Systematic Search for Tetrahedral and Octahedral Symmetries*) in Subatomic Physics: Follow-up of the First-Discovery Case#) of ^{152}Sm

Jerzy DUDEK

UdS/IN$_2$P$_3$/CNRS, France and UMCS, Poland

*) *Also called: Platonic Symmetries – or – High-Rank Symmetries*
or: Hidden Symmetry, in Greek: κρυπτοσυμμετρία

#) *JD and collaborators, PHYS. REV. C 97, 021302(R) (2018)*

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• We found the first experimental confirmation: ^{152}Sm nucleus

• Presence of an unprecedented class of nuclear isomeric states

FURTHER CONSEQUENCES FOR SUBATOMIC PHYSICS:

• New highway towards new exotic nuclei: Long-lived Isomers

• Astrophysics: New magic numbers for the nucleosynthesis

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Content of This Presentation

- **From Mean-Field and Group-Representation Theories to High-Rank Symmetries in Subatomic Physics**

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- **Symmetries: Recent and Near Future Developments**
  - **Experiment: Collaboration GSI-Strasbourg-Lublin**
  - **Theory: Excited Configurations & Excited Bands**

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Part 1

From the Nuclear Mean-Field Theory to High-Rank Symmetries in Subatomic Physics

Tetrahedral Symmetry – Shape Representation

Only special combinations of spherical harmonics may form a basis for surfaces with tetrahedral symmetry and only odd-order except 5

Three Lowest Order Solutions: Rank $\leftrightarrow$ Multipolarity λ

$$\boldsymbol{\lambda = 3:} \quad \alpha_{3,\pm 2} \equiv t_3$$

$$\boldsymbol{\lambda = 5:} \quad \text{no solution possible}$$

$$\boldsymbol{\lambda = 7:} \quad \alpha_{7,\pm 2} \equiv t_7; \quad \alpha_{7,\pm 6} \equiv -\sqrt{\tfrac{11}{13}} \cdot t_7$$

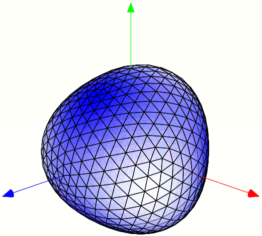
$$\boldsymbol{\lambda = 9:} \quad \alpha_{9,\pm 2} \equiv t_9; \quad \alpha_{9,\pm 6} \equiv +\sqrt{\tfrac{28}{198}} \cdot t_9$$

- Problem presented in detail in:

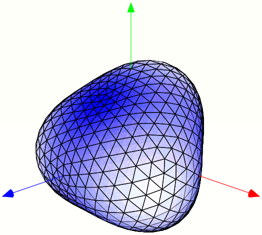
JD, J. Dobaczewski, N. Dubray, A. Góźdź, V. Pangon and N. Schunck, Int. J. Mod. Phys. E16, 516 (2007) [516-532].

Nuclear Tetrahedral Shapes – 3D Examples

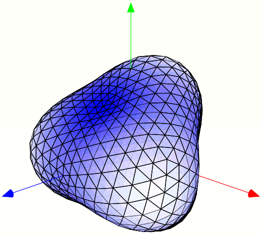
Illustrations below show the tetrahedral-symmetric surfaces at three increasing values of rank $\lambda = 3$ deformations α_{32}: 0.1, 0.2 and 0.3



$\alpha_{32} \equiv t_3 = 0.1$



$\alpha_{32} \equiv t_3 = 0.2$



$\alpha_{32} \equiv t_3 = 0.3$

Observations:

- There are infinitely many tetrahedral-symmetric surfaces
- Nuclear 'pyramids' do not resemble pyramids very much!

OBSERVATION:

Tetrahedral symmetry group, T_d,
is a sub-group of the octahedral one, O_h

A Basis for the Octahedral Symmetry

Only special combinations of spherical harmonics may form a basis for surfaces with octahedral symmetry and only in even-orders:

Three Lowest Order Solutions: Rank $\leftrightarrow$ Multipolarity λ

$$\lambda = 4: \quad \alpha_{40} \equiv o_4; \quad \alpha_{4,\pm 4} \equiv \pm\sqrt{\tfrac{5}{14}} \cdot o_4$$

$$\lambda = 6: \quad \alpha_{60} \equiv o_6; \quad \alpha_{6,\pm 4} \equiv -\sqrt{\tfrac{7}{2}} \cdot o_6$$

$$\lambda = 8: \quad \alpha_{80} \equiv o_8; \quad \alpha_{8,\pm 4} \equiv \sqrt{\tfrac{28}{198}} \cdot o_8; \quad \alpha_{8,\pm 8} \equiv \sqrt{\tfrac{65}{198}} \cdot o_8$$

