

Fragment is univocally determined by Z and A

performed @200 MeV/u (hadrontherapy) and 700 MeV/u (radioprotection)

## Charge determination (Z):

$$-\frac{dE}{dx} = \frac{\rho \cdot Z}{A} \frac{4\pi N_A m_e c^2}{M_U} \left( \frac{e^2}{4\pi\epsilon_0 m_e c^2} \right)^2 \frac{z^2}{\beta^2} \left[ \ln \left( \frac{2m_e c^2 \beta^2}{I \cdot (1 - \beta^2)} \right) - \beta^2 \right]$$

The diagram shows two boxes: 'SCN' and 'TOF'. Arrows point from the term  $\frac{\rho \cdot Z}{A}$  to the 'SCN' box, and from the term  $\frac{z^2}{\beta^2}$  to the 'TOF' box.

## Number of mass determination (A):

### Reconstruction

$$A_1 = \frac{m}{U} = \frac{\mathbf{p}}{U \beta \gamma}$$

The diagram shows a box labeled 'TRACKER' above a box labeled 'TOF'. An arrow points from 'TRACKER' to 'TOF'.

$$A_2 = \frac{m}{U} = \frac{E_{kin}}{U(\gamma - 1)}$$

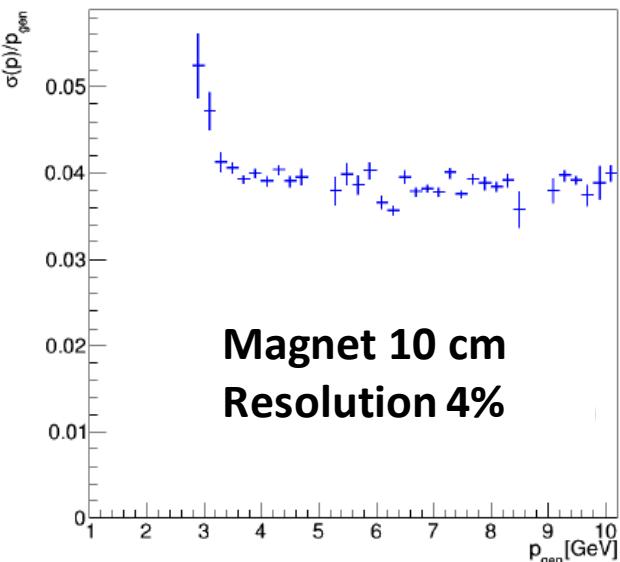
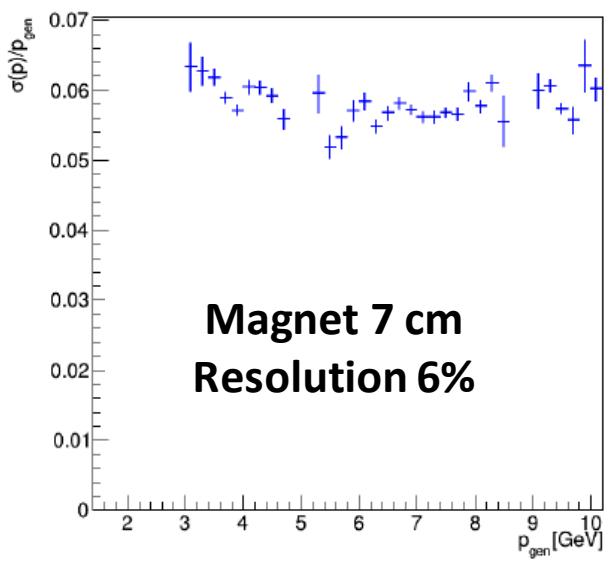
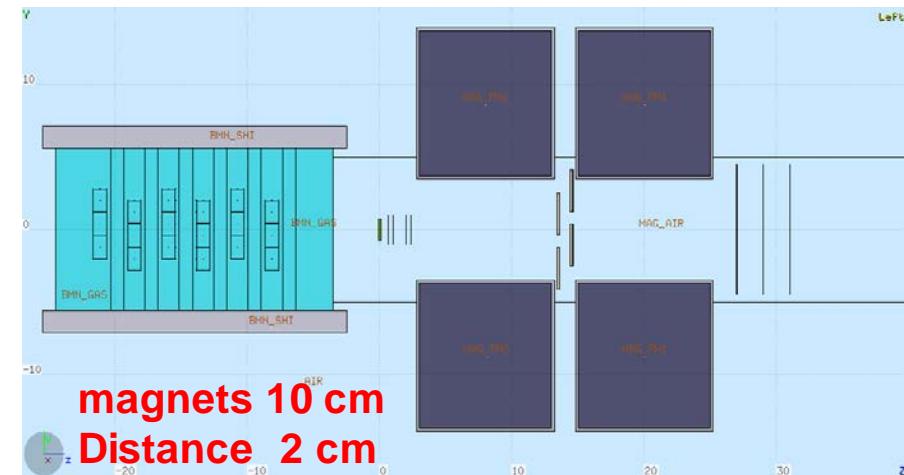
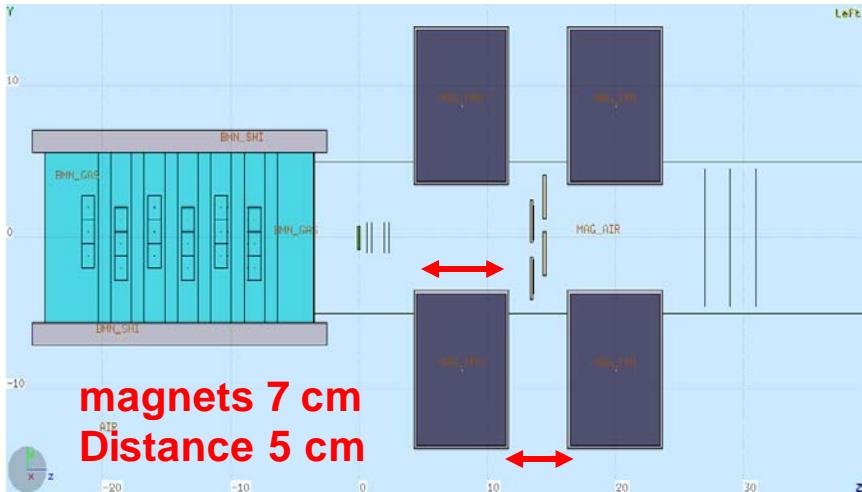
The diagram shows a box labeled 'CALO' above a box labeled 'TOF'. An arrow points from 'CALO' to 'TOF'.

$$A_3 = \frac{m}{U} = \frac{\mathbf{p}^2 - E_{kin}^2}{2E_{kin}}$$

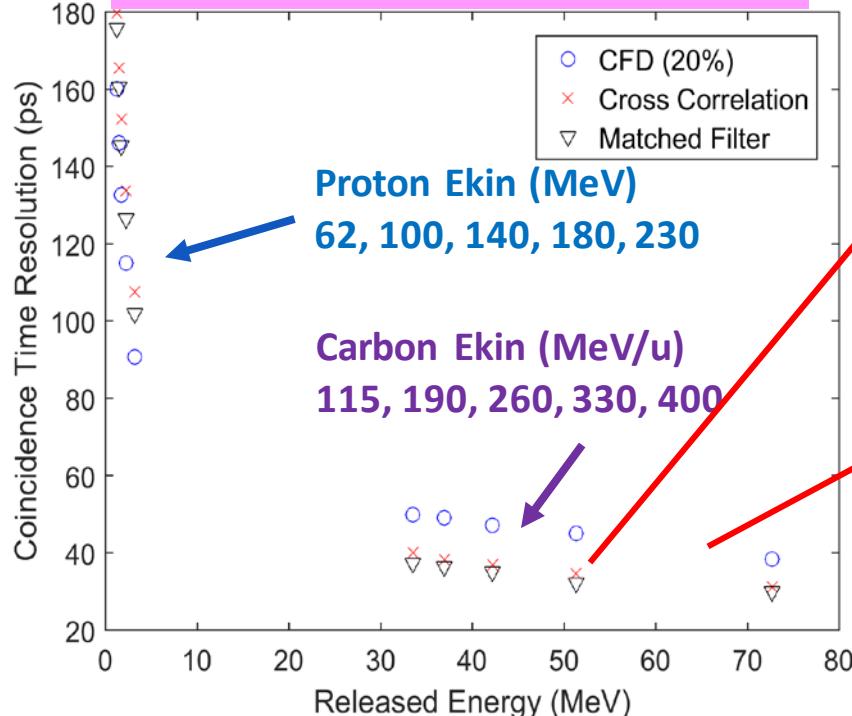
The diagram shows a box labeled 'TRACKER' above a box labeled 'CALO'. An arrow points from 'TRACKER' to 'CALO'.

