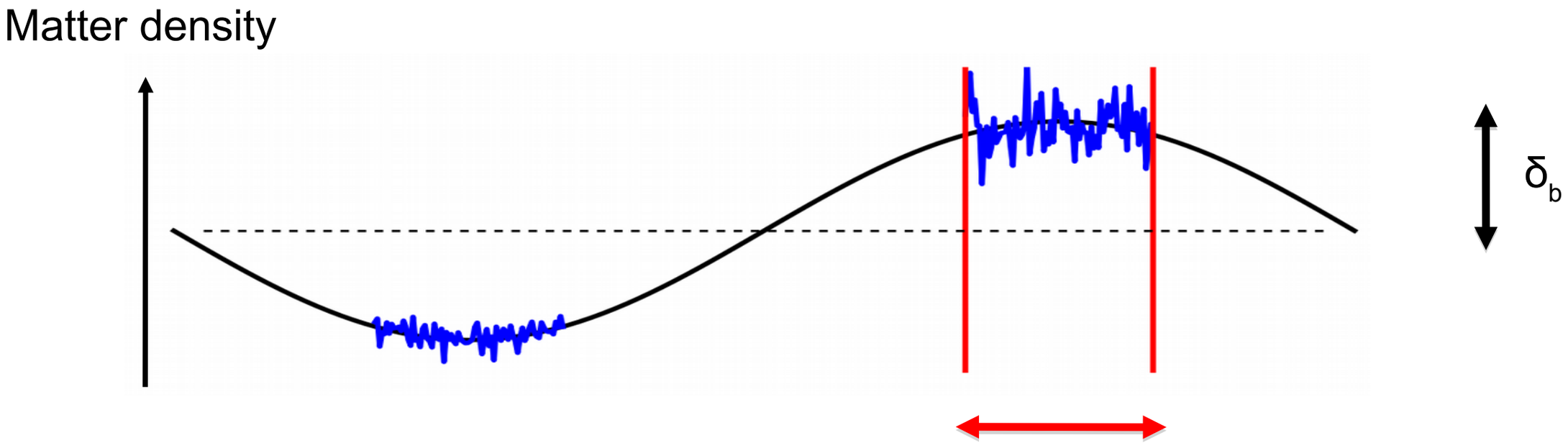
Easy super-sample covariance

Fabien Lacasa
University of Geneva
fabien.lacasa@unige.ch

Based on : arXiv:1809.05437

Super-sample covariance (SSC)



Power spectrum : all scales probes react to δ_b
→ more important when more modes

All probes react → more important when more probes

Separate universe (e.g. Wagner et al. 2015) :
can mimick δ_b with a change of cosmology

Is SSC important ?

I for galaxy surveys

- Spectroscopic galaxy clustering : **meh**

Table 4. Standard deviations of the super-sample errors on various parameters, for the 3 redshift bins of BOSS DR12 NGC.

	σ_{D_A}/D_A	σ_H/H	σ_{D_V}/D_V	$\sigma_{F_{AP}}/F_{AP}$	$\sigma_{b_1\sigma_8}/b_1\sigma_8$	$\sigma_{f\sigma_8}/f\sigma_8$
$0.2 < z < 0.5$	0.10%	0.29%	0.14%	0.27%	0.4%	1.2%
$0.4 < z < 0.6$	0.09%	0.27%	0.13%	0.24%	0.3%	1.1%
$0.5 < z < 0.75$	0.08%	0.26%	0.12%	0.23%	0.3%	1.0%

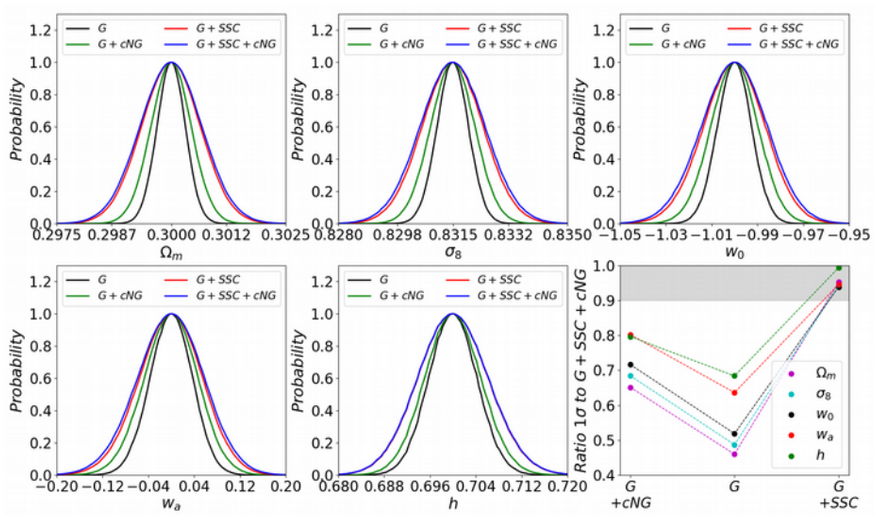
Li et al. 2017 arXiv:1711.00018

Effect negligible on BAO
Fraction of current error bars on RSD

- Cluster counts : **yes** (when pushing to small mass)