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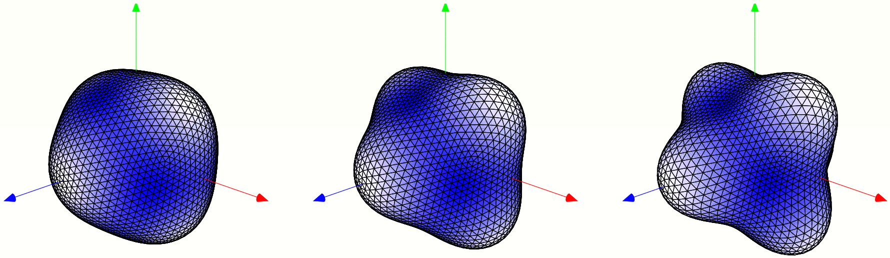


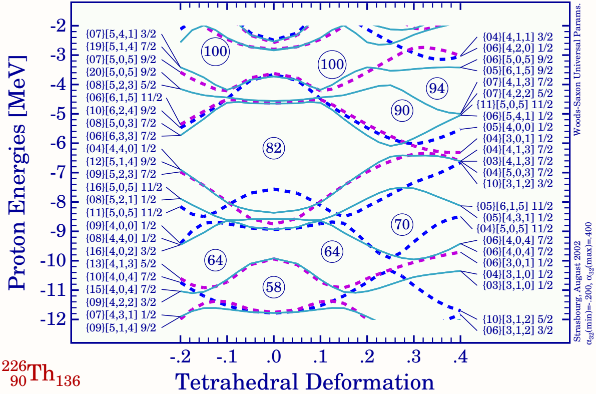
$o_4 = 0.1$ $o_4 = 0.2$ $o_4 = 0.3$

Observations:

- There are infinitely many octahedral-symmetric surfaces
- Nuclear 'diamonds' do not resemble diamonds very much!

Mean Field Theory: Tetrahedral Gaps

Double group T_d^D has two 2-dimensional - and one 4-dimensional irreducible representations: Three distinct families of nucleon levels



Full lines $\leftrightarrow$ 4-dimensional irreducible representations - marked with double Nilsson labels. Observe huge gaps at N=64, 70, 90-94, 100.

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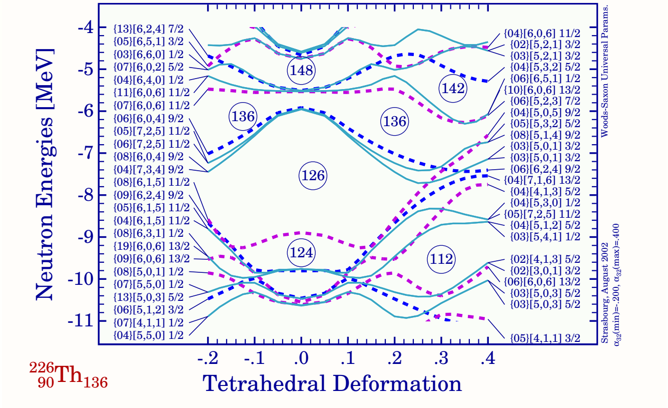
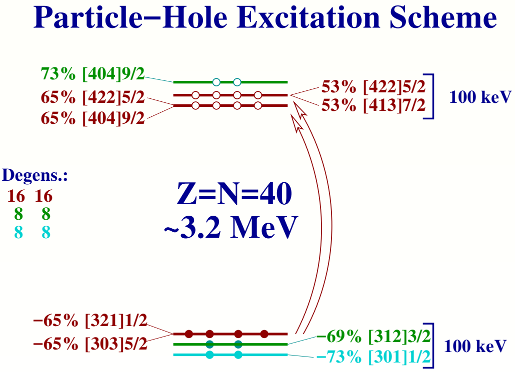


