Status of NLO Monte Carlo for μe scattering

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MUonE collaboration meeting

Bologna, September 12-13, 2018

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μe scattering kinematics for leading order ($2 \to 2$, elastic process)

p_1, p_2 initial state μ and e p_3, p_4 final state μ and e

In the lab

$$
\begin{aligned}
p_1 &= (E_\mu^{beam}, 0, 0, p) \\
p_2 &= (m_e, 0, 0, 0) \\
p_3 &= p_1 + p_2 - p_4 \\
p_4 &= (E_e, p_e \sin\theta_e, 0, p_e \cos\theta_e)
\end{aligned}
$$

In the center of mass

$$
\begin{aligned}
p_1 &= (E_{CM}^\mu, 0, 0, p_{CM}) \\
p_2 &= (E_{CM}^e, 0, 0, -p_{CM}) \\
p_3 &= (E_{CM}^\mu, p_{CM} \sin\theta, 0, p_{CM} \cos\theta) \\
p_4 &= (E_{CM}^e, -p_{CM} \sin\theta, 0, -p_{CM} \cos\theta)
\end{aligned}
$$

Invariants:

$$
\begin{aligned}
s &= (p_1 + p_2)^2 = (p_3 + p_4)^2 \\
&= m_e^2 + m_\mu^2 + 2E_{CM}^\mu E_{CM}^e + 2p_{CM}^2 \\
&= m_e^2 + m_\mu^2 + 2E_\mu^{beam} m_e
\end{aligned}
$$

$$
\begin{aligned}
t &= (p_1 - p_3)^2 = (p_2 - p_4)^2 \\
&= -2p_{CM}^2 (1 - \cos\theta) \\
&= 2m_e^2 - 2E_e m_e
\end{aligned}
$$

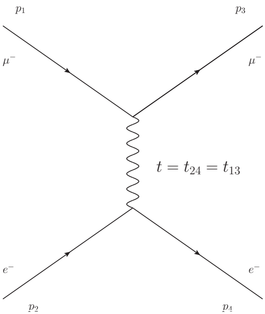
$$p_{CM} = \frac{1}{2}\sqrt{\frac{\lambda(s, m_\mu^2, m_e^2)}{s}}$$

$$t = m_\mu^2 \frac{x^2}{x-1} \propto E_e$$

$$E_e = m_e \frac{1 + r^2 \cos^2\theta_e}{1 - r^2 \cos^2\theta_e}$$

$$r \equiv \frac{\sqrt{\left(E_\mu^{beam}\right)^2 - m_\mu^2}}{E_\mu^{beam} + m_e}$$

LO cross section



- analytical expression

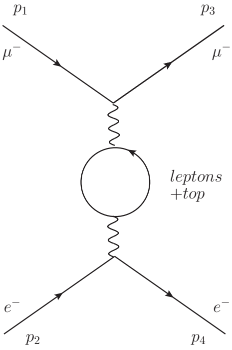
$$\frac{d\sigma}{dt} = \frac{4\pi\alpha^2}{\lambda(s, m_\mu^2, m_e^2)} \left[\frac{(s - m_\mu^2 - m_e^2)^2}{t^2} + \frac{s}{t} + \frac{1}{2} \right]$$

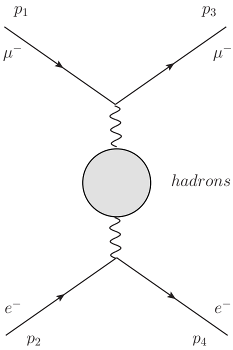
- VP gauge invariant subset of NLO rad. corr.
- factorized over tree-level: $\alpha \to \alpha(t)$

- first step towards higher precision and to estimate of th. uncertainties:

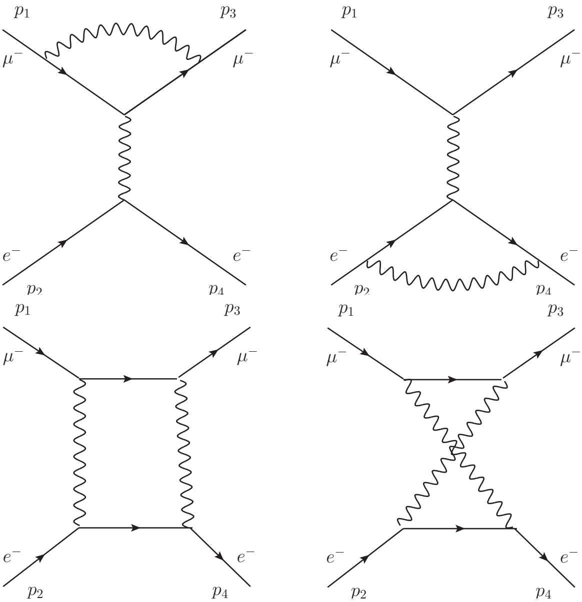
NLO Monte Carlo fully differential predictions

