Continuum QCD

Craig Roberts

Problem: Nature chooses to build (most) things, us included, from matter fields instead of gauge fields.

Quarks & QCD

In perturbation theory, quarks don't seem to do much, just a little bit of very-normal charge screening.

- Quarks are the problem with QCD
- Pure-glue QCD is far simpler
  - Bosons are the only degrees of freedom
    - Bosons have a classical analogue – see Maxwell's formulation of electrodynamics
  - Generating functional can be formulated as a discrete probability measure that is amenable to direct numerical simulation using Monte-Carlo methods
    - No perniciously nonlocal fermion determinant

- Pure-glue QCD has the Area Law & Linearly Rising Potential between static sources, so long identified with confinement

K.G. Wilson, formulated lattice-QCD in 1974 paper: "Confinement of quarks".

Wilson Loop

Nobel Prize (1982): *"for his theory for critical phenomena in connection with phase transitions"*.

Contrast with Minkowksi metric: infinitely many four-vectors satisfy $p^2 = p^0 p^0 - p^i p^i = 0$; e.g., $p = \mu\,(1,0,0,1)$, μ any number

Formulating QCD Euclidean Metric

- In order to translate QCD into a computational problem, Wilson had to employ a *Euclidean Metric*

  $x^2 = 0$ possible if and only if $x=(0,0,0,0)$

  because Euclidean-QCD action defines a probability measure, for which many numerical simulation algorithms are available.
- However, working in Euclidean space is more than simply pragmatic:
  - Euclidean lattice field theory is currently a primary candidate for the rigorous definition of an interacting quantum field theory.
  - This relies on it being possible to define the generating functional via a proper limiting procedure.

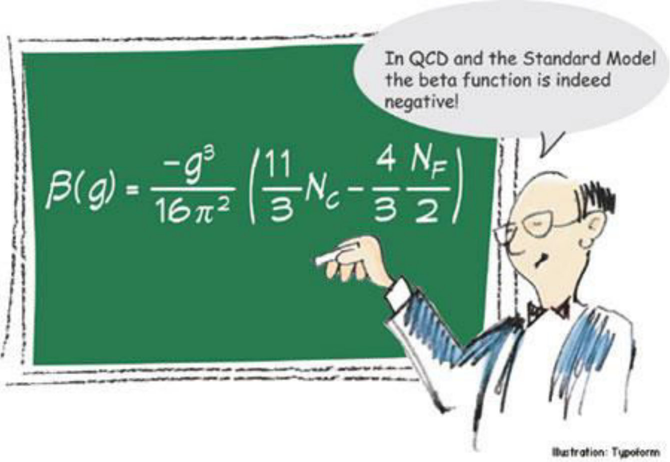
Formulating QCD Euclidean Metric

- The moments of the measure; i.e., “vacuum expectation values” of the fields, are the n-point Schwinger functions; and the quantum field theory is completely determined once all its Schwinger functions are known.
- The time-ordered Green functions of the associated Minkowski space theory can be obtained in a formally well-defined fashion from the Schwinger functions.

This is all *formally* true.

Formulating Quantum Field Theory Euclidean Metric

- Constructive Field Theory Perspectives:
  - Symanzik, K. (1963) in *Local Quantum Theory (Academic, New York) edited* by R. Jost.
  - Streater, R.F. and Wightman, A.S. (1980), *PCT, Spin and Statistics, and All That (Addison-Wesley, Reading, Mass, 3rd edition).*
  - Glimm, J. and Jaffee, A. (1981), *Quantum Physics. A Functional Point of View* (Springer-Verlag, New York).
  - Seiler, E. (1982), *Gauge Theories as a Problem of Constructive Quantum Theory and Statistical Mechanics (Springer-Verlag, New York).*
- For some theorists, interested in essentially nonperturbative QCD, this is always in the back of our minds

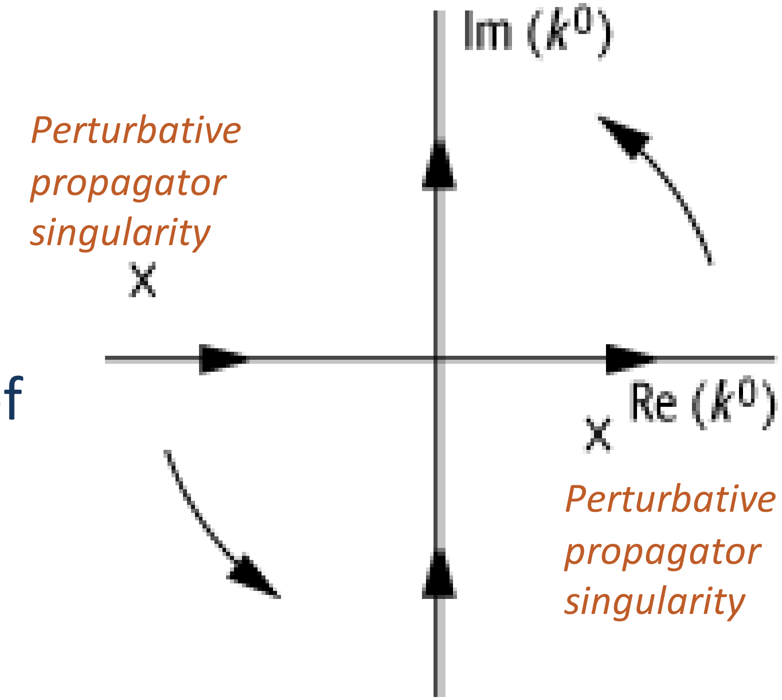


Formulating QCD Euclidean Metric

- However, there is another very important reason to work in Euclidean space; viz.,
  Owing to asymptotic freedom, all results of perturbation theory are strictly valid only at spacelike-momenta.
  - The set of spacelike momenta correspond to a Euclidean vector space
- The continuation to Minkowski space rests on many assumptions about Schwinger functions that are demonstrably valid only in perturbation theory.

Euclidean Metric & Wick Rotation

- It is assumed that a Wick rotation is valid; namely, that QCD dynamics don't nonperturbatively generate anything *unnatural*
- This is a brave assumption, which turns out to be ***FALSE*** in the case of coloured states.
- Hence, QCD *MUST* be defined in Euclidean space.
- The properties of the real-world are then determined only from a continuation of colour-singlet quantities.



Aside: QED is only defined perturbatively. It possesses an infrared stable fixed point; and masses and couplings are regularised and renormalised in the vicinity of $k^2=0$. *Wick rotation is always valid in this context.*

The Problem with QCD

- This is a RED FLAG in QCD because
  nothing elementary is a colour singlet
- Must somehow solve real-world problems
  – the spectrum and interactions of complex two- and three-body bound-states

  before returning to the real world
- This is going to require a little bit of imagination and a very good set of tools …

Euclidean Metric Conventions

- To make clear our conventions: for 4-vectors a, b: $a \cdot b := a_\mu\, b_\nu\, \delta_{\mu\nu} := \sum_{i=1}^{4} a_i\, b_i$,

  Hence, a spacelike vector, Q_μ, has $Q^2 > 0$.

- Dirac matrices:
  - Hermitian and defined by the algebra $\{\gamma_\mu, \gamma_\nu\} = 2\, \delta_{\mu\nu}$;
  - we use $\gamma_5 := -\gamma_1\gamma_2\gamma_3\gamma_4$, so that $\mathrm{tr}\,[\gamma_5\gamma_\mu\gamma_\nu\gamma_\rho\gamma_\sigma] = -4\,\varepsilon_{\mu\nu\rho\sigma}$, $\varepsilon_{1234} = 1$.
  - The Dirac-like representation of these matrices is:

$$\vec{\gamma} = \begin{pmatrix} 0 & -i\vec{\tau} \\ i\vec{\tau} & 0 \end{pmatrix},\ \gamma_4 = \begin{pmatrix} \tau^0 & 0 \\ 0 & -\tau^0 \end{pmatrix}, \tag{2}$$

where the 2×2 Pauli matrices are:

$$\tau^0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix},\ \tau^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\ \tau^2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\ \tau^3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{3}$$

Euclidean Transcription Formulae

It is possible to derive every equation of Euclidean QCD by assuming certain analytic properties of the integrands. However, the derivations can be sidestepped using the following *transcription rules*:

Configuration Space

1. $\int^M d^4x^M \rightarrow -i \int^E d^4x^E$
2. $\not\partial \rightarrow i\gamma^E \cdot \partial^E$
3. $\not A \rightarrow -i\gamma^E \cdot A^E$
4. $A_\mu B^\mu \rightarrow -A^E \cdot B^E$
5. $x^\mu \partial_\mu \rightarrow x^E \cdot \partial^E$

Momentum Space

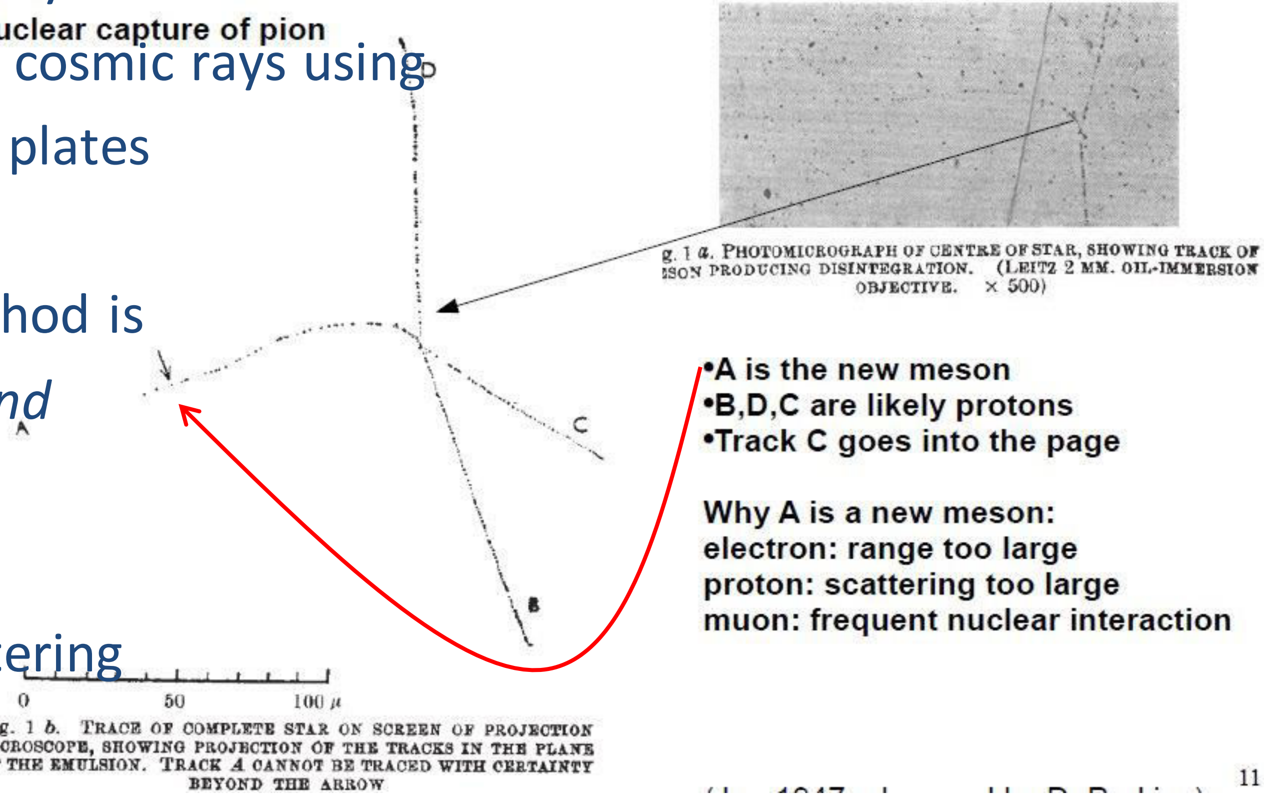
1. $\int^M d^4k^M \rightarrow i \int^E d^4k^E$
2. $\not k \rightarrow -i\gamma^E \cdot k^E$
3. $\not A \rightarrow -i\gamma^E \cdot A^E$
4. $k_\mu q^\mu \rightarrow -k^E \cdot q^E$
5. $k_\mu x^\mu \rightarrow -k^E \cdot x^E$

These rules are valid in perturbation theory; i.e., the correct Minkowski space integral for a given diagram will be obtained by applying these rules to the Euclidean integral: they take account of the change of variables and rotation of the contour. However, for diagrams that represent DSEs which involve dressed n-point functions, whose analytic structure is not known *a priori*, the Minkowski space equation obtained using this prescription will have the right appearance but it's solutions may bear no relation to the analytic continuation of the solution of the Euclidean equation. Any such differences will be nonperturbative in origin.

Never before seen by the human eye

Nature's strong messenger – The Pion

- 1947 – Pion discovered by Cecil Frank Powell
- Studied tracks made by cosmic rays using photographic emulsion plates
- Despite the fact that Cavendish Lab said method is incapable of "*reliable and reproducible precision measurements.*"
- Mass measured in scattering ≈ $250\text{-}350\ m_e$



g. 1 a. PHOTOMICROGRAPH OF CENTRE OF STAR, SHOWING TRACK OF ESON PRODUCING DISINTEGRATION. (LEITZ 2 MM. OIL-IMMERSION OBJECTIVE. × 500)

Fig. 1 b. TRACE OF COMPLETE STAR ON SCREEN OF PROJECTION MICROSCOPE, SHOWING PROJECTION OF THE TRACKS IN THE PLANE OF THE EMULSION. TRACK A CANNOT BE TRACED WITH CERTAINTY BEYOND THE ARROW

Nature's strong messenger - Pion

- The beginning of Particle Physics
- Then came
  - Disentanglement of confusion between $_{(1937)}$ muon and pion – similar masses
  - Discovery of particles with "strangeness" (e.g., kaon$_{1947\text{-}1953}$)
- Subsequently, a complete spectrum of mesons and baryons with mass below ≈1 GeV
  - 28 states

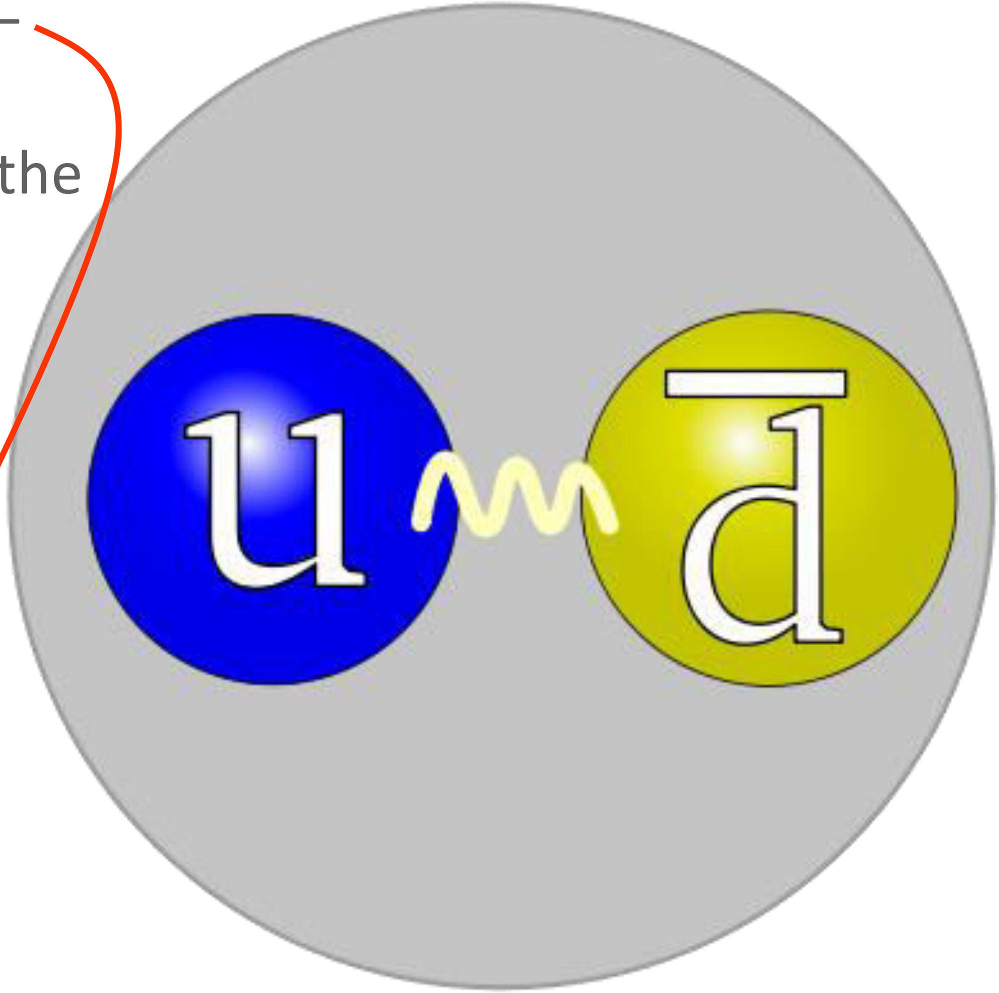
π	140 MeV
ρ	780 MeV
P	940 MeV

- Became clear that pion is "too light"

  \- *hadrons* supposed to be heavy, yet …

Simple picture - Pion

- Gell-Mann and Ne'eman:
  - Eightfold way$_{(1961)}$ – a picture based on group theory: *SU(3)*
  - Subsequently, quark model – where the *u*-, *d*-, *s*-quarks became the basis vectors in the fundamental representation of *SU(3)*
- Pion = Two *quantum-mechanical* constituent-quarks - particle+antiparticle - interacting via a *potential*

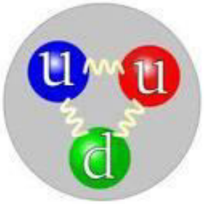


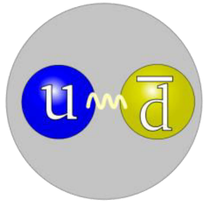
Some of the Light Mesons

LIGHT UNFLAVORED MESONS ($S = C = B = 0$)

For $I=1$ (π, b, ρ, a): $u\bar{d}, (u\bar{u}-d\bar{d})/\sqrt{2}, d\bar{u}$;
for $I=0$ ($\eta, \eta', h, h', \omega, \varphi, f, f'$): $c_1(u\bar{u}+d\bar{d}) + c_2(s\bar{s})$

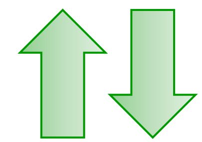
Meson		$I^G(J^{PC})$	Meson	$I^G(J^{PC})$	Meson	$I^G(J^{PC})$
$\pi^\pm$	140 MeV	$1^-(0^-)$	$\eta(1475)$	$0^+(0^{-+})$	• $f_2(1910)$	$0^+(2^{++})$
π^0		$1^-(0^{-+})$	$f_0(1500)$	$0^+(0^{++})$	$f_2(1950)$	$0^+(2^{++})$
η		$0^+(0^{-+})$	• $f_1(1510)$	$0^+(1^{++})$	• $\rho_3(1990)$	$1^+(3^{--})$
$f_0(600)$ or σ		$0^+(0^{++})$	$f_2'(1525)$	$0^+(2^{++})$	$f_2(2010)$	$0^+(2^{++})$
$\rho(770)$	780 MeV	$1^+(1^{--})$	• $f_2(1565)$	$0^+(2^{++})$	• $f_0(2020)$	$0^+(0^{++})$
$\omega(782)$		$0^-(1^{--})$	• $\rho(1570)$	$1^+(1^{--})$	$a_4(2040)$	$1^-(4^{++})$
$\eta'(958)$		$0^+(0^{-+})$	• $h_1(1595)$	$0^-(1^{+-})$	$f_4(2050)$	$0^+(4^{++})$
$f_0(980)$		$0^+(0^{++})$	$\pi_1(1600)$	$1^-(1^{-+})$	• $\pi_2(2100)$	$1^-(2^{-+})$
$a_0(980)$		$1^-(0^{++})$	• $a_1(1640)$	$1^-(1^{++})$	• $f_0(2100)$	$0^+(0^{++})$
$\varphi(1020)$		$0^-(1^{--})$	• $f_2(1640)$	$0^+(2^{++})$	• $f_2(2150)$	$0^+(2^{++})$
$h_1(1170)$		$0^-(1^{+-})$	$\eta_2(1645)$	$0^+(2^{-+})$	• $\rho(2150)$	$1^+(1^{--})$
$b_1(1235)$		$1^+(1^{+-})$	$\omega(1650)$	$0^-(1^{--})$	$\varphi(2170)$	$0^-(1^{--})$
$a_1(1260)$		$1^-(1^{++})$	$\omega_3(1670)$	$0^-(3^{--})$	• $f_0(2200)$	$0^+(0^{++})$
$f_2(1270)$		$0^+(2^{++})$	$\pi_2(1670)$	$1^-(2^{-+})$	• $f_J(2220)$	$0^+(2^{++}$ or $4^{++})$
$f_1(1285)$		$0^+(1^{++})$	$\varphi(1680)$	$0^-(1^{--})$		
$\eta(1295)$		$0^+(0^{-+})$	$\rho_3(1690)$	$1^+(3^{--})$	• $\eta(2225)$	$0^+(0^{-+})$
$\pi(1300)$		$1^-(0^{-+})$	$\rho(1700)$	$1^+(1^{--})$	• $\rho_3(2250)$	$1^+(3^{--})$
$a_2(1320)$		$1^-(2^{++})$	• $a_2(1700)$	$1^-(2^{++})$	$f_2(2300)$	$0^+(2^{++})$
$f_0(1370)$		$0^+(0^{++})$	$f_0(1710)$	$0^+(0^{++})$	• $f_4(2300)$	$0^+(4^{++})$
• $h_1(1380)$		$?^-(1^{+-})$	• $\eta(1760)$	$0^+(0^{-+})$	• $f_0(2330)$	$0^+(0^{++})$
$\pi_1(1400)$		$1^-(1^{-+})$	$\pi(1800)$	$1^-(0^{-+})$	$f_2(2340)$	$0^+(2^{++})$
$\eta(1405)$		$0^+(0^{-+})$	• $f_2(1810)$	$0^+(2^{++})$	• $\rho_5(2350)$	$1^+(5^{--})$
$f_1(1420)$		$0^+(1^{++})$	• $X(1835)$	$?^?(?^{-+})$	• $a_6(2450)$	$1^-(6^{++})$
$\omega(1420)$		$0^-(1^{--})$	$\varphi_3(1850)$	$0^-(3^{--})$	• $f_6(2510)$	$0^+(6^{++})$
• $f_2(1430)$		$0^+(2^{++})$	• $\eta_2(1870)$	$0^+(2^{-+})$	• — OMITTED FROM SUMMARY TABLE	
$a_0(1450)$		$1^-(0^{++})$	$\pi_2(1880)$	$1^-(2^{-+})$		
$\rho(1450)$		$1^+(1^{--})$	• $\rho(1900)$	$1^+(1^{--})$		



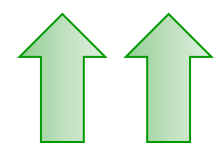


Modern Miracles in Hadron Physics

- proton = three constituent quarks
  - $M_{proton} \approx$ 1GeV
  - Therefore guess $M_{constituent-quark} \approx ⅓ \times$ GeV $\approx$ 350MeV
- pion = constituent quark + constituent antiquark



  - Guess $M_{pion} \approx ⅔ \times M_{proton} \approx$ 700MeV
- WRONG . . . . M_{pion} = 140MeV
- Rho-meson



  - Also *constituent quark + constituent antiquark* – just pion with spin of one constituent flipped
  - $M_{rho} \approx 770MeV \approx 2 \times M_{constituent-quark}$

What is "wrong" with the pion?