### Fit Methods

## Fluka Simulation: $^{16}\text{O}$ (200 MeV/u) on $\text{C}_2\text{H}_4$ target

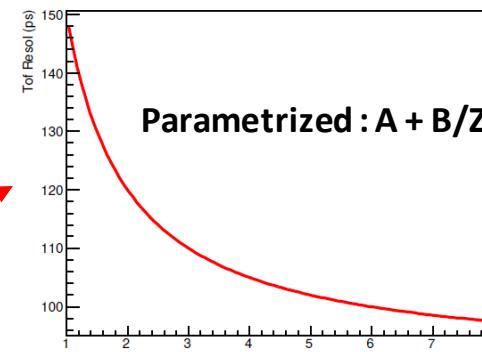


**Test beam @CNAO:  
Tof resolution (heavy fragments)  $\sim 50$  ps**

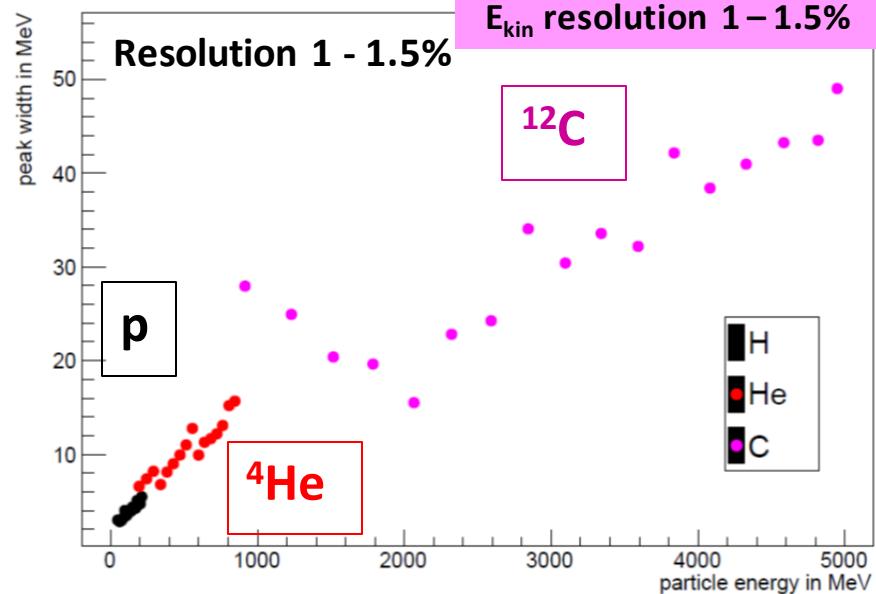


**Tof &  $E_{kin}$  resolution**

**Test on a single SCN, supposing the same resolution in the SC  $\rightarrow$  Tof Resolution = 70 ps**



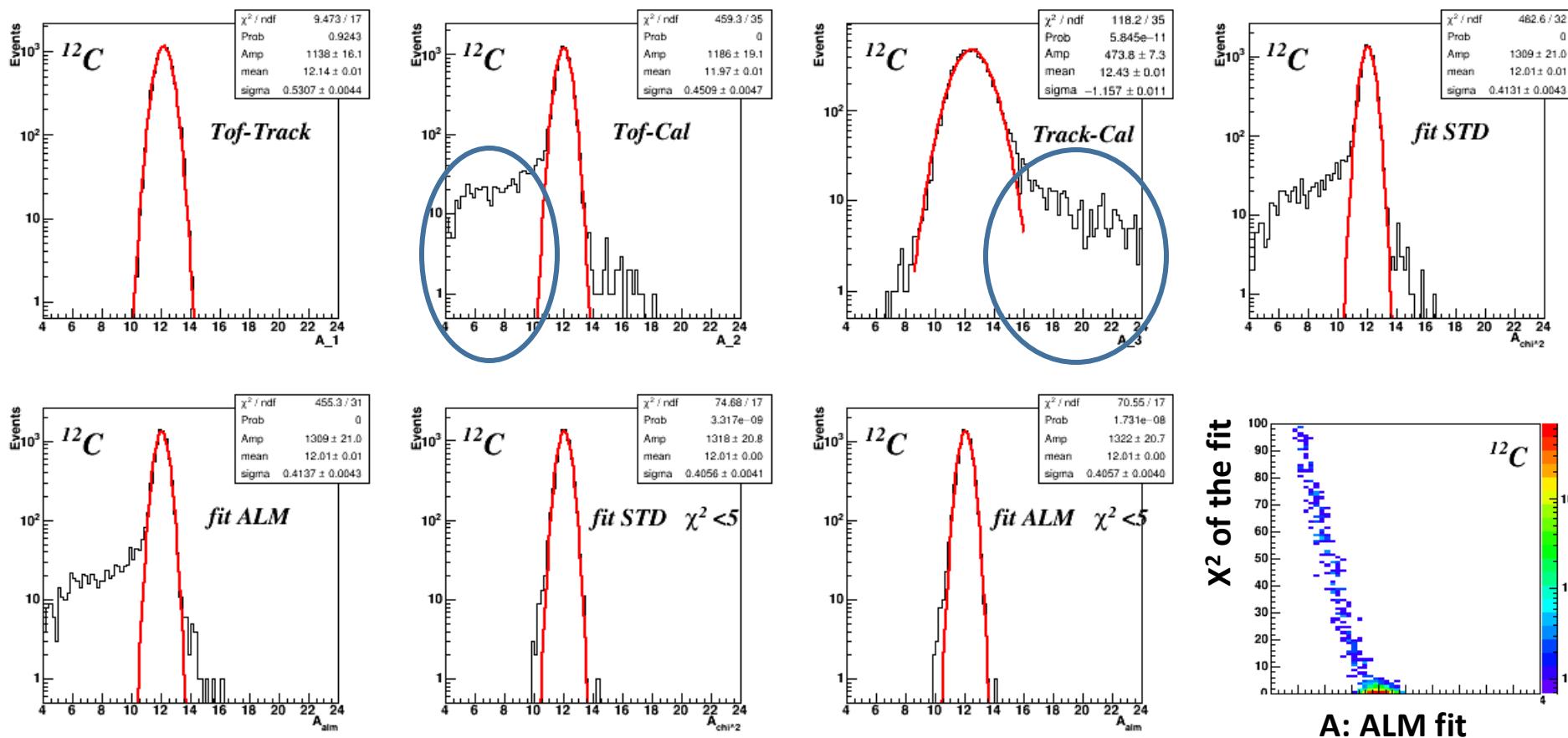
**Test beam @GSI:  
 $E_{kin}$  resolution 1 – 1.5%**



Quantity	Resolution Magnet 7 cm	Resolution Magnet 10 cm
Tof (ps)	70 (C) – 140 (H)	70 (C) – 140 (H)
$E_{kin}$ (%)	1.5	1.5
p (%)	6	4

**Standard Setup (SCN & CAL at 1m)**

# A determination @200 MeV with Magnet 10 cm (example of Carbon)



A: ALM fit

17% of the produced Carbon fragments on CALO → neutron production  
 → Energy loose → wrong A determination (same for Magnet with 7 cm)



FOOT Redundancy

Possibility to select and remove these events with a  $\chi^2$  cut on the Fit methods

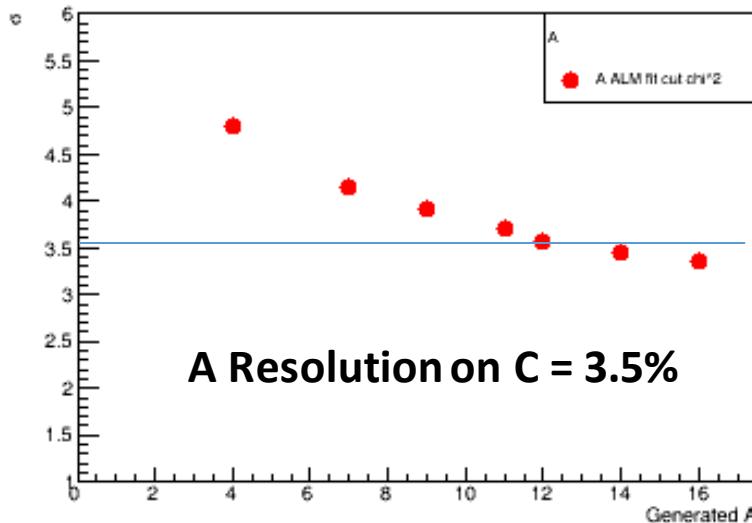
# Number of Mass Resolution @200 MeV (example of Carbon)

Magnet 7 cm ( $\Delta p/p=6.0\%$ )

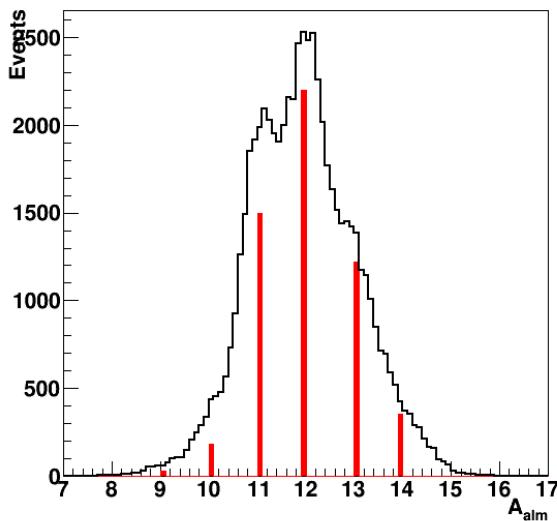
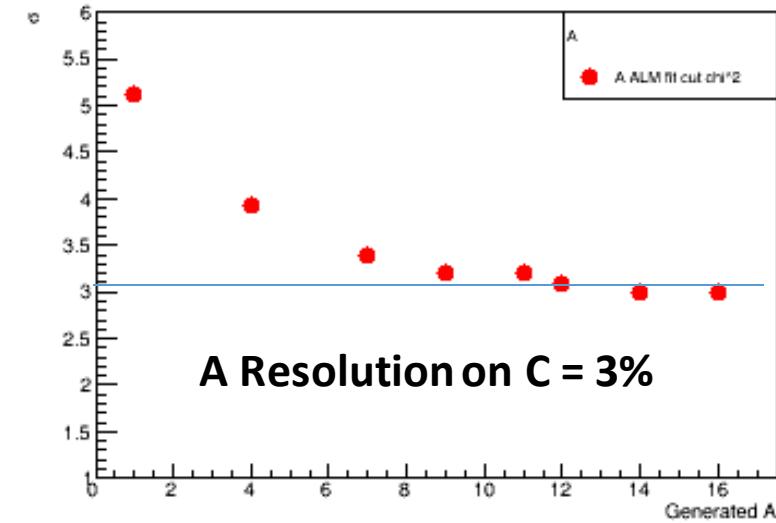
ALM Fit Method

Magnet 10 cm ( $\Delta p/p=4.0\%$ )

