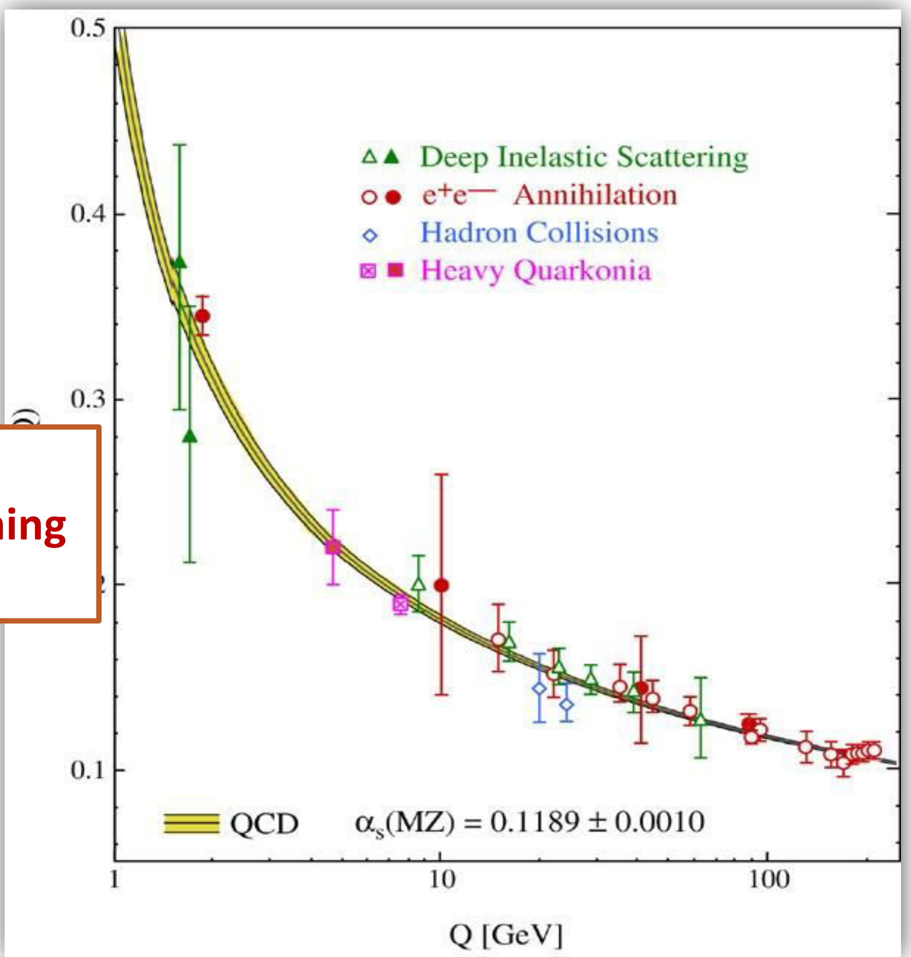
Continuum QCD

Craig Roberts

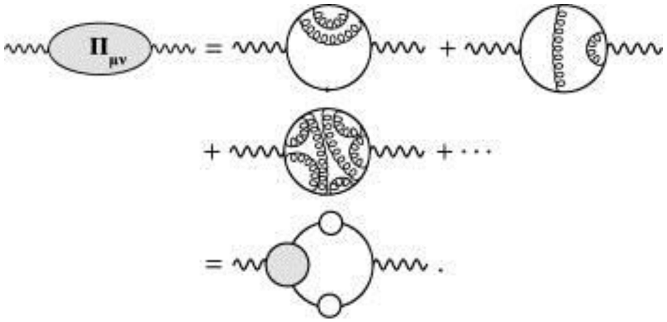


⇐ What's happening out here?!

QCD's Running Coupling

QED Running Coupling

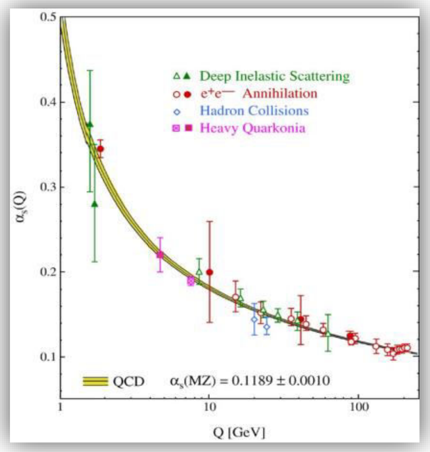
- Quantum gauge field theories defined in four spacetime dimensions,
  - Lagrangian couplings and masses come to depend on a mass scale
  - Can often be related to the energy or momentum at which a given process occurs.
- Archetype is QED, for which there is a sensible perturbation theory.
- QED, owing to the Ward identity:
  - a single running coupling
  - measures strength of the photon-charged-fermion vertex
  - can be obtained by summing the virtual processes that dress the bare photon, *viz*. by computing the photon vacuum polarisation.



- QED's running coupling is known to great accuracy and the running has been observed directly.

QCD Running Coupling

- At first sight, addition of QCD to Standard Model does not qualitatively change anything, despite presence of four possibly distinct strong-interaction vertices in the renormalized theory
  - gluon-ghost, three-gluon, four-gluon and gluon-quark.
- An array of Slavnov-Taylor identities (STIs) implementing BRST symmetry – generalisation of non-Abelian gauge invariance for the quantised theory – ensures that a single running coupling characterises all four interactions on perturbative domain.
- New Feature:
  - QCD is asymptotically free and extant evidence suggests that perturbation theory is valid at large momentum scales
  - But all dynamics is nonperturbative at scales typical of everyday strong-interaction phenomena, *e.g.* $\zeta \leq m_p$

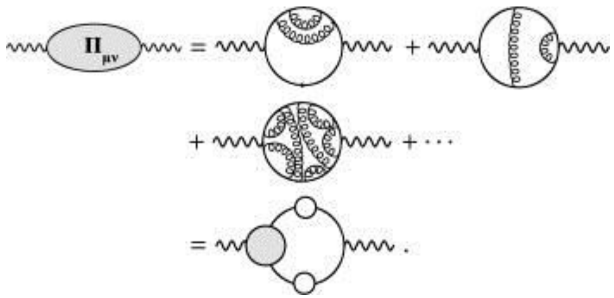


QCD Running Coupling

- Four individual, apparently UV-divergent interaction vertices in perturbative QCD ⇒ possibly four distinct IR couplings.
  - Naturally, if nonperturbatively there are two or more couplings, they must all become equivalent on the perturbative domain.
- Questions:
  - How many distinct running couplings exist in nonperturbative QCD?
  - How can they be computed?
  - If defined using a 3- or 4-point vertex, which arrangement of momenta defines the running? (Infinitely many choices.)
- ***Claim*: Nonperturbatively, too,**
  QCD possesses a unique running coupling.
- Alternative
  - Possibly an essentially different RGI intrinsic mass-scale for each coupling
  - Then BRST symmetry irreparably broken by nonperturbative dynamics
  - Conclusion: QCD non-renormalisable owing to IR dynamics.
- No empirical evidence to support such a conclusion: QCD does seem to be a well-defined theory at all momentum scales, possibly owing to dynamical generation of gluon and quark masses, which are large at IR momenta.

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling



- There is a particular simplicity to QED:
  - Unique running coupling
  - Process-independent effective charge
  - Obtained simply by computing the photon vacuum polarisation.
- This is because ghost-fields decouple in Abelian theories; and, consequently, one has the Ward identity

  $$Z_1 = Z_2$$

  which guarantees that the electric-charge renormalisation constant is equivalent to that of the photon field.
- Physically: impact of dressing the interaction vertices is absorbed into the vacuum polarisation.
- Not generally true in QCD because ghost-fields do not decouple.

QCD Running Coupling

- There is *one* approach to analysing QCD's Schwinger functions that preserves some of QED's simplicity
  - Combination of pinch technique (PT) & background field method (BFM)
- Means by which QCD can be made to "look" Abelian:
  - Systematically rearrange classes of diagrams and their sums in order to obtain modified Schwinger functions that satisfy linear STIs.
- In the gauge sector, this produces a modified gluon dressing function from which one can compute the QCD running coupling
  - So this polarisation captures all required features of the renormalisation group.
- Furthermore, the coupling is process independent: one obtains precisely the same result, independent of the scattering process considered, whether $gg \rightarrow gg$, $qq \rightarrow qq$, etc.

QCD Running Coupling

PT-BFM vacuum polarisation

ghost-gluon vertex

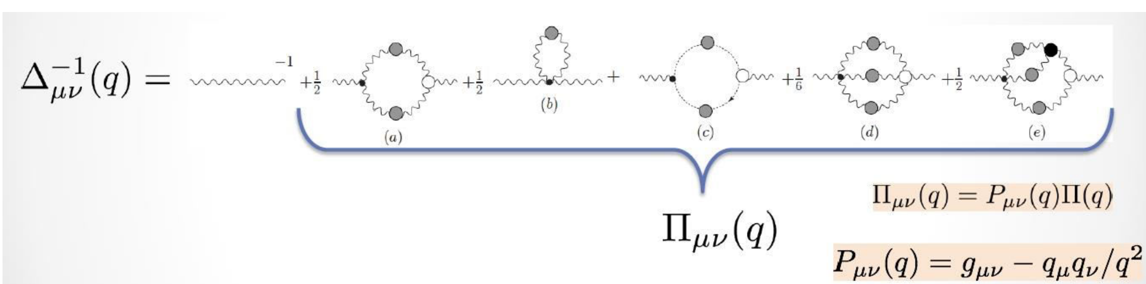
- Formulae:

$$\alpha(\zeta^2) D^{\rm PB}_{\mu\nu}(k;\zeta) = \widehat{d}(k^2) T_{\mu\nu}(k)\,,$$

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2;\zeta^2) F(k^2;\zeta^2)]^2}\,,$$

ghost-gluon vacuum polarisation

– $\alpha(\zeta^2)$: scale-dependent renormalized coupling
– $D^{PB}_{\mu\nu}$: PT-BFM gluon two-point function
– $\hat{d}(k^2)$: renormalisation-group-invariant (RGI) running-interaction that unifies top-down and bottom-up approaches to gauge and matter sectors of QCD
– F : dressing function for the ghost propagator;
– L : a longitudinal piece of the gluon-ghost vacuum polarisation that vanishes at $k^2=0$ and as $k^2 \to \infty$



QCD Running Coupling

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  - F : dressing function for the ghost propagator;
  - L : a longitudinal piece of the gluon-ghost vacuum polarisation that vanishes at $k^2=0$ and as $k^2 \to \infty$
- Gap Equation: $S^{-1}(p) = Z_2\,(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p)\,,$

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2)\, T_{\mu\nu}(k) \gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p)$$

Process independent strong running coupling
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arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

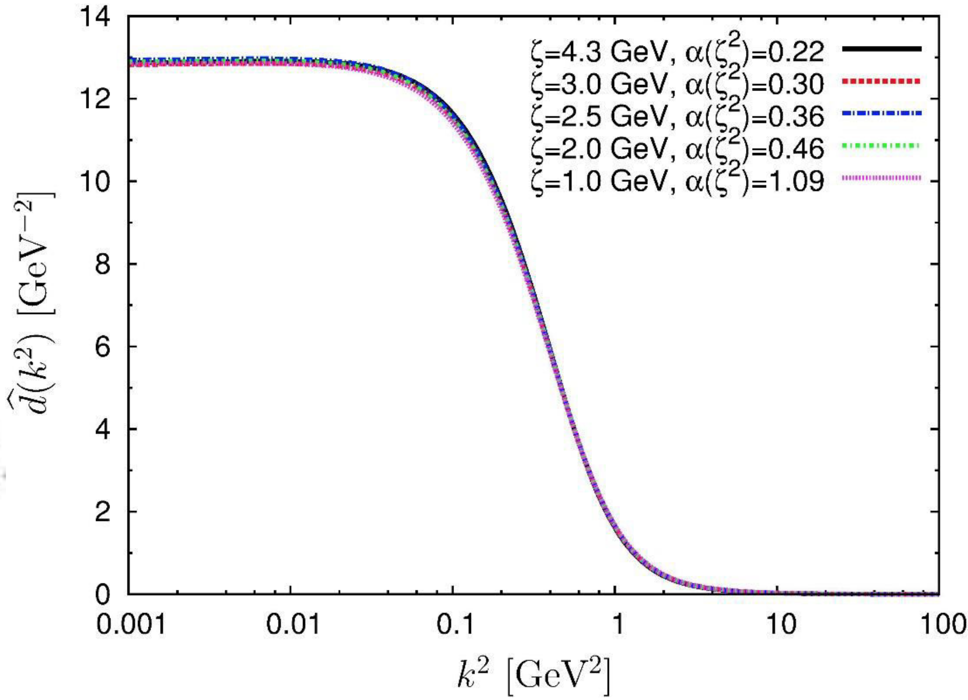
- RGI interaction, $\hat{d}(k^2)$ has been computed.
- Establishes a remarkable feature of QCD; namely, the interaction saturates at infrared momenta:

$$\widehat{d}(k^2=0)=\alpha(\zeta^2)/m_g^2(\zeta)=\alpha_0/m_0^2$$

where

  - $\alpha_0 := \alpha(0) \approx 1.0\pi$
  - $m_0 := m_g(0) \approx ½\, m_p$

- Gluon sector of QCD is characterised by a nonperturbatively-generated infrared mass-scale …
  CANNIBALISM



Bridging a gap between continuum-QCD
& ab initio predictions of hadron observables
Binosi, Chang, Papavassiliou, Roberts,
arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

QCD Effective Charge

- Define a RGI product: $\mathcal{D}(k^2) = \Delta_{\rm F}(k^2;\zeta)\, m_g^2(\zeta^2)/m_0^2$
  - $\Delta_F(k^2;\zeta)$ is parametrisation of continuum- and/or lattice-QCD calculations of the canonical gluon two-point function
    - Preserves IR behaviour of calculations
    - $1/\Delta_F(k^2;\zeta) = k^2 + O(1)$ on $k^2 \gg m_0^2$
- Gap equation becomes:

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{\alpha}_{\rm PI}(k^2) \mathcal{D}_{\mu\nu}(k^2) \gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p)\,,$$

where $\mathcal{D}_{\mu\nu} = \mathcal{D} T_{\mu\nu}$ and the dimensionless product

$$\boxed{\widehat{\alpha}_{\rm PI}(k^2) = \widehat{d}(k^2)/\mathcal{D}(k^2)}$$

is a RGI running-coupling (effective charge)

by construction, $\hat{\alpha}(k^2) = \mathcal{I}(k^2)$ on $k^2 \gg m_0^2$

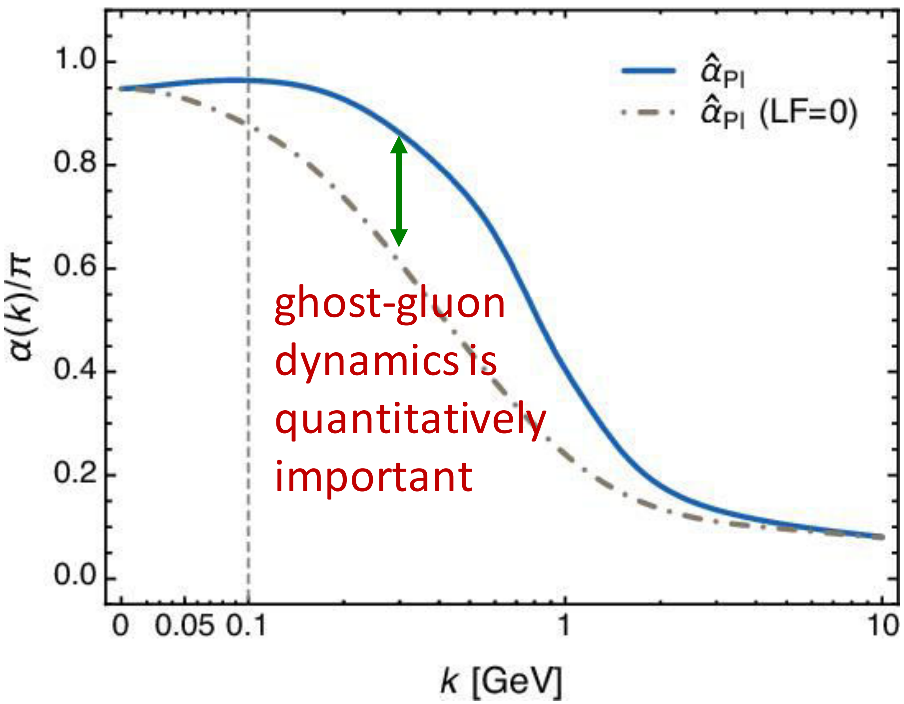
QCD Effective Charge

- $\hat{\alpha}_{PI}(k^2) = \hat{d}(k^2) / \mathcal{D}(k^2)$
  - Process independent: as noted above, the same function appears irrespective of the initial and final parton systems.
  - Unifies a diverse and extensive array of hadron observables
    - evident in fact that dressed-quark self-energy serves as generating functional for the Bethe-Salpeter kernel in all meson channels
    - and $\hat{\alpha}_{PI}(k^2)$ is untouched by the generating procedure in all flavoured systems
  - Sufficient to know $\hat{\alpha}_{PI}(k^2)$ in Landau gauge
    - form-invariant under gauge transformations
    - and gauge covariance ensures that such transformations produce nothing but an overall “phase” in the gap equation's solution, which may be absorbed into S(p)

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Effective Charge

- Parameter-free prediction: curve is completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.
- Physical, in the sense that there is no Landau pole, and saturates in the IR: $\hat{\alpha}(0) \approx 1.0\,\pi$, *i.e*. the coupling possesses an infrared fixed point



- Prediction is equally sound at all spacelike momenta, connecting the IR and UV domains, with no need for an *ad hoc* "matching procedure," such as that employed in models
- Essentially nonperturbative: combination of self-consistent solutions of gauge-sector gap equations with lattice simulations

Process-dependent (emergent)Effective Charge

- [Grunberg:1982fw]: process-dependent procedure

$$\int_0^{1^-} dx_{Bj}\left(g_1^p\left(x_{Bj},Q^2\right)-g_1^n\left(x_{Bj},Q^2\right)\right)\equiv\frac{g_A}{6}\left[1-\frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

- an effective running coupling defined to be completely fixed by leading-order term in the perturbative expansion of a given observable in terms of the canonical running coupling.
  - Obvious difficulty/drawback = process-dependence itself.
  - Effective charges from different observables can in principle be algebraically connected to each other via an expansion of one coupling in terms of the other.
  - But, any such expansion contains infinitely many terms; and connection doesn’t provide a given process-dependent charge with ability to predict another observable, since the expansion is only defined after both effective charges are independently constructed.

Process-dependent Effective Charge

S.J. Brodsky, H.J. Lu, Phys. Rev. D 51 (1995) 3652
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J. Lu, Phys. Lett. B 372 (1996) 133
A. Deur, V. Burkert, Jian-Ping Chen, Phys.Lett. B 650 (2007) 244-248

- α_{g1} – Bjorken sum rule

$$\int_0^{1^-} dx_{Bj}\left(g_1^p\left(x_{Bj},Q^2\right) - g_1^n\left(x_{Bj},Q^2\right)\right) \equiv \frac{g_A}{6}\left[1 - \frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

$g_1^{p,n}$ are spin-dependent proton and neutron structure functions

g_A is the nucleon flavour-singlet axial-charge

- Merits, *e.g.*
  - Existence of data for a wide range of k^2
  - Tight sum-rules constraints on the behaviour of the integral at the IR and UV extremes of k^2
  - isospin non-singlet ⇒ suppression of contributions from numerous processes that are hard to compute and hence might muddy interpretation of the integral in terms of an effective charge
    - Δ resonance
    - Disconnected (gluon mediated) diagrams

Process independent strong running coupling
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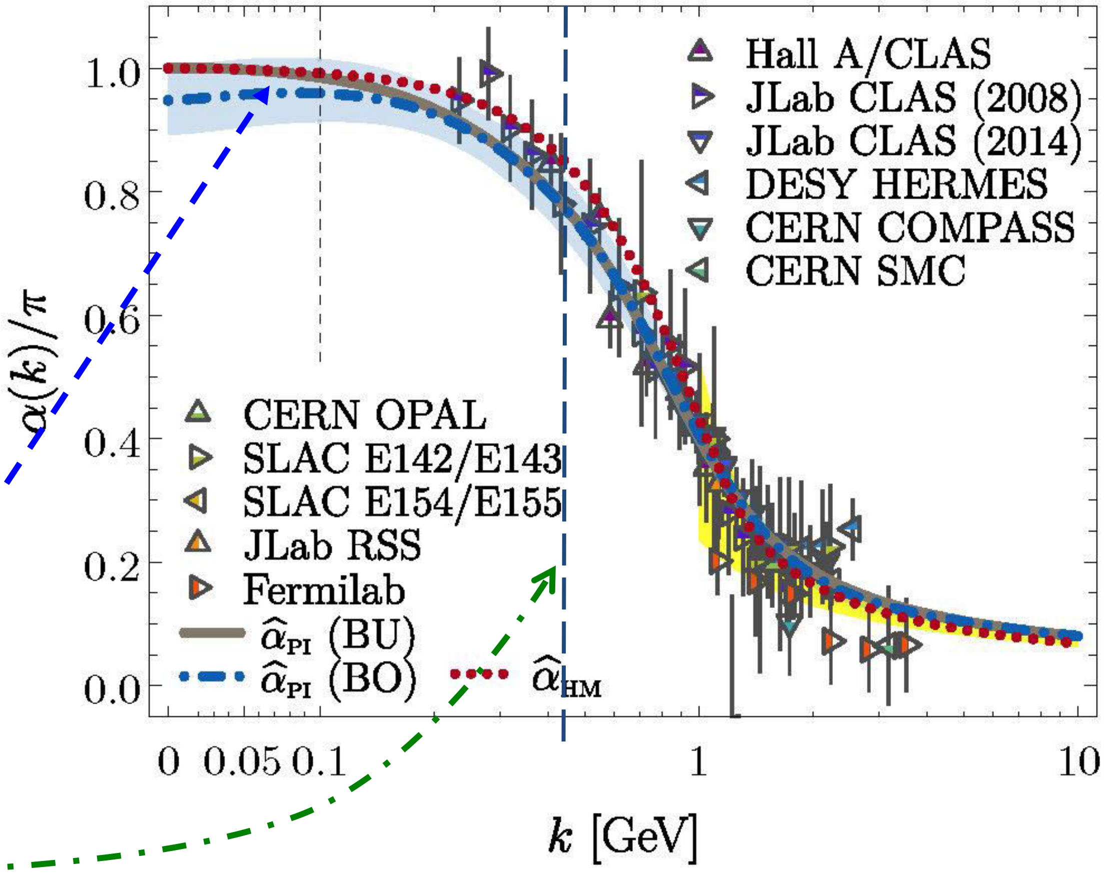
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

Process-independent effective-charge in QCD

- Modern continuum & lattice methods for analysing gauge sector enable

  "Gell-Mann – Low"

  running charge to be defined in QCD
- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction
- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$



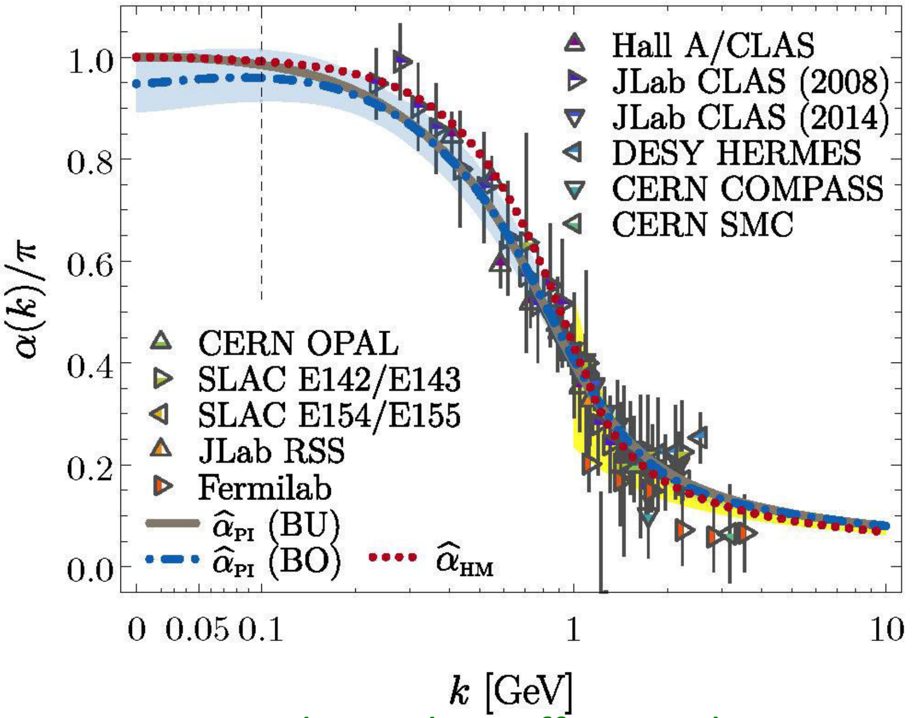
QCD Effective Charge

- Near precise agreement between process-independent $\hat{\alpha}_{PI}$ and α_{g1}
- Perturbative domain:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.14\,\alpha_{\overline{\rm MS}}(k^2) + \ldots),$$
$$\widehat{\alpha}_{\rm PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\,\alpha_{\overline{\rm MS}}(k^2) + \ldots),$$

Just 4% difference

- Parameter-free prediction:
  - curve completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.