"for future surveys [..] sample variance is generally comparable to or greater than shot noise [..] For example, sample variance is usually **more important** than shot variance in constraints on w_{DE} from z<1 clusters." Hu & Kravtsov 2003

- Weak lensing : **yes** (when pushing to small scales)



Barreira et al. 2018 arXiv:1807.04266

Euclid : error bars increase +30% to +110%

DE, σ_8 and Ω_m particularly affected

- Photometric galaxy clustering : **YES**

Is SSC important ?
II for CMB

We don't know

Expectations for CMB x CMB : not much
Reasons : high z, large volume, large scales

- iSW : negligible
- CMB lensing :
  no for Planck, maybe when pushing to smaller scales
- tSZ : maybe if we cut the high masses

For Euclid x CMB : maybe
cross-correlation → lower z, lower volume

Easy SSC

Problems with SSC modeling :

- Complex literature, many NL effects, quickly need full non-linear model (e.g. HM)
- 4-5 codes do it, only 1 public to my knowledge (cosmolike)

Need something easily usable by the community, flexible, can see the impact wrt to Gaussian case

↓

$$\mathrm{Cov}_{\mathrm{SSC}}\left(C_\ell^{AB}(i_z,j_z),C_{\ell'}^{CD}(k_z,l_z)\right) \approx \; R_\ell^{AB}\, C_\ell^{AB}(i_z,j_z)\; R_{\ell'}^{CD}\, C_{\ell'}^{CD}(k_z,l_z) \times S^{A,B;C,D}_{i_z,j_z;k_z,l_z}$$

Lacasa & Grain 2018
arXiv:1809.05437

Just needs :

- S_{ij} : integral of linear P(k) and survey window — Computable in < 1s on laptop
- R_l : probe's response (contains non-linear physics) — Can take simple ansatz, see later

Extendable to correlation function, cluster counts, bispectrum...

Easy **and fast** SSC

Form of the SSC approx → inverse covariance is correction to Gaussian case

→ easy computation of S/N, Fisher, $\ln \mathcal{L}$

and correction is computable **as fast as Gaussian case**

Analytical application for a C_l analysis

$$(S/N)^2 = (S/N)^2_G \left(1 - \frac{Y}{1+Y}\right) \qquad Y = \frac{(S/N)^2_G}{(S/N)^2_{\max}}$$

$$(S/N)^2_{\max} = \frac{1}{R^2\ S_{i,i}} \qquad \ell_{\rm SSC} = \sqrt{\frac{2}{R^2 S_{i,i}}}$$

Fisher : $$F_{\alpha,\alpha} = F^G_{\alpha,\alpha}\left(1 - \cos^2\theta_\alpha \frac{Y}{1+Y}\right)$$

cos θ_α : how parameter α is correlated with background change

SSC relevant if cosθ_α=O(1) **and** l_{max}=O(l_{SSC})

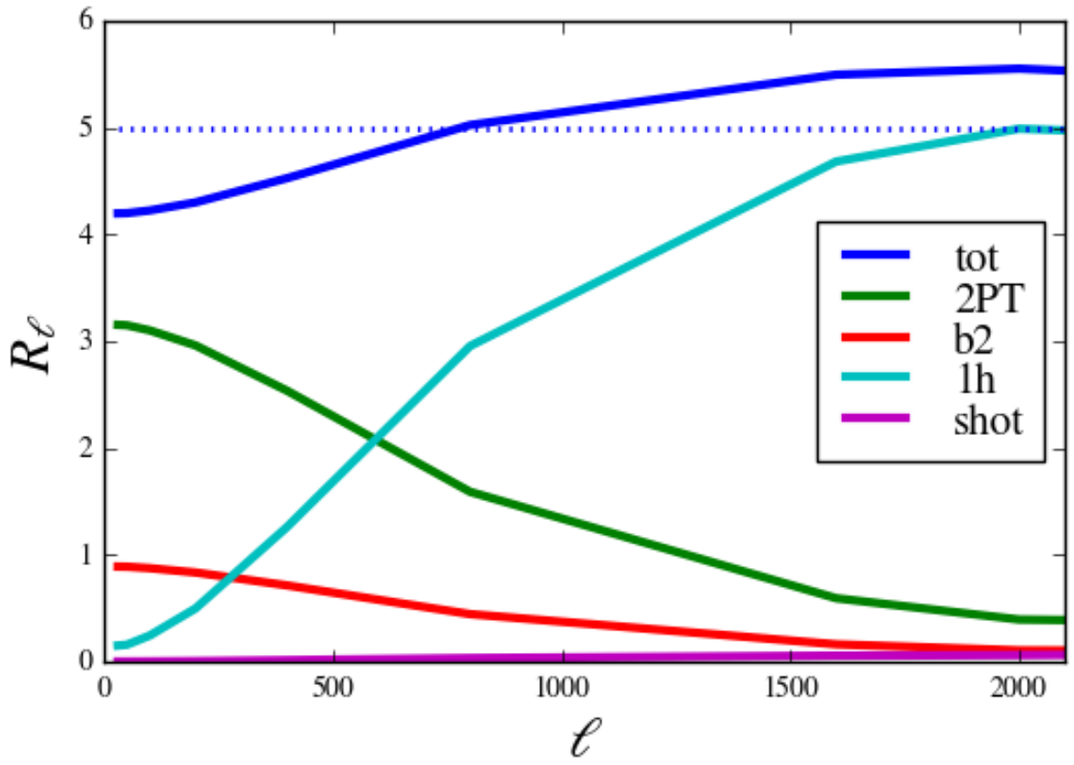
Application I : relevance

Forecast of GCphot C_l with Euclid-like specs :