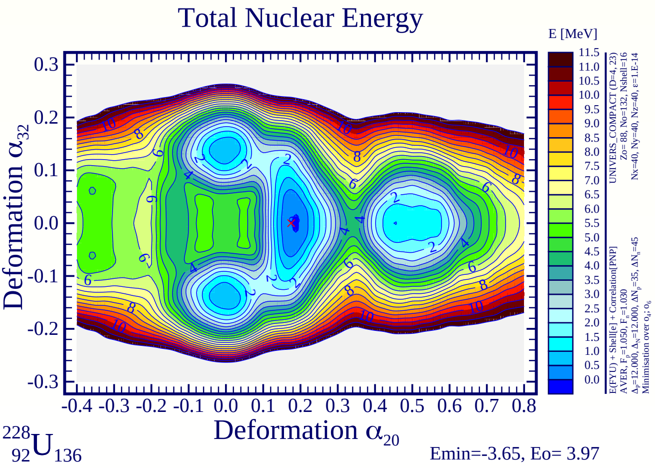
Full lines $\leftrightarrow$ 4-dimensional irreducible representations - marked with double Nilsson labels. Observe huge gaps at N=112, 136.

- Unprecedented degeneracies of nucleonic levels that are neither equal to $(2j+1)$ nor to 2 (time-up, time-down)

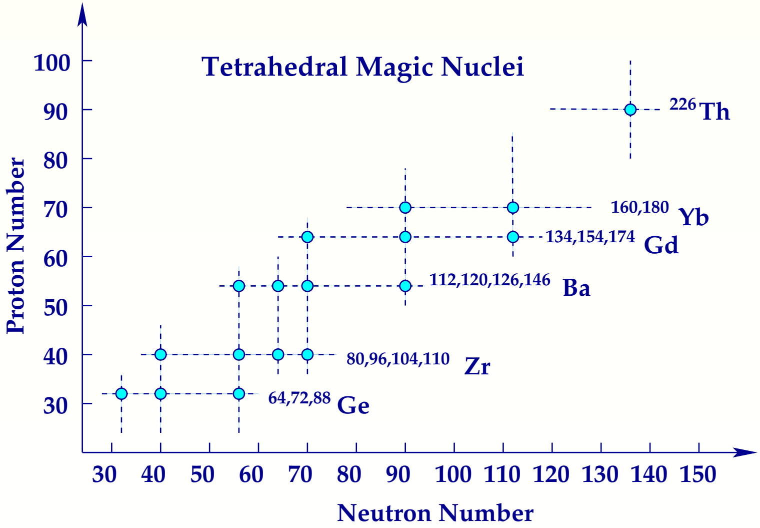


Mean Field Theory: Tetrahedral Minima

- Potential energy surfaces manifest well pronounced tetrahedral minima importantly enriching the shape coexistence phenomena



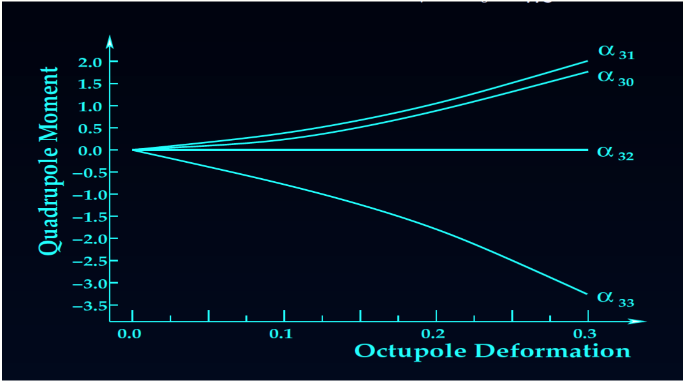
Numerous Tetrahedral Doubly-Magic Nuclei



It may be instructive to recall that in the exact symmetry limit tetrahedral nuclei emit neither E2 nor E1 transitions → *ISOMERS*

Indeed, for microscopically calculated quadrupole moments (W.S.)

$$Q_{20}(\alpha_{3\mu}) = \int \Psi^*_{WS}(\tau)\hat{Q}_{20}\Psi_{WS}(\tau)d\tau$$



Observe that $Q_{20}(\alpha_{32})$ vanishes identically in the W.S. mean-field

OBSERVATION:

Tetrahedral symmetry generates rotational bands without rotational electromagnetic transitions

