

$a_\mu^{2\pi}$ in spacelike region
(work in collaboration with M. Della Gatta, F.
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GV

12/9/18

- Comparison of $a_\mu^{2\pi}$ for KLOE, BaBar, CMD2 in spacelike region
- Sensitivity of MUonE to different dataset (to be done)

Relevant formulae

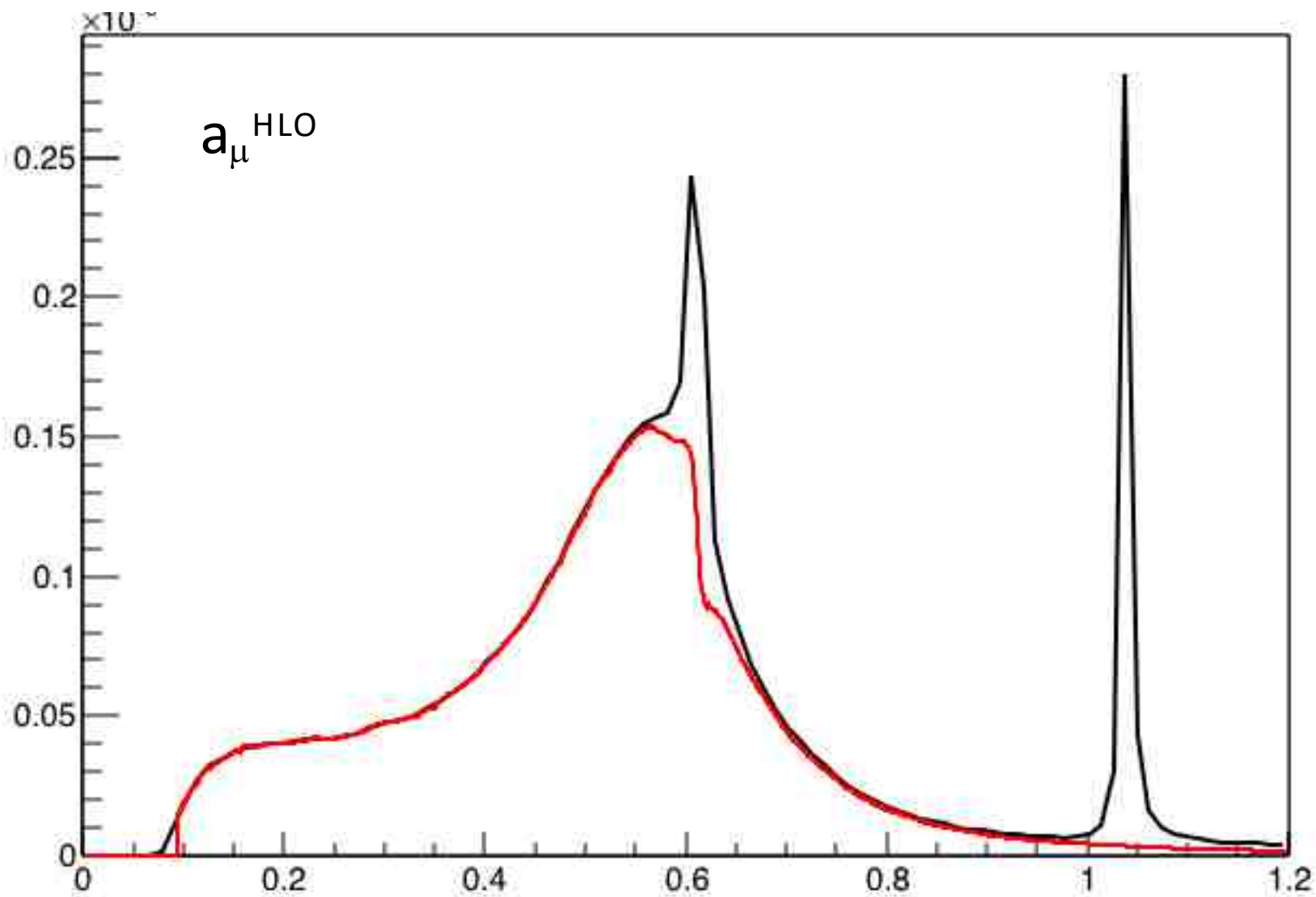
$$a_{\mu}^{HLO} = \frac{\alpha}{\pi} \int_0^1 dx (1-x) \cdot \Delta\alpha_{had} \left(-\frac{x^2 m_{\mu}^2}{1-x} \right)$$

$$\Delta\alpha_{HAD}(t) = -\frac{\alpha t}{3\pi} \int_{4m_{\pi}^2}^{\infty} \frac{R(s)}{s(s-t)} ds$$

