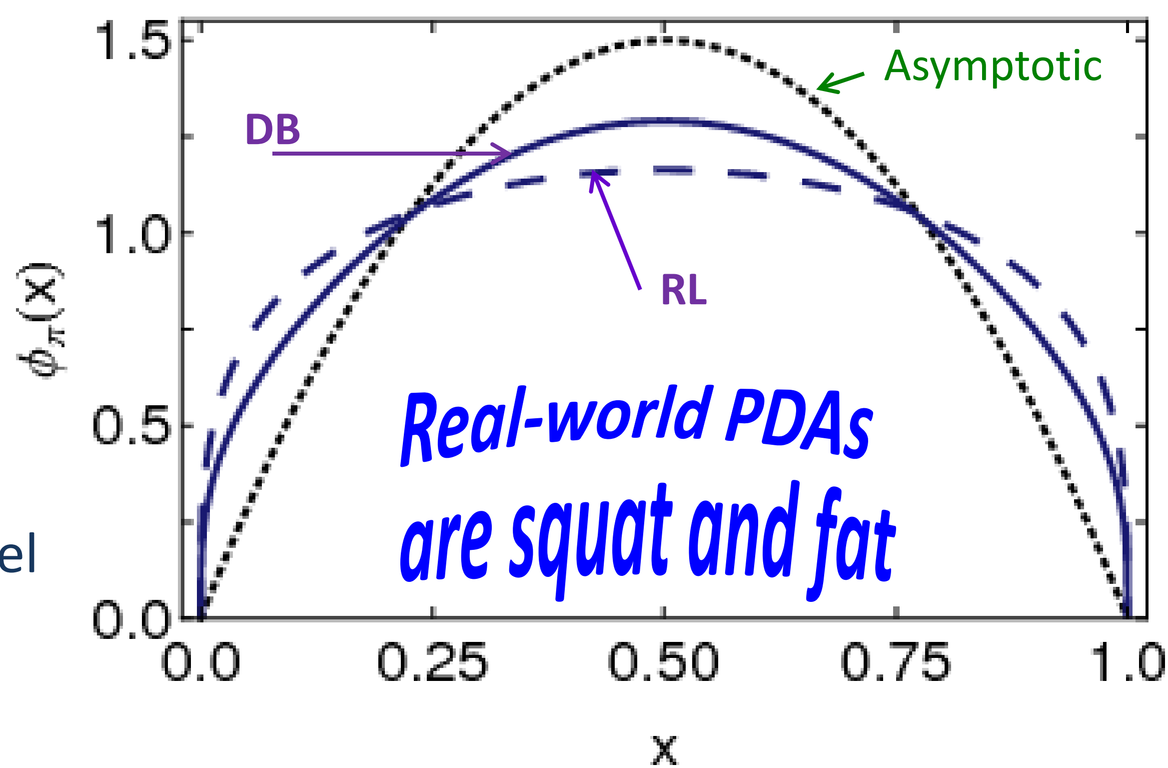
$x^\alpha\,(1-x)^\alpha$, with $\alpha=0.5$

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Both kernels agree: marked broadening of $\varphi_\pi(x)$, which owes to DCSB

- This may be claimed because PDA is computed at a low renormalisation scale in the chiral limit, whereat the quark mass function owes entirely to DCSB.
- Difference between RL and DB results is readily understood: $B(p^2)$ is more slowly varying with DB kernel and hence a more *balanced* result



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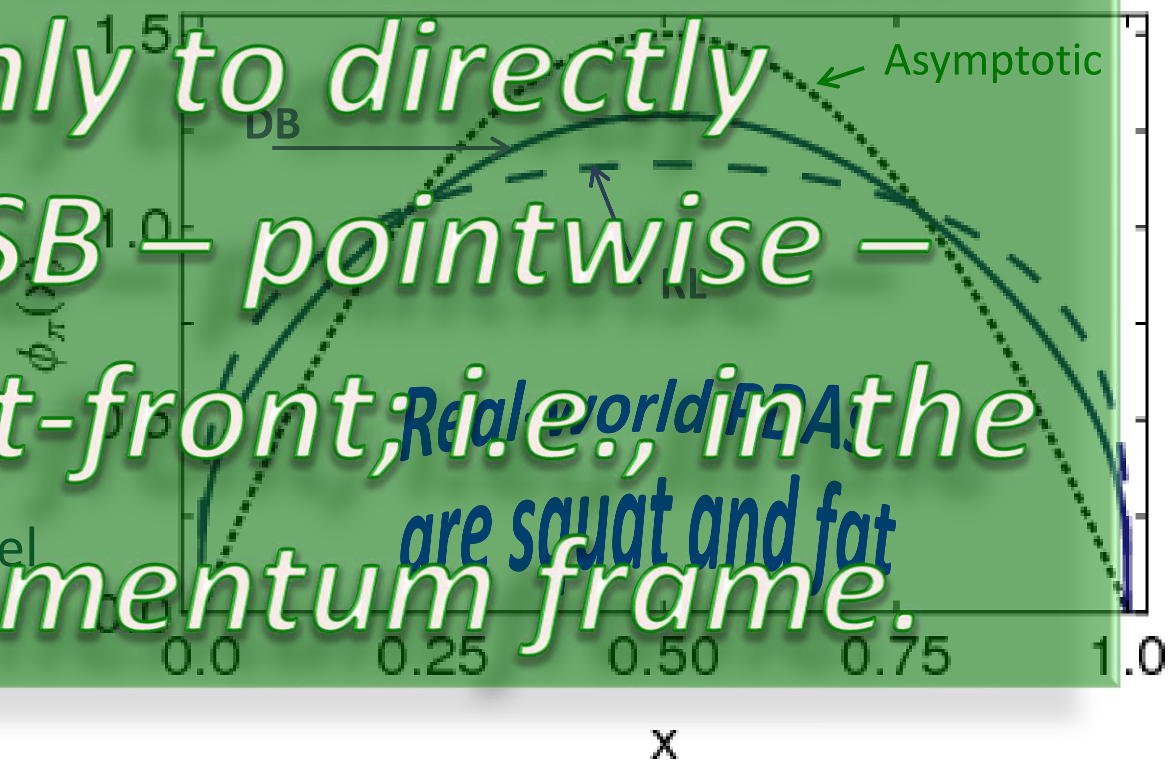
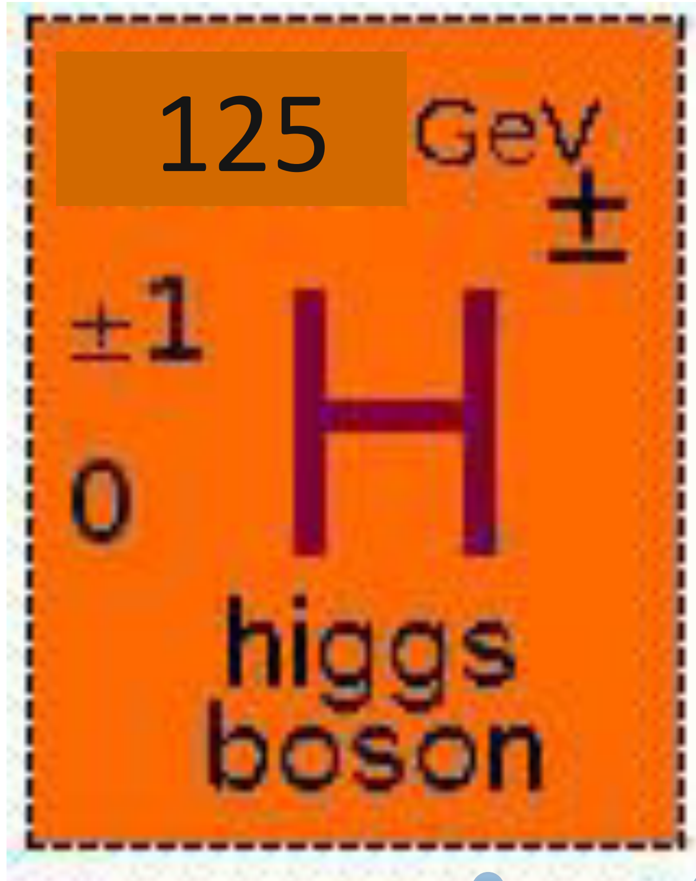


These computations are the first and only to directly expose DCSB – pointwise – on the light-front; i.e., in the infinite momentum frame.

Leading-twist PDAs of *S*-wave light-quark mesons

- End of a <u>*long*</u> story (longer than 30 years war)
- Continuum predictions that pion and kaon PDAs are broad, concave functions confirmed by simulations of lattice-regularised QCD
  - *Pion Distribution Amplitude from Lattice QCD*, Jian-Hui Zhang *et al.*, Phys.Rev. D95 (2017) 094514; 1702.00008
  - *Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry*, Jiunn-Wei Chen *et al.*, arXiv:1712.10025 [hep-ph]
  - *Pion and kaon valence-quark parton quasidistributions*, S.-S. Xu, L. Chang *et al.* Phys. Rev. D**97** (2018) 094014; arXiv:1802.09552 [nucl-th]
- Continuum analyses predict that these properties <u>characterise</u> the leading-twist PDAs of ***all*** *S*-wave light-quark mesons
- ***Many empirically verifiable predictions***

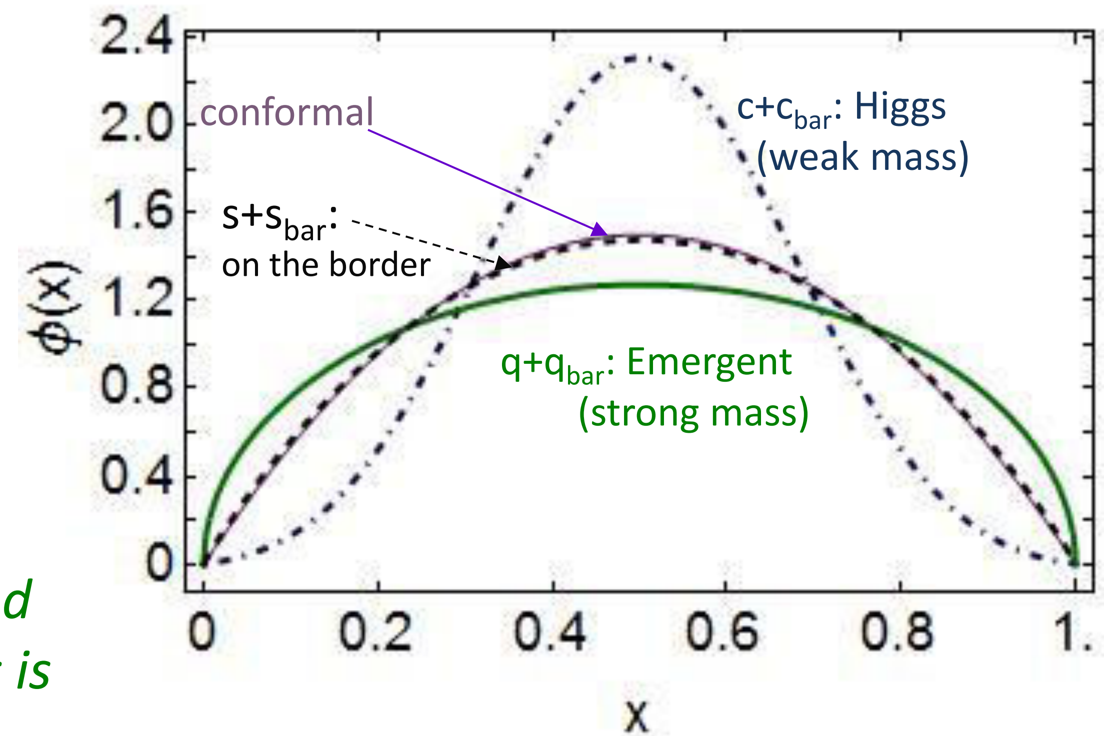


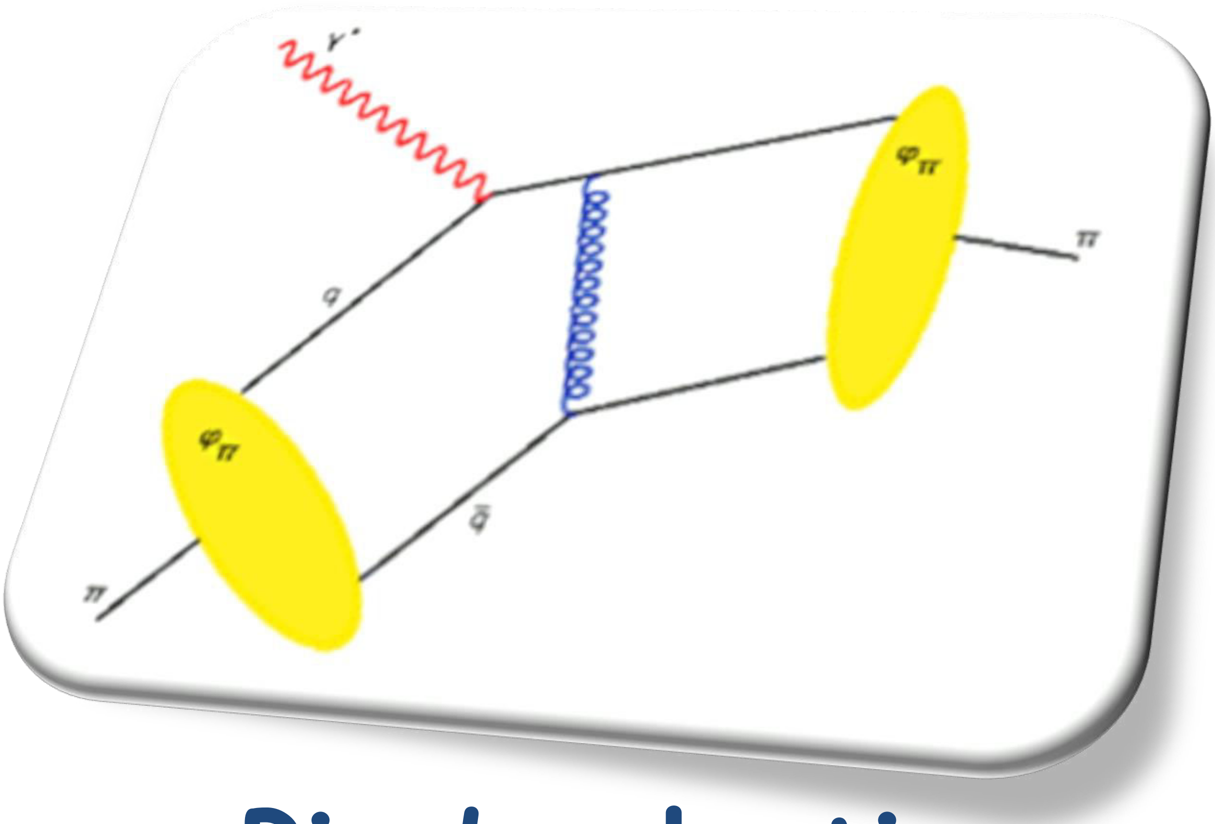
Where is it?

Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1511.04943 [nucl-th], Phys. Lett. B **753** (2016) pp. 330-335

Emergent Mass *vs.* Higgs Mechanism

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{\text{quark}} \to \infty$
  $\varphi(x) \to \delta(x-½)$
- limit $\mathrm{m}_{\text{quark}} \to 0$
  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above m_{strange}
- *Comparison between distributions of light-quarks and those involving strange-quarks is good place to seek signals for strong-mass generation*

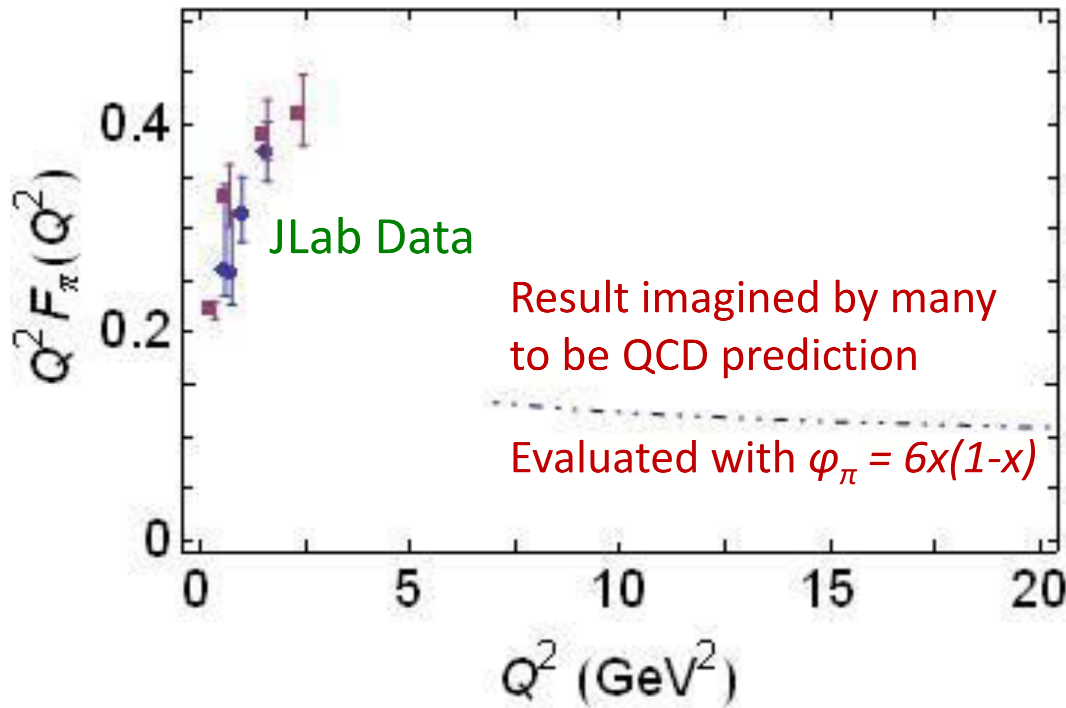




Pion's elastic electromagnetic form factor

Pion electromagnetic form factor

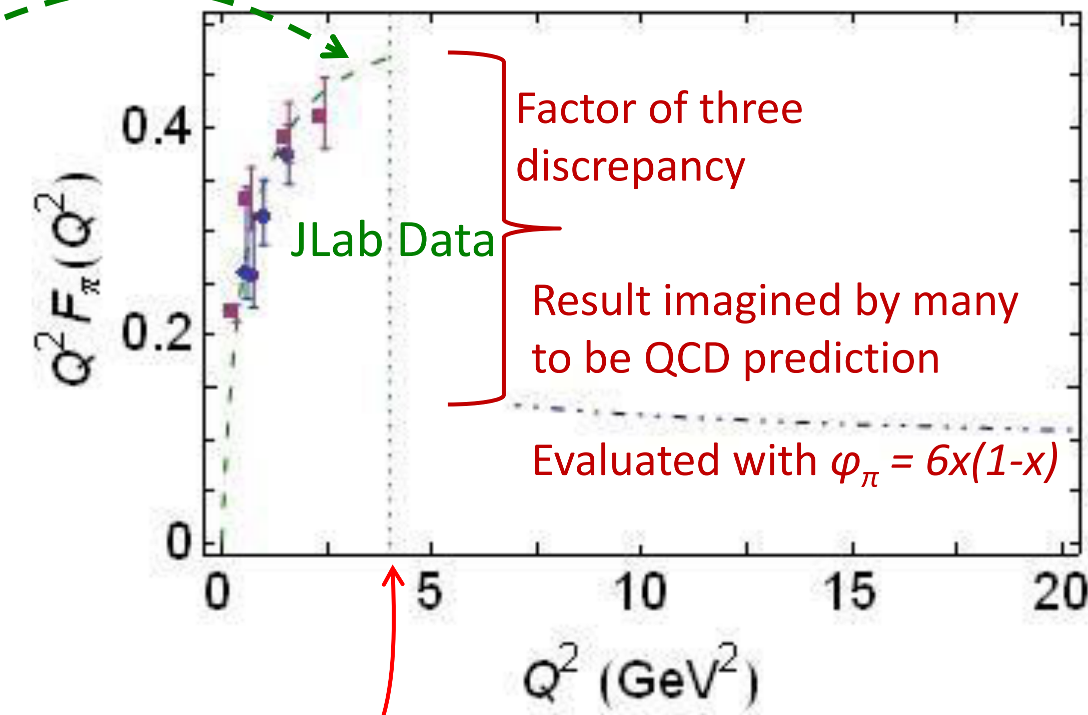
- In 2001 – seven years after beginning operations, Jefferson Lab provided the first high precision pion electroproduction data for F_π between Q^2 values of 0.6 and 1.6 (GeV/c)^2.



- 2006 & 2007 – new result, at $Q^2=2.45$ (GeV/c)^2
- Authors of the publications stated: “still far from the transition to the Q^2 region where the pion looks like a simple quark-antiquark pair”
  - disappointment and surprise

Pion electromagnetic form factor

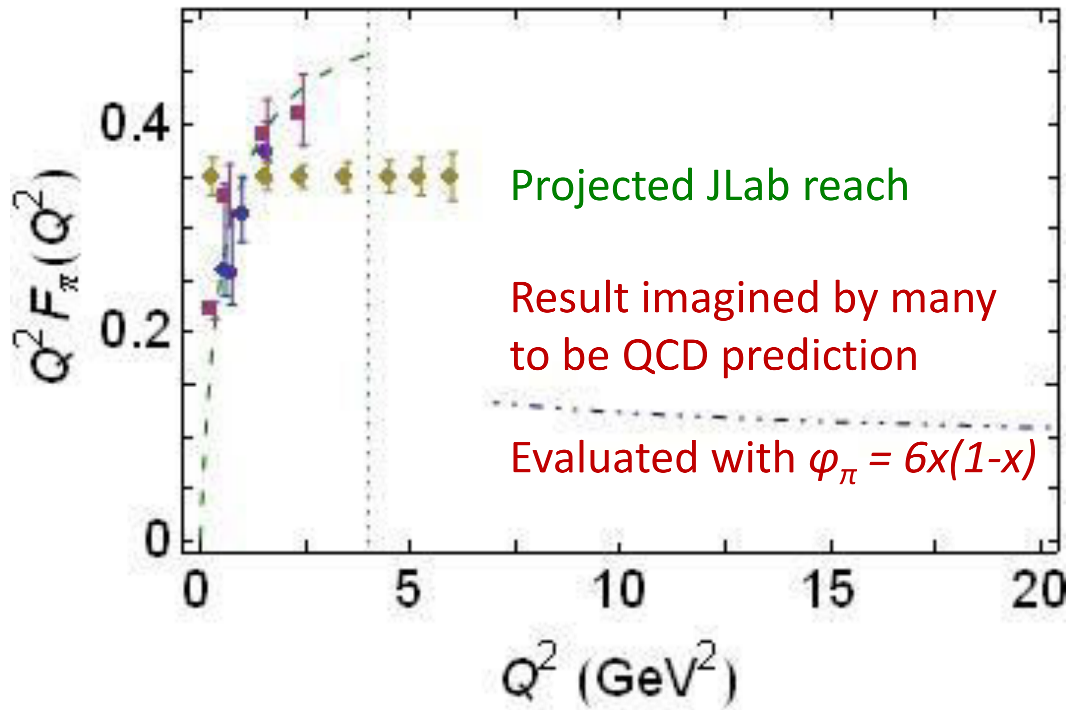
- Year 2000 *prediction* for $F_\pi(Q^2)$
  - *P.Maris & P.C. Tandy, Phys.Rev. C62 (2000) 055204*
- Problem … used brute-force computational method … unable to compute for $Q^2 > 4\,\text{GeV}^2$



- Shape of prediction suggested to many that one might *never* see parton model scaling and QCD scaling violations

Pion electromagnetic form factor

- Plans were made and an experiment approved that use the higher-energy electron beam at the 12 GeV Upgrade at Jefferson Lab.
- The Upgrade will allow an extension of the F_π measurement up to a value of Q^2 of about 6 $(GeV/c)^2$, which will probe the pion at double the resolution.

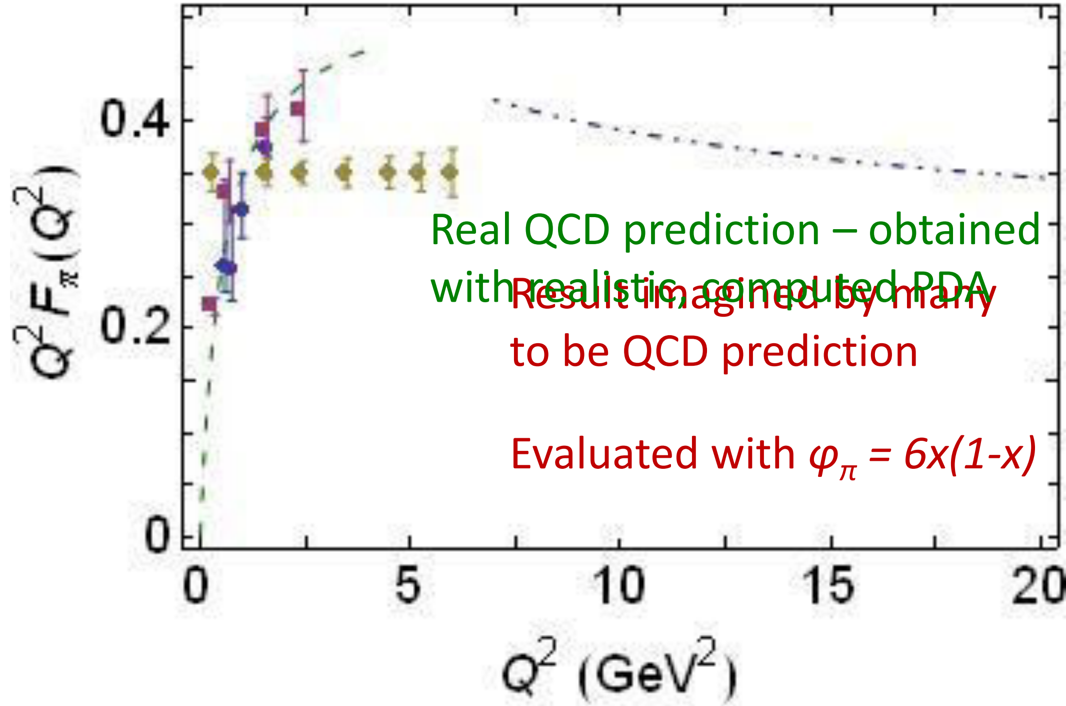


- Will there be any hint of a trend toward the asymptotic pQCD prediction?