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Tetrahedral symmetry generates rotational bands without rotational electromagnetic transitions thus possibly "bands of isomers"

$$\boxed{[E_I \propto I(I+1), \;\; B(E2) = 0, \;\; B(E1) = 0]}$$

SUMMARISING
THIS PART of DISCUSSION

Part I: Special Interests in High-Rank Symmetries

- Theory predicts whole families of nuclear states in many regions of the Periodic Table compatible with exotic, new symmetries
- These symmetries may lead to well pronounced potential energy minima and unprecedented, attractive new nuclear mechanisms
- For instance: unprecedented degeneracies of nucleonic levels that are neither equal to $(2j+1)$ nor to 2 (time-up, time-down)
- For instance: exotic (16-fold) degeneracies of 2p-2h excitations
- For instance: unprecedented degeneracies of rotational states
- For instance: unprecedented forms of the nuclear rotational behaviour - rotational bands without 'rotational (E2) transitions'
- One shows that the high-rank symmetries generate no collective E1 transitions either what combined with the vanishing of the E2 transitions brings us to the notion of the "Isomeric Bands"

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Part II: The Notion of Isomeric Bands

• **Once tetrahedral nuclei are populated one may expect the presence of numerous isomers since B(E2) and B(E1) at the exact tetrahedral and/or octahedral symmetry limits – vanish!**

• In particular, one expects series of long living (isomeric) states with unprecedented parabolic energy-spin relation

$$\boxed{\text{Isomers at: } E_I \propto I(I+1) \leftarrow \text{ Isomeric Bands}}$$

• Such states may live much longer than the ground-states what opens the new... highways towards the new areas of exotic nuclei in the (Z,N)-plane

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Microscopic Calculations of the Bands: Projected HFB

• After obtaining the constrained HFB state $|\Phi\rangle$, we perform the full quantum number projection from it to obtain the the projected wave function:

$$|\Psi^{INZ(\pm)}_{M;\alpha}\rangle = \sum_K g^{INZ(\pm)}_{K,\alpha}\, \hat{P}^I_{MK} \hat{P}_\pm \hat{P}^N \hat{P}^Z |\Phi\rangle,$$

• The amplitude $g^{INZ(\pm)}_{K,\alpha}$ and the energy eigenvalue $E^{INZ(\pm)}_\alpha$ are obtained by the so-called Hill-Wheeler relation

$$\sum_{K'} \mathcal{H}^{INZ(\pm)}_{K,K}\; g^{INZ(\pm)}_{K',\alpha} = E^{INZ(\pm)}_\alpha \sum_{K'} \mathcal{N}^{INZ(\pm)}_{K,K'}\; g^{INZ(\pm)}_{K',\alpha},$$

• The kernels are defined by

$$\left\{ \begin{array}{c} \mathcal{H}^{INZ(\pm)}_{K,K'} \\ \mathcal{N}^{INZ(\pm)}_{K,K'} \end{array} \right\} = \langle\Phi| \left\{ \begin{array}{c} \hat{H} \\ 1 \end{array} \right\} \hat{P}^I_{KK'} \hat{P}^N \hat{P}^Z \hat{P}_\pm |\Phi\rangle.$$

Part 2

Rotating High-Rank Symmetric Nuclei Seen Through Group-Representation Theory

[Symmetry Properties of Quantum Rotors]

Simple Theorems of Group-Representation Theory

- Let G be the symmetry group of the quantum rotor Hamiltonian
- Let $\{D_i,\ i = 1, 2, \ldots M\}$ be the irreducible representations of G
- The representation $D^{(I\pi)}$ of the rotor states with the definite spin-parity $I\pi$, can be decomposed in terms of D_i with multiplicities $a_i^{(I\pi)}$:

$$\boxed{\boldsymbol{D^{(I\pi)} = \sum_{i=1}^{M} a_i^{(I\pi)} D_i}}$$

- Multiplicities [M. Hamermesh, *Group Theory*, 1962] are given by:

$$\boxed{a_i^{(I\pi)} = \frac{1}{N_G} \sum_{R \in G} \chi_{I\pi}(R)\chi_i(R) = \frac{1}{N_G} \sum_{\alpha=1}^{M} g_\alpha \chi_{I\pi}(R_\alpha)\chi_i(R_\alpha);}$$

N_G: order of the group G; $\{\chi_{I\pi}(R), \chi_i(R)\}$: characters of $\{D^{(I\pi)}, D_i\}$
R: group element; g_α: the number of elements in the class α, whose representative element is R_α.