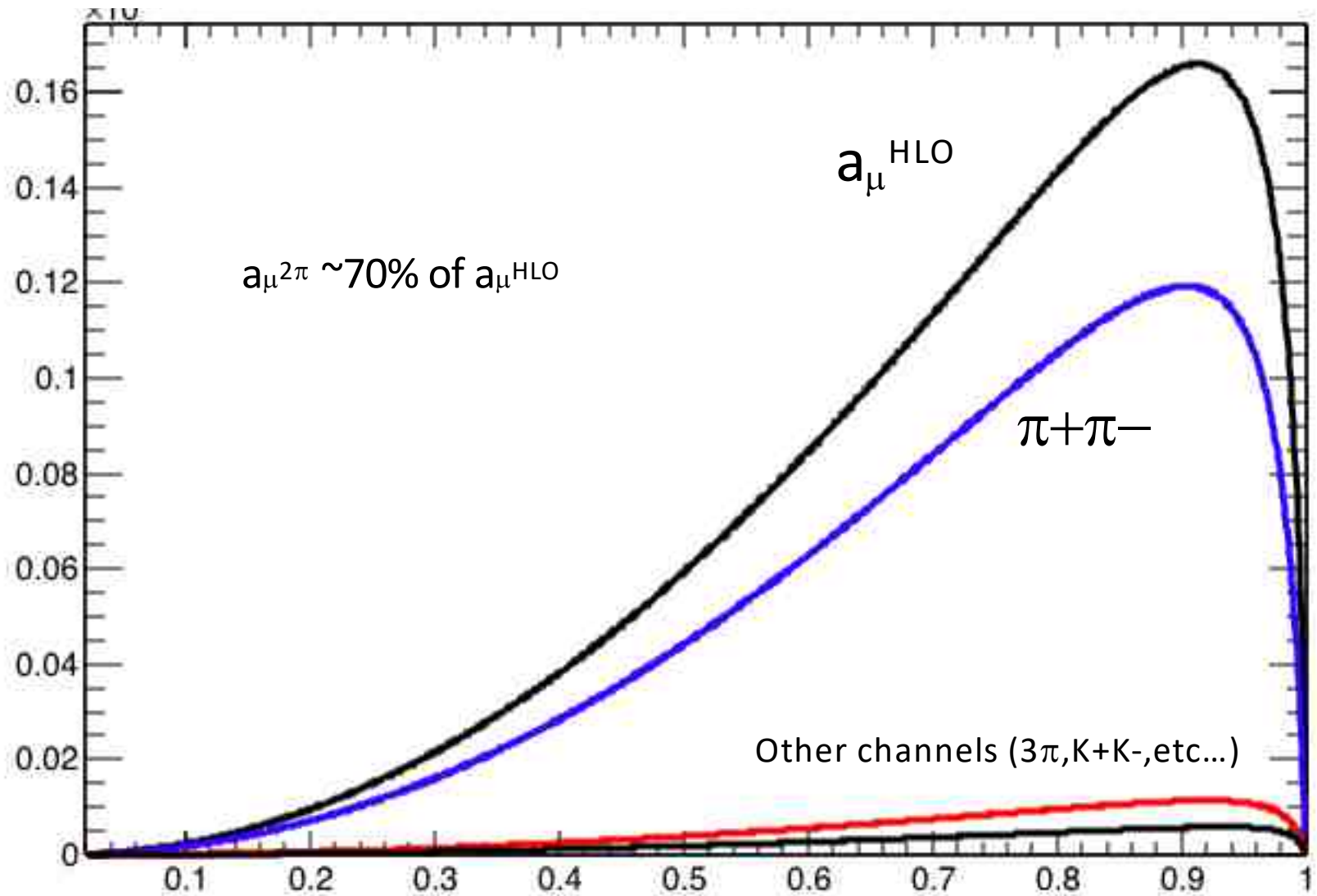
We consider now the 2pi contribution

$$R(s) = \frac{\sigma_{TOT}^0(e^-e^+ \longrightarrow \pi^-\pi^+)}{\sigma_{TOT}^0(e^-e^+ \longrightarrow \mu^-\mu^+)} = \frac{\frac{\alpha^2 \pi}{3s} \left(1 - \frac{4m_{\pi}^2}{s}\right)^{\frac{3}{2}} |F_{\pi}^0(q^2)|^2}{\frac{4\pi\alpha^2}{3s}} = \frac{1}{4} \left(1 - \frac{4m_{\pi}^2}{s}\right)^{\frac{3}{2}} |F_{\pi}^0(q^2)|^2$$

2π contribution to a_μ^{HLO} timelike



2π contribution to a_μ^{HLO} spacelike



Contribution of different channels to a_μ^{HLO}

Table 2. Contributions to a_μ in the energy region from 0.305 to 1.8 GeV from exclusive channels (Table 4 from [10]).

Channel	HLMNT (11) [10]	DHMZ (10) [1]	Difference
$\eta\pi^+\pi^-$	0.88 ± 0.10	1.15 ± 0.19	-0.27
K^+K^-	22.09 ± 0.46	21.63 ± 0.73	0.46
$K_S^0 K_L^0$	13.32 ± 0.16	12.96 ± 0.39	0.36
$\omega\pi^0$	0.76 ± 0.03	0.89 ± 0.07	-0.13
$\pi^+\pi^-$	505.65 ± 3.09	507.80 ± 2.84	-2.15
$2\pi^+2\pi^-$	13.50 ± 0.44	13.35 ± 0.53	0.15
$3\pi^+3\pi^-$	0.11 ± 0.01	0.12 ± 0.01	-0.01
$\pi^+\pi^-\pi^0$	47.38 ± 0.99	46.00 ± 1.48	1.38
$\pi^+\pi^-2\pi^0$	18.62 ± 1.15	18.01 ± 1.24	0.61
$\pi^0\gamma$	4.54 ± 0.14	4.42 ± 0.19	0.12
$\eta\gamma$	0.69 ± 0.02	0.64 ± 0.02	0.05
$\eta2\pi^+2\pi^-$	0.02 ± 0.00	0.02 ± 0.01	0.00
$\eta\omega$	0.38 ± 0.06	0.47 ± 0.06	-0.09
$\eta\phi$	0.33 ± 0.03	0.36 ± 0.03	-0.03
$\phi(\rightarrow \text{unaccounted})$	0.04 ± 0.04	0.05 ± 0.00	-0.01
Sum of isospin channels	5.98 ± 0.42	6.06 ± 0.46	-0.08
Total	634.28 ± 3.53	633.93 ± 3.61	0.35

Parametrization of $\sigma_{\pi\pi}$

$$\sigma_{\pi\pi} = C \cdot F^2(0) \cdot (1 + \delta_{\text{FSR}}) = C \cdot \text{GS} \cdot (1 + \delta_{\text{FSR}})$$