New Algorithm
New Insights

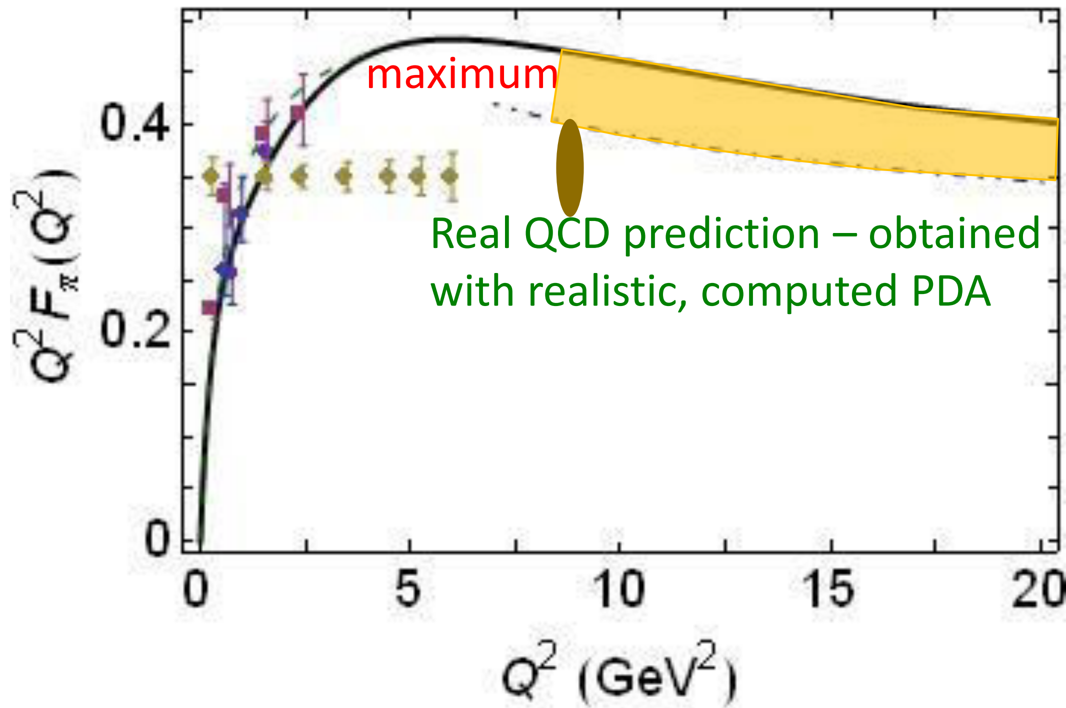
Pion electromagnetic form factor

- Solution – Part 1
  - Compare data with the real QCD prediction; i.e. the result calculated using the broad pion PDA predicted by modern analyses of continuum QCD



Pion electromagnetic form factor

Agreement within 15%



- Solution – Part 1
  - Compare data with the real QCD prediction; i.e. the result calculated using the broad pion PDA predicted by modern analyses of continuum QCD
- Solution – Part 2
  - Algorithm used to compute the PDA can also be employed to compute $F_\pi(Q^2)$ directly, to arbitrarily large Q^2

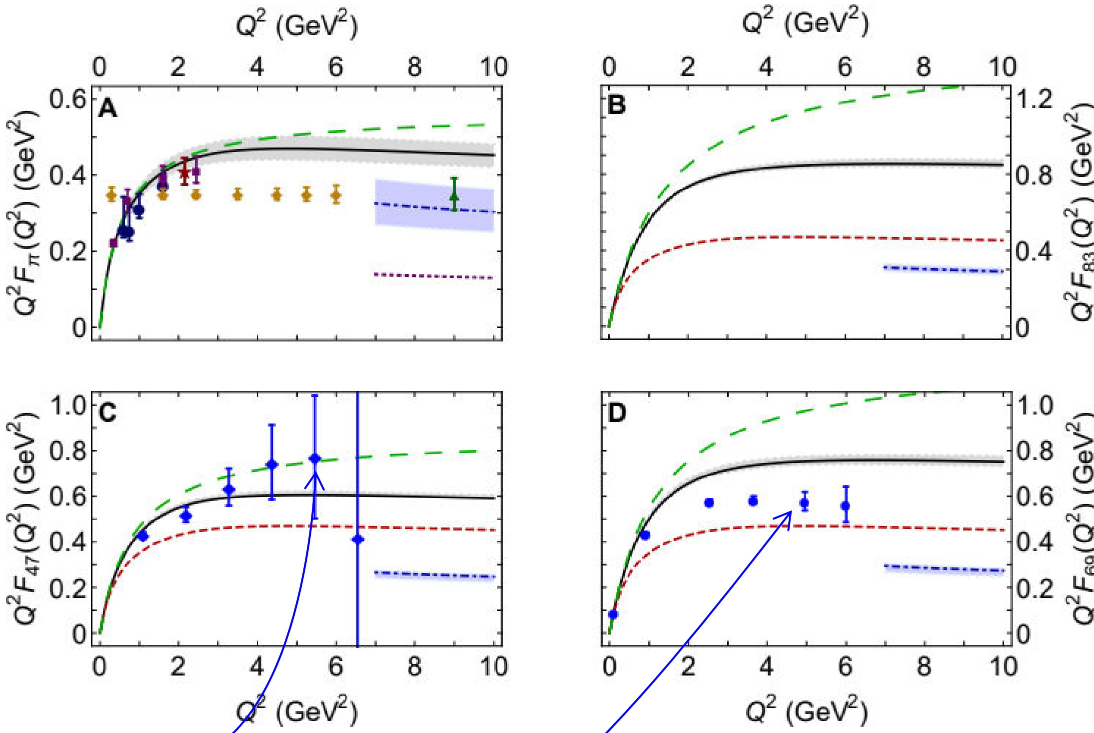
- Predictions:
  - JLab will see maximum
  - Experiments to 8GeV^2 will see parton model scaling and QCD scaling violations for the *first time* in a hadron form factor

Pion electromagnetic form factor at spacelike momenta
L. Chang et al., arXiv:1307.0026 [nucl-th], Phys. Rev. Lett. 111, 141802 (2013)

Mass-dependence of pseudoscalar meson elastic form factors,
Muyang Chen, Minghui Ding, Lei Chang and Craig D. Roberts,
arXiv:1808.09461 [nucl-th], Phys. Rev. D Rapid Comm. (*in press*)

Matching lattice-QCD

- ✓ lQCD cannot compute pion form factor at physical mass owing to competing demands, *e.g.*
  - ✓ large lattice volume to represent light pions
  - ✓ small lattice spacing to reach large Q^2
  - ✓ high statistics to compensate for decaying signal-to-noise ratio as form factors drop rapidly with increasing Q^2
- ✓ lQCD computations available at larger, unphysical pion masses



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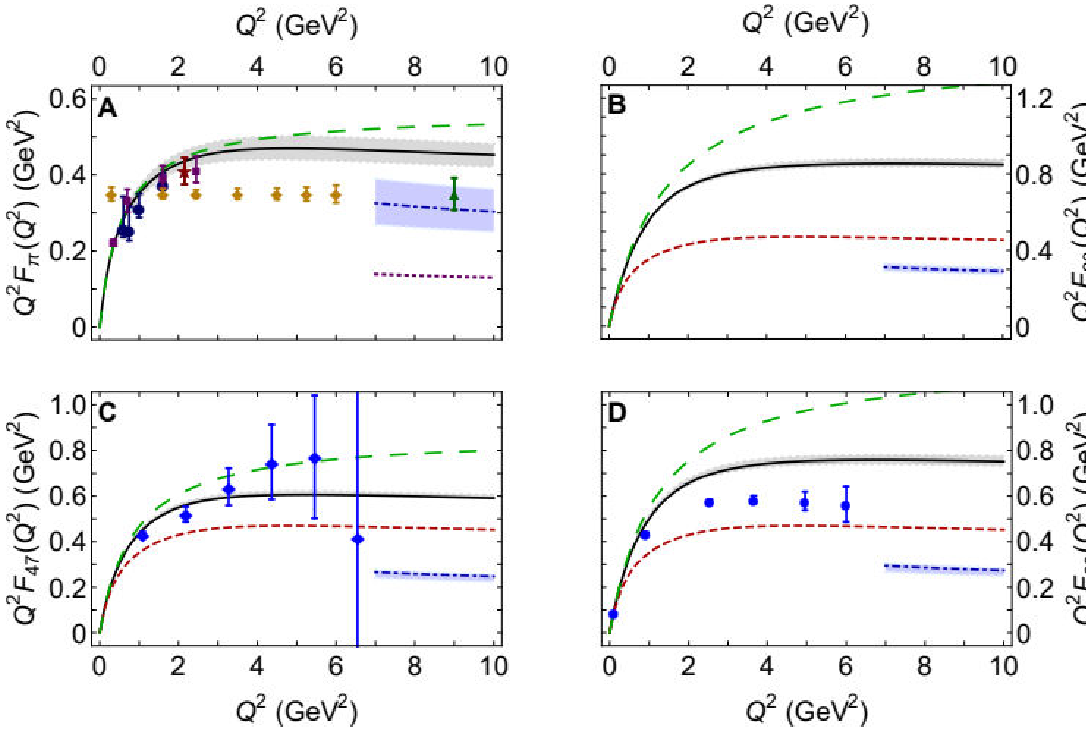
Matching lattice-QCD

- ✓ DSE RL approach to quark-antiquark bound-states
  - ✓ Used to determine electromagnetic form factors of pion-like mesons with masses m_{0-}/GeV=0.14, 0.47, 0.69, 0.83
  - ✓ Spacelike domain that extends to $Q^2 \lesssim 10$ GeV^2.
- ✓ Results enable direct comparisons with contemporary lattice-QCD calculations of heavy-pion form factors at large values of momentum transfer and aid in understanding them.
- ✓ Reveal, *inter alia*,
  - – that form factor of the physical pion provides the best opportunity for verification of the factorised hard-scattering formula relevant to this class of exclusive processes
  - – This capacity diminishes steadily as the meson mass increases.



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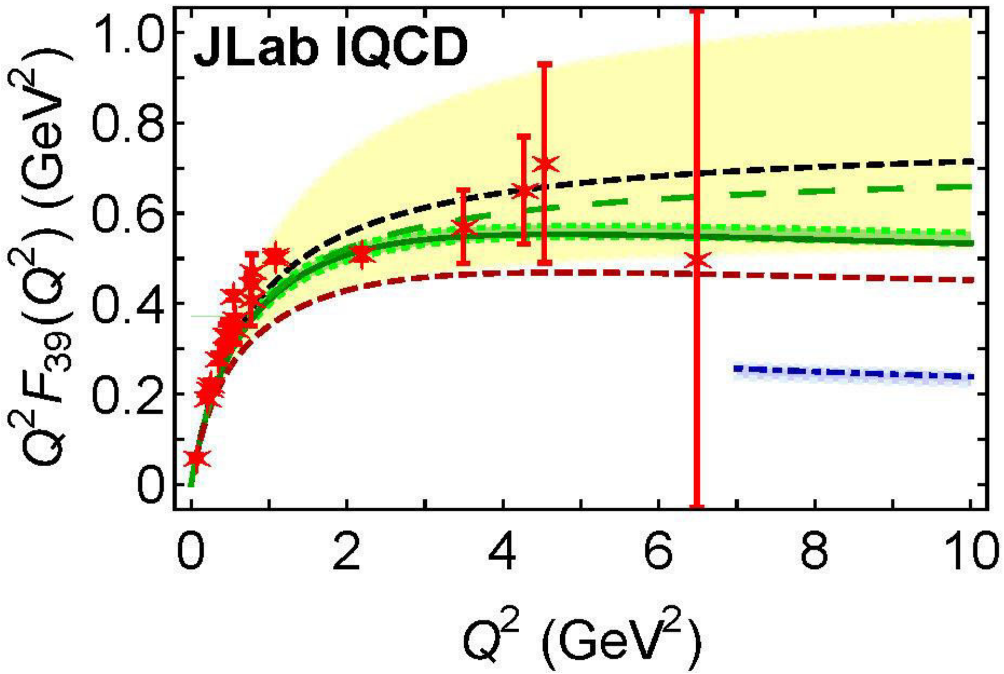

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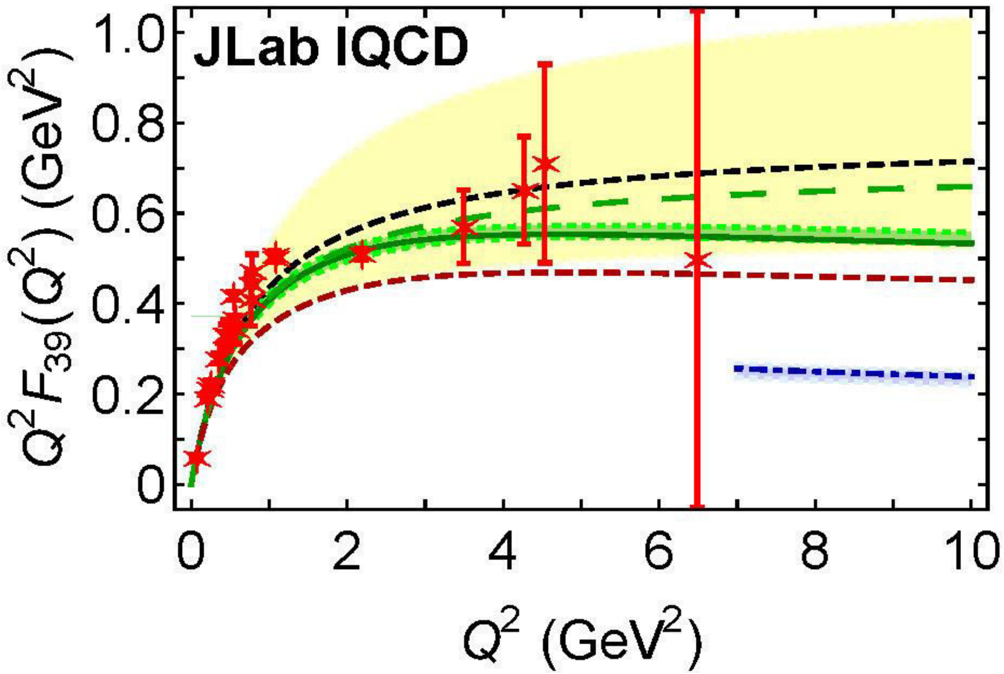
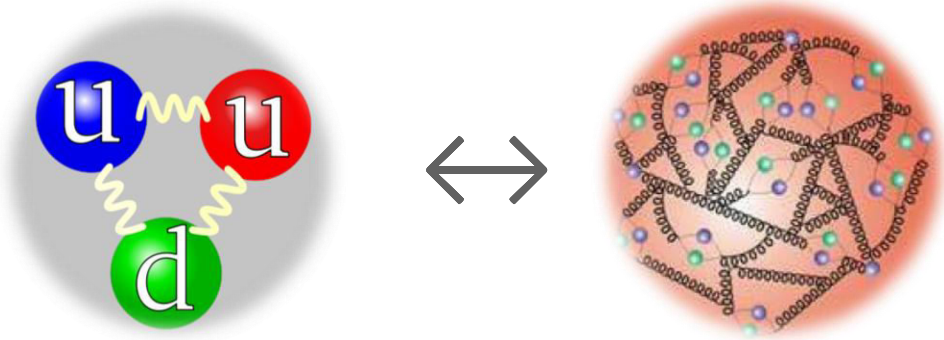


In successfully unifying all lattice results obtained at $m_\pi > m_\pi^{physical}$, the continuum method serves as a physical extrapolation, mapping extant lattice results for $F_\pi(Q^2; m_\pi > m_\pi^{physical})$ into concrete predictions for real-world experiments.

In achieving this, it is demonstrated that modern lattice results can be reinterpreted as confirming the continuum prediction at $m_\pi^{physical}$, providing even greater motivation for the extended, new generation experiments.

Implications

- Verify the theory of factorisation in hard exclusive processes, with dominance of hard contributions to the pion form factor for Q^2>8GeV^2.
- Notwithstanding that, normalisation of $F_\pi(Q^2)$ is fixed by a pion wave-function whose dilation with respect to $\varphi_\pi^{asy}(x)=6x(1-x)$ is a definitive signature of DCSB
  - ***Empirical measurement of the strength of DCSB in the Standard Model – the origin of visible mass***
- Close the book on a story that began thirty-five years ago
- Paves the way for a dramatic reassessment of pictures of proton & neutron structure, which is already well underway

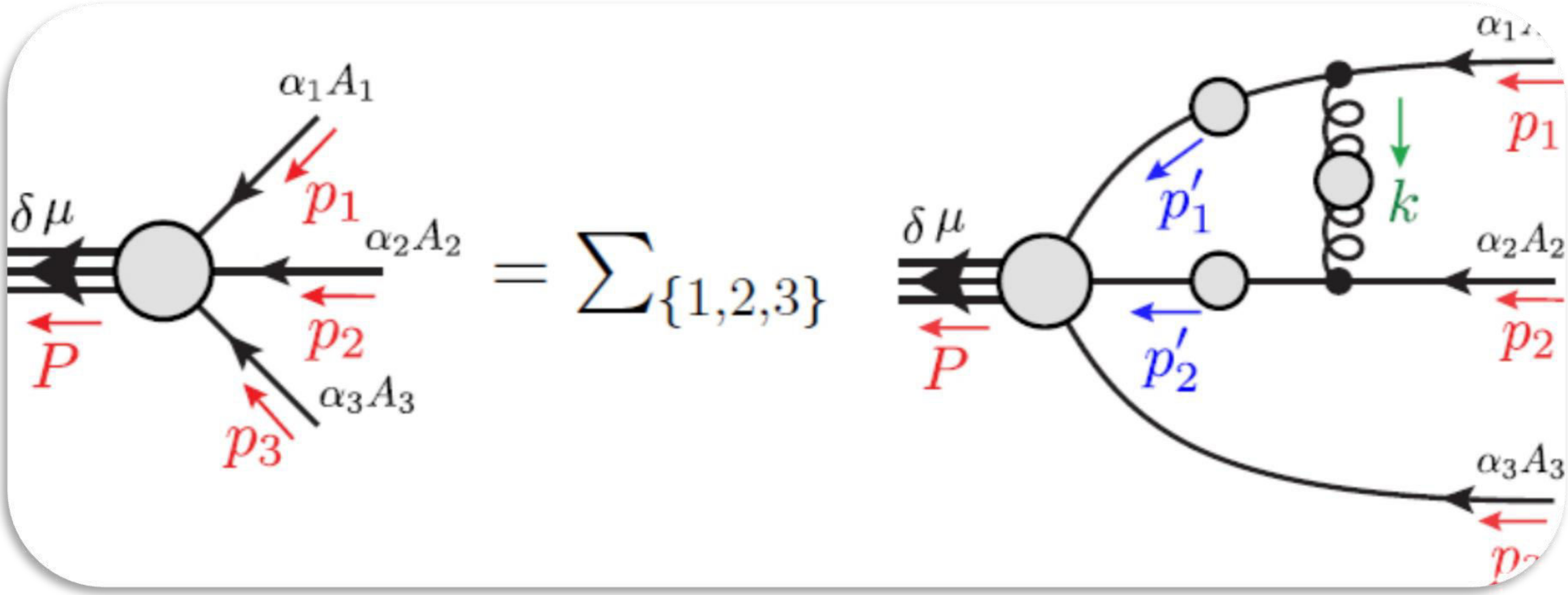


New Challenge

- Three valence-body problem
- Baryons in QCD
  - Three valence quarks
- Spectrum and properties of hybrid and exotic mesons

  exotic mesons: quantum numbers not possible for quantum mechanical quark-antiquark systems
  hybrid mesons: normal quantum numbers but non-quark-model decay pattern
  BOTH suspected of having "constituent gluon" content

  - Valence-quark + valence-antiquark+valence-gluon(?)



Baryons as a 3-valence-body problem

Unification of Meson & Baryon Properties

- Correlate the properties of meson and baryon ground- and excited-states within a *single, symmetry-preserving framework*
  - Symmetry-preserving means:

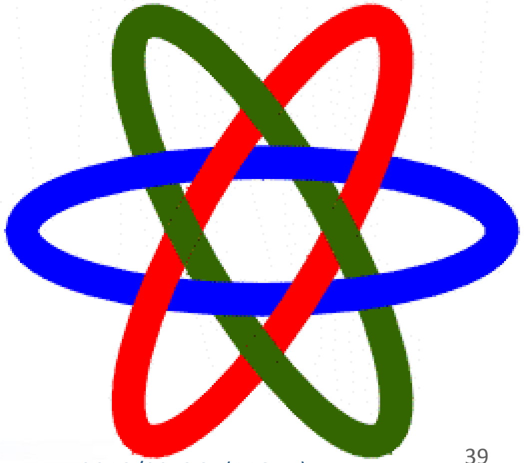
    Poincaré-covariant & satisfy relevant Ward-Takahashi identities

Understanding the nucleon as a Borromean bound-state,
J. Segovia, C.D. Roberts, S.M. Schmidt,
arXiv: 1506.05112 [nucl-th], Phys.Lett. B ***750*** *(2015) pp. 100-106*

Proton

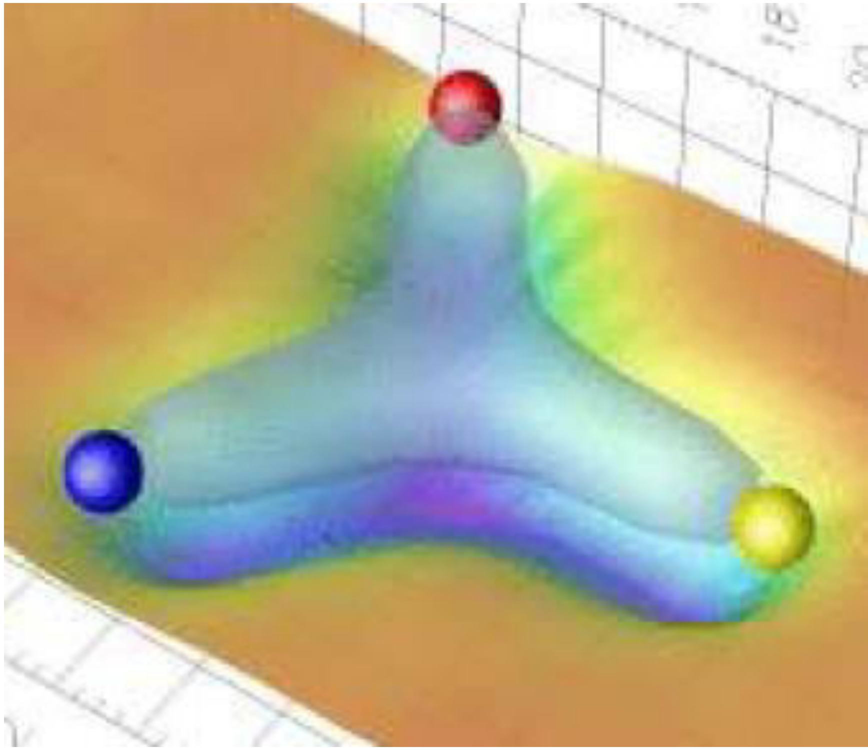
- Proton can be viewed as Borromean bound-state, *viz*. system constituted from three bodies, no two of which can combine to produce an independent two-body bound-state.
- Naturally, in QCD the complete picture of the proton is more complicated owing, in large part, to the loss of particle number conservation in quantum field theory.
- Notwithstanding that, the Borromean analogy provides an instructive perspective from which to consider both quantum mechanical models and continuum treatments of the nucleon bound-state problem in QCD.
- Borromean perspective poses a crucial question:

  Whence binding between the valence quarks in the proton?



Whence binding in the proton?

- Numerical simulations of lattice-regularised QCD (lQCD) that use static sources to represent the proton's valence-quarks produce a ``Y-junction'' flux-tube picture of nucleon structure
- Might be viewed as originating in the three-gluon vertex, which signals the non-Abelian character of QCD and is the source of asymptotic freedom
- Such results and notions would suggest a key role for the three-gluon vertex in nucleon structure *if* they were equally valid in real-world QCD wherein light dynamical quarks are ubiquitous.
- As we have seen, however, they are not; and so a different explanation of binding within the nucleon must be found.

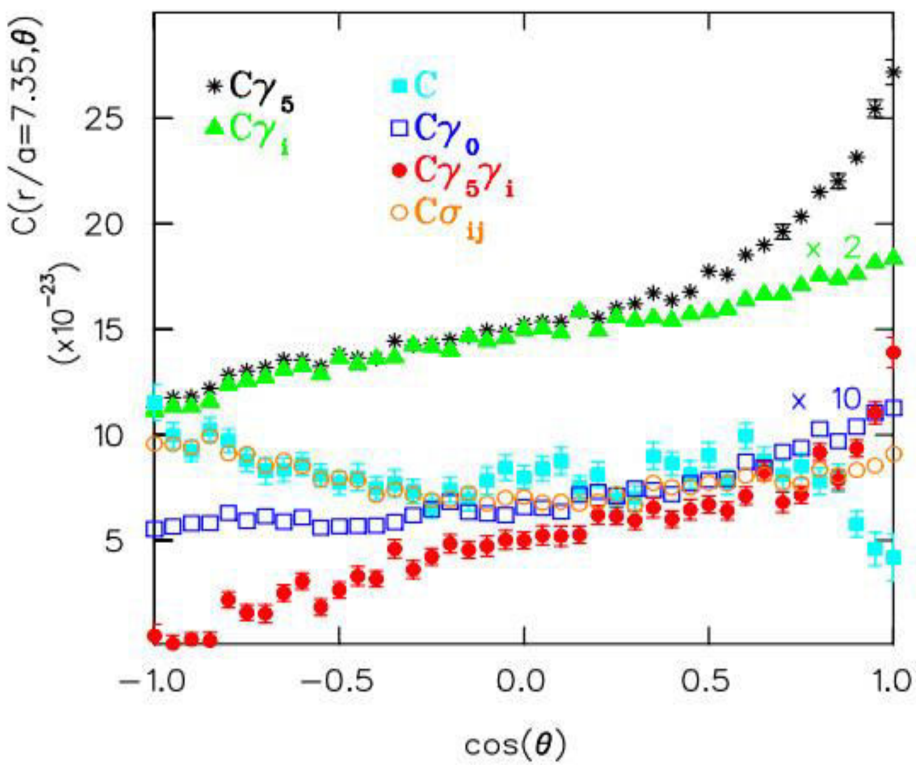


Corollary of DCSB (*little-known*)

- Any interaction that is capable of creating pseudo-Goldstone modes as bound-states of a light dressed-quark and -antiquark, and reproduce the measured value of their leptonic decay constant, will necessarily also generate strong correlations between any two dressed quarks contained within a nucleon.
- This assertion is based on an accumulated body of evidence gathered in nearly two decades of studying two- and three-body bound-state problems in hadron physics
- No realistic counter examples are known.