Dichotomy of the pion

- How does one make an almost massless particle from two massive constituent-quarks?
- Naturally, one *could* always tune a potential in quantum mechanics so that the ground-state is massless

  – but some are still making this mistake

- However: $$m_\pi^2 \propto m$$

  current-algebra (1968)

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-state} \propto m_{constituent}$

Dichotomy of the pion
Goldstone mode and bound-state

- The *correct understanding* of pion observables; e.g. mass, decay constant and form factors, requires an approach to contain a
  - well-defined and valid chiral limit;
  - and an accurate realisation of dynamical chiral symmetry breaking.

HIGHLY NONTRIVIAL
Impossible in quantum mechanics
Impossible in QED
Only possible in asymptotically-free gauge theories

Chiral QCD

- Action, in terms of local Lagrangian density:

$$S[G^a_\mu,\bar{q},q] = \int d^4x \, \{\frac{1}{4} G^a_{\mu\nu}(x) G^a_{\mu\nu}(x) + \frac{1}{2\xi} \partial_\mu G^a_\mu(x) \, \partial_\nu G^a_\nu(x) + \bar{q}(x)[\gamma_\mu D_\mu + M] q(x)\}$$

  - Chromomagnetic Field Strength Tensor:
    $\partial_\mu G^a_\nu(x) - \partial_\nu G^a_\mu(x) + g f^{abc} G^b_\mu(x) G^c_\nu(x)$
  - Covariant Derivative: $D_\mu = \partial_\mu - ig \frac{\lambda^a}{2} G^a_\mu(x)$
  - Current-quark Mass matrix: $\begin{pmatrix} m_u & 0 & 0 & \ldots \\ 0 & m_d & 0 & \ldots \\ 0 & 0 & m_s & \ldots \\ \vdots & \vdots & \vdots & \end{pmatrix}$

$m_t = 40{,}000 \, m_u$

Why?

- Understanding Hadron Physics means knowing all that this Action predicts.

- Current-quark masses
  - External paramaters in QCD
  - Generated by the Higgs boson, within the Standard Model
  - Raises more questions than it answers

Chiral Symmetry

- Interacting gauge theories, in which it makes sense to speak of massless fermions, have a nonperturbative chiral symmetry
- A related concept is *Helicity,* which is the projection of a particle's spin, *J*, onto it's direction of motion:

$$\lambda \propto J \bullet p$$

- For a massless particle, helicity is a Lorentz-invariant *spin-observable* $\lambda = \pm$; i.e., it's parallel or antiparallel to the direction of motion
  - Obvious:
    - massless particles travel at speed of light
    - hence no observer can overtake the particle and thereby view its momentum as having changed sign

Chiral Symmetry

- Chirality operator is γ_5
  - Chiral transformation: $\Psi(x) \to \exp(i\, \gamma_5\, \vartheta)\, \Psi(x)$
  - Chiral rotation through $\vartheta = \frac{1}{4}\pi$
    - Composite particles: $J^{P=+} \leftrightarrow J^{P=-}$
    - Equivalent to the operation of parity conjugation
- *Therefore, a prediction of chiral symmetry is the existence of degenerate parity partners in the theory's spectrum*

Chiral Symmetry

- Perturbative QCD: *u*- & *d*- quarks are very light

  $m_u/m_d \approx 0.5$ & $m_d \approx 4$ MeV

  (a generation of high-energy experiments)

  H. Leutwyler, *0911.1416 [hep-ph]*

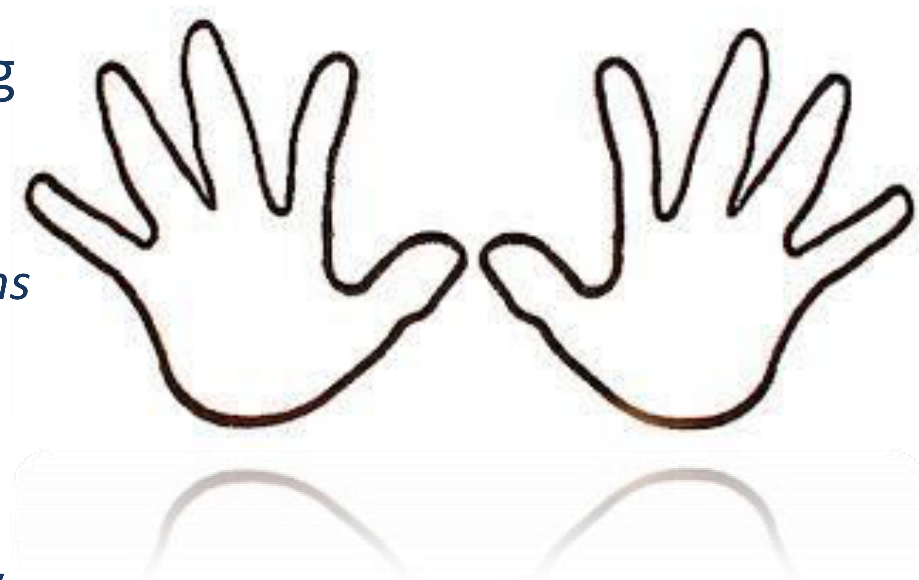
- However, splitting between parity partners is greater-than 100-times this mass-scale; e.g.,

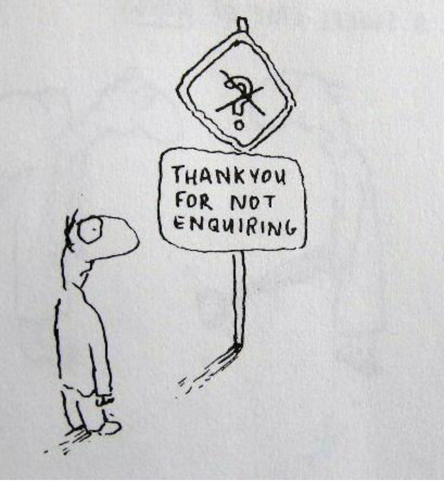
J^P	$\frac{1}{2}^+$ (p)	$\frac{1}{2}^-$
Mass	940 MeV	1535 MeV

Dynamical
Chiral Symmetry Breaking

- Something is happening in QCD
  - some inherent dynamical effect is dramatically changing the pattern by which the Lagrangian's chiral symmetry is expressed
- Qualitatively different from spontaneous symmetry breaking *aka* the Higgs mechanism
  - *Nothing is added to the theory*
  - *Have only fermions & gauge-bosons*

  Yet, the mass-operator generated by the theory produces a spectrum with no sign of chiral symmetry





QCD's Challenges

Understand emergent phenomena

- *Quark and Gluon Confinement*
  No matter how hard one strikes the proton,
  one cannot liberate an individual quark or gluon
- *Dynamical Chiral Symmetry Breaking*
  Very unnatural pattern of bound state masses;
  e.g., Lagrangian (pQCD) quark mass is small but
  . . . no degeneracy between $J^P=+$ and $J^P=-$ *(parity partners)*
- *Neither of these phenomena is apparent in QCD's Lagrangian*
  Yet they are the dominant determining characteristics of real-world QCD.
- QCD
  – Complex behaviour arises from apparently simple rules.

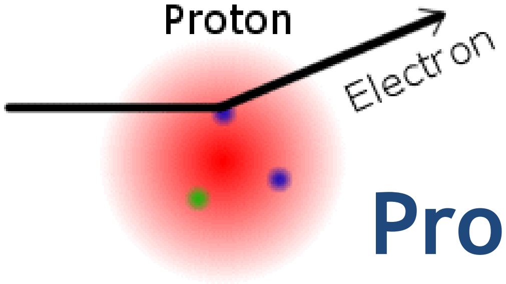
The study of nonperturbative QCD is the puriew of ...

Hadron Physics

Nucleon … Two Key Hadrons
Proton and Neutron

- Fermions – two static properties:
  proton electric charge = +1; and magnetic moment, μ_p
- Magnetic Moment discovered by Otto Stern and collaborators in 1933; Stern awarded Nobel Prize (1943): *"for his contribution to the development of the molecular ray method and his discovery of the magnetic moment of the proton"*.
- Dirac (1928) – pointlike fermion: $\mu_p = \frac{e\hbar}{2M}$
- Stern (1933) – $\mu_p = (1 + 1.79)\frac{e\hbar}{2M}$
- Big Hint that Proton is not a point particle
  - Proton has constituents
  - These are Quarks and Gluons
- Quark discovery via e-p-scattering at SLAC in 1968
  - the elementary quanta of QCD

Friedman, Kendall, Taylor, Nobel Prize (1990): "for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"



Nucleon Structure
Probed in scattering experiments

- Electron is a good probe because it is structureless

  Electron's relativistic current is

$$
\begin{aligned}
j_\mu(P',P) &= ie\,\bar{u}_e(P')\,\Lambda_\mu(Q,P)\,u_e(P)\,,\quad Q = P'-P \\
&= ie\,\bar{u}_e(P')\,\gamma_\mu(-1)\,u_e(P)
\end{aligned}
$$

Structureless fermion, or simply-structured fermion, $F_1=1$ & $F_2=0$, so that $G_E=G_M$ and hence distribution of charge and magnetisation within this fermion are identical

- Proton's electromagnetic current

$$
\begin{aligned}
J_\mu(P',P) &= ie\,\bar{u}_p(P')\,\Lambda_\mu(Q,P)\,u_p(P)\,, \\
&= ie\,\bar{u}_p(P')\left(\gamma_\mu F_1(Q^2) + \frac{1}{2M}\,\sigma_{\mu\nu}\,Q_\nu\,F_2(Q^2)\right)u_p(P)
\end{aligned}
$$

F_1 = Dirac form factor

F_2 = Pauli form factor

$$
G_E(Q^2) = F_1(Q^2) - \frac{Q^2}{4M^2}F_2(Q^2)\,,\quad G_M(Q^2) = F_1(Q^2) + F_2(Q^2)
$$

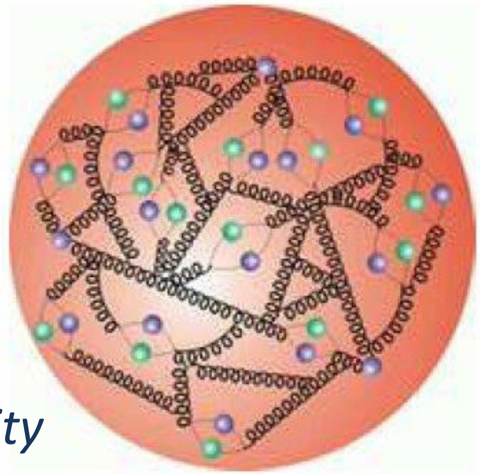
G_E = Sachs Electric form factor
If a nonrelativistic limit exists, this relates to the charge density

G_M = Sachs Magnetic form factor
If a nonrelativistic limit exists, this relates to the magnetisation density

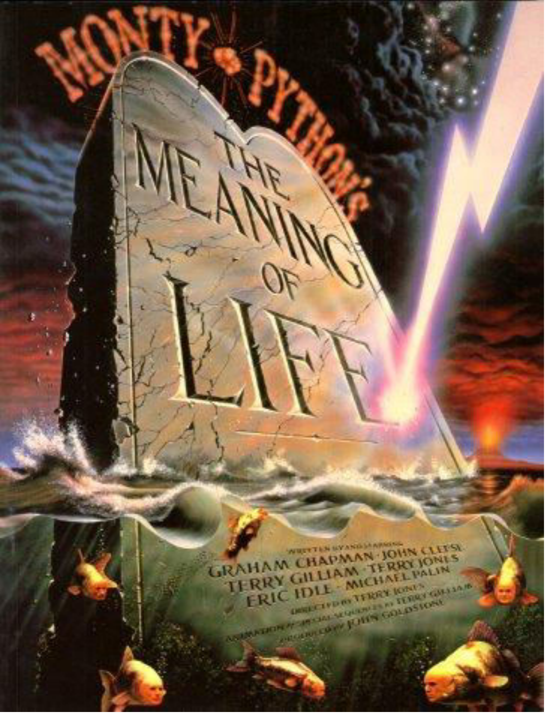
Modern Nuclear Physics

- A central goal of nuclear physics is to understand the structure and properties of protons and neutrons, and ultimately atomic nuclei, in terms of the quarks and gluons of QCD
- So, what's the problem?

  They are legion …
  - *Confinement*
  - *Dynamical chiral symmetry breaking*
  - *A fundamental theory of unprecedented complexity*



- QCD defines an *interaction zone* between nuclear and particle physics:
  - Need the expertise of both communities to solve this theory



Understanding the Questions

- *What are the quarks and gluons of QCD?*
- *Is there such a thing as a constituent quark, a constituent-gluon?*

  After all, these are the concepts for which Gell-Mann won the Nobel Prize.

- *Do they – can they – correspond to well-defined quasi-particle degrees-of-freedom?*
- *If not, with what should they be replaced?*

What is the meaning of the Modern Nuclear/Hadro-Particle Physics Challenge?

Recall the dichotomy of the pion

- How does one make an almost massless particle from two massive constituent-quarks?
- One can always tune a potential in quantum mechanics so that the ground-state is massless

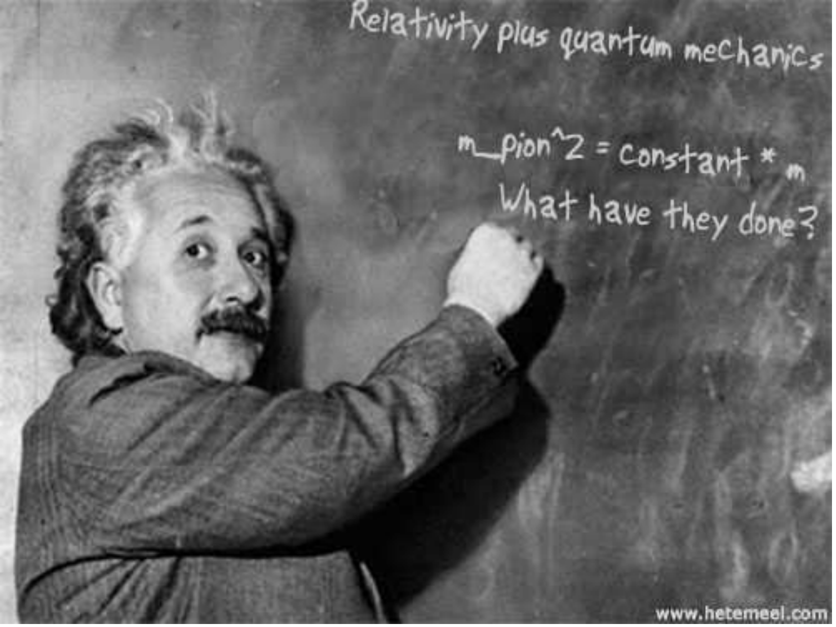
  – *and some are still making this mistake*

- However: $m_\pi^2 \propto m$

  current-algebra (1968)

Models based on constituent-quarks cannot produce this outcome. They must be fine tuned in order to produce the empirical splitting between the π & ρ mesons

- This is *impossible in quantum mechanics*, for which one always finds: $m_{bound-state} \propto m_{constituent}$



What is the meaning of all this?

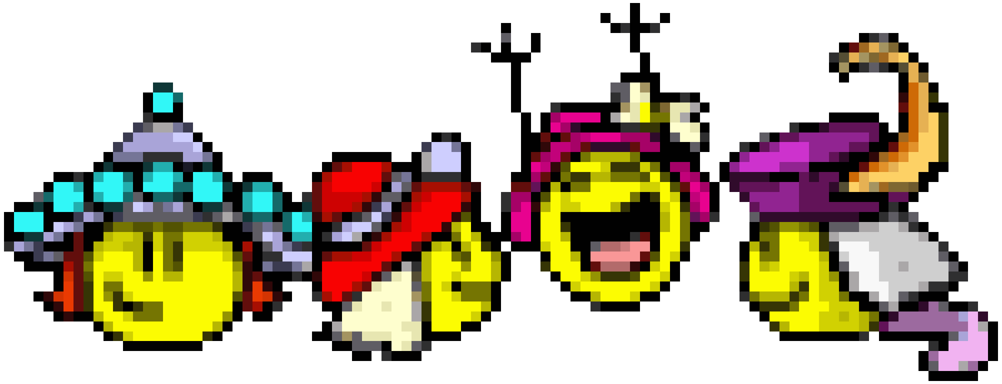
If $m_\pi = m_\rho$, then repulsive and attractive forces in the Nucleon-Nucleon potential have the SAME range and there is NO intermediate range attraction.

Under these circumstances:

- *Can ^{12}C be produced, can it be stable?*
- *Is the deuteron stable; can Big-Bang Nucleosynthesis occur?*

  (Many more existential questions …)

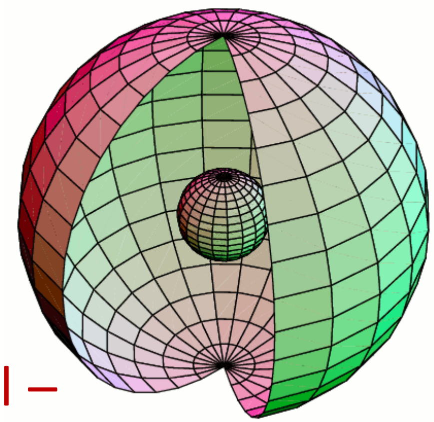
Probably not … but it wouldn't matter because we wouldn't exist to worry about it.



Why don't we just stop talking and solve the problem?

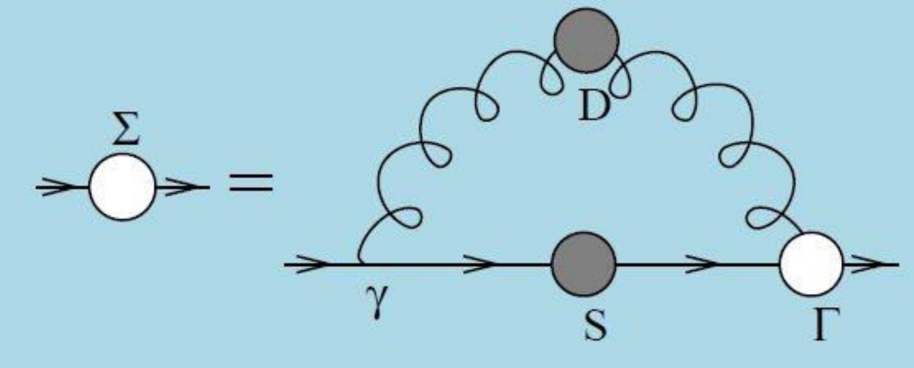
Just get on with it!

- But … QCD's emergent phenomena can't be studied using perturbation theory
  - *So what? Same is true of bound-state problems in quantum mechanics!*
- Differences:
  - Here relativistic effects are crucial – *virtual particles*
    Quintessence of Relativistic Quantum Field Theory
  - Interaction between quarks – the Interquark Potential –
    Unknown throughout > 98% of the pion's/proton's volume!
- Understanding requires *ab initio* nonperturbative solution of fully-fledged interacting relativistic quantum field theory, something which Mathematics and Theoretical Physics are a long way from achieving.



How can we tackle the SM's Strongly-interacting piece?

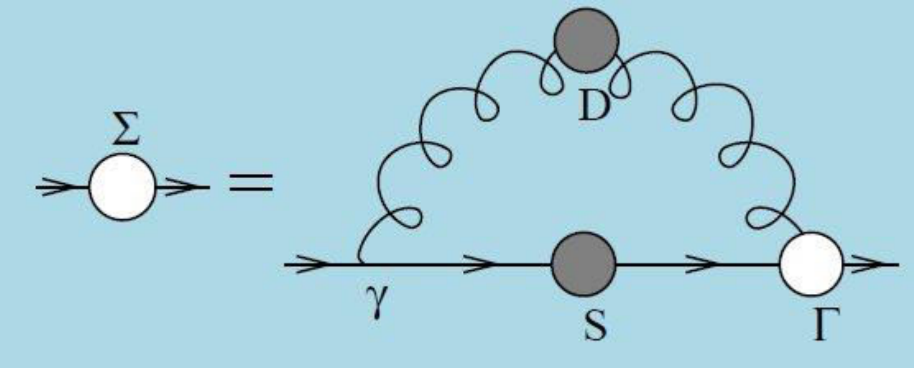
- *Use everything at our disposal*
  - *Constituent-quark and algebraic models*
  - *Dyson-Schwinger equations (continuum functional methods)*
  - *Lattice regularised QCD (discrete functional methods)*
  - *Reaction models and theories*
  - *Sum rules*
  - …
  - The methods discussed at this school!



Dyson-Schwinger Equations

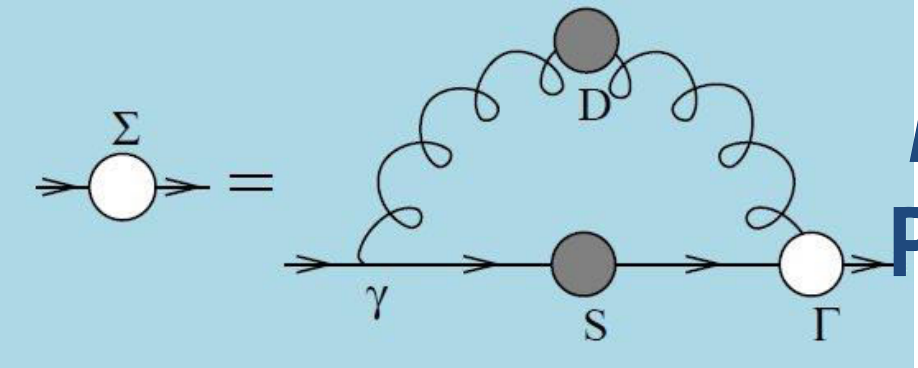
- Well suited to Relativistic Quantum Field Theory
- Simplest level: Generating Tool for Perturbation Theory . . . Materially Reduces Model-Dependence … Statement about long-range behaviour of quark-quark interaction
- NonPerturbative, Continuum approach to QCD
- Hadrons as Composites of Quarks and Gluons
- Qualitative and Quantitative Importance of:
  - Dynamical Chiral Symmetry Breaking
    - Generation of fermion mass from *nothing*
  - Quark & Gluon Confinement
    - Coloured objects not detected, *Not detectable?*

- Approach yields Schwinger functions; i.e., propagators and vertices
- Cross-Sections built from Schwinger Functions
- Hence, method connects observables with long-range behaviour of the running coupling
- Experiment ↔ Theory comparison leads to an understanding of long-range behaviour of strong running-coupling



Dyson-Schwinger Equations

- Useful because they provide symmetry-preserving (hence Poincaré covariant) framework with traceable connection to QCD Lagrangian
- Known limitation: need to employ a truncation in order to define a tractable continuum bound-state problem.
- Concerning truncation, much has been learnt in past twenty years, so that one may now separate DSE predictions into three classes:
  - ✓ (A) model-independent statements about QCD;
  - ✓ (B) illustrations of such statements using well-constrained model elements and possessing a traceable connection to QCD;
  - ✓ (C) analyses that can be described as QCD-based but whose elements have not been computed using a truncation that preserves a systematically-improvable connection with QCD.