$$\delta_{\text{FSR}} = \frac{\alpha}{\pi} \Lambda(s)$$

GS=Gounaris-Sakurai parametrization

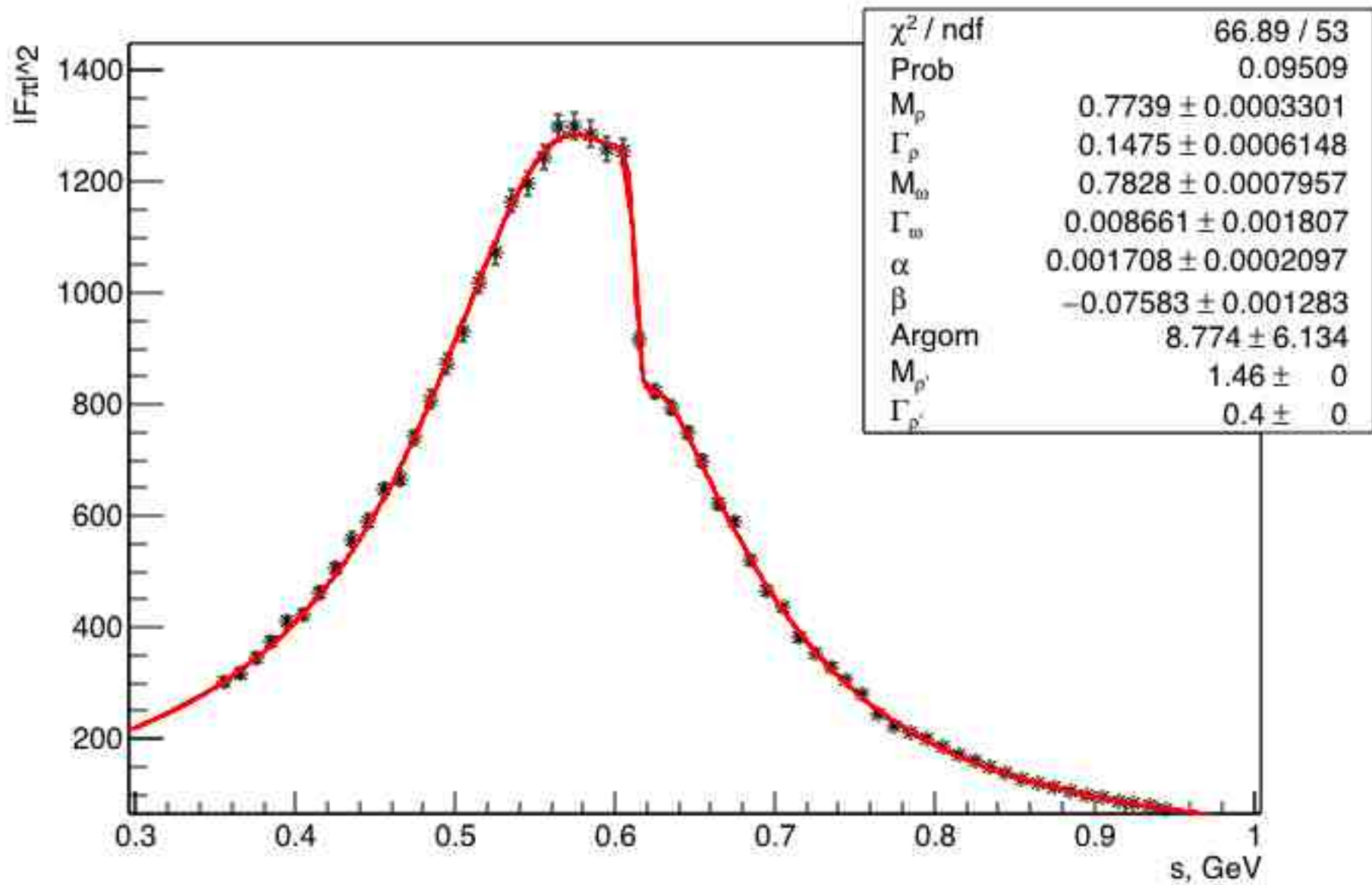
$$F_{\pi}(s) = \left(\text{BW}_{\rho(770)}^{\text{GS}}(s) \cdot \left(1 + \delta \frac{s}{M_{\omega}^2} \text{BW}_{\omega}(s) \right) + \beta \cdot \text{BW}_{\rho(1450)}^{\text{GS}}(s) \right) (1 + \beta)^{-1}, \quad (8)$$

FSR

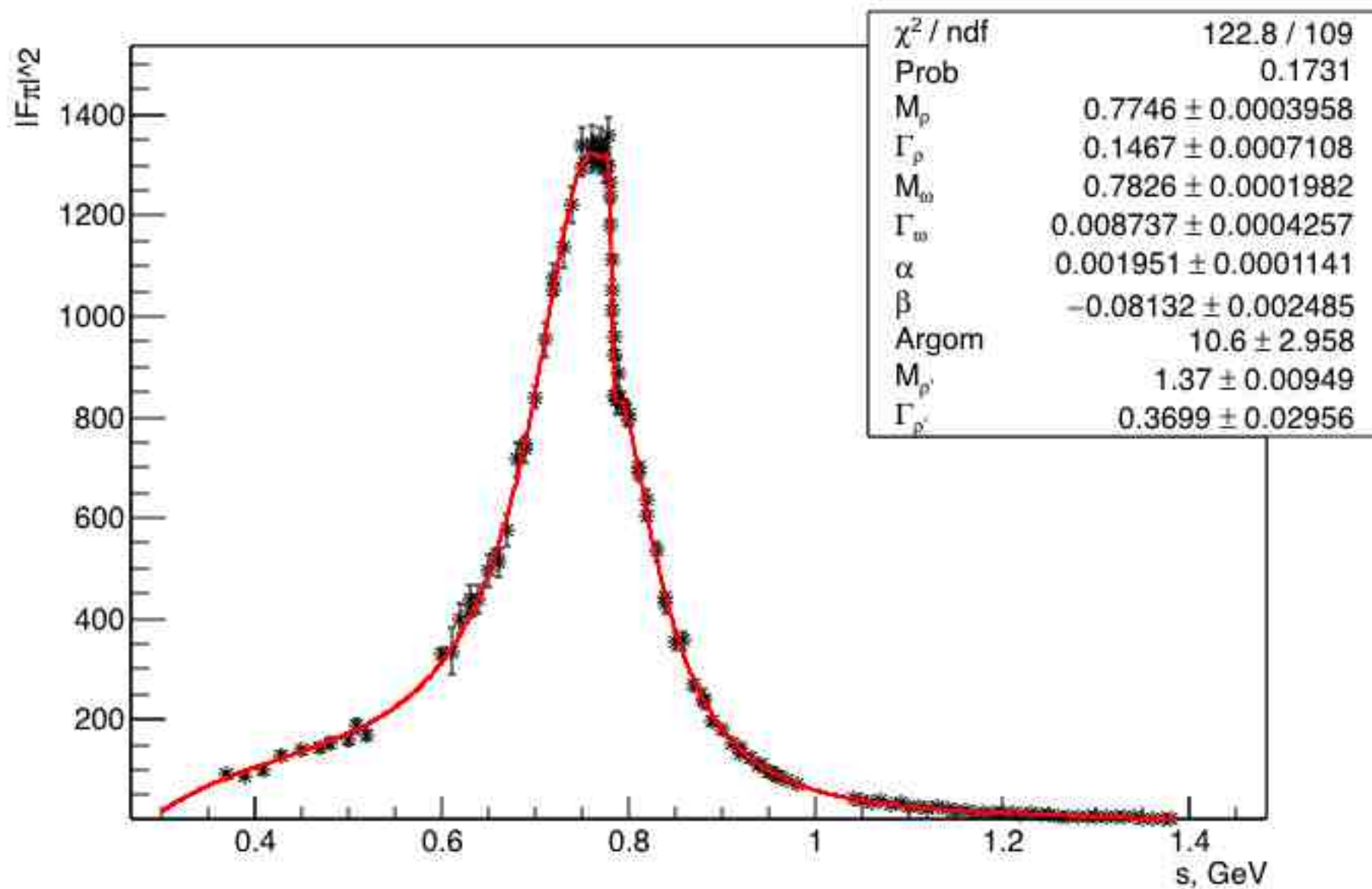
$$\begin{aligned} \Lambda(s) = \frac{1 + \beta_{\pi}^2}{\beta_{\pi}} & \left\{ 4 \text{Li}_2 \left(\frac{1 - \beta_{\pi}}{1 + \beta_{\pi}} \right) + 2 \text{Li}_2 \left(-\frac{1 - \beta_{\pi}}{1 + \beta_{\pi}} \right) \right. \\ & \left. - \left[3 \ln \left(\frac{2}{1 + \beta_{\pi}} \right) + 2 \ln \beta_{\pi} \right] L_{\beta_{\pi}} \right\} \\ & - 3 \ln \left(\frac{4}{1 - \beta_{\pi}^2} \right) - 4 \ln \beta_{\pi} \\ & + \frac{1}{\beta_{\pi}^3} \left[\frac{5}{4} (1 + \beta_{\pi}^2)^2 - 2 \right] L_{\beta_{\pi}} + \frac{3}{2} \frac{1 + \beta_{\pi}^2}{\beta_{\pi}^2}, \end{aligned} \quad (6)$$