Corollary of DCSB (*little-known*)

- The existence of such diquark correlations is supported, too, by numerical simulations of lQCD
  - Alexandrou:2006cq
  - Babich:2007ah
- Correlations in scalar and pseudo vector diquark channels are strong
- Correlations in pseudoscalar and vector channels are smaller by a factor of 10



SU(2)-colour

- In a dynamical theory based on SU(2)-colour, diquarks would be colour-singlets.
- They would exist as asymptotic states and form mass-degenerate multiplets with mesons composed from like-flavoured quarks.
- Consequently, the $[ud]_{0+}$ diquark would be massless in the presence of DCSB, matching the pion.
- These properties are a manifestation of Pauli-Gürsey symmetry
- Such identities are lost in changing the gauge group to SU(3)-colour; but strong and instructive similarities between mesons and diquarks remain

Understanding the nucleon as a Borromean bound-state,
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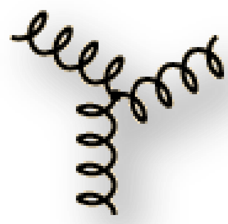
*arXiv: 1506.05112 [nucl-th], Phys.Lett. B **750** (2015) pp. 100-106*

Strong running coupling - α_s

- Bulk of QCD's particular features can be traced to evolution of α_S
- Characteristics are primarily determined by three-gluon vertex:
  - four-gluon vertex doesn't contribute dynamically at leading order in perturbative analyses of matrix elements;
  - nonperturbative continuum analyses of gauge sector indicate that satisfactory agreement with lQCD gluon propagator is typically obtained without reference to dynamical contributions from four-gluon vertex



- Three-gluon vertex is therefore the dominant factor in producing the class of renormalisation-group-invariant running interactions that have provided both successful descriptions of and predictions for many hadron observables
- This class of interactions generates strong attraction between two quarks ⇒ tight diquark correlations in analyses of the three valence-quark scattering problem.

Faddeev Equation

- Faddeev equation was introduced almost sixty years ago

  Faddeev:1960su: Faddeev, L. D., *Scattering theory for a three particle system*, Sov. Phys. JETP **12** (1961) pp. 1014-1019

- Treats the quantum mechanical problem of three-bodies interacting via pairwise potentials
  - reducing scattering problem to a sum of three terms
  - each of which describes a solvable scattering problem in distinct two-body subsystems
- The Faddeev formulation of that three-body problem has a unique solution.

Faddeev equation … mentioned more than 47,000 times in scientific literature

Faddeev Equation

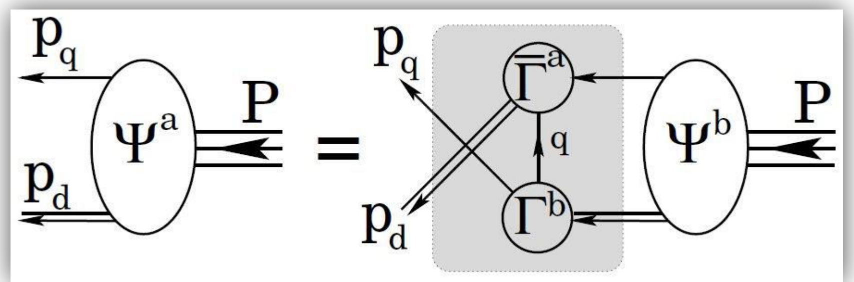
- Analogous approach to the three-valence-quark (baryon) bound-state problem in quantum chromodynamics (QCD) was explained in
  R.T. Cahill *et al*., Austral. J. Phys. 42 (1989) 129-145
- In this case, owing to
  - dynamical mass generation, expressed most simply in QCD's one-body Schwinger functions in the gauge and matter sectors
  - and importance of symmetries

  a Poincaré-covariant quantum field theory generalisation of the Faddeev equation is required

R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

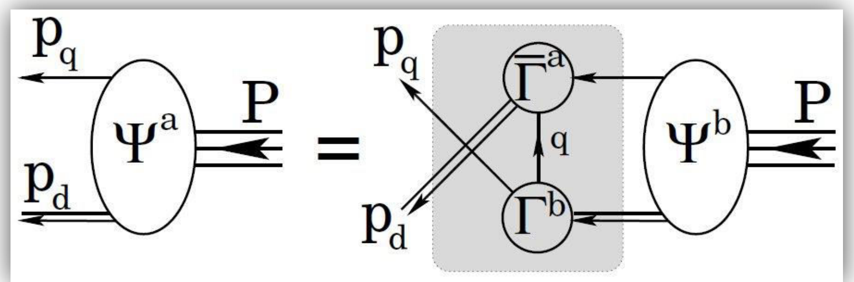
DSEs & Baryons

- *Dynamical chiral symmetry breaking* (DCSB)
  – has enormous impact on meson properties.
  - *Must be included in description and prediction of baryon properties*.
- *DCSB* is essentially a quantum field theoretical effect. In quantum field theory
  - Meson appears as pole in four-point quark-antiquark Green function → Bethe-Salpeter Equation
  - *Nucleon appears as a pole in a six-point quark Green function* → *Faddeev Equation*.
- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks



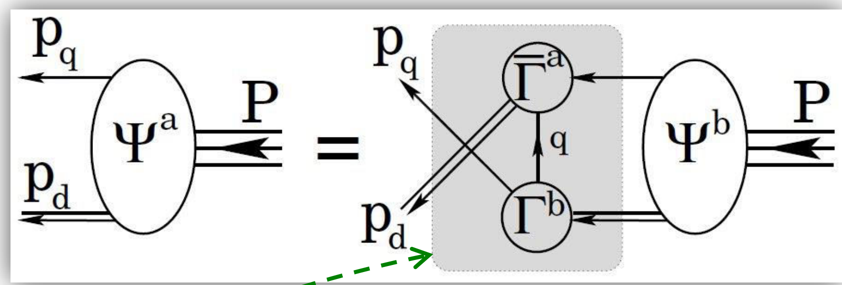
Baryon Structure

- Existence of tight diquark correlations simplifies analyses of baryon bound states … reduces task to solving Poincaré covariant Faddeev equation
- Three gluon vertex … not explicitly part of bound-state kernel
- Instead, one uses fact that phase-space factors and combinatorics enhance 2-body interactions over $n\geq3$-body interactions & exploits dominant role played by diquark correlations in the 2-body subsystems
- The dominant effect of non-Abelian multi-gluon vertices is expressed in the formation of diquark correlations
- Baryon is then a compound system whose observable properties and interactions are primarily determined by the quark+diquark structure

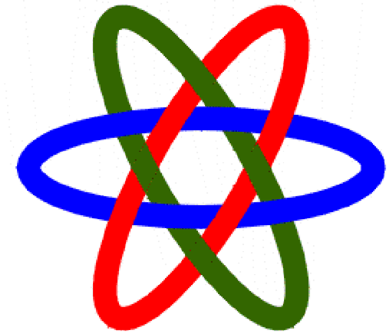


Baryon Structure

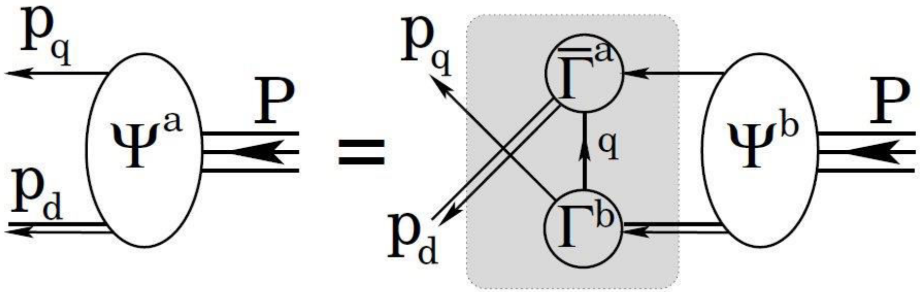
- Both scalar-isoscalar and pseudovector-isotriplet diquark correlations feature within a nucleon.
- Any study that neglects pseudovector diquarks is unrealistic because no self-consistent solution of the Faddeev equation can produce a nucleon constructed solely from a scalar diquark
- The relative probability of scalar versus pseudovector diquarks in a nucleon is a dynamical statement.
  - Realistic computations predict a scalar diquark strength of approximately 60%
  - This prediction can be tested by contemporary experiments.



Baryon Structure



- A nucleon (and kindred baryons) described by the Poincaré-covariant Faddeev equation is a Borromean bound-state, the binding within which has two contributions:
  - One part is expressed in the formation of tight diquark correlations, originating in non-Abelian nature of QCD
  - That is augmented, by attraction generated by the quark exchange depicted in the shaded area
  - This exchange ensures that diquark correlations within the nucleon are fully dynamical: no quark holds a special place because each one participates in all diquarks to the fullest extent allowed by its quantum numbers.
  - The continual rearrangement of the quarks guarantees, *inter alia*, that the nucleon's dressed-quark wave function complies with Pauli statistics.

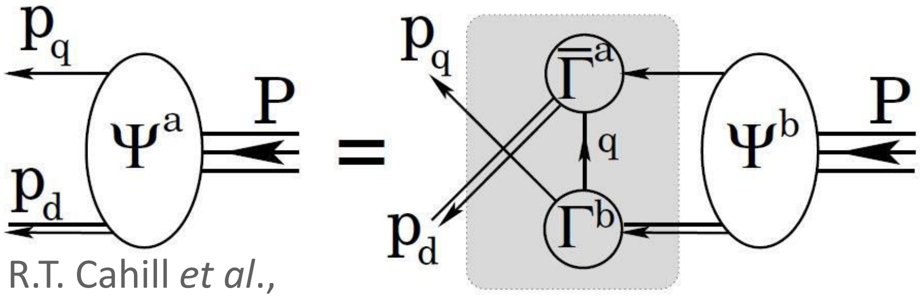


Diquarks

- **Not your grandfather's diquarks!**
- Dynamically generated correlations
- Two particle sub-cluster is not frozen
  - There is a predicted probability for each given cluster within a given J^P baryon
  - Nucleon: $1^+/0^+ \approx 60\%$

Other clusters are negligible in J^+ states

- Faddeev equation baryon spectrum must have significant overlap with that of the three-constituent quark model and no relation to the Lichtenberg-Tassie quark+diquark model



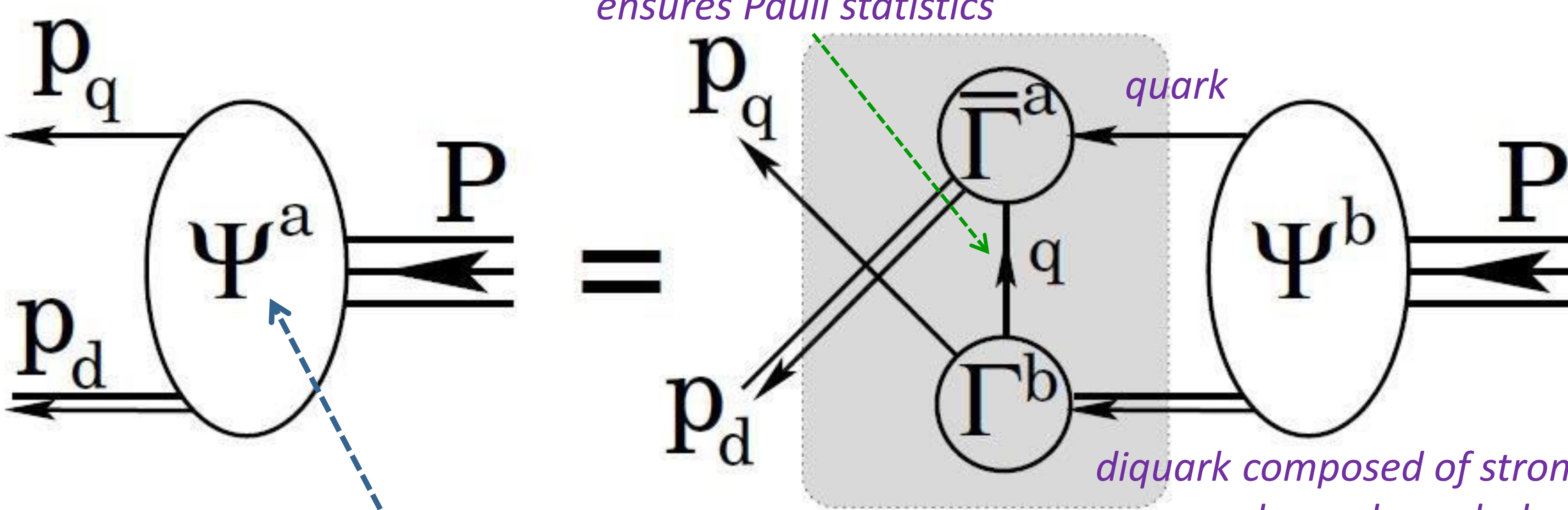
R.T. Cahill *et al.*,
[Austral. J. Phys. 42 (1989) 129-145]

Baryon Structure

- Poincaré covariant Faddeev equation sums all possible exchanges and interactions that can take place between three dressed-quarks
- Confinement and DCSB are readily expressed
- ***Prediction***: owing to *DCSB in QCD, strong diquark correlations exist within baryons*
- Diquark correlations are not pointlike
  - Typically, $r_{0+} \sim r_\pi$ & $r_{1+} \sim r_\rho$ (actually 10% larger)
  - They have soft form factors

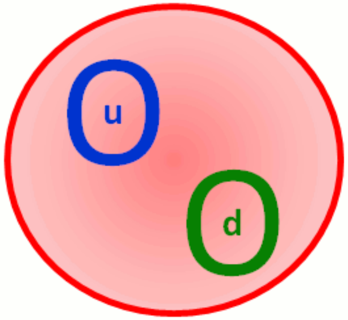
R.T. Cahill *et al.*,
Austral. J. Phys. 42 (1989) 129-145

Faddeev Equation



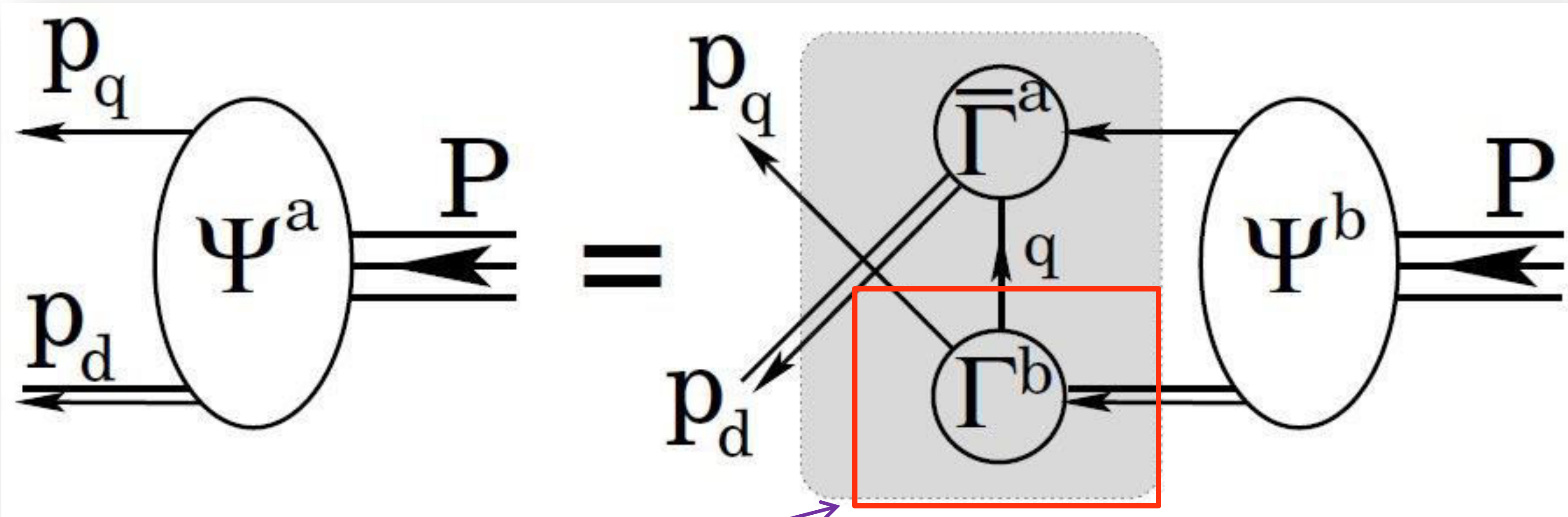
- Linear, Homogeneous Matrix equation
  - Yields *wave function (Poincaré Covariant Faddeev* Amplitude) that describes quark-diquark relative motion within the nucleon
- Scalar and Axial-Vector Diquarks . . .
  - Both have "*correct*" parity and "*right*" masses
  - In Nucleon's Rest Frame Amplitude has

    s−, p− & d−wave correlations

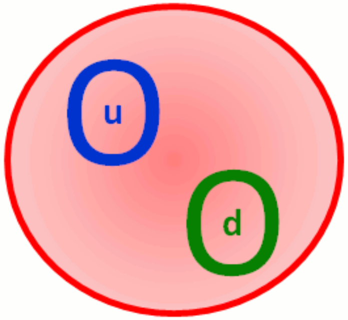


Faddeev Equation

- Why should a pole approximation produce reliable results?



quark-quark scattering matrix
- a pole approximation is used to arrive at the Faddeev-equation



Calculation of diquark masses in QCD
R.T. Cahill, C.D. Roberts and J. Praschifka
[Phys.Rev. D**36** (1987) 2804]

Diquarks

Consider the rainbow-gap and ladder-Bethe-Salpeter equations

$$S(p)^{-1} = i\gamma \cdot p + m + \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q) \frac{\lambda^a}{2} \gamma_\nu(q,p),$$

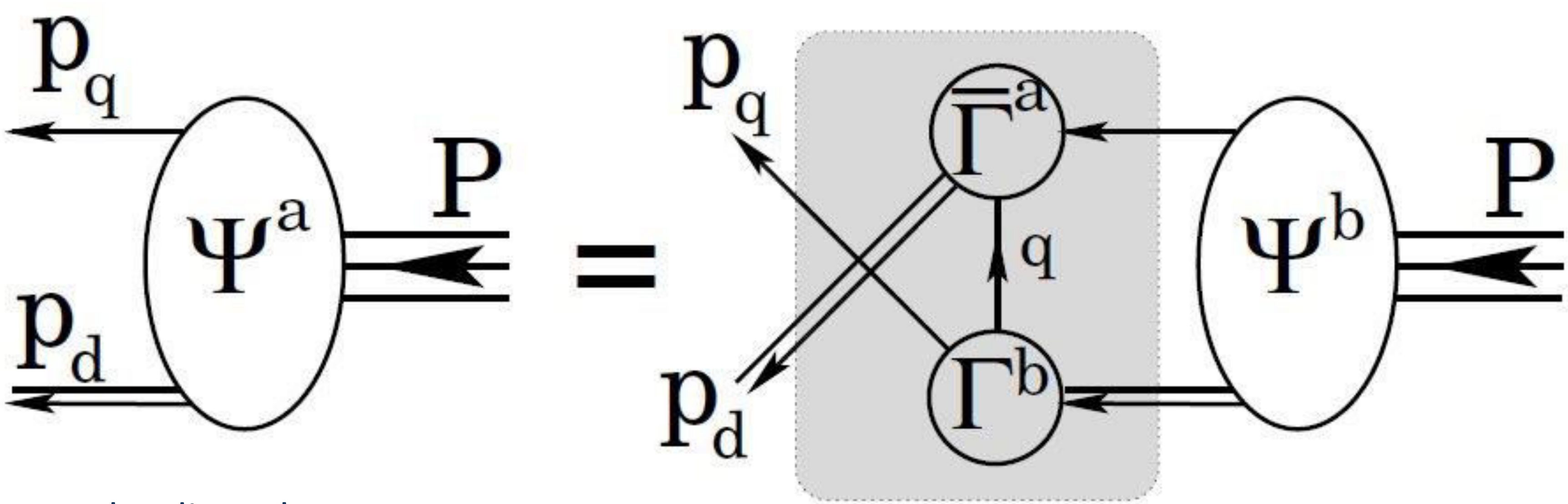
$$\Gamma(k;P) = -\int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q+P) \Gamma(q;P) S(q) \frac{\lambda^a}{2} \gamma_\nu .$$

- In this symmetry-preserving truncation, colour-antitriplet quark-quark correlations (diquarks) are described by a very similar homogeneous Bethe-Salpeter equation

$$\Gamma_{qq}(k;P)C^\dagger = -\frac{1}{2} \int \frac{d^4q}{(2\pi)^4} g^2 D_{\mu\nu}(p-q) \frac{\lambda^a}{2} \gamma_\mu S(q+P) \Gamma_{qq}(q;P) C^\dagger S(q) \frac{\lambda^a}{2} \gamma_\nu$$

- Only difference is factor of ½
- Hence, an interaction that describes mesons also generates diquark correlations in the colour-antitriplet channel

Faddeev Equation



scalar diquark component

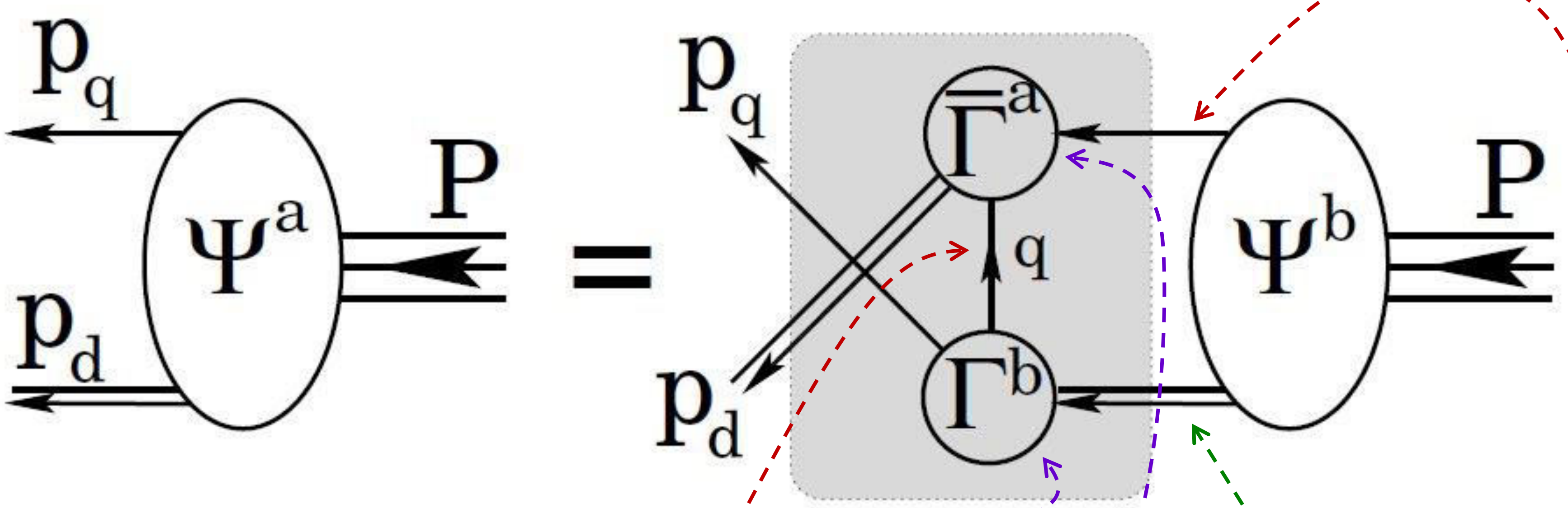
$$\begin{bmatrix} \mathcal{S}(k;P)\,u(P) \\ \mathcal{A}^i_\mu(k;P)\,u(P) \end{bmatrix} = -4\int \frac{d^4\ell}{(2\pi)^4}\,\mathcal{M}(k,\ell;P) \begin{bmatrix} \mathcal{S}(\ell;P)\,u(P) \\ \mathcal{A}^j_\nu(\ell;P)\,u(P) \end{bmatrix}$$

axial-vector diquark component

$$\mathcal{M}(k,\ell;P) = \begin{bmatrix} \mathcal{M}_{00} & (\mathcal{M}_{01})^j_\nu \\ (\mathcal{M}_{10})^i_\mu & (\mathcal{M}_{11})^{ij}_{\mu\nu} \end{bmatrix}$$

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

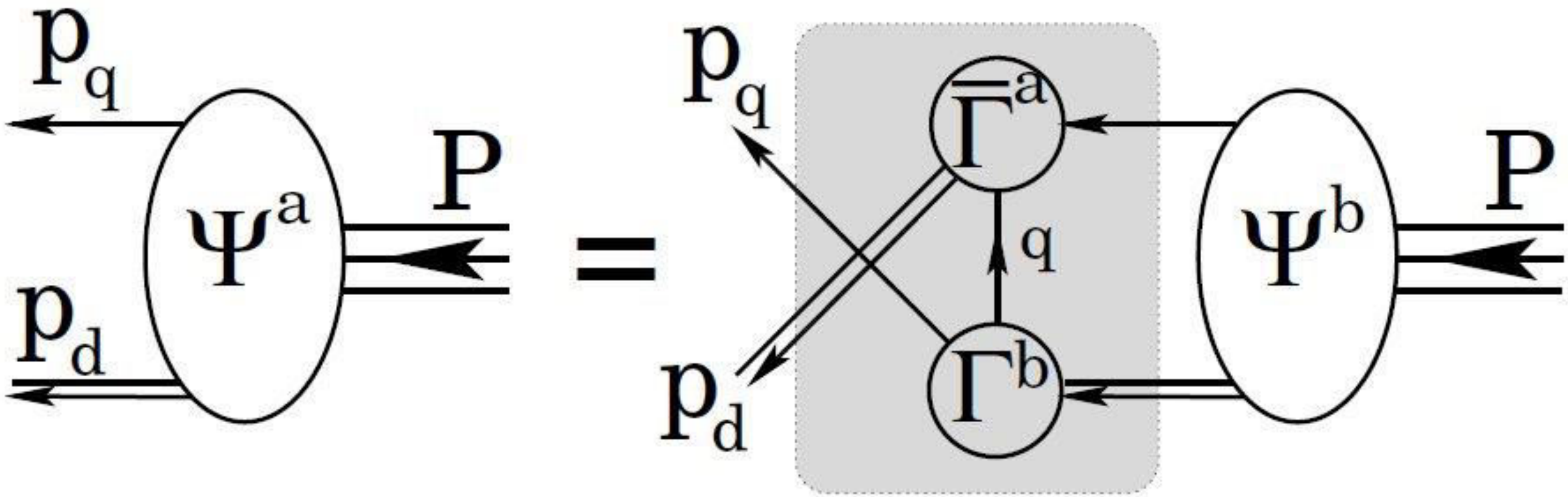
Faddeev Equation



$$\mathcal{M}_{00} = \Gamma^{0^+}(k_q - \ell_{qq}/2; \ell_{qq})\, S^{\rm T}(\ell_{qq} - k_q)$$
$$\times\, \bar\Gamma^{0^+}(\ell_q - k_{qq}/2; -k_{qq})\, S(\ell_q)\, \Delta^{0^+}(\ell_{qq})$$
$$\ell_q = \ell + P/3,\; k_q = k + P/3,$$
$$\ell_{qq} = -\ell + 2P/3,\; k_{qq} = -k + 2P/3$$

Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Faddeev Equation



- Every one of these entries has a simple matrix structure
- Similar form for the kernel entries that involve axial-vector diquark correlations
- Combining everything, one arrives at a linear homogeneous matrix equation for the amplitudes

  S(k;P)u(P), A(k;P)u(P)

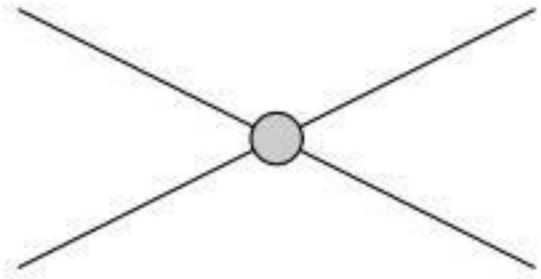
Contact Interaction

- Symmetry-preserving treatment of vector × vector contact interaction is useful tool for the study of phenomena characterised by probe momenta less-than the dressed-quark mass.
- Whilst this interaction produces form factors which are too hard, interpreted carefully, even they can be used to draw valuable insights, *e.g*. concerning relationships between different hadrons.
- Studies employing a symmetry-preserving regularisation of the contact interaction serve as a useful surrogate, exploring domains which analyses using interactions that more closely resemble those of QCD are as yet unable to enter.
- They provide crucial benchmarks for analyses that use data as tool for charting the quark-quark interaction at long-range; i.e., identifying signals of the running of couplings and masses in QCD.