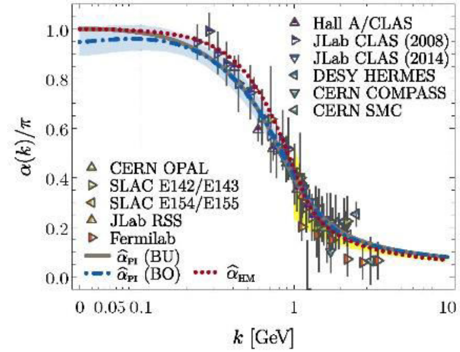
Data = process dependent effective charge [Grunberg:1982fw]:
α_{g1}, defined via Bjorken Sum Rule

- Ghost-gluon scattering contributions are critical for agreement between the two couplings at intermediate momenta … omit them, and disagreement by factor of ~ 2 at intermediate momenta

QCD Effective Charge

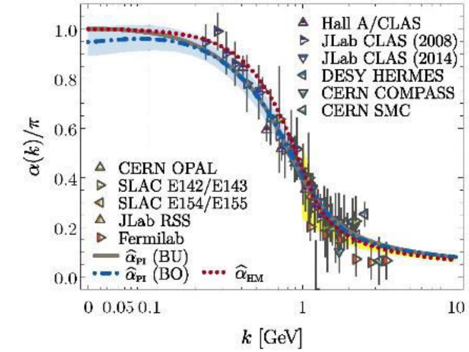
- Why are these two apparently unrelated definitions of a QCD effective charge can so similar?
  - Bjorken sum rule is an isospin non-singlet relation & hence contributions from many hard-to-compute processes are suppressed
  - these same processes are omitted in DSE computation of $\hat{\alpha}_{PI}$
- Unification of two vastly different approaches to understanding the infrared behaviour of QCD
  - one essentially phenomenological: data-based, process-dependent
  - the other, deliberately computational, embedded within QCD.
- ***Bjorken sum rule is a near direct means by which to gain empirical insight into QCD's "Gell-Mann – Low effective charge"***

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - appears in every one of QCD’s dynamical equations of motion
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale, which also serves to eliminate the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

QCD Effective Charge



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- $\hat{\alpha}_{PI}$ is
  - pro
  - app
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- $\hat{\alpha}_{PI}$ posse
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- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

Conceivably, therefore, QCD can serve as a basis for theories that take physics beyond the Standard Model

Spontaneous(Dynamical) Chiral Symmetry Breaking = Mass Generation

The 2008 Nobel Prize in Physics was divided, one half awarded to Yoichiro Nambu

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

Nambu - Jona-Lasinio Model

Dynamical Model of Elementary Particles
Based on an Analogy with Superconductivity. I
Y. Nambu and G. Jona-Lasinio, Phys. Rev. 122 (1961) 345–358
Dynamical Model Of Elementary Particles
Based On An Analogy With Superconductivity. II
Y. Nambu, G. Jona-Lasinio, Phys.Rev. 124 (1961) 246-254

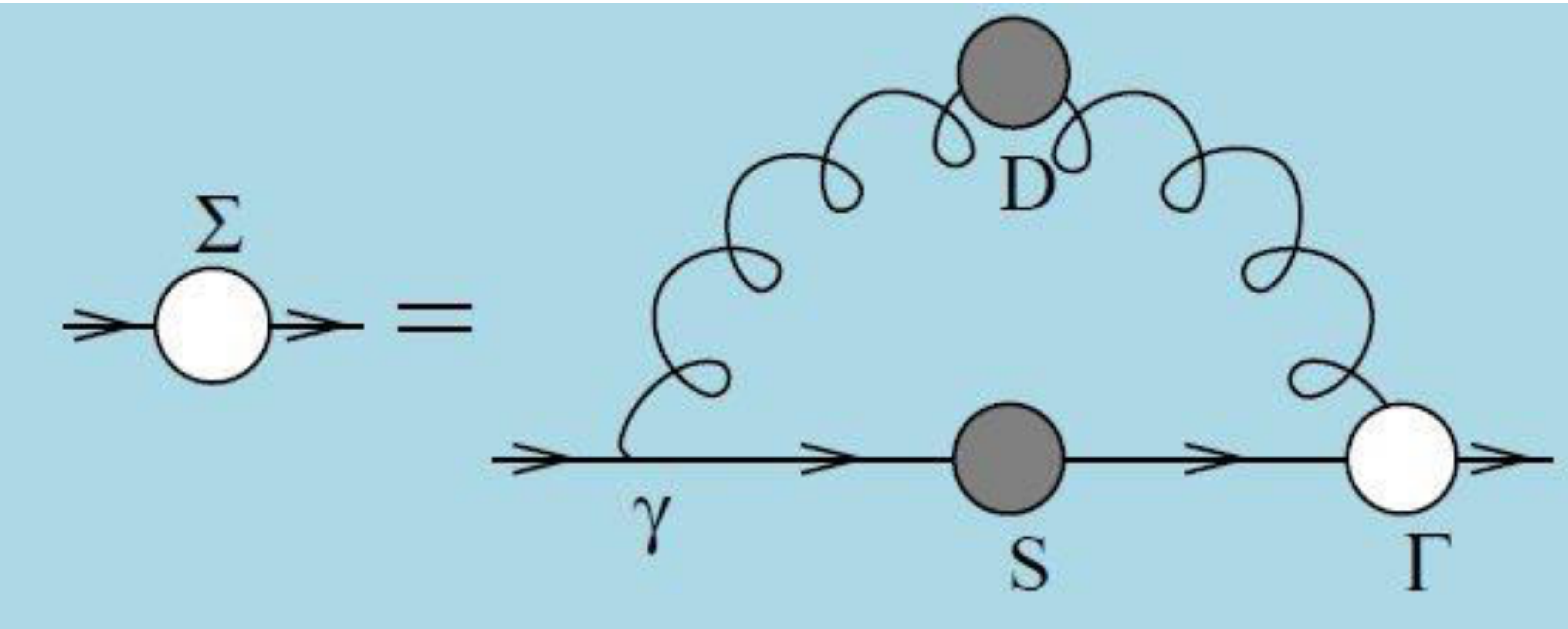
- Treats a massless (chirally-invariant) four-fermion Lagrangian & solves the gap equation in Hartree-Fock approximation (analogous to rainbow truncation)

The following Lagrangian density will be assumed ($\hbar = c = 1$):

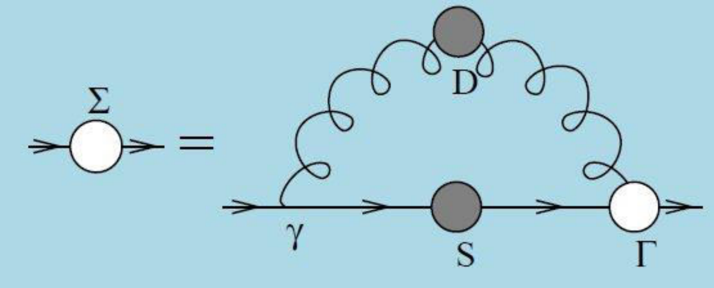
$$L = -\bar{\psi}\gamma_\mu\partial_\mu\psi + g_0[(\bar{\psi}\psi)^2 - (\bar{\psi}\gamma_5\psi)^2]. \qquad (2.6)$$

The coupling parameter g_0 is positive, and has dimensions $[\text{mass}]^{-2}$. The γ_5 invariance property of the interaction is evident from Eq. (2.5). According to the

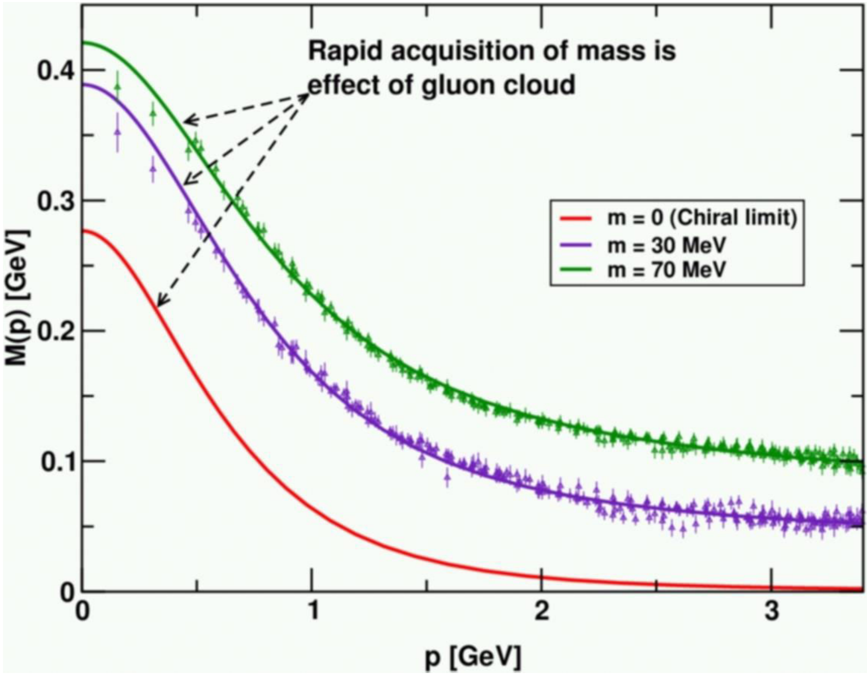
$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



Quark Gap Equation



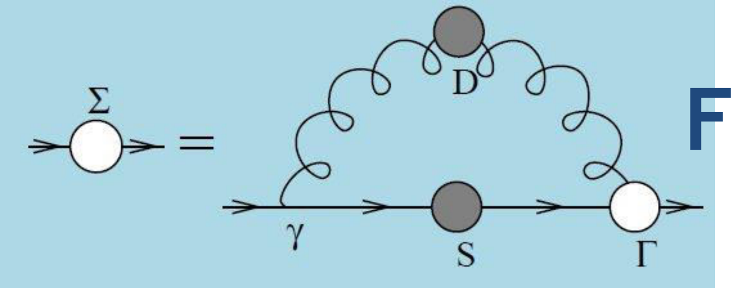
Dynamical Chiral Symmetry Breaking



- *DCSB is a fact in QCD*
  - ***Dynamical***, not spontaneous
    - Add nothing to *QCD* ,

      No Higgs field, nothing!

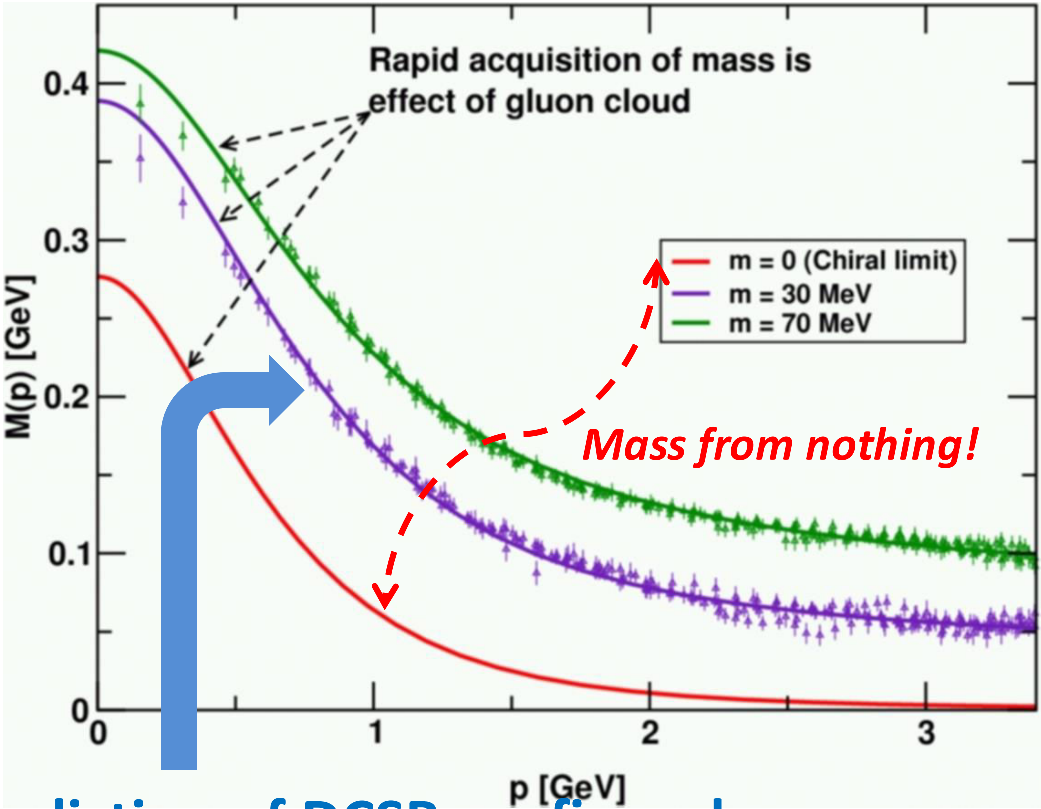
      Effect achieved purely through quark+gluon dynamics.
  - It's the most important mass generating mechanism for visible matter in the Universe.
    - Responsible for ≈98% of the proton's mass.
    - Higgs mechanism is (*almost*) irrelevant to light-quarks.



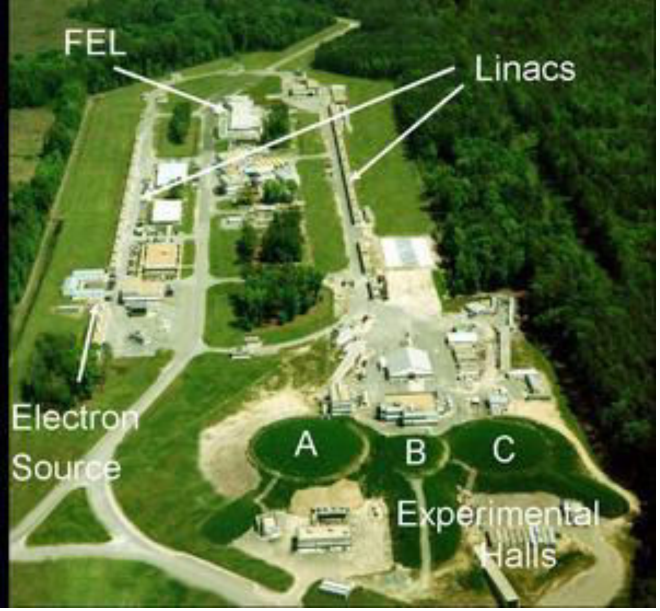
Frontiers of Nuclear Science: Theoretical Advances

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

In QCD a quark's effective mass depends on its momentum. The function describing this can be calculated and is depicted here. Numerical simulations of lattice QCD (data, at two different bare masses) have confirmed model predictions (solid curves) that the vast bulk of the constituent mass of a light quark comes from a cloud of gluons that are dragged along by the quark as it propagates. In this way, a quark that appears to be absolutely massless at high energies (m =0, red curve) acquires a large constituent mass at low energies.

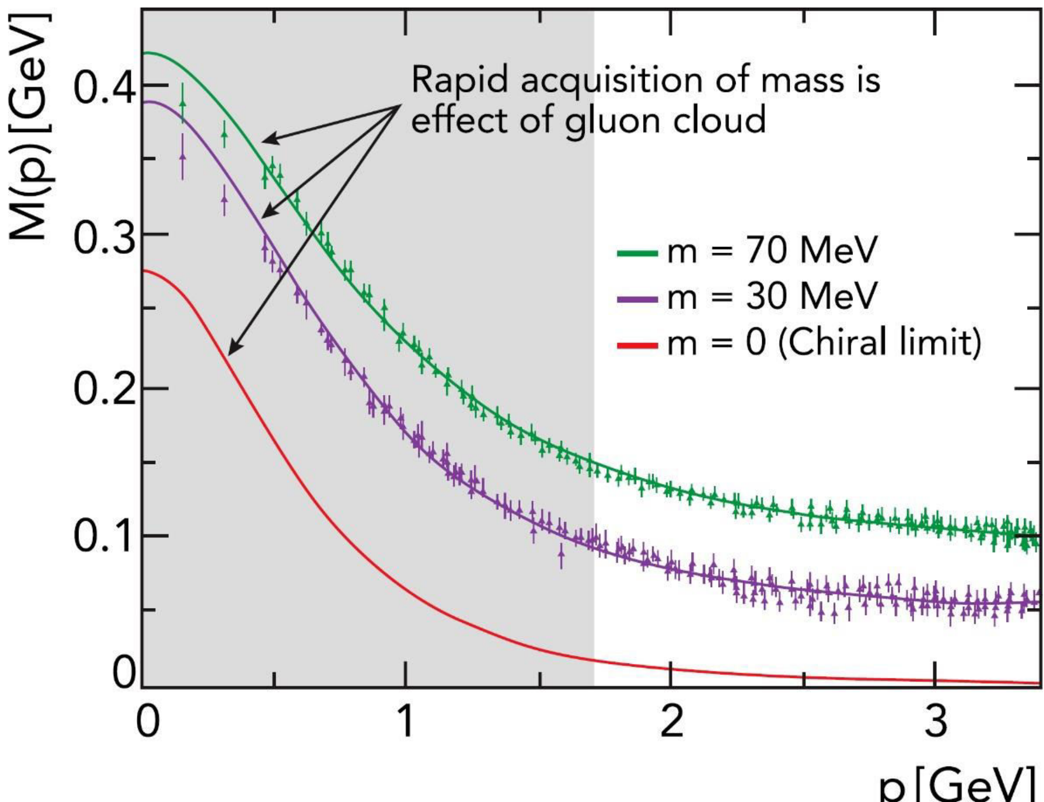


DSE prediction of DCSB confirmed



12GeV
The Future of JLab

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$



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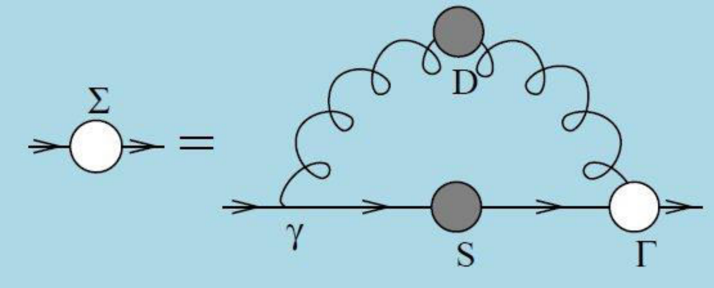
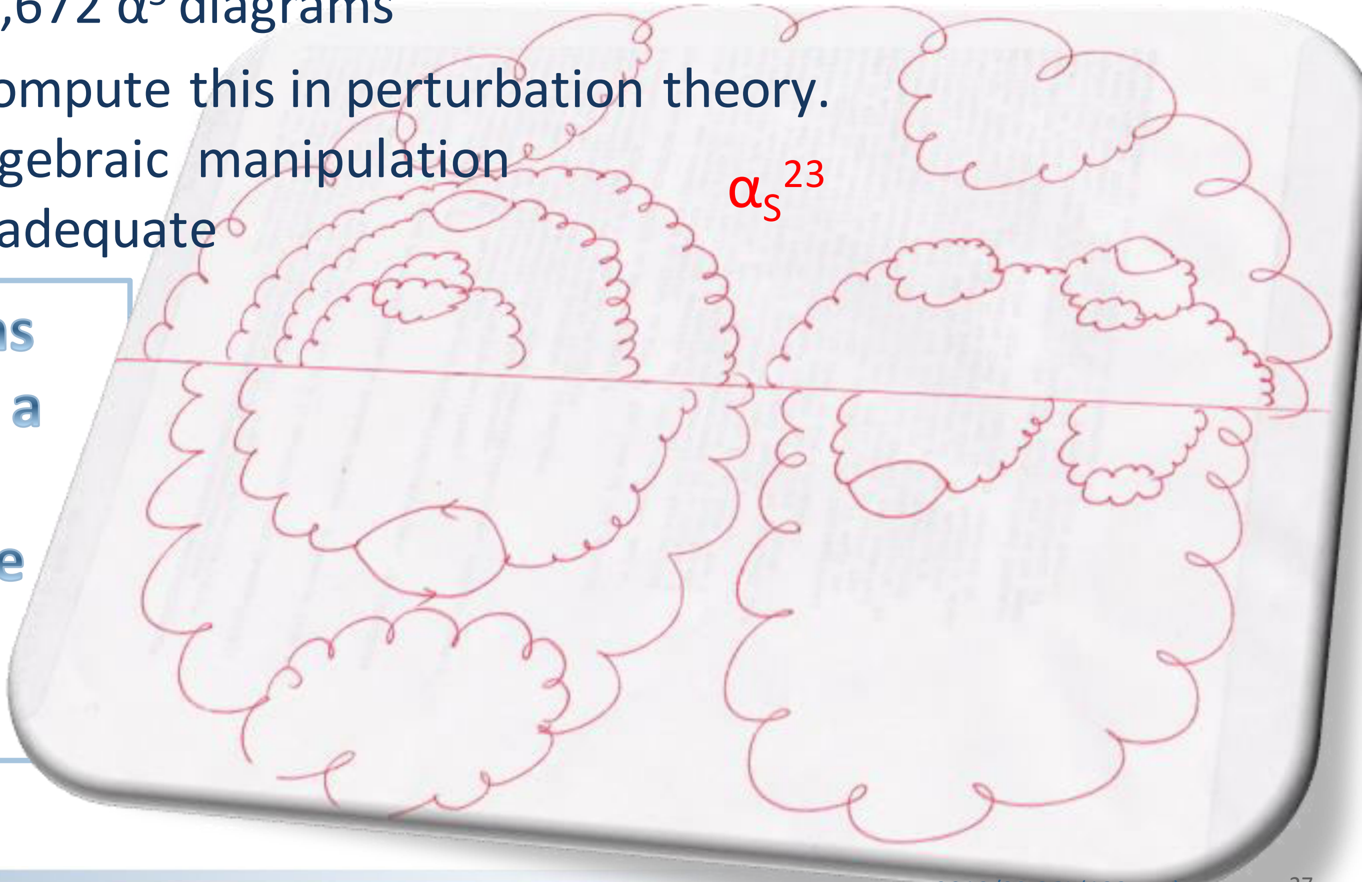
Jlab 12GeV: Scanned by $2<Q^2<9$ GeV^2 elastic & transition form factors.