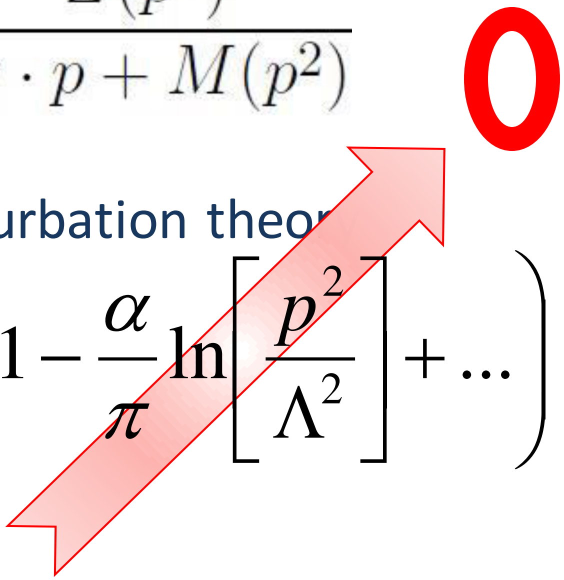


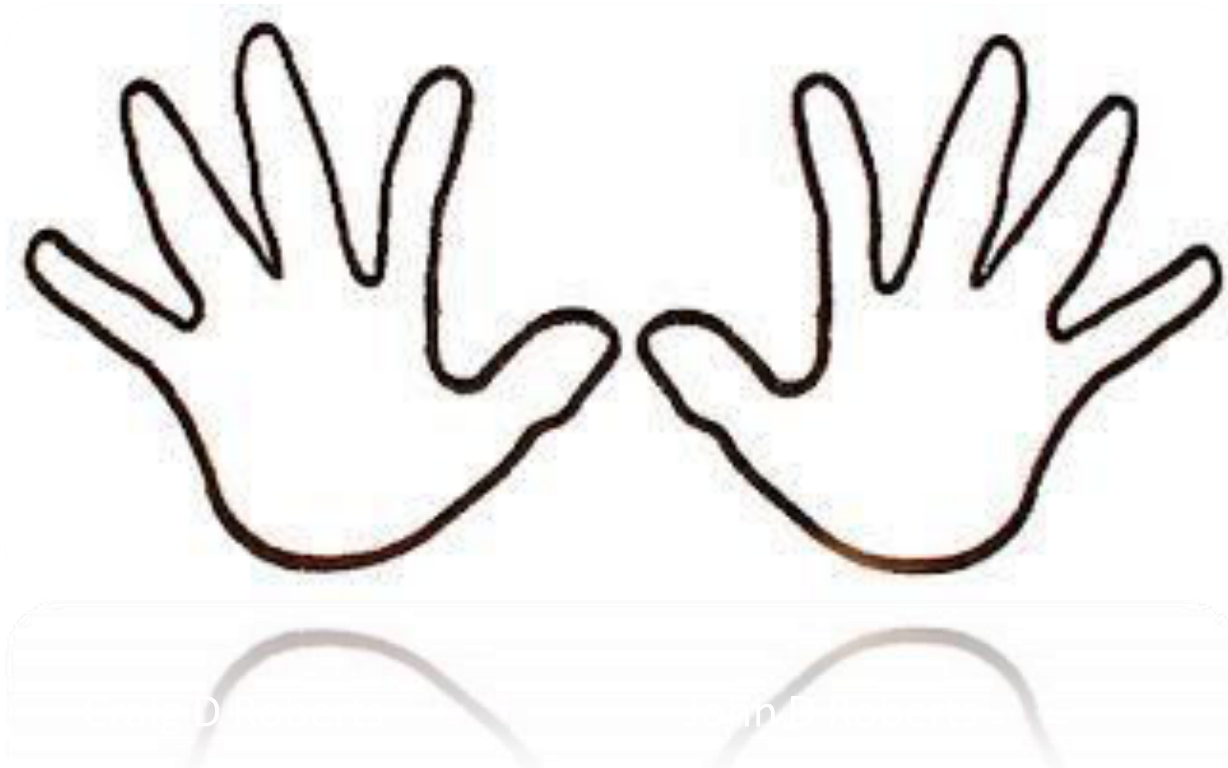
Mass from Nothing?! Perturbation Theory

- QCD is asymptotically-free (2004 Nobel Prize)
  - Chiral-limit is well-defined;
    i.e., one can truly speak of a massless quark.
  - NB. This is nonperturbatively *impossible* in QED.

- Dressed-quark propagator: $$S(p) = \frac{Z(p^2)}{i\gamma\cdot p + M(p^2)}$$
- Weak coupling expansion of gap equation yields every diagram in perturbation theory
- In perturbation theory:
  If *m=0*, then *M(p²)=0*
  Start with no mass,
  Always have no mass.

$$M(p^2) = m\left(1 - \frac{\alpha}{\pi}\ln\left[\frac{p^2}{\Lambda^2}\right] + \ldots\right)$$





Dynamical Chiral Symmetry Breaking

DCSB à la Nambu

- Recall the gap equation

$$S^{-1}(p) = i\gamma\cdot p\,A(p^2) + B(p^2) = i\gamma\cdot p + m$$
$$+ \int^{\Lambda} \frac{d^4\ell}{(2\pi)^4}\, g^2 D_{\mu\nu}(p-\ell)\,\gamma_\mu \frac{\lambda^a}{2} \frac{1}{i\gamma\cdot\ell A(\ell^2) + B(\ell^2)} \Gamma^a_\nu(\ell,p)$$

NJL: $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \dfrac{\lambda^a}{2};$

$$g^2 D_{\mu\nu}(p-\ell) \to \delta_{\mu\nu}\,\frac{1}{m_G^2}\,\theta(\Lambda^2-\ell^2)$$

- Model is not renormalisable

  $\Rightarrow$ regularisation parameter (Λ) plays a dynamical role.

- Contact-interaction gap equation

$$i\gamma\cdot p\,A(p^2) + B(p^2)$$
$$= \quad i\gamma\cdot p + m + \frac{4}{3}\,\frac{1}{m_G^2} \int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\gamma_\mu\,\frac{-i\gamma\cdot\ell A(\ell^2) + B(\ell^2)}{\ell^2 A^2(\ell^2) + B^2(\ell^2)}\,\gamma_\mu$$

DCSB à la Nambu

- Multiply the gap equation by ($-i\gamma\cdot p$); trace over Dirac indices:

$$p^2 A(p^2) = p^2 + \frac{8}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,p\cdot\ell\,\frac{A(\ell^2)}{\ell^2 A^2(\ell^2)+B^2(\ell^2)}$$

  - Angular integral vanishes, therefore $A(p^2) = 1$.
  - This owes to the fact that the model is defined by a four-fermion contact-interaction in configuration space, which entails a momentum-independent interaction in momentum space.

- Simply take Dirac trace of model's gap equation:

$$B(p^2) = m + \frac{16}{3}\frac{1}{m_G^2}\int \frac{d^4\ell}{(2\pi)^4}\,\theta(\Lambda^2-\ell^2)\,\frac{B(\ell^2)}{\ell^2+B^2(\ell^2)}$$

  - Integrand is p^2-independent, therefore the only solution is $B(p^2)$ = constant = M.

- General form of the propagator for a fermion dressed by the contact interaction: $S(p) = 1/[\,i\gamma\cdot p + M\,]$

Critical coupling for dynamical mass generation?

Contact interaction & a mass gap?

- Evaluate the integrals

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, \Lambda^2),$$
$$\mathcal{C}(M^2, \Lambda^2) = \Lambda^2 - M^2 \ln\left[1 + \Lambda^2/M^2\right].$$

- Λ defines the model's mass-scale. Henceforth set $\Lambda = 1$, then all other dimensioned quantities are given in units of this scale, in which case the gap equation can be written

$$M = M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

- Chiral limit, *m=0*
  - Solutions?
    - One is obvious; viz., *M=0*
      This is the *perturbative result*

    … start with no mass, end up with no mass

- Chiral limit, *m=0*
  - Suppose, on the other hand that *M≠0*, and thus may be cancelled
    - This nontrivial solution can exist if-and-only-if one may satisfy

$$3\pi^2\, m_G^2 = \mathcal{C}(M^2, 1)$$

Critical coupling for dynamical mass generation!

Contact interaction & a mass gap?

- Can one satisfy $3\pi^2 m_G^2 = C(M^2,1)$?
  - $C(M^2, 1) = 1 - M^2 \ln [1 + 1/M^2]$
    - Monotonically decreasing function of M
    - Maximum value at $M = 0$; viz., $C(M^2=0, 1) = 1$
- Consequently, there is a solution *iff* $3\pi^2 m_G^2 < 1$
  - Typical scale for hadron physics: $\Lambda = 1$ GeV
    - There is a M≠0 solution iff $m_G^2 < (\Lambda/(3\pi^2)) = (0.2\ \text{GeV})^2$
- Interaction strength is proportional to $1/m_G^2$
  - Hence, if interaction is strong enough, then one can start with no mass but end up with a massive, perhaps very massive fermion

Dynamical Chiral Symmetry Breaking

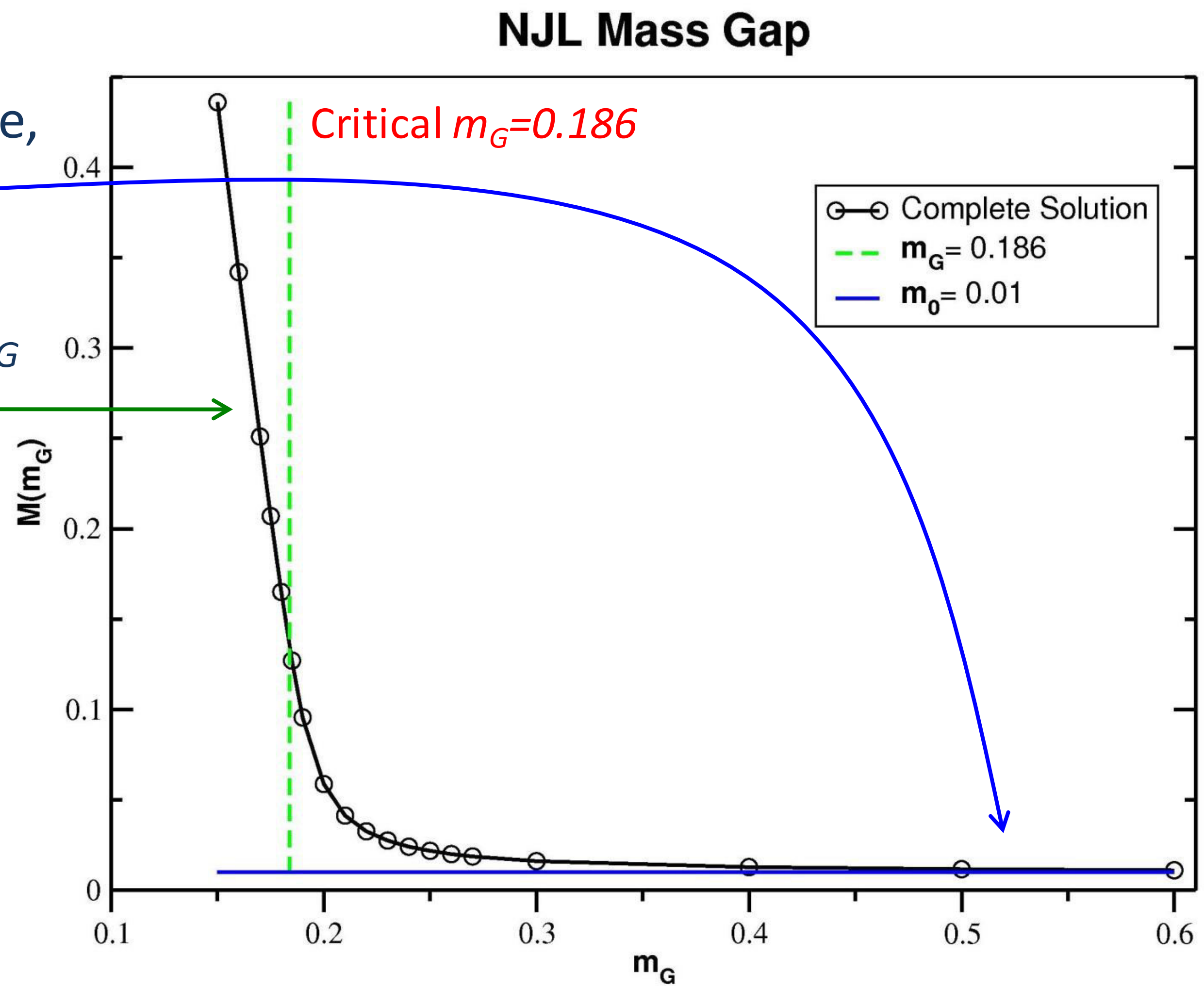
Solution of gap equation

$$M = m + M \frac{1}{3\pi^2} \frac{1}{m_G^2} \mathcal{C}(M^2, 1)$$

Contact Interaction & Dynamical Mass

- Weak coupling corresponds to m_G large, in which case $M \approx m$
- On the other hand, strong coupling; i.e., m_G small, $M >> m$

This is the defining characteristic of dynamical chiral symmetry breaking



NJL Model and Confinement?

- **Confinement**: no free-particle-like quarks
- Fully-dressed NJL propagator

$$S(p)^{\rm NJL} = \frac{1}{i\gamma\cdot p[A(p^2)=1]+[B(p^2)=M]} = \frac{-i\gamma\cdot p+M}{p^2+M^2}$$

- This is merely a free-particle-like propagator with a shifted mass

  $p^2 + M^2 = 0 \rightarrow$ Minkowski-space mass = M

- Hence, whilst **NJL model** exhibits dynamical chiral symmetry breaking it **does not confine**.

NJL-fermion still propagates as a plane wave

Munczek-Nemirovsky Model

- Munczek, H.J. and Nemirovsky, A.M. (1983),
  "The Ground State q-q.bar Mass Spectrum In QCD,"
  Phys. Rev. ***D 28, 181.***

Antithesis of NJL model; viz.,
Delta-function in momentum space
NOT in configuration space.

In this case, G sets the mass scale

- $\Gamma^a_\mu(k,p)_{\rm bare} = \gamma_\mu \frac{\lambda^a}{2};$

$$g^2 D_{\mu\nu}(k) \to (2\pi)^4\, G\, \delta^4(k) \left[\delta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}\right]$$

- MN Gap equation

$$i\gamma\cdot p\, A(p^2) + B(p^2) = i\gamma\cdot p + m + G\,\gamma_\mu \frac{-i\gamma\cdot p\, A(p^2) + B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,\gamma_\mu$$

MN Model's Gap Equation

- The gap equation yields the following pair of coupled, algebraic equations (set $G = 1$ GeV^2)

$$A(p^2) = 1 + 2\,\frac{A(p^2)}{p^2 A^2(p^2) + B^2(p^2)}$$

$$\boxed{B(p^2) = 4\,\frac{B(p^2)}{p^2 A^2(p^2) + B^2(p^2)}\,,}$$

- Consider the chiral limit form of the equation for $B(p^2)$
  - Obviously, one has the trivial solution $B(p^2) = 0$
  - However, is there another?

MN model and DCSB

- The existence of a $B(p^2) \neq 0$ solution; i.e., a solution that dynamically breaks chiral symmetry, requires (in units of G)

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

- Substituting this result into the equation for $A(p^2)$ one finds

  $$A(p^2) - 1 = \tfrac{1}{2} A(p^2) \;\rightarrow\; A(p^2) = 2,$$

  which in turn entails

  $$B(p^2) = 2\,(1 - p^2)^{1/2}$$

- Physical requirement: quark self-energy is real on the domain of spacelike momenta → complete chiral limit solution

$$A(p^2) = \begin{cases} 2\,; & p^2 \leq 1 \\ \frac{1}{2}\left(1 + \sqrt{1 + 8/p^2}\right)\,; & p^2 > 1 \end{cases}$$

$$B(p^2) = \begin{cases} \sqrt{1 - p^2}\,; & p^2 \leq 1 \\ 0\,; & p^2 > 1\,. \end{cases}$$

NB. Self energies are momentum-dependent because the interaction is momentum-dependent. Should expect the same in QCD.

MN Model and Confinement?

- Solution we've found is continuous and defined for all p^2, even $p^2 < 0$; namely, timelike momenta
- Examine the propagator's denominator

  $$p^2 A^2(p^2) + B^2(p^2) = 4$$

  This is greater-than zero for all p^2 …
  - There are no zeros
  - So, the *propagator has no pole*
- This is nothing like a free-particle propagator. It can be interpreted as describing a **confined degree-of-freedom**
- Note that, in addition there is no critical coupling: The nontrivial solution exists so long as $\boldsymbol{G > 0}$.
- Conjecture: ***All confining theories exhibit DCSB***
  - NJL model demonstrates that converse is not true.

Massive solution in MN Model

- In the chirally asymmetric case the gap equation yields

$$A(p^2) = \frac{2\,B(p^2)}{m + B(p^2)}\,,$$
$$B(p^2) = m + \frac{4\,[m + B(p^2)]^2}{B(p^2)([m + B(p^2)]^2 + 4p^2)}\,.$$

- Second line is a quartic equation for $B(p^2)$.

  Can be solved algebraically with four solutions, available in a closed form.

- Only one solution has the correct $p^2 \to \infty$ limit; viz.,

  $B(p^2) \to m$.

  This is the *unique physical* solution.

- NB. The equations and their solutions always have a smooth $m \to 0$ limit, a result owing to the persistence of the DCSB solution.

Munczek-Nemirovsky Dynamical Mass

- Large-*s*: $M(s) \sim m$
- Small-*s*: $M(s) \gg m$

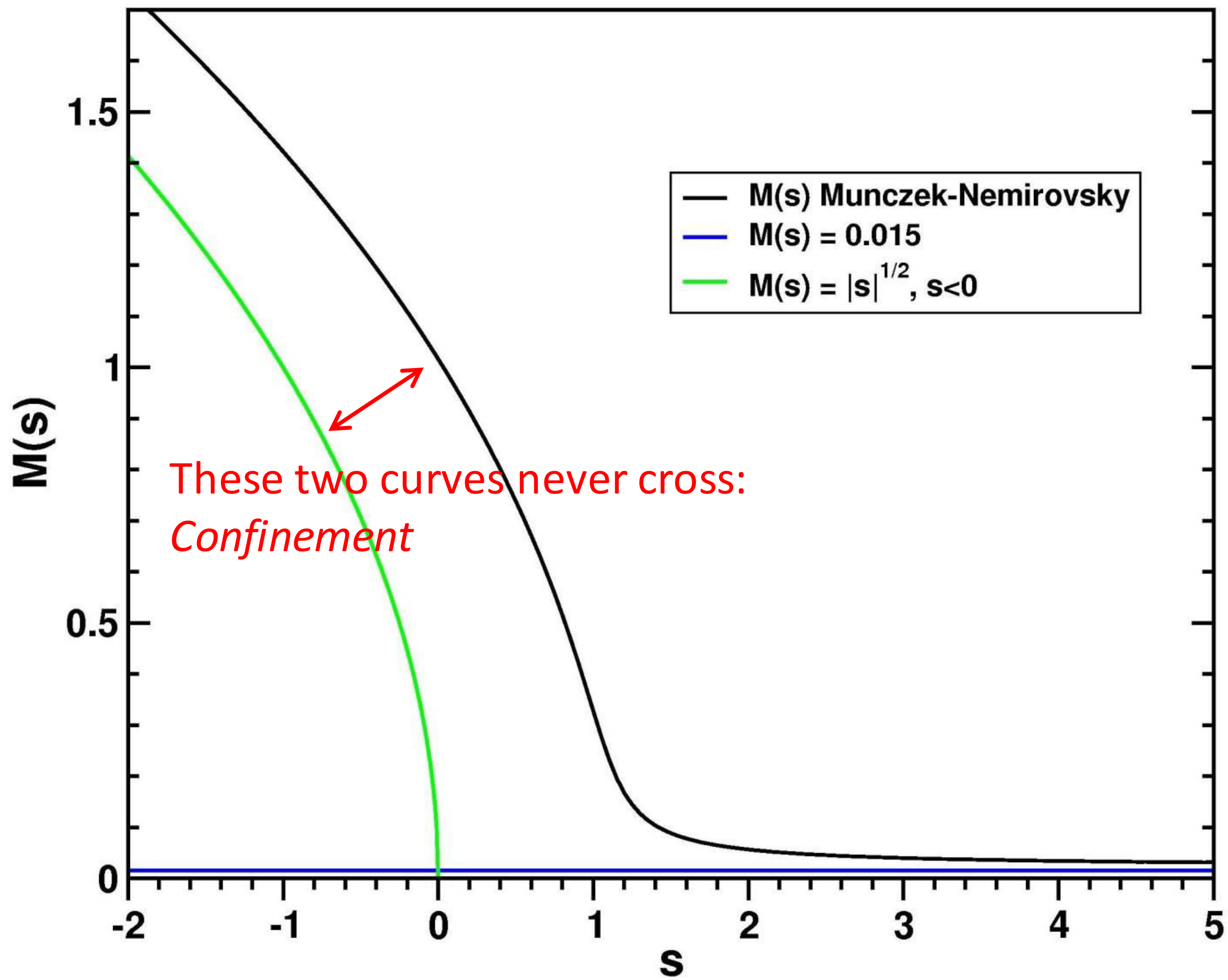
  This is the essential characteristic of DCSB

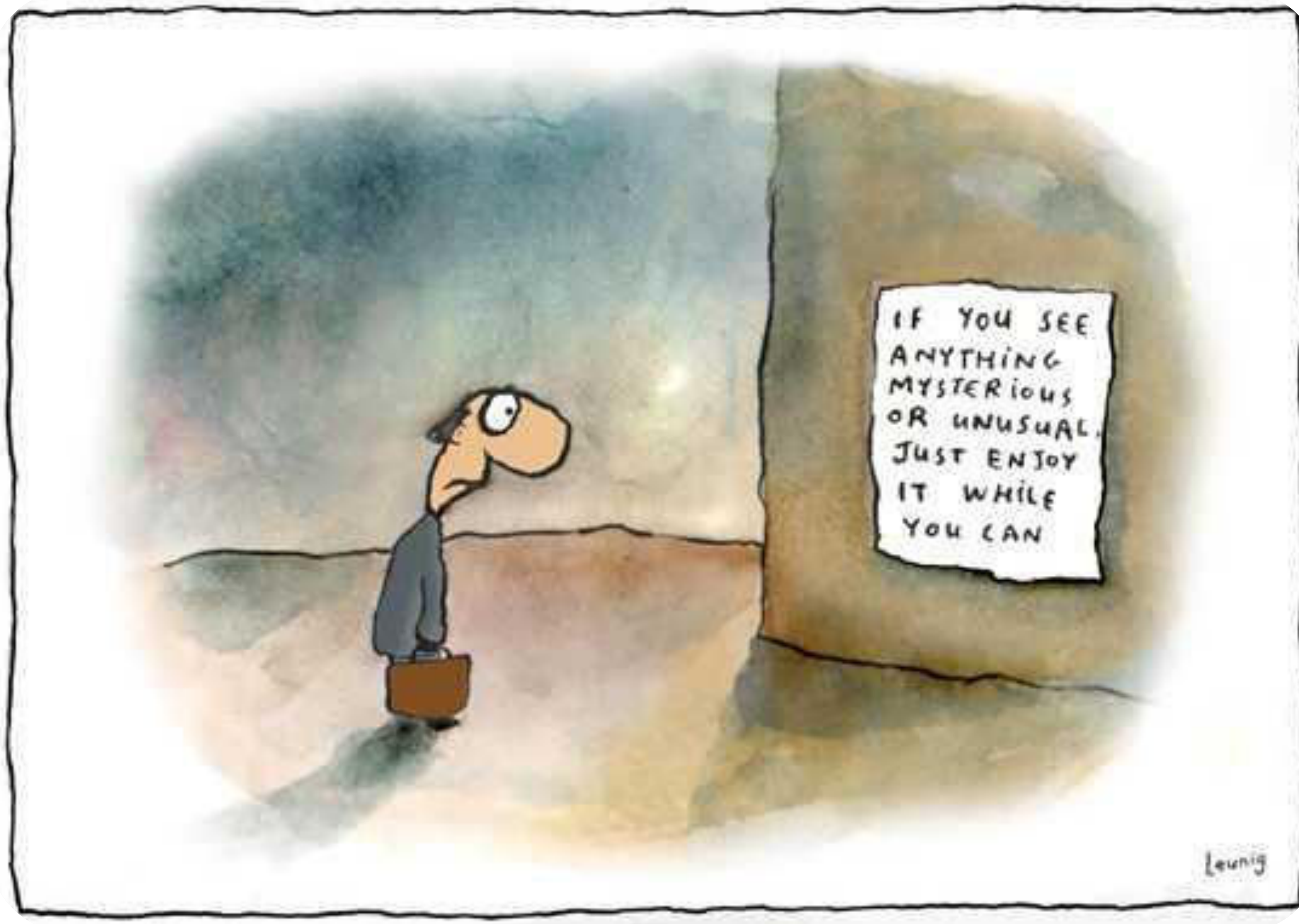
- We will see that p^2-dependent mass-functions are a quintessential feature of QCD.
- No solution of

  $$s + M(s)^2 = 0$$

  → *No plane-wave propagation*

  Confinement?!