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons*, Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

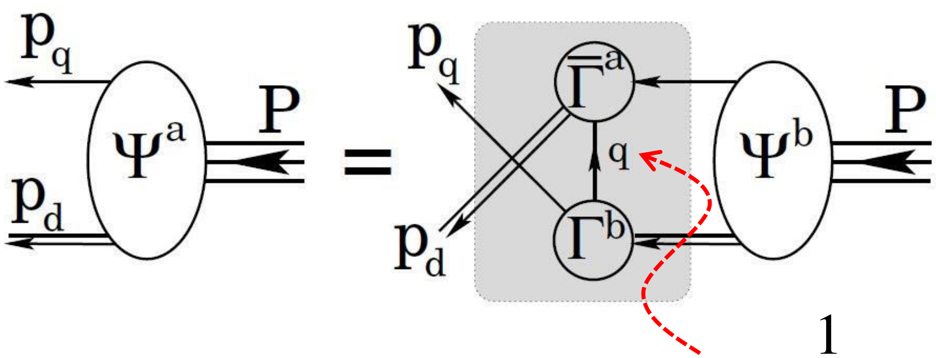
Δ-Baryon

- Simpler case: proton's first spin-flip excitation = *Δ*-Baryon
- $J=3/2^+$ & Isospin = $3/2^+$
  - Scalar diquark has isospin = 0^+ … cannot combine 0^+ & $½^+$ to obtain 3/2+ … no scalar diquarks in decuplet baryons!
  - Axial-vector diquark has isospin = 1+ … $1^+ \otimes ½^+$ can yield 3/2+
- Flavour-symmetric ground-state baryons contain only axial-vector diquarks
- Now … Illustration of Faddeev equation
  - Contact interaction
  - Static approximation:
    - Adjunct to contact interaction,
      improving internal consistency



H.L.L. Roberts *et al.*, arXiv:1101.4244 [nucl-th], Few Body Syst. **51** (2011) pp. 1-25

Spectrum of Baryons



$$\frac{1}{i\gamma\cdot p + M} \to \frac{g_B^2}{M}$$

Variant of:
A. Buck, R. Alkofer & H. Reinhardt, Phys. Lett. ***B286*** *(1992) 29.*

- Static "approximation"
  - Implements analogue of contact interaction in Faddeev-equation
- In combination with contact-interaction diquark-correlations, generates Faddeev equation kernels which themselves are momentum-independent
- The merit of this truncation is the *dramatic simplifications* which it produces
- Enables many reliable insights to be drawn

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons,* Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

Δ-Baryon

- Δ-baryon has four charge states:
  - uuu = +2; uud = + 1; udd = 0; ddd = -1
- Assume isospin symmetry (good to better than 3%):
  - mu = md & strong interactions are flavour-blind

  Then all charge states are degenerate, so can focus on the uuu^{++} system, which is maximally symmetric
- Contact interaction Faddeev amplitude for Δ^{++}:

$$\Psi^\Delta(P) = \mathrm{t}^+ \Gamma_\mu^{1+}(K) \Delta_{\mu\nu}(K) \Delta_\nu(P)$$

$$\Gamma_\mu^{1+}(K) = \gamma_5 \gamma_\mu^{\mathrm{T}} E_{1+}(K) \qquad \Delta_\nu(P) = \mathcal{S}^\Delta(P) u_\nu(P)$$

$$\Delta_{\mu\nu}(K) = \left(\delta_{\mu\nu} + \frac{K_\mu K_\nu}{m_{1+}^2} \right) \frac{1}{K^2 + m_{1+}^2}$$

  - t+ = diagonal[√2 , 0 , 0]

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons*, Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

Δ-Baryon

- u_ν … Rarita-Schwinger spinor is necessary to unambiguously represent a covariant J=3/2.
- The positive-energy spinor is defined by the following equations:

$$(i\gamma \cdot P + M)\, u_\mu(P;r) = 0\,,\ \gamma_\mu u_\mu(P;r) = 0\,,\ P_\mu u_\mu(P;r) = 0\,,$$

where $r = -3/2, -1/2, 1/2, 3/2$ are the spin projections. It is normalised:

$$\bar{u}_\mu(P;r')\, u_\mu(P;r) = 2M\,,$$

and satisfies a completeness relation

Positive-energy projection operator

$$\frac{1}{2M}\sum_{r=-3/2}^{3/2} u_\mu(P;r)\, \bar{u}_\nu(P;r) = \Lambda_+(P)\, R_{\mu\nu}\,,$$

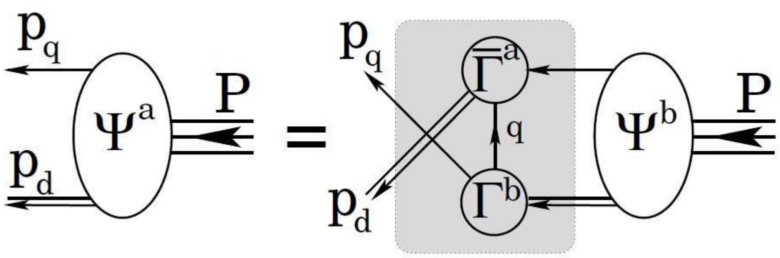
where

$$R_{\mu\nu} = \delta_{\mu\nu} I_{\rm D} - \frac{1}{3}\gamma_\mu\gamma_\nu + \frac{2}{3}\hat{P}_\mu\hat{P}_\nu I_{\rm D} - i\frac{1}{3}[\hat{P}_\mu\gamma_\nu - \hat{P}_\nu\gamma_\mu]\,,$$

with $I_{\rm D}$ the 4×4 identity matrix in Dirac space and $\hat{P}^2 = -1$. This identity is very useful in simplifying the positive-energy Δ's Faddeev equation.

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons*, Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

Δ-Baryon



- Faddeev amplitudes
  - contact interaction
  - Static approximation

  Dependence on relative momentum = forbidden
- Δ-baryon Faddeev amplitude: $\mathcal{S}^{\Delta}(P) = f_1^{\Delta}(P) I_{\rm D}\,,$
- Faddeev equation:

$$\mathcal{S}^{\Delta}(P) u_\nu(P) = 4 \int \frac{d^4 l}{(2\pi)^4} \mathcal{M}^{\Delta}_{\mu\nu}(l,P)\, \mathcal{S}^{\Delta}(P) u_\nu(P)$$

$$\mathcal{M}^{\Delta}_{\mu\nu}(l,P) = \mathbf{t}^+ \Gamma^{1^+}_{\rho}(K_1)\, S^{\rm T}(l)\, \mathbf{t}^+ \bar{\Gamma}^{1^+}_{\mu}(-K_1)\, S(l)\, \Delta^{1^+}_{\rho\nu}(-l+P)$$

$$S^{\rm T}(l) \to \frac{g_\Delta^2}{M}$$ static approximation

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons*, Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

Δ-Baryon

- Δ-baryon Faddeev equation:
  - use standard (Euclidean) Dirac algebra relations
  - employ quantities defined above, including in discussion of mesons
- Contact interaction Faddeev amplitude for Δ^{++}:

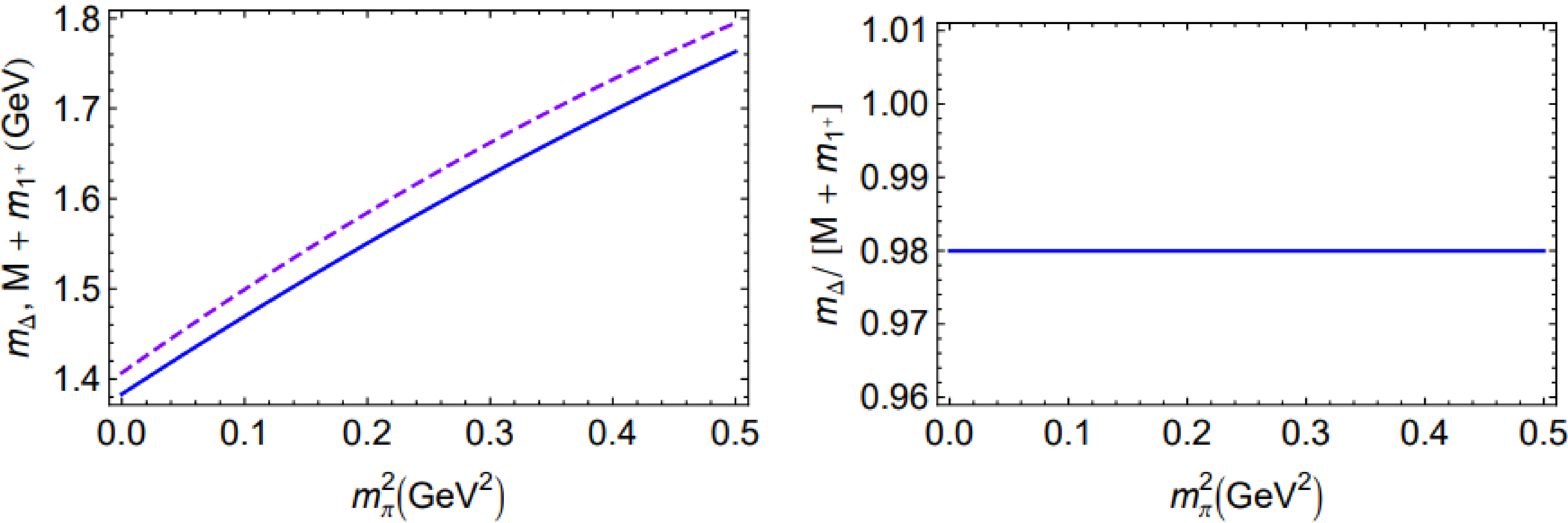
$$1 = \frac{1}{4\pi^2}\frac{2}{M_Q}\frac{1}{M_{1^+}^2}\int_0^1 d\alpha\, 4\int_k^\Lambda (\Gamma_{1^+}^C)^2 \frac{(M_{1^+}^2 + (1-\alpha)^2 M_\Delta^2)(\alpha M_\Delta + M_Q)}{(q^2 + (1-\alpha)M_Q^2 + \alpha M_{1^+}^2 - \alpha(1-\alpha)M_\Delta^2)^2}$$

- Features:
  - Coupling decreases as dressed-quark and 1^+ diquark masses increase
    - ∴ mass of bound-state must increase as a result
  - Coupling depends on strength of the qq → Γ_{1+} coupling
    - ∴ mass of bound-state decreases as correlations become stronger

Roberts, H. L. L. *et al.*, *Masses of ground and excited-state hadrons*, Few Body Syst. **51** (2011) pp. 1-25; arXiv:1101.4244 [nucl-th]

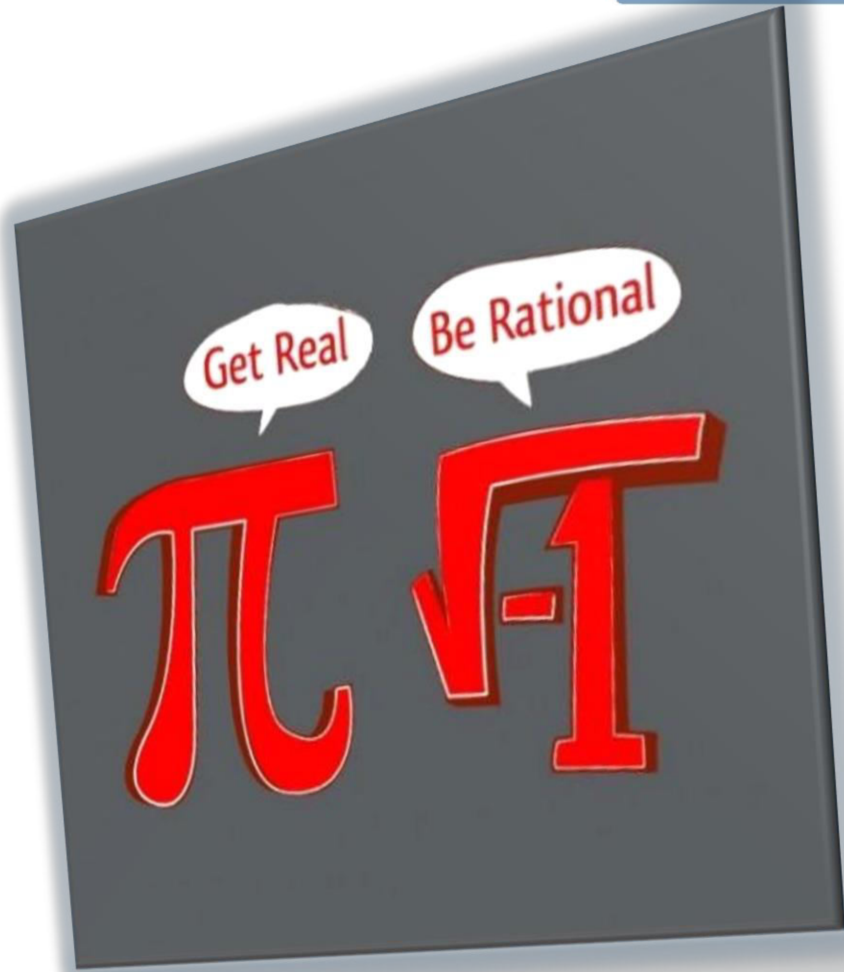
Δ-Baryon

➢ Solution to Δ-baryon Faddeev equation:

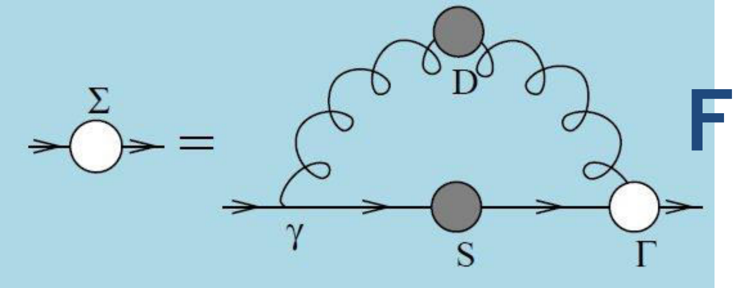


➢ Δ-baryon is a simple system:
- mass rises linearly with the sum of the mass of the constituents
- sum of masses of constituents is good approximation to mass of the bound state

➢ These are real features of the physical baryon spectrum



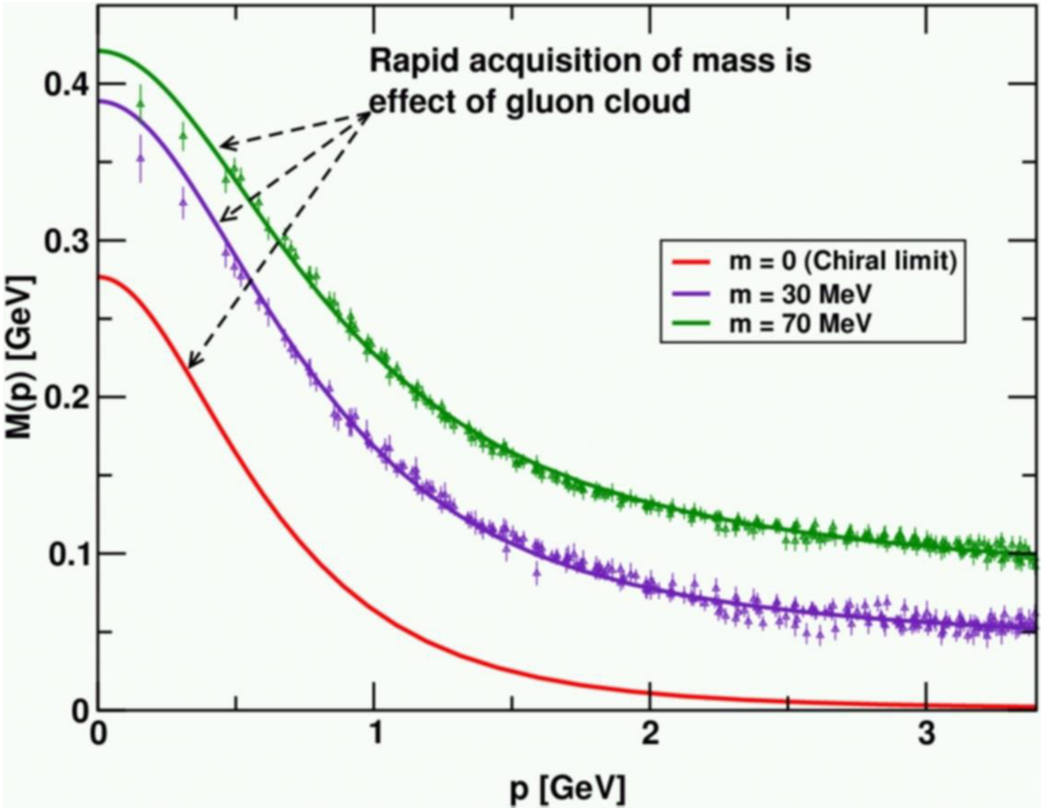
Getting real



Frontiers of Nuclear Science: Theoretical Advances

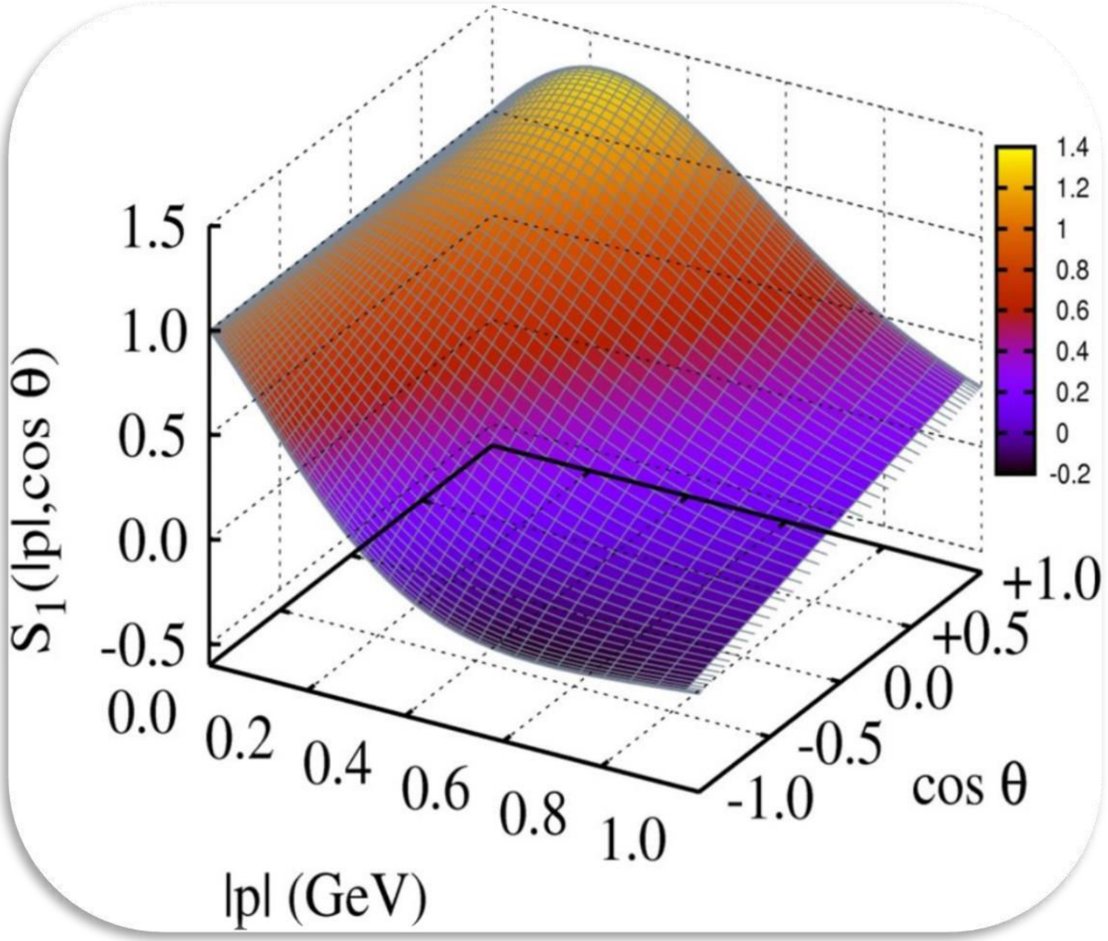
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. **Numerical simulations of lattice QCD** (data, at two different bare masses) **have confirmed model predictions** (solid curves) **that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates**. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.