NLO amplitudes: vacuum polarization



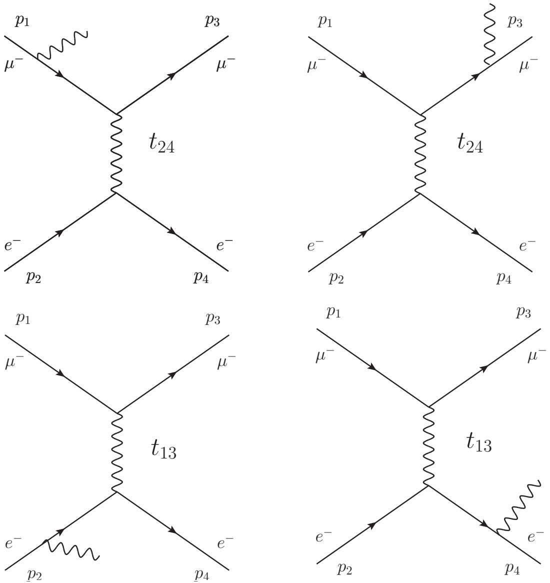


NLO QED virtual diagrams $\mathcal{A}_{NLO}^{virtual}$ (dependent on λ)



+ counterterms

NLO real diagrams $\mathcal{A}^{1\gamma}_{NLO}$



Next-to-leading order calculation

$$\sigma_{NLO} = \sigma_{2\to 2} + \sigma_{2\to 3} = \sigma_{\mu e\to\mu e} + \sigma_{\mu e\to\mu e\gamma}$$

- ↦ IR singularities are regularized with a vanishingly small photon mass $\boldsymbol{\lambda}$
- ↦ $[2\to 2]/[2\to 3]$ phase space splitting at an arbitrarily small γ-energy cutoff $\boldsymbol{\omega_s}$
  - $\mu e \to \mu e$

$$\sigma_{2\to 2} = \sigma_{LO} + \sigma_{NLO}^{virtual} = \frac{1}{F}\int d\Phi_2 \left(|\mathcal{A}_{LO}|^2 + 2\Re[\mathcal{A}_{LO}^{*} \times \mathcal{A}_{NLO}^{virtual}(\boldsymbol{\lambda})]\right)$$

  - $\mu e \to \mu e\gamma$

$$\sigma_{2\to 3} = \frac{1}{F}\int_{\omega>\boldsymbol{\lambda}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 = \frac{1}{F}\left(\int_{\boldsymbol{\lambda}<\omega<\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2 + \int_{\omega>\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2\right)$$

$$= \Delta_s(\boldsymbol{\lambda}, \boldsymbol{\omega_s}) \int d\sigma_{LO} + \frac{1}{F}\int_{\omega>\boldsymbol{\omega_s}} d\Phi_3 |\mathcal{A}_{NLO}^{1\gamma}|^2$$

  - the integration over the 2/3-particles phase space is done with MC techniques and fully-exclusive events are generated

Method and cross-checks

- Calculation performed in the on-shell renormalization scheme
- Full mass dependency kept everywhere, fermions' helicities kept explicit
- Diagrams manipulated with the help of **FORM**, independently by at least two of us [perfect agreement]

  J. Vermaseren, **https://www.nikhef.nl/~form**

- 1-loop tensor coefficients and scalar 2-3-4 points functions evaluated with **LoopTools** and **Collier** libraries [perfect agreement]

  T. Hahn, **http://www.feynarts.de/looptools**

  A. Denner, S. Dittmaier, L. Hofer, **https://collier.hepforge.org**

- UV finiteness and λ independence verified with high numerical accuracy
- 3 body phase-space cross-checked with 3 independent implementations [perfect agreement]
- Comparison with past (and present) independent results