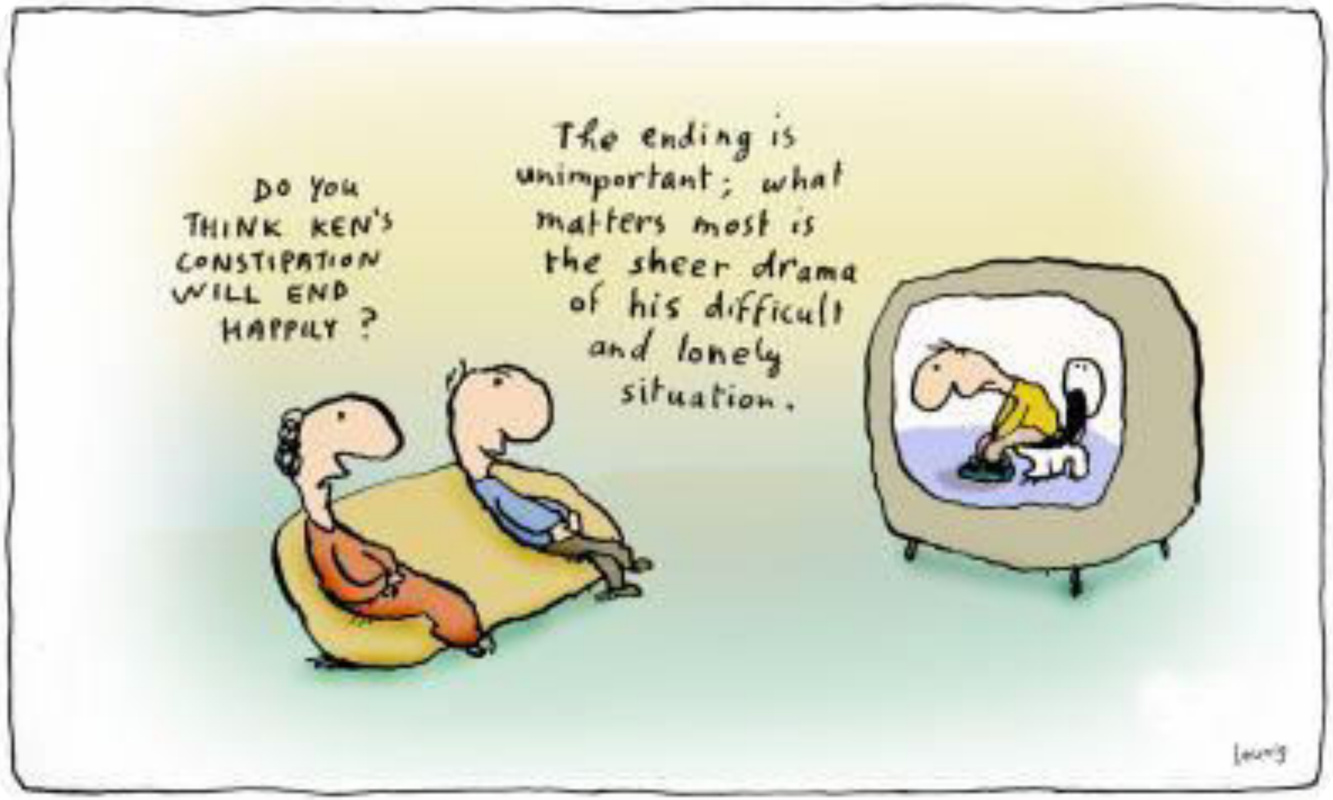

IF YOU SEE ANYTHING MYSTERIOUS OR UNUSUAL. JUST ENJOY IT WHILE YOU CAN
Leunig

Big Picture

Overview

- Confinement and Dynamical Chiral Symmetry Breaking are Key Emergent Phenomena in QCD
- Understanding requires Nonperturbative Solution of Fully-Fledged Relativistic Quantum Field Theory
  - Mathematics and Physics still far from being able to accomplish that
- Confinement and DCSB are expressed in QCD's propagators and vertices
  - Nonperturbative modifications should have observable consequences
- Dyson-Schwinger Equations are a useful analytical and numerical tool for nonperturbative study of relativistic quantum field theory
- Simple models (NJL) can exhibit DCSB
  - DCSB $\not\Rightarrow$ Confinement
- Simple models (MN) can exhibit Confinement
  - Confinement $\Rightarrow$ DCSB

What's the story in QCD?



Confinement

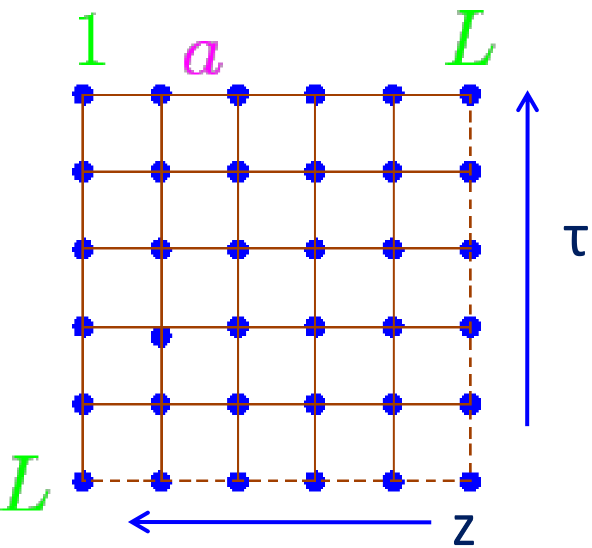
Confinement of quarks*

Kenneth G. Wilson
Laboratory of Nuclear Studies, Cornell University, Ithaca, New York 14850
(Received 12 June 1974)

A mechanism for total confinement of quarks, similar to that of Schwinger, is defined which requires the existence of Abelian or non-Abelian gauge fields. It is shown how to quantize a gauge field theory on a discrete lattice in Euclidean space-time, preserving exact gauge invariance and treating the gauge fields as angular variables (which makes a gauge-fixing term unnecessary). The lattice gauge theory has a computable strong-coupling limit; in this limit the binding mechanism applies and there are no free quarks. There is unfortunately no Lorentz (or Euclidean) invariance in the strong-coupling limit. The strong-coupling expansion involves sums over all quark paths and sums over all surfaces (on the lattice) joining quark paths. This structure is reminiscent of relativistic string models of hadrons.

Wilson Loop & the Area Law

$$W_C := \mathrm{Tr}\left(\mathcal{P}\exp i \oint_C A_\mu dx^\mu\right)$$



periodic lattice

- *C* is a closed curve in space,
  P is the path order operator
- Now, place static (infinitely heavy) fermionic sources of colour charge at positions

  $z_0=0$ & $z=\frac{1}{2}L$
- Then, evaluate $\langle W_C(z,\tau)\rangle$ as a functional integral over gauge-field configurations
- In the strong-coupling limit, the result can be obtained algebraically; viz.,

  $\langle W_C(z,\tau)\rangle = \exp(-V(z)\,\tau)$

  where *V(z)* is the potential between the static sources, which behaves as $V(z) = \sigma z$

Linear potential
σ = String tension

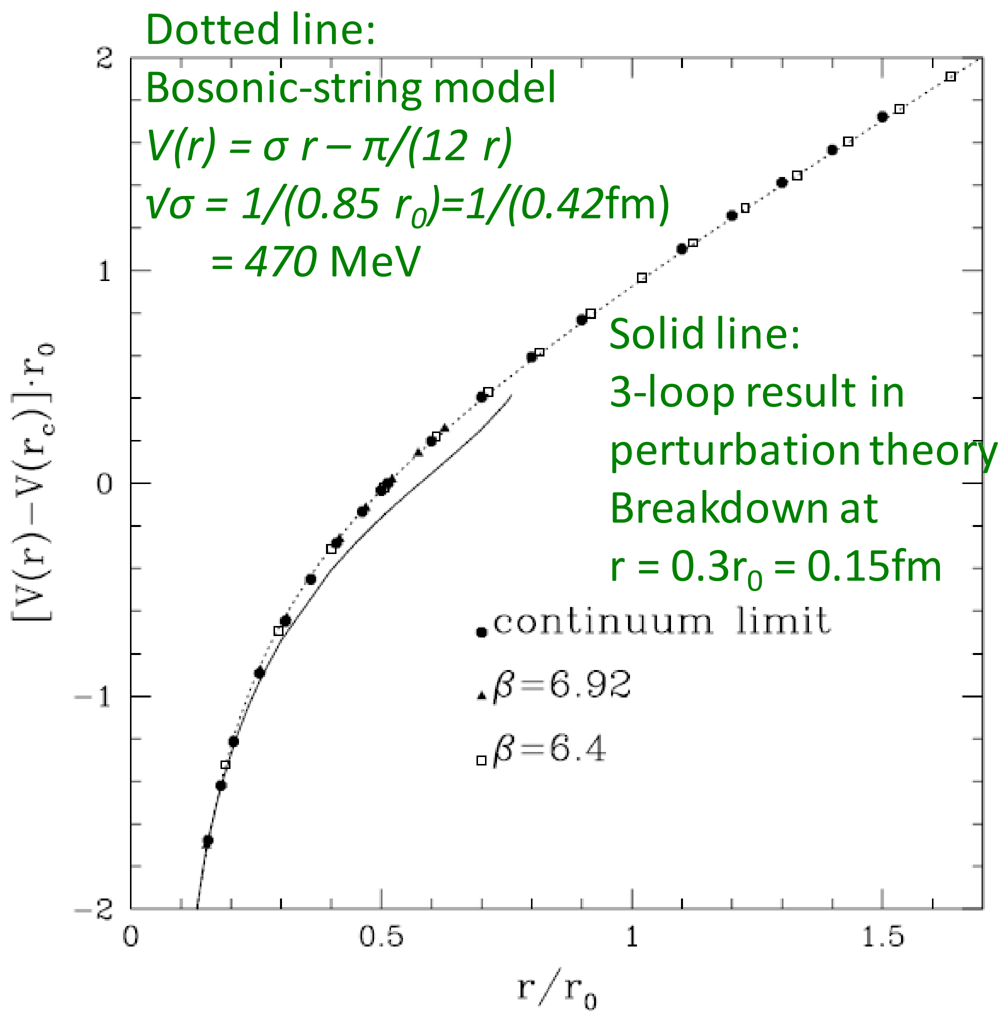
Wilson Loop & Area Law

- Typical result from a numerical simulation of pure-glue QCD (hep-lat/0108008)
- r_0 is the Sommer-parameter, which relates to the force between static quarks at intermediate distances.
- The requirement

  $r_0^2\, F(r_0) = 1.65$

  provides a connection between pure-glue QCD and potential models for mesons, and produces

  $r_0 \approx 0.5$ fm



Excerpt from the top-10

The New York Times

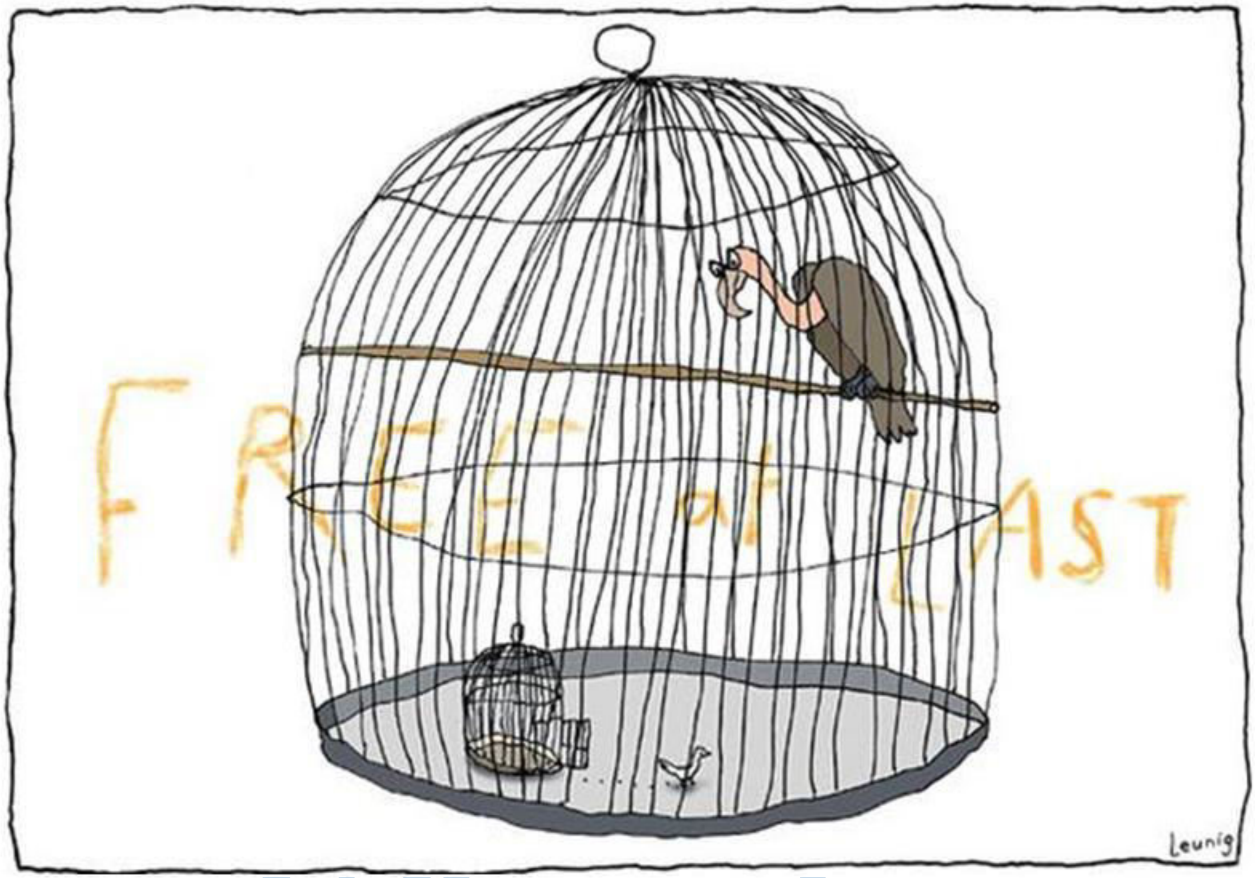
WORLD | U.S. | N.Y. / REGION | BUSINESS | TECHNOLOGY | SCIENCE | HEALTH | SPORTS | OPINION

10 Physics Questions to Ponder for a Millennium or Two

By George Johnson
Published: August 15, 2000

- ***Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?***

  Quantum chromodynamics is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.



What is Confinement?

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Millennium prize of $1,000,000 for proving that $SU_c(3)$ gauge theory is mathematically well-defined, which will necessarily prove or disprove a confinement conjecture

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on* $\mathbb{R}^4$ *and has a mass gap* $\Delta > 0$. *Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle \Omega, \mathcal{O}\Omega \rangle = 0$, one has

$$(2) \qquad |\langle \Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega \rangle| \leq \exp(-C|\vec{x}-\vec{y}|),$$

as long as $|\vec{x}-\vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to

Confinement?

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Yang–Mills Existence and Mass Gap. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on* $\mathbb{R}^4$ *and has a mass gap* $\Delta > 0$. *Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

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Confinement?

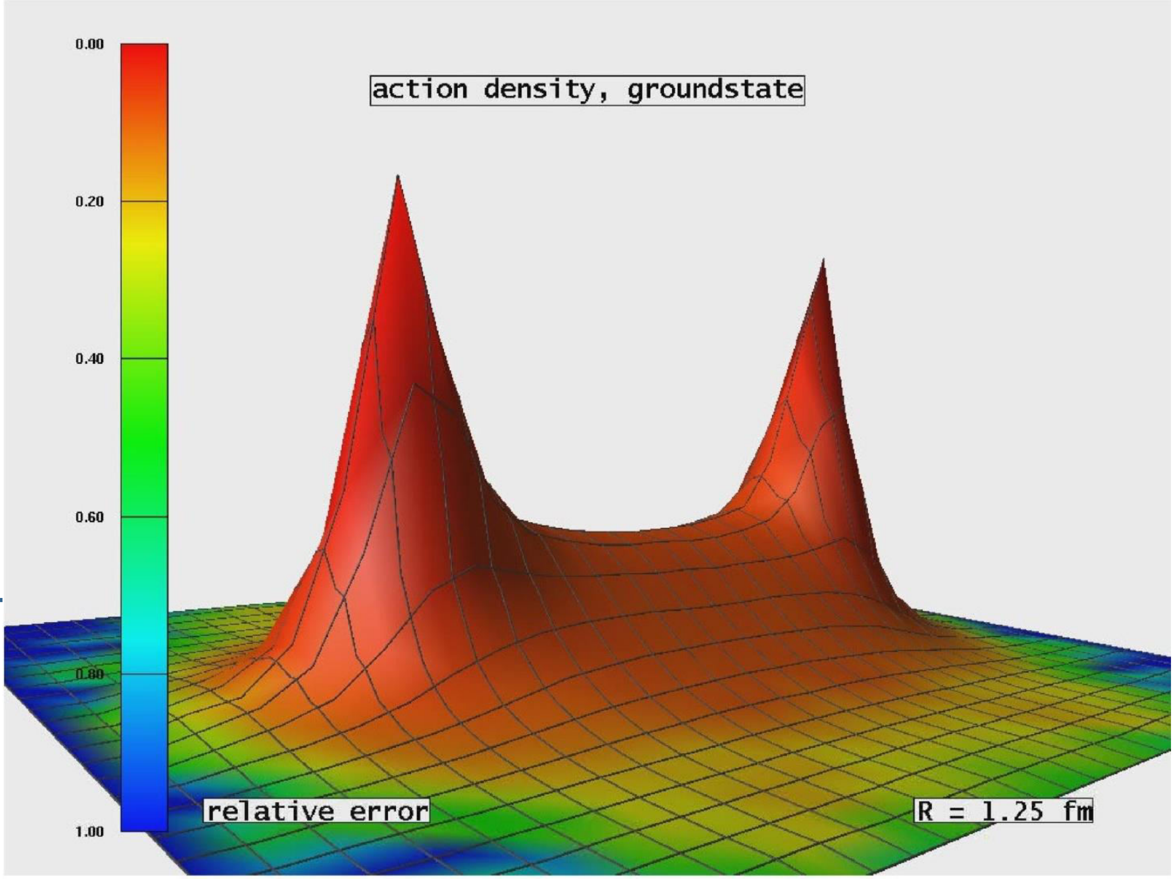
Light quarks & Confinement

- Folklore … *Hall-D Conceptual Design Report(5)*

"The color field lines between a quark and an anti-quark form flux tubes.

A unit area placed midway between the quarks and perpendicular to the line connecting them intercepts a constant number of field lines, independent of the distance between the quarks.

This leads to a constant force between the quarks – and a large force at that, equal to about 16 metric tons."



Light quarks & Confinement

- *Static* picture of confinement

4 × 10^{-27} g

16 × 10^{+6} g

8 × 10^{-27} g

Light quarks & Confinement

- Problem:

  16 tonnes of force

  makes a lot of pions.

Light quarks & Confinement

➢Problem:

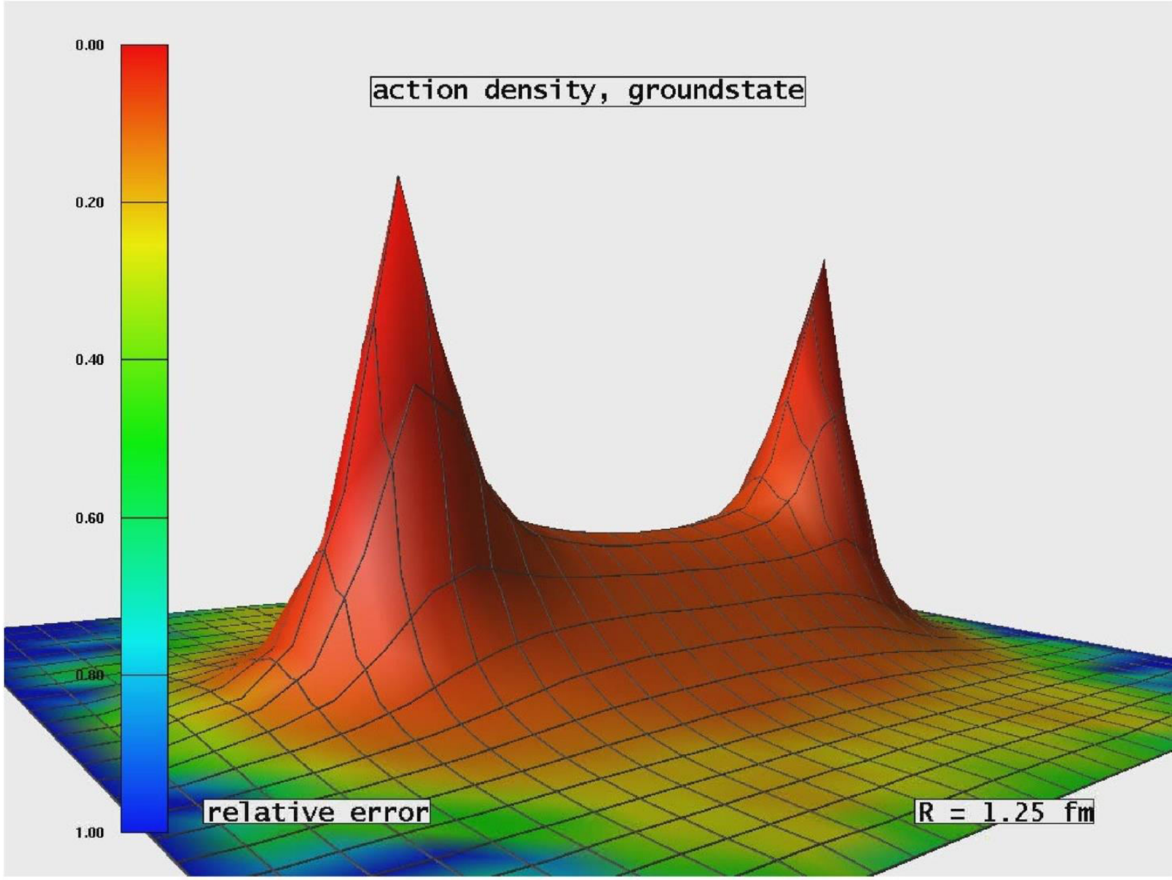
16 tonnes of force

makes a lot of pions.