C.D. Roberts, Prog. Part. Nucl. Phys. 61 (2008) 50

M. Bhagwat & P.C. Tandy, AIP Conf.Proc. 842 (2006) 225-227



Proton's Wave Function

Light-cone distribution amplitudes of the nucleon and negative parity nucleon resonances from lattice QCD
V. M. Braun *et al*., Phys. Rev. D 89 (2014) 094511

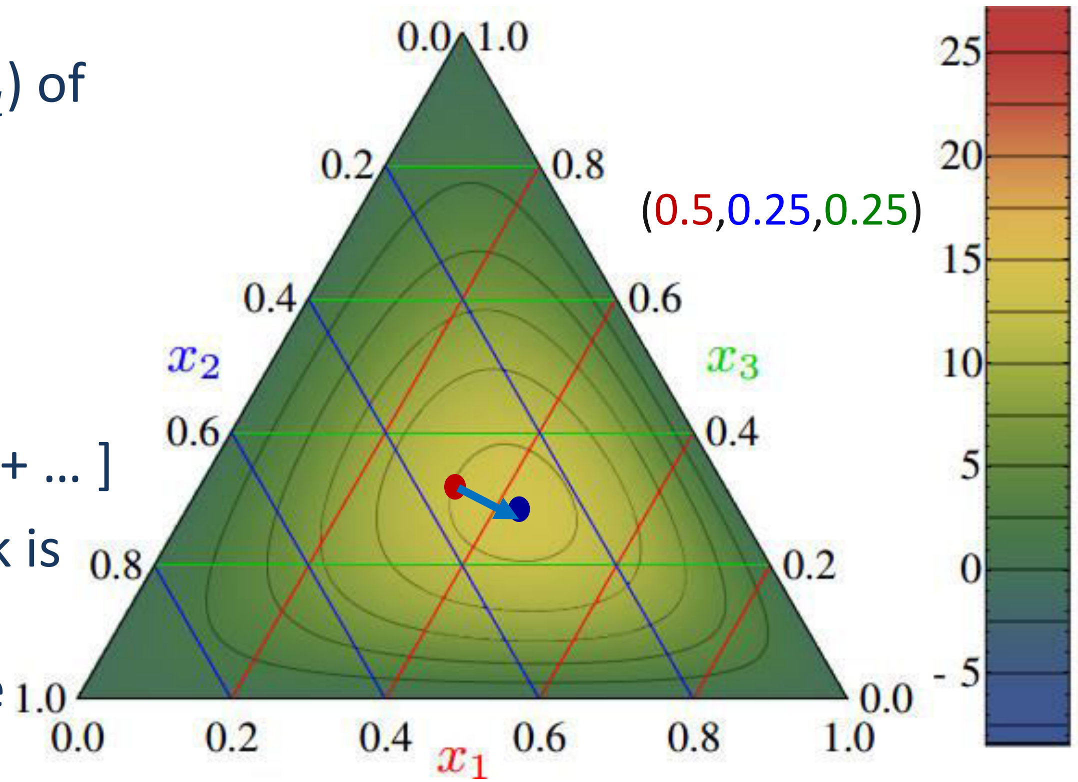
Light-cone distribution amplitudes of the baryon octet
G. S. Bali *et al*. JHEP 1602 (2016) 070

Nucleon PDAs & lQCD

- First lQCD results for n=0, 1 moments of the leading twist PDA of the nucleon are available
- Used to constrain strength (a_{11}) of the leading-order term in a conformal expansion of the nucleon's PDA:

$$\Phi(x_1,x_2,x_3) = 120\, x_1\, x_2\, x_3\, [\, 1 + a_{11}\, P_{11}(x_1,x_2,x_3) + \ldots\,]$$

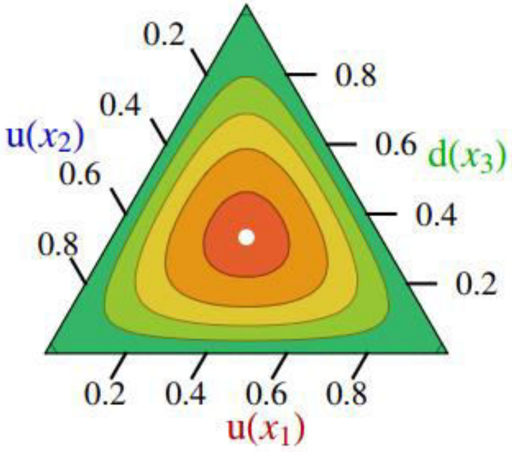
- Shift in location of central peak is *consistent* with existence of diquark correlations within the nucleon



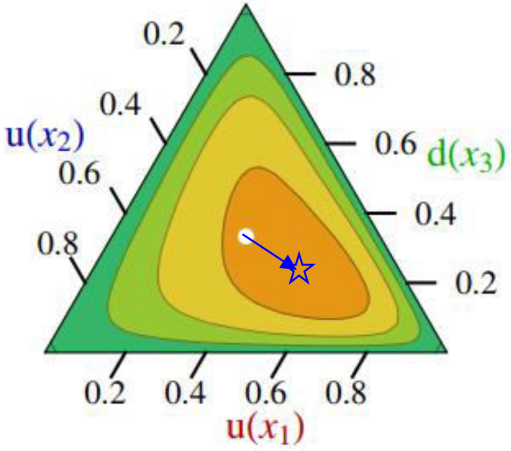
Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
arXiv:1711.09101 [nucl-th]

PDAs of Nucleon & its 1st Radial Excitation

- Methods used for mesons can be extended to compute pointwise behaviour of baryon PDAs



conformal



nucleon

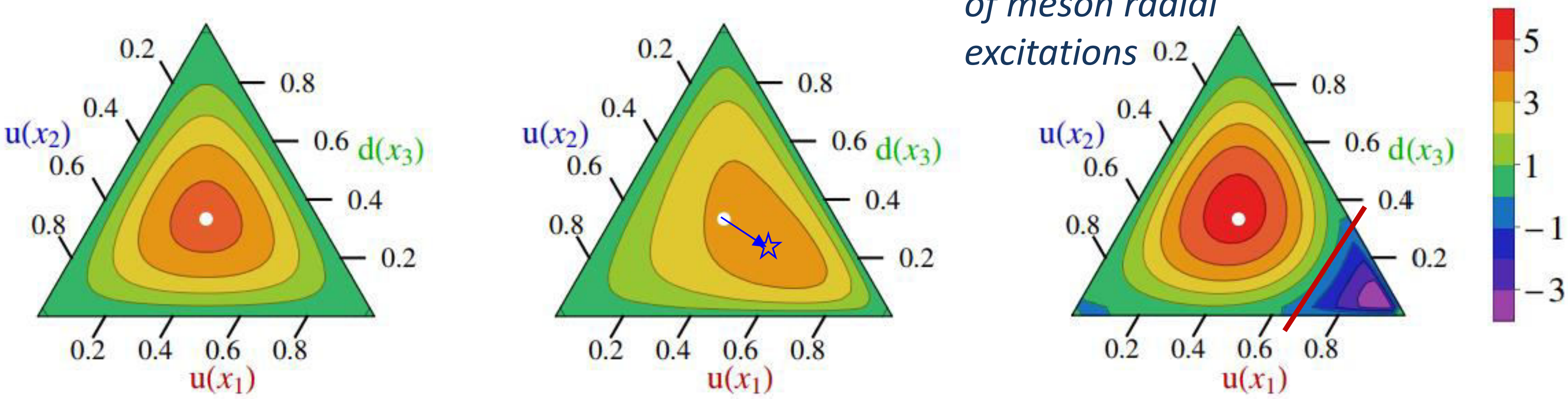
Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
[arXiv:1711.09101 [nucl-th]]

PDAs of Nucleon & its 1st Radial Excitation

- Methods used for mesons can be extended to compute pointwise behaviour of baryon PDAs

Just like QM & PDAs of meson radial excitations



conformal

nucleon

Diquark clustering skews the distribution toward the dressed-quark bystander, which therefore carries more of the proton's light-front momentum

Roper's quark core

Excitation's PDA is not positive definite … there is a prominent locus of zeros in the lower-right corner of the barycentric plot

Diquark correlations in the nucleon

- Agreement between continuum and lattice results

  ONLY when nucleon contains scalar & axial-vector diquark correlations

- Nucleon with only a scalar-diquark, omitting the axial-vector diquark, ruled-out by this confluence between continuum and lattice results

TABLE I. *A* – Eq. (13) interpolation parameters for the proton and Roper PDAs in Fig. 2. *B* – Computed values of the first four moments of the PDAs. Our error on f_N reflects a scalar diquark content of $65 \pm 5\%$; and values in rows marked with "$\not\supset$ av" were obtained assuming the baryon is constituted solely from a scalar diquark. (All results listed at $\zeta = 2\,\text{GeV}$.)

A	$n_{\hat{\varphi}}$	α	β	w_{01}	w_{11}	w_{02}	w_{12}	w_{22}
p	65.8	1.47	1.28	0.096	0.094	0.15	−0.053	0.11
R	14.4	1.42	0.78	−0.93	0.22	−0.21	−0.057	−1.24

B	$10^3 f_N/\text{GeV}^2$	$\langle x_1\rangle_u$	$\langle x_2\rangle_u$	$\langle x_3\rangle_d$
conformal PDA		0.333	0.333	0.333
lQCD [17]	2.84(33)	0.372(7)	0.314(3)	0.314(7)
lQCD [18]	3.60(6)	0.358(6)	0.319(4)	0.323(6)
herein proton	3.78(14)	0.379(4)	0.302(1)	0.319(3)
herein ~~proton $\not\supset$ av~~	~~2.97~~	~~0.412~~	~~0.295~~	~~0.293~~
herein Roper	5.17(32)	0.245(13)	0.363(6)	0.392(6)
herein Roper $\not\supset$ av	2.63	0.010	0.490	0.500

Parton distribution amplitudes: revealing diquarks in the proton and Roper resonance, Cédric Mezrag, Jorge Segovia, Lei Chang and Craig D. Roberts
arXiv:1711.09101 [nucl-th]

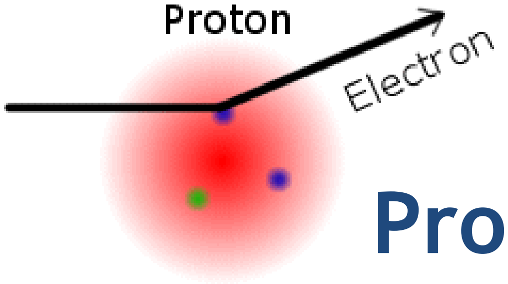
Nucleon and Roper PDAs

No humps or bumps in leading-twist PDAs of ground-state S-wave baryons

- The proton's PDA is a broad, concave function
  - maximum shifted relative to peak in QCD's conformal limit expression
  - Magnitude of shift signals presence of

    both scalar & axial-vector diquark correlations in the nucleon
    - scalar generates around 60% of the proton's normalisation.
- The radial-excitation (Roper) is constituted similarly
  - Pointwise form of its PDA
    - Negative on a material domain
  - Is result of marked interferences between the contributions from both scalar and axial-vector diquarks
    - particularly, the locus of zeros, which

      highlights its character as a radial excitation.
- These features originate with the emergent phenomenon of dynamical chiral symmetry breaking in the Standard Model.



Nucleon Structure
Probed in scattering experiments

- Electron is a good probe because it is structureless

Structureless fermion, or simply structured fermion, F_1=1 & F_2=0, so that G_E=G_M and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$J_\mu(P',P) = ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,,$$
$$= \quad ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)$$

G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

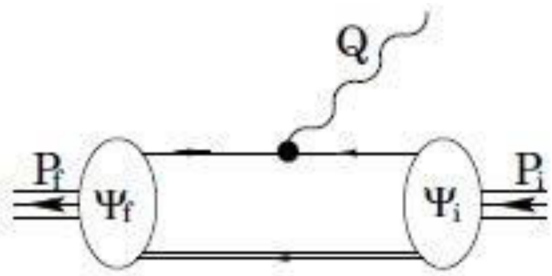
G_M = Sachs Magntic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

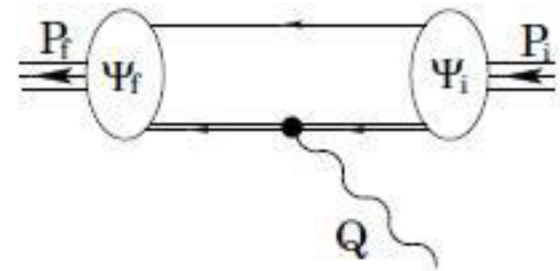
Nucleon form factors

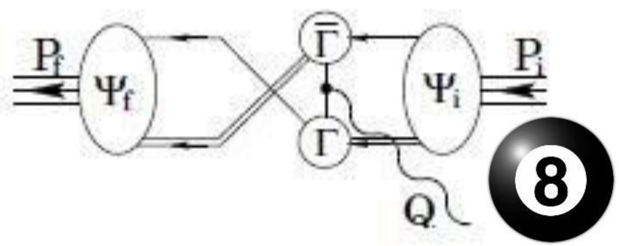
- For the nucleon & *Δ*-baryon and Roper-resonance, studies of the Faddeev equation exist that are based on the 1-loop renormalisation-group-improved interaction that was used efficaciously in the study of mesons
  - *Toward unifying the description of meson and baryon properties*
    G. Eichmann, I.C. Cloët, R. Alkofer, A. Krassnigg and C.D. Roberts
    arXiv:0810.1222 [nucl-th], Phys. Rev. C **79** (2009) 012202(R) (5 pages)
  - *Nucleon electromagnetic form factors from the Faddeev equation*
    G. Eichmann, arXiv:1104.4505 [hep-ph]
  - *Nucleon and Δ elastic and transition form factors,* Jorge Segovia, Ian C. Cloët, Craig D. Roberts and Sebastian M. Schmidt
    arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) pp. 1185-1222
- Analyses retain the scalar and axial-vector diquark correlations, known to be necessary and sufficient for reliable description

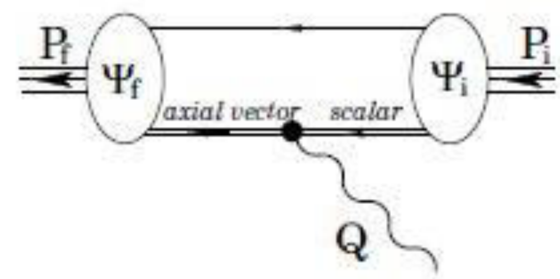
Photon-nucleon current

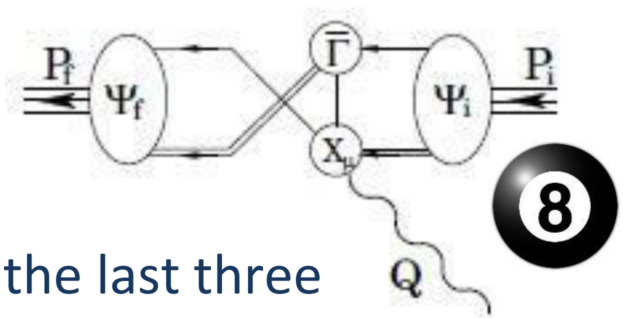
- To compute form factors, one needs a photon-nucleon current
- Composite nucleon must interact with photon via nontrivial current constrained by Ward-Green-Takahashi identities
- DSE → BSE → Faddeev equation plus current → nucleon form factors
- In a realistic calculation, the last three diagrams represent 8-dimensional integrals, which can be evaluated using Monte-Carlo techniques

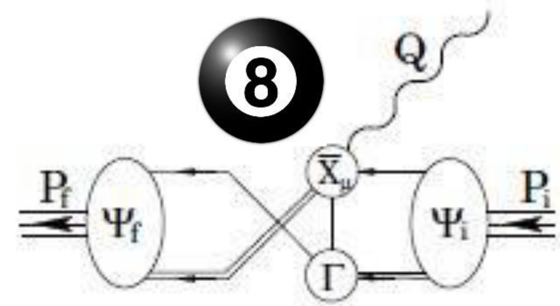












Oettel, Pichowsky, Smekal
Eur.Phys.J. A8 (2000) 251-281

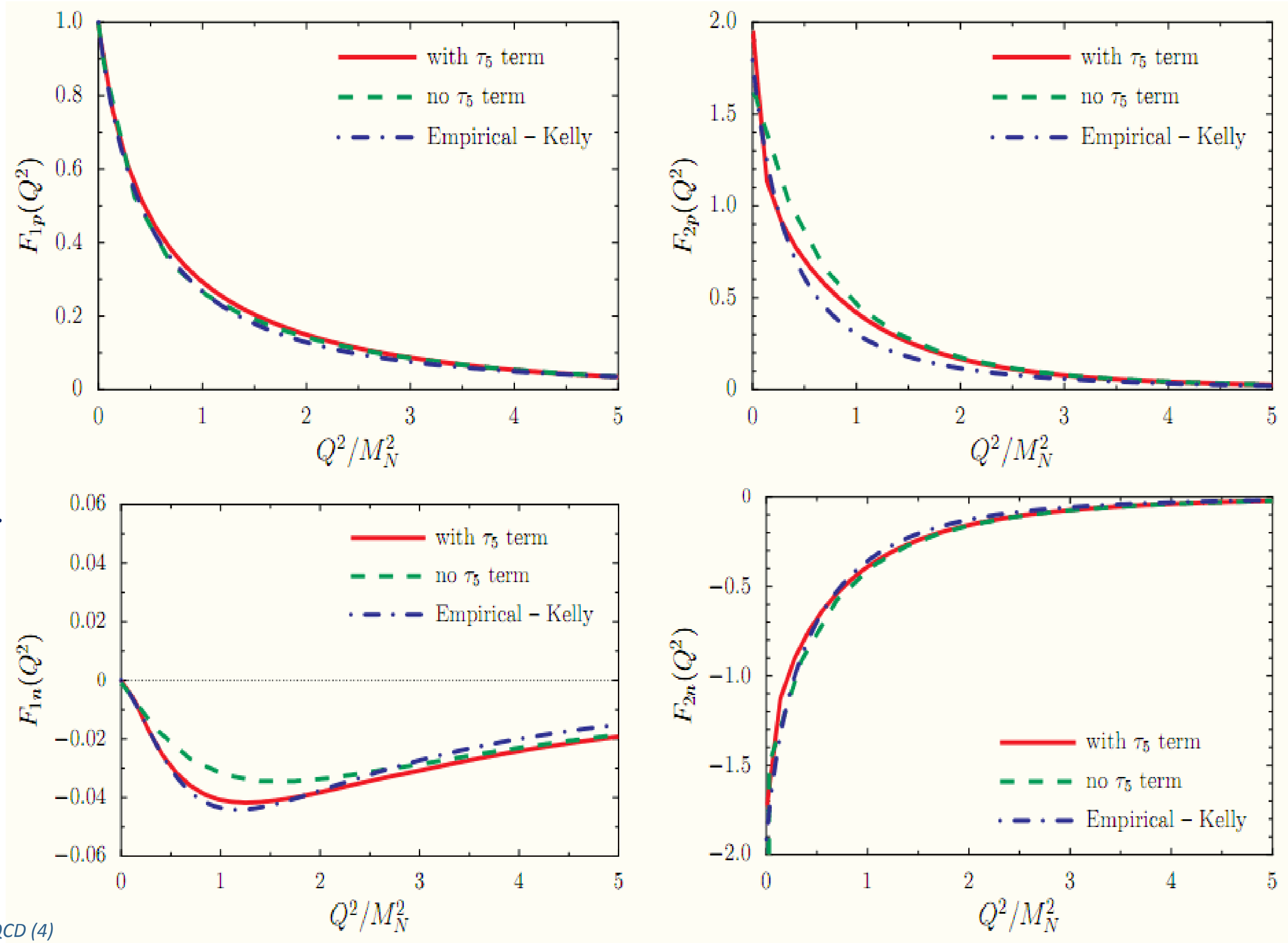
Survey of nucleon electromagnetic form factors
I.C. Cloët et al, arXiv:0812.0416 [nucl-th],
Few Body Syst. **46** (2009) pp. 1-36

Nucleon Form Factors

Unification of meson and nucleon form factors.

Very good description.

Quark's momentum-dependent anomalous magnetic moment has observable impact & materially improves agreement in all cases.



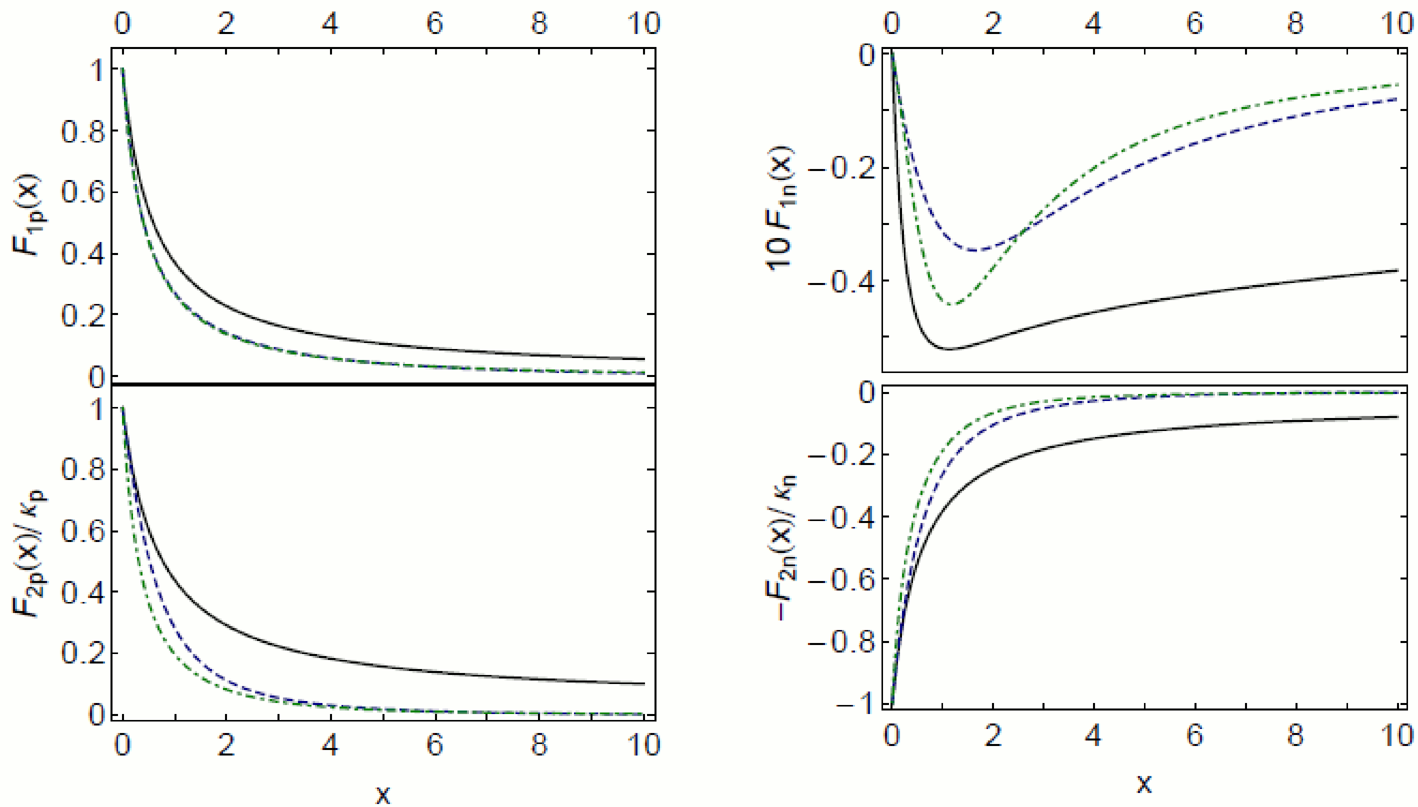
Nucleon Form Factors

Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, [arXiv:1112.2212 [nucl-th]], [Phys. Rev. C**85** (2012) 025205 [21 pages]]

Black solid curve = contact interaction

Blue dashed curve = momentum-dependent interaction

Green dot-dashed curve = parametrisation of experimental data



Momentum independent Faddeev amplitudes, paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

Nucleon and Roper electromagnetic elastic and transition form factors, D. J. Wilson, I. C. Cloët, L. Chang and C. D. Roberts, [arXiv:1112.2212 [nucl-th]](), [Phys. Rev. C**85** (2012) 025205 [21 pages]]()

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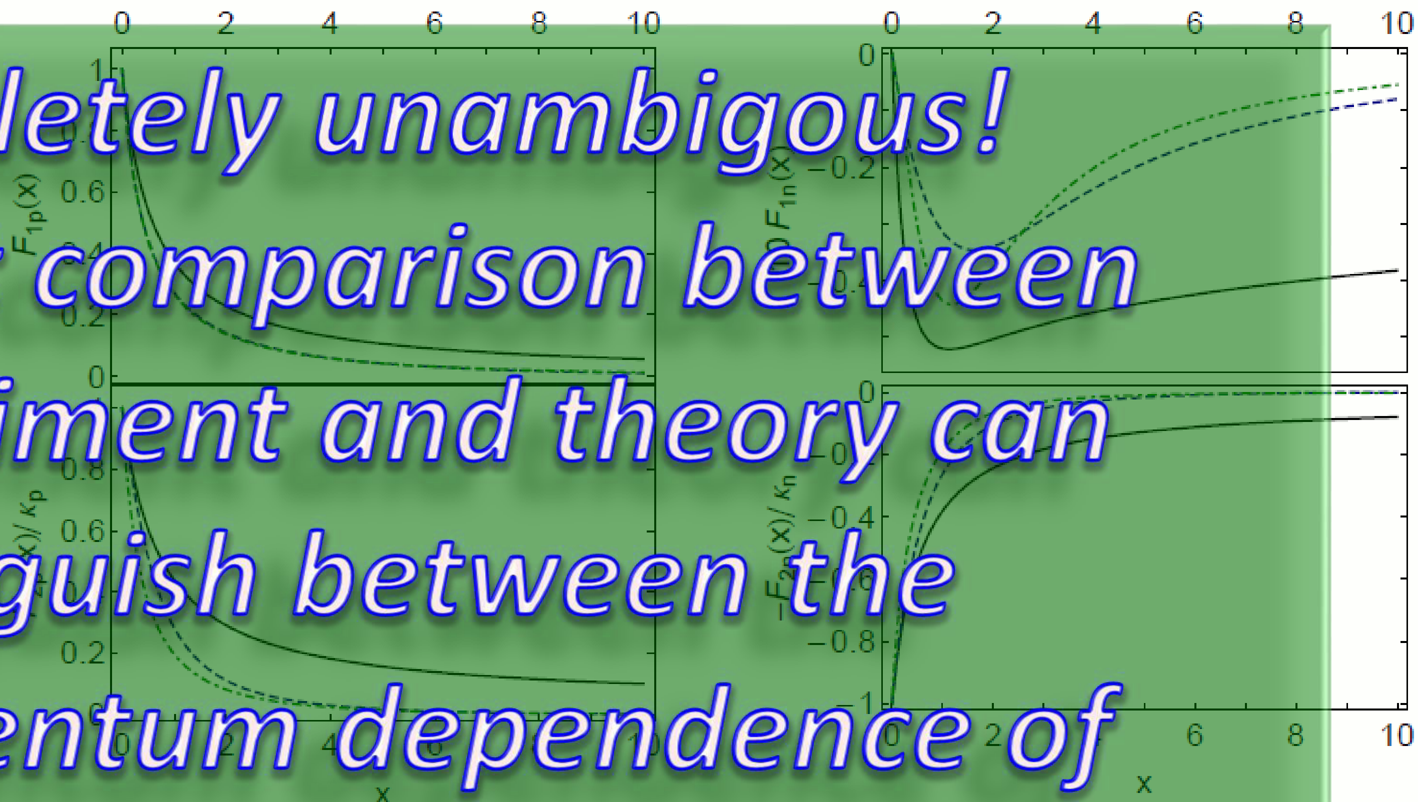


Momentum independent Faddeev amplitudes paired with momentum-independent dressed-quark mass and diquark Bethe-Salpeter amplitudes, produce harder form factors, which are readily distinguished from experiment

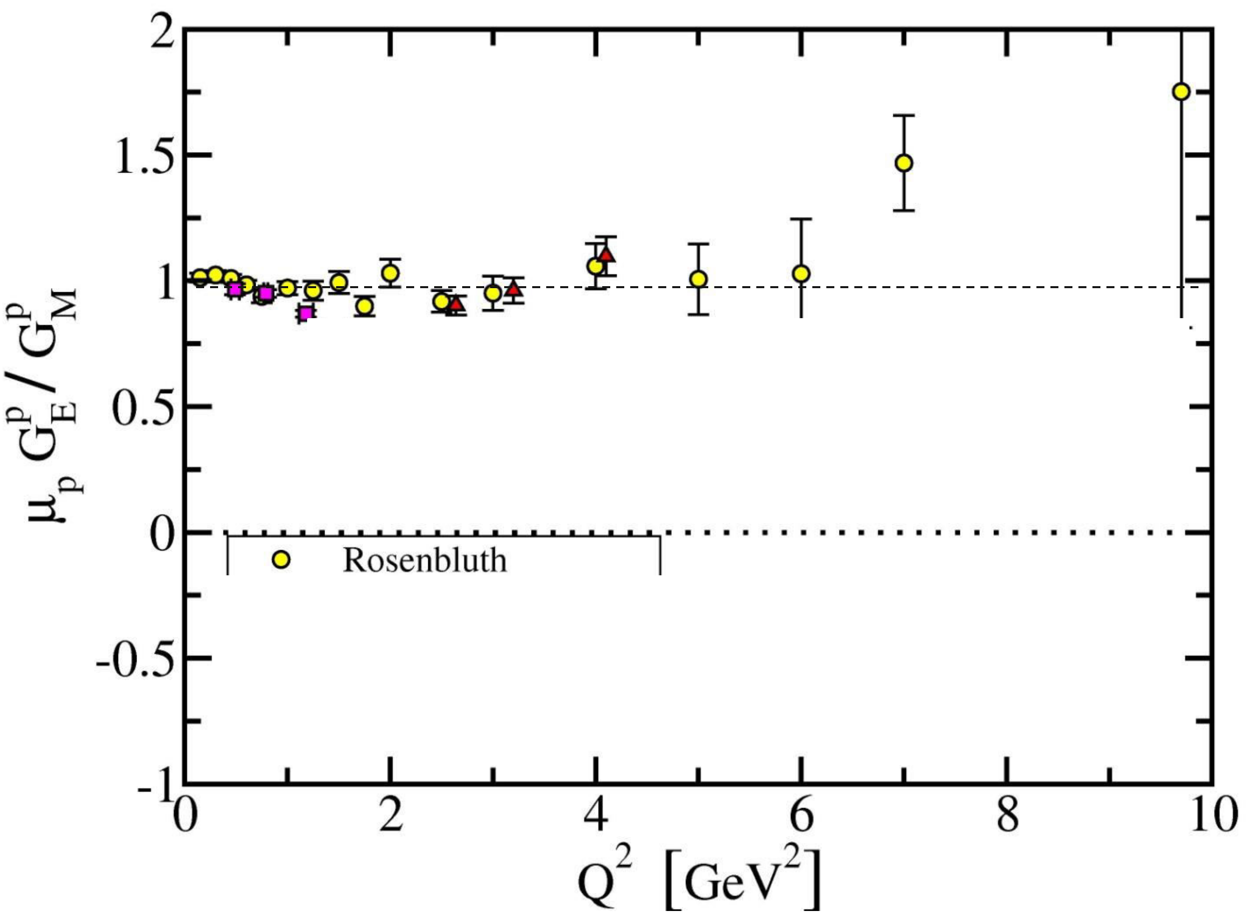
Completely unambigous!