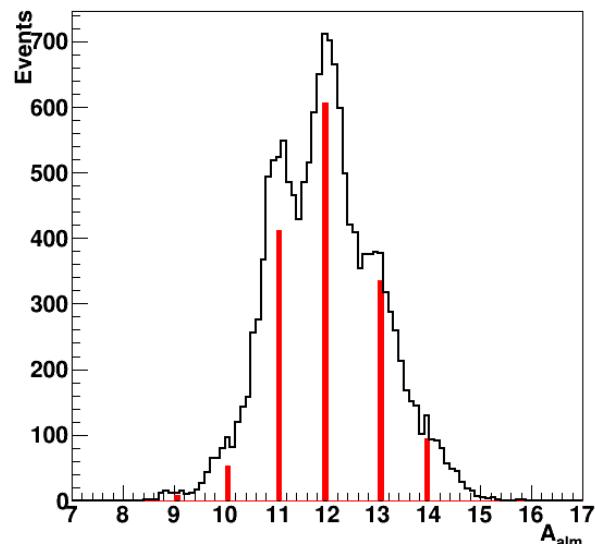
PERCENTAGE RESOLUTION



PERCENTAGE RESOLUTION



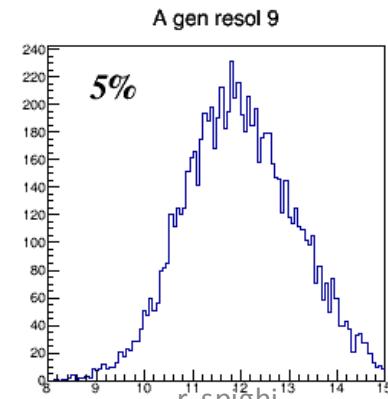
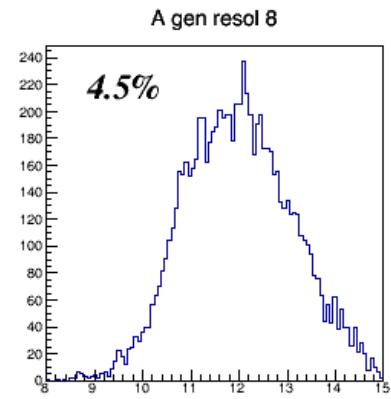
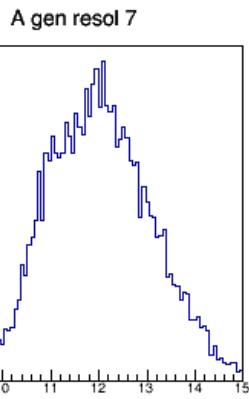
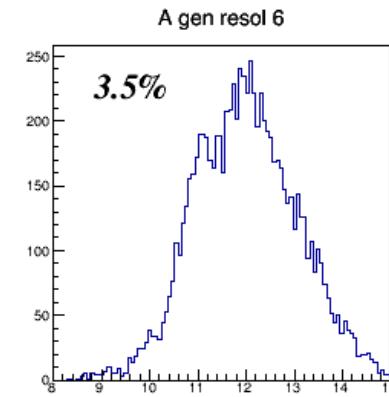
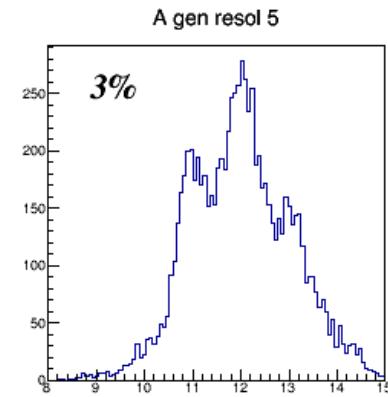
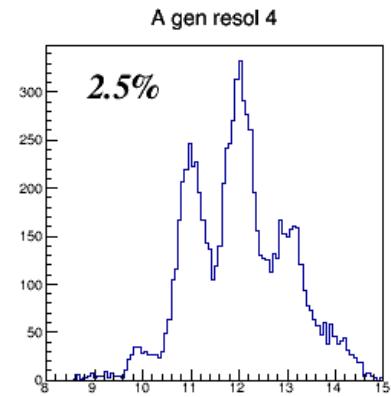
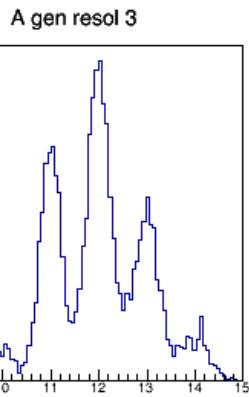
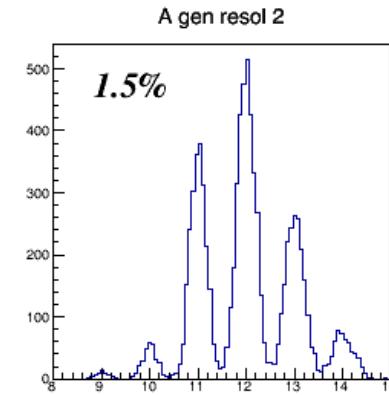
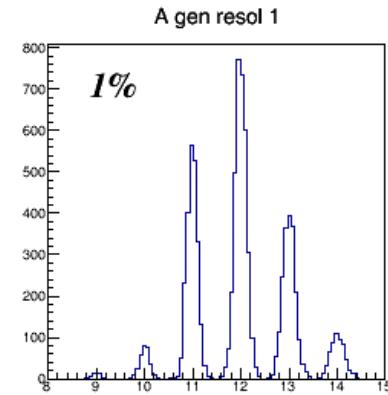
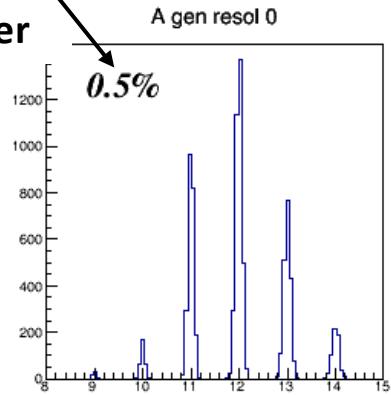
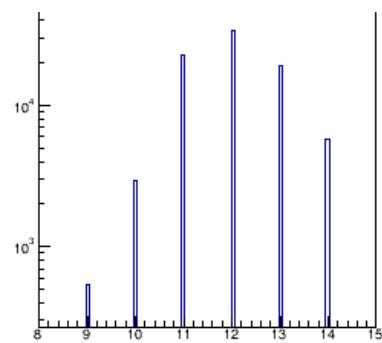
Better Isotope separation  
with magnets of 10 cm



# disentangle Carbon isotopes for $^{16}\text{O}$ (200 MeV) $\rightarrow \text{C}_2\text{H}_4$

Resolution

Isotopes position & number

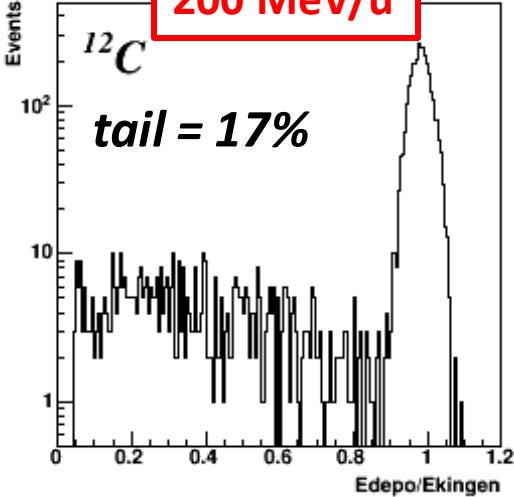


To disentangle Carbon Isotope  $\rightarrow$  A resolution of 2.5%

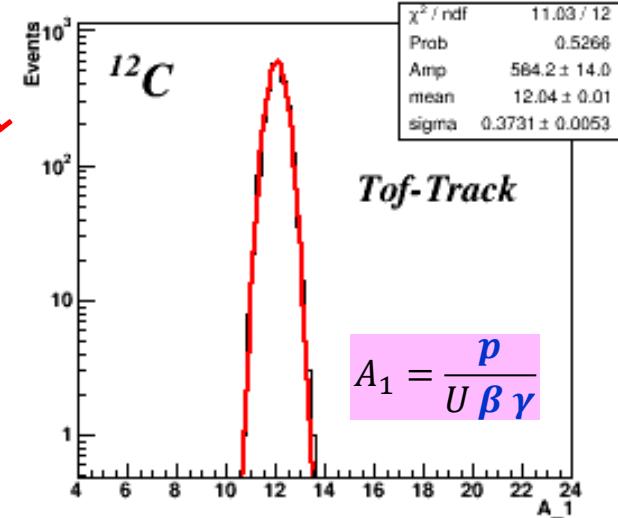
# 700 MeV/u: Problem 1 (example of $^{12}\text{C}$ )

Fraction of deposited energy

200 MeV/u

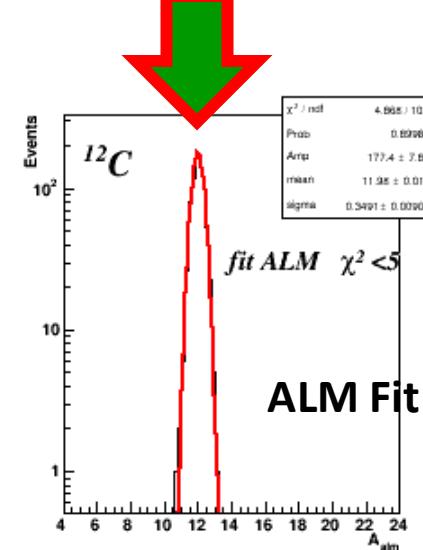
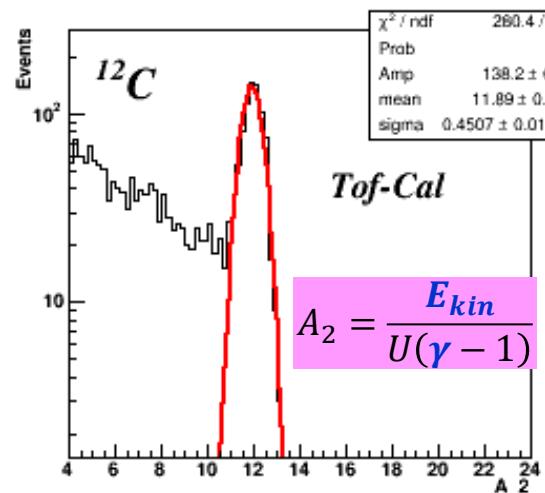
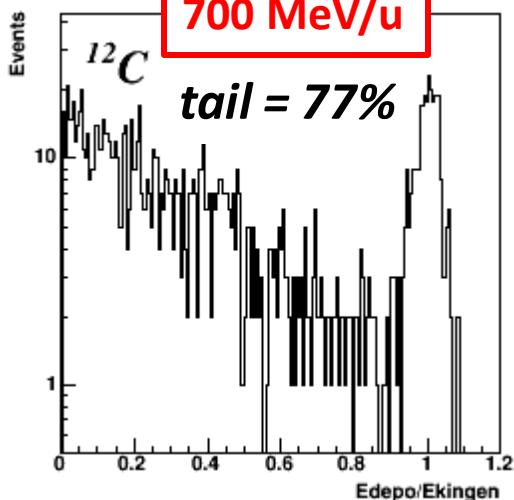