  P. Van Nieuwenhuizen, Nucl. Phys. B **28** (1971) 429

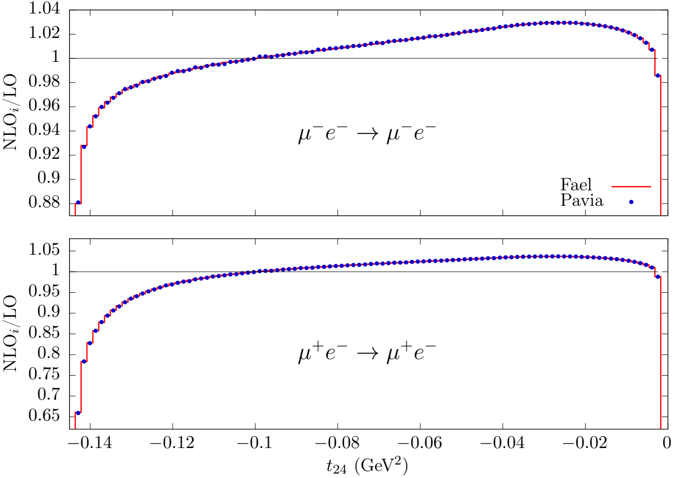
  T. V. Kukhto, N. M. Shumeiko and S. I. Timoshin, J. Phys. G **13** (1987) 725

  D. Y. Bardin and L. Kalinovskaya, DESY-97-230, **hep-ph/9712310**

  N. Kaiser, J. Phys. G **37** (2010) 115005

  Fael, Passera *et al.*, see next slide

  G. D'Ambrosio, Lettere al Nuovo Cimento, Vol. 38, n. 18, 1983

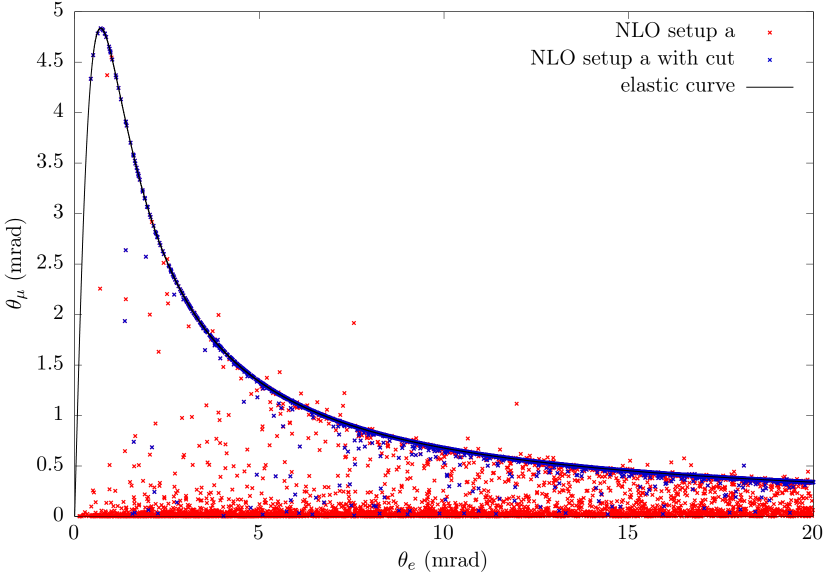


1.04
1.02
1
0.98
0.96
0.94
0.92
0.9
0.88
NLO$_i$/LO
$\mu^- e^- \to \mu^- e^-$
Fael
Pavia
1.05
1
0.95
0.9
0.85
0.8
0.75
0.7
0.65
NLO$_i$/LO
$\mu^+ e^- \to \mu^+ e^-$
−0.14
−0.12
−0.1
−0.08
−0.06
−0.04
−0.02
0
t_{24} (GeV^2)

preliminary results

- in 4 typical event selections
  1. 0 mrad $\leq \vartheta_{\mu,e} \leq 100$ mrad, $E_e \geq 200$ MeV
  2. 0 mrad $\leq \vartheta_{\mu,e} \leq 100$ mrad, $E_e \geq 1$ GeV
  3. as $1+$ acoplanarity cut $|\pi - (\phi_e - \phi_\mu)| \leq 3.5$ mrad
  4. as $2+$ acoplanarity cut $|\pi - (\phi_e - \phi_\mu)| \leq 3.5$ mrad
- size of three separate contributions at NLO:
  1. QED radiation (virtual and real) from μ line
  2. QED radiation (virtual and real) from electron line
  3. QED interference between μ and e line
- size of genuine electroweak effects
  - Z exchange diagram in addition to the γ exchange
  - Z and W in the virtual loops (including QED real rad. from tree-level with Z exchange)
- size of finite electron mass effects in the NLO corrections (in progress)