Direct comparison between experiment and theory can distinguish between the momentum dependence of strong-interaction theory

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

Ratio of proton's electromagnetic form factors

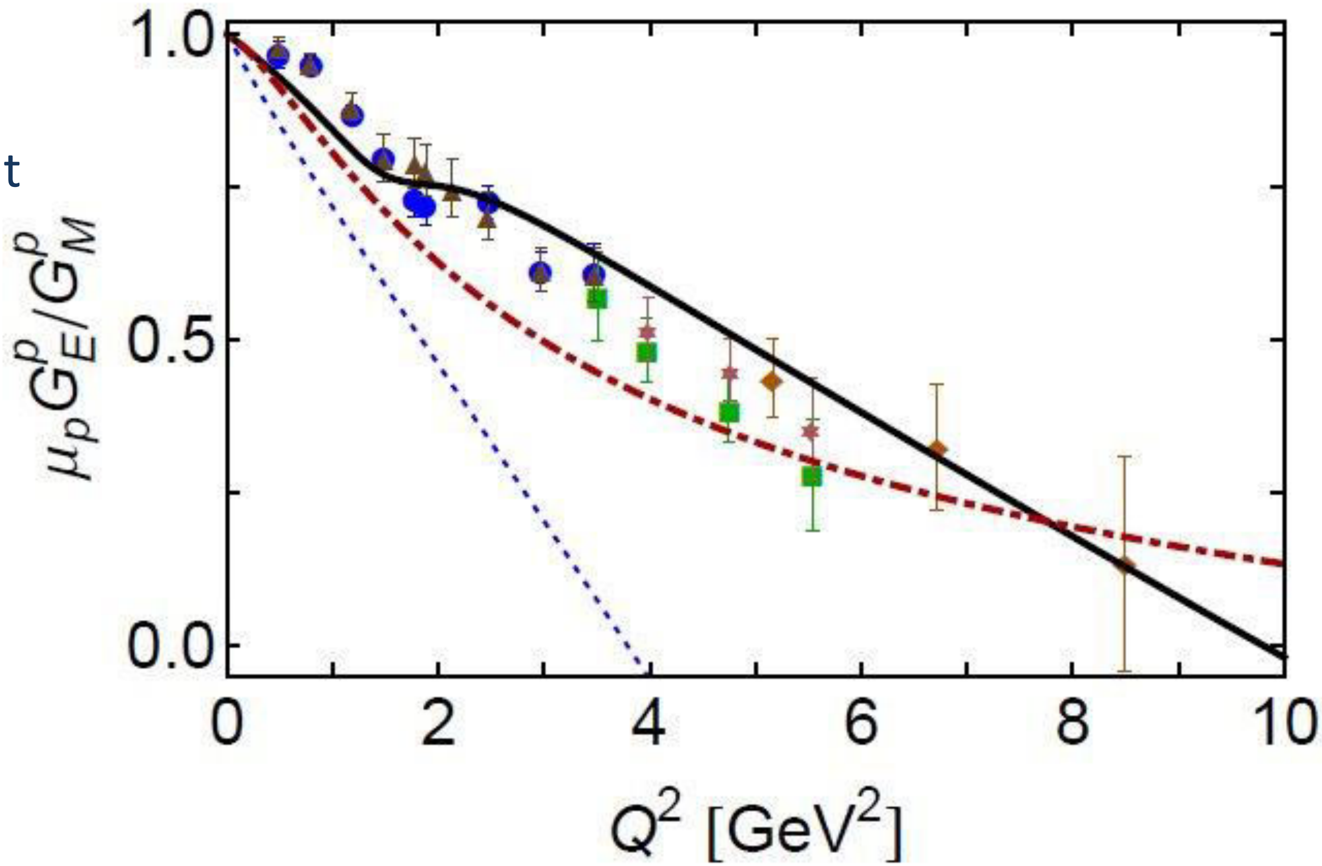
- Data before 1999
  - Looks like the structure of the proton is simple
- The properties of JLab (high luminosity) enabled a new technique to be employed.
- First data released in 1999 and paint a VERY DIFFERENT PICTURE



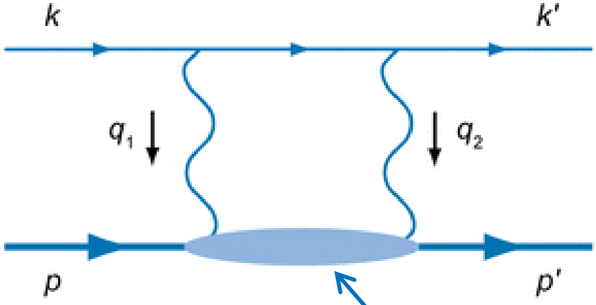
Nucleon and Δ elastic and transition form factors,
Jorge Segovia, Ian C. Cloët, Craig D. Roberts and Sebastian M. Schmidt
arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) pp. 1185-1222

$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

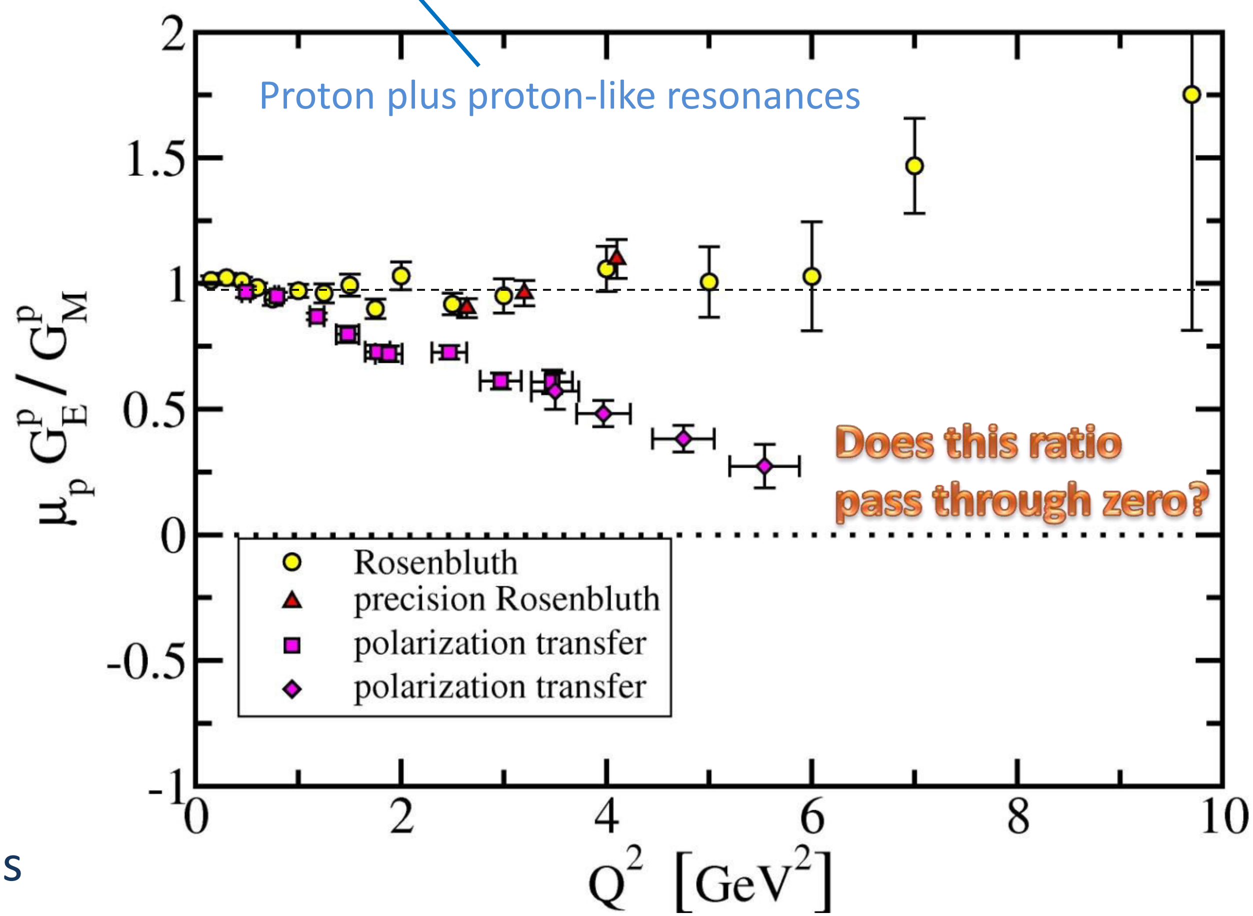
- DSE
  - Solid: $M(p^2)$ result
  - Dashed: M constant
- Dot-dashed = 2004 parametrisation of data



- DSE studies indicate that the *proton has a very rich internal structure*
- The JLab data, obtained using the polarisaton transfer method, are an accurate indication of the behaviour of this ratio
- The pre-1999 data (Rosenbluth) receive large corrections from so-called 2-photon exchange contributions



$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$



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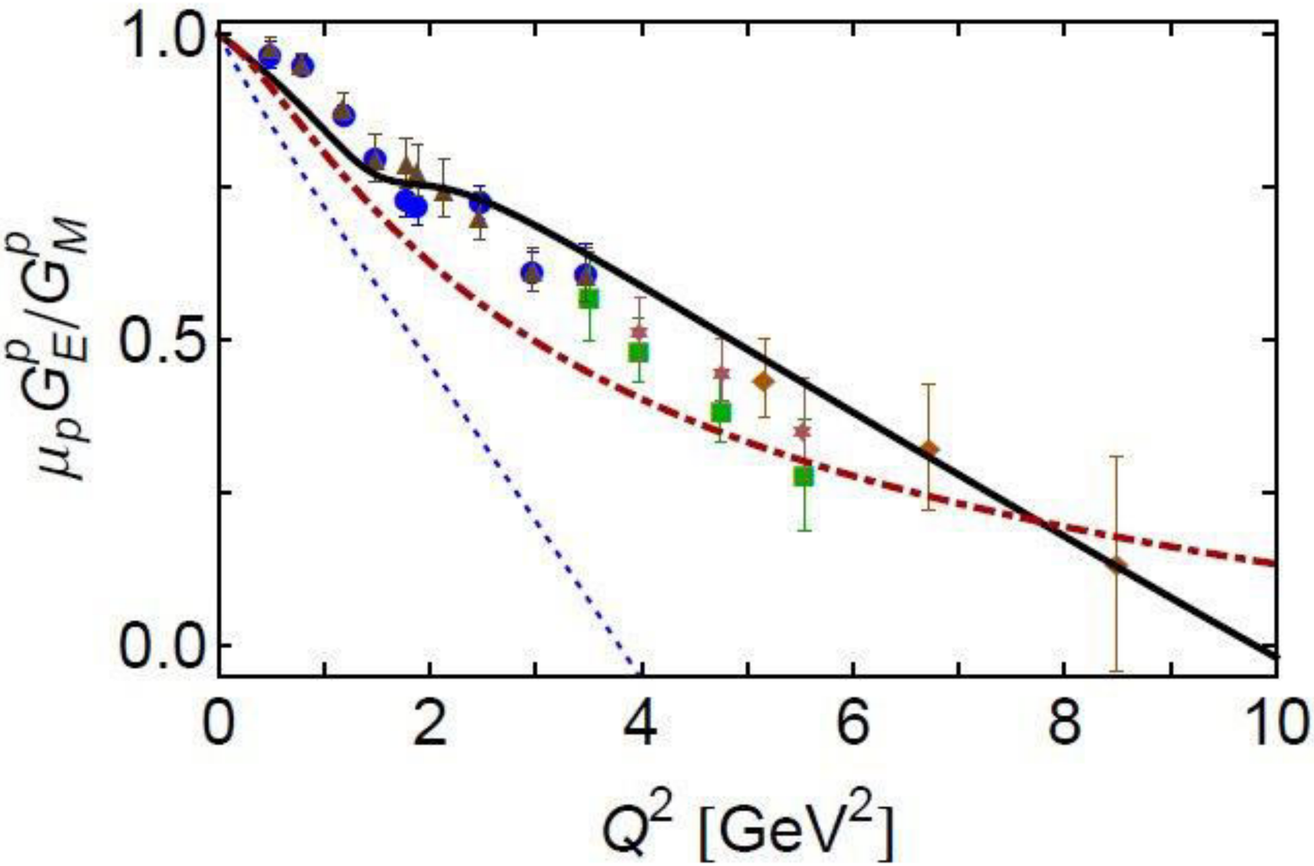
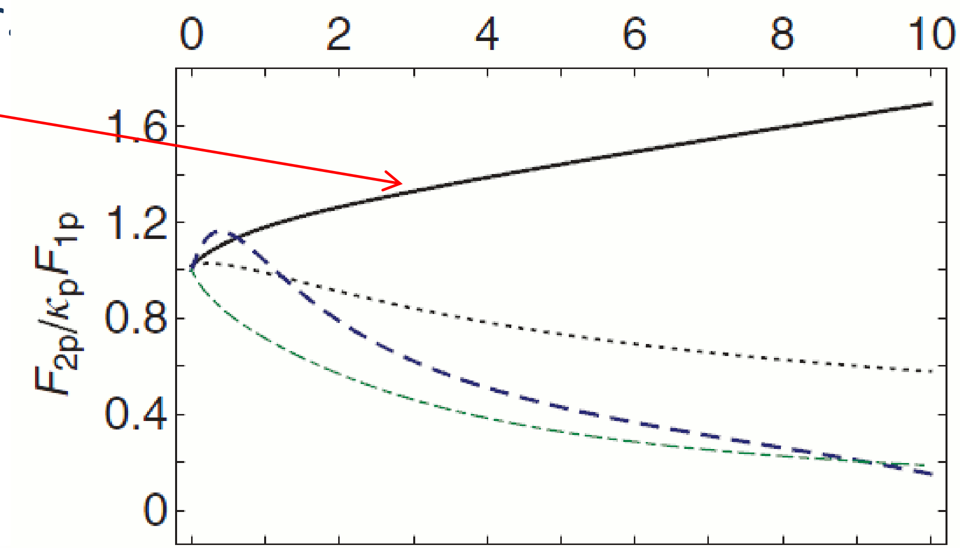
$$\frac{\mu_p G_E^p(Q^2)}{G_M^p(Q^2)}$$

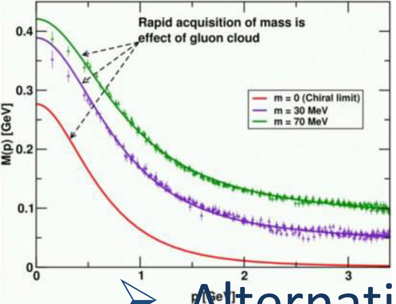
- DSE: there is plainly a chance that G_E can theoretically pass through zero
- But, is a zero unavoidable?



Origin of the zero & its location

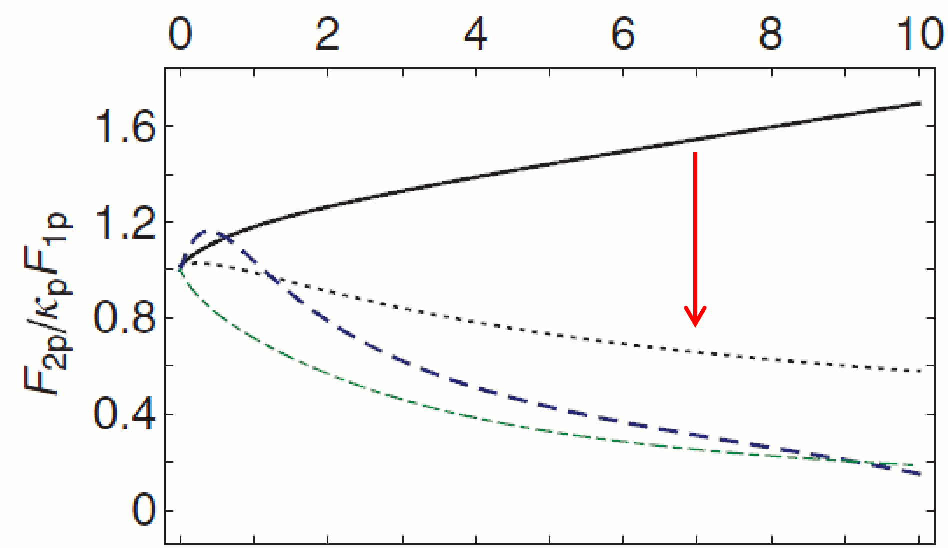
- The Pauli form factor is a gauge of the distribution of magnetization within the proton. Ultimately, this magnetisation is carried by the dressed quarks and influenced by correlations amongst them, which are expressed in the Faddeev wave function.
- If the dressed quarks are described by a momentum-independent mass function, *M*=constant, then they behave as Dirac particles with constant Dirac values for their magnetic moments and produce a hard Pauli form factor.





Origin of the zero & its location

- Alternatively, suppose that the dressed quarks possess a momentum-dependent mass function, $M=M(p^2)$, which is large at infrared momenta but vanishes as their momentum increases.
- At small momenta they will then behave as constituent-like particles with a large magnetic moment, but their mass and magnetic moment will drop toward zero as the probe momentum grows. (Massless fermions cannot possess a measurable magnetic moment in Wigner phase.)
- Such dressed quarks produce a proton Pauli form factor that is large for $Q^2 \sim 0$ but drops rapidly on the domain of transition between nonperturbative and perturbative QCD, to give a very small result at large Q^2.



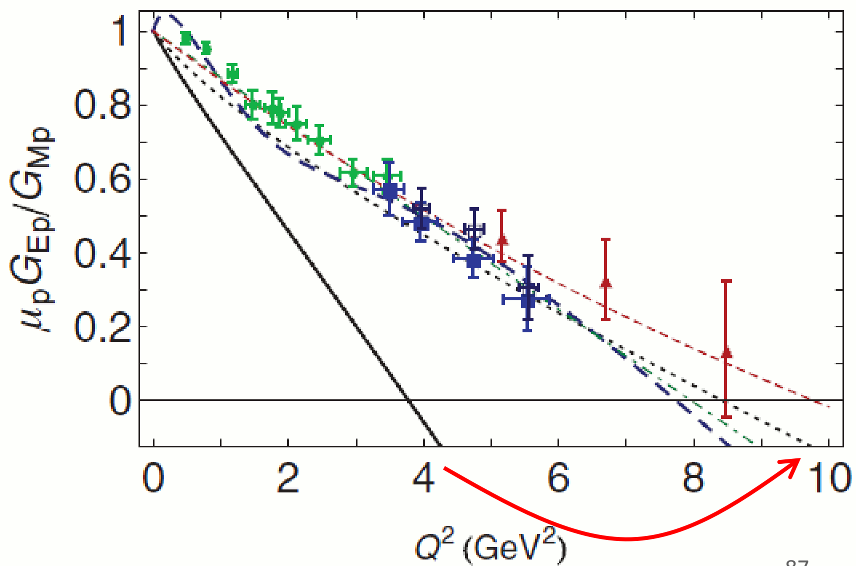
Origin of the zero & its location

- The precise form of the Q^2 dependence will depend on the evolving nature of the angular momentum correlations between the dressed quarks.
- From this perspective, existence, and location if so, of the zero in

  $\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)$

  are a fairly direct measure of the location and width of the transition region between the nonperturbative and perturbative domains of QCD as expressed in the momentum dependence of the dressed-quark mass function.
- Hard, M=constant

  $\rightarrow$ Soft, $M=M(p^2)$

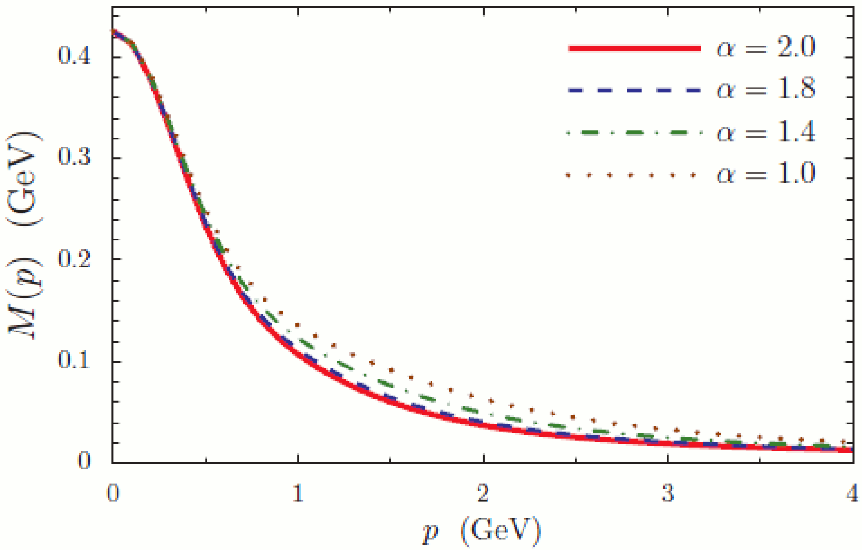


Origin of the zero & its location

- One can anticipate that a mass function which rapidly becomes partonic—namely, is very soft—will not produce a zero
- We've seen that a constant mass function produces a zero at a small value of Q^2
- And also seen and know that a mass function which resembles that obtained in the best available DSE studies and via lattice-QCD simulations produces a zero at a location that is consistent with extant data.
- There is opportunity here for very constructive feedback between future experiments and theory.

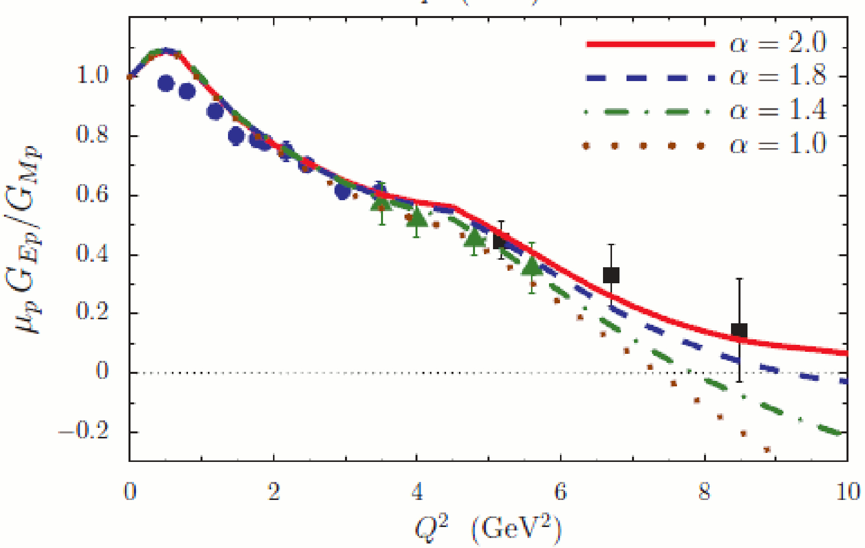
I.C. Cloët, C.D. Roberts, A.W. Thomas: Revealing dressed-quarks via the proton's charge distribution,
arXiv:1304.0855 [nucl-th], Phys. Rev. Lett. **111** (2013) 101803

Visible Impacts of DCSB



$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

- Apparently small changes in M(p) within the domain 1<p(GeV)<3 have striking effect on the proton's electric form factor
- The possible existence and location of the zero is determined by behaviour of $Q^2F_2^p(Q^2)$, proton's Pauli form factor
- Like the pion's PDA, $Q^2F_2^p(Q^2)$ measures the rate at which dressed-quarks become parton-like:
  - ✓ $F_2^p=0$ for bare quark-partons
  - ✓ Therefore, G_E^p can't be zero on the bare-parton domain



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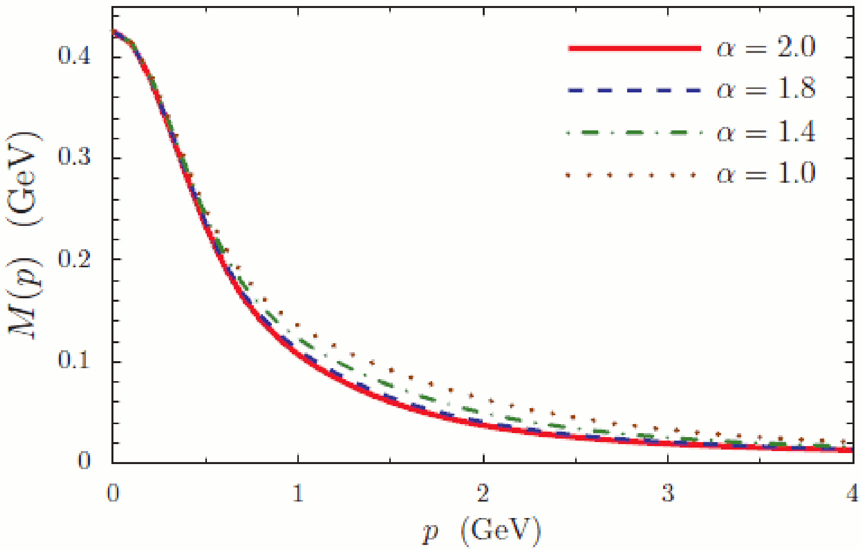
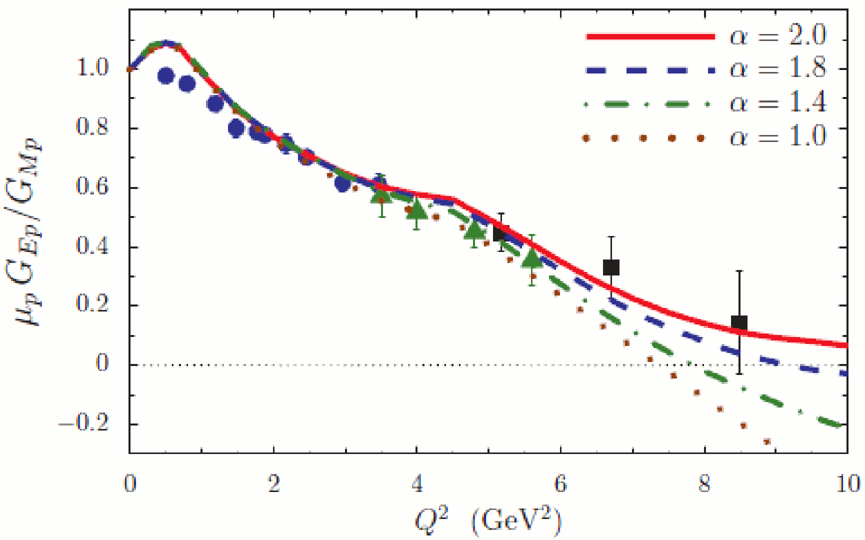




- Follows that the
  - ✓ possible existence
  - ✓ and location

of a zero in the ratio of proton elastic form factors

$$[\mu_p G_{Ep}(Q^2)/G_{Mp}(Q^2)]$$

are a direct measure of the nature of the quark-quark interaction in the Standard Model.