Where does the mass come from?

- Deceptively simply picture
- Corresponds to the sum of a countable infinity of diagrams.
  NB. QED has 12,672 α^5 diagrams
- Impossible to compute this in perturbation theory.
  The standard algebraic manipulation tools are just inadequate



Just one of the terms that are summed in a solution of the simplest, reasonable truncation of QCD's gap equation

Nambu-Goldstone Bosons

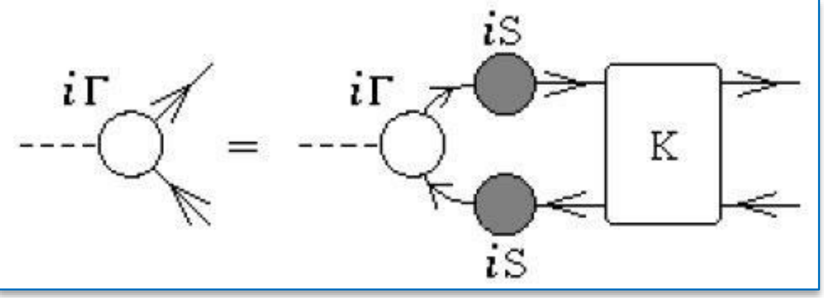

- Suppose one has a vector $\otimes$ vector interaction, strong enough to produce a dressed-quark mass in the absence of Higgs-coupling:
  - How do Nambu-Goldstone bosons emerge?
  - Is their emergence fine-tuned in any way?
  - How does their mass vary with current-quark mass?
  - What do their properties reveal about the origin of mass?

Pseudoscalar Mesons as Bound-States

- Poincaré covariance entails that the bound-state (Bethe-Salpeter) amplitude for an isovector pseudoscalar meson (valence-quark and valence-antiquark) takes the form

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\Big[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\Big]$$

  – $\{\tau^{i=1,2,3}\}$ = Pauli matrices
  – P = total momentum of system
  – k = relative momentum between valence degrees-of-freedom



Pseudoscalar Mesons as Bound-States

- Bound-State amplitude is determined from the homogeneous Bethe-Salpeter equation (BSE):

$$[\Gamma_\pi^j(k;P)]_{tu} = \int \frac{d^4q}{(2\pi)^4} [\chi_\pi^j(q;P)]_{sr} K_{tu}^{rs}(q,k;P)$$

  - χ(q;P) = S(q) Γ(q;P) S(q+P) ,   S = dressed-quark propagator
  - r,s,t,u represent colour, isospin, spinor indices
  - *K* = quark-antiquark scattering amplitude
- A generalisation to quantum field theory of the Lippmann-Schwinger equation = one of the most-used equations in quantum mechanics … to describe scattering and bound states

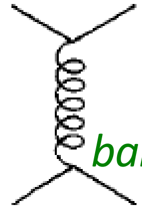
Pseudoscalar Mesons as Bound-States

dressed propagators

BS kernel

O(G):

gap eq.



bare vertices

Leading-order: rainbow- ladder truncation

- Leading-order approximation to Bethe-Salpeter kernel (rainbow-ladder truncation):

$$\mathcal{K}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \mathcal{G}_{\mu\nu}(k)[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\nu]_{\alpha_2\alpha_2'}\,,$$

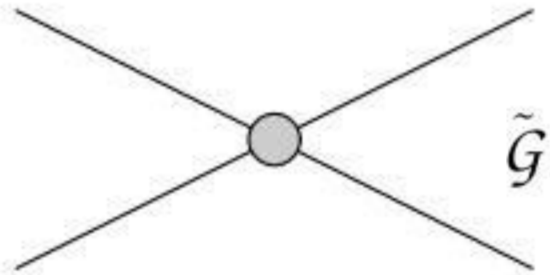
$$\mathcal{G}_{\mu\nu}(k) = \tilde{\mathcal{G}}(k^2)T_{\mu\nu}(k)\,,$$

$$k^2T_{\mu\nu}(k) = k^2\delta_{\mu\nu} - k_\mu k_\nu .$$

- Recall preceding discussion: owing to the dynamical generation of a nonzero gluon mass-scale in QCD [65–74], $\mathcal{G}$ saturates at infrared momenta; hence, one may write

$$\tilde{\mathcal{G}}(k^2) \overset{k^2\simeq 0}{=} \frac{4\pi\alpha_{IR}}{m_G^2}$$

- In QCD
  - $m_G \approx 0.5$ GeV and $\alpha \approx \pi$



$$\tilde{\mathcal{G}}(k^2) \stackrel{k^2\simeq 0}{=} \frac{4\pi\alpha_{IR}}{m_G^2}$$

Pseudoscalar Mesons as Bound-States

- This equation defines the contact interaction
- Since a contact interaction cannot support relative momentum between bound-state constituents
  - – no momentum exchanged between scatterers

tensor structure simplifies: contact-interaction RL kernel is

$$\mathcal{K}^{\rm CI}_{\alpha_1\alpha_1',\alpha_2\alpha_2'} = \frac{4\pi\alpha_{IR}}{m_G^2}[i\gamma_\mu]_{\alpha_1\alpha_1'}[i\gamma_\mu]_{\alpha_2\alpha_2'}$$

- No relative momentum, then Bethe-Salpeter amplitude simplifies

$$\Gamma_{0^-}(Q) = \gamma_5\left[iE_{0^-} + \frac{1}{M}\gamma\cdot P F_{0^-}\right]$$

  - – M = dressed-mass of quarks in bound-state
  - – E_{0-} & F_{0-} independent of relative momentum

Pseudoscalar Mesons as Bound-States

- BSE becomes (total momentum now = Q, relative momentum = t)

$$\Gamma_{0^-}(Q) = -\frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4t}{(2\pi)^4}\gamma_\mu S_f(t+Q)\Gamma_{0^-}(Q)S_g(t)\gamma_\mu$$

- Integral is obviously quadratically divergent:
  - $\int d^4t \sim \int dt^2\, t^2$
  - Integrand $\sim 1/t^2$

  Regularisation needed.

- Same true of contact interaction gap equation:

$$S_f^{-1}(p) = i\gamma\cdot p + m_f + \frac{16\pi}{3}\frac{\alpha_{\rm IR}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\gamma_\mu S_f(q)\gamma_\mu$$

  Recall Lecture II, contact-interaction solution has form:

$$S(p) = 1/[i\gamma\cdot p + M]$$

- What shall we do to complete definition of theory?

Pseudoscalar Mesons as Bound-States

- With $S(p) = 1/[i\gamma\cdot p + M]$, only dynamical content of CI gap equation is determination of dressed-quark's mass (f = flavour label):

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \left[\int_0^\infty ds\, s\, \frac{1}{s+M_f^2} \right]_{\rm reg}$$

$$\frac{1}{s+M^2} = \int_0^\infty d\tau\, {\rm e}^{-\tau(s+M^2)}$$
$$\to \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, {\rm e}^{-\tau(s+M^2)}$$
$$= \frac{{\rm e}^{-(s+M^2)\tau_{\rm uv}^2} - {\rm e}^{-(s+M^2)\tau_{\rm ir}^2}}{s+M^2}$$

- Follow [inSPIRE: Ebert:1996vx]

$$M_f = m_f + M_f \frac{4\alpha_{\rm IR}}{3\pi m_G^2}\, \mathcal{C}_0^{\rm iu}(M_f^2)\,,$$

where

$$\mathcal{C}_0^{\rm iu}(\sigma) = \int_0^\infty ds\, s \int_{\tau_{\rm uv}^2}^{\tau_{\rm ir}^2} d\tau\, {\rm e}^{-\tau(s+\sigma)}$$
$$= \sigma\big[\Gamma(-1,\sigma\tau_{\rm uv}^2) - \Gamma(-1,\sigma\tau_{\rm ir}^2)\big],$$

with $\Gamma(\alpha,y)$ being the incomplete gamma-function.

$\tau_{\rm uv} = 1/\Lambda_{\rm uv}$ & $\tau_{\rm ir} = 1/\Lambda_{\rm ir}$ are ultraviolet and infrared cutoffs.

$\tau_{\rm ir}$ not necessary, but very useful

Pseudoscalar Mesons as Bound-States

- Why $\Lambda_{ir} \neq 0$, *i.e.* $\tau_{ir} < \infty$?
- Answer:

  with $\frac{1}{s+M^2} \to \frac{e^{-(s+M^2)\tau_{uv}^2} - e^{-(s+M^2)\tau_{ir}^2}}{s+M^2}$

  – then for $s + M^2 \simeq 0$, propagator becomes = $\tau_{ir}^2 - \tau_{uv}^2$
  – free-particle pole is avoided!
- Using the IR cutoff, one has a rudimentary expression of quark confinement
- A natural choice is $\Lambda_{ir} = \Lambda_{QCD} = 0.24$ GeV
- Use this scheme for every integrand, *e.g.* those in the BSE
  – If one has $1/[s+M^2]^2$, then use $1/[s+M^2]^2 = - (d/dM^2)\, 1/[s+M^2]$

Pseudoscalar Mesons as Bound-States

- Preserving the vector and axial-vector Ward-Green-Takahashi identities is crucial when studying NG-bosons!
  - It is nonsense to study emergent systems in a theory where the conditions for emergence are explicitly violated!
- The m = 0 axial-vector WGT identity states:

$$P_\mu \Gamma_{5\mu}(k_+, k) = S^{-1}(k_+) i\gamma_5 + i\gamma_5 S^{-1}(k)$$

  - ($k_+ = k+P$)
- $\Gamma_{5\mu}$ is the axial-vector vertex

$$\Gamma_{5\mu}(k_+, k) = \gamma_5\gamma_\mu - \frac{16\pi}{3}\frac{\alpha_{IR}}{m_G^2}\int \frac{d^4q}{(2\pi)^4}\gamma_\alpha \chi_{5\mu}(q_+, q)\gamma_\alpha$$

- (How) Is this identity satisfied using $S(p) = 1/[i\gamma{\cdot}p + M]$?

Pseudoscalar Mesons as Bound-States

- Insert propagator into AV WGT identity … satisfied if, and only if:

$$M = \frac{16}{3}\frac{4\pi\,\alpha_{IR}}{m_G^2}\int\frac{d^4q}{(2\pi)^4}\frac{M}{[q^2+M^2]},$$

$$0 = \int\frac{d^4q}{(2\pi)^4}\frac{\frac{1}{2}q^2+M^2}{[q^2+M^2]^2}.$$

A. Line 1 = chiral-limit quark gap equation … so the identity can only be satisfied so long as the gap equation is satisfied

B. Line 2 = entirely new:

   i. AV WGT identity is satisfied if, and only if, the model is regularised so as to ensure there are no quadratic or logarithmic divergence

   Unsurprisingly, these are the just the circumstances under which a shift in integration variables is permitted, an operation required in order to prove the AV WGT identity

Pseudoscalar Mesons as Bound-States

- Lesson: the contact interaction can be defined sensibly

  if, and only if, one contains the symmetry constraints in the regularisation prescription
- For example,

  Line 2 $\Rightarrow \int_0^1 d\alpha\, \left[C_0^{iu}\big(w(\alpha)\big) + C_1^{iu}\big(w(\alpha)\big)\right] = 0$

  where $C_1^{iu}(\sigma) = -\sigma \frac{d}{d\sigma} C_1^{iu}(\sigma)$
- Using this identity, one obtains NG-boson BSE in chiral limit

$$\begin{bmatrix} E_{0^-}(Q) \\ F_{0^-}(Q) \end{bmatrix} = \frac{4\alpha_{\rm IR}}{3\pi m_G^2} \begin{bmatrix} \mathcal{K}_{EE}^{0^-} & \mathcal{K}_{EF}^{0^-} \\ \mathcal{K}_{FE}^{0^-} & \mathcal{K}_{FF}^{0^-} \end{bmatrix} \begin{bmatrix} E_{0^-}(Q) \\ F_{0^-}(Q) \end{bmatrix}$$

$$\begin{aligned} \mathcal{K}_{EE} &= \mathcal{C}(M^2; \tau_{\rm ir}^2, \tau_{\rm uv}^2)\,, \quad \mathcal{K}_{EF} = 0\,, \\ 2\mathcal{K}_{FE} &= \mathcal{C}_1(M^2; \tau_{\rm ir}^2, \tau_{\rm uv}^2)\,, \quad \mathcal{K}_{FF} = -2\mathcal{K}_{FE} \end{aligned}$$

Pseudoscalar Mesons as Bound-States

- BSE … first line

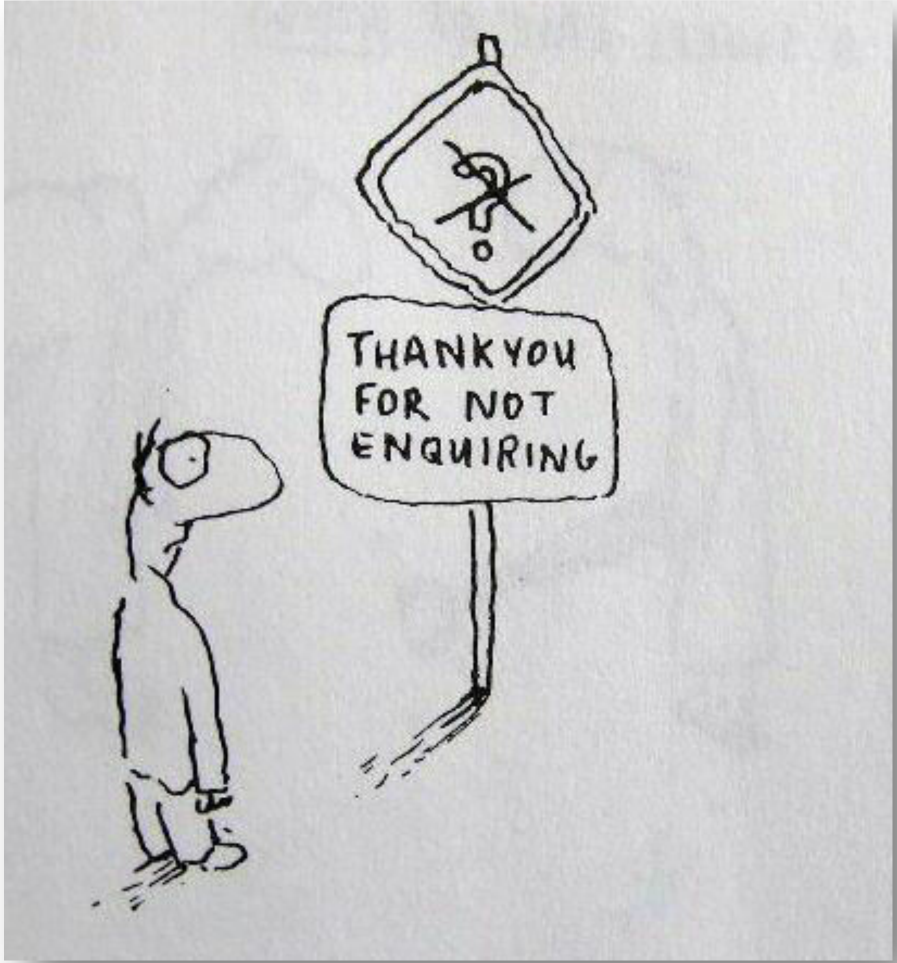
$$E_{0^-} = \frac{4\,a_{IR}}{3\pi m_G^2}\; C_0(M^2)\, E_{0^-}$$

But this is exactly the chiral-limit gap equation!

- Plainly:
  - ✓ If ∃ M ≠ 0 solution of the gap equation, then the NG-boson exists because E_{0^-} is forced to be nonzero
  - ✓ Equally if NG-boson exists, *i.e.* $E_{0^-} \neq 0$, then chiral-limit gap equation must support M ≠ 0 solution.

- NG Boson exists if, and only if, chiral symmetry is dynamically broken
  - ✓ No fine tuning involved
  - ✓ $E_{0^-} \propto M$

2-body ⇔ 1-body



Enigma of Mass

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Pion's Goldberger -Treiman relation

- Pion's Bethe-Salpeter amplitude

  Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k;P) = \tau^{\pi^j}\gamma_5\left[iE_\pi(k;P) + \gamma\cdot P F_\pi(k;P) + \gamma\cdot k\, k\cdot P\, G_\pi(k;P) + \sigma_{\mu\nu}\, k_\mu P_\nu\, H_\pi(k;P)\right]$$

- Dressed-quark propagator $S(p) = \dfrac{1}{i\gamma\cdot p\, A(p^2) + B(p^2)}$

- Axial-vector Ward-Takahashi identity entails

$$f_\pi E_\pi(k;P=0) = B(k^2)$$

Owing to DCSB & Exact in Chiral QCD

***Miracle*: two body problem solved, almost completely, once solution of one body problem is known**

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) = B(p^2)$$

The most fundamental expression of Goldstone's Theorem and DCSB

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

This is why $m_\pi=0$ in the absence of a Higgs mechanism

Rudimentary version of this relation is apparent in Nambu's Nobel Prize work

Model independent
Gauge independent
Scheme independent

$$f_\pi E_\pi(p^2) \Leftrightarrow B(p^2)$$

Pion exists if, and only if, mass is dynamically generated

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

Enigma of mass

- The quark level Goldberger-Treiman relation shows that DCSB has a very deep and far reaching impact on physics within the strong interaction sector of the Standard Model; viz.,

  Goldstone's theorem is fundamentally an expression of equivalence between the one-body problem and the two-body problem in the pseudoscalar channel.

  $$f_\pi E_\pi(p^2) = B(p^2)$$

- This emphasises that Goldstone's theorem has a pointwise expression in QCD
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.
- Thus, enigmatically, the properties of the *massless* pion are the cleanest expression of the mechanism that is responsible for almost all the visible mass in the universe.

Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5} m_{H_5}^2 = \rho_{H_5}^\zeta \mathcal{M}_{H_5}^\zeta$$

- Mass-squared of the pseudscalar hadron
- Sum of the current-quark masses of the constituents;
  e.g., pion = $m_u^\varsigma + m_d^\varsigma$, where "ς" is the renormalisation point

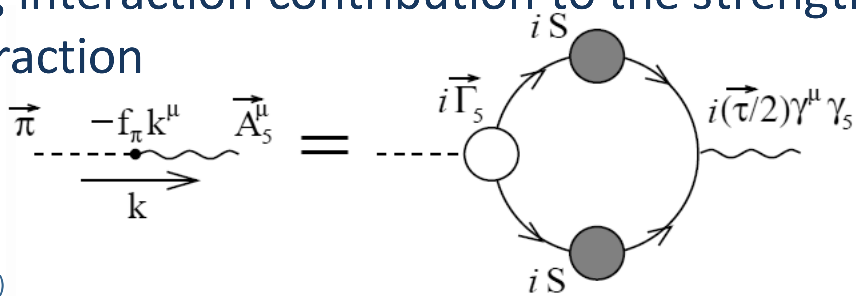
Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^\zeta_{H_5} \mathcal{M}^\zeta_{H_5}$$

$$f_{H_5} P_\mu = Z_2 \mathrm{tr} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^{\mathrm{t}} \gamma_5 \gamma_\mu \boxed{S(q+\frac{1}{2}P)\Gamma_{H_5}(q;P)S(q-\frac{1}{2}P)}$$

- Pseudovector projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, the pseudoscalar meson's leptonic decay constant, which is the strong interaction contribution to the strength of the meson's weak interaction



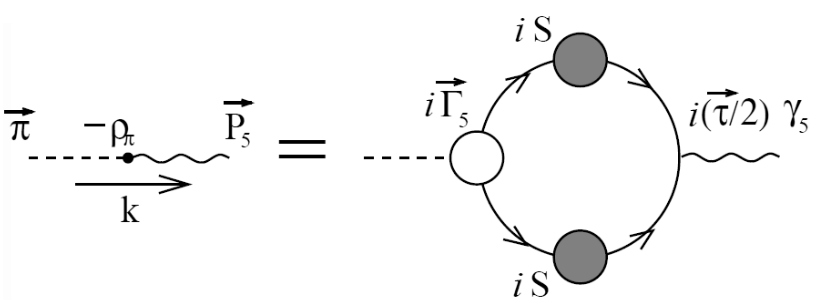
Maris, Roberts and Tandy
nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m_{H_5}^2 = \rho_{H_5}^{\zeta} \mathcal{M}_{H_5}^{\zeta}$$

$$i\rho_{H_5} = Z_4 \mathrm{tr} \int \frac{d^4q}{(2\pi)^4} \frac{1}{2} (T^{H_5})^{t} \gamma_5 \boxed{S(q+\frac{1}{2}P)\Gamma_{H_5}(q;P)S(q-\frac{1}{2}P)}$$

- Pseudoscalar projection of the Bethe-Salpeter wave function onto the origin in configuration space
  - Namely, a pseudoscalar analogue of the meson's leptonic decay constant



Maris, Roberts and Tandy
*nucl-th/9707003, Phys.Lett. B**420** (1998) 267-273*

Dichotomy of the pion
Mass Formula for 0⁻ Mesons

$$f_{H_5}\, m^2_{H_5} = \rho^{\zeta}_{H_5} \mathcal{M}^{\zeta}_{H_5}$$

- Consider the case of light quarks; namely, $m_q \approx 0$
  - If chiral symmetry is dynamically broken, then
    - $f_{H5} \to f_{H5}^{\,0} \neq 0$
    - $\rho_{H5} \to -\langle q\text{-bar } q\rangle / f_{H5}^{\,0} \neq 0$

    both of which are independent of m_q
- Hence, one arrives at the corollary

The so-called "vacuum quark condensate." It's actually contained within hadrons.