SOLUTION  
Use Tof-Tracker method: NO TAIL



Only 20% of the fragments well reconstructed

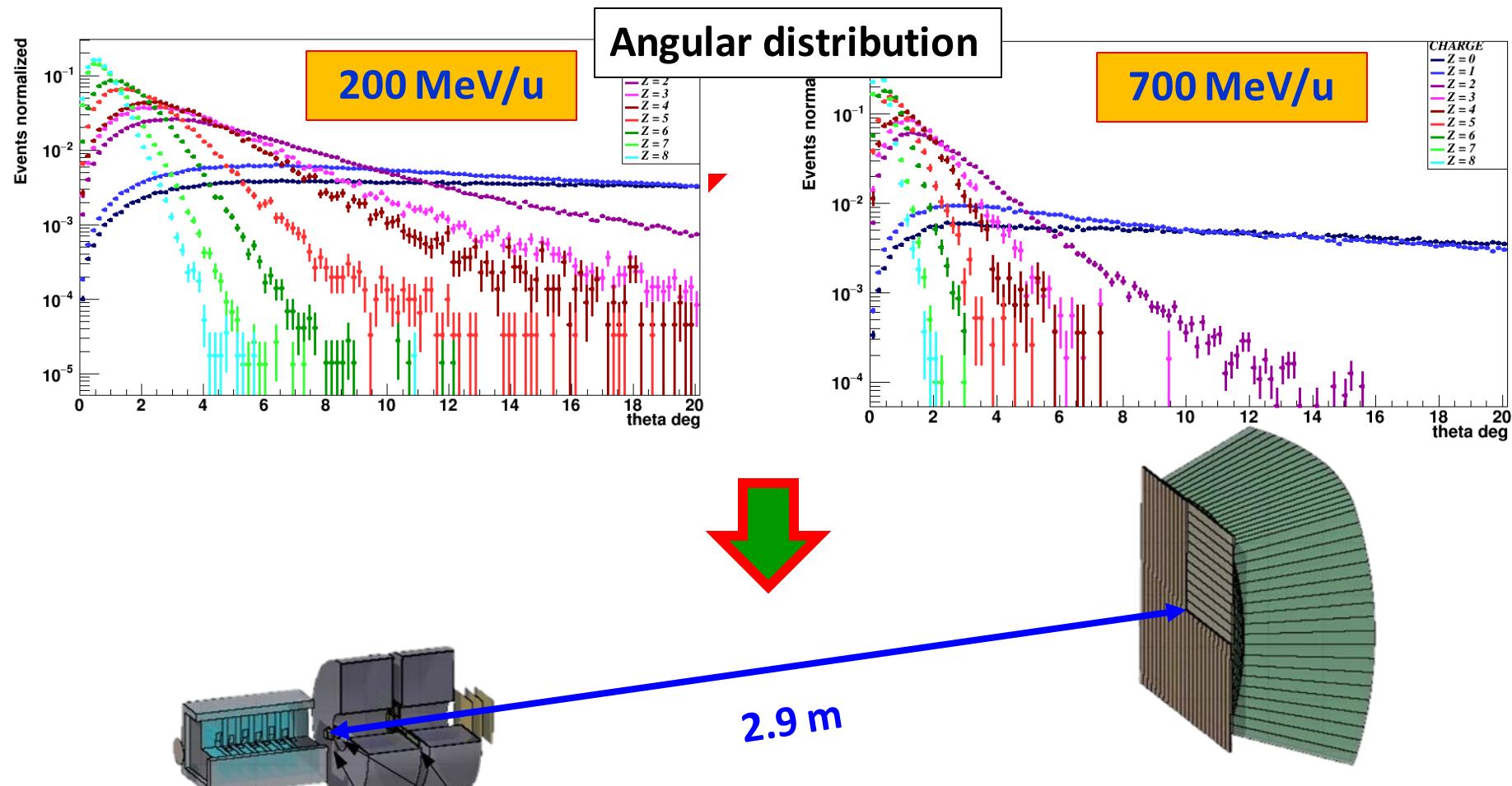
700 MeV/u



## 700 MeV/u: Problem 2

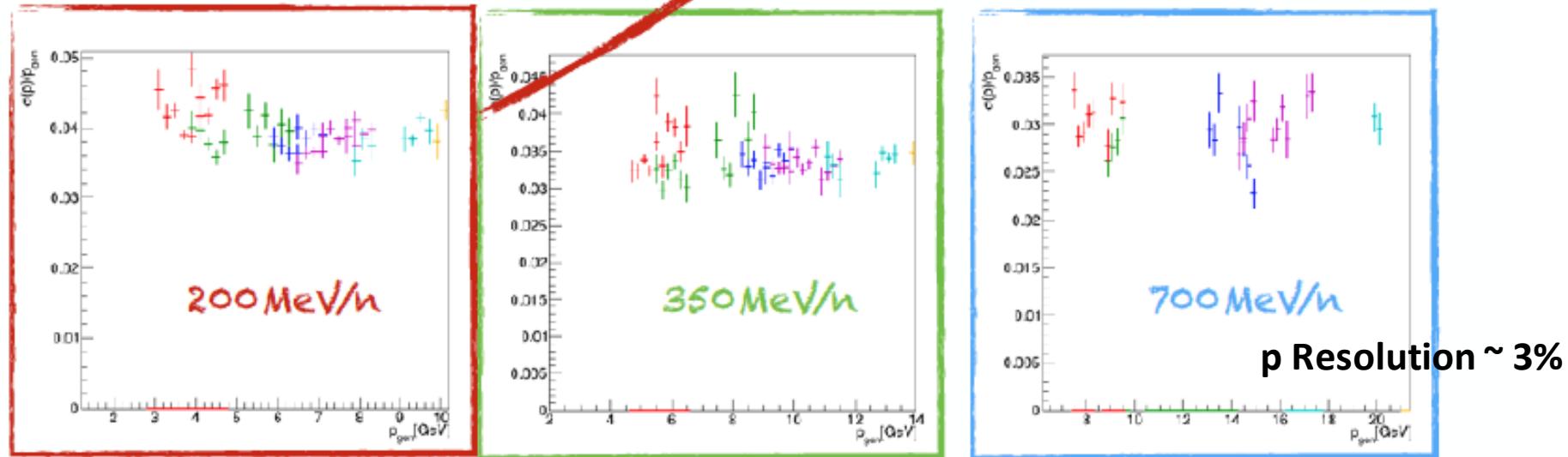
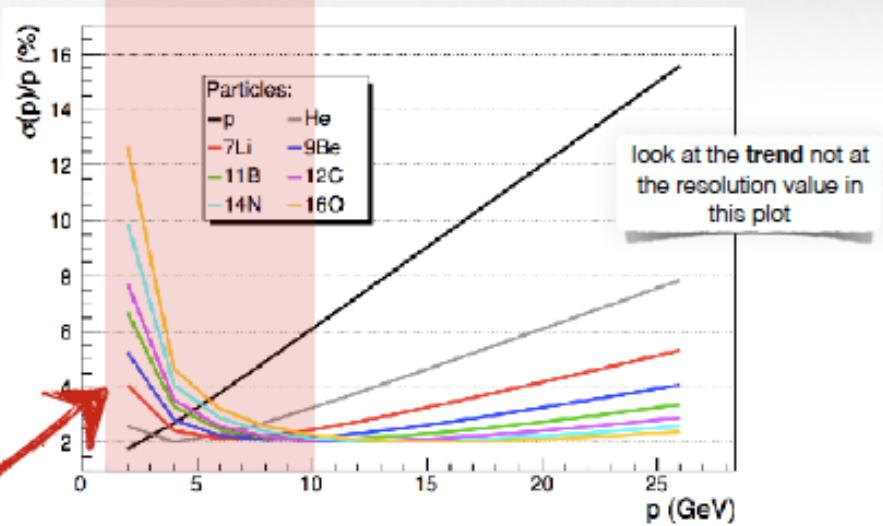
higher energy fragment  $\rightarrow$  lower Energy deposition but similar tof resolution (see pag 3)

Fragments with higher  $\beta \rightarrow$  lower Tof  $\rightarrow$  lower relative resolution



## p Resolution @ 700 MeV/u

- At 200MeV/n, all elements are in the “decreasing” part, dominated by MS contribution;
  - heavy-elements have high p so almost at minimum (that is at lower-p), MS not so high anymore;
- At 700MeV/n, all in region dominated by spatial resolution;
  - light elements (steeper growth, lower-p minimum) have low p; still close to the minimum
  - heavy elements (grows slower, higher-p minimum) have higher p but still close to the minimum