where $L_{\beta_{\pi}} = \ln \frac{1 + \beta_{\pi}}{1 - \beta_{\pi}}$, $\text{Li}_2(z) = -\int_0^z \frac{dx}{x} \ln(1 - x)$,

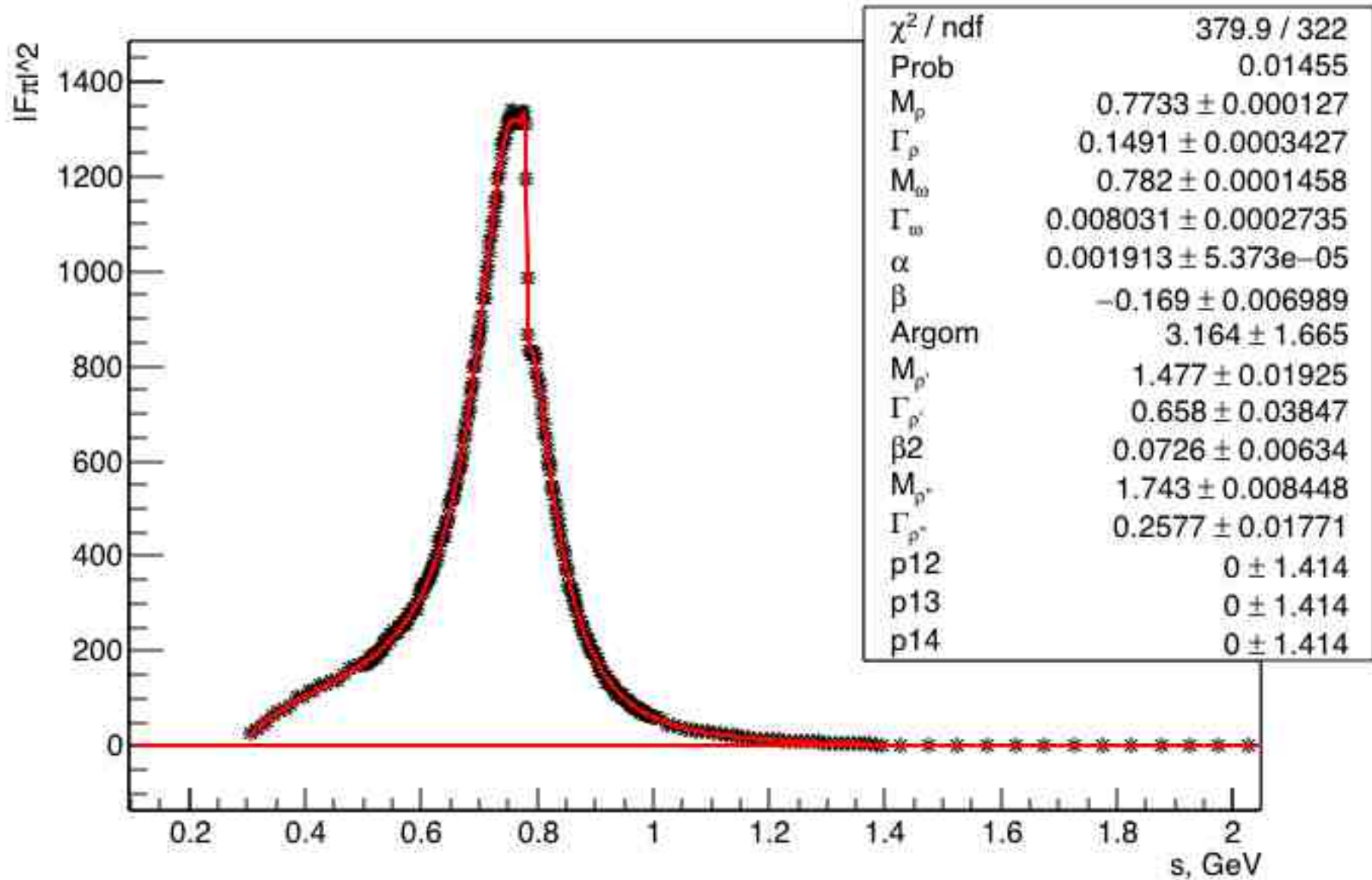