Gell-Mann, Oakes, Renner relation 1968

$$\boxed{m^2_{H_5} = 2 m_q \frac{-\langle \bar{q} q\rangle}{f^0_{H_5}}}$$

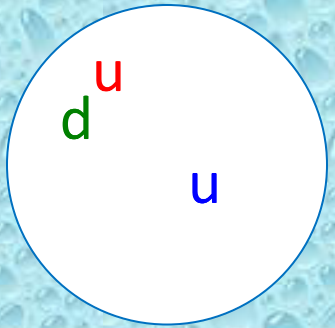
$$m_\pi^2 \propto m$$

Dynamical Chiral Symmetry Breaking

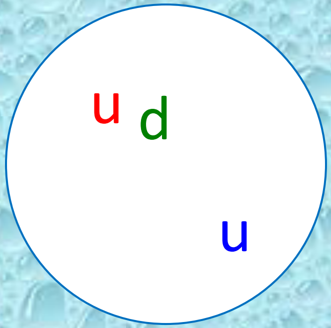
Vacuum Condensates?

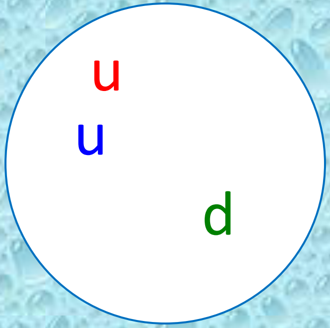
“Orthodox Vacuum”

- Vacuum = “frothing sea”
- Hadrons = bubbles in that “sea”,



containing nothing but quarks & gluons interacting perturbatively, unless they’re near the bubble’s boundary, whereat they feel they’re trapped!





PHYSICAL REVIEW VOLUME 175, NUMBER 5 25 NOVEMBER 1968

Behavior of Current Divergences under $SU_3 \times SU_3$*†

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(Received 22 July 1968)

GMOR Relation

Gell-Mann - Oakes - Renner Relation

Behavior of current divergences under SU(3) x SU(3).
Murray Gell-Mann, R.J. Oakes , B. Renner
Phys.Rev. 175 (1968) 2195-2199

- This paper derives a relation between

  m_π^2 and the expectation-value $< \pi | u_0 | \pi >$,

  where u_o is an operator that is linear in the putative Hamiltonian's explicit chiral-symmetry breaking term
  - NB. QCD's current-quarks were not yet invented, so u_0 was not expressed in terms of current-quark fields
- PCAC-hypothesis (partial conservation of axial current) is used in the derivation
- Subsequently, the concepts of soft-pion theory
  - Operator expectation values do not change as $t=m_\pi^2 \to t=0$

  to take $< \pi | u_0 | \pi > \to < 0 | u_0 | 0 >$ … *in-pion → in-vacuum*

Gell-Mann - Oakes - Renner Relation

Behavior of current divergences under SU(3) x SU(3).
Murray Gell-Mann, R.J. Oakes , B. Renner
Phys.Rev. 175 (1968) 2195-2199

- PCAC hypothesis; *viz*., pion field dominates the divergence of the axial-vector current

Zhou Guangzhao 周光召
Born 1929 Changsha, Hunan province

$$\partial_\mu A_\mu \propto \phi_\pi$$

- Soft-pion theorem

$$\begin{aligned} & \langle \alpha | \mathcal{O} | \beta \pi(q) \rangle \approx \langle \alpha | [Q_5, \mathcal{O}] | \beta \rangle \\ \Rightarrow \; & \langle \pi | \mathcal{O} | \pi \rangle \approx \langle 0 | [Q_5, [Q_5, \mathcal{O}]] | 0 \rangle \\ \Rightarrow \; & \langle \pi(q) | \mathcal{H} | \pi(q) \rangle \approx \langle 0 | [Q_5, [Q_5, \mathcal{H}]] | 0 \rangle \\ \propto \; & \langle 0 | \mathcal{H}_{\rm chiral-symmetry-breaking} | 0 \rangle \end{aligned}$$

Commutator is chiral rotation
Therefore, isolates explicit chiral-symmetry breaking term in the putative Hamiltonian

- In QCD,

this is $m\bar{q}q$ and one therefore has $m_\pi^2 \propto m \langle 0 | \bar{q} q | 0 \rangle$

Gell-Mann - Oakes - Renner Relation

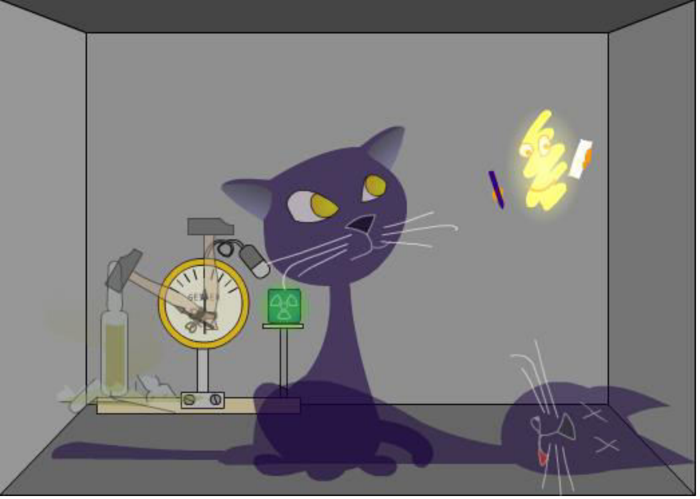
$$m_\pi^2 \propto m\langle 0|\bar{q}q|0\rangle$$

$- (0.25\text{GeV})^3$

- Theoretical physics at its best.
- But no one is thinking about how properly to consider or define what will come to be called the

vacuum quark condensate

- So long as the condensate is just a mass-dimensioned constant, which approximates another well-defined matrix element, there is no problem.
- Problem arises if one over-interprets this number, which textbooks have been doing for a *VERY LONG TIME*.



Note of Warning

Chiral Magnetism (or Magnetohadrochironics)

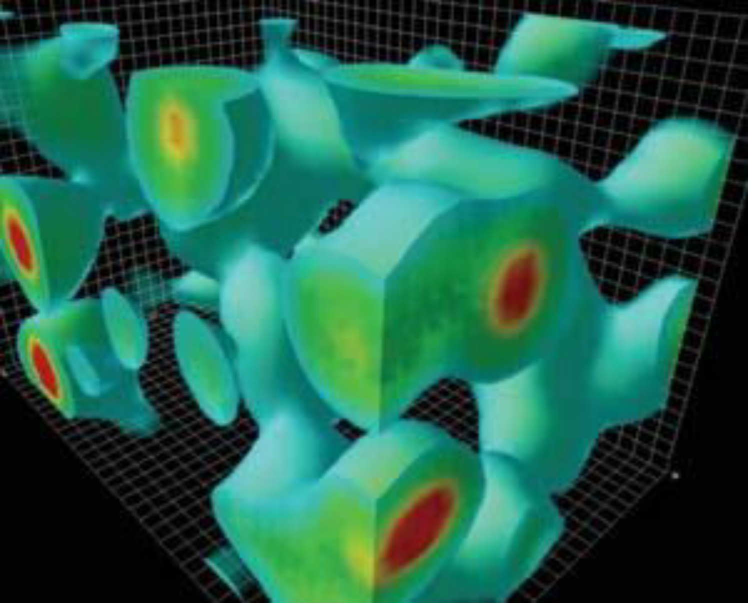
A. Casher and L. Susskind, Phys. Rev. D9 (1974) 436

The spontaneous breakdown of chiral symmetry in hadron dynamics is generally studied as a vacuum phenomenon.[1] Because of an instability of the chirally invariant vacuum, the real vacuum is "aligned" into a chirally asymmetric configuration. On the other hand an approach to quantum field theory exists in which the properties of the vacuum state are not relevant. This is the parton or constituent approach formulated in the infinite-momentum frame.[2] A number of investigations

- *These authors argue that dynamical chiral-symmetry breaking can be realised as a property of hadrons, instead of via a nontrivial vacuum exterior to the measurable degrees of freedom*

The essential ingredient required for a spontaneous symmetry breakdown in a composite system is the existence of a divergent number of constituents

– DIS provided evidence for divergent sea of low-momentum partons – parton model.



QCD Sum Rules

QCD and Resonance Physics. Sum Rules.
M.A. Shifman, A.I. Vainshtein, and V.I. Zakharov
Nucl.Phys. B147 (1979) 385-447; citations: 3713

- Introduction of the gluon vacuum condensate

$$\frac{\alpha}{\pi}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle = (0.33\,\mathrm{GeV})^4$$

and development of “sum rules” relating properties of low-lying hadronic states to vacuum condensates

QCD Sum Rules

QCD and Resonance Physics. Sum Rules.
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- Introduction of the gluon vacuum condensate

$$\frac{\alpha}{\pi}\langle 0|G_{\mu\nu}G^{\mu\nu}|0\rangle = (0.33\,\mathrm{GeV})^4$$

  and development of "sum rules" relating properties of low-lying hadronic states to vacuum condensates

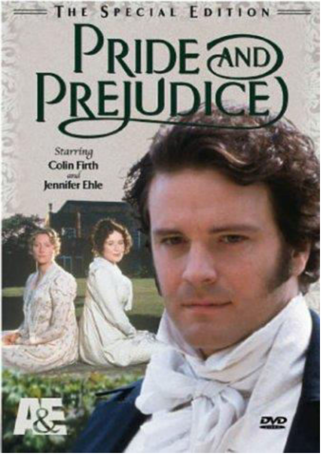
- *At this point (1979), the cat was out of the bag: a physical reality was seriously attributed to a plethora of vacuum condensates*

“quark condensate” 1960-1980

- Instantons in non-perturbative QCD vacuum,
  MA Shifman, AI Vainshtein… - Nuclear Physics B, 1980
- Instanton density in a theory with massless quarks,
  MA Shifman, AI Vainshtein… - Nuclear Physics B, 1980
- Exotic new quarks and dynamical symmetry breaking,
  WJ Marciano - Physical Review D, 1980
- The pion in QCD
  J Finger, JE Mandula… - Physics Letters B, 1980

No references to this phrase before 1960

17,900+ REFERENCES TO THIS PHRASE SINCE 1980

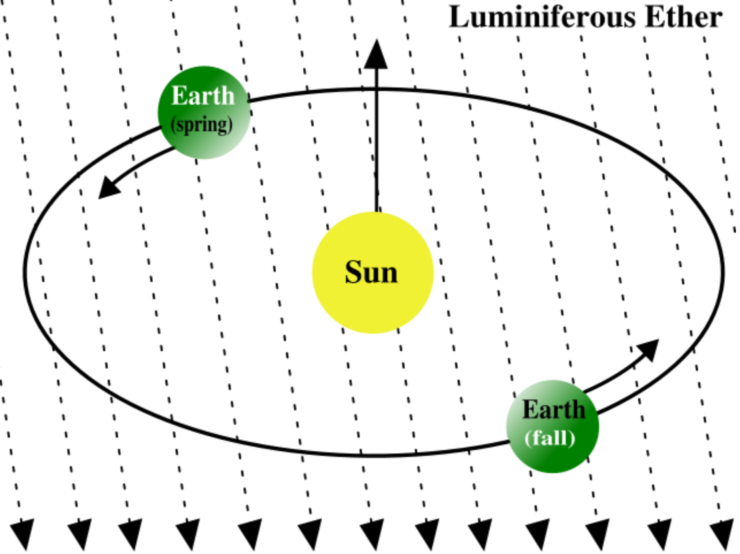


Universal *Conventions*

- Wikipedia: [(http://en.wikipedia.org/wiki/QCD_vacuum)](http://en.wikipedia.org/wiki/QCD_vacuum)

  "The QCD vacuum is the vacuum state of quantum chromodynamics (QCD). It is an example of a non-perturbative vacuum state, characterized by many non-vanishing condensates such as the gluon condensate or the quark condensate. These condensates characterize the normal phase or the confined phase of quark matter."

Precedent?



Precedent- *Luminiferous Aether*

Pre-1887

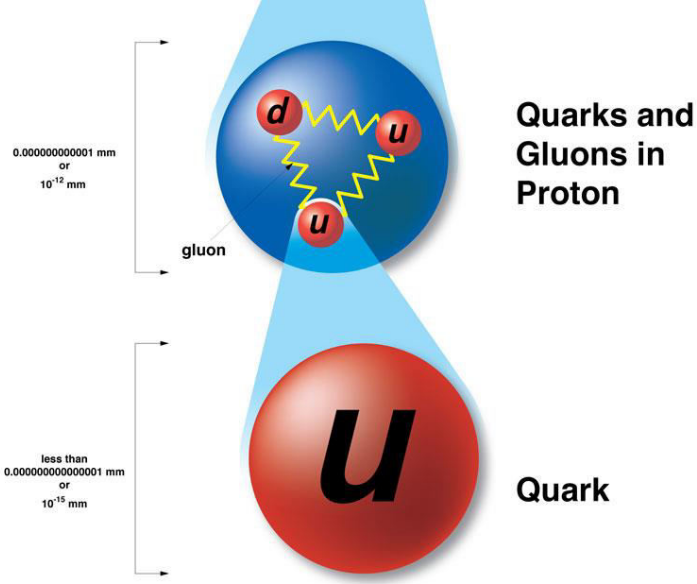
~~Since the Earth is in motion, the flow of aether across the Earth should produce a detectable "aether wind"~~

- Physics theories of the late 19th century postulated that, just as water waves must have a medium to move across (water), and audible sound waves require a medium to move through (such as air or water), so also light waves require a medium, the "*luminiferous aether*".

Apparently unassailable logic

- Until, of course, "… *the most famous failed experiment to date.*"

On the Relative Motion of the Earth and the Luminiferous Ether
Michelson, Albert Abraham & Morley, Edward Williams
American Journal of Science 34 (1887) 333–345.



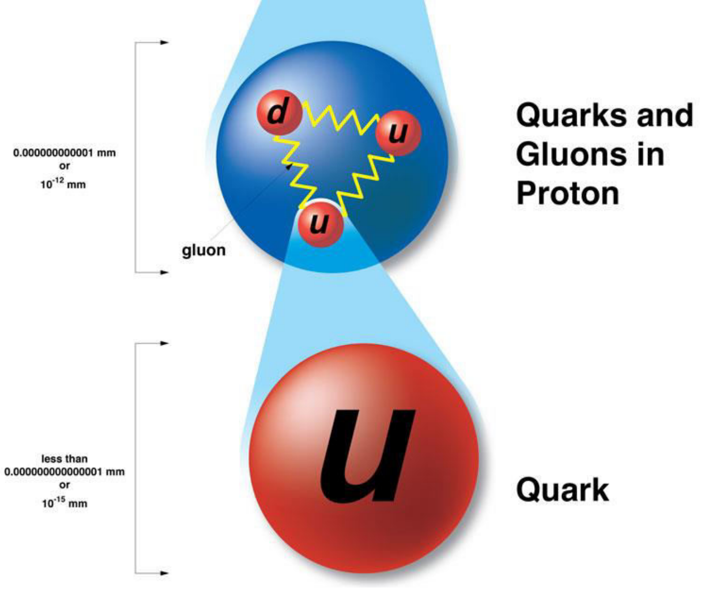
QCD

1973-1974

$$\langle 0 | \bar{q}q | 0 \rangle$$

- How should one approach this problem, understand it, within Quantum ChromoDynamics?

1) Are the quark and gluon "condensates" theoretically well-defined?
2) Is there a physical meaning to this quantity or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?



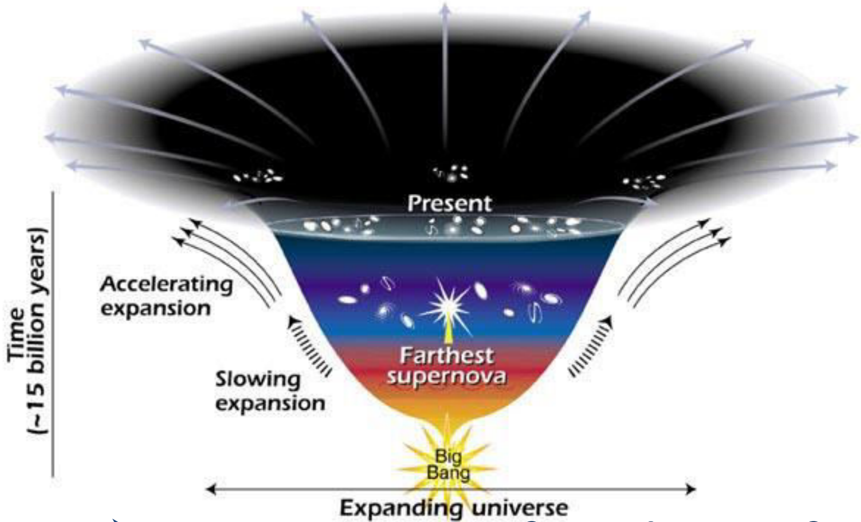
QCD

1973-1974

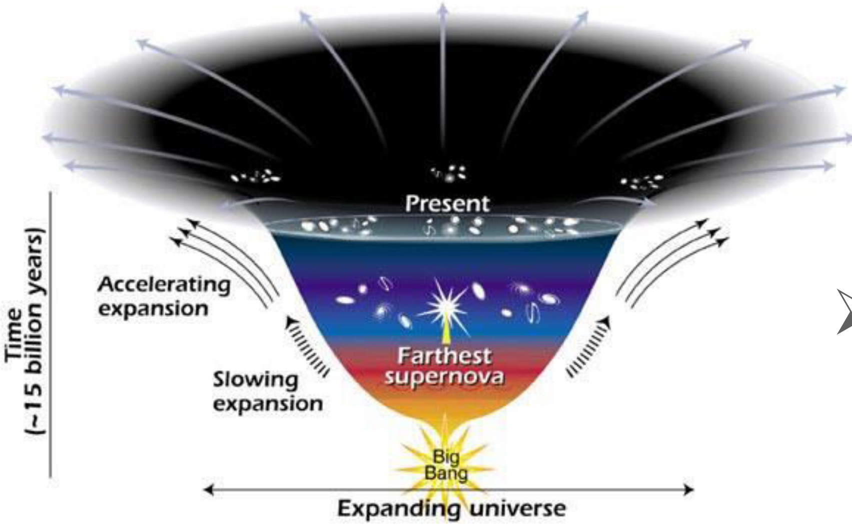
$$\langle 0 | \bar{q}q | 0 \rangle$$

Why does it matter?

"Dark Energy"



- Two pieces of evidence for an accelerating universe
  1) Observations of type Ia supernovae
     → *the rate of expansion of the Universe is growing*
  2) Measurements of the composition of the Universe point to a missing energy component with negative pressure:
     CMB anisotropy measurements indicate that the Universe is at
     $$\Omega_0 = 1 \,^{+}\!/\!_{-}\, 0.04.$$
     In a flat Universe, the matter density and energy density must sum to the critical density. However, matter only contributes about ⅓ of the critical density,
     $$\Omega_M = 0.33 \,^{+}\!/\!_{-}\, 0.04.$$
     Thus, *⅔ of the critical density is missing*.

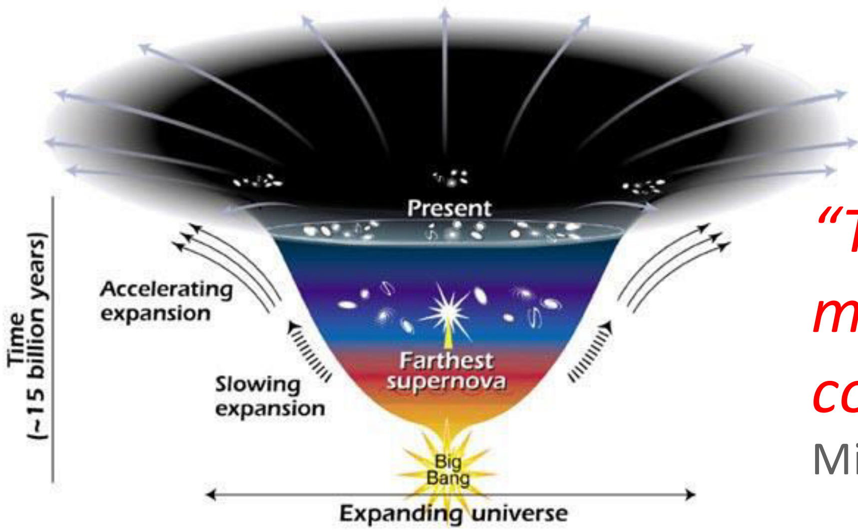


"Dark Energy"

- In order to have escaped detection, the missing energy must be smoothly distributed.
- In order not to interfere with the formation of structure (by inhibiting the growth of density perturbations) the energy density in this component must change more slowly than matter (so that it was subdominant in the past).
- Accelerated expansion can be accommodated in General Relativity through the Cosmological Constant, *Λ*.
  - Contemporary cosmological observations mean:

$$\rho_{\Lambda}^{obs} = \frac{\Lambda}{8\pi G} \leq (10^{-12}\, GeV)^4$$

  - Einstein introduced the repulsive effect of the cosmological constant in order to balance the attractive gravity of matter so that a static universe was possible. He promptly discarded it after the discovery of the expansion of the Universe.



"Dark Energy"

"The advent of quantum field theory made consideration of the cosmological constant obligatory not optional."