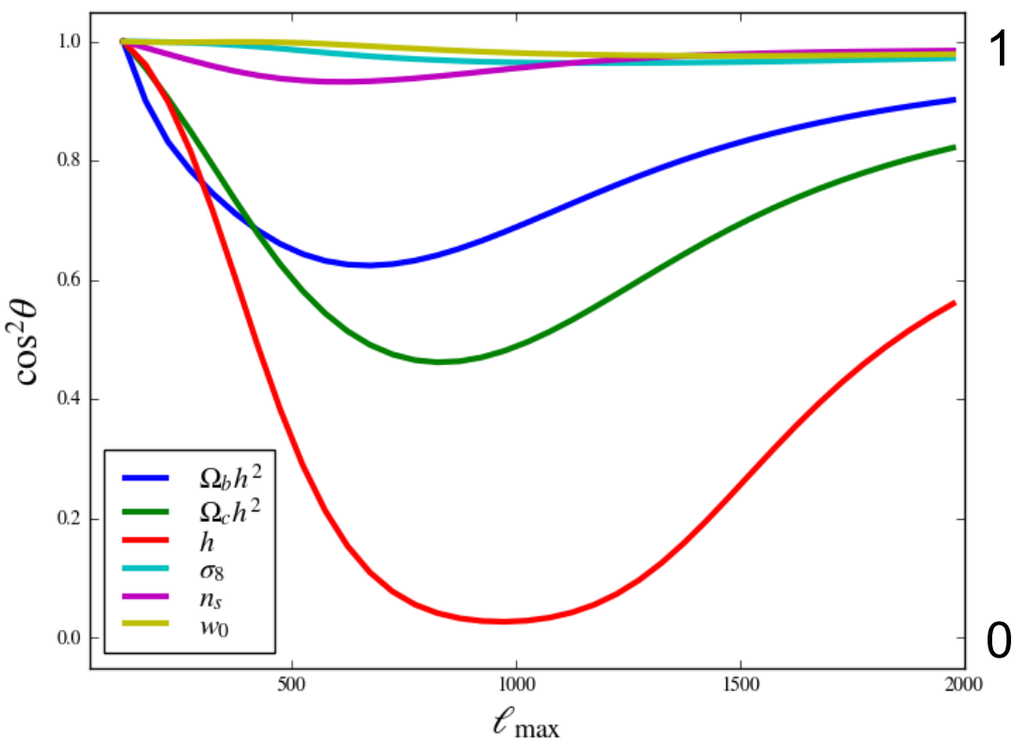
- Euclid n(z). $0.9<z<1 \rightarrow 2.5$ gal/arcmin^2
- $l_{max} = 2000$. Bins Dl=50. Full-sky
- Planck 2013 cosmology, HOD fitted to n(z)