KLOE



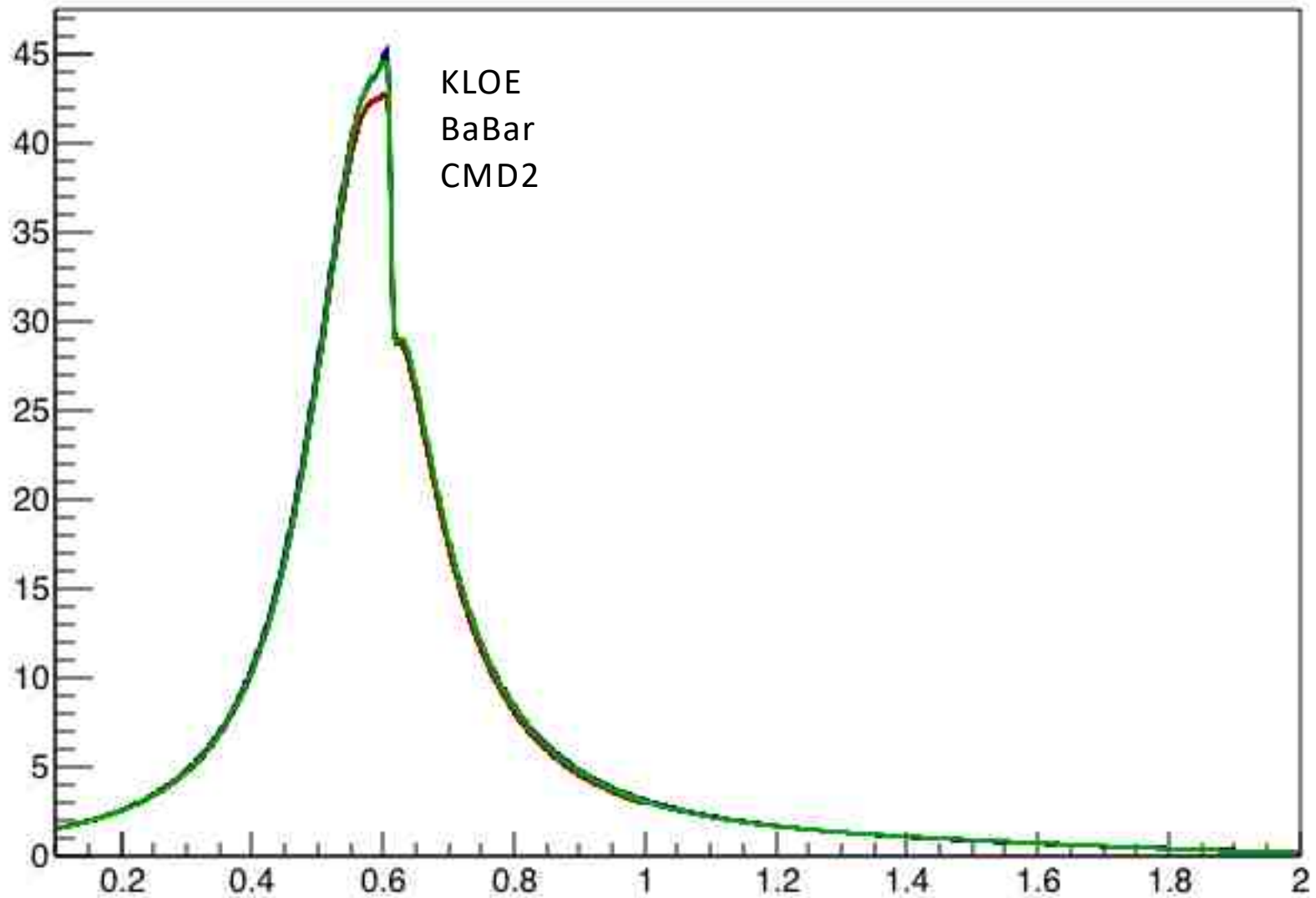
CMD2



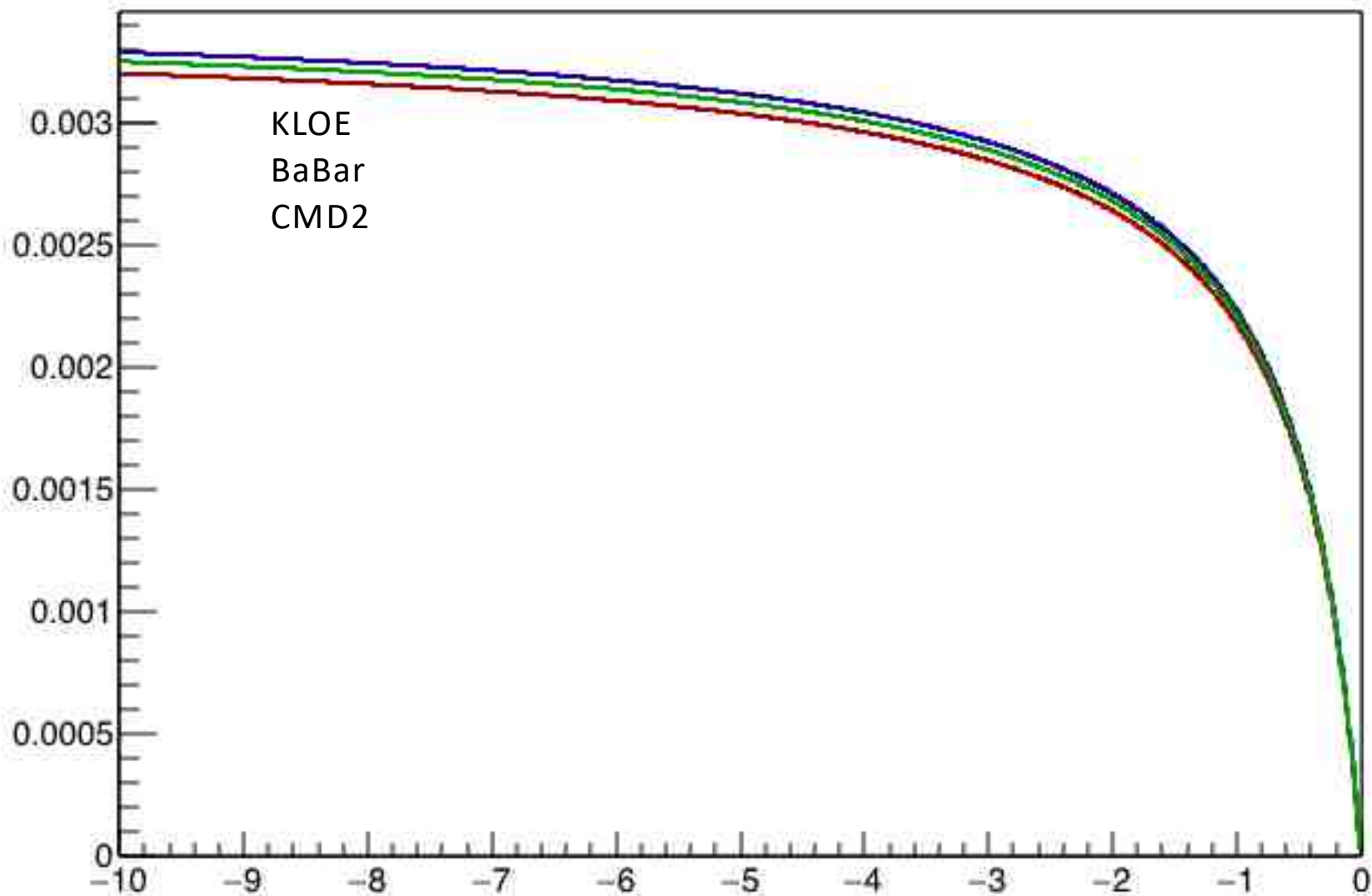
BaBar



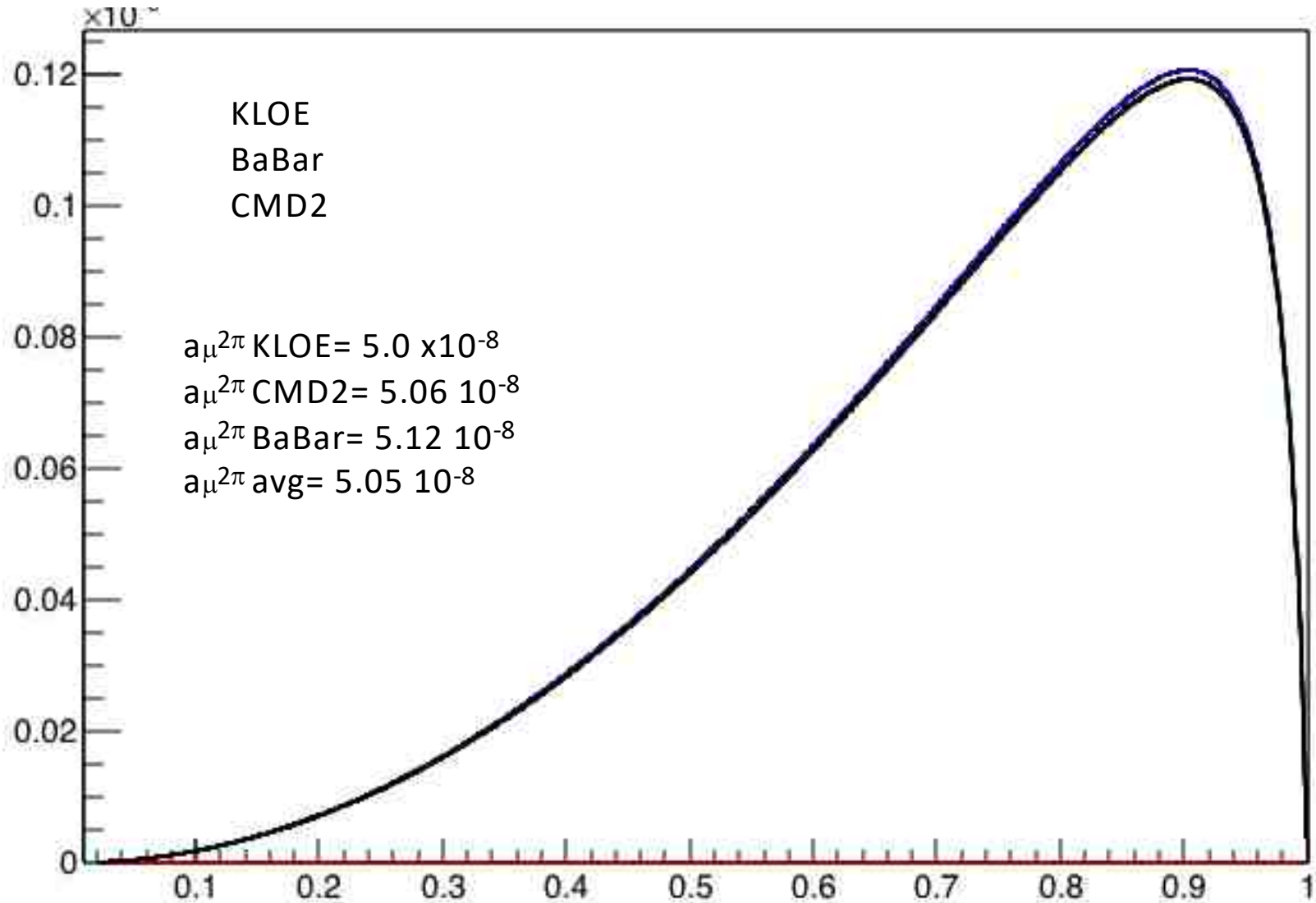