Michael Turner, "Dark Energy and the New Cosmology"

- The only possible covariant form for the energy of the (quantum) vacuum; viz.,

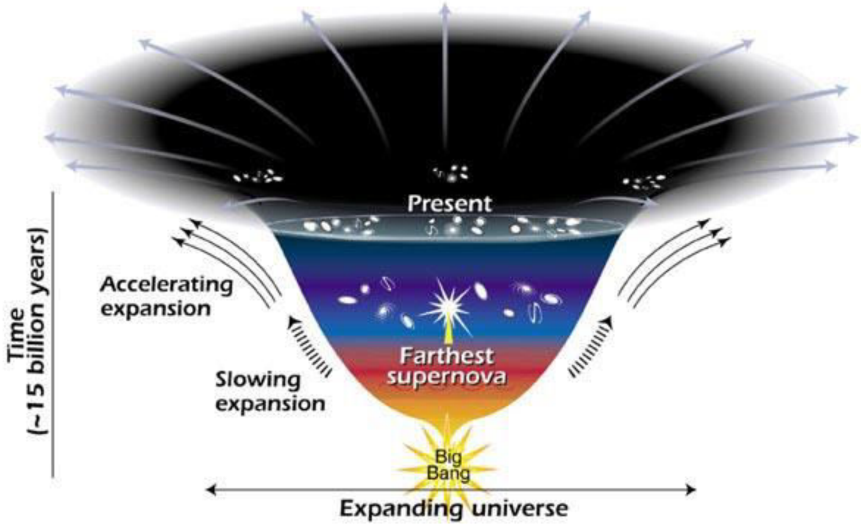
$$T^{\mu\nu}_{\rm VAC} = \rho_{\rm VAC}\, g^{\mu\nu}$$

is mathematically equivalent to the cosmological constant.

"It is a perfect fluid and precisely spatially uniform"

"Vacuum energy is almost <u>the perfect candidate</u> for dark energy."

“Dark Energy”



$$T^{\mu\nu}_{\mathrm{VAC}} = \rho_{\mathrm{VAC}} g^{\mu\nu}$$

Enormous and even greater contribution from Higgs VEV!

- *QCD vacuum contribution*
  - If chiral symmetry breaking is expressed in a nonzero expectation value of the quark bilinear, then the energy difference between the symmetric and broken phases is of order

$M_{\mathrm{QCD}} \approx 0.3$ GeV

Mass-scale generated by spacetime-independent condensate

  - One obtains therefrom:

$$\rho_\Lambda^{QCD} = 10^{46}\,\rho_\Lambda^{obs}$$

“The biggest embarrassment in theoretical physics.”


IT'S OFFICIAL
THE SYSTEM IS COLLAPSING INTO CHAOS
AT LAST! NOW I CAN RELATE TO THE SYSTEM...
Leunig

Resolution?

$$\langle 0|\bar{q}q|0\rangle$$

QCD

1973-1974

Are the condensates real?

- Is there a physical meaning to the vacuum quark condensate (and others)?
- Or is it merely just a mass-dimensioned parameter in a theoretical computation procedure?

S. Weinberg, Physica 96A (1979)
Elements of truth in this perspective

What is measurable?

This remark is based on a "theorem", which as far as I know has never been proven, but which I cannot imagine could be wrong. The "theorem" says that although individual quantum field theories have of course a good deal of content, quantum field theory itself has no content beyond analyticity, unitarity, cluster decomposition, and symmetry. This can be put more precisely in the context of perturbation theory: if one writes down the most general possible Lagrangian, including *all* terms consistent with assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order of perturbation theory, the result will simply be the most general possible S-matrix consistent with analyticity, perturbative unitarity, cluster decomposition and the assumed symmetry principles. As I said, this has not been proved, but any counterexamples would be of great interest, and I do not know of any.

PHYSICAL REVIEW C **85**, 065202 (2012)

Confinement contains condensates

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(Received 27 February 2012; published 21 June 2012)

Dynamical chiral symmetry breaking and its connection to the generation of hadron masses has historically been viewed as a vacuum phenomenon. We argue that confinement makes such a position untenable. If quark-hadron duality is a reality in QCD, then condensates, those quantities that have commonly been viewed as constant empirical mass scales that fill all space-time, are instead wholly contained within hadrons; i.e., they are a property of hadrons themselves and expressed, e.g., in their Bethe-Salpeter or light-front wave functions. We explain that this paradigm is consistent with empirical evidence and incidentally expose misconceptions in a recent Comment.

DOI: 10.1103/PhysRevC.85.065202 PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

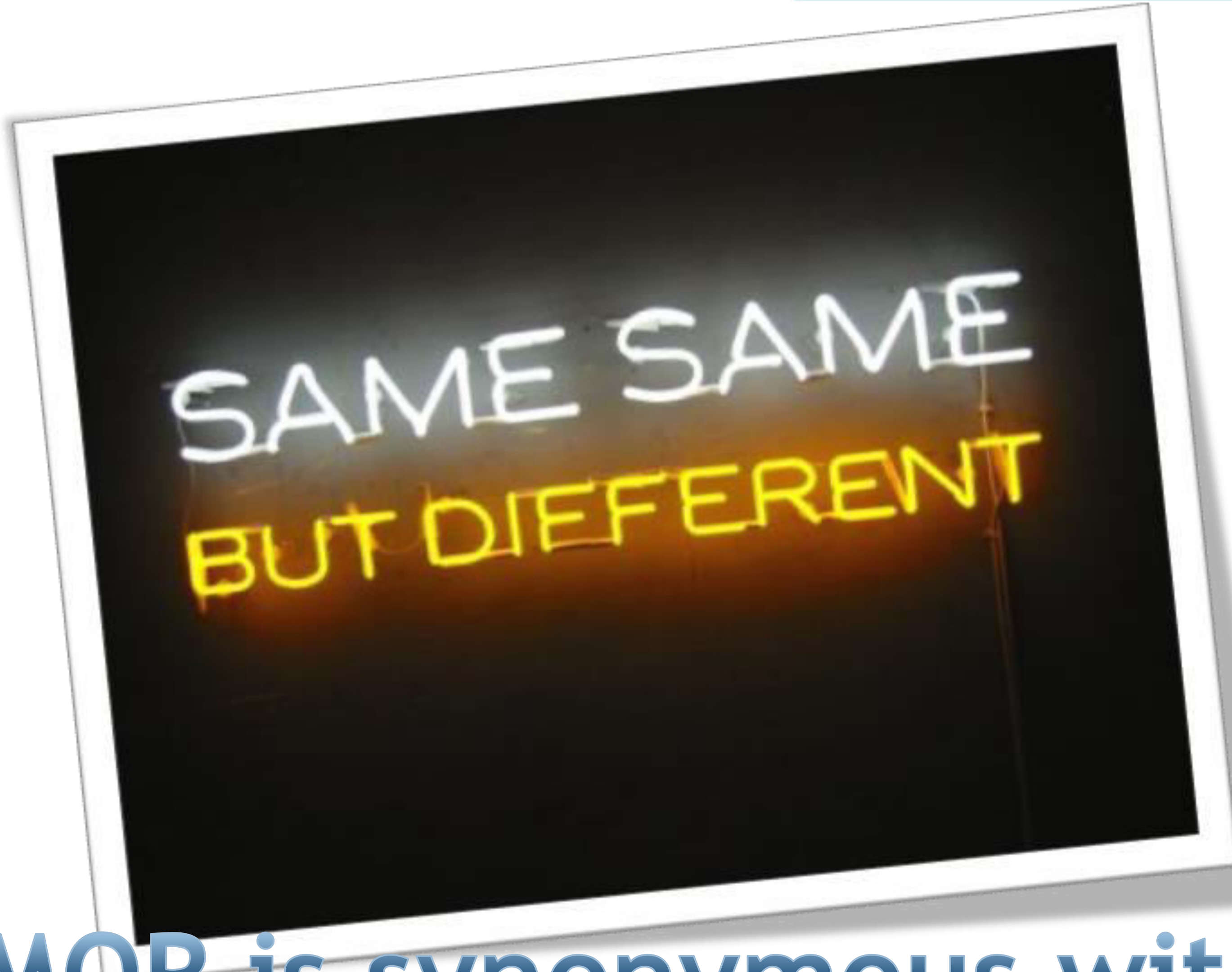
Confinement contains condensates

GMOR Relation

- Valuable to highlight the precise form of the Gell-Mann–Oakes–Renner (GMOR) relation: Eq. (3.4) in Phys.Rev. 175 (1968) 2195

$$m_\pi^2 = \lim_{P' \to P \to 0} \langle \pi(P') | \mathcal{H}_{\chi \mathrm{sb}} | \pi(P) \rangle$$

  - *m_π is the pion's mass*
  - *$H_{\chi sb}$ is that part of the* hadronic Hamiltonian density which explicitly breaks chiral symmetry.
- The operator expectation value in this equation is evaluated between pion states.
- Un-approximated form of the GMOR relation doesn't make any reference to a vacuum condensate



GMOR is synonymous with "Vacuum Quark Condensate"

RAPID COMMUNICATIONS

PHYSICAL REVIEW C **85**, 012201(R) (2012)

Expanding the concept of in-hadron condensates

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(Received 13 September 2011; published 23 January 2012)

The in-pseudoscalar-meson condensate can be represented through the pseudoscalar meson's scalar form factor at zero-momentum transfer. With the aid of a mass formula for scalar mesons, revealed herein, the analog is shown to be true for in-scalar-meson condensates. The concept is readily extended to all hadrons so that, via the zero-momentum-transfer value of any hadron's scalar form factor, one can readily extract the value for a quark condensate in that hadron which is a measure of dynamical chiral symmetry breaking.

DOI: 10.1103/PhysRevC.85.012201 PACS number(s): 12.38.Aw, 11.30.Rd, 11.15.Tk, 24.85.+p

Concept of in-hadron condensates

Gell-Mann Oakes Renner Relation

- Demonstrated algebraically that the so-called Gell-Mann – Oakes – Renner relation is the following statement

$$\forall m_{ud} \sim 0\,,\; m_{\pi^\pm}^2 = m_{ud}^\zeta\, \mathcal{S}_\pi^\zeta(0)\,,$$
$$\mathcal{S}_\pi^\zeta(0) \;=\; -\langle \pi(P)|\tfrac{1}{2}(\bar{u}u + \bar{d}d)|\pi(P)\rangle$$

Namely, the mass of the pion is completely determined by the pion's scalar form factor at zero momentum transfer $Q^2 = 0$.

viz., by ***the pion's scalar charge***

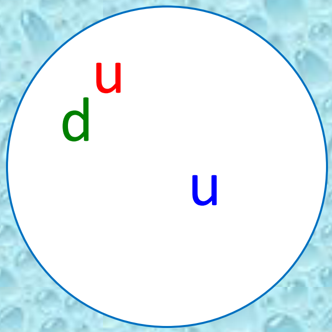
$$\langle H(p')|\bar{q}\mathcal{O}q|H(p)\rangle$$

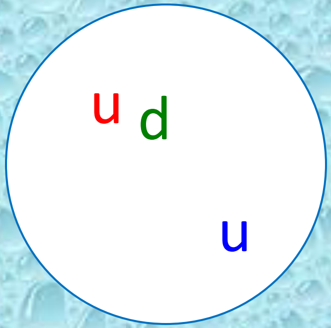
Hadron Charges

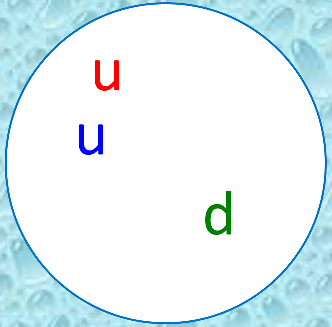
- Matrix elements associated with hadron form factors
- *Scalar charge of a hadron is an intrinsic property of that hadron … no more a property of the vacuum than the hadron’s electric charge, axial charge, tensor charge, etc. …*

“Orthodox Vacuum”

- Vacuum = “frothing sea”
- Hadrons = bubbles in that “sea”, containing nothing but quarks & gluons interacting perturbatively, unless they’re near the bubble’s boundary, whereat they feel they’re trapped!

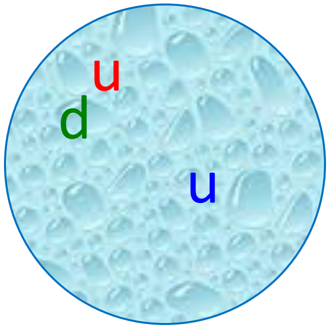


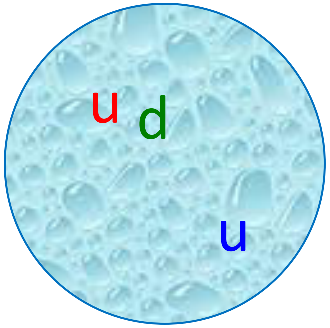


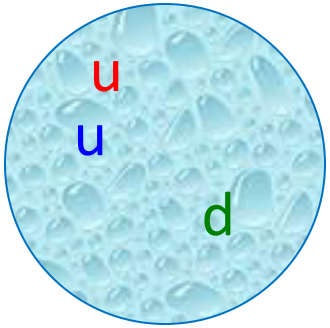


New Paradigm

- Vacuum = perturbative hadronic fluctuations
  but no nonperturbative condensates
- Hadrons = complex, interacting systems
  within which perturbative behaviour is
  restricted to just 2% of the interior







Paradigm shift: In-Hadron Condensates

"Void that is truly empty solves dark energy puzzle"
Rachel Courtland, New Scientist 4th Sept. 2010

"The biggest embarrassment in theoretical physics."

"EMPTY space may really be empty. Though quantum theory suggests that a vacuum should be fizzing with particle activity, it turns out that this paradoxical picture of nothingness may not be needed. A calmer view of the vacuum would also help resolve a nagging inconsistency with dark energy, the elusive force thought to be speeding up the expansion of the universe."

$$\Omega_{QCD-condensates} = 8\pi G_N \frac{\Lambda_{QCD}^4}{3H_0^2} \cong 10^{46} * \text{experiment}$$

Cosmological Constant:

- ✓ ***Putting QCD condensates back into hadrons reduces the mismatch between experiment and theory by a factor of 10^{46}***
- ✓ ***Possibly by far more, if technicolour-like theories are the correct paradigm for extending the Standard Model***


Why are we here?

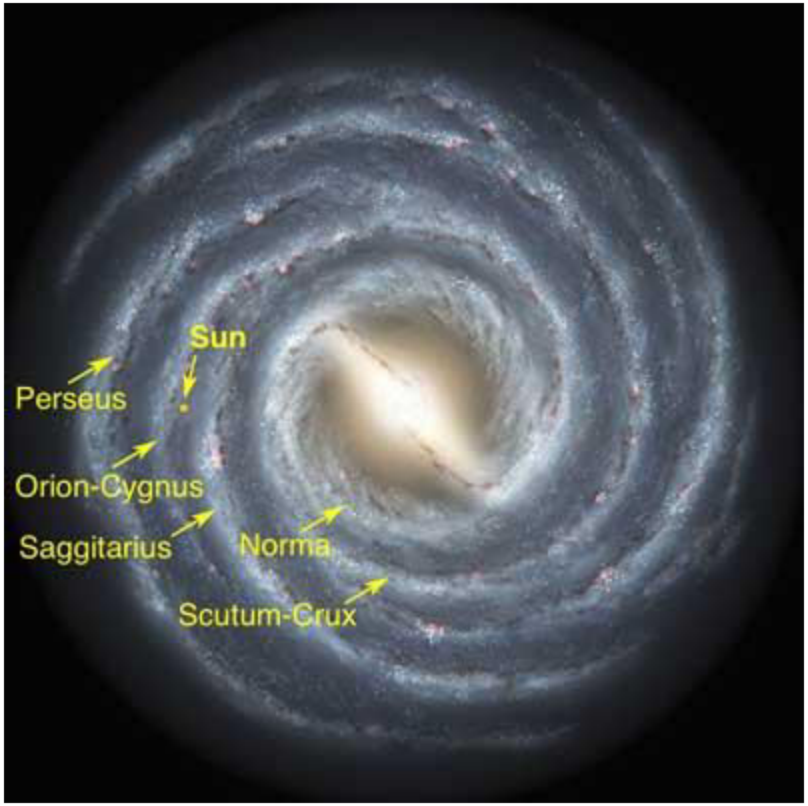
Why are we here?

- *Suppose* quantum field theories
  are the correct paradigm for understanding Nature
- We find ourselves in a Universe where
  time and space give us Four Dimensions ($D = 4$)
- $D = 4$ is a critical point
  - Quantum Field Theories with $D \neq 4$ possess an explicit mass-dimension
  - Couplings are mass-dimensioned, setting scale for all quantities
    - $D > 4$ … uncontrollable ultraviolet divergences
    - $D < 4$ … super-convergent, but hierarchy problem with dynamical effects being < 10% of explicit scale
- Standard Model is built from scale-invariant classical field theories
  (Ignoring Higgs couplings)
  - Such theories are renormalizable
  - Procedure introduces a mass scale
  - The size of the mass-scale is not determined by the theory
- What determines the natural mass-scale for visible matter?
- We know it is $m_{Nature} \approx m_p \approx 1$ GeV
  - How much tolerance exists? … We can exist so long as $m_{Nature} = 1 \pm ?$ GeV

Emergent Phenomena in the Standard Model

Existence of the Universe as we know it depends critically on the following empirical facts:

- Proton is massive, *i.e*. the mass-scale for strong interactions is vastly different to that of electromagnetism
- Proton is absolutely stable, despite being a composite object constituted from three valence quarks
- Pion is unnaturally light (not massless, but lepton-like mass), despite being a strongly interacting composite object built from a valence-quark and valence antiquark



Emergence: low-level rules producing high-level phenomena, with enormous apparent complexity

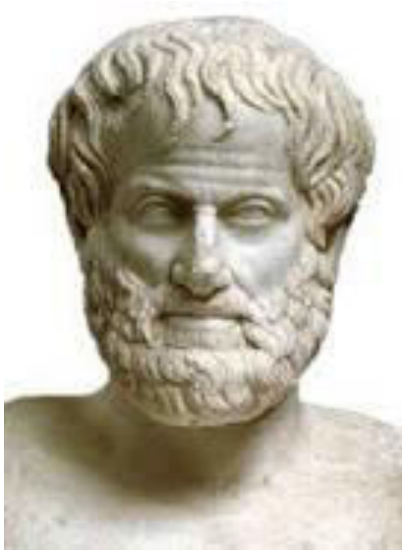
L'ANIMAL-MACHINE

L'**animal-machine** est une hypothèse éthologique selon laquelle les animaux sont des machines. Comme les machines, les animaux seraient des assemblages de pièces et rouages, dénués de conscience ou de pensée. D'un point de vue religieux, l'application du mécanisme à la vie revient à nier l'âme des bêtes qui périssent donc entièrement au moment de leur mort. Poussée à l'extrême, notamment par Nicolas Malebranche, cette conception implique que leurs cris et gémissements ne peuvent être que le reflet de dysfonctionnements dans les « rouages » plutôt que l'expression d'une souffrance. Même si cette vision du problème est complètement décalée par rapport à la vision moderne, elle peine à être délogée par des conceptions plus en adéquation avec les avancées scientifiques récentes.

In the nineteenth century, Descartes was revered for his mechanistic physiology and theory that animal bodies are machines (that is, are constituted by material mechanisms, governed by the laws of matter alone).

Reductionist Perspective

Holism = Emergentism



- Aristotle: 384-322 BC
- Holism:
  - the idea that items can have properties,

    (emergent properties), as a whole that are not explainable from the sum of their parts.
- Summarised concisely (Aristotle):
  - *The whole is more than the sum of its parts*
- The modern discussion of emergence is a return to an ancient debate between partisans of reductionism and holism
- Hegel (Stuttgart 1770 – Berlin 1831):
  - *Das Wahre ist das Ganze (The true is the whole)*

Strong Interactions in the Standard Model

$$\mathcal{L}_{\rm QCD} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} - m\, \delta_{ij}\right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

- Only apparent scale in chromodynamics is mass of the quark field
- Quark mass is said to be generated by Higgs boson.
- In connection with everyday matter, that mass is 1/250$^{\rm th}$ of the natural (empirical) scale for strong interactions,
  viz. more-than two orders-of-magnitude smaller
- Plainly, the Higgs-generated mass is very far removed from the natural scale for strongly-interacting matter
- *Nuclear physics mass-scale* – 1 GeV – is an *emergent feature of the Standard Model*
  - No amount of staring at $L_{\rm QCD}$ can reveal that scale
- Contrast with quantum electrodynamics, *e.g*. spectrum of hydrogen levels measured in units of m_e, which appears in $L_{\rm QED}$

What & where is mass?

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i (\gamma^\mu D_\mu)_{ij} \qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence Mass?

- Classical chromodynamics … non-Abelian local gauge theory
- Remove the current mass … there's no energy scale left
- *No dynamics in a scale-invariant theory*; only kinematics … the theory looks the same at all length-scales … there can be no clumps of anything … *hence bound-states are impossible*.
- *Our Universe can't exist*
- *Higgs boson doesn't solve this problem* …
  - normal matter is constituted from light-quarks
  - the mass of protons and neutrons, the kernels of all visible matter, are 100-times larger than anything the Higgs can produce
- *Where did it all begin?*

  … becomes … Where did it all come from?

Whence Mass?

- Lorentz-invariance is *not* enough … must add the effect of space-time translations, invariance under which guarantees energy and momentum conservation ⇐ *Poincaré invariance*
- Poincaré invariance entails that the Energy-Momentum Tensor is divergence-free, *i.e*. it defines a conserved current:

$$\partial_\mu T_{\mu\nu} = 0$$

$T_{\mu\nu}$ can *always* be made symmetric

- Noether current associated with a global scale transformation:

  $x \to e^{-\sigma} x$

  is the dilation current: $D_{\mu\nu} = T_{\mu\nu} x_\nu$
- In a scale invariant theory, the dilation current is conserved

$$\begin{aligned}\partial_\mu \mathcal{D}_\mu = 0 &= [\partial_\mu T_{\mu\nu}] x_\nu + T_{\mu\nu}\delta_{\mu\nu} \\ &= T_{\mu\mu} \,,\end{aligned}$$

- Consequently, in a scale invariant theory

  the *energy-momentum tensor must be traceless*.