Results : $S_{i,i} = 6.2 \times 10^{-7}$ maximum S/N = 250 $l_{SSC} = 360$

Response

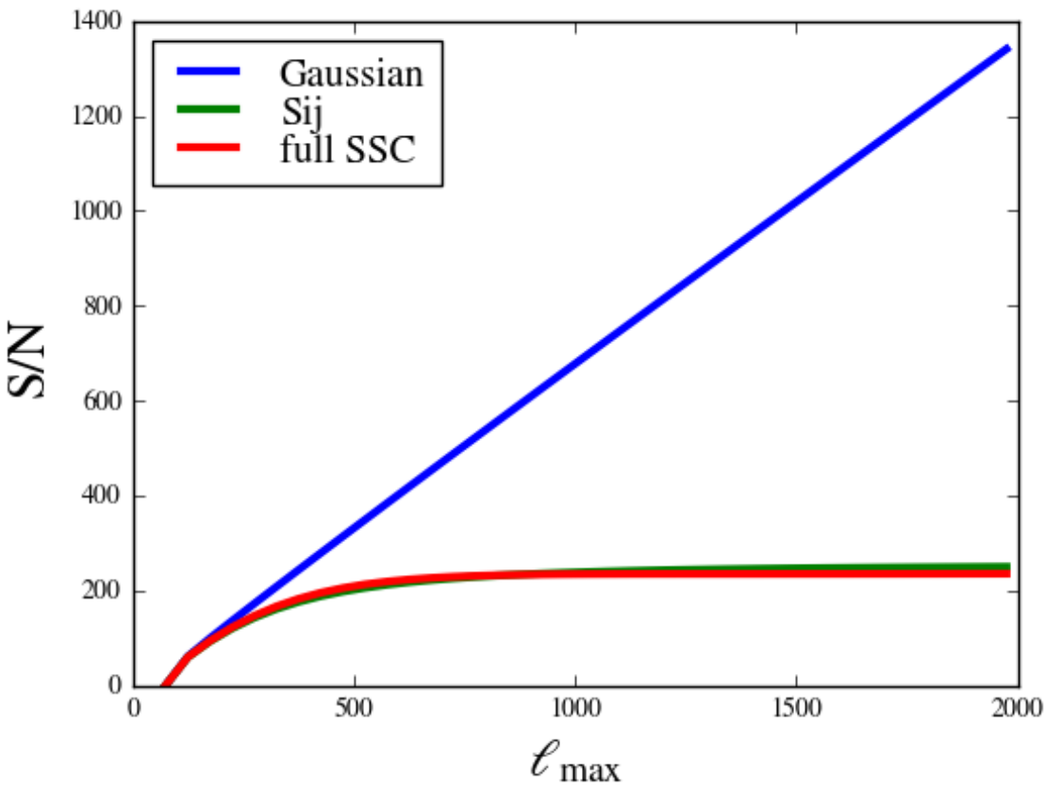


$\cos^2 \theta_\alpha$

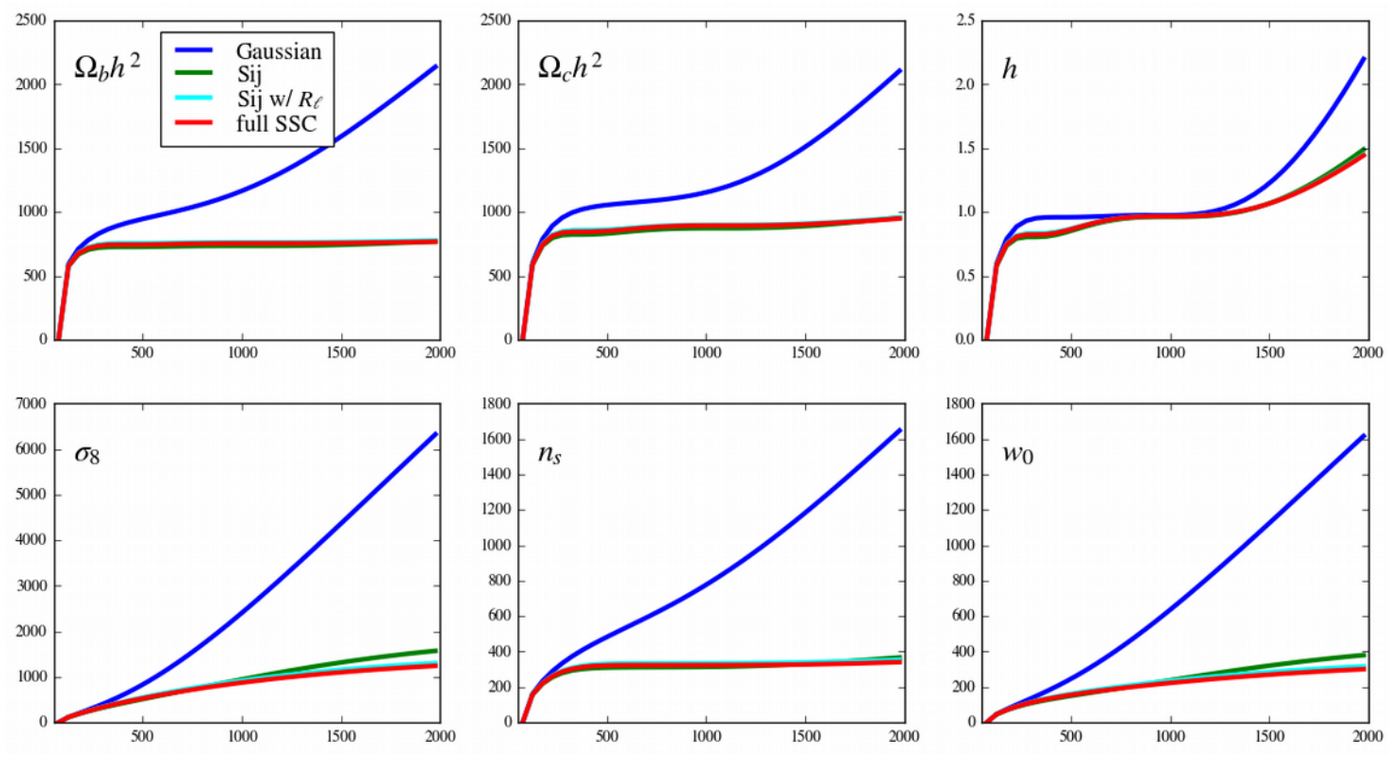


Application II : comparison with full SSC

Cumulative S/N vs lmax



Cumulative (square root of) Fisher element, for each cosmo parameter



Lacasa & Grain 2018 arXiv:1809.05437

Applicable to XCMB ?

- Directly applicable to iSW and CMB lensing
  equations are in the article

- IST forecasts assume a multipole cut to mimick the SSC effect
  lmax=750 for GCphot ; lmax=1500 for WL
  we can do better (more realistic) thanks to this approximation !