μ-e angle correlation in the lab

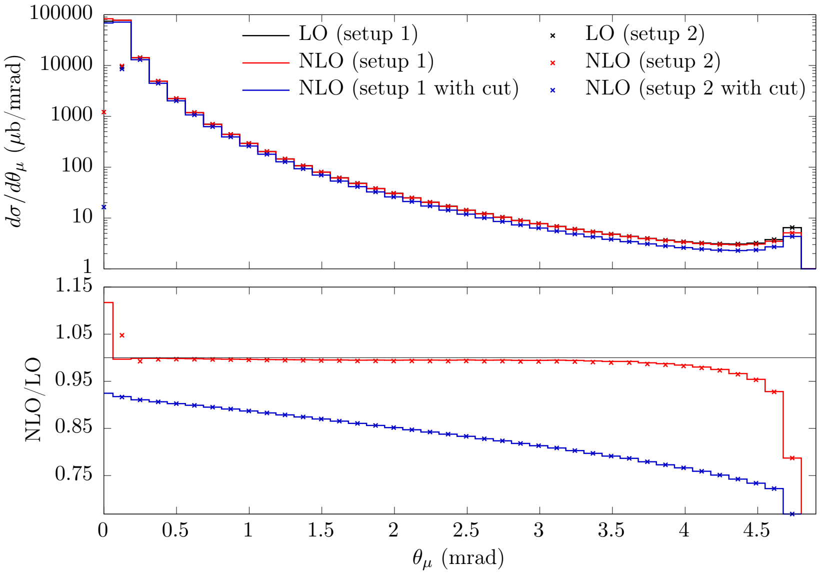


Integrated cross sections (μbarn)

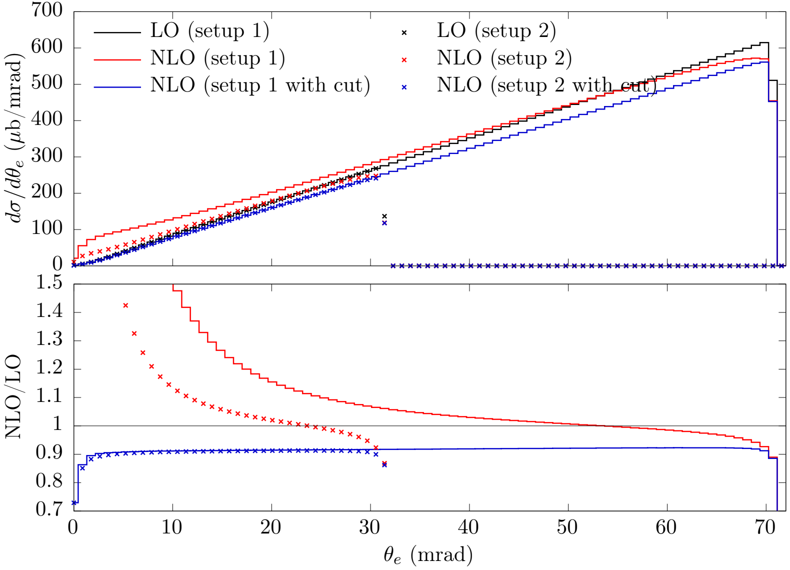
Cross section	Setup 1	Setup 2	Setup 3	Setup 4
$\mu^+e^- \to \mu^+e^-$				
σ_{NLO}^{QED}	1325.216(15)	255.8439(27)	1162.445(10)	222.7727(20)
$\sigma_{NLO}^{electron}$	1324.443(27)	255.5486(22)	1163.120(34)	223.4353(27)
σ_{NLO}^{muon}	1264.96276(75)	244.97022(68)	1264.0745(75)	244.41301(68)
$\sigma_{NLO}^{electron-muon}$	0.871(43)	0.3640(57)	0.312(90)	-0.0367(54)
$\mu^-e^- \to \mu^-e^-$				
σ_{NLO}^{QED}	1323.485(18)	255.1194(26)	1161.879(10)	222.8511(18)
$\sigma_{NLO}^{electron}$	1324.438(24)	255.5487(26)	1163.143(25)	223.4351(27)
σ_{NLO}^{muon}	1264.9630(76)	244.97058(46)	1264.0747(75)	244.41333(45)
$\sigma_{NLO}^{electron-muon}$	-0.855(50)	-0.3610(56)	-0.254(66)	+0.0415(50)

Relative correction	Setup 1	Setup 2	Setup 3	Setup 4
$\mu^+e^- \to \mu^+e^-$				
δ_{NLO}^{QED}	0.04755(1)	0.04409(1)	-0.081114(37)	-0.0908681(81)
$\delta_{NLO}^{electron}$	0.04694(2)	0.04289(1)	-0.080582(27)	-0.088164(11)
δ_{NLO}^{muon}	-0.000075(6)	-0.000280(3)	-0.0007794(59)	-0.0025543(28)
$\delta_{NLO}^{electron-muon}$	0.000686(40)	0.001485(23)	0.000247(69)	-0.000150(22)
$\mu^-e^- \to \mu^-e^-$				
δ_{NLO}^{QED}	0.04618(1)	0.04114(1)	-0.081543(27)	-0.0905482(75)
$\delta_{NLO}^{electron}$	0.04694(2)	0.04289(1)	-0.080563(20)	-0.088165(11)
δ_{NLO}^{muon}	-0.000077(6)	-0.000279(2)	-0.0007793(60)	-0.0025529(18)
$\delta_{NLO}^{electron-muon}$	-0.000676(40)	-0.001473(23)	-0.000200(52)	0.000169(20)