## Number of Mass Resolution @700 MeV

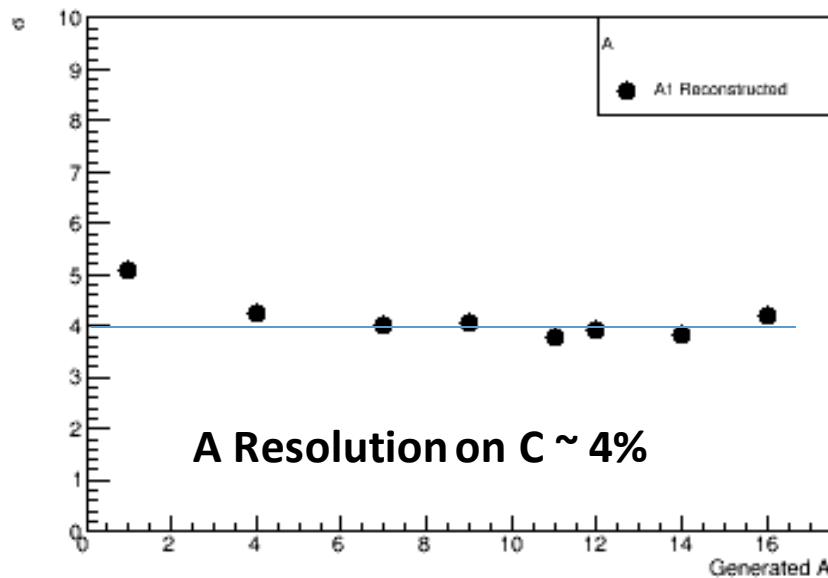
Quantity	Resolution Magnet 7 cm	Resolution Magnet 10 cm
Tof (ps)	70 (C) – 140 (H)	70 (C) – 140 (H)
$E_{kin}$ (%)	1.5	1.5
p (%)	3.5	2.5
Setup (SCN & CAL at 2.9 m)		

Same as @200 MeV/u (see pag 3)

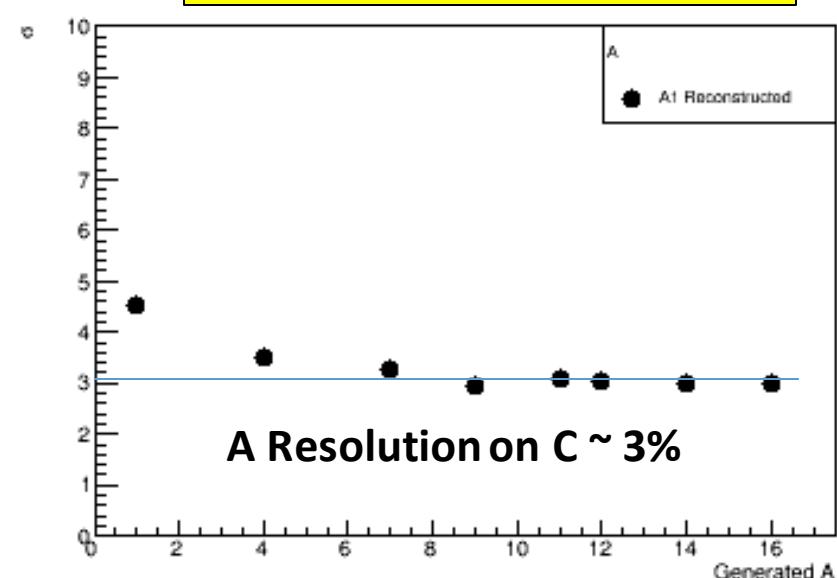
Setup (SCN & CAL at 2.9 m)

Mass reconstructed with Tof + Tracker Method

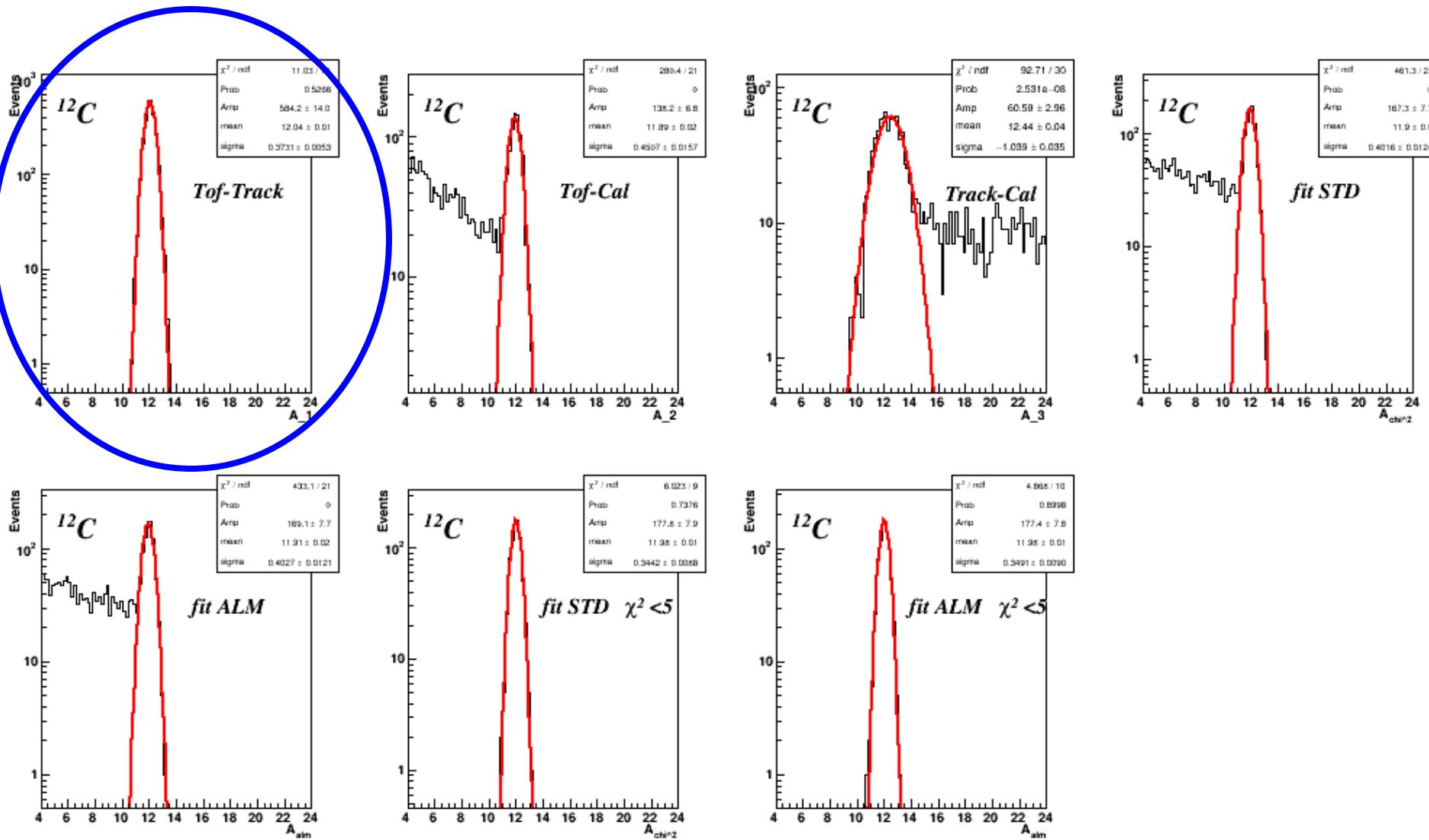
Magnet 7 cm ( $\Delta p/p = 3.5\%$ )



Magnet 10 cm ( $\Delta p/p = 2.5\%$ )

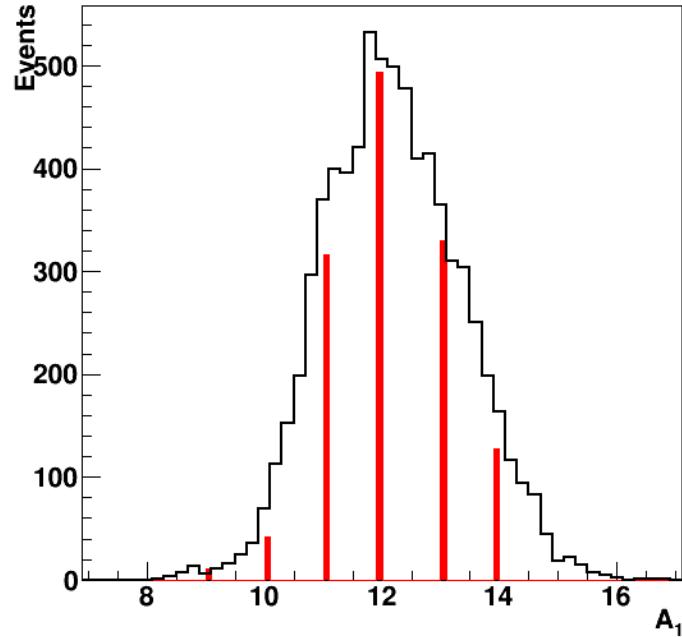


# A determination @700 MeV with Magnet 10 cm (example of Carbon)

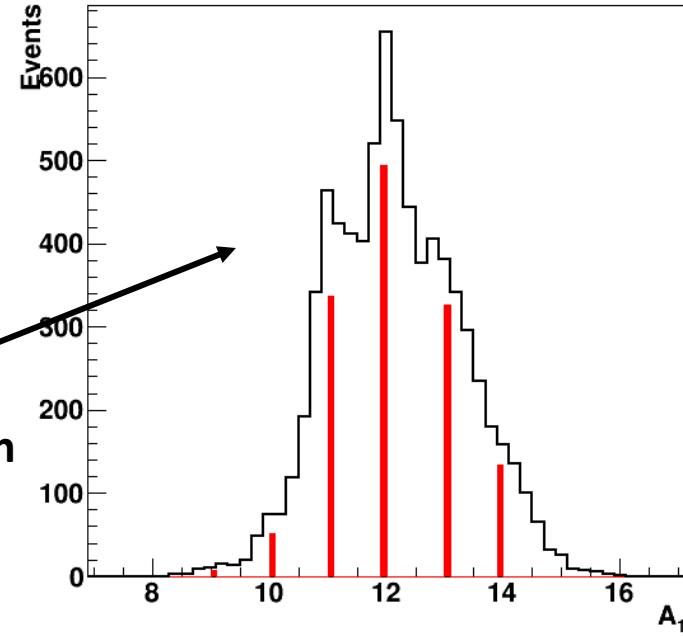


# *Carbon Isotopes separation @ 700 MeV*

Magnet 7 cm ( $\Delta p/p = 3.5\%$ )