- I will make a **public python code** to compute the approx
  (though it's already easy to implement in a pipeline)

- I will get in contact with code developers
  (Stéphane, Isaac already interested) to **help** implement it

Lacasa & Grain 2018 arXiv:1809.05437

Conclusions / perspectives

- Non-Gaussian covariances are important, in part. SSC
- Have developed easy to use SSC approximation
- Relevant for Euclid
- Deal with SSC at the likelihood level
  sketched in Lacasa & Grain 2018 1809.05437
- Relevance for XCMB

Thanks for the attention

Additional slides

SSC : technical stuff

$$\mathrm{Cov}\left(C_\ell, C_{\ell'}\right) = \underbrace{\frac{2C_\ell^2}{2\ell+1}\delta_{\ell\ell'}}_{\text{Gaussian part}} + \underbrace{\frac{T_{\ell\ell}^{\ell'\ell'}}{4\pi}}_{\text{NG part}}$$

Late time non-linearity → NG → trispectrum $\quad T(\mathbf{k}_1, \mathbf{k}_2, \mathbf{k}_3, \mathbf{k}_4)$

SSC : part of the NG cov due to trispectrum terms $\propto P(|\mathbf{k}_1 + \mathbf{k}_2|)$

Exact SSC equation
(density modes only, no tidal fields)

$$\mathrm{Cov}_{\mathrm{SSC}}\left(\mathcal{O}_1, \mathcal{O}_2\right) = \int \mathrm{d}V_{12}\, \frac{\partial \mathfrak{o}_1}{\partial \delta_b}\, \frac{\partial \mathfrak{o}_2}{\partial \delta_b}\, \sigma^2(z_1, z_2)$$

probe's 3D **response**
to a change of background

(co)variance of the **background**
(in infinitesimal redshift slices)

Can be computed with arbitrary mask, rewritten more numerically efficiently :
Lacasa, Lima & Aguena 2018 1612.05958 ; Barreira et al. 2018

Accurate NL covariances : why ?

- Not to underestimate cosmological errors

ex : if we underestimate error by factor 3,
then a 1σ fluctuation become a 3σ discovery
→ “ruling out” Λ ...

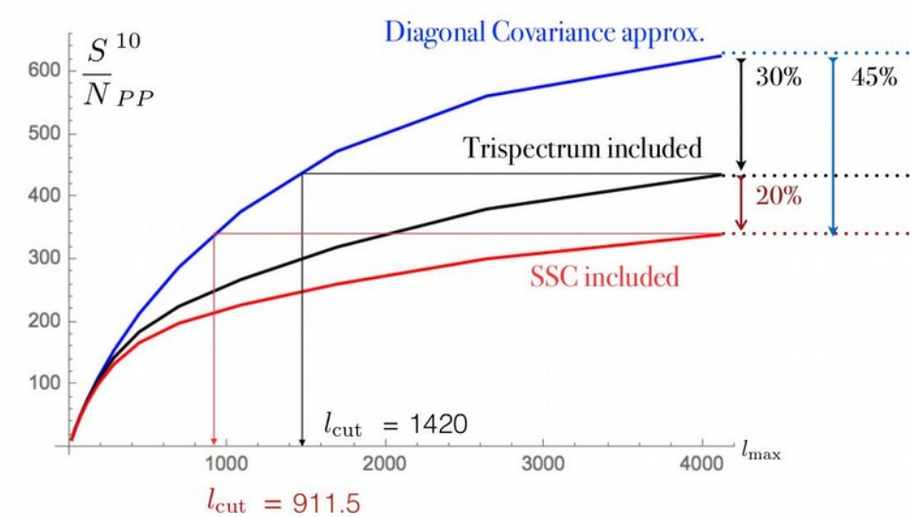
- Bias on cosmological parameters

ex : KiDS-450 analysis (Hildebrandt+ 2017) tried different approaches to the covariance. Impact :