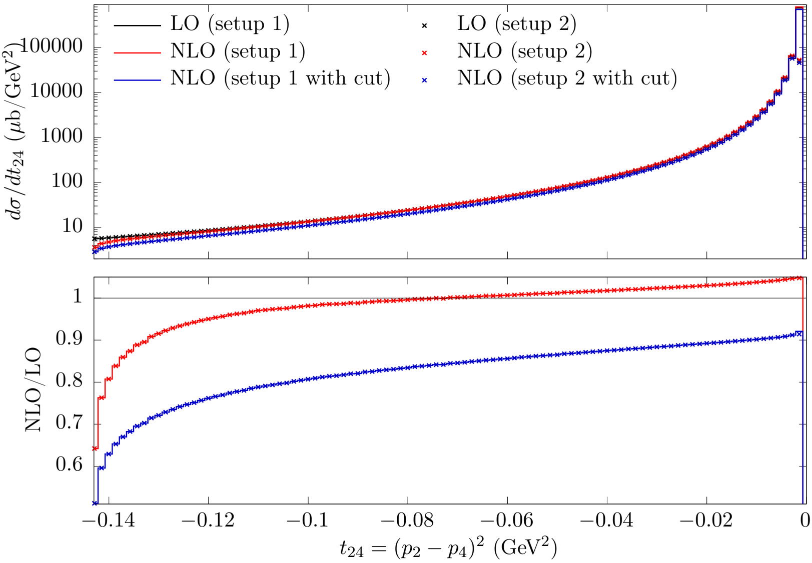
θ_μ distribution and corrections in the lab



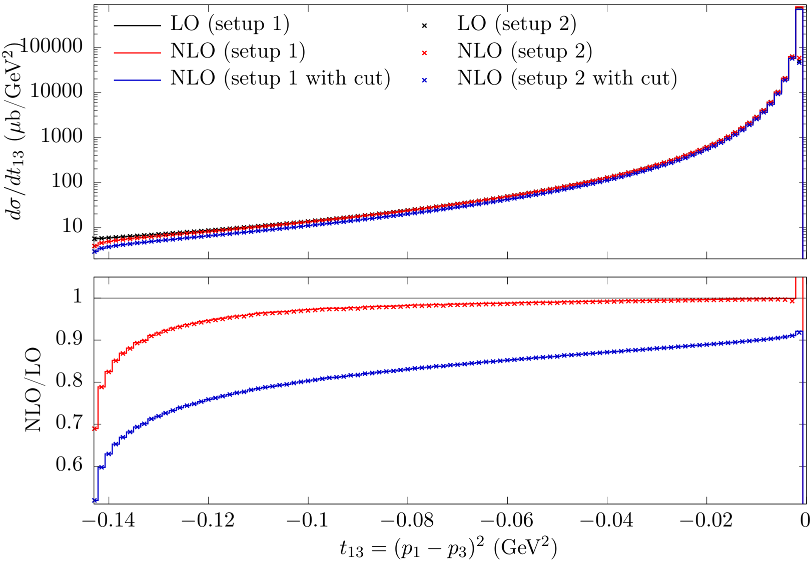
θ_e distribution and corrections in the lab



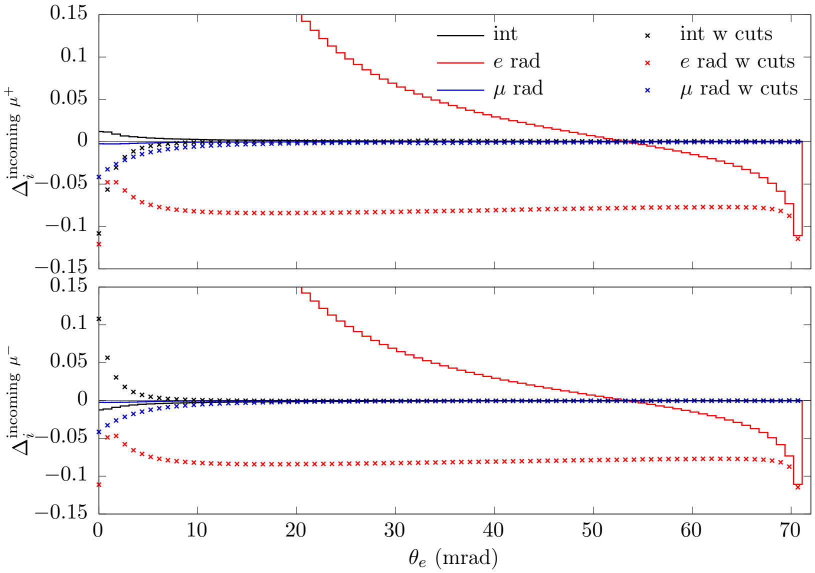
t_{24} (t on the “electron line”) distribution and corrections