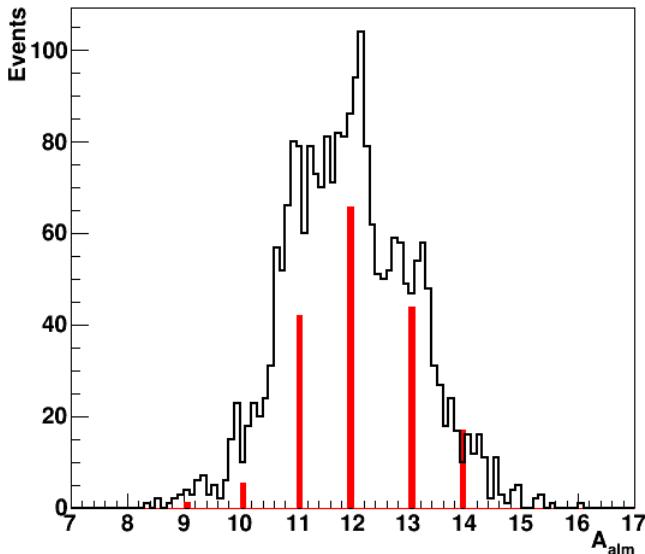


Magnet 10 cm ( $\Delta p/p = 2.5\%$ )



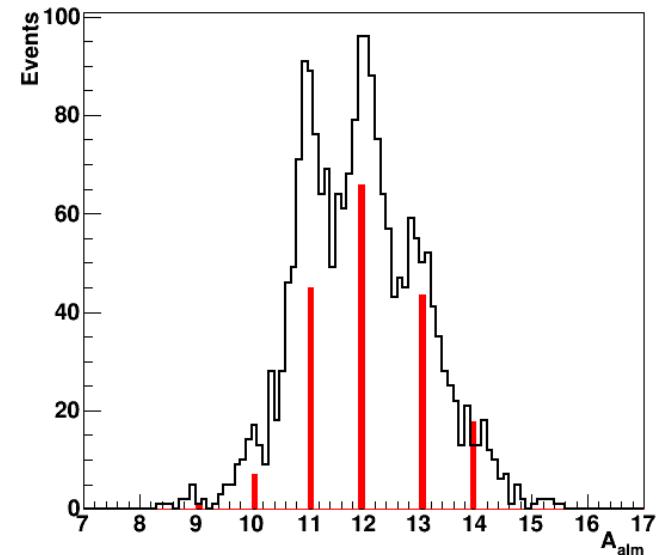
Tof + Tracker Method

Better isotope separation



ALM Fit Method

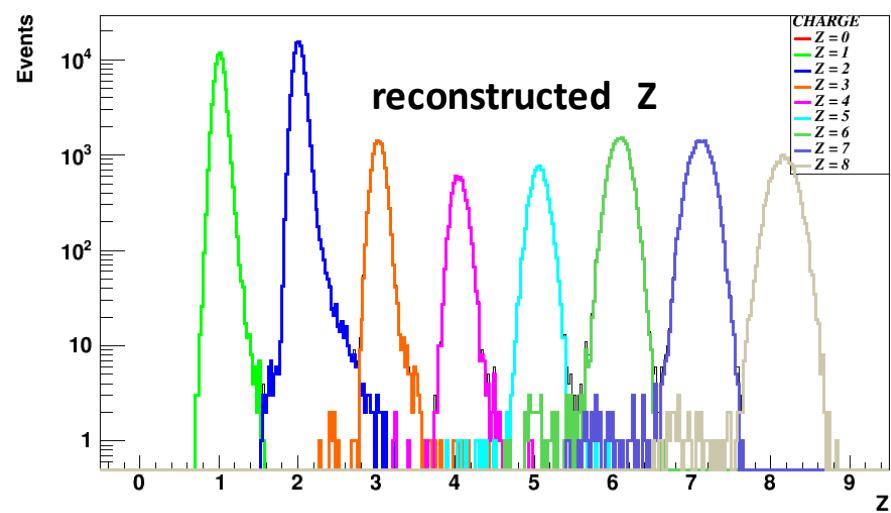
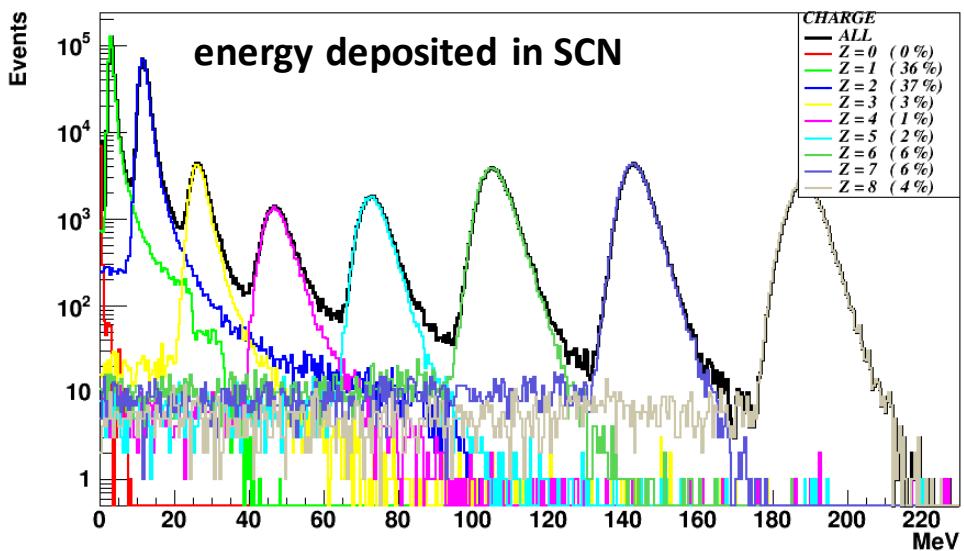
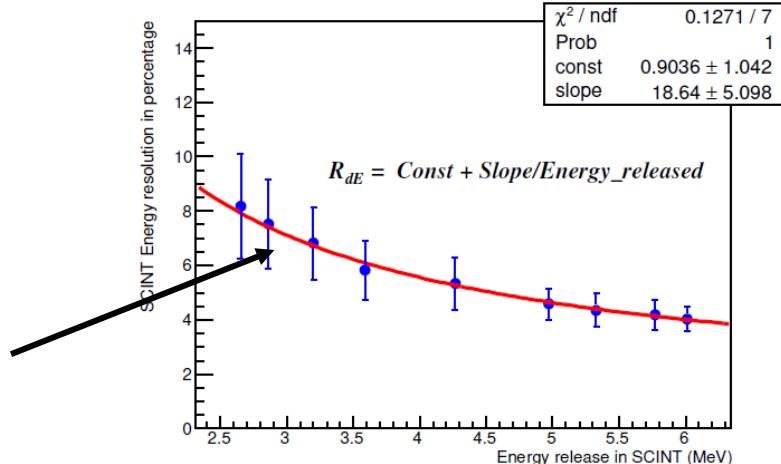
Only for well  
reconstructed fragments  
(20% of the statistics)



# Z Reconstruction @200 MeV

Fluka simulation  $^{16}\text{O}$  (200 MeV/u)  $\rightarrow \text{C}_2\text{H}_4$

SCN energy resolution  
parametrization from test beam



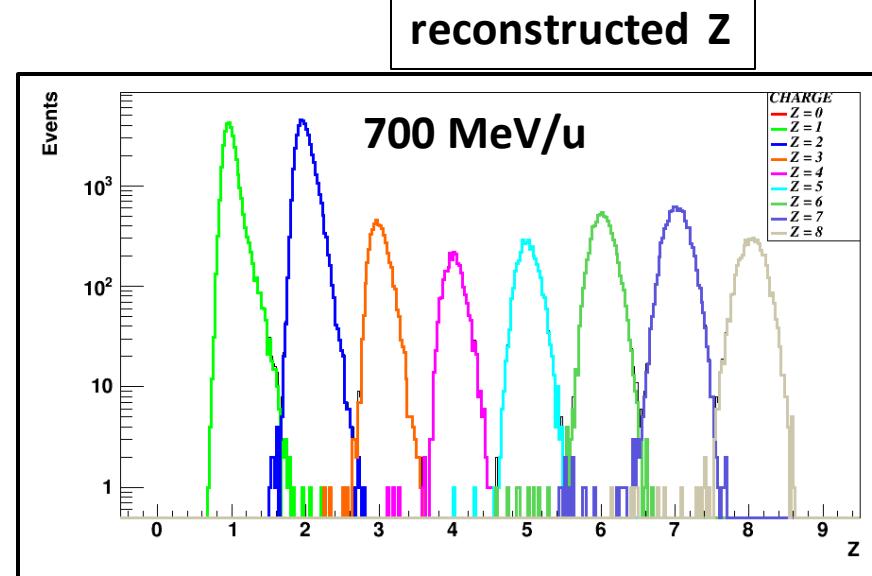
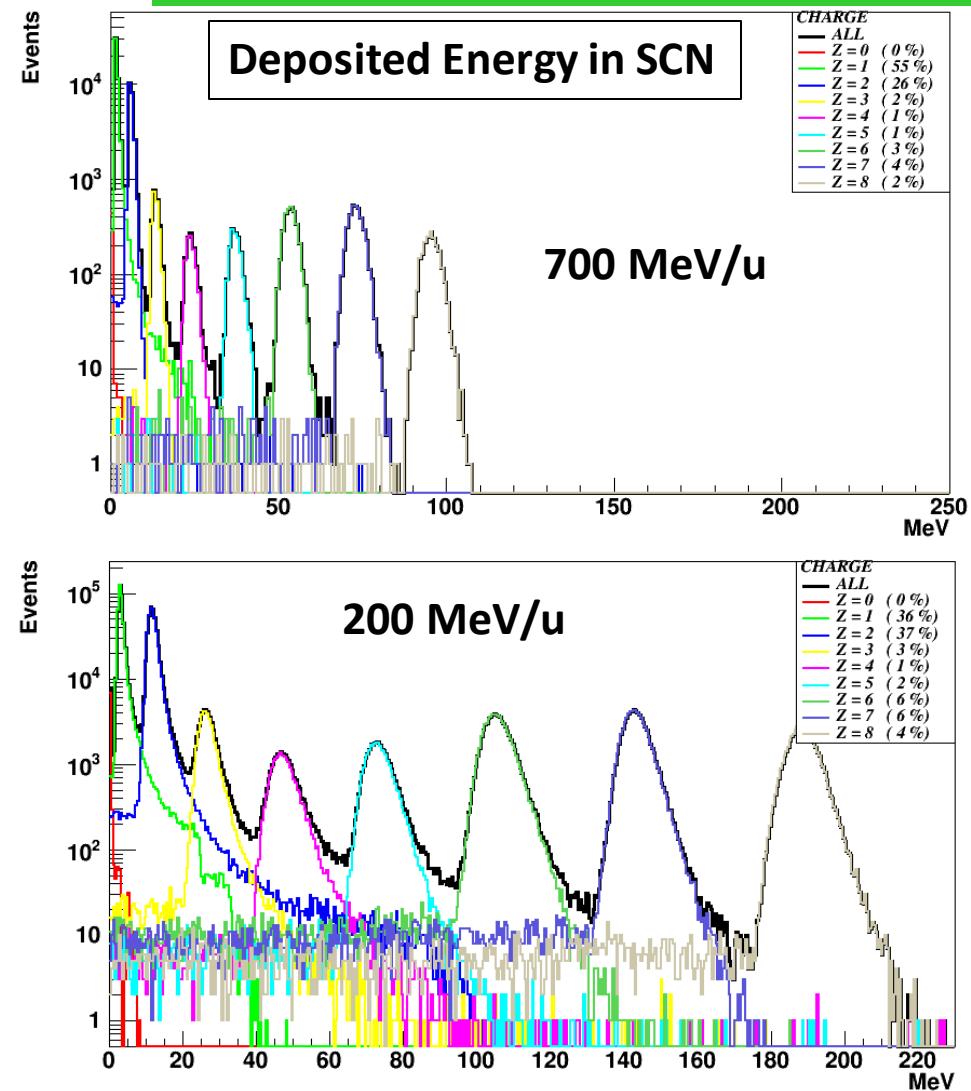
$^1\text{H}$	$^4\text{He}$	$^7\text{Li}$	$^9\text{Be}$	$^{11}\text{B}$	$^{12}\text{C}$	$^{14}\text{N}$	$^{16}\text{O}$
1	2	3	4	5	6	7	8
$1.01 \pm 0.09$	$2.01 \pm 0.06$	$3.03 \pm 0.08$	$4.05 \pm 0.09$	$5.06 \pm 0.10$	$6.09 \pm 0.12$	$7.11 \pm 0.14$	$8.15 \pm 0.15$