Elementary $\mathrm{T_d}$-Group Properties: Part I

- Tetrahedral group has 5 irreducible representations and 5 classes
- The representative elements $\{R\}$ are: $E,\ C_2\,(=S_4^2),\ C_3,\ \sigma_d,\ S_4$
- The characters of irreducible representation of T_d are listed below

T_d	E	$C_3(8)$	$C_2(3)$	$\sigma_d(2)$	$S_4(6)$
A_1	1	1	1	1	1
A_2	1	1	1	-1	-1
E	2	-1	2	0	0
$F_1(T_1)$	3	0	-1	-1	1
$F_2(T_2)$	3	0	-1	1	-1

- The characters $\chi_{I\pi}(R_\alpha)$ for the rotor representations are as follows:

$$\chi_{I\pi}(E)=2I{+}1,\ \chi_{I\pi}(C_n)=\sum_{K=-I}^{I} e^{\frac{2\pi K}{n}i},\ \chi_{I\pi}(\sigma_d)=\pi\times\chi_{I\pi}(C_2),\ \chi_{I\pi}(S_4)=\pi\times\chi_{I\pi}(C_4)$$

- From these relations we obtain ‘employing the pocket calculator’:

$$\boxed{a_i^{(I\pi)}=\frac{1}{N_G}\sum_{\alpha=1}^{M} g_\alpha\chi_{I\pi}(R_\alpha)\chi_i(R_\alpha)\ \leftrightarrow\ a_{A_1}^{(I\pm)}=a_{A_2}^{(I\mp)},\ a_E^{(I+)}=a_E^{(I-)},\ a_{F_1}^{(I\pm)}=a_{F_2}^{(I\mp)}}$$

Elementary $\mathrm{T_d}$-Group Properties: Part II

- The number of states $a_i^{(I\pi)}$ within five irreducible representations.

If $a_i^{(I\pi)} = 0 \;\rightarrow$ states not allowed; $a_i^{(I\pi)} = 2 \;\rightarrow$ doubly degenerate

I^+	0^+	1^+	2^+	3^+	4^+	5^+	6^+	7^+	8^+	9^+	10^+
A_1	1	0	0	0	1	0	1	0	1	1	1
A_2	0	0	0	1	0	0	1	1	0	1	1
E	0	0	1	0	1	1	1	1	2	1	2
$F_1(T_1)$	0	1	0	1	1	2	1	2	2	3	2
$F_2(T_2)$	0	0	1	1	1	1	2	2	2	2	3

I^-	0^-	1^-	2^-	3^-	4^-	5^-	6^-	7^-	8^-	9^-	10^-
A_1	0	0	0	1	0	0	1	1	0	1	1
A_2	1	0	0	0	1	0	1	0	1	1	1
E	0	0	1	0	1	1	1	1	2	1	2
$F_1(T_1)$	0	0	1	1	1	1	2	2	2	2	3
$F_2(T_2)$	0	1	0	1	1	2	1	2	2	3	2

- In this way we find the spin-parity sequence for A_1-representation

$$\boxed{\mathrm{A_1}: \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, 11^-, 2\times 12^+, 12^-, \cdots}$$

The bottom line for an experimentalist:

The tetrahedral ground-state band $I^\pi = 0^+$ is composed of the following states:

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Its structure has not much in common with the "usual" one(s), e.g.:

$$\boxed{I^\pi = 0^+, 2^+, 4^+, \; \ldots}$$

and implies a new way of thinking (and acting)

The Era of New Spectroscopy of Quantum Rotation in Subatomic Physics

[Symmetry Properties of Quantum Rotors]

The Era of New Spectroscopy of Quantum Rotation in Subatomic Physics

[Symmetry Properties of Quantum Rotors]

Or:
How to think about future experiments

Elementary $\mathrm{T_d}$-Group-Theory Band-Properties

- **Representations A_1 and A_2 $\to$ band-heads $I^\pi = 0^\pm$ and opposite parity sequences**

$$\boxed{\mathrm{A}_1: \quad 0^+, 3^-, 4^+, 6^+, 6^-, 7^-, 8^+, 9^+, 9^-, 10^+, 10^-, \cdots}$$

$$\boxed{\mathrm{A}_2: \quad 0^-, 3^+, 4^-, 6^-, 6^+, 7^+, 8^-, 9^-, 9^+, 10^-, 10^+, \cdots}$$