G. Bali et al., *PoS LAT2005 (2006) 308*

Light quarks & Confinement

- In the presence of light quarks, *pair creation seems to occur non-localized and instantaneously*
- No flux tube in a theory with light-quarks.
- ***Flux-tube is not the correct paradigm for confinement in hadron physics***



G. Bali et al., *PoS LAT2005 (2006) 308*

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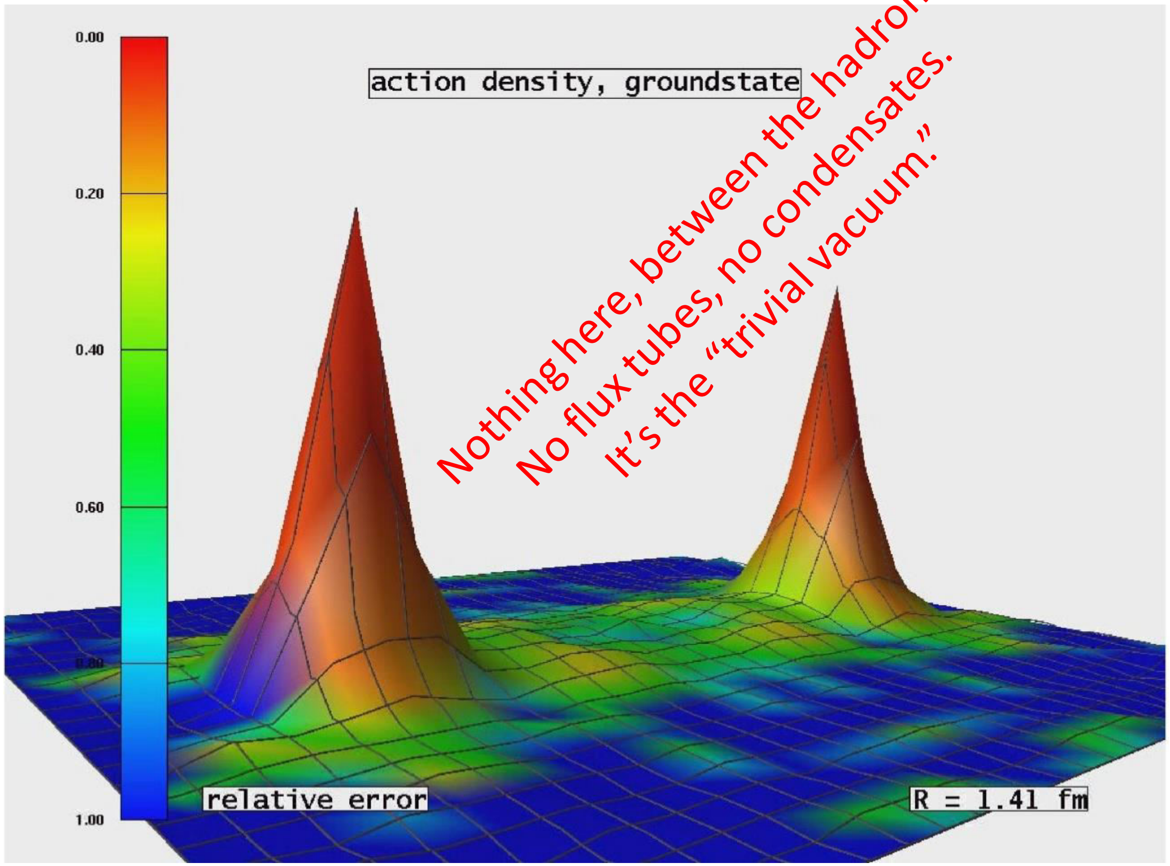
Confinement contains condensates
Brodsky, Roberts, Shrock, Tandy
arXiv:1202.2376 [nucl-th], Phys. Rev. C**85** (2012) 065202



- *Existence of mass-gap in pure-gauge theory*
- Strong evidence supporting this conjecture: lQCD predicts $\Delta \sim 1.5$ GeV
- But $\Delta^2/m_\pi^2 > 100$,

  So, can mass-gap in pure Yang-Mills play any role in understanding confinement when dynamical chiral symmetry breaking (DCSB) ensures existence of an almost-massless strongly-interacting excitation in our Universe?
- Conjecture: If *answer is* not *simply **no***, then it is probable that one cannot claim to provide an understanding of confinement without simultaneously explaining its connection with DCSB.
- Conjecture: *Pion must play critical role in any explanation of real-world confinement. Any discussion that omits reference to the pion's role is possibly irrelevant.*

YANG–MILLS EXISTENCE AND MASS GAP. *Prove that for any compact simple gauge group G, a non-trivial quantum Yang–Mills theory exists on $\mathbb{R}^4$ and has a mass gap $\Delta > 0$. Existence includes establishing axiomatic properties at least as strong as those cited in* [**45, 35**].

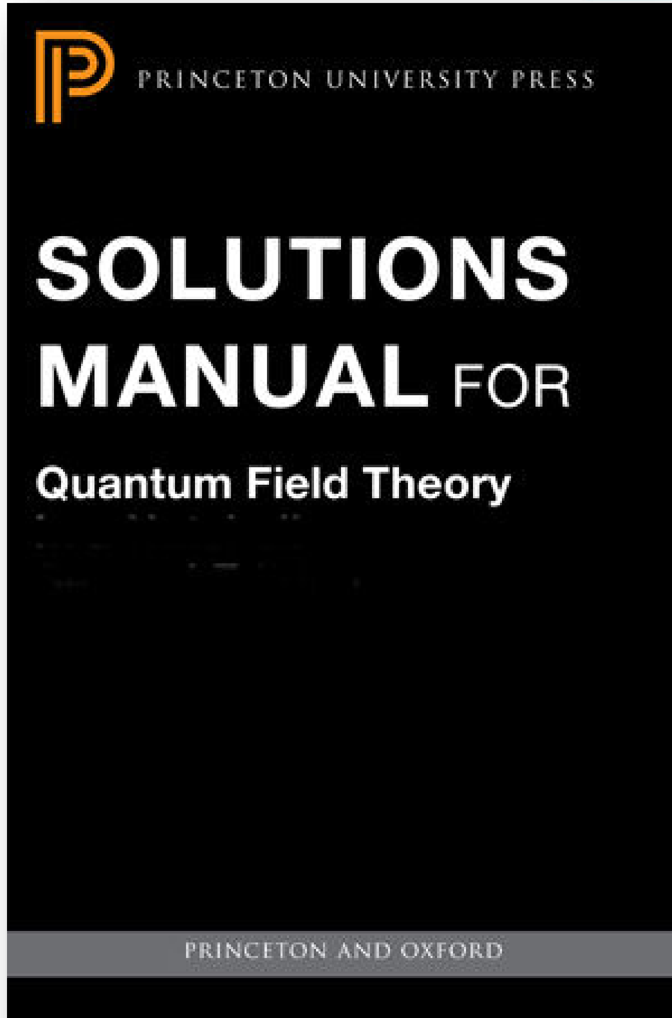
5. Comments

An important consequence of the existence of a mass gap is clustering: Let $\vec{x} \in \mathbb{R}^3$ denote a point in space. We let H and $\vec{P}$ denote the energy and momentum, generators of time and space translation. For any positive constant $C < \Delta$ and for any local quantum field operator $\mathcal{O}(\vec{x}) = e^{-i\vec{P}\cdot\vec{x}}\mathcal{O}e^{i\vec{P}\cdot\vec{x}}$ such that $\langle\Omega, \mathcal{O}\Omega\rangle = 0$, one has

$$|\langle\Omega, \mathcal{O}(\vec{x})\mathcal{O}(\vec{y})\Omega\rangle| \leq \exp(-C|\vec{x}-\vec{y}|), \tag{2}$$

as long as $|\vec{x}-\vec{y}|$ is sufficiently large. Clustering is a locality property that, roughly speaking, may make it possible to apply mathematical results established on $\mathbb{R}^4$ to any 4-manifold, as argued at a heuristic level (for a supersymmetric extension of four-dimensional gauge theory) in [**49**]. Thus the mass gap not only has a physical significance (as explained in the introduction), but it may also be important in mathematical applications of four-dimensional quantum gauge theories to geometry. In addition the existence of a uniform gap for finite-volume approximations may play a fundamental role in the proof of existence of the infinite-volume limit.

There are many natural extensions of the Millennium problem. Among other things, one would like to prove the existence of an isolated one-particle state (an upper gap, in addition to the mass gap), to prove confinement, to



Theoretical Answers

Textbook definition: Gauge Boson

- A gauge boson is a force carrier, mediating one of Nature's fundamental interactions
- All known gauge bosons have spin "1", *i.e*. all are vector bosons.
- Owing to gauge invariance, no term of the form

$$m^2 B_\mu B_\mu$$

can appear in the gauge theory Lagrangian.

- Thus, all gauge bosons are massless in the absence of a Higgs mechanism:
  - Photon … known to be massless
  - W and Z bosons … begin life massless, but known to become massive, owing to Higgs mechanism, which is abundantly clear in the Lagrangian
  - Gluon … there is no Higgs coupling and textbooks describe them as massless

Particle Data Group

Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

g or gluon

$$I(J^P) = 0(1^-)$$

SU(3) color octet

Mass $m = 0$. Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

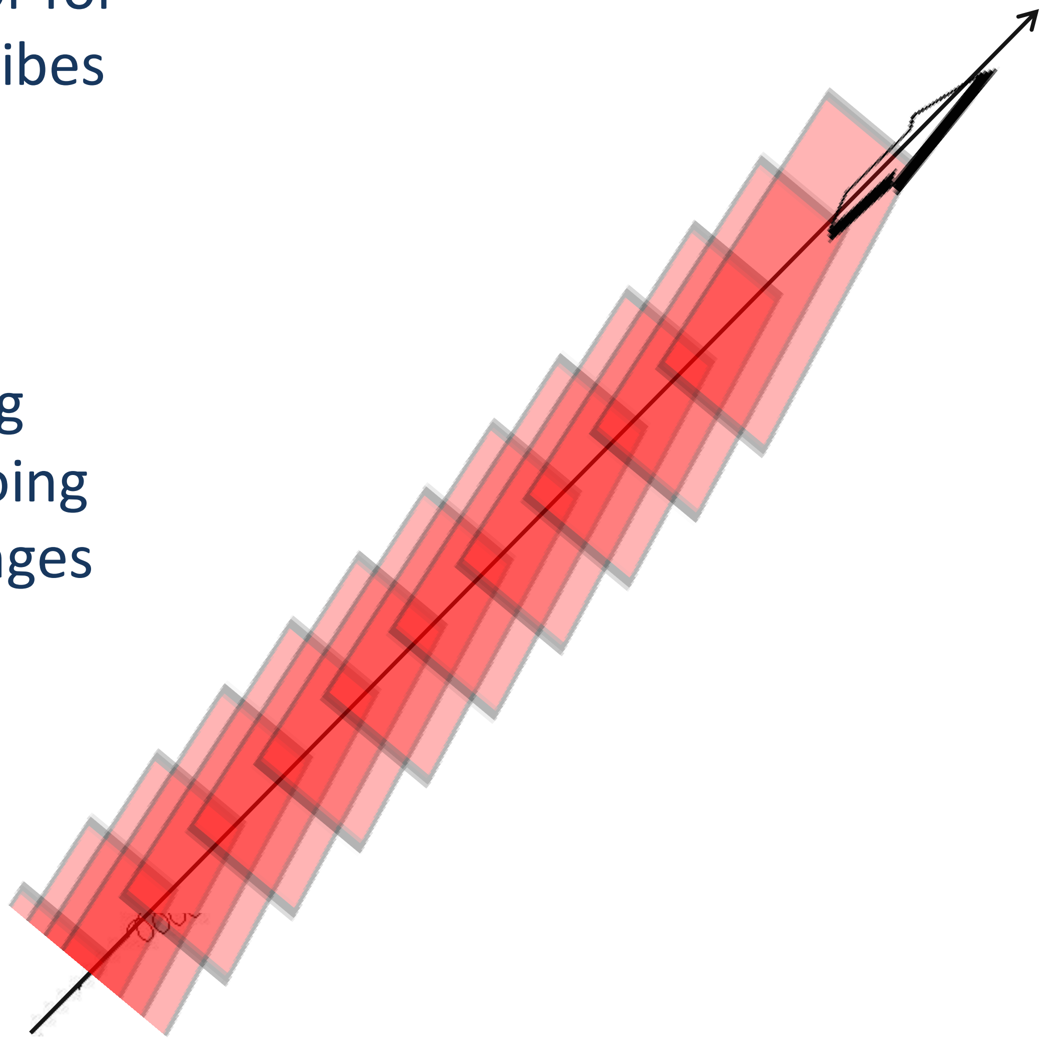
VALUE	*DOCUMENT ID*		*TECN*	*COMMENT*
• • • We do not use the following data for averages, fits, limits, etc. • • •				
	ABREU	92E	DLPH	Spin 1, not 0
	ALEXANDER	91H	OPAL	Spin 1, not 0
	BEHREND	82D	CELL	Spin 1, not 0
	BERGER	80D	PLUT	Spin 1, not 0
	BRANDELIK	80C	TASS	Spin 1, not 0

gluon REFERENCES

YNDURAIN	95	PL B345 524	F.J. Yndurain	(MADU)
ABREU	92E	PL B274 498	P. Abreu *et al.*	(DELPHI Collab.)
ALEXANDER	91H	ZPHY C52 543	G. Alexander *et al.*	(OPAL Collab.)
BEHREND	82D	PL B110 329	H.J. Behrend *et al.*	(CELLO Collab.)
BERGER	80D	PL B97 459	C. Berger *et al.*	(PLUTO Collab.)
BRANDELIK	80C	PL B97 453	R. Brandelik *et al.*	(TASSO Collab.)

Plane wave propagation

- Feynman propagator for a free particle describes a Plane Wave
- A particle begins to propagate
- It can proceed a long way before undergoing any qualitative changes

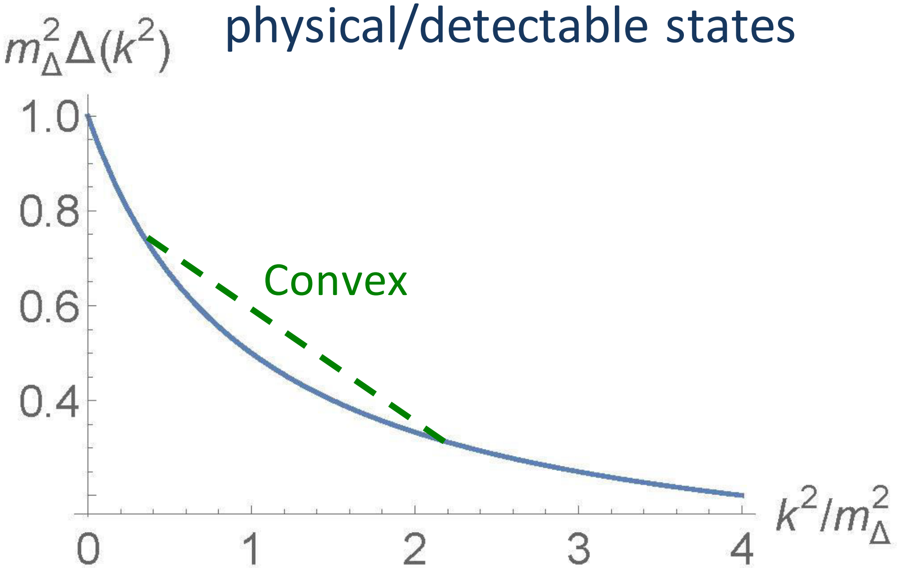


Normal Particle

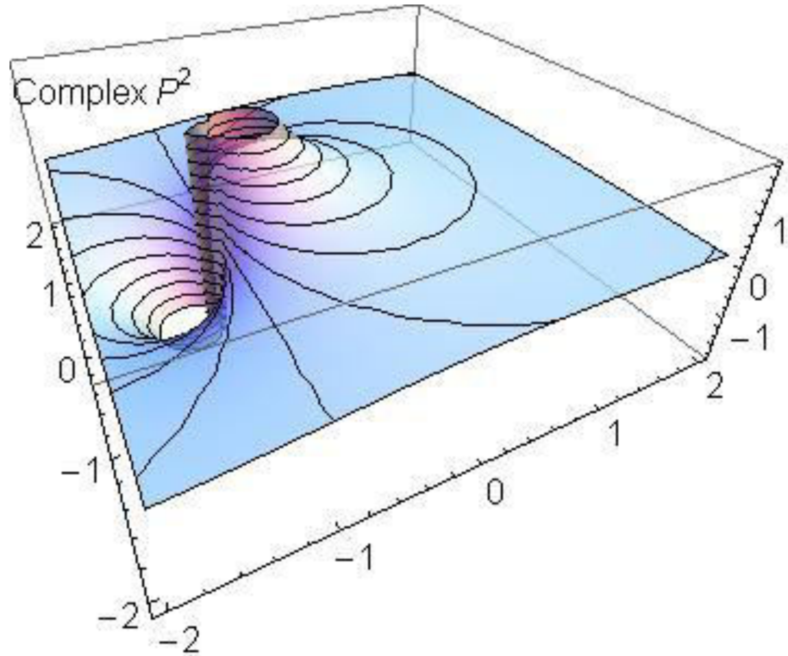
- Free-particle propagator
  - Convex function
- Spectral function is positive

$$\Delta(k^2) = \int_0^\infty ds \, \frac{\rho(s)}{s+k^2}$$

  - $\rho(s)>0$
- Corresponds to a state with positive norm
  - Will appear in the space of physical/detectable states

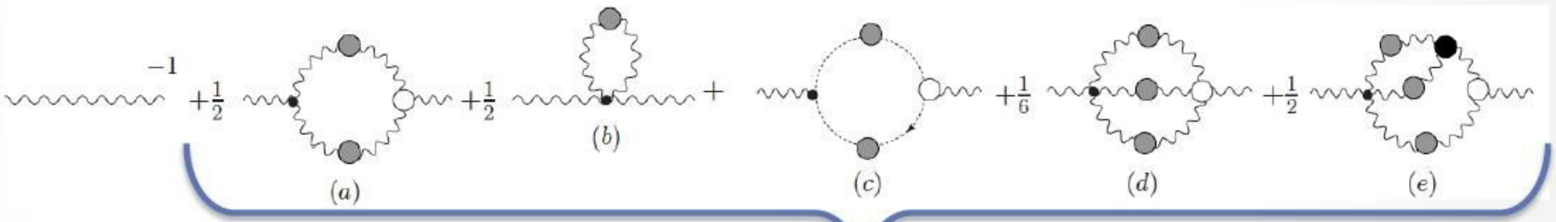


- Exhibits a simple pole on the timelike axis



Pinch Technique: Theory and Applications
Daniele Binosi & Joannis Papavassiliou
Phys. Rept. 479 (2009) 1-152

$$\Delta^{-1}_{\mu\nu}(q) =$$

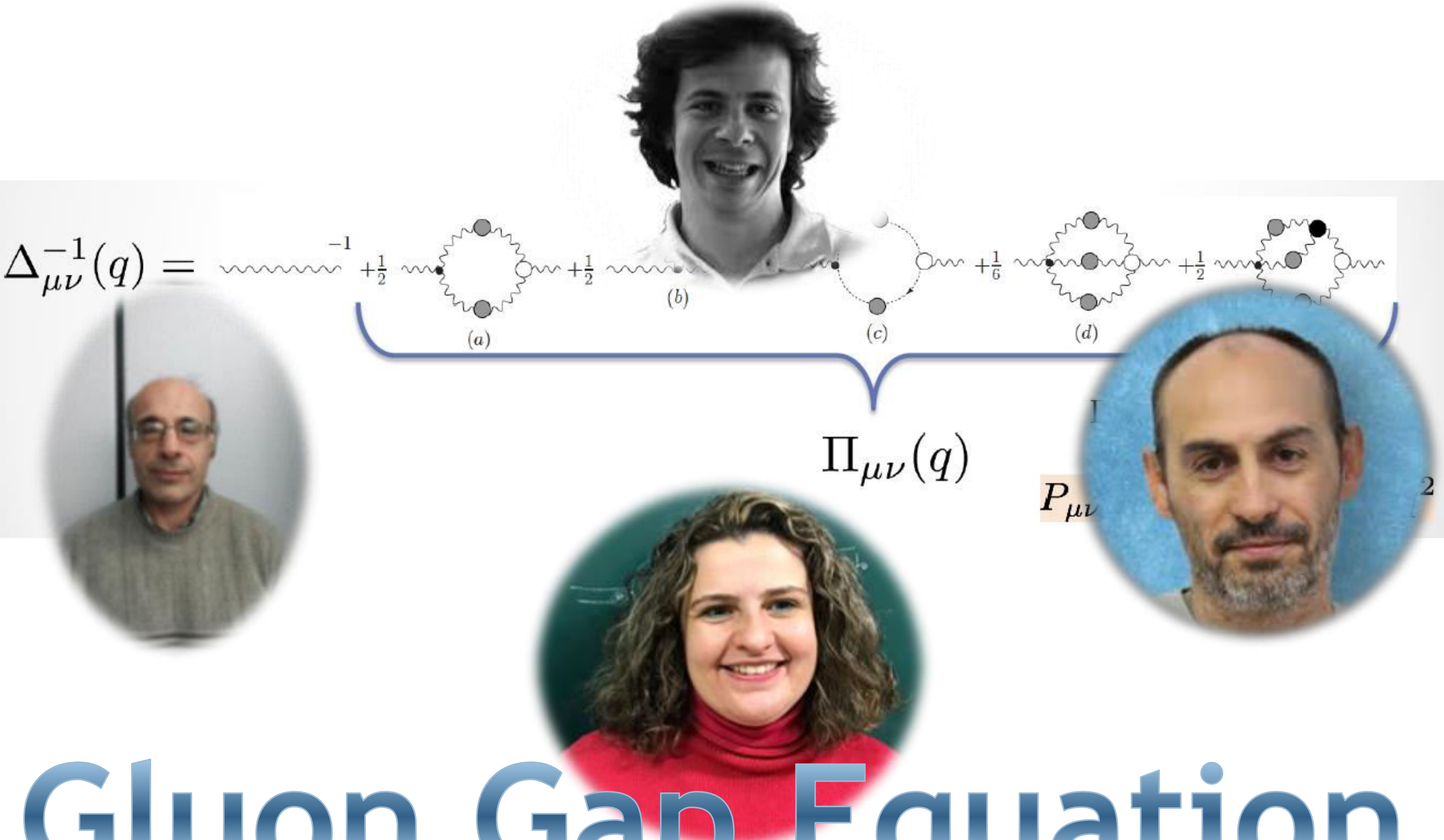


$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

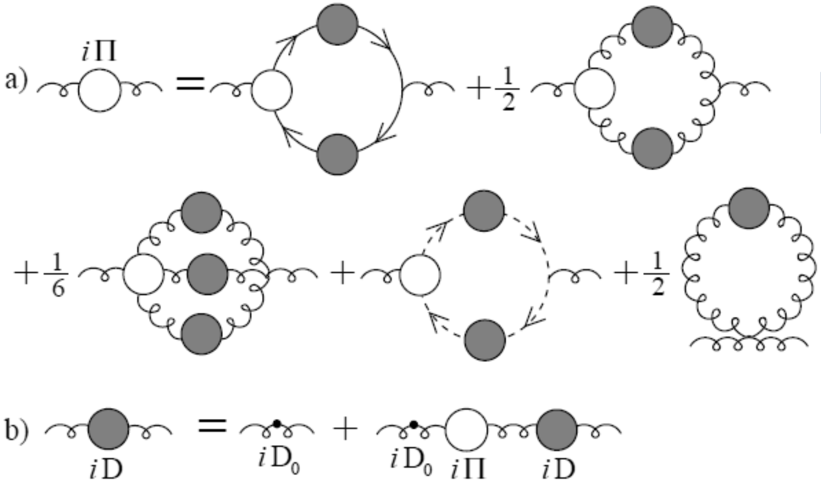
$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