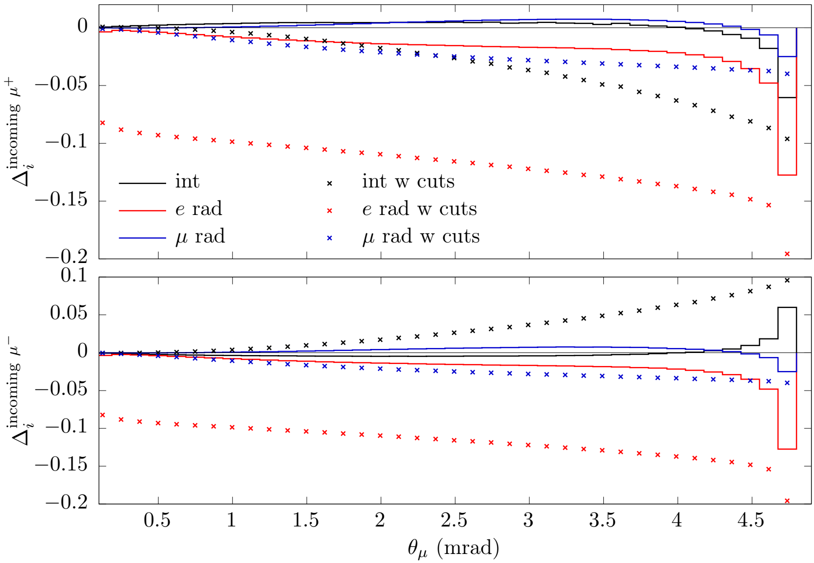
t_{13} (t on the "μ line") distribution and corrections



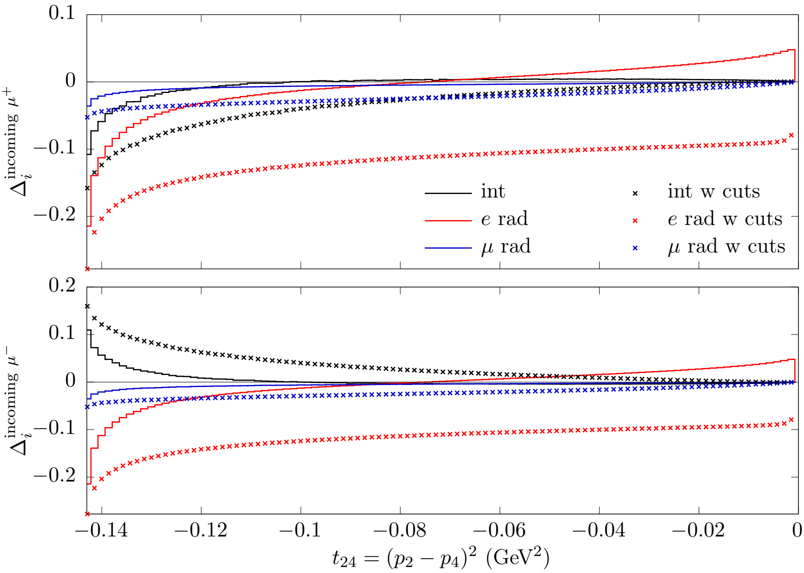
in terms of gauge invariant radiation contributions