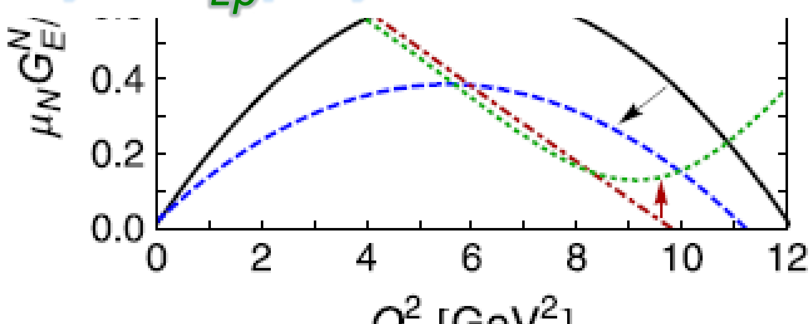
J. Segovia, I.C. Cloët, C.D. Roberts, S.M. Schmidt: Nucleon and Δ Elastic and Transition Form Factors, arXiv:1408.2919 [nucl-th], Few Body Syst. **55** (2014) 1185 [on-line]

Electric Charge

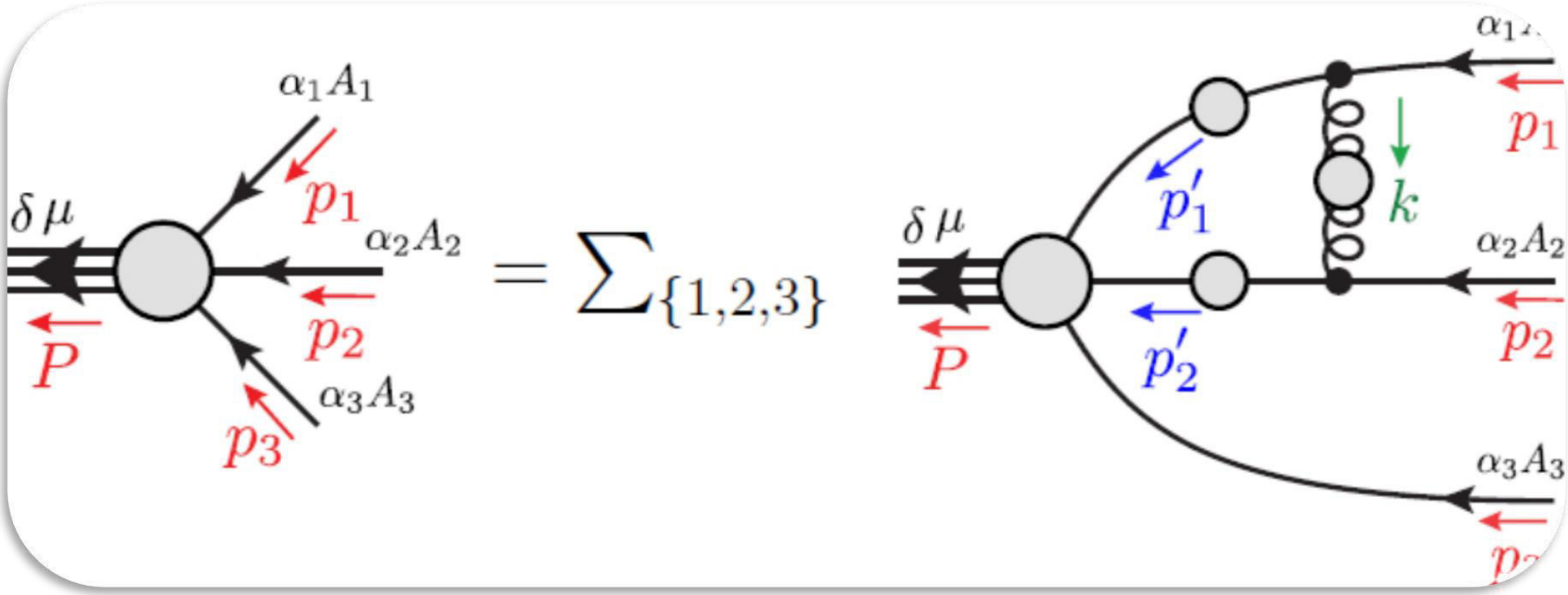
- Proton: if one accelerates the rate at which the dressed-quark sheds its cloud of gluons to become a parton, then zero in G_{ep} is pushed to larger Q^2
- Opposite for neutron!
- Explained by presence of diquark correlations

Leads to *Prediction neutron:proton*

$G_{En}(Q^2) > G_{Ep}(Q^2)$ at $Q^2 > 4GeV^2$



- These features entail that at x≈ 5 the electric form factor of the neutral neutron will become larger than that of the unit-charge proton!
- JLab12 will probe this prediction



Baryons as a 3-valence-body problem

Faddeev Equation

- Poincaré-covariant Faddeev equation for baryons is typically treated in a quark-diquark approximation, where the diquark correlations are nonpointlike and dynamical.
- Important: the predictions of the quark-diquark Faddeev equation framework are consistent with experiments
- Further, the use of such methods, with largely unfettered application to a wide range of static and dynamic hadron properties, is of growing importance, considering recent advances in charting the spectrum of excited nucleons using electromagnetic probes.
  - For instance: a recent global multi-channel analysis of exclusive meson photoproduction revealed evidence for several new baryon states [Anisovich:2017bsk];
  - Combined studies of charged double-pion photo- and electro-production data provide strong indications for another new baryon, i.e. N'(1730) $3/2^+$ [Ripani:2002ss, Mokeev:2015moa, Golovatch:2018hjk}.

Faddeev Equation

- First treatment of the Poincaré-covariant Faddeev equation for the nucleon to eschew the quark-diquark approximation is described in [Eichmann:2009qa].
  - Regarding the nucleon mass, it revealed that the quark-diquark picture is accurate at the level of 5%.
- Numerous applications to light-quark baryons ensued
- Significant algebraic and computational effort is required to complete these studies
- Results are instructive and promising
  - indicating that the framework is potentially capable of drawing a traceable connection between QCD and the many baryon observables that are being made accessible by modern facilities.

Poincaré-covariant analysis of heavy-quark baryons
S.-X. Qin, C. D. Roberts & S. M. Schmidt,
*arXiv:1801.09697 [nucl-th], Phys.Rev. D **97** (2018) 114017/1-13*
Spectrum of light- and heavy-baryons,
Si-xue Qin, Craig D Roberts, Sebastian M Schmidt
arXiv:1902.00026 [nucl-th] … dedicated to Ludwig Faddeev

Baryon Spectrum

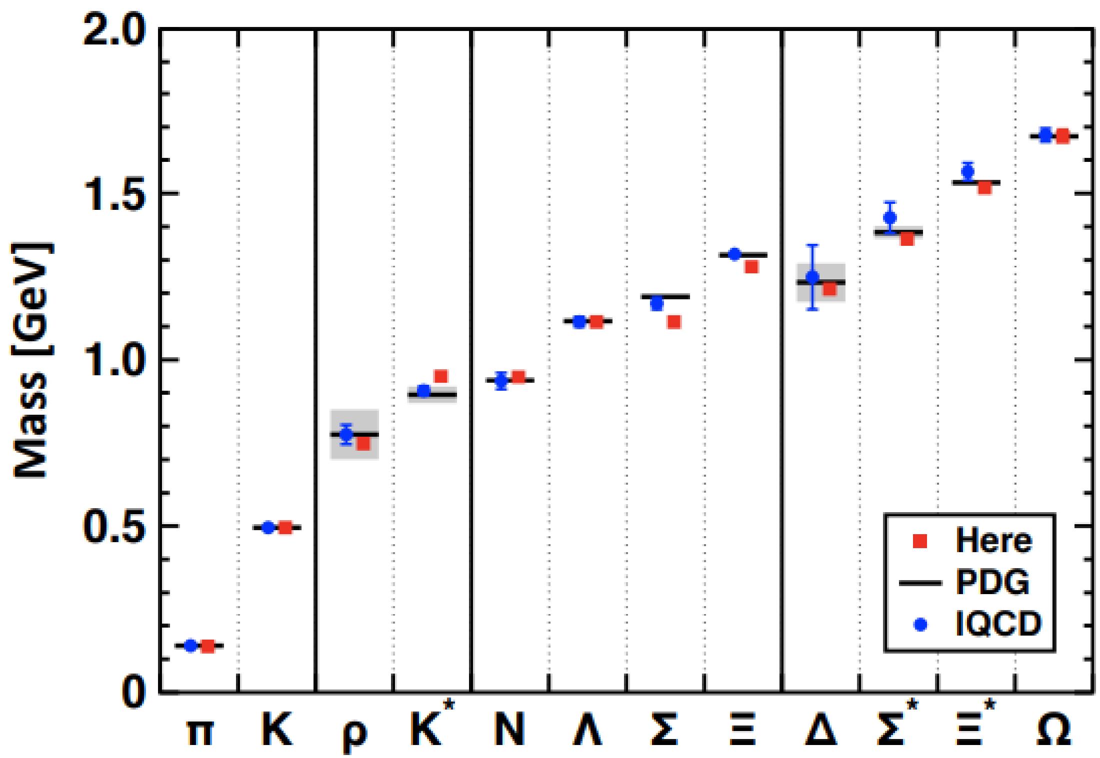
- Unified study of mesons and baryons built from light- and heavy-quarks
  - Symmetry-preserving rainbow-ladder truncation of all relevant bound-state equations:
    - Gap equations
    - Bethe-Salpeter equations
    - and Faddeev-equations

Only two people in the world can do this.
Si-Xue Qin, was at Argonne
Now a Professor at Chongqing U.

- No diquark approximation to the quark-quark scattering kernel
  - Reverse engineering … searching for dynamical emergence of diquark correlations and their effects
- Delivered spectrum and decay constants of ground-state pseudoscalar- and vector-mesons
- Concerning $J=1/2^+$, $3/2^+$ (qq′q″)-baryons, $q,q',q'' \in \{u,d,s,c,b\}$,
  - description of the known spectrum of 39 such states is obtained, with relative accuracy 3.6(2.7)%.
  - Framework subsequently used to predict masses of 90 states not yet seen empirically.

Baryon Spectrum

- ✓ Light-baryon spectrum computed using rainbow-ladder truncation in fully consistent implementation
- ✓ lQCD = Durr, S. et al. (2008) *Ab-Initio Determination of Light Hadron Masses*. Science, 322, 1224–1227.
- ✓ Understanding the hadron spectrum and connecting it with the QCD Lagrangian is possible *without* massive computational resources



Spectrum of light- and heavy-baryons,
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Special Issue dedicated to Ludwig Faddeev

Baryon Spectrum

- ✓ And the continuum approach is more versatile, so that *predictions* are possible
- ✓ Spectrum of 3/2+ baryons, their first radial excitations and their negative parity partners
- ✓ Crucially … spectrum of symmetry-preserving Faddeev equation is just as rich as that of the constititent quark model
- ✓ ***THERE ARE MISSING BARYONS***

Spectrum of light- and heavy-baryons,
Si-xue Qin, Craig D Roberts, Sebastian M Schmidt
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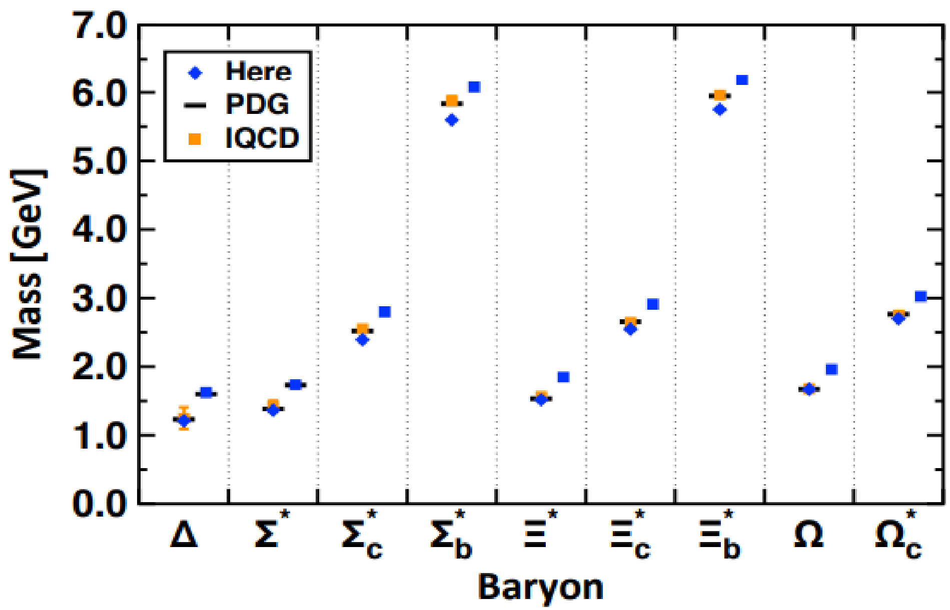

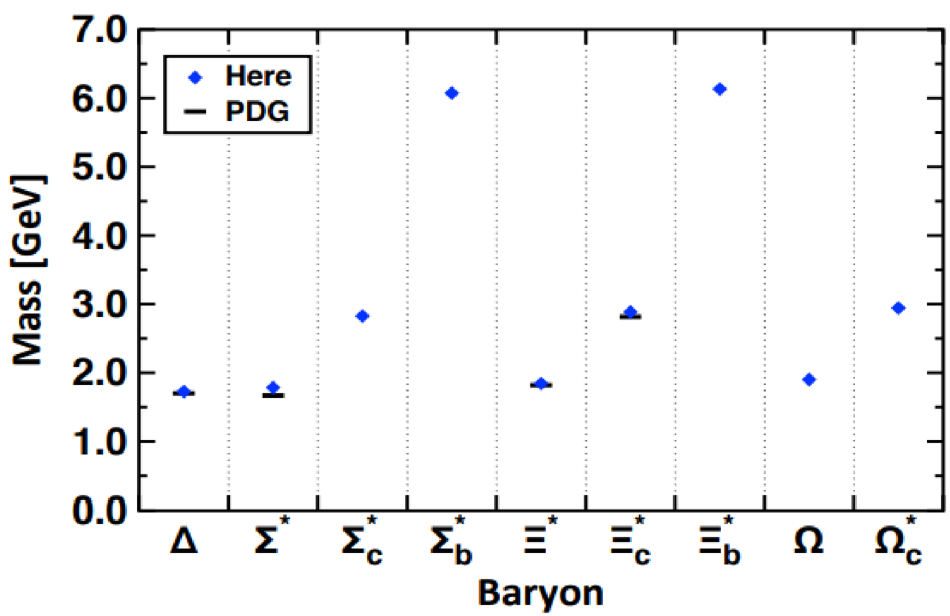

Spectrum of light- and heavy-baryons,
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Special Issue dedicated to Ludwig Faddeev

Baryon Spectrum

- Approach also yields Poincaré-covariant wave functions for these states
- Existing analyses of this type have provided insights that, e.g.
  - reveal which of those structural perspectives provided by constituent-quark potential models are qualitatively robust,
  - and also enrich the understanding of all these systems because Poincaré-covariance brings many new freedoms & constraints
- Furthermore, with wave functions in hand, one can also compute an array of dynamical observables, including, *inter alia*:
  - electroweak couplings and form factors;
  - and strong transition form factors.
- Such quantities provide connections with observables that are particularly sensitive to the internal structure of these basic yet complex strong-interaction bound-states.
- High-performance computing coupled with intelligent algorithms might be useful with these observables

Hybrids & Exotics

Spectrum of light hadrons

- Known spectrum of light hadrons is simple
  - Qualitatively matches the pattern established by the constituent-quark models of Gell-Mann and Zweig (1964)
    - Mesons built from a constituent-quark-antiquark ($Q\bar{Q}$) pair
    - Baryons constituted from three constituent quarks (QQQ)

    where *Q* is associated with any one of the light *u*-, *d*-, *s*-quarks.

- Gell-Mann and Zweig also raised possibility that more complicated bound-states are possible, *e.g*.
  - $QQ\bar{Q}\bar{Q}$ & $Q\bar{Q}QQQ$ (they didn't know about glue)

  No candidates were then known

  But after ~ 50 years, in systems involving the heavier *c*- and *b*-quarks, that has now changed

  X, Y, Z … pentaquarks

Spectrum of light hadrons

- Early '70s … "discovery" of quantum chromodynamics (QCD)
  - Non-Abelian, relativistic quantum gauge field theory
  - 8 self-interacting gauge bosons (gluons) mediate the interactions between current quarks
- New possibilities arose, *viz*. systems with valence glue,
  - hybrid (& exotic) mesons – $GQ\bar{Q}$
  - hybrid baryons – QQQG,
  - even *glueballs* – GG.

Distinction is lost between force and matter fields

  "G" is a "constituent gluon" degree of freedom
  - Unknown quantity
  - Character will only become known once such systems are detected
- Today's tabulations of hadron masses identify at least three plausible hybrid-meson candidates below 2 GeV
  - Dedicated searches for such states are underway at modern facilities (*e.g*. COMPASS @ CERN, GlueX @ JLab)

Models & Hybrids

- Over time, numerous models have been employed to calculate spectrum of light hybrid mesons
- Approaches are distinguished by, *inter alia*:
  - Disparate treatments/definitions of G
- Resulting spectra disagree
- Nevertheless
  - Development of a reliable continuum method for calculating hybrid meson properties would be very valuable
    - For interpretation of empirical observations
    - Provide insights into results obtained via the numerical simulation of lattice regularised QCD (lQCD)

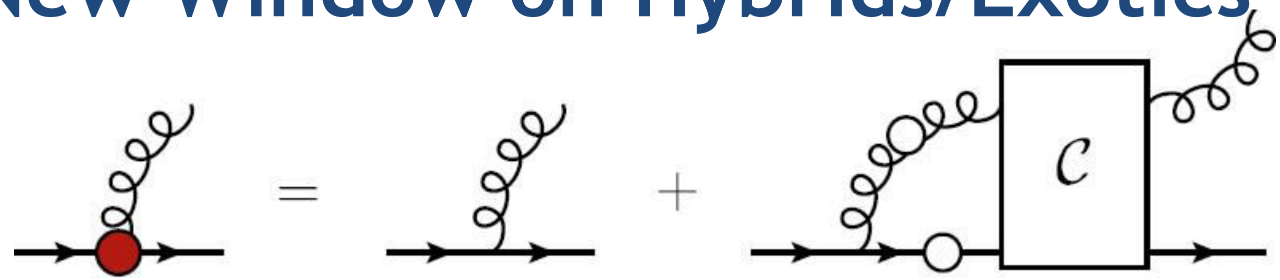
Basic Hypothesis

- *A hybrid meson is not qualitatively different from any other strong interaction bound-state, viz. it can be described by a Poincaré-covariant bound-state equation built with the dressed-parton degrees of freedom that are generated by solutions of gap equations in the matter and gauge sectors*.
- Within quantum field theory, there is no alternative to this position. Stated simply, we search for a bound-state solution in the gluon-quark-antiquark scattering problem.
- Lattice-QCD analyses of hybrids and glueballs formulate the problem in the same way.
- The only difference between the lattice starting point and ours is that we work in momentum space, whereas lattice studies are in configuration space.

Continuum Bound-State Problem

- $Q\bar{Q}$ mesons in quantum mechanics can't possess following (exotic) quantum numbers: $J^{PC} = 0^{+-}, 0^{--}, 1^{-+}$, etc.
- Not so in Poincaré-covariant treatments of two-valence-body bound states owing to existence of additional degree of freedom
  - relative time between the valence-quark and –antiquark => $k\cdot P \neq 0$
- However, extant studies of exotic mesons using simple *Ansätze* or truncations for Bethe-Salpeter kernel produce unrealistic spectra
  - exotic mesons with masses so light that they should already have been seen empirically when, in fact, signals for such states are currently weak and lie at significantly higher masses.
- Furthermore, 2-body Bethe-Salpeter equation does not readily distinguish between regular mesons and hybrids with same J^{PC}.
- Weaknesses: not remedied by using more sophisticated kernels
- Strong signal that hybrid mesons must contain explicit valence-gluon degree-of-freedom

New Window on Hybrids/Exotics



C = 1PI gluon-quark scattering amplitude

- Question:

  Does QCD support bound-states with valence gluons?

  Exotic/Hybrid meson = g q $\bar{q}$

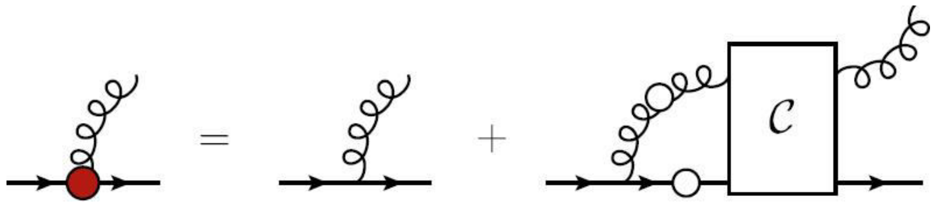
If so, then distinction is lost between force and matter fields

- Three valence-body problem in quantum field theory:

  Novel formulation based on observation gluon-quark vertex can be represented in terms of a gluon-quark scattering amplitude

  - Described in *Symmetry preserving truncations of the gap and Bethe-Salpeter equations,* Binosi, Chang, Papavassiliou, Qin, Roberts, arXiv:1601.05441 [nucl-th], Phys. Rev. D **93** (2016) 096010/1-7

New Perspective on Hybrid Mesons



- Recall two things …
  - Textbook derivations of the two-body Bethe-Salpeter equation in analyses of two-particle scattering and relationship between the scattering matrix and kernel
  - Role that coloured quark-quark (diquark) correlations play in simplifying the baryon three-body problem
- Then, reinterpretation of gluon-quark vertex suggests that

  gluon-quark [q_g=gq] & degenerate gluon-antiquark [$\bar{q}_g$=g$\bar{q}$]

  correlations play important role in solving 3-body problem for hybrids
- Conjecture: Hybrids = highly-correlated $q_g\bar{q} \leftrightarrow q\bar{q}_g$ bound-states

New Perspective on Hybrids

- ***Suppose*** strong q_g and $\bar{q}_g$ correlations exist, then …

  Hybrids mesons explained by:

  ✓ Coupled-channels Faddeev-like bound-state equation

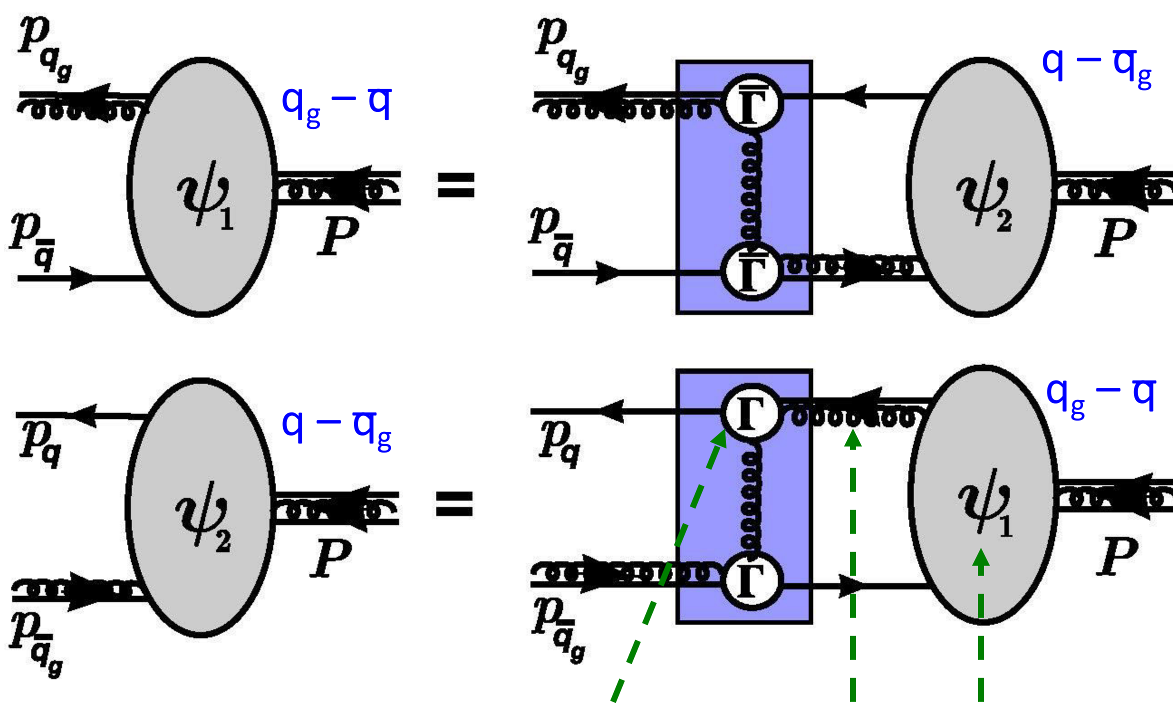
  $\Psi = \Psi_1 + \Psi_2$,

  $\Psi_1 = q_g\bar{q}$ & $\Psi_2 = q\bar{q}_g$

- Challenges:

  ✓ Confirm existence of tight gluon-quark correlations

  ✓ Determine their properties



$\Psi_1 = \Gamma^a_\mu(l;p_{qg})\, S_{gq}(p_{qg})\, \psi_1(p;P)$

gq correlation amplitude

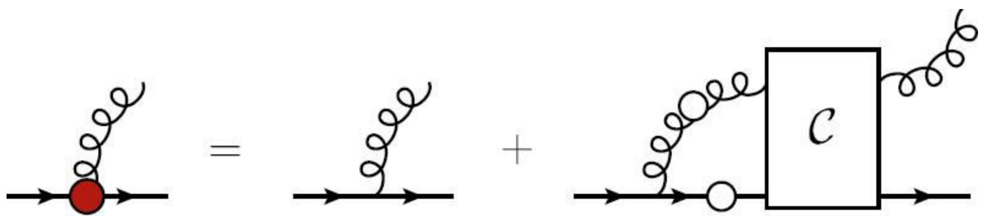
gq correlation propagator

bystander+correlation Faddeev amplitude

Gluon-Quark Correlations

- Adapt logic used to establish existence and properties of diquark correlations:

  Search for a pole solution to a leading-order (rainbow-ladder) truncation of vertex equation



- i.e. for a solution of the following homogeneous Bethe-Salpeter equation, $\Gamma^a_\mu = t^a\Gamma_\mu$, $k=p-\ell$:

$$t^a\Gamma_\mu(p;Q)\Lambda_+ = -\int_{d\ell} \mathcal{G}(k^2)t^b\gamma_\rho S(\ell_+)$$
$$\times\, t^c\Gamma_\lambda(\ell;Q)D_{\lambda\tau}(\bar\ell_-)\boxed{f_{3g}(k^2)\,{}_0V^{bca}_{\rho\tau\mu}(k,\bar\ell_-,\bar p_-)}\Lambda_+$$

valence quark

bare 3-gluon vertex

valence gluon

3g vertex dressing factor

continuum & lattice: 3g vertex greatly suppressed on $k^2<1$ GeV^2

Gluon-Quark Correlations

- Any kernel that provides good description of π- and ρ-meson properties (masses, decay constants, etc.):
  - Generates quark+quark correlations in all possible J^{PC} channels
    - Diquarks play crucial role in determining structure and interactions of baryons
  - Generates gluon+quark correlations
    - Dressed valence gluon and valence quark both have running masses, large in infrared
      - $M_g^{IR} \approx \frac{1}{2}\, m_{proton}$
      - $M_q^{IR} \approx \frac{1}{3}\, m_{proton}$
    - $Mass_{(g+q)} \approx m_{proton} \approx 1$ GeV

Hybrid Meson Spectrum

	0^{-+}	1^{-+}	1^{--}	0^{+-}	0^{--}
RL direct	1.28(9)	1.80(4)	1.64(10)	1.73(13)	1.74(3)
ACM improved	1.62(6)	1.75(8)	1.86(10)	1.87(14)	1.90(3)
lQCD$_R$ - 16^3	1.72(2)	1.73(2)	1.84(2)	2.03(1)	
lQCD$_R$ - 20^3	1.69(2)	1.72(2)	1.77(6)	1.99(2)	
lQCD - 16^3	2.14(1)	2.15(2)	2.26(2)	2.45(1)	
lQCD - 20^3	2.12(2)	2.16(2)	2.21(6)	2.43(2)	

lQCD. Rows 5, 6: $m_\pi > 0.4$ GeV … Dudek *et al*.: arXiv:1004.4930 [hep-ph]

These simulations overestimate mass of pion's first radial excitation by $\delta_{\pi 1} = 0.43$ GeV

lQCD. Rows 3, 4: = Rows 5, 6 – $\delta_{\pi 1}$

Hybrid Meson Spectrum

	0^{-+}	1^{-+}	1^{--}	0^{+-}	0^{--}
RL direct	1.28(9)	1.80(4)	1.64(10)	1.73(13)	1.74(3)
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Faddeev Equation with [gq] correlations

- ✓ Bound-states exist in all channels
- ✓ Notably: 0^{-+} & 1^{--} hybrids are structurally distinct from those accessible using the 2-body Bethe-Salpeter equation in these channels, as in all such previous studies

However, in comparison with lQCD predictions:

- ❖ All states too light, especially 0^{-+}, and 1^{-+}-1^{--} ordering is reversed.
- ❖ Wide variations of model parameters do not alter this outcome.

Hitherto, such problems typical of continuum studies

Hybrid Meson Spectrum

- Mismatch between RL-direct (Row 1) and lQCD results
  - Reconsider each element in our formulation of hybrid meson problem
- Analyses of improvements to RL truncation indicate origin:
  - [gq] correlation amplitude was computed in RL truncation
  - RL truncation underestimates DCSB in bound-state amplitudes
- Consequently, anomalous chromomagnetic moment (ACM) associated with this correlation is underestimated
  - ACM enhancement essential to explain, *e.g.* $a_1 - \rho$ splitting
- Introduce correction factor
  - Multiplication of ACM term in [gq] correlation by constant, κ_{gq}
- *Ask question: Can any value of κ_{gq} yield match with lQCD?*

Hybrid Meson Spectrum

	0^{-+}	1^{-+}	1^{--}	0^{+-}	0^{--}
RL direct	1.28(9)	1.80(4)	1.64(10)	1.73(13)	1.74(3)
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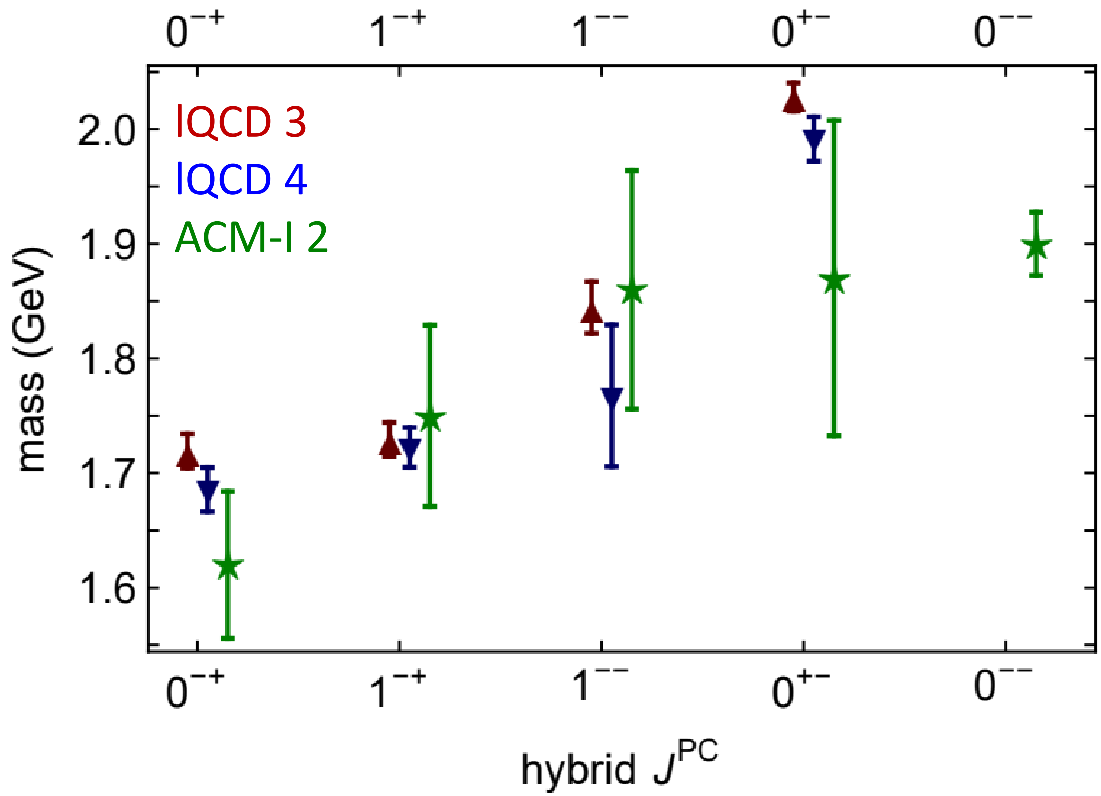
- ***YES***: κ_{gq} … RL = 1 → 2.4 = ACM
  - Magnification typical of result obtained with DCSB-improved kernels
- ACM-improved calculations in Row 2:
  - Level ordering identical to lQCD (3, 4)
  - Absolute values of the masses are commensurate.

Hybrid Spectrum

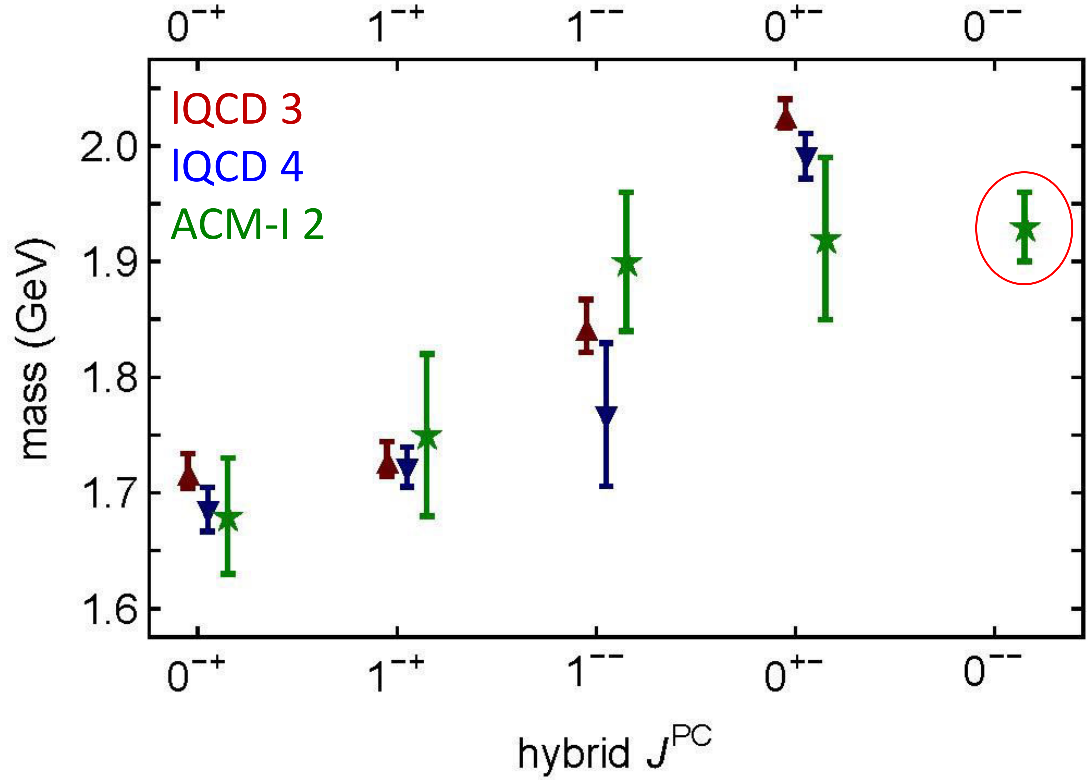
New Perspective on Hybrid Mesons
Shu-Sheng Xu, *et al*.
arXiv:1805.06430 [nucl-th]

- Beyond-RL essential to agreement with lQCD
- Agreement is non-trivial
  - lQCD masses are rescaled by subtraction of $\delta_{\pi 1}$, a number which is completely unrelated to our calculations.
- No single lQCD mass was used as a constraint when fitting κ_{gq}
- Magnitude of our results set by
  - infrared values of the running gluon and quark masses
  - determined by π- and ρ-meson properties
  - unrelated to hybrid channels.



S.-S. Xu, Z.-F. Cui, L. Chang, J. Papavassiliou, C. D. Roberts, H.-S Zong

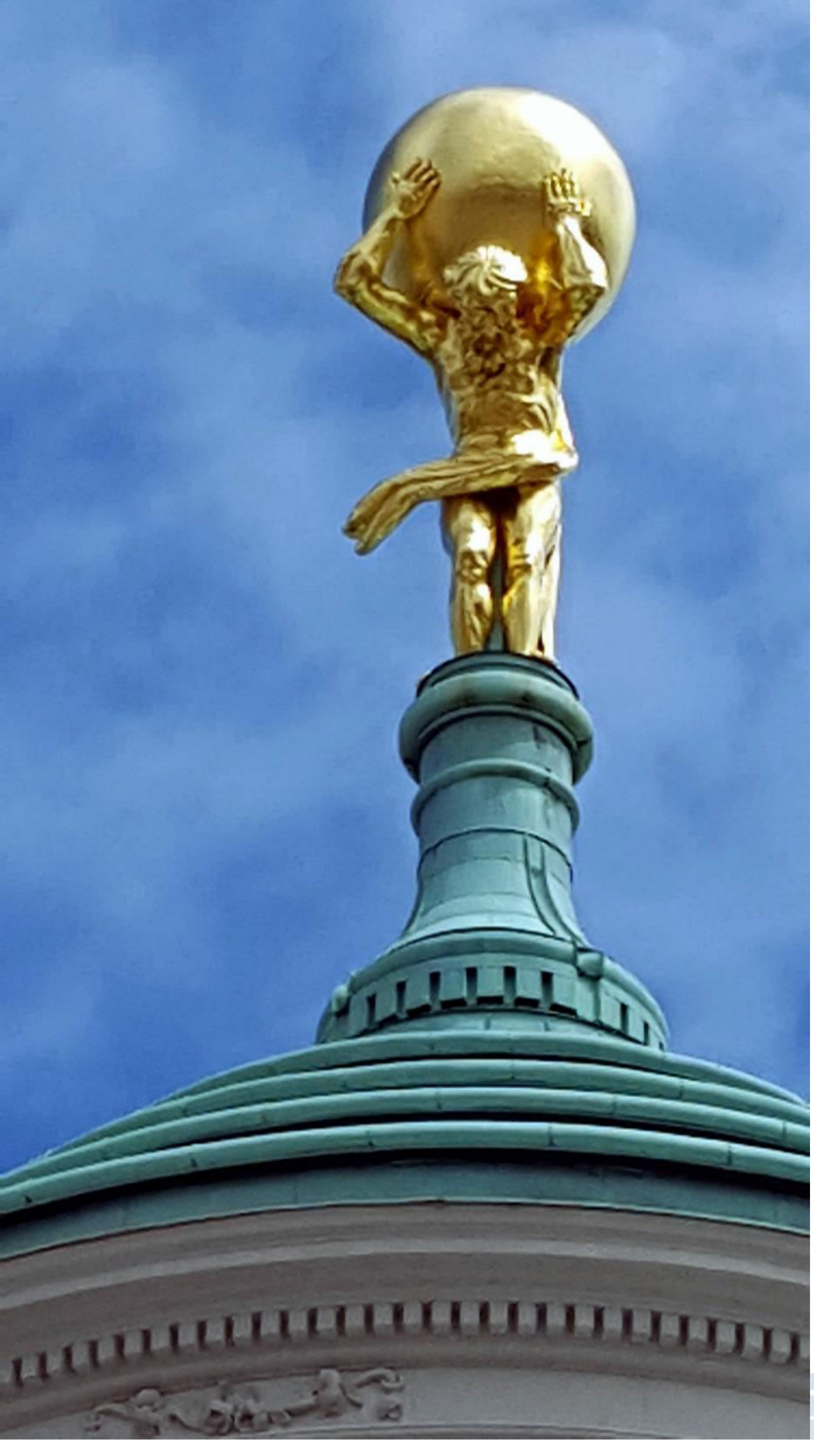
Hybrid Spectrum



- 0^{--} … deserves special attention
- lQCD predicts lightest state in this channel above m_ρ+2GeV
- [gq] Faddeev equation confirms 0^{--} is ground-state heaviest hybrid
  - Corrects defect of RL-truncation analyses of exotics using the two-body Bethe-Salpeter equation
- Computed 0^{--} mass nevertheless probably too light
  - Such a system is likely to possess large amount of angular momentum
  - Leads to significant DCSB-enhanced repulsion within the bound-state
  - Simple expedient for correcting associated defects of RL truncation may not be completely adequate.
  - Approach we have described will always produce a heavy 0^{-} state, but precise location must await future, more sophisticated analyses.

New Perspective on Hybrids

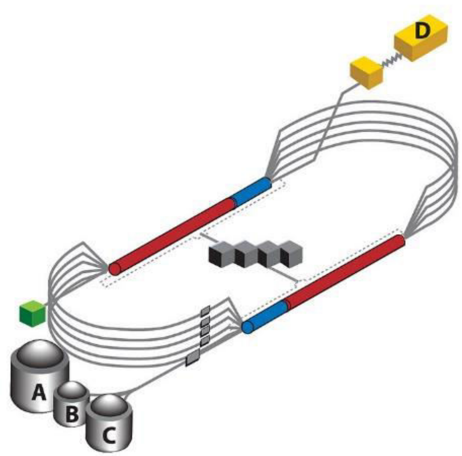
- Faddeev equation approach to the valence-gluon+quark+antiquark bound-state problem in relativistic quantum field theory.
  - Strong correlations exist in the [q_g=gq] & [$\bar{q}_g$ = g$\bar{q}$] channels
  - Hybrid mesons appear as highly-correlated $q_g\bar{q} \leftrightarrow q\bar{q}_g$
  - Since diquark correlations basic to determining baryon properties, existence & importance of kindred correlations in hybrids appears credible
- Described a first analysis of hybrids from this new perspective
  - Established plausibility
  - More sophisticated treatments necessary before the validity of the formulation can be firmly established
- Meanwhile:
  - Serve as a guide for subsequent continuum treatments of hybrid-meson three-body problem
  - Computed, highly-correlated wave functions can be used to predict a range of hybrid decays and other processes
    - Elucidate empirical signatures for the presence and role of q_g & $\bar{q}_g$

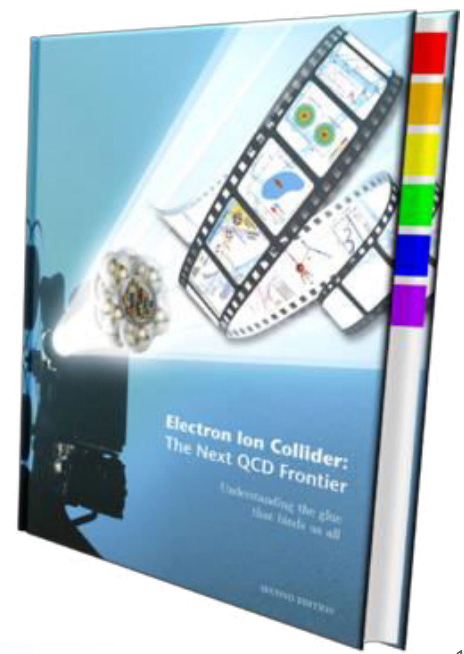


Epilogue

Epilogue

- Challenge: Explain and Understand
  the Origin and Distribution of the Vast Bulk of Visible Mass
- Current Paradigm: Quantum Chromodynamics
- QCD is plausibly a mathematically well-defined quantum field theory,
  The only one we've ever produced
  - Consequently, it is a worthwhile paradigm for developing Beyond-SM theories
- Challenge is to reveal the content of strong-QCD
- **Tough Problem**
- *Progress* and *Insights*
  being delivered by amalgai
  - Experiment
  - Phenomenology
  - Theory
- Must continue into eras of



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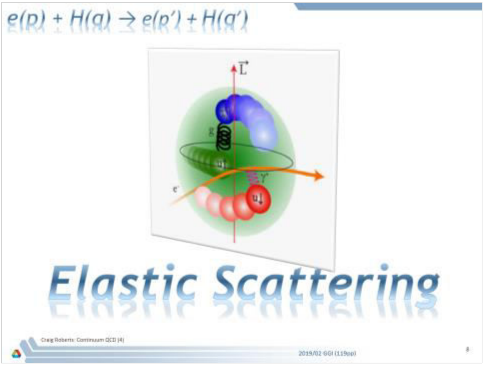
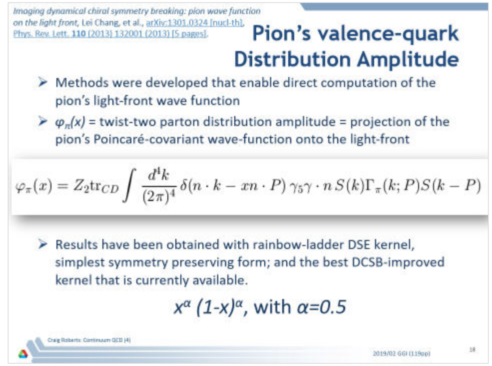
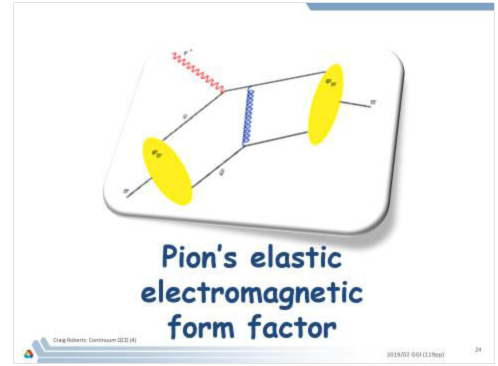
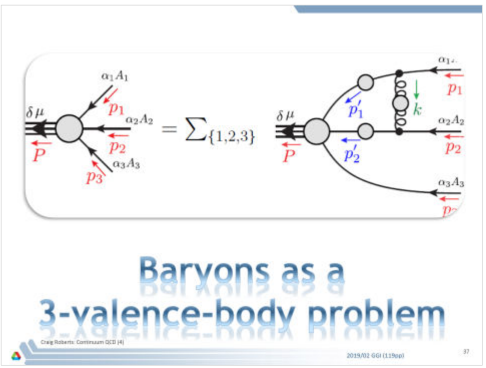
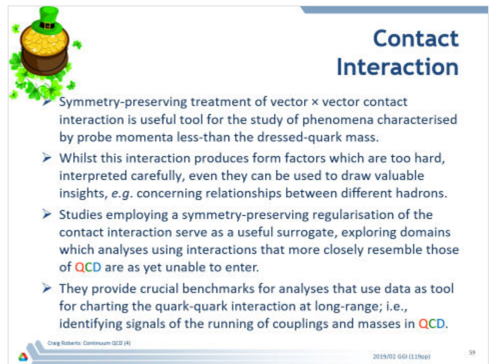
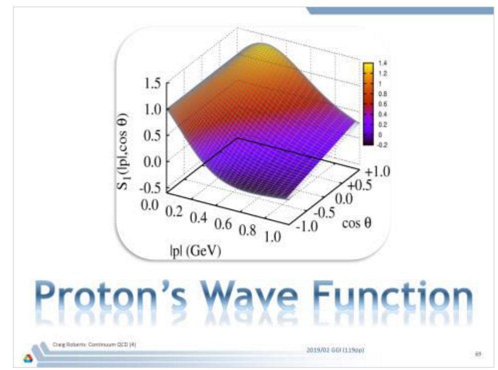

$e(p) + H(q) \to e(p') + H(q')$
Elastic Scattering

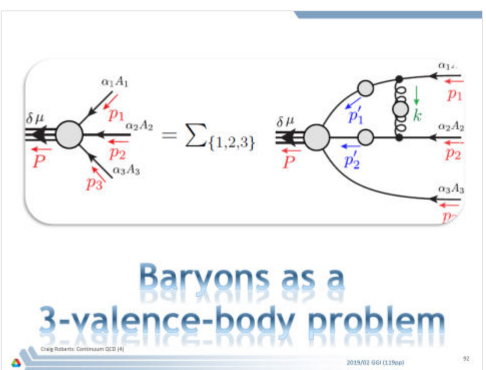

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. 110 (2013) 132001 (2013) [5 pages].
Pion's valence-quark Distribution Amplitude
Methods were developed that enable direct computation of the pion's light-front wave function
$\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front
$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n\cdot k - x n \cdot P)\, \gamma_5 \gamma \cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$
Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.
$x^\alpha (1-x)^\alpha$, with $\alpha=0.5$


Pion's elastic electromagnetic form factor

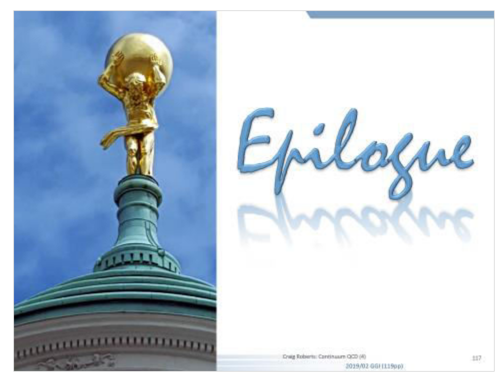

$= \Sigma_{\{1,2,3\}}$
Baryons as a
3-valence-body problem


Contact Interaction
Symmetry-preserving treatment of vector × vector contact interaction is useful tool for the study of phenomena characterised by probe momenta less-than the dressed-quark mass.
Whilst this interaction produces form factors which are too hard, interpreted carefully, even they can be used to draw valuable insights, e.g. concerning relationships between different hadrons.
Studies employing a symmetry-preserving regularisation of the contact interaction serve as a useful surrogate, exploring domains which analyses using interactions that more closely resemble those of QCD are as yet unable to enter.
They provide crucial benchmarks for analyses that use data as tool for charting the quark-quark interaction at long-range; i.e., identifying signals of the running of couplings and masses in QCD.


$S_1(|p|,\cos\theta)$
|p| (GeV)
cos θ
Proton's Wave Function


$= \Sigma_{\{1,2,3\}}$
Baryons as a
3-valence-body problem


GLUEX citations periment
Hybrids & Exotics


Epilogue