Gluon Gap Equation



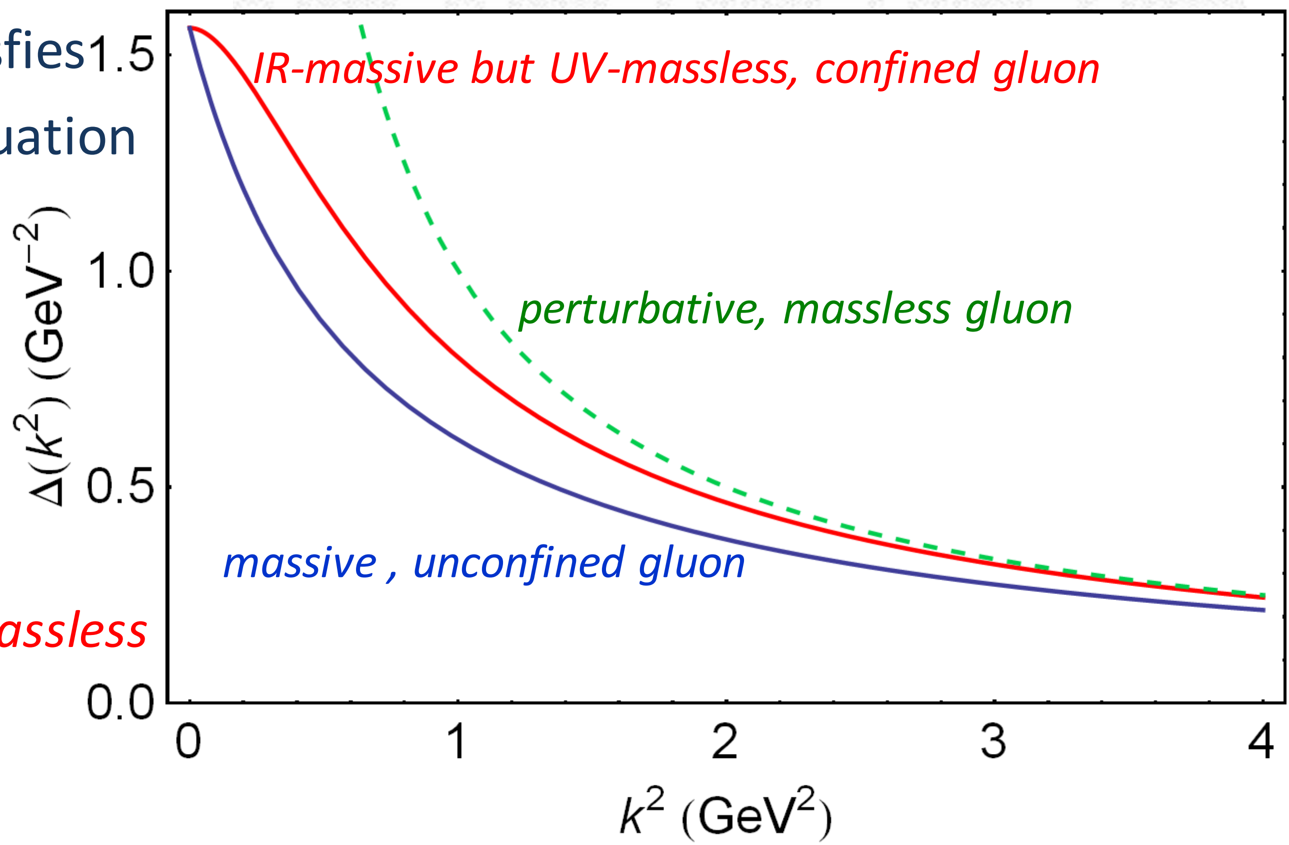
Gluon Gap Equation

Dressed-gluon propagator



A.C. Aguilar et al., *Phys.Rev. D80 (2009) 085018*

- Gluon propagator satisfies a Dyson-Schwinger Equation
- Plausible possibilities for the solution
- DSE and lattice-QCD agree on the result
  - *Confined gluon*
  - *IR-massive but UV-massless*
  - $m_G \approx 2\text{-}4\ \Lambda_{QCD}$



Bridging a gap between continuum-QCD and ab initio predictions of hadron observables, D. Binosi *et al.*, arXiv:1412.4782 [nucl-th], Phys. Lett. B742 (2015) 183-188

In QCD: Gluons become massive!

- Running gluon mass

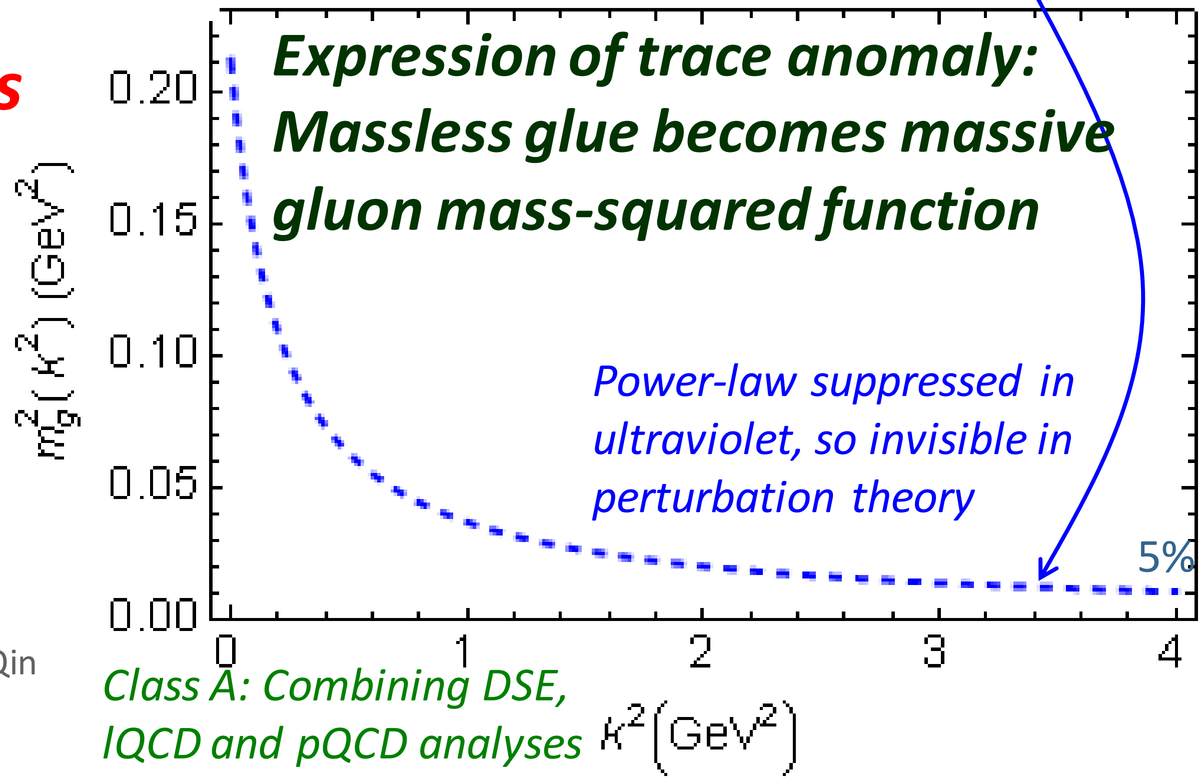
$$d(k^2) = \frac{\alpha(\zeta)}{k^2 + m_g^2(k^2;\zeta)}$$

$$\alpha_s(0) = 2.77 \approx 0.9\pi\,,\; m_g^2(0) = (0.46\,\mathrm{GeV})^2$$

- Gluons are ***cannibals*** – a particle species whose members become massive by eating each other!

$\mu_g \approx ½\, m_p$

$$m_g^2(k^2) \approx \frac{\mu_g^4}{\mu_g^2 + k^2}$$



Expression of trace anomaly: Massless glue becomes massive gluon mass-squared function

Power-law suppressed in ultraviolet, so invisible in perturbation theory

Class A: Combining DSE, lQCD and pQCD analyses of QCD's gauge sector

Interaction model for the gap equation, S.-x.Qin *et al.*, arXiv:1108.0603 [nucl-th], Phys. Rev. C **84** (2011) 042202(R) [5 pages]

Confined particle

- All QCD solutions for gluon & quark propagators exhibit an inflection point in k^2 … consequence of the running-mass function

⇒ Spectral function is NOT positive

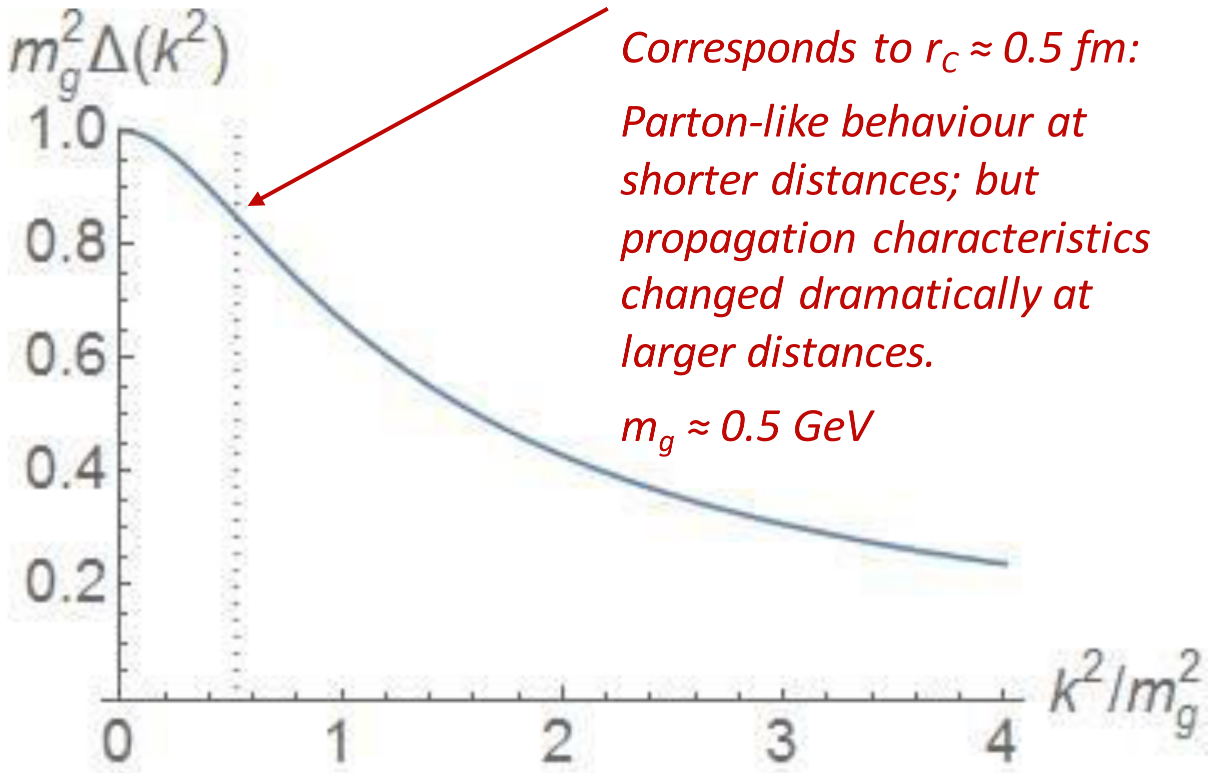
⇒ Such states have negative norm (negative probability)

⇒ Negative norm states are not observable

⇒ This object is confined!

$$\Delta(k^2) = \int_0^\infty ds\, \frac{\rho(s)}{s+k^2}$$

Sum of "probabilities"



Inflexion point

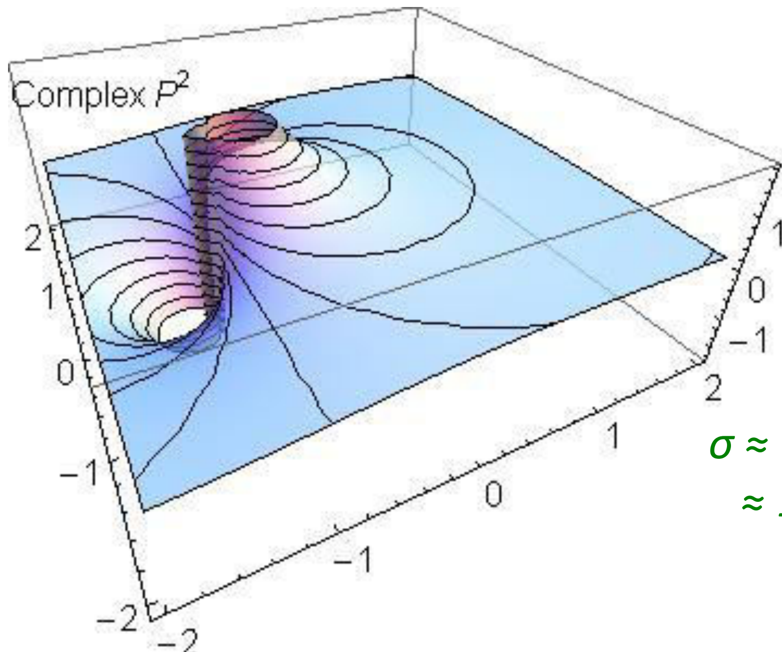
Corresponds to $r_C \approx 0.5$ fm:

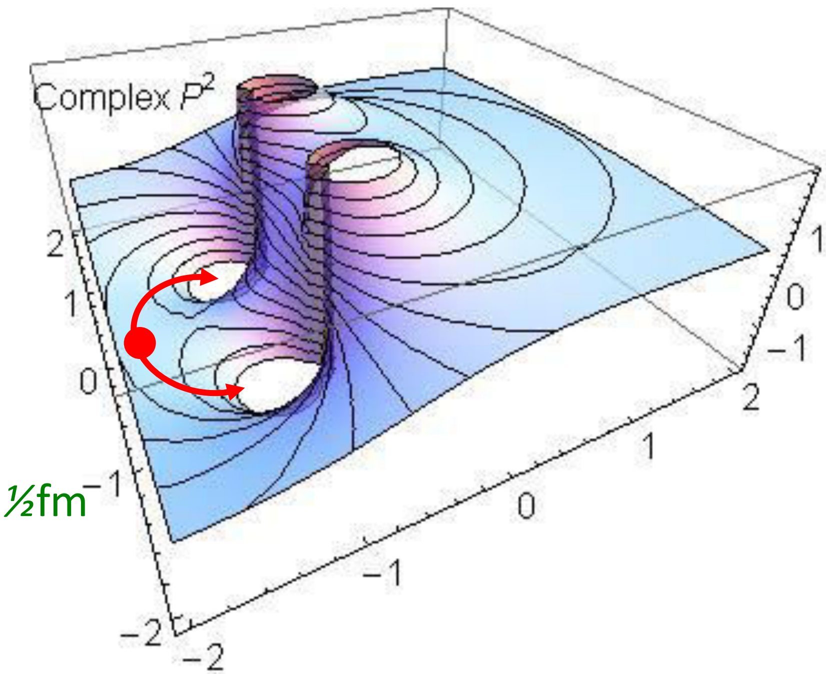
Parton-like behaviour at shorter distances; but propagation characteristics changed dramatically at larger distances.

$m_g \approx 0.5$ GeV

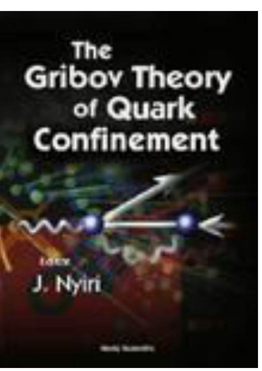
Confinement

➢ Meaning …





$\sigma \approx 1/\text{Im}(m)$
$\approx 1/2\Lambda_{QCD} \approx$ ½fm

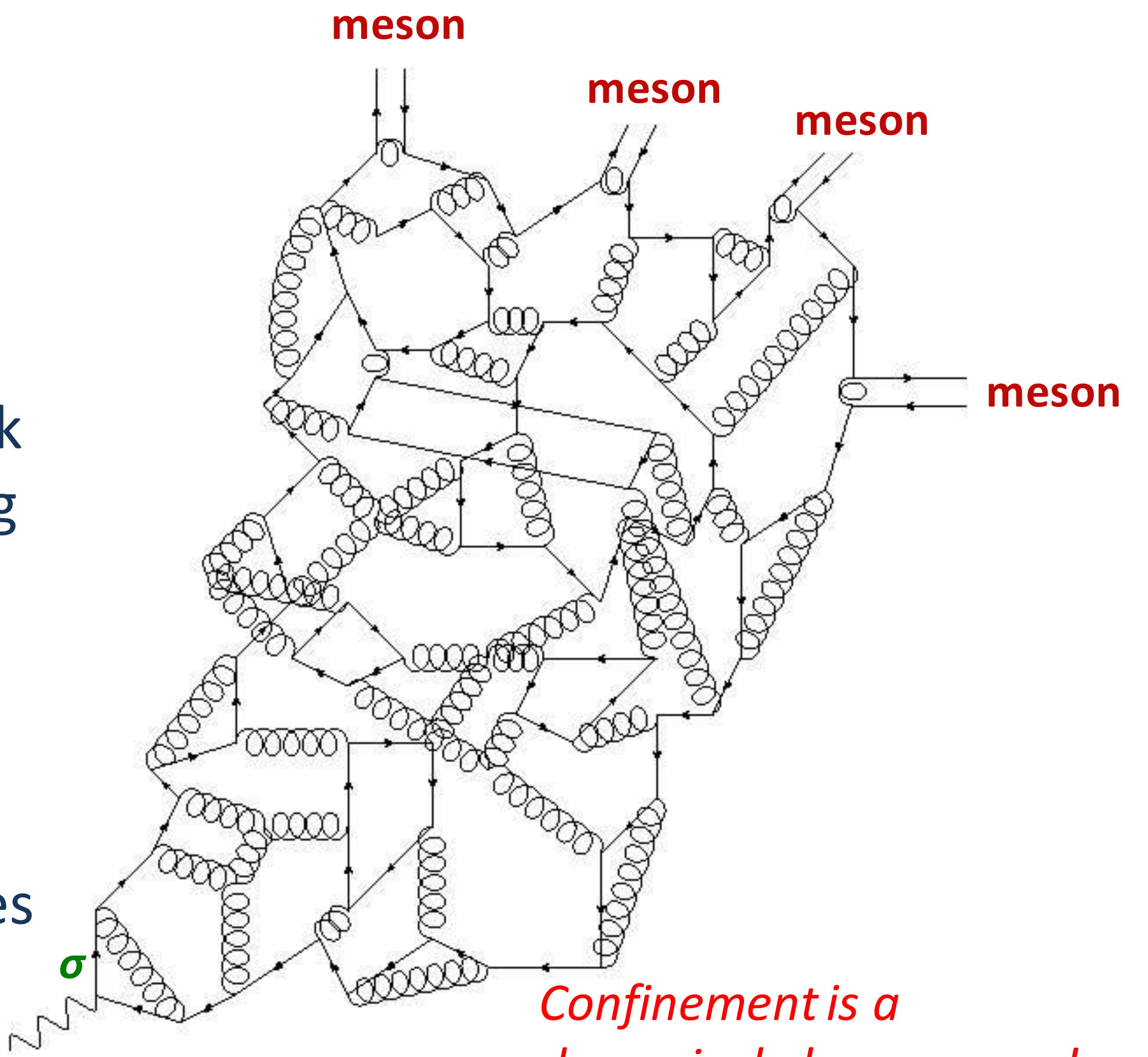


Real-particle mass-pole splits, moving into pair(s) of complex conjugate singularities, (or qualitatively analogous structures chracterised by a dynamically generated mass-scale)

Propagation described by rapidly damped wave & hence state cannot exist in observable spectrum

Quark Fragmentation

- A quark begins to propagate
- But after each "step" of length $\sigma \approx 1/m_g$, on average, an interaction occurs, so that the quark *loses* its identity, sharing it with other partons
- Finally, a cloud of partons is produced, which coalesces into colour-singlet final states



Confinement is a dynamical phenomenon!

Charting the interaction between light-quarks

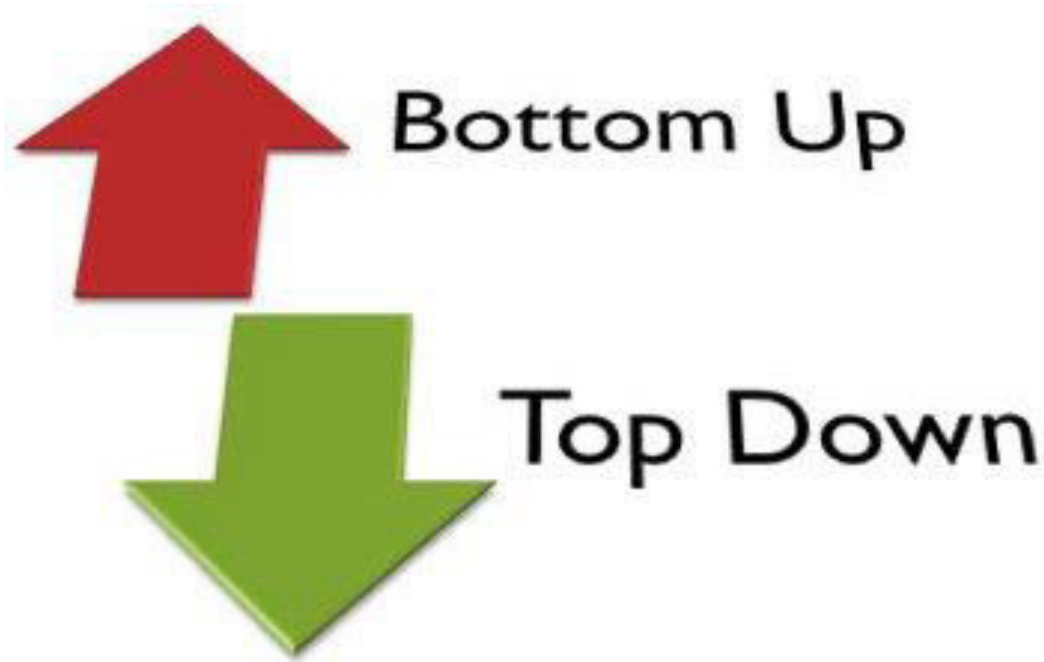
This is a well-posed problem whose solution is an elemental goal of modern hadron physics. The answer provides QCD's running coupling.