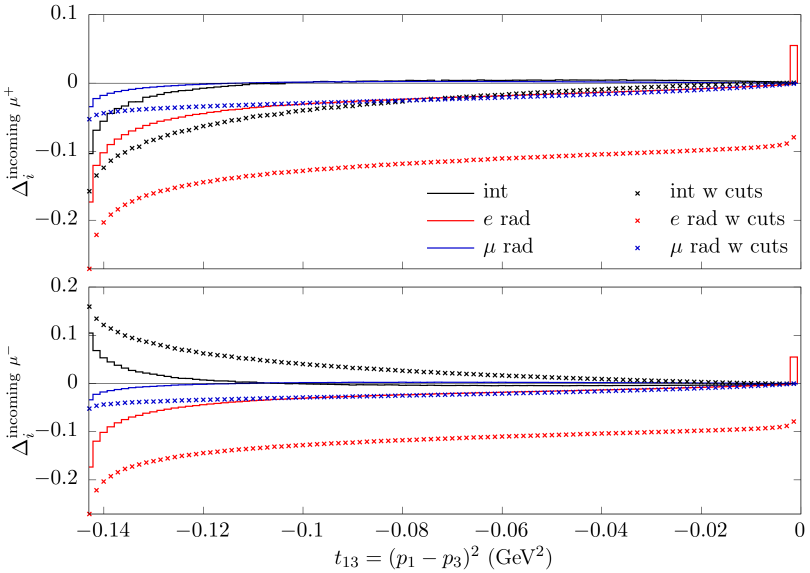
in terms of gauge invariant radiation contributions



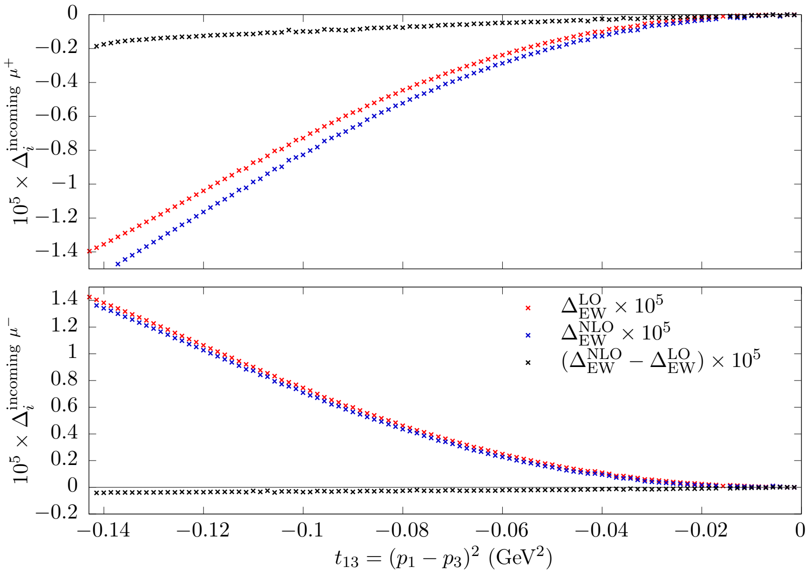
in terms of gauge invariant radiation contributions



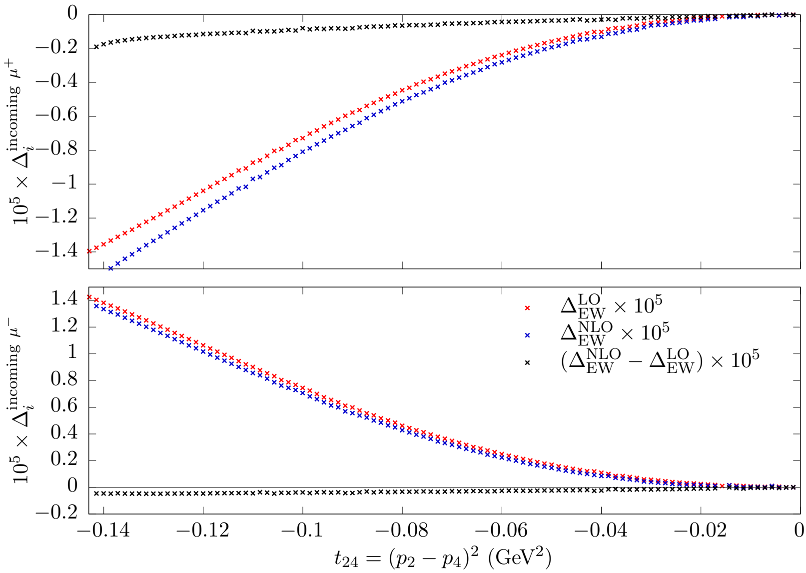
in terms of gauge invariant radiation contributions