Trace Anomaly

- Classically, in a scale invariant theory

  the *energy-momentum tensor must be traceless:* $T_{\mu\mu} \equiv 0$
- Classical chromodynamics is meaningless … must be quantised
- Regularisation and renormalisation of (ultraviolet) divergences introduces a mass-scale

  … *dimensional transmutation*: mass-dimensionless quantities become dependent on a mass-scale, ζ
- $\alpha \to \alpha(\zeta)$ in QCD's (massless) Lagrangian density, `L(m=0)`

  QCD β *function*

  Under a scale transformation $\zeta \to e^{\sigma} \zeta$, then $\alpha \to \sigma\, \alpha\beta(\alpha)$

  Trace anomaly

  $$\mathbf{L} \to \sigma\, \alpha\beta(\alpha)\, d\mathbf{L}/d\alpha$$

  $$\Rightarrow \partial_\mu \mathbf{D}_\mu = \delta\mathbf{L}/\delta\sigma = \alpha\beta(\alpha)\, d\mathbf{L}/d\alpha = \boxed{\beta(\alpha)\, \tfrac{1}{4} G_{\mu\nu} G_{\mu\nu} = T_{\rho\rho} =: \Theta_0}$$
- Straightforward, nonperturbative derivation, without need for diagrammatic analysis …

  Quantisation of renormalisable four-dimensional theory forces nonzero value for trace of energy-momentum tensor

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- The trace anomaly is not peculiar to QCD
- Quantum electrodynamics (QED) … the first real quantum field theory … also exhibits a trace anomaly
  - Form is the same
  - But, empirically, the scale is vastly different

- Can we understand Why?
  - Is there a theoretical explanation?
  - Can it be verified empirically?

$$\mathcal{L}_{\mathrm{QCD}} = \bar{\psi}_i \left(i(\gamma^\mu D_\mu)_{ij} \qquad\qquad \right) \psi_j - \frac{1}{4} G^a_{\mu\nu} G^{\mu\nu}_a$$

Whence?

- Classical chromodynamics … non-Abelian local gauge theory
- Local gauge invariance; but there is no confinement without a mass-scale
  - Three quarks can still be colour-singlet
  - Colour rotations will keep them colour singlets
  - But they need have no proximity to one another
    … proximity is meaningless in a scale-invariant theory
- Whence mass … equivalent to whence a mass-scale … equivalent to whence a confinement scale
- *Understanding the origin of mass in QCD is quite likely inseparable from the task of understanding confinement.*

Where is the mass?

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

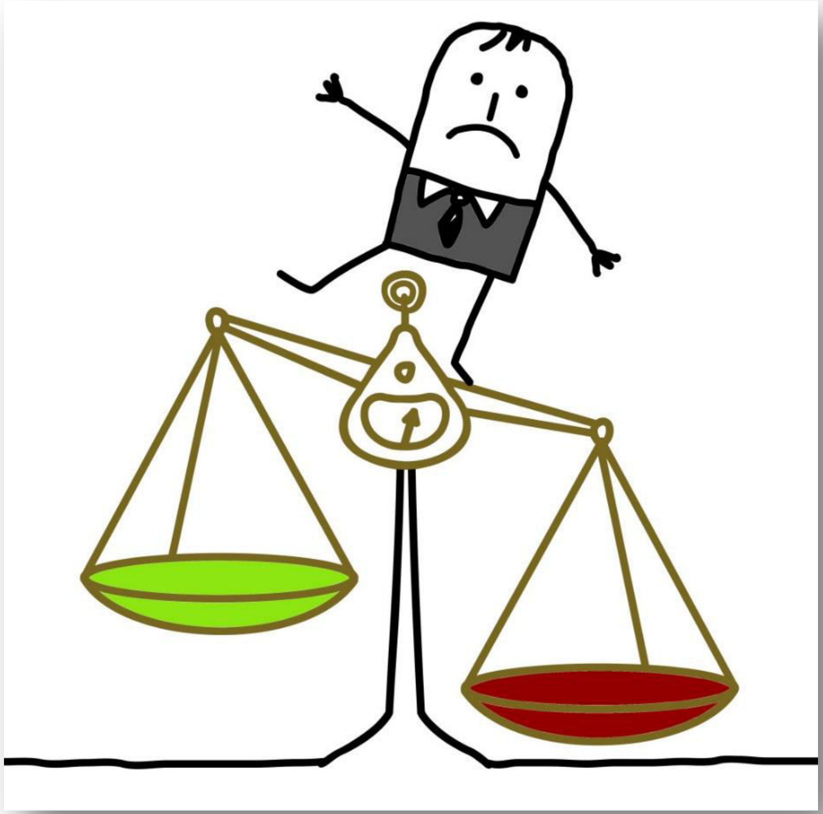
- Knowing that a trace anomaly exists does not deliver a great deal
  … Indicates only that a mass-scale must exist
- Can one compute and/or understand the magnitude of that scale?
- One can certainly *measure* the magnitude … consider proton:

$$\langle p(P)|T_{\mu\nu}|p(P)\rangle = -P_\mu P_\nu$$

$$\langle p(P)|T_{\mu\mu}|p(P)\rangle = -P^2 = m_p^2$$

$$= \langle p(P)|\Theta_0|p(P)\rangle$$

- In the chiral limit the entirety of the proton's mass is produced by the trace anomaly, Θ_0
  … In QCD, Θ_0 measures the strength of gluon self-interactions
  … so, from one perspective, m_p is completely generated by glue.



On the other hand …

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Trace Anomaly

- In the chiral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- Does this mean that the scale anomaly vanishes trivially in the pion state, *i.e*. gluons contribute nothing to the pion mass?
- Difficult way to obtain "zero"!
- Easier to imagine that "zero" owes to cancellations between different operator contributions to the expectation value of Θ_0.
- Of course, such precise cancellation should not be an accident.

  It could only arise naturally because of some symmetry and/or symmetry-breaking pattern.

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2 \,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "How does the mass of the proton arise?"

  is complete without the additional clause

  "How does the pion remain **massless***?"*

- Natural visible-matter mass-scale must emerge simultaneously with apparent preservation of scale invariance in related systems
  - Expectation value of Θ_0 in pion is always zero, irrespective of the size of the natural mass-scale for strong interactions = m_p

Whence "1" and yet "0" ?

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \qquad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- *No statement of the question*

  "How does the mass of the proton arise?"

  is complete without the additional clause

  *"How does the pion remain **massless**?"*

- Natu … usly
  with … stems
  - Ex … ve of
    th … $= m_p$

Elucidate the entire array
of empirical consequences
of the mechanism responsible
so that the theory can be validated

$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

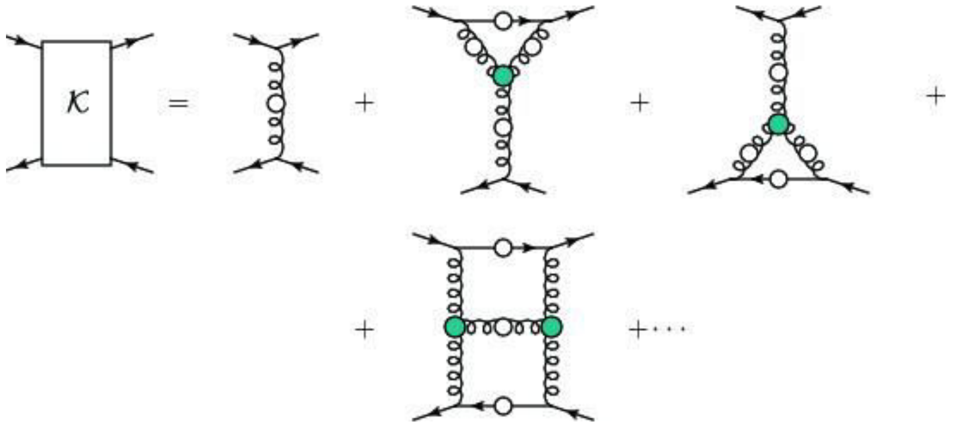
$$\langle p(P)|\Theta_0|p(P)\rangle = m_p^2\,, \quad \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

Whence "0" ?

The answer is algebraic

Pion masslessness

- Pion's Poincaré-invariant mass and Poincaré-covariant wave function are obtained by solving a Bethe-Salpeter equation.
- This is a scattering problem
- In chiral limit
  - two massless fermions interact via exchange of massless gluons
    … initial system is massless;
    … and it remains massless at every order in perturbation theory
- But, complete the calculation using an enumerable infinity of dressings and scatterings



- Then …

Munczek, H. J., Phys. Rev. D ***52*** *(1995) pp. 4736-4740*
Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B ***380*** *(1996) pp. 7-12*
Maris, P. , Roberts, C.D. and Tandy, P.C., Phys. Lett. B ***420*** *(1998) pp. 267-273*
Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D ***93*** *(2016) 096010/1-7*

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations
  - Bethe-Salpeter Kernel:
    - valence-quarks with a momentum-dependent running mass produced by self-interacting gluons, which have given themselves a running mass
    - Interactions of arbitrary but enumerable complexity involving these “basis vectors”
  - Chiral limit:
    - Algebraic proof
      - at any & each finite order in symmetry-preserving construction of kernels for
        - » the gap (quark dressing)
        - » and Bethe-Salpeter (bound-state) equations,
      - there is a precise cancellation between
        - » mass-generating effect of dressing the valence-quarks
        - » and attraction introduced by the scattering events
    - Cancellation guarantees that
      - simple system, which began massless,
      - becomes a complex system, with
        - » a nontrivial bound-state wave function
        - » attached to a pole in the scattering matrix, which remains at $P^2=0$ …
    - Interacting, bound system remains massless!

*Munczek, H. J., Phys. Rev. D **52** (1995) pp. 4736-4740*
*Bender, A., Roberts, C.D. and von Smekal, L., Phys. Lett. B **380** (1996) pp. 7-12*
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*Binosi, Chang, Papavassiliou, Qin, Roberts, Phys. Rev. D **93** (2016) 096010/1-7*

Pion masslessness

- Obtain a coupled set of gap- and Bethe-Salpeter equations

Quantum field theory statement:

In the pseudsocalar channel, the dynamically generated mass of the two fermions is precisely cancelled by the attractive interactions between them – iff –

$$f_\pi E_\pi(p^2) = B(p^2)$$

- Cancellation guarantees that

$$\Rightarrow 2 M_q + U_g \equiv 0$$

at $P^2=0$ …

- Interacting, bound system remains massless

Pion masslessness

DCSB Paradigm

$$\langle\pi(q)|\theta_0|\pi(q)\rangle \stackrel{\zeta\gg\zeta_2}{=} \langle\pi(q)|\tfrac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}|\pi(q)\rangle$$

$$\stackrel{\zeta\simeq\zeta_2}{\to} \langle\tilde{\pi}(q)|\sum_{f=u,d} M_f(\zeta)\,\bar{\mathcal{Q}}_f(\zeta)\mathcal{Q}_f(\zeta) + \tfrac{1}{4}[\beta(\alpha(\zeta))\mathcal{G}^a_{\mu\nu}\mathcal{G}^a_{\mu\nu}]_{2PI}|\tilde{\pi}(q)\rangle$$

- Parton-basis chiral-limit expression of the expectation-value of the trace-anomaly in the pion at $\zeta \gg \zeta_2$
- Metamorphoses into a new expression, written in terms of a nonperturbatively-dressed quasi-particle basis and associated, evolved wave functions
  - 1st term = *positive* = one-body dressing content of the trace anomaly … Plainly, a massless valence-quark acquiring a large mass through interactions with its own gluon field is an expression of the trace-anomaly in the one-body subsector of the complete pion wave function
  - 2nd term = *negative* (attraction) = 2-particle-irreducible scattering event content of the scale-anomaly … Plainly, acquires a scale because the couplings, and the gluon- and quark-propagators in the 2PI processes have all acquired a mass-scale
- Away from the chiral limit, and in other channels, the cancellation is incomplete.

$$T_{\mu\mu} = \frac{1}{4}\beta(\alpha(\zeta))G^a_{\mu\nu}G^a_{\mu\nu}$$

Massless Pion

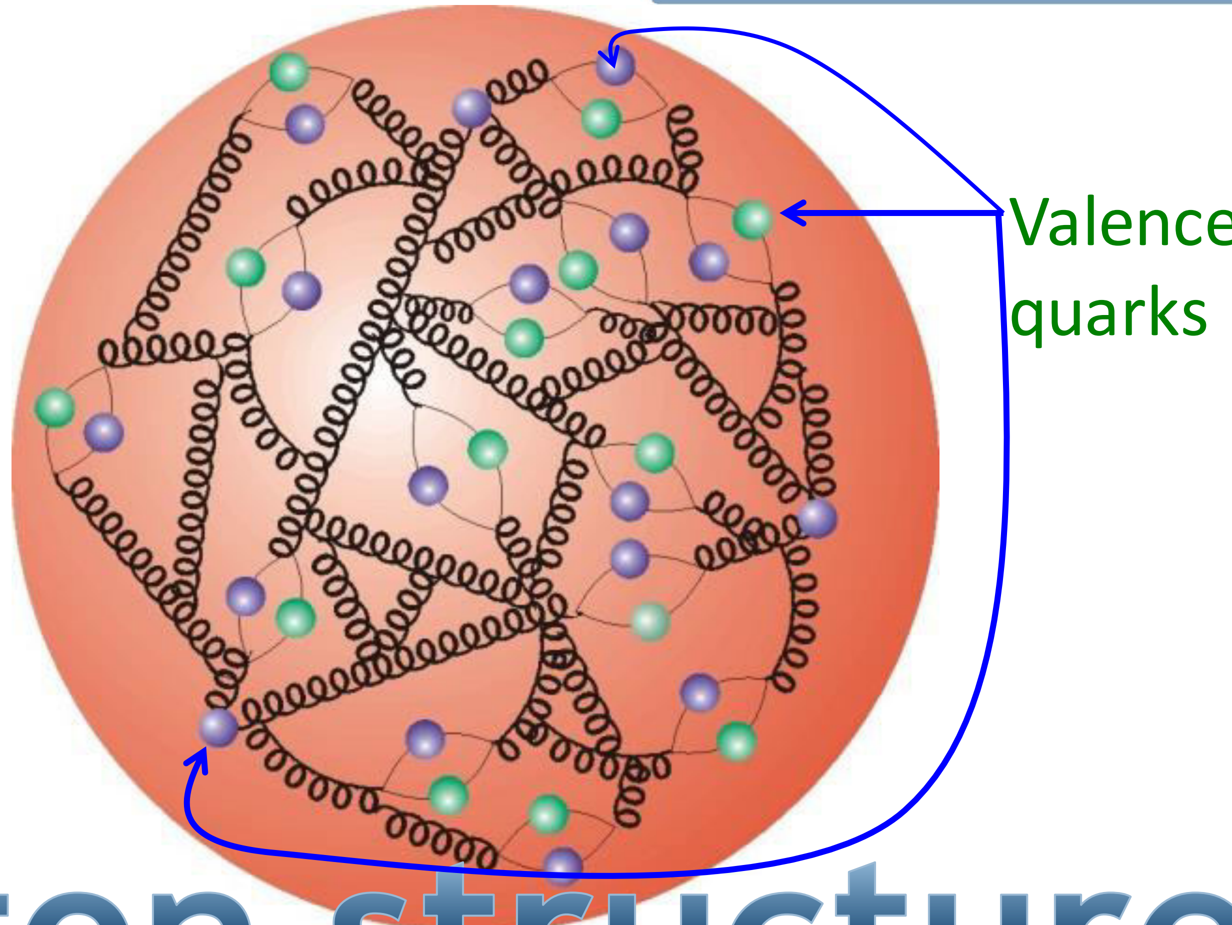
- Chiral limit

$$\langle \pi(q)|T_{\mu\nu}|\pi(q)\rangle = -q_\mu q_\nu \;\Rightarrow\; \langle \pi(q)|\Theta_0|\pi(q)\rangle = 0$$

- "Zero" owes to cancellations between different operator-component contributions to the expectation value of Θ_0.
- The cancellations are precise

  Arising naturally because chiral symmetry – the apparent masslessness of the QCD action – is broken by strong dynamics in a very particular manner.

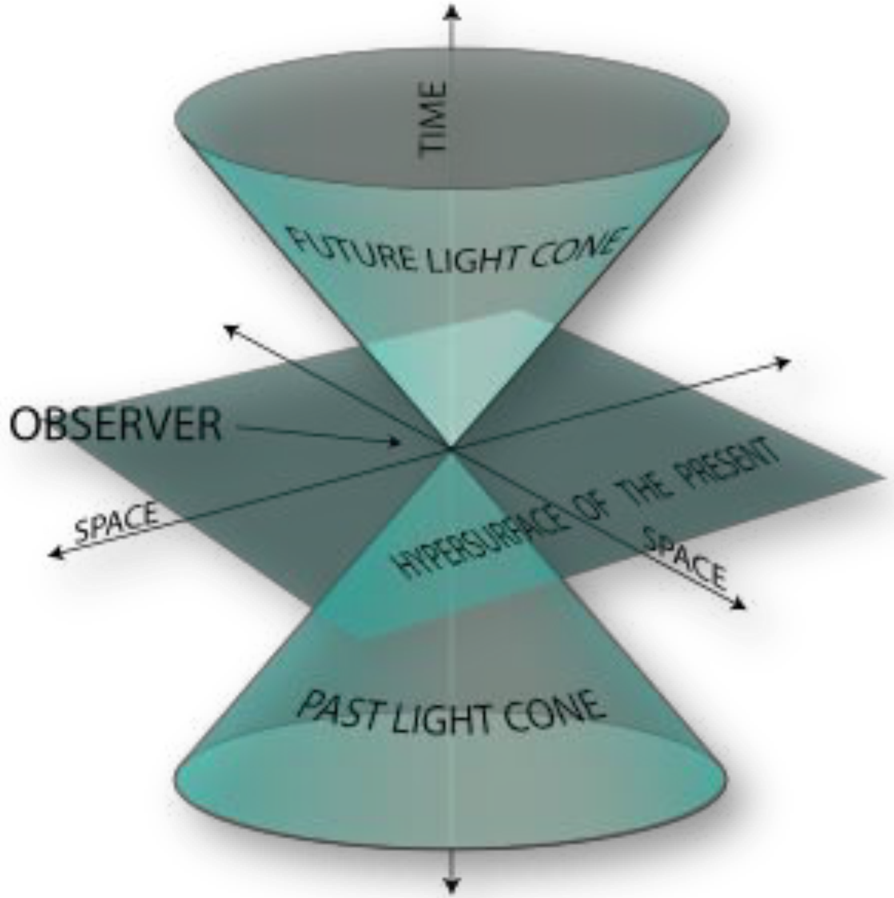
- In the chiral limit, the pion is massless irrespective of the magnitude of any other hadron's mass



Parton structure of hadrons

Parton Structure of Hadrons

- Valence-quark structure of hadrons
  - Definitive of a hadron.
    After all, it's how we distinguish a proton from a neutron
  - Expresses charge; flavour; baryon number; and other Poincaré-invariant macroscopic quantum numbers
  - Via evolution, determines background at LHC
- Foreseeable future will bring precision experimental study of (far) valence region, and theoretical computation of distribution functions and distribution amplitudes
  - *Computation is critical*
  - *Without it, no amount of data will reveal anything about the theory underlying the phenomena of strong interaction physics*



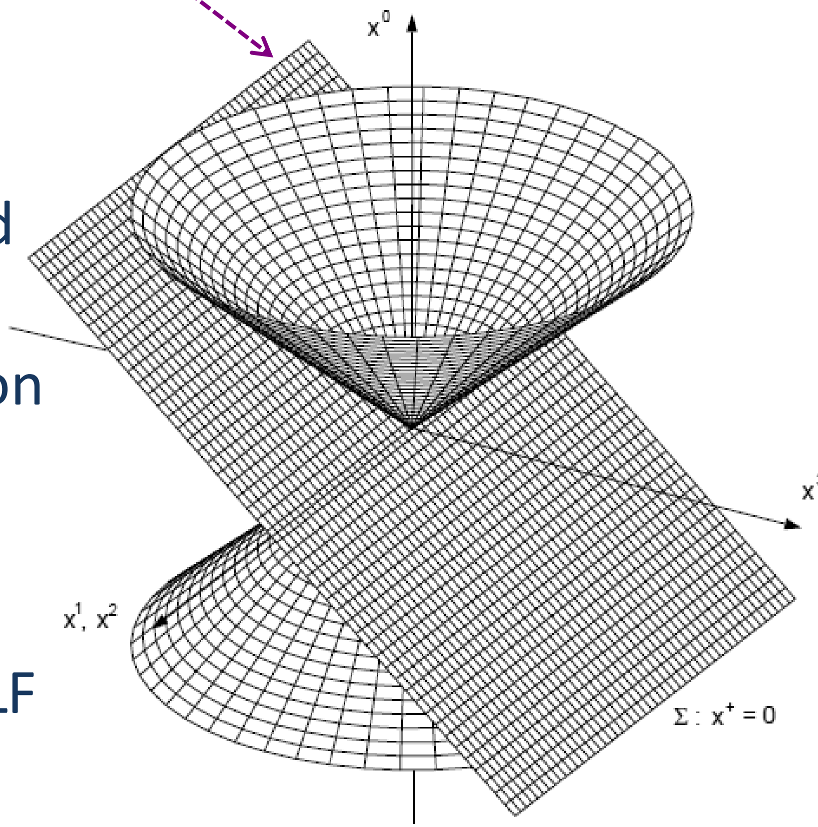
Why light-front?