- Confinement can be related to the analytic properties of QCD's Schwinger functions.
- Question of light-quark confinement can be translated into the challenge of charting the infrared behavior of QCD's universal β-function
  - This function may depend on the scheme chosen to renormalise the quantum field theory but it is unique within a given scheme.
  - Of course, the behaviour of the β-function on the perturbative domain is well known.

Charting the interaction between light-quarks

- Through QCD's Dyson-Schwinger equations (DSEs) the pointwise behaviour of the β-function determines the pattern of chiral symmetry breaking.
- DSEs connect β-function to experimental observables. Hence, comparison between computations and observations of
  - Hadron mass spectrum
  - Elastic and transition form factors
  - Parton distribution functions

  can be used to chart β-function's long-range behaviour.
- Extant studies show that the properties of hadron excited states are a great deal more sensitive to the long-range behaviour of the β-function than those of the ground states.



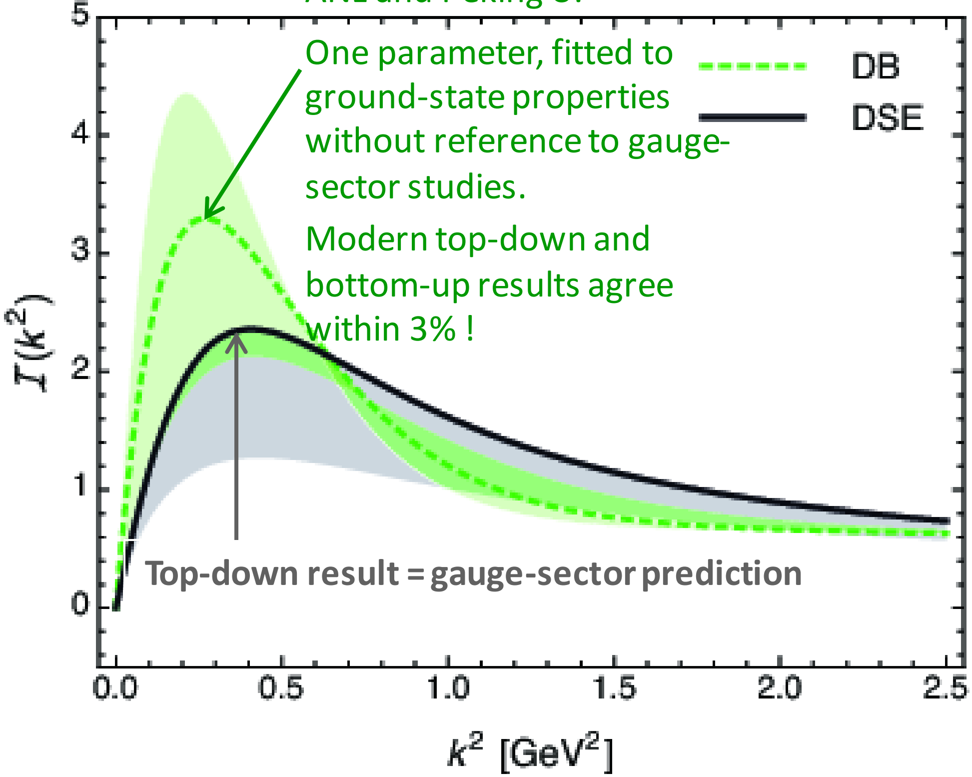
Continuum-QCD & *ab initio* predictions

Bridging a gap between continuum-QCD & ab initio predictions of hadron observables

Top down & Bottom up

D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain), C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

- *Top-down approach* – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *Bottom-up scheme – infer interaction by fitting data within a well-defined truncation of the matter sector DSEs that are relevant to bound-state properties.*
- *Serendipitous collaboration, conceived at one-week ECT* Workshop on* DSEs in Mathematics and Physics, *has united these two approaches*



– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation

Bridging a gap between continuum-QCD & ab initio *predictions of hadron observables*

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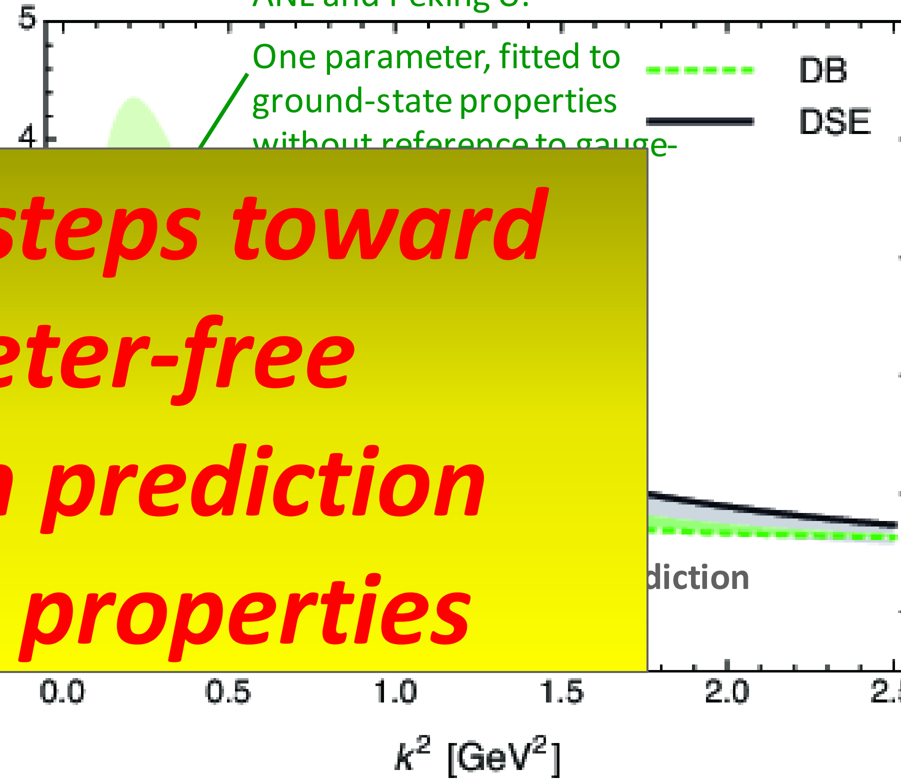
D. Binosi (Italy), L. Chang (Australia), J. Papavassiliou (Spain),
C. D. Roberts (US), arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

- *Top-down approach* – ab initio *computation of the interaction via direct analysis of the gauge-sector gap equations*
- *Bottom-up sc… by fitting data… truncation of… that are relev… properties.*
- *Serendipitous… at one-week E…* in Mathematics and Physics, *has united these two approaches*

Modern kernels and interaction, developed at ANL and Peking U.

One parameter, fitted to ground-state properties without reference to gauge-

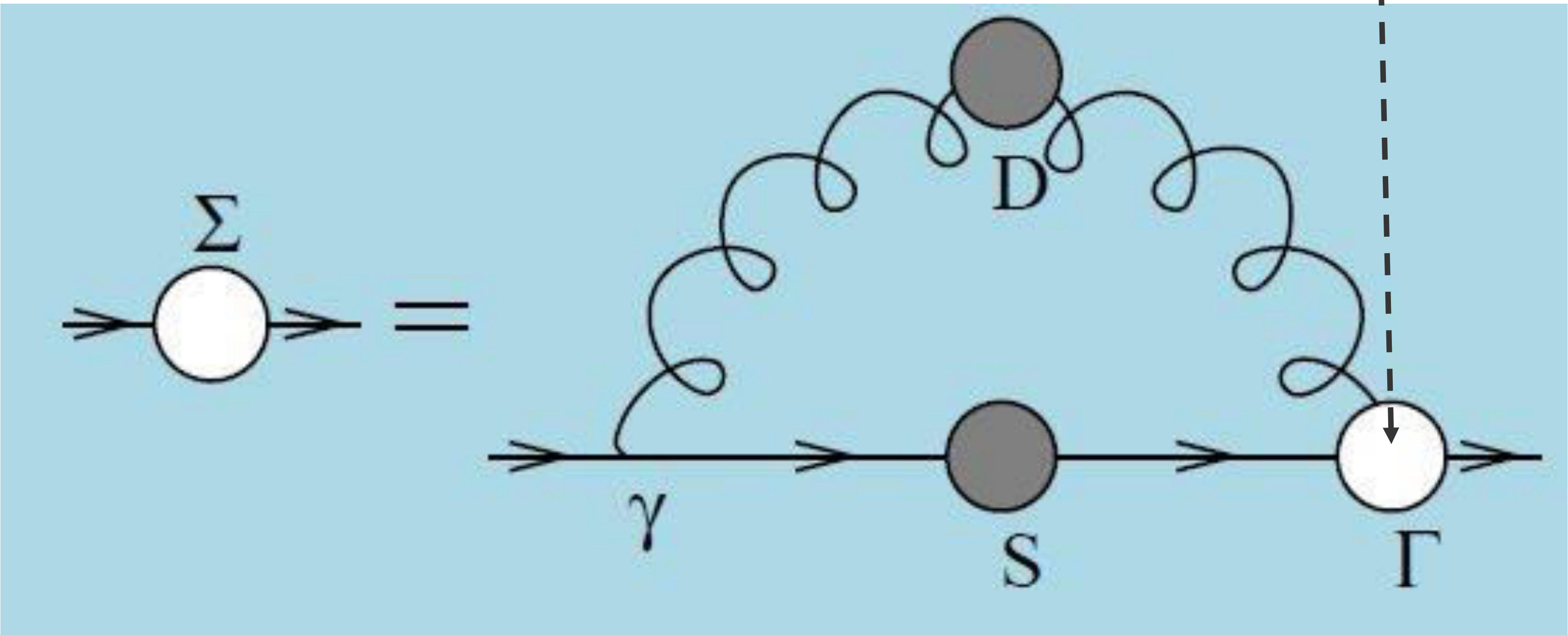


Significant steps toward parameter-free continuum prediction of hadron properties

– Interaction predicted by modern analyses of QCD's gauge sector coincides with that required to describe ground-state observables using the sophisticated matter-sector ANL-PKU DSE truncation

$$S(p) = \frac{Z(p^2)}{i\gamma \cdot p + M(p^2)}$$

Dressed gluon-quark vertex



Quark Gap Equation

Natural constraints on the gluon-quark vertex, Binosi, Chang, Papavassiliou, Qin & Roberts, *arXiv:1609.02568 [nucl-th]*, Phys. Rev. D 95 (2017) 031501(R)/1-7

Reconciliation demands dressed-gluon-quark vertex

- Significant progress since 2009:
  dressed Γ_μ in gap- and Bethe-Salpeter equations …
- In principle, ∃ unique form of Γ_μ , but it's still obscure.
- To improve this situation, used the top-down/bottom-up RGI running-interaction
  - Computed gap equation solutions with
    1,660,000 distinct *Ansätze* for Γ_μ
- Each one of the solutions tested for compatibility with three physical criteria
- Remarkably, merely 0.55% of solutions survive the test

⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

$$\tau_1^{qk} = a_1 \frac{\Delta_B^{qk}}{q^2 + k^2} \qquad \tau_3^{qk} = -a_3 2\Delta_A^{qk},$$

$$\tau_4^{qk} = a_4 \frac{4\Delta_B^{qk}}{t^{\mathrm{T}} \cdot t^{\mathrm{T}}}, \quad \tau_5^{qk} = a_5 \Delta_B^{qk},$$

$$\tau_8^{qk} = a_8 \Delta_A^{qk},$$

$$T_\nu^1 = \tfrac{i}{2} t_\nu^{\mathrm{T}}, \qquad T_\nu^3 = \gamma_\nu^{\mathrm{T}},$$

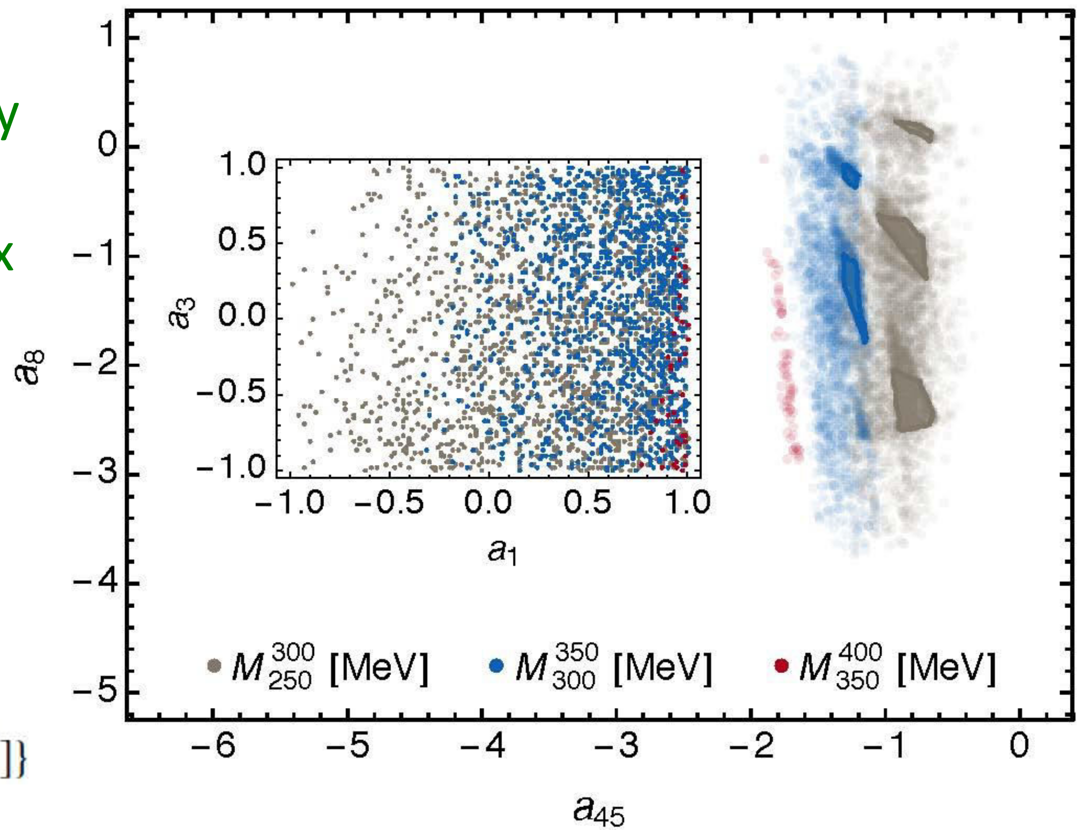
$$T_\nu^4 = -i T_\nu^1 \sigma_{\alpha\beta} q_\alpha k_\beta, \quad T_\nu^5 = \sigma_{\nu\rho} p_\rho,$$

$$T_\nu^8 = q_\nu \gamma \cdot k - k_\nu \gamma \cdot q + i \gamma_\nu \sigma_{\alpha\beta} q_\alpha k_\beta, \tag{A1}$$

⇒ Even a small selection of observables places extremely tight bounds on the domain of acceptable, realistic vertex *Ansätze*

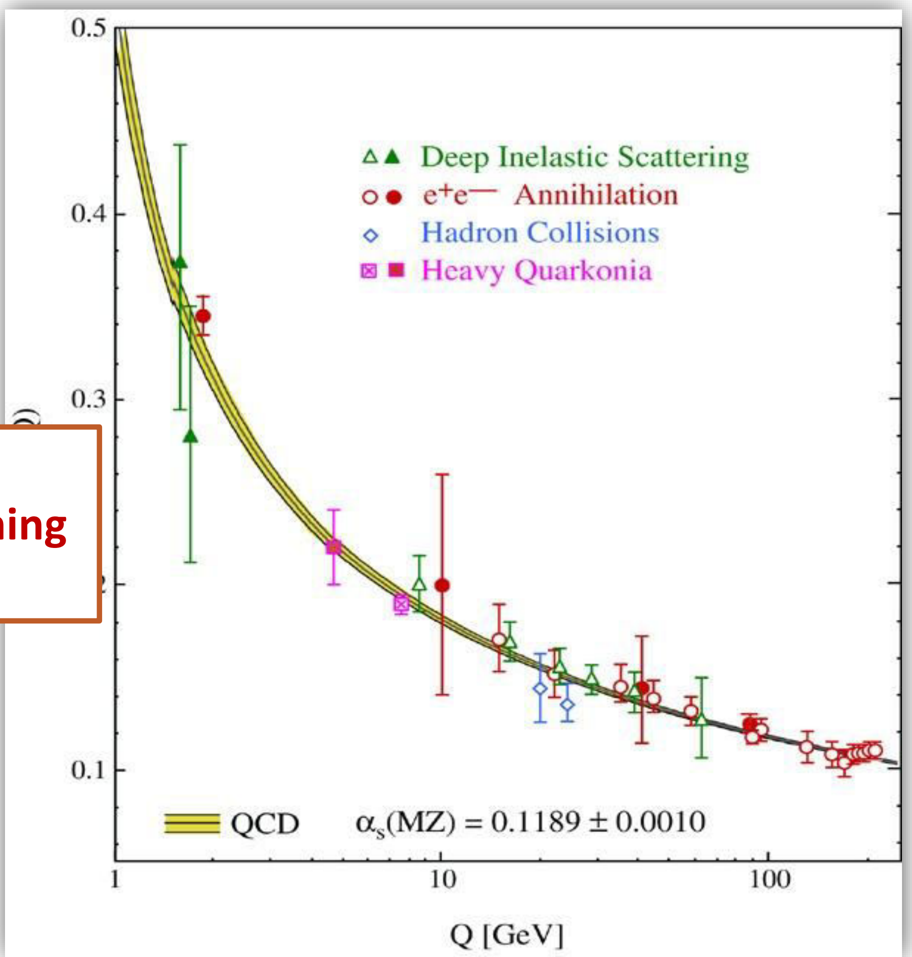
- Meson spectrum ⇒ $a_{2,6,7} = 0$
  (Sixue Qin *et al.*)
- In $\mathbf{R}^4$ … subset of (almost) zero measure

$$\mathbb{G}_4 \subset \{(a_1, a_3, a_{\hat{45}}, a_8) \,|\, a_1 \in [-0.5, 1], \; a_3 \in [-1, 1], a_{\hat{45}} \in [-2, -0.4], a_8 \in [-4, 1]\}$$



Dressed-gluon-quark vertex

Gap equation only "feels" $a_{45} = a_4 - 3a_5$

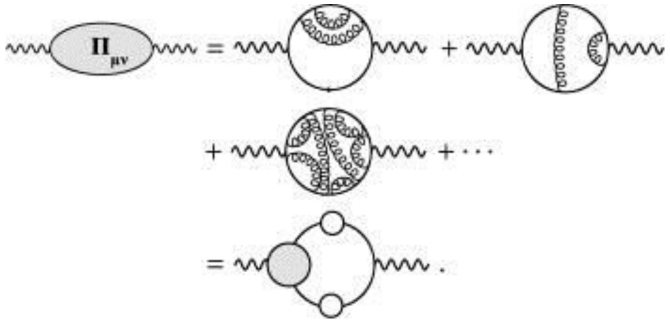


⇐
What's happening out here?!

QCD's Running Coupling

QED Running Coupling

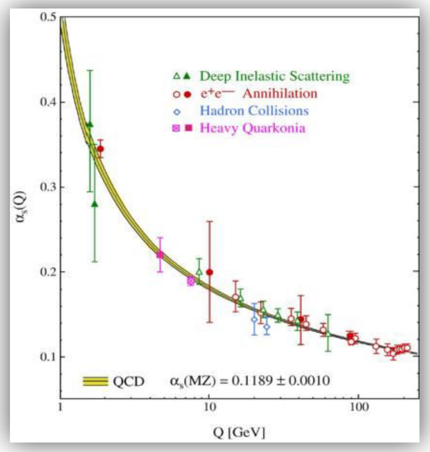
- Quantum gauge field theories defined in four spacetime dimensions,
  - Lagrangian couplings and masses come to depend on a mass scale
  - Can often be related to the energy or momentum at which a given process occurs.
- Archetype is QED, for which there is a sensible perturbation theory.
- QED, owing to the Ward identity:
  - a single running coupling
  - measures strength of the photon-charged-fermion vertex
  - can be obtained by summing the virtual processes that dress the bare photon, *viz*. by computing the photon vacuum polarisation.



- QED's running coupling is known to great accuracy and the running has been observed directly.

QCD Running Coupling

- At first sight, addition of QCD to Standard Model does not qualitatively change anything, despite presence of four possibly distinct strong-interaction vertices in the renormalized theory
  - gluon-ghost, three-gluon, four-gluon and gluon-quark.
- An array of Slavnov-Taylor identities (STIs) implementing BRST symmetry – generalisation of non-Abelian gauge invariance for the quantised theory – ensures that a single running coupling characterises all four interactions on perturbative domain.
- New Feature:
  - QCD is asymptotically free and extant evidence suggests that perturbation theory is valid at large momentum scales
  - But all dynamics is nonperturbative at scales typical of everyday strong-interaction phenomena, *e.g.* $\zeta \leq m_p$

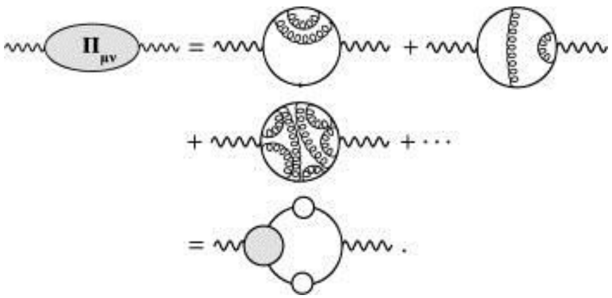


Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

- Four individual, apparently UV-divergent interaction vertices in perturbative QCD ⇒ possibly four distinct IR couplings.
  - Naturally, if nonperturbatively there are two or more couplings, they must all become equivalent on the perturbative domain.
- Questions:
  - How many distinct running couplings exist in nonperturbative QCD?
  - How can they be computed?
  - If defined using a 3- or 4-point vertex, which arrangement of momenta defines the running? (Infinitely many choices.)
- ***Claim*: Nonperturbatively, too,**
  QCD possesses a unique running coupling.
- Alternative
  - Possibly an essentially different RGI intrinsic mass-scale for each coupling
  - Then BRST symmetry irreparably broken by nonperturbative dynamics
  - Conclusion: QCD non-renormalisable owing to IR dynamics.
- No empirical evidence to support such a conclusion: QCD does seem to be a well-defined theory at all momentum scales, possibly owing to dynamical generation of gluon and quark masses, which are large at IR momenta.

QCD Running Coupling



- There is a particular simplicity to QED:
  - Unique running coupling
  - Process-independent effective charge
  - Obtained simply by computing the photon vacuum polarisation.
- This is because ghost-fields decouple in Abelian theories; and, consequently, one has the Ward identity

$$Z_1 = Z_2$$

which guarantees that the electric-charge renormalisation constant is equivalent to that of the photon field.