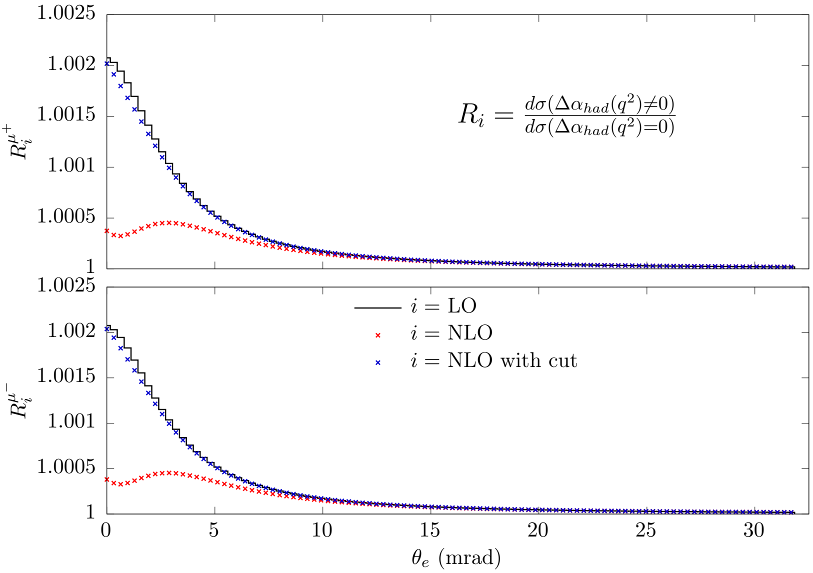
Electroweak (LO and NLO) contributions



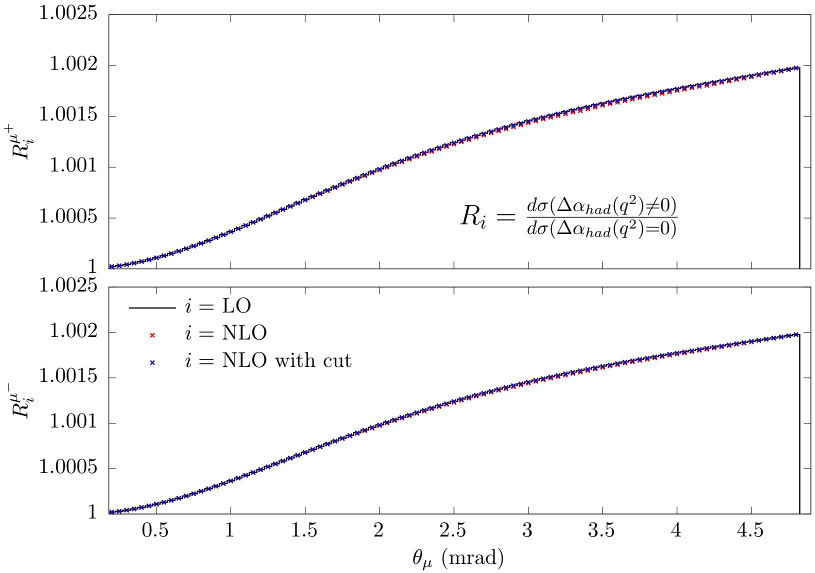
Electroweak (LO and NLO) contributions



$\Delta\alpha$ measured on top of LO or NLO



$\Delta\alpha$ measured on top of LO or NLO



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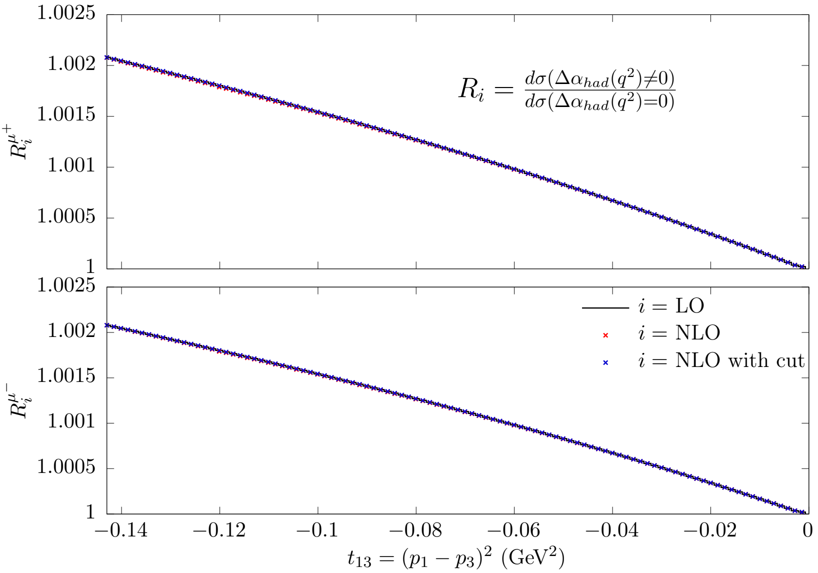

$\Delta\alpha$ measured on top of LO or NLO