Comparison for different data: F_{π}^2



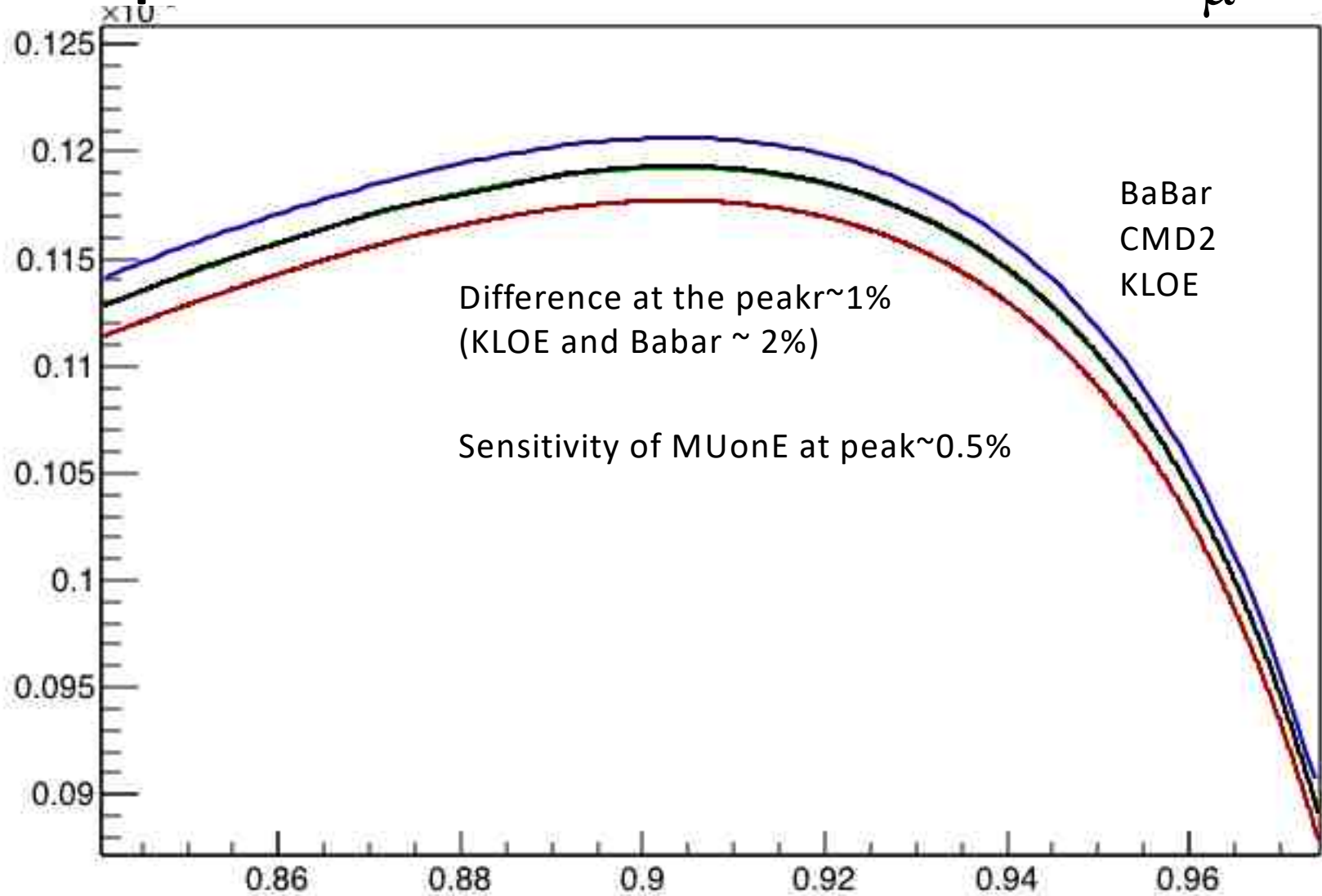
Comparison for different data: $\Delta\alpha_{2\pi}(t)$