Z Resol :    9%            3%            2.0%

wrong charge assignment < %

# Z Reconstruction @700 MeV

Fluka simulation  $^{16}\text{O}$  (700 MeV/u)  $\rightarrow \text{C}_2\text{H}_4$



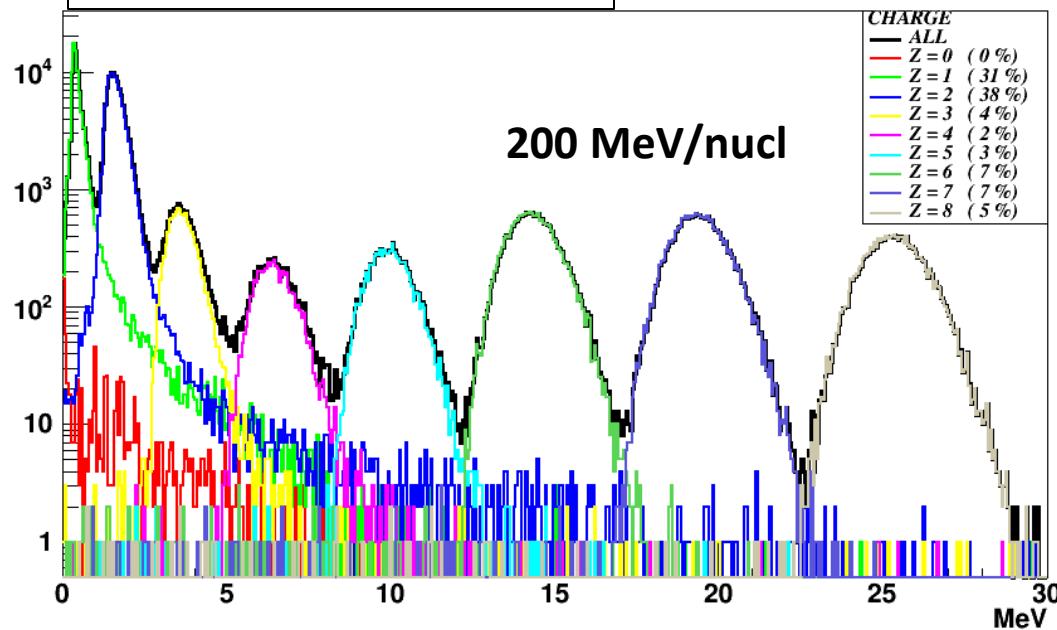
Z:	1	2	3	4	8
Resol (200 MeV):	9%	3.0%	2.0%	2.0%	2.0%
Resol (700 MeV):	9%	4.5%	3.6%	3.0%	2.1%

wrong charge assignment < %

$^1\text{H}$	$^4\text{He}$	$^7\text{Li}$	$^9\text{Be}$	$^{11}\text{B}$	$^{12}\text{C}$	$^{14}\text{N}$	$^{16}\text{O}$
1	2	3	4	5	6	7	8
$0.97 \pm 0.08$	$1.99 \pm 0.09$	$3.00 \pm 0.11$	$4.01 \pm 0.12$	$5.01 \pm 0.13$	$6.03 \pm 0.14$	$7.03 \pm 0.16$	$8.04 \pm 0.17$

# Z Reconstruction @200 & 700 MeV with MSD

Deposited Energy in MSD



200 MeV/nucl

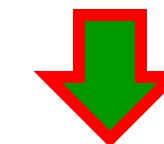
Fluka simulation:



450  $\mu\text{m}$  of Silicon

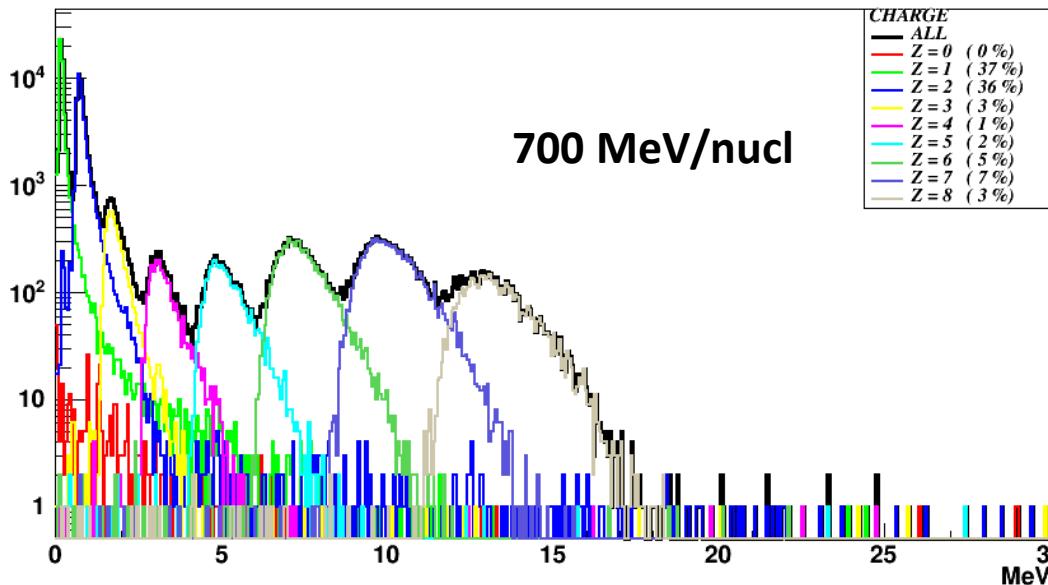
Peaks resolved @200 & 700 MeV/u

No smearing included



Possibility to combine the Z determination with the SCN

Events



700 MeV/nucl

## Summary

- **Z determination:**
  - $^{16}\text{O}$  (200 MeV/u & 700 MeV/u): Z Resolution in the range [2-5]% → wrong charge identification less than percent (SCN + MSD)
- **A determination**
  - @200 MeV/u: **FIT METHOD** with SCN, TRACKER and CALO (Tof, p,  $E_{\text{kin}}$ )

200 MeV/u	$\Delta p/p$	$\Delta A/A$ (example of C)
Magnet 7 cm	6%	3.5%
Magnet 10 cm	4%	3.0%

→ Good Isotope separation

- @700 MeV/u: FIT method possible only on 20% of statistics (missed energy)