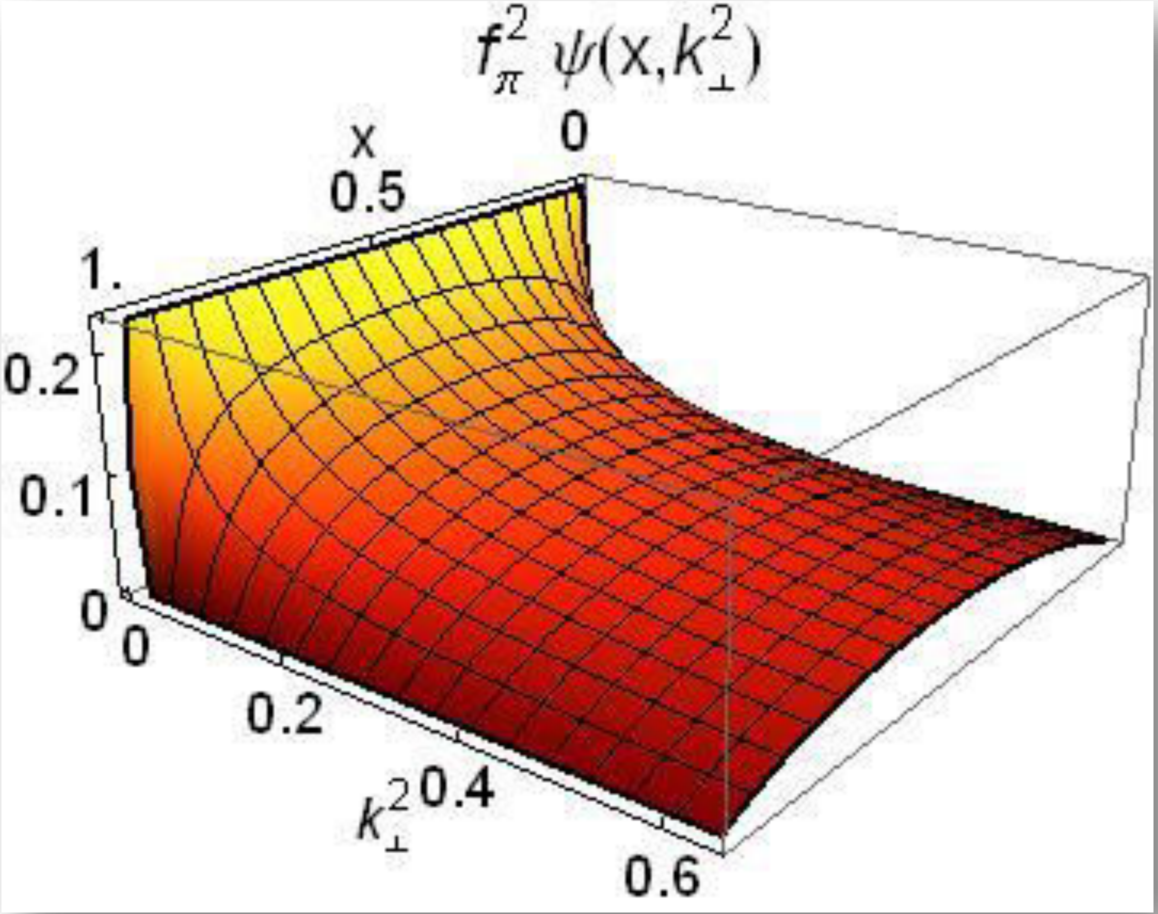
Light-front Quantisation

- Hamiltonian formulation of quantum field theory.
  - Fields are specified on a particular initial surface:

    Light front $x^+ = x^0 + x^3 = 0$

- Using LF quantisation:
  - ✓ quantum mechanics-like wave functions can be defined;
  - ✓ quantum-mechanics-like expectation values can be defined and evaluated
  - ✓ Parton distributions are correlation functions at equal LF-time x^+ ; namely, within the initial surface $x^+ = 0$, and can thus be expressed directly in terms of ground state LF wavefunctions





Pion's Wave Function

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Methods were developed that enable direct computation of the pion's light-front wave function
- $\varphi_\pi(x)$ = twist-two parton distribution amplitude = projection of the pion's Poincaré-covariant wave-function onto the light-front

$$\varphi_\pi(x) = Z_2 \mathrm{tr}_{CD} \int \frac{d^4k}{(2\pi)^4} \delta(n \cdot k - x n \cdot P)\, \gamma_5 \gamma \cdot n\, S(k) \Gamma_\pi(k;P) S(k-P)$$

- Results have been obtained with rainbow-ladder DSE kernel, simplest symmetry preserving form; and the best DCSB-improved kernel that is currently available.

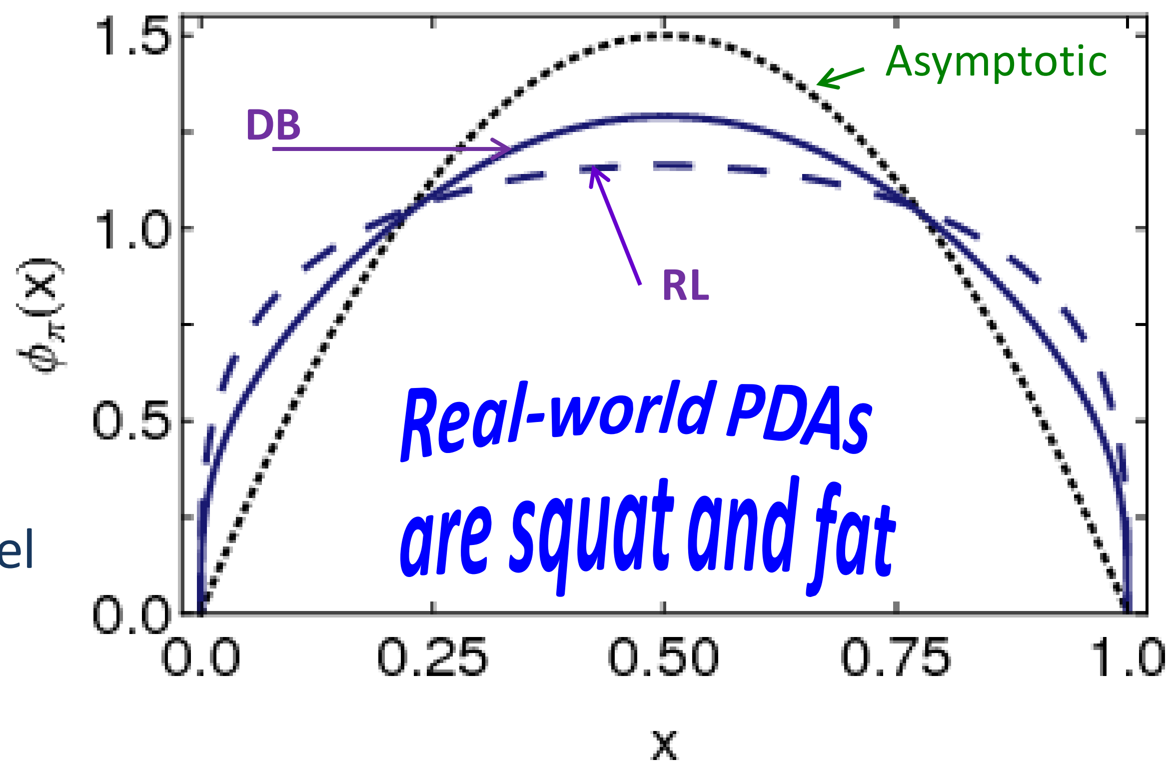
$x^\alpha\,(1-x)^\alpha$, with $\alpha=0.5$

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

- Both kernels agree: marked broadening of $\varphi_\pi(x)$, which owes to DCSB

- This may be claimed because PDA is computed at a low renormalisation scale in the chiral limit, whereat the quark mass function owes entirely to DCSB.
- Difference between RL and DB results is readily understood: $B(p^2)$ is more slowly varying with DB kernel and hence a more *balanced* result



Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, et al., arXiv:1301.0324 [nucl-th], Phys. Rev. Lett. **110** (2013) 132001 (2013) [5 pages].

Pion's valence-quark Distribution Amplitude

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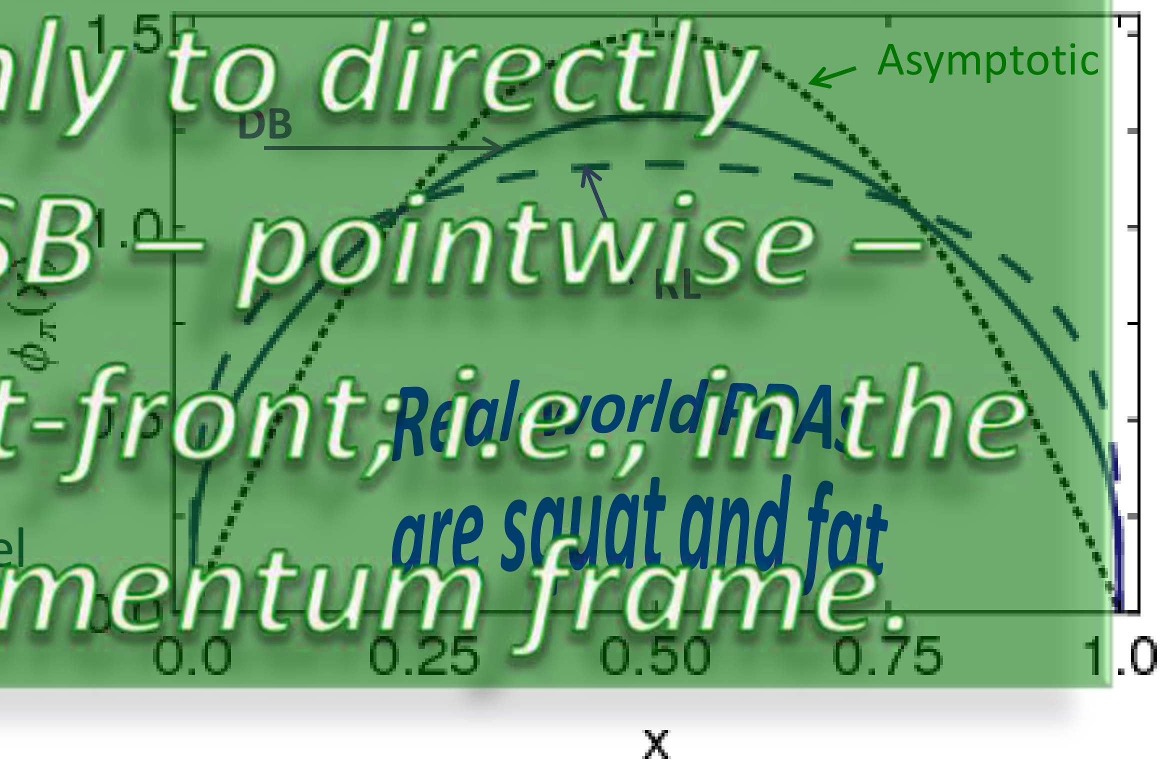
These computations are the first and only to directly expose DCSB – pointwise – on the light-front; i.e., in the infinite momentum frame.

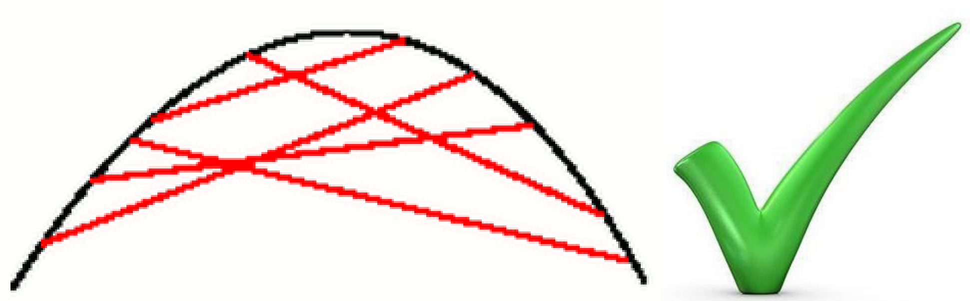


arXiv:1301.0324 [nucl-th], arXiv:1306.2645 [nucl-th], arXiv:1311.1390 [nucl-th], arXiv:1405.0289 [nucl-th], arXiv:1406:3353 [nucl-th]

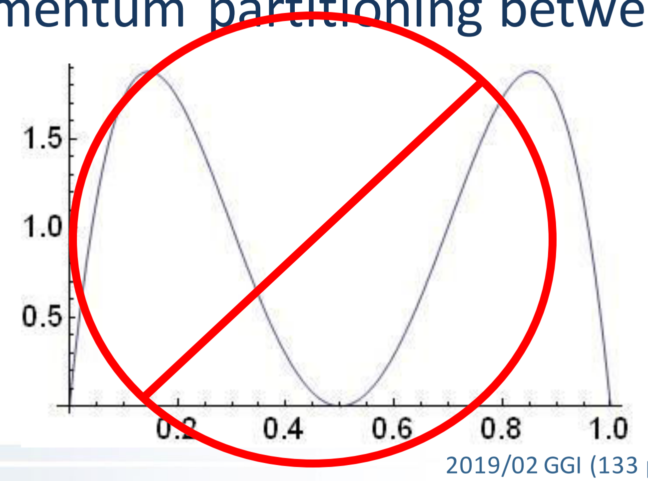
Features of Ground-state PDAs

- A diverse array of studies since Caraguatatuba (2012) have shown that ground-state meson PDAs are broad, concave functions

Concave function: no line segment lies above any point on the graph

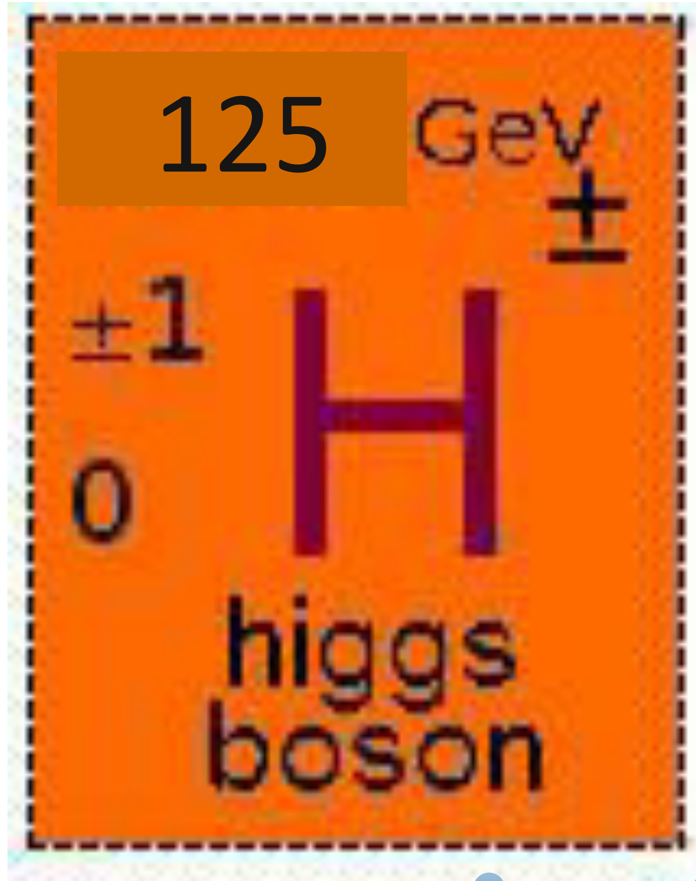


- Camel-humped distributions – popular for many years – are physically unreasonable because they correspond to bound-state amplitudes that disfavour equal momentum partitioning between valence-quark degrees of freedom



Leading-twist PDAs of *S*-wave light-quark mesons

- End of a *long* story (longer than 30 years war)
- Continuum predictions that pion and kaon PDAs are broad, concave functions confirmed by simulations of lattice-regularised QCD
  - *Pion Distribution Amplitude from Lattice QCD*, Jian-Hui Zhang *et al.*, Phys.Rev. D95 (2017) 094514; 1702.00008
  - *Kaon Distribution Amplitude from Lattice QCD and the Flavor SU(3) Symmetry*, Jiunn-Wei Chen *et al.*, arXiv:1712.10025 [hep-ph]
  - *Pion and kaon valence-quark parton quasidistributions*, S.-S. Xu, L. Chang *et al*. Phys. Rev. D**97** (2018) 094014; arXiv:1802.09552 [nucl-th]
- Continuum analyses predict that these properties characterise the leading-twist PDAs of ***all*** *S*-wave light-quark mesons
- ***Many empirically verifiable predictions***

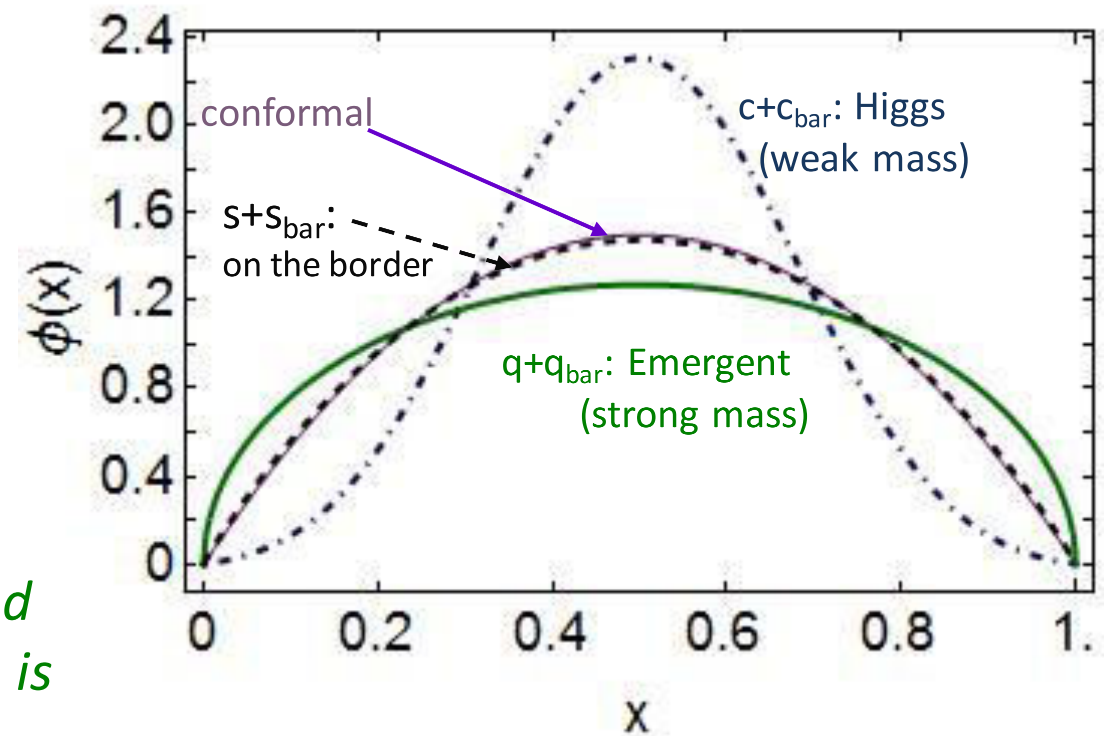


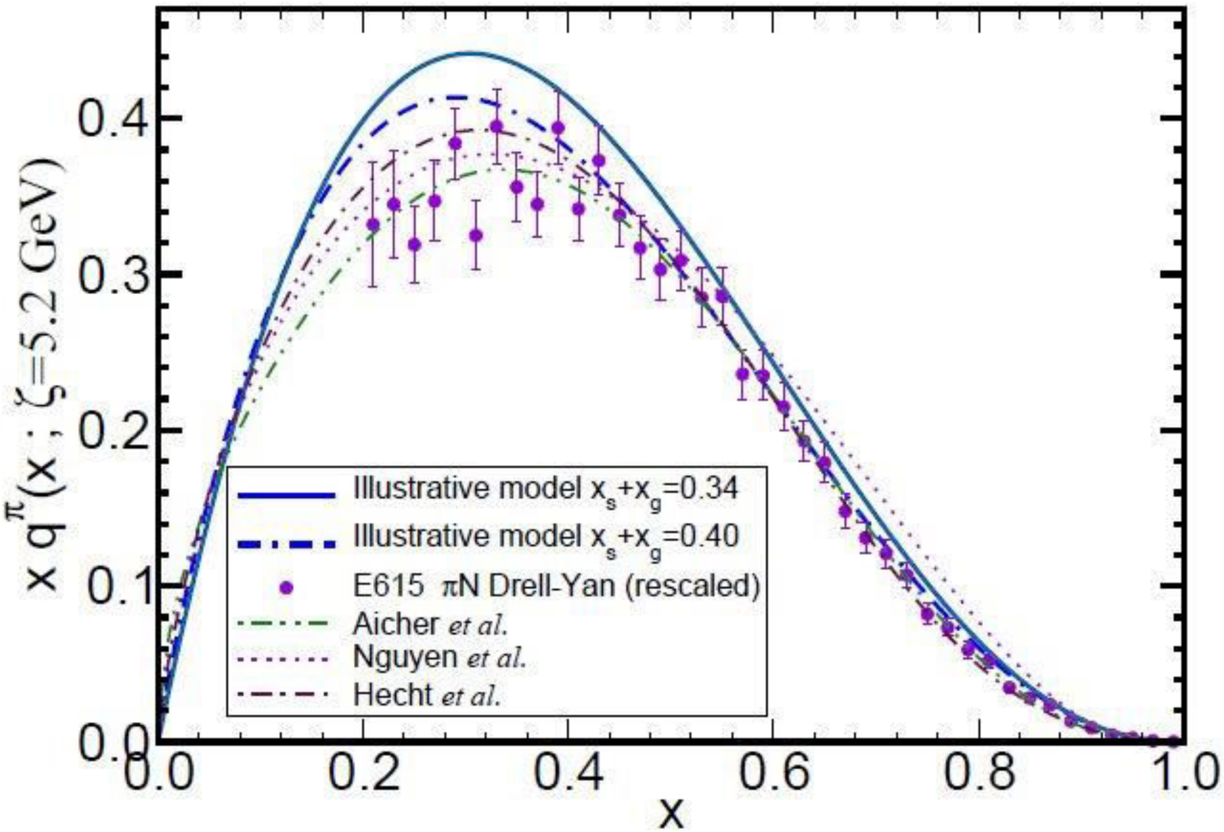
Where is it?