Comparison for different data: $a_\mu^{2\pi}$ SL



Comparison for different data: $a_\mu^{2\pi}$ SL



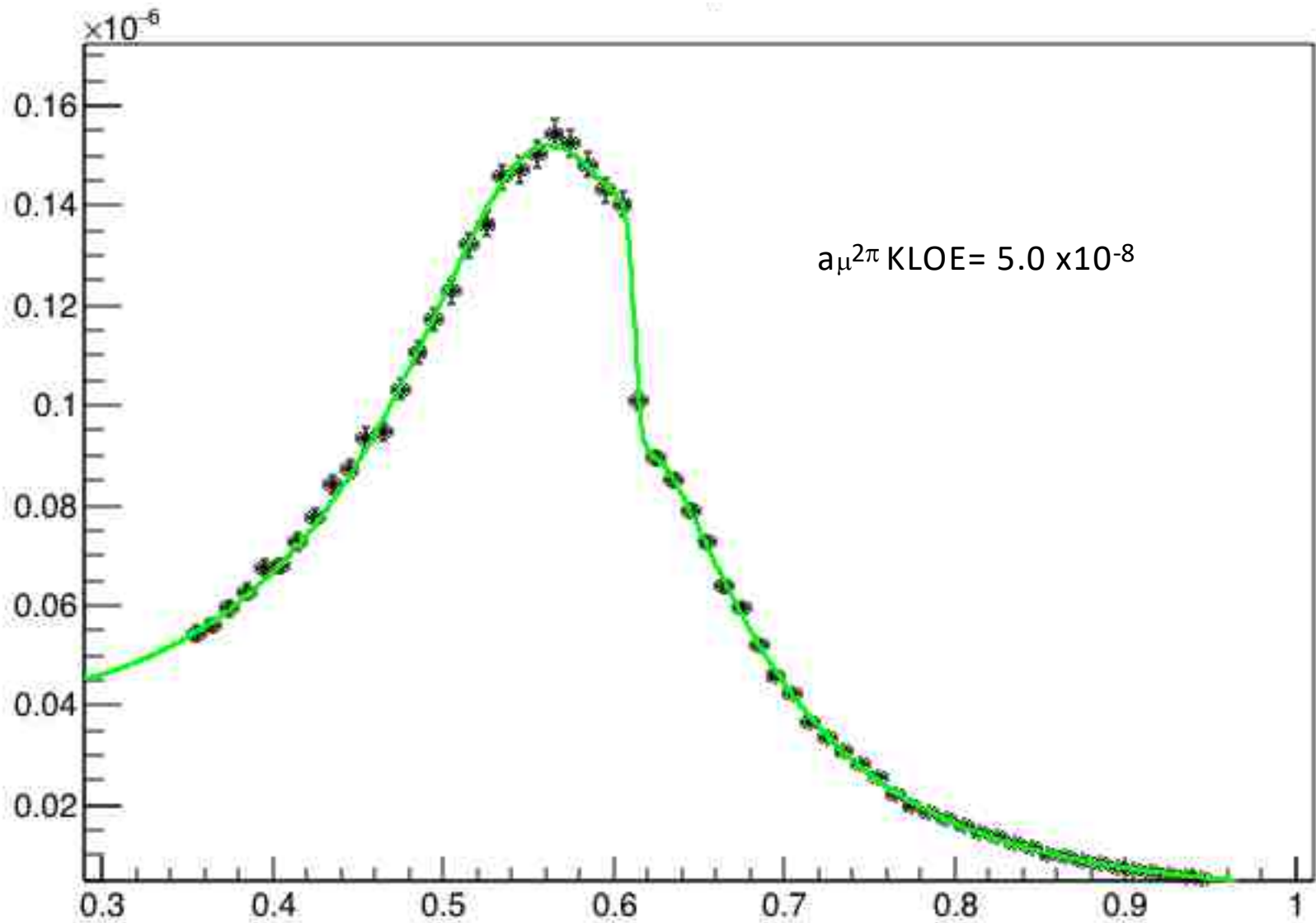
Conclusion

- Comparison of $a_\mu^{2\pi}$ for KLOE, BaBar, CMD2 in spacelike region show $\sim 1\%$ difference (2% btw KLOE and BaBar)
- Sensitivity of MUonE at peak (should be) $\sim 0.5\%$. Sufficient to distinguish amongst the different data?

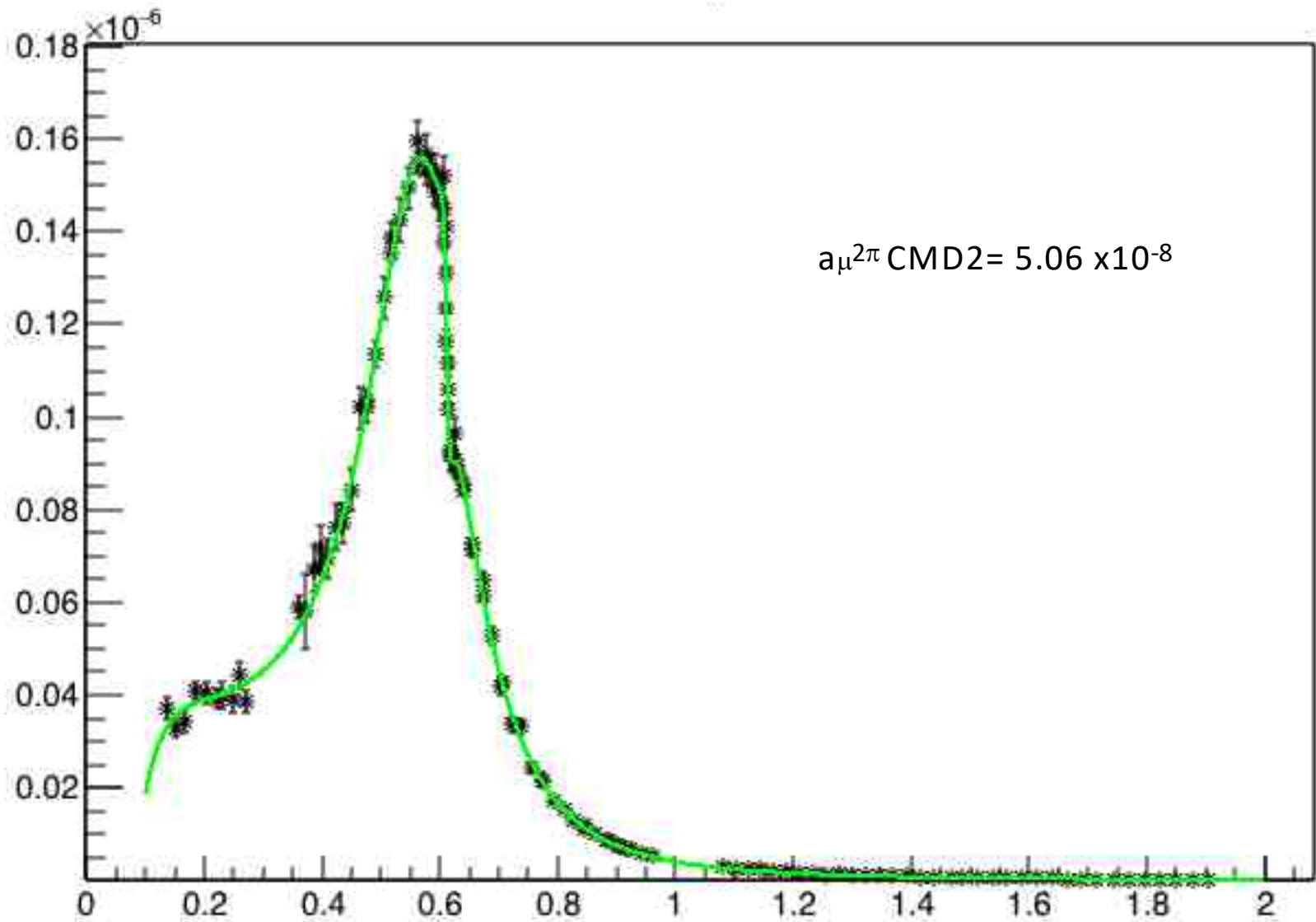
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Backup

$a_\mu^{2\pi}$ TIMELIKE KLOE



$a_\mu^{2\pi}$ TIMELIKE CMD2



$a_\mu^{2\pi}$ TIMELIKE BaBar