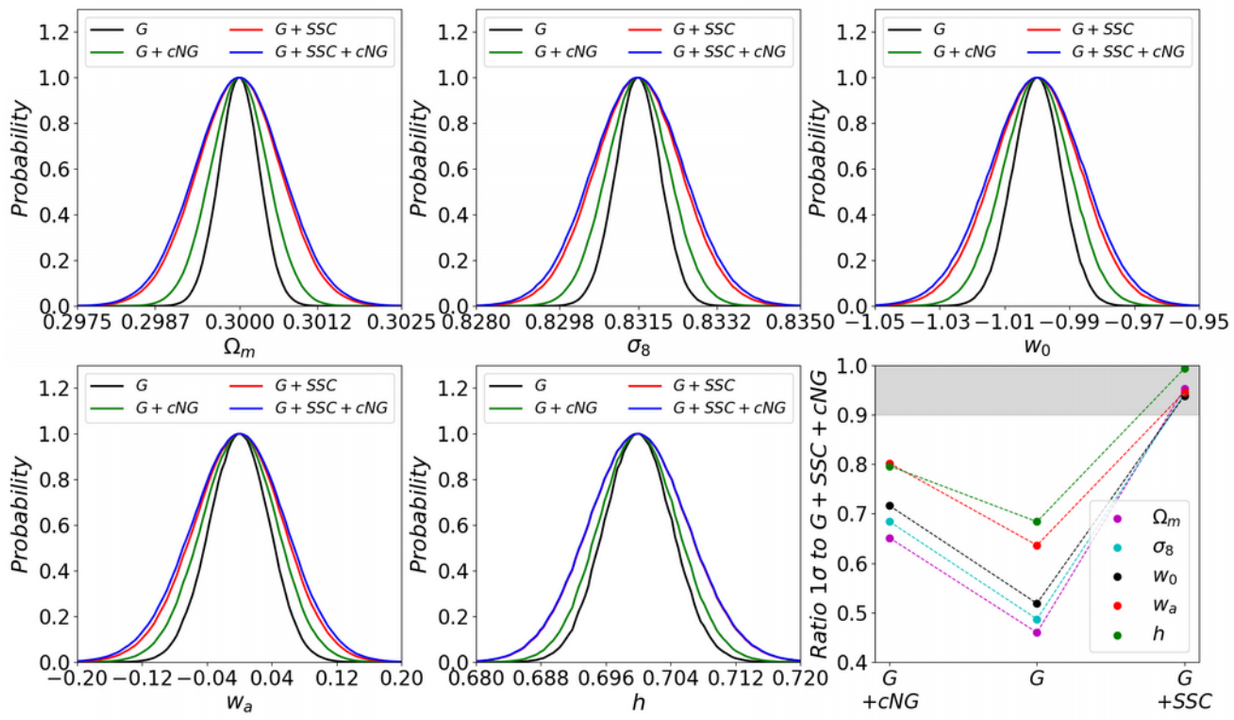
“There is however a shift in the central values of the best-fit parameters [...] **This shift is equivalent to the size of the 1σ error on S_8** […]
We attribute these shifts to super-sample-covariance terms […] ”

NL impact on weak lensing

- Impact on S/N (courtesy of M. Rizzato, IAP)
  - 10-bins tomographic WL power spectrum with Euclid-like specifications
  - NG impact wrt Gaussian cov : equivalent to cutting the data from lmax=5000 down to lmax=1400 (w/o SSC) or lmax=910 (w/ SSC)

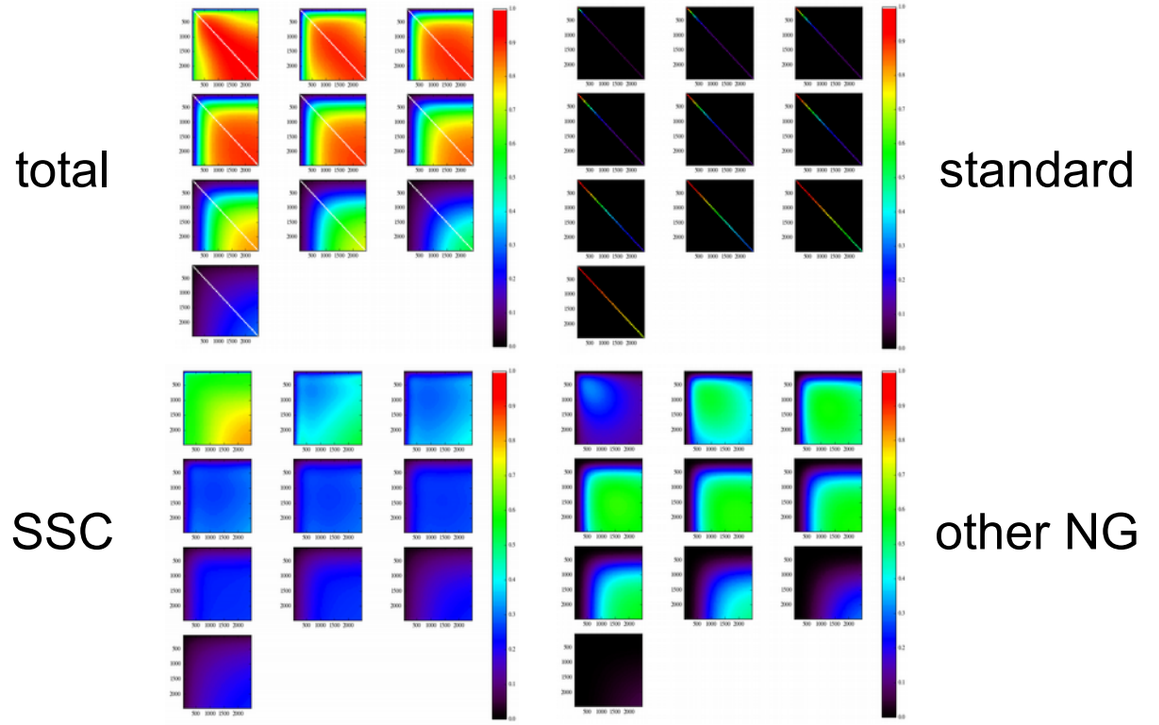


- Impact on param constraints : Barreira+ 2018
  - Error bars increased by +30% to +110%
  - DE heavily affected (as σ_8 & Ω_m)
  - SSC is dominant beyond Gauss, and with ~5% error on errors we can forget other trispectrum terms (really true ? Not sure for other cosmo params because impact on cov mat is ~15% median)