- Physically: impact of dressing the interaction vertices is absorbed into the vacuum polarisation.
- Not generally true in QCD because ghost-fields do not decouple.

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

- There is *one* approach to analysing QCD's Schwinger functions that preserves some of QED's simplicity
  - Combination of pinch technique (PT) & background field method (BFM)
- Means by which QCD can be made to “look” Abelian:
  - Systematically rearrange classes of diagrams and their sums in order to obtain modified Schwinger functions that satisfy linear STIs.
- In the gauge sector, this produces a modified gluon dressing function from which one can compute the QCD running coupling
  - So this polarisation captures all required features of the renormalisation group.
- Furthermore, the coupling is process independent: one obtains precisely the same result, independent of the scattering process considered, whether gg→gg, qq→qq, etc.

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

- There is one approach to analysing QCD's Schwinger functions that preserves some of QED's simplicity
  - Combination of pinch technique (PT) & background field method (BFM)
- The clean connection between the coupling and the gluon vacuum polarisation relies on another particular feature of QCD:
  - In Landau gauge the renormalisation constant of the gluon-ghost vertex is not only finite but unity
- Consequently, effective charge obtained from the PT-BFM gluon vacuum polarisation is directly connected with that deduced from the gluon-ghost vertex:
  - "Taylor coupling", α_T

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

PT-BFM vacuum polarisation

ghost-gluon vertex

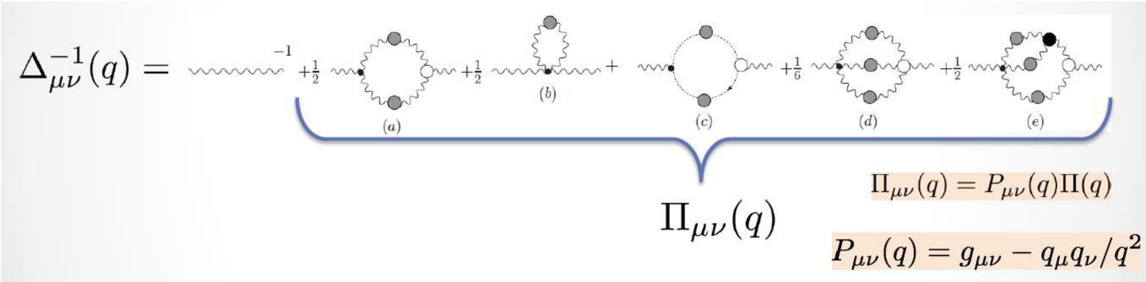
- Formulae:

$$\alpha(\zeta^2) D^{\rm PB}_{\mu\nu}(k;\zeta) = \widehat{d}(k^2) T_{\mu\nu}(k)\,,$$

$$I(k^2) := k^2 \widehat{d}(k^2) = \frac{\alpha_{\rm T}(k^2)}{[1 - L(k^2;\zeta^2)F(k^2;\zeta^2)]^2}\,,$$

ghost-gluon vacuum polarisation

  - $\alpha(\zeta^2)$: scale-dependent renormalized coupling
  - $D^{PB}{}_{\mu\nu}$: PT-BFM gluon two-point function
  - $\hat{d}(k^2)$: renormalisation-group-invariant (RGI) running-interaction that unifies top-down and bottom-up approaches to gauge and matter sectors of QCD
  - F : dressing function for the ghost propagator;
  - L : a longitudinal piece of the gluon-ghost vacuum polarisation that vanishes at $k^2=0$ and as $k^2 \to \infty$



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QCD Running Coupling

PT-BFM vacuum polarisation

ghost-gluon vertex

- Formulae:

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ghost-gluon vacuum polarisation

  - $\alpha(\zeta^2)$: scale-dependent renormalized coupling
  - $D^{PB}_{\mu\nu}$: PT-BFM gluon two-point function
  - $\hat{d}(k^2)$: renormalisation-group-invariant (RGI) running-interaction that unifies top-down and bottom-up approaches to gauge and matter sectors of QCD
  - F : dressing function for the ghost propagator;
  - L : a longitudinal piece of the gluon-ghost vacuum polarisation that vanishes at $k^2=0$ and as $k^2 \to \infty$

- Gap Equation: $S^{-1}(p) = Z_2\,(i\gamma\cdot p + m^{\rm bm}) + \Sigma(p)\,,$

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{d}(k^2)\, T_{\mu\nu}(k) \gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p)$$

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Running Coupling

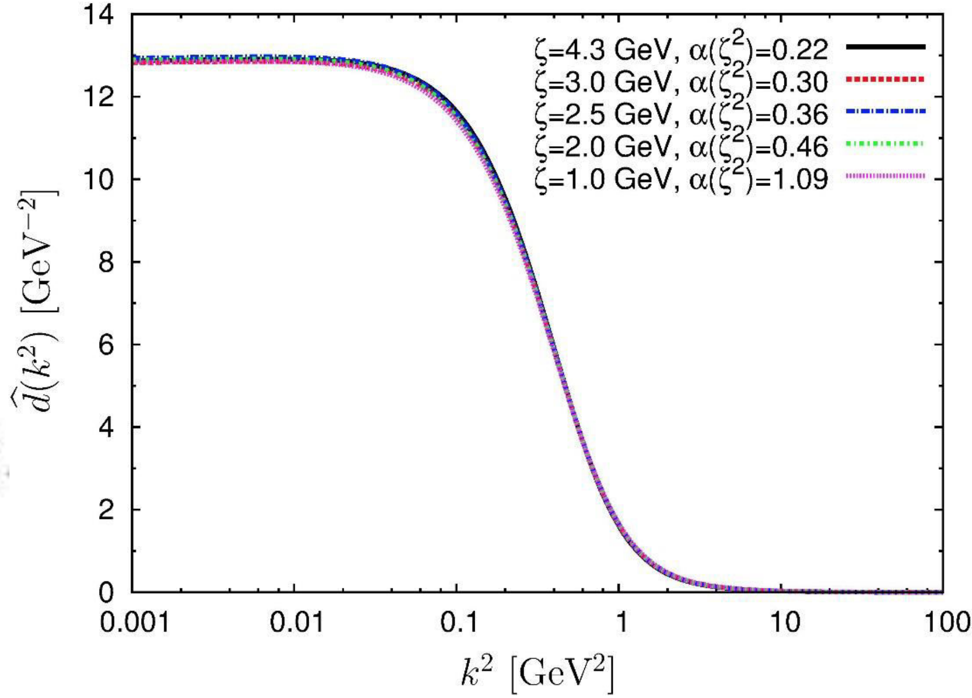
- RGI interaction, $\hat{d}(k^2)$ has been computed.
- Establishes a remarkable feature of QCD; namely, the interaction saturates at infrared momenta:

$$\widehat{d}(k^2=0)=\alpha(\zeta^2)/m_g^2(\zeta)=\alpha_0/m_0^2$$

where

  - $\alpha_0 := \alpha(0) \approx 1.0\pi$
  - $m_0 := m_g(0) \approx ½\, m_p$

- Gluon sector of QCD is characterised by a nonperturbatively-generated infrared mass-scale …
CANNIBALISM



Bridging a gap between continuum-QCD & ab initio predictions of hadron observables
Binosi, Chang, Papavassiliou, Roberts,
arXiv:1412.4782 [nucl-th], *Phys. Lett. B* ***742*** *(2015) 183*

QCD Effective Charge

- Define a RGI product: $\mathcal{D}(k^2) = \Delta_F(k^2;\zeta)\, m_g^2(\zeta^2)/m_0^2$
  - $\Delta_F(k^2;\zeta)$ is parametrisation of continuum- and/or lattice-QCD calculations of the canonical gluon two-point function
    - Preserves IR behaviour of calculations
    - $1/\Delta_F(k^2;\zeta) = k^2 + O(1)$ on $k^2 \gg m_0^2$
- Gap equation becomes:

$$\Sigma(p) = Z_2 \int_{dq}^{\Lambda} 4\pi \widehat{\alpha}_{\rm PI}(k^2) \mathcal{D}_{\mu\nu}(k^2) \gamma_\mu S(q) \hat{\Gamma}^a_\nu(q,p)\,,$$

where $\mathcal{D}_{\mu\nu} = \mathcal{D}T_{\mu\nu}$ and the dimensionless product

$$\widehat{\alpha}_{\rm PI}(k^2) = \widehat{d}(k^2)/\mathcal{D}(k^2)$$

is a RGI running-coupling (effective charge)

by construction, $\hat{\alpha}(k^2) = \mathcal{I}(k^2)$ on $k^2 \gg m_0^2$

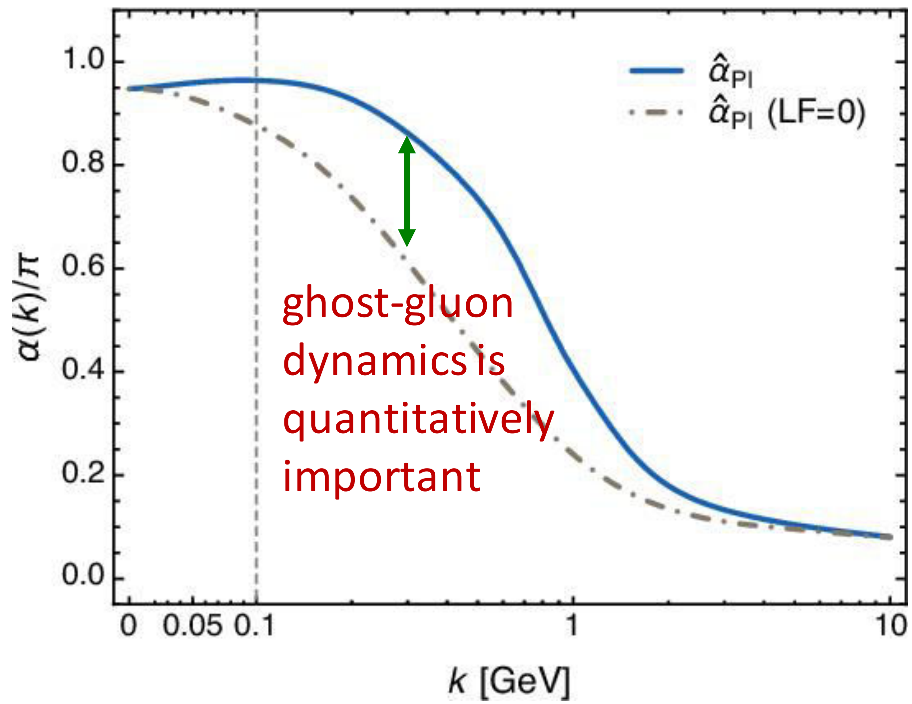
QCD Effective Charge

- $\hat{\alpha}_{PI}(k^2) = \hat{d}(k^2) / \mathcal{D}(k^2)$
  - Process independent: as noted above, the same function appears irrespective of the initial and final parton systems.
  - Unifies a diverse and extensive array of hadron observables
    - evident in fact that dressed-quark self-energy serves as generating functional for the Bethe-Salpeter kernel in all meson channels
    - and $\hat{\alpha}_{PI}(k^2)$ is untouched by the generating procedure in all flavoured systems
  - Sufficient to know $\hat{\alpha}_{PI}(k^2)$ in Landau gauge
    - form-invariant under gauge transformations
    - and gauge covariance ensures that such transformations produce nothing but an overall "phase" in the gap equation's solution, which may be absorbed into S(p)

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

QCD Effective Charge

- Parameter-free prediction: curve is completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.
- Physical, in the sense that there is no Landau pole, and saturates in the IR: *$\hat{\alpha}(0)$* ≈ 1.0 π, *i.e*. the coupling possesses an infrared fixed point



- Prediction is equally sound at all spacelike momenta, connecting the IR and UV domains, with no need for an *ad hoc* "matching procedure," such as that employed in models
- Essentially nonperturbative: combination of self-consistent solutions of gauge-sector gap equations with lattice simulations

Process-dependent (emergent)Effective Charge

- [Grunberg:1982fw]: process-dependent procedure

$$\int_0^{1^-} dx_{Bj}\left(g_1^p\left(x_{Bj},Q^2\right) - g_1^n\left(x_{Bj},Q^2\right)\right) \equiv \frac{g_A}{6}\left[1-\frac{\alpha_{g_1}\left(Q^2\right)}{\pi}\right]$$

- an effective running coupling defined to be completely fixed by leading-order term in the perturbative expansion of a given observable in terms of the canonical running coupling.
  - Obvious difficulty/drawback = process-dependence itself.
  - Effective charges from different observables can in principle be algebraically connected to each other via an expansion of one coupling in terms of the other.
  - But, any such expansion contains infinitely many terms; and connection doesn’t provide a given process-dependent charge with ability to predict another observable, since the expansion is only defined after both effective charges are independently constructed.

Process independent strong running coupling
Binosi, Mezrag, Papavassiliou, Roberts, Rodriguez-Quintero
arXiv:1612.04835 [nucl-th], Phys. Rev. D 96 (2017) 054026/1-7

Process-dependent Effective Charge

S.J. Brodsky, H.J. Lu, Phys. Rev. D 51 (1995) 3652
S.J. Brodsky, G.T. Gabadadze, A.L. Kataev, H.J. Lu, Phys. Lett. B 372 (1996) 133
A. Deur, V. Burkert, Jian-Ping Chen, Phys.Lett. B 650 (2007) 244-248

- α_{g1} – Bjorken sum rule

$$\int_0^{1^-} dx_{Bj} \left(g_1^p\left(x_{Bj}, Q^2\right) - g_1^n\left(x_{Bj}, Q^2\right) \right) \equiv \frac{g_A}{6}\left[1 - \frac{\alpha_{g_1}(Q^2)}{\pi}\right]$$

  $g_1^{p,n}$ are spin-dependent proton and neutron structure functions

  g_A is the nucleon flavour-singlet axial-charge

- Merits, *e.g.*
  - Existence of data for a wide range of k^2
  - Tight sum-rules constraints on the behaviour of the integral at the IR and UV extremes of k^2
  - isospin non-singlet ⇒ suppression of contributions from numerous processes that are hard to compute and hence might muddy interpretation of the integral in terms of an effective charge
    - Δ resonance
    - Disconnected (gluon mediated) diagrams

Process-independent effective-charge in QCD

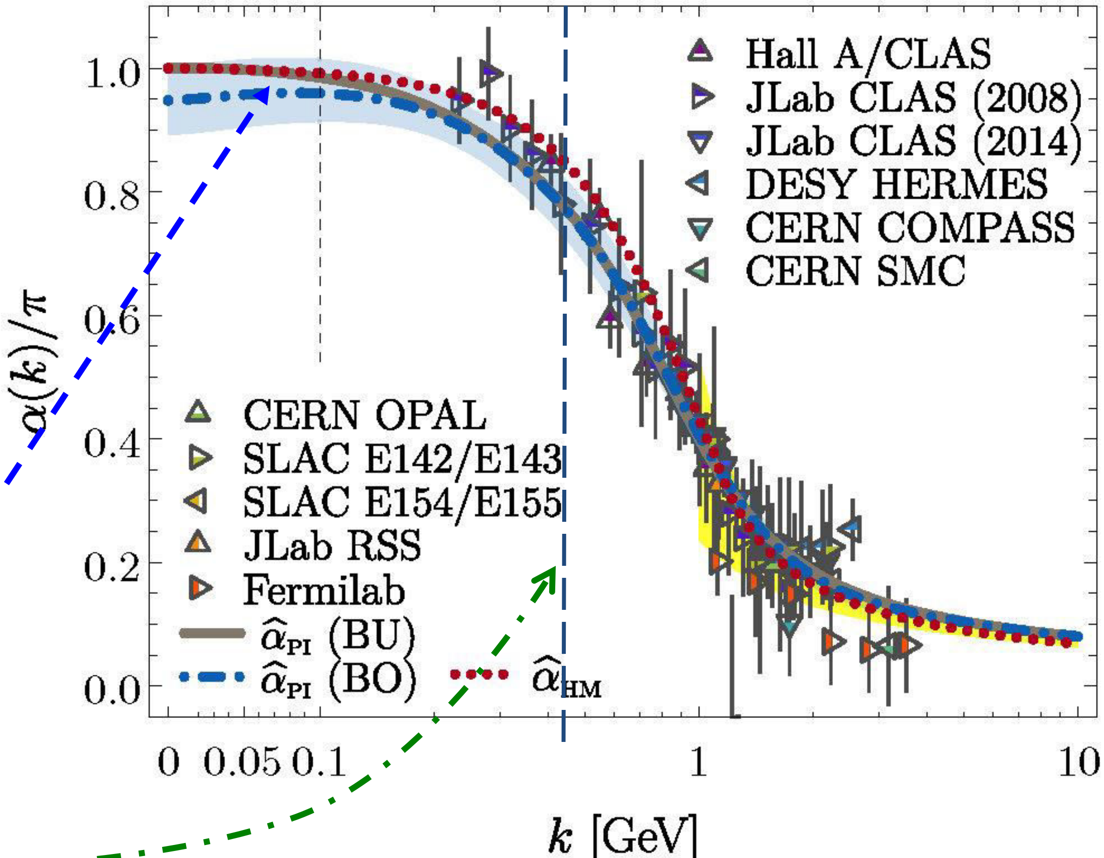
- Modern continuum & lattice methods for analysing gauge sector enable

  "Gell-Mann – Low"

  running charge to be defined in QCD

- Combined continuum and lattice analysis of QCD's gauge sector yields a parameter-free prediction

- N.B. Qualitative change in $\hat{\alpha}_{PI}(k)$ at $k \approx ½\, m_p$



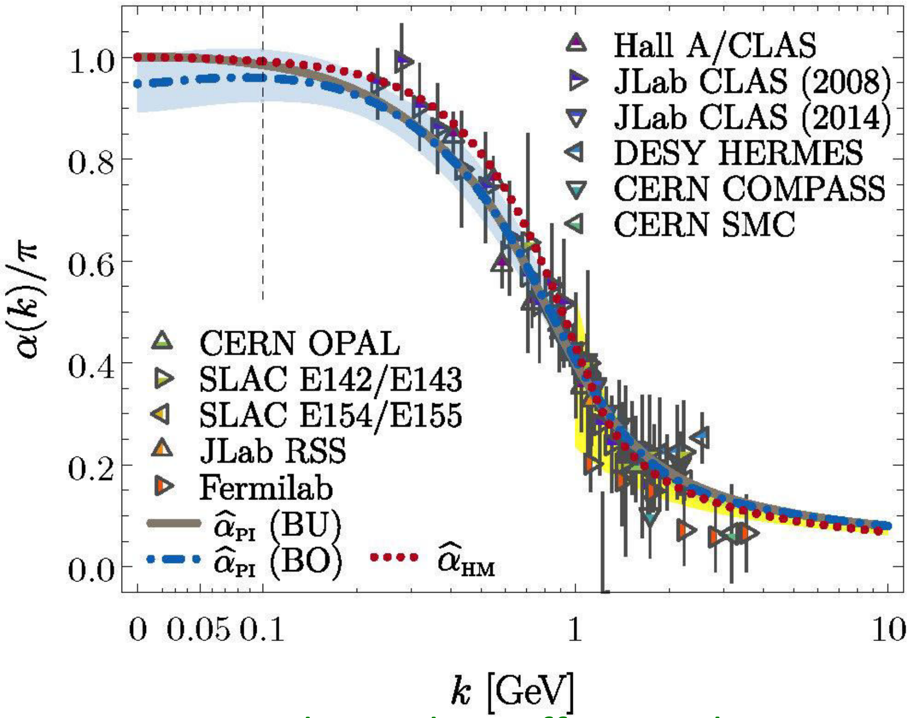
QCD Effective Charge

- Near precise agreement between process-independent $\hat{\alpha}_{PI}$ and α_{g1}
- Perturbative domain:

$$\alpha_{g_1}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.14\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$
$$\widehat{\alpha}_{\rm PI}(k^2) = \alpha_{\overline{\rm MS}}(k^2)(1 + 1.09\,\alpha_{\overline{\rm MS}}(k^2) + \ldots)\,,$$

Just 4% difference

- Parameter-free prediction:
  - curve completely determined by results obtained for gluon and ghost two-point functions using continuum and lattice-regularised QCD.



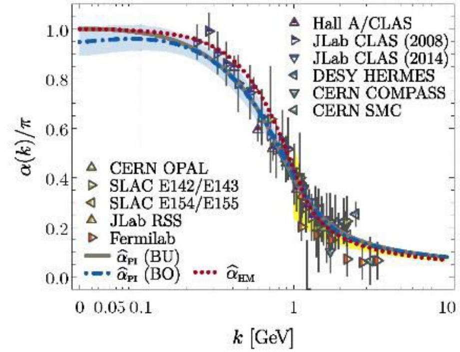
Data = process dependent effective charge [Grunberg:1982fw]:
α_{g1}, defined via Bjorken Sum Rule

- Ghost-gluon scattering contributions are critical for agreement between the two couplings at intermediate momenta … omit them, and disagreement by factor of ~ 2 at intermediate momenta

QCD Effective Charge

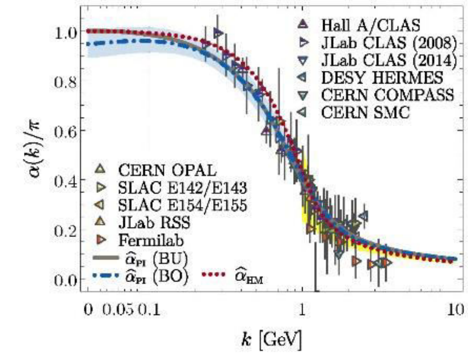
- Why are these two apparently unrelated definitions of a QCD effective charge can so similar?
  - Bjorken sum rule is an isospin non-singlet relation & hence contributions from many hard-to-compute processes are suppressed
  - these same processes are omitted in DSE computation of $\hat{\alpha}_{PI}$
- Unification of two vastly different approaches to understanding the infrared behaviour of QCD
  - one essentially phenomenological: data-based, process-dependent
  - the other, deliberately computational, embedded within QCD.
- ***Bjorken sum rule is a near direct means by which to gain empirical insight into QCD's "Gell-Mann – Low effective charge"***

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e*. completely determined by the gauge-boson two-point function.
- $\hat{\alpha}_{PI}$ is
  - process-independent
  - appears in every one of QCD's dynamical equations of motion
  - known to unify a vast array of observables
- $\hat{\alpha}_{PI}$ possesses an infrared-stable fixed-point
  - Nonperturbative analysis demonstrating absence of a Landau pole in QCD
- QCD is IR finite, owing to dynamical generation of gluon mass-scale, which also serves to eliminate the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

QCD Effective Charge



- $\hat{\alpha}_{PI}$ is a new type of effective charge
  - direct analogue of the Gell-Mann–Low effective coupling in QED, *i.e.* completely determined by the gauge ... tion.
- $\hat{\alpha}_{PI}$ is
  - pro
  - app
  - know
- $\hat{\alpha}_{PI}$ posse
  - Nonp ... n QCD
- QCD is IR ... of gluon mass-scale, which als ... the Gribov ambiguity
- Asymptotic freedom ⇒ QCD is well-defined at UV momenta
- ***QCD is therefore unique amongst known 4D quantum field theories***
  - ***Potentially, defined & internally consistent at all momenta***

Conceivably, therefore, QCD can serve as a basis for theories that take physics beyond the Standard Model