**TOF + TRACKER METHOD & different FOOT Setup**

700 MeV/u	$\Delta p/p$	$\Delta A/A$ (example of C)
Magnet 7 cm	3.5%	4.0%
Magnet 10 cm	2.5%	3.0%

→ Isotope separation

*Backup slides*

# Fraction on deposited energy @ 200 MeV/u

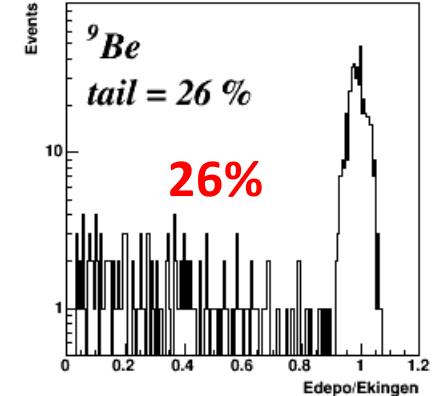
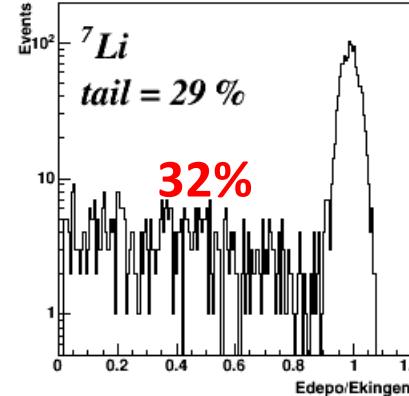
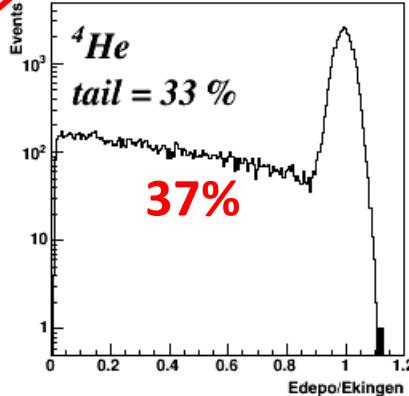
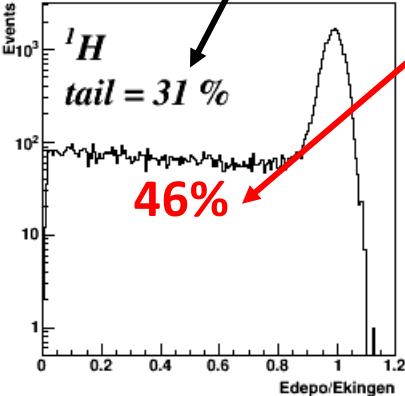
% Events outside peak (ratio<0.9), calo 21 cm

Energy deposited SCINT + CALO

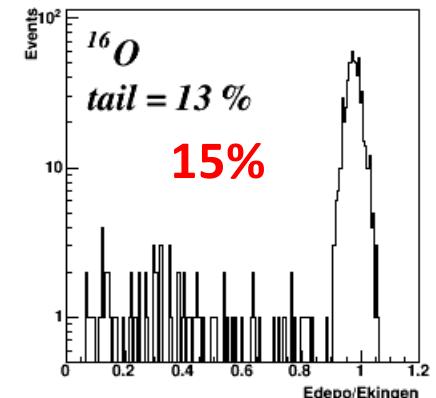
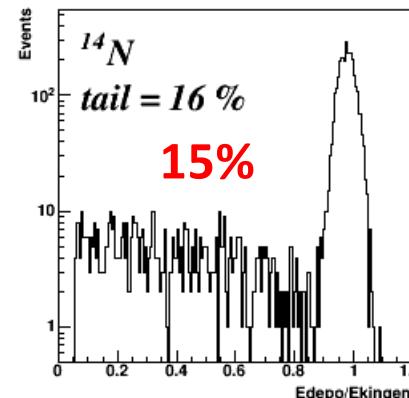
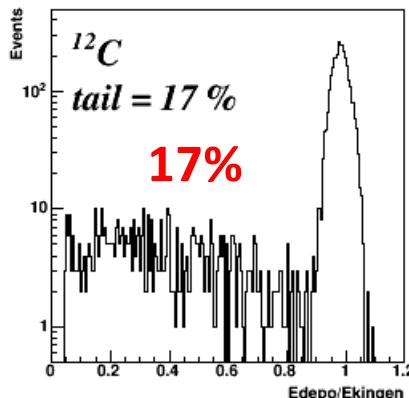
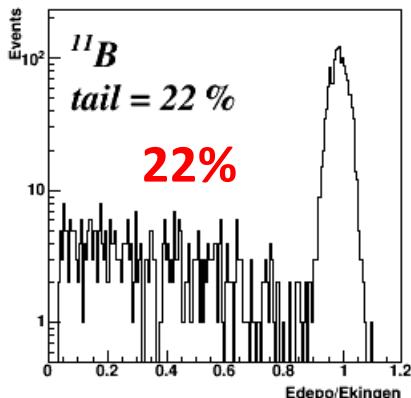
Kinetic Energy generated

calo 7 cm

46%



22%



$^1\text{H}$   
0.985

$^4\text{He}$   
0.990

$^7\text{Li}$   
0.988

$^9\text{Be}$   
0.985

$^{11}\text{B}$   
0.981

$^{12}\text{C}$   
0.975

$^{14}\text{N}$   
0.972

$^{16}\text{O}$   
0.968

r. spighi

## A Reconstruction and fit

**TOF ( $\beta$ ) – TRACKER ( $p$ )**

$$A_1 = \frac{m}{U} = \frac{p}{U \beta \gamma}$$

**TOF ( $\beta$ ) – CALO ( $E_{kin}$ )**

$$A_2 = \frac{m}{U} = \frac{E_{kin}}{U(\gamma - 1)}$$

**TRACKER ( $p$ ) – CALO ( $T$ )**

$$A_3 = \frac{m}{U} = \frac{p^2 - E_{kin}^2}{2E_{kin}}$$

- **Standard  $\chi^2$  Fit**

- Taking into account the correlation between  $A_1$ ,  $A_2$  and  $A_3$

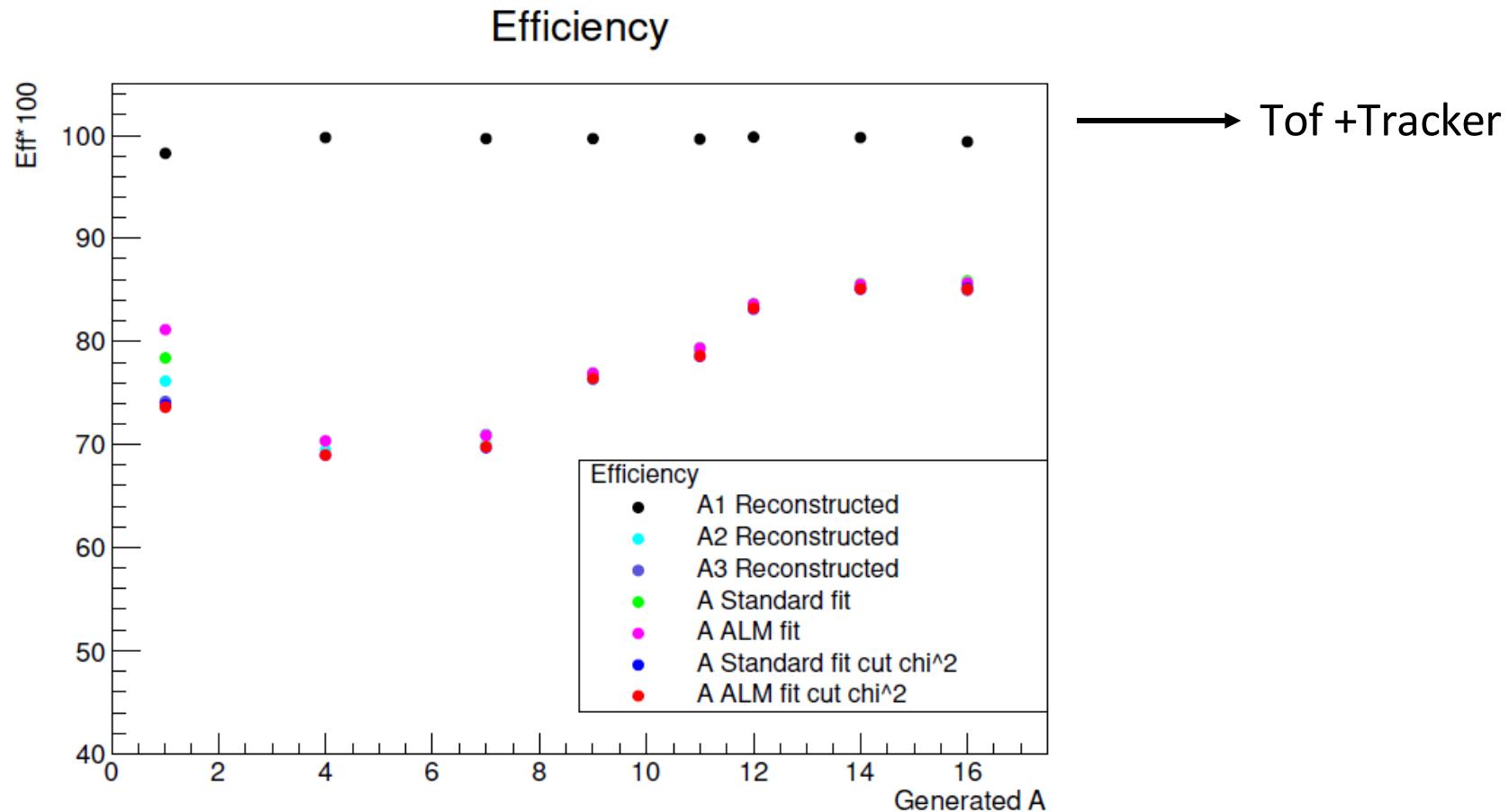
$$f = \left( \frac{(tof_{reco} - t)}{\sigma tof_{reco}} \right)^2 + \left( \frac{(p_{reco} - p)}{\sigma p_{reco}} \right)^2 + \left( \frac{(T_{reco} - T)}{\sigma T_{reco}} \right)^2 + (A_1 - A) \begin{pmatrix} C_{00} & C_{01} & C_{02} \\ C_{10} & C_{11} & C_{12} \\ C_{20} & C_{21} & C_{22} \end{pmatrix} \begin{pmatrix} A_1 - A \\ A_2 - A \\ A_3 - A \end{pmatrix}$$

$$C = (A \cdot A^T)^{-1} \quad A = \begin{pmatrix} \frac{\partial A_1}{\partial t} dt & \frac{\partial A_1}{\partial p} dp & 0 \\ \frac{\partial A_2}{\partial t} dt & 0 & \frac{\partial A_2}{\partial T} dT \\ 0 & \frac{\partial A_3}{\partial p} dp & \frac{\partial A_3}{\partial T} dT \end{pmatrix}$$

- **Augmented LagrangianFit (ALM)**

$$\tilde{\mathcal{L}}(\vec{x}; \lambda, \mu) \equiv f(\vec{x}) - \sum_a \lambda_a c_a(\vec{x}) + \frac{1}{2\mu} \sum_a c_a^2(\vec{x}).$$

## *A reconstruction efficiency*



Reconstruction efficiency ~ 70-80 % depending on the fragment