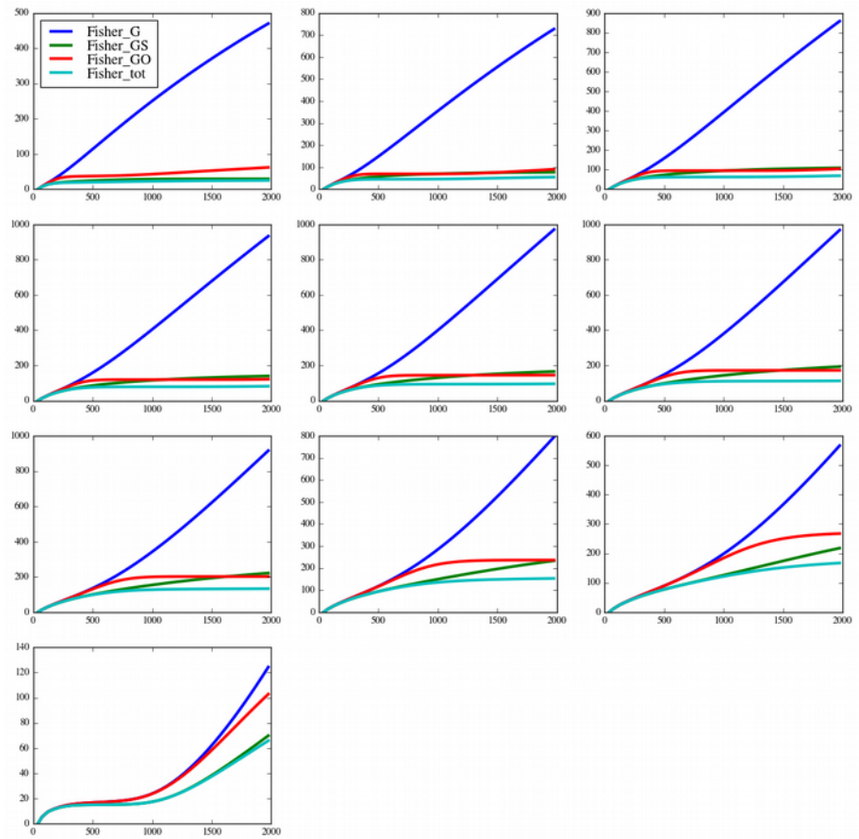


NL impact on galaxy clustering

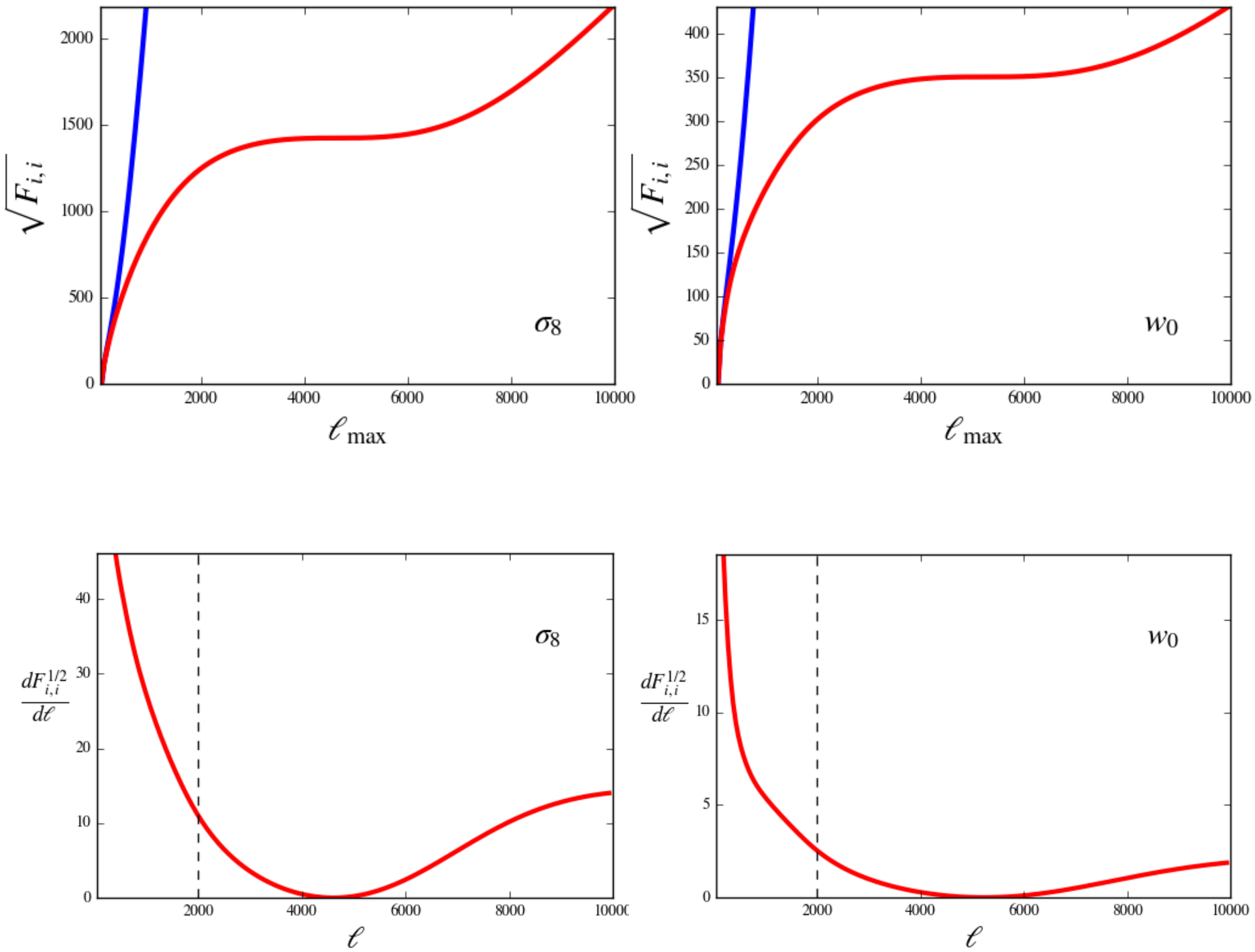
- Impact on cov matrix for Euclid-like GCphot