Parton distribution amplitudes of S-wave heavy-quarkonia
Minghui Ding, Fei Gao, Lei Chang, Yu-Xin Liu and Craig D. Roberts
arXiv:1511.04943 [nucl-th], Phys. Lett. B **753** (2016) pp. 330-335

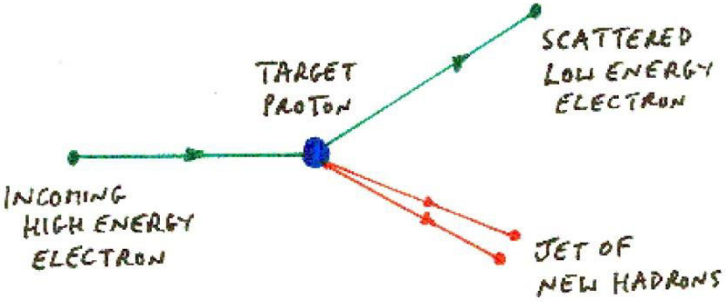
Emergent Mass *vs.* Higgs Mechanism

- When does Higgs mechanism begin to influence mass generation?
- limit $m_{quark} \to \infty$
  $\varphi(x) \to \delta(x-½)$
- limit $m_{quark} \to 0$
  $\varphi(x) \sim (8/\pi)\,[x(1-x)]^{½}$
- Transition boundary lies just above $m_{strange}$
- *Comparison between distributions of light-quarks and those involving strange-quarks is good place to seek signals for strong-mass generation*





π & K Valence-quark Distribution Functions

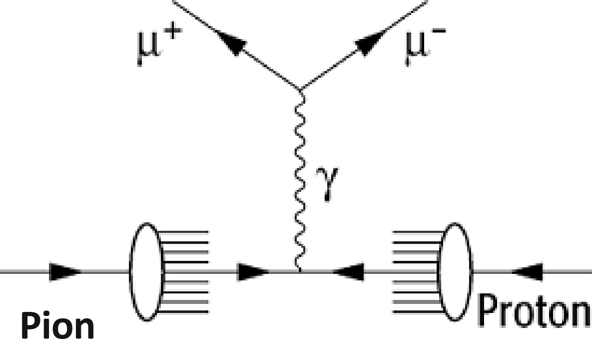


Deep inelastic scattering

- Quark discovery experiment at SLAC
  (1966-1978, Nobel Prize in 1990)
- Completely different to elastic scattering
  - *Blow the target to pieces instead of keeping only those events where it remains intact.*
- Cross-section is interpreted as a measurement of the momentum-fraction probability distribution for quarks and gluons within the target hadron: $q(x)$, $g(x)$

Probability that a quark/gluon within the target will carry a fraction x of the bound-state's light-front momentum

Distribution Functions of the Nucleon and Pion in the Valence Region, Roy J. Holt and Craig D. Roberts, arXiv:1002.4666 [nucl-th], Rev. Mod. Phys. **82** (2010) pp. 2991-3044



Empirical status of the Pion's valence-quark distributions

- Owing to absence of pion targets, the pion's valence-quark distribution functions are measured via the Drell-Yan process:

  $$\pi\, p \rightarrow \mu^+ \mu^- X$$

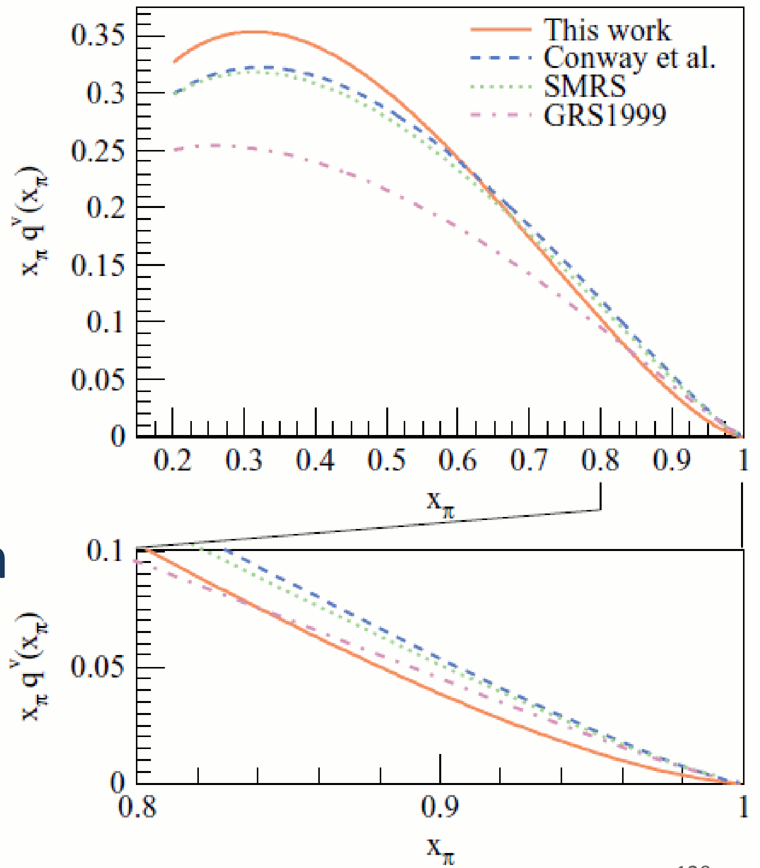
- Three experiments: CERN (1983 & 1985) and FNAL (1989). No more recent experiments because theory couldn't even explain these!
- Problem

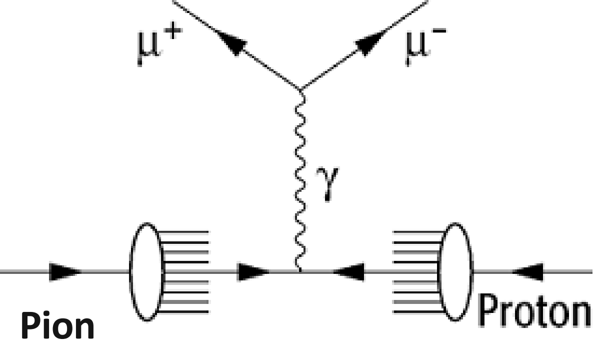
  Conway *et al*. Phys. Rev. D **39**, 92 (1989)
  Wijesooriya *et al*. Phys.Rev. C **72** (2005) 065203

  Behaviour at large-*x* inconsistent with pQCD; viz,

  $$\text{expt. } (1-x)^{1+\varepsilon}$$

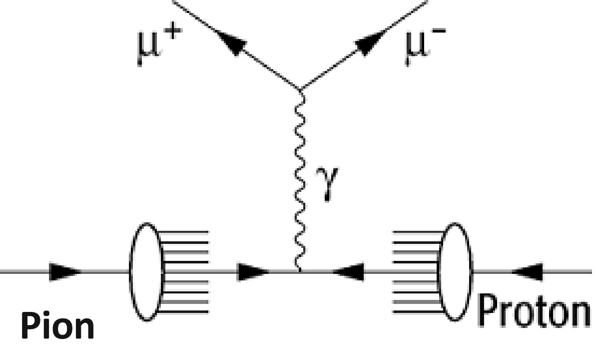
  $$\text{cf. QCD } (1-x)^{2+\gamma}$$





Models of the Pion's valence-quark distributions

- *$(1-x)^\beta$ with $\beta=0$ (i.e., a* constant – any fraction is equally probable!)
  - Nambu–Jona-Lasinio models, when a translationally invariant regularization is used
- *$(1-x)^\beta$ with $\beta=1$*
  - *Nambu*–Jona-Lasinio NJL models with a hard cutoff
  - AdS/QCD models using light-front holography
  - Duality arguments produced by some theorists
- *$(1-x)^\beta$ with $0<\beta<2$*
  - Relativistic constituent-quark models, with power-law depending on the form of model wave function
- *$(1-x)^\beta$ with $1<\beta<2$*
  - Instanton-based models, all of which have incorrect large-k^2 behaviour



Models of the Pion's valence-quark distributions

- $(1-x)^\beta$ *with* $\beta=0$ (*i.e., a* constant – any fraction is equally probable!)
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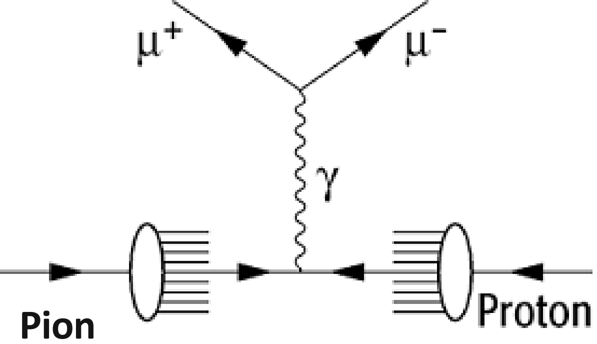
Completely unsatisfactory. Impossible to suggest that there's even qualitative agreement!



DSE prediction of the Pion's valence-quark distributions

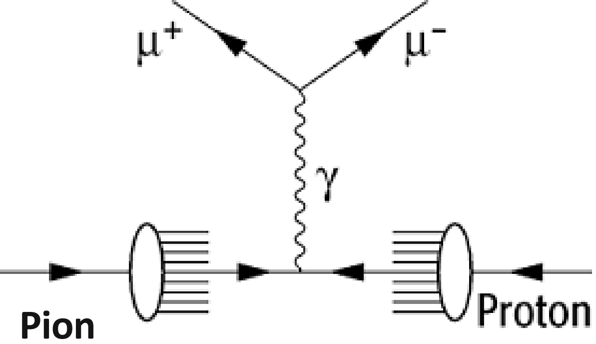
- Consider a theory in which quarks scatter via a vector-boson exchange interaction whose $k^2 >> m_G^2$ behaviour is $(1/k^2)^\beta$,
- Then at a resolving scale Q_0

$$u_\pi(x;Q_0) \sim (1-x)^{2\beta}$$

namely, the large-x behaviour of the quark distribution function is a direct measure of the momentum-dependence of the underlying interaction.

- In QCD, β=1 and hence

$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$



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- In QCD, β=1 and hence

$$^{QCD}u_\pi(x;Q_0) \sim (1-x)^2$$

Completely unambigous! Direct connection between experiment and theory, empowering both as tools of discovery.

π & K PDFs

- Extant data on π & K PDFs (mesonic Drell-Yan) is old: 1980-1989
- New data would be welcome:
  - persistent doubts about the Bjorken-$x \simeq 1$ behaviour of the pion's valence-quark PDF
  - single modest-quality measurement of $u^K(x)/u^\pi(x)$ cannot be considered definitive.
- Approved experiments, using tagged DIS at JLab 12, should contribute to a resolution of pion question and will also address the kaon PDFs
- Future:
  - new mesonic Drell-Yan measurements at modern facilities could yield valuable information on π and K PDFs (COMPASS),
  - as could two-jet experiments at the large hadron collider;
  - EIC would be capable of providing access to π and K PDFs through measurements of forward nucleon structure functions.
- Gribov-Lipatov reciprocity (crossing symmetry) entails connection between PDFs and fragmentation functions on $z \simeq 1$ ($z \geq 0.75$)

$$D_{H/q}(z) \approx z\, q^H(z)$$

Reliable information on meson fragmentation functions is critical if the worldwide programme aimed at determining TMDs is to be successful

Basic features of the pion valence-quark distribution function,
Lei Chang, Cédric Mezrag, et al.,
arXiv:1406:5450 [nucl-th], Phys. Lett. B **737** (2014)pp. 23–29

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang et al.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

Valence-quark PDFs within mesons

- Compute PDFs from imaginary part of virtual-photon – pion forward Compton scattering amplitude:

  $\gamma\,\pi \to \gamma\,\pi$

- Handbag diagram is insufficient. Doesn't even preserve global symmetries. Exists a class of leading-twist corrections that remedies this defect ⇒

Projection onto light-front

$$u_V^\pi(x) = N_c \mathrm{tr} \int_{dk} \delta_n^x(k_\eta^\pi)$$

Partial derivative wrt relative momentum

$$\times\, \partial_{k_\eta^\pi} \left[\Gamma_\pi(k_\eta^\pi, -P_\pi) S(k_\eta^\pi)\right] \Gamma_\pi(k_{\bar\eta}^\pi, P_\pi)\, S(k_{\bar\eta}^\pi)\,,$$

Similar expressions for $u_V^K(x)$, $s_V^K(x)$

Measurable quantities
Directly related to
dynamically generated quark masses
& bound-state wave functions

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Valence-quark PDFs within mesons

- Formulae guarantee that valence-quark PDFs satisfy, independent of any and all structural details:

$$\langle x\rangle_u^0 = \int_0^1 dx\, x\, u_V^0(x) = \frac{1}{2}$$

$$\int_0^1 dx\, x[u_V^K(x) + \bar{s}_V^K(x)] = 1$$

- Algebraic proof that at an hadronic scale $\zeta \approx 0.5$ GeV

$$q_V^M(x \simeq 1) \propto (1-x)^{2n}$$

in any theory with $(1/k^2)^n$ vector-boson exchange interaction

Pion:
DSE comparison with lQCD moments

- lQCD studies agree with each other, within errors

  [66]: Brommel et al. (2007)

  [67]: Best *et al*. (1997)

  [68]: Detmold *et al*. (2003)
- DSE and lQCD agree, within errors; and DSE at level of 4% with lQCD-average
- *On light-front, just 52% of the pion's momentum is carried by valence-quarks at ζ_2 = 2GeV, down from 65% at ζ_H= 0.51GeV*

moments. Such results are available for $u^\pi(x)$, *e.g.* a contemporary simulation [66], using two dynamical fermion flavours, $m_\pi \gtrsim 0.34\,\text{GeV}$ and nonperturbative renormalisation at $\zeta_2 = 2\,\text{GeV}$, produces the first row here:

	$\langle x\rangle_u^\pi$	$\langle x^2\rangle_u^\pi$	$\langle x^3\rangle_u^\pi$
[66]	0.27(1)	0.13(1)	0.074(10)
[67]	0.28(8)	0.11(3)	0.048(20)
[68]	0.24(2)	0.09(3)	0.053(15)
average	0.26(8)	0.11(4)	0.058(27)
herein	0.26	0.11	0.052

(37)

The results in Ref. [66] agree with those obtained in earlier estimates based on simulations of quenched lQCD [67, 68] and are consistent with the values obtained from our computed distribution, which are reported in the last row of Eq. (37).

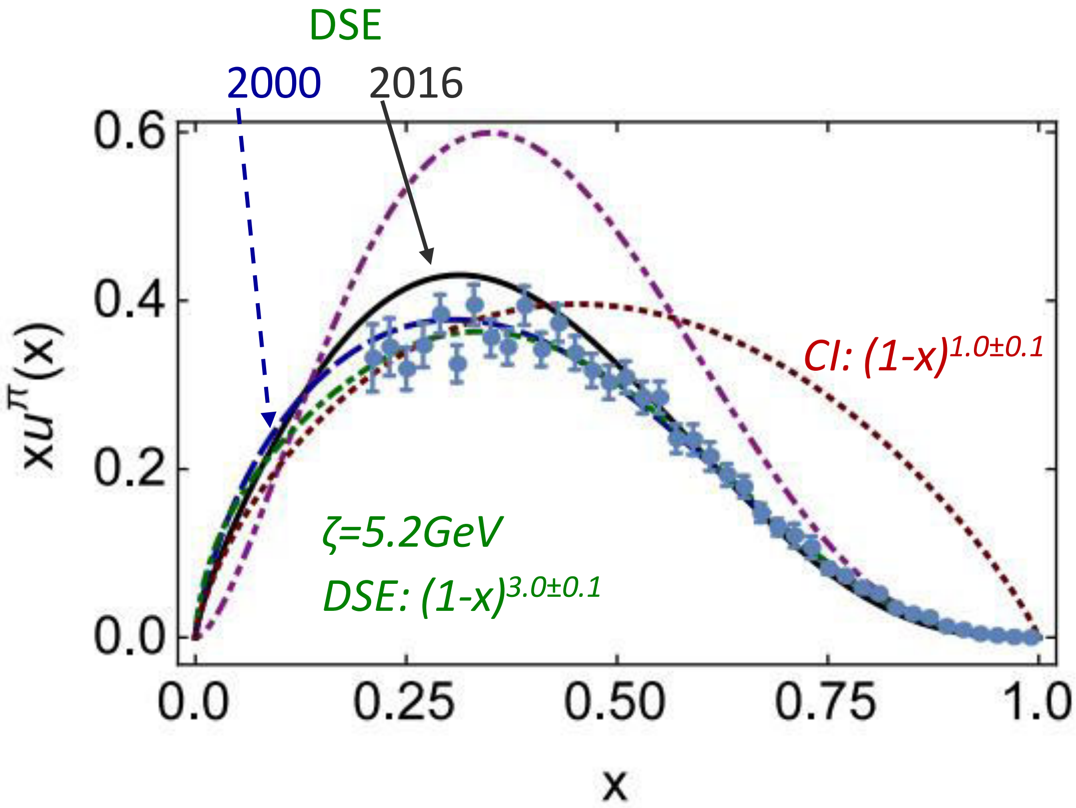


FIG. 3. $xu^{\pi}(x;\zeta_{5.2})$. Solid (black) curve, our prediction, expressed in Eqs. (32), (33); dot-dot-dashed (purple) curve, result obtained when sea-quark and gluon contributions are neglected at ζ_H, *i.e.* using $u_V^{\pi}(x)$ from Eqs. (14), (17); dashed (blue) curve first DSE prediction [38]; and data, Ref. [4], rescaled according to the reanalysis described in Ref. [40], from which the dot-dashed (green) curve is drawn. The dotted (red) curve is the result obtained using a Poincaré-covariant regularisation of a contact interaction, Eq. (36).

Pion PDF

- Purple dot-dot-dash = prediction at ζ_H
- Data = modern reappraisal of E615: NLO analysis plus ***soft-gluon resummation*** (ASV)

QCD demands it
& phenomenology should respect that

- Solid black curve, prediction evolved to ζ=5.2GeV, the scale associated with the experiments
- Blue dashed curve = first DSE prediction, in 2000 (ζ=5.2GeV)
- Dotted red curve = result obtained with momentum-independent gluon exchange (contact interaction, ζ=5.2GeV)

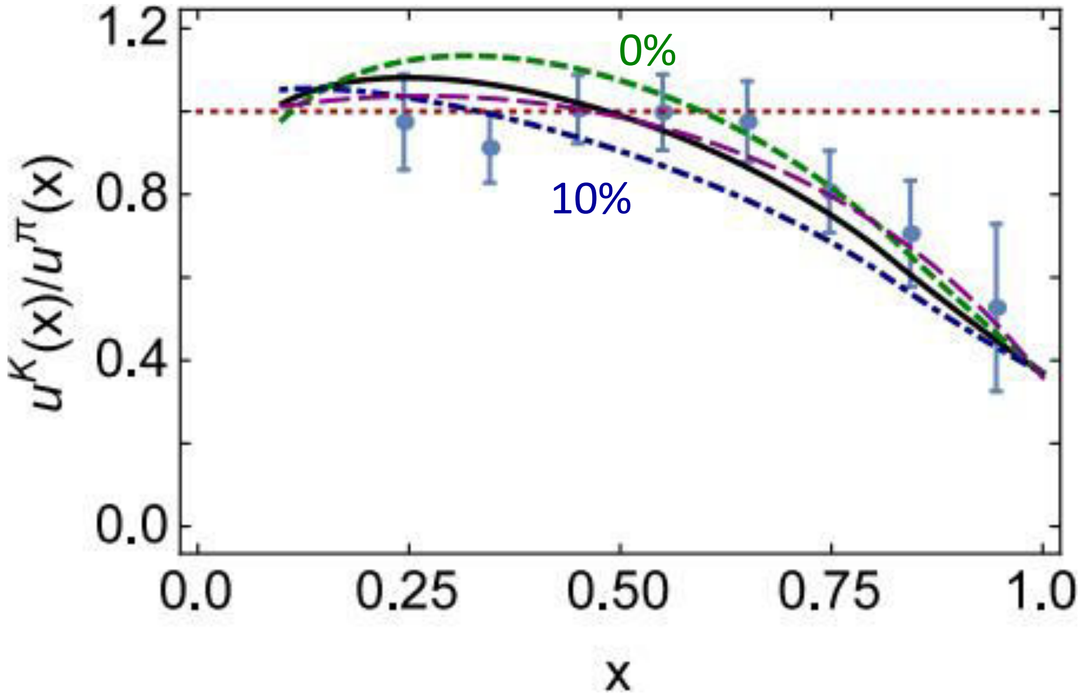
Kaon's gluon content

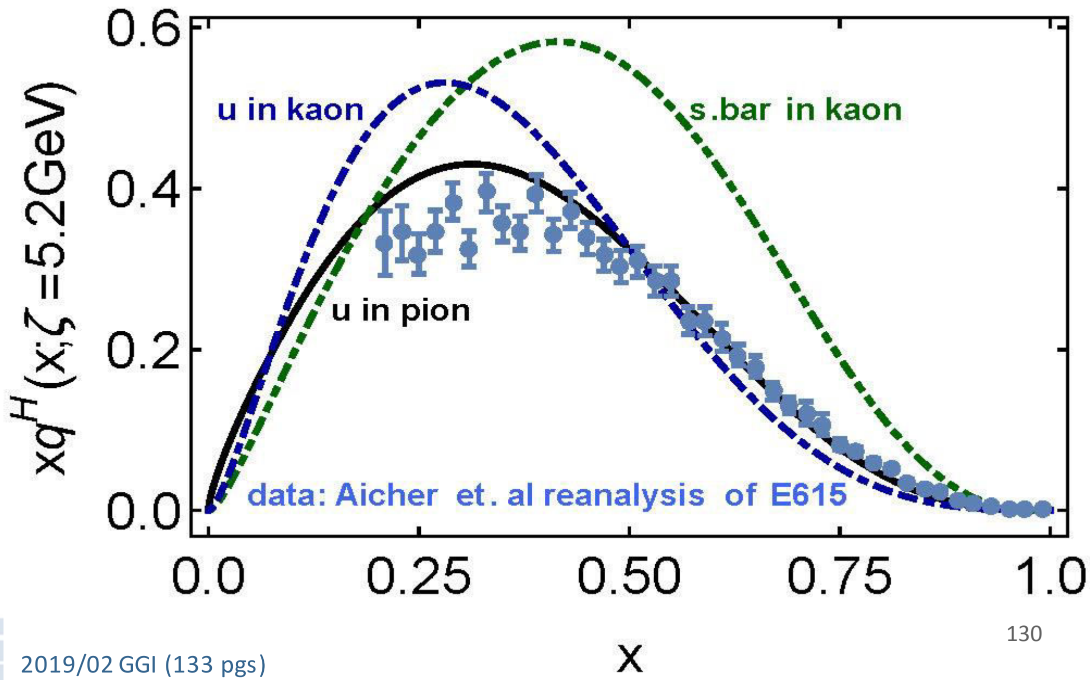
- $\langle x\rangle_g^K(\zeta_H) = 0.05 \pm 0.05$
  $\Rightarrow$ Valence quarks carry 95% of kaon's momentum at ζ_H
- DGLAP-evolved to ζ_2

q	$\langle x\rangle_q^K$	$\langle x^2\rangle_q^K$	$\langle x^3\rangle_q^K$
u	0.28	0.11	0.048
$\bar{s}$	0.36	0.17	0.092

Valence-quarks carry ⅔ of kaon's light-front momentum

Cf. Only ½ for the pion





π & K PDFs

- Marked differences between π & K gluon content
  - ζ_H:
    - Whilst $\frac{1}{3}$ of pion's light-front momentum carried by glue
    - $Only$ $\frac{1}{20}$ of the kaon's light-front momentum lies with glue
  - $\zeta_2^{\,2} = 4$ GeV^2
    - Glue carries $\frac{1}{2}$ of pion's momentum and $\frac{1}{3}$ of kaon's momentum
  - Evident in differences between large-x behaviour of valence-quark distributions in these two mesons
- Signal of Nambu-Goldstone boson character of π
  - Nearly complete cancellation between one-particle dressing and binding attraction in this almost massless pseudoscalar system

$$2\,\mathrm{Mass}_Q + U_g \approx 0$$

π & *K* PDFs

- Existing textbook description of Goldstone's theorem via pointlike modes is *simplistic*

Valence-quark distribution functions in the kaon and pion,
Chen Chen, Lei Chang *et al*.
arXiv:1602.01502 [nucl-th], Phys. Rev. D**93** (2016) 074021/1-11

π & K PDFs

- The appearance of Nambu-Goldstone modes in the Standard Model is far more interesting
  - Nambu-Goldstone modes are nonpointlike!
  - Intimately connected with origin of mass!
  - Possibly/Probably(?) inseparable from expression of confinement!

- Difference between gluon content of π & K is measurable … using well-designed EIC
- Write a definitive new chapter in future textbooks on the Standard Model

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

- There is one approach to analysing QCD's Schwinger functions that preserves some of QED's simplicity
  - Combination of pinch technique (PT) & background field method (BFM)
- The clean connection between the coupling and the gluon vacuum polarisation relies on another particular feature of QCD:
  - In Landau gauge the renormalisation constant of the gluon-ghost vertex is not only finite but unity
- Consequently, effective charge obtained from the PT-BFM gluon vacuum polarisation is directly connected with that deduced from the gluon-ghost vertex:
  - "Taylor coupling", α_T