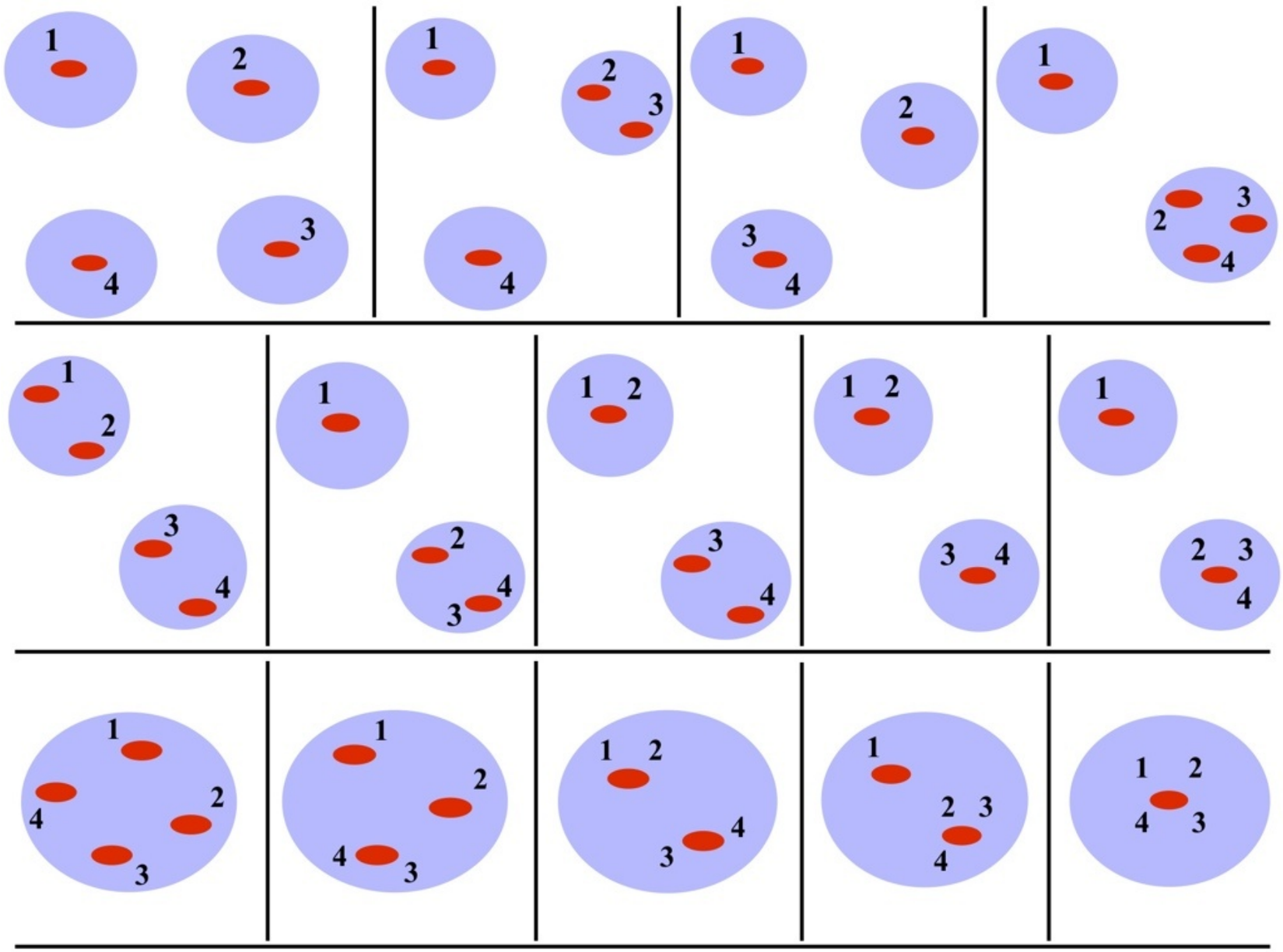
- Information content on DE cumulative F_{ww} vs lmax in the 10 redshift bins
  (no marginalisation on any other parameter, just to show the qualitative importance of the covariance terms)



Hope ? The small scale miracle



Covariance of the galaxy power spectrum : diagrammatic approach



Lacasa 2018 arXiv:1711.07372