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$$\boxed{\mathrm{A}_2: \quad 0^-, 3^+, 4^-, 6^-, 6^+, 7^+, 8^-, 9^-, 9^+, 10^-, 10^+, \cdots}$$

- **Representation E → degenerate band-heads $I^\pi = 2^\pm$ and various parity doublets**

$$\boxed{\mathrm{E}: \quad 2^\pm, 4^\pm, 5^\pm, 6^\pm, 7^\pm, 2\times 8^\pm, 9^\pm, 2\times 10^\pm, \cdots}$$

Elementary T_d-Group-Theory Band-Properties

- **Representations A_1 and A_2 $\longrightarrow$ band-heads $I^\pi = 0^\pm$ and opposite parity sequences**

$$\boxed{A_1: \quad 0^+,\, 3^-,\, 4^+,\, 6^+,\, 6^-,\, 7^-,\, 8^+,\, 9^+,\, 9^-,\, 10^+,\, 10^-, \cdots}$$

$$\boxed{A_2: \quad 0^-,\, 3^+,\, 4^-,\, 6^-,\, 6^+,\, 7^+,\, 8^-,\, 9^-,\, 9^+,\, 10^-,\, 10^+, \cdots}$$

- **Representation E $\longrightarrow$ degenerate band-heads $I^\pi = 2^\pm$ and various parity doublets**

$$\boxed{E: \quad 2^\pm,\, 4^\pm,\, 5^\pm,\, 6^\pm,\, 7^\pm,\, 2\times 8^\pm,\, 9^\pm,\, 2\times 10^\pm,\, \cdots}$$

- **Representations F_1 and F_2 $\longrightarrow$ band-heads $I^\pi = 1^\pm$ and parity doublets, triplets...**

$$\boxed{F_1: \quad 1^+,\, 2^-,\, 3^\pm,\, 4^\pm,\, (2\times 5^+, 5^-), (6^+, 2\times 6^-),\, 2\times 7^\pm,\, 2\times 8^\pm,\, (3\times 9^+, 2\times 9^-), \cdots}$$

$$\boxed{F_2: \quad 1^-,\, 2^+,\, 3^\pm,\, 4^\pm,\, (5^+, 2\times 5^-), (2\times 6^+, 6^-),\, 2\times 7^\pm,\, 2\times 8^\pm,\, (2\times 9^+, 3\times 9^-), \cdots}$$

Part 3

About the Experimental Evidence*) for the First Tetrahedral Rotor Case: ^{152}Sm

*) **J. Dudek, D. Curien, I. Dedes, K. Mazurek, S. Tagami, Y. R. Shimizu and T. Bhattacharjee; PHYSICAL REVIEW C 97, 021302(R) (2018)**

About the Experimental Evidence*) for the First Tetrahedral Rotor Case: ^{152}Sm

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About the Experimental Evidence*) for the First Tetrahedral Rotor Case: ^{152}Sm

or

A reminder: What and How do We Look For?

*) J. Dudek, D. Curien, I. Dedes, K. Mazurek, S. Tagami, Y. R. Shimizu and T. Bhattacharjee; PHYSICAL REVIEW C 97, 021302(R) (2018)

Quantum Rotors: Tetrahedral vs. Octahedral

- The tetrahedral symmetry group has 5 irreducible representations
- The ground-state $I^\pi = 0^+$ belongs to A_1 representation given by:

$$\underbrace{\mathrm{A}_1: \quad 0^+,\, 3^-,\, 4^+,\, \underbrace{(6^+, 6^-)}_{\text{doublet}},\, 7^-,\, 8^+,\, \underbrace{(9^+, 9^-)}_{\text{doublet}},\, \underbrace{(10^+, 10^-)}_{\text{doublet}},\, 11^-,\, \underbrace{2 \times 12^+,\, 12^-}_{\text{triplet}},\, \cdots}_{\textbf{Forming a common parabola}}$$

- There are no states with spins $I = 1, 2$ and 5. We have parity doublets: $I = 6, 9, 10 \ldots$, at energies: $E_{6^-} = E_{6^+}$, $E_{9^-} = E_{9^+}$, etc.

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- One shows that the analogue structure in the octahedral symmetry

$$\underbrace{A_{1g}: \; 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \; \ldots, \;\; I^\pi = I^+}_{\textbf{Forming a common parabola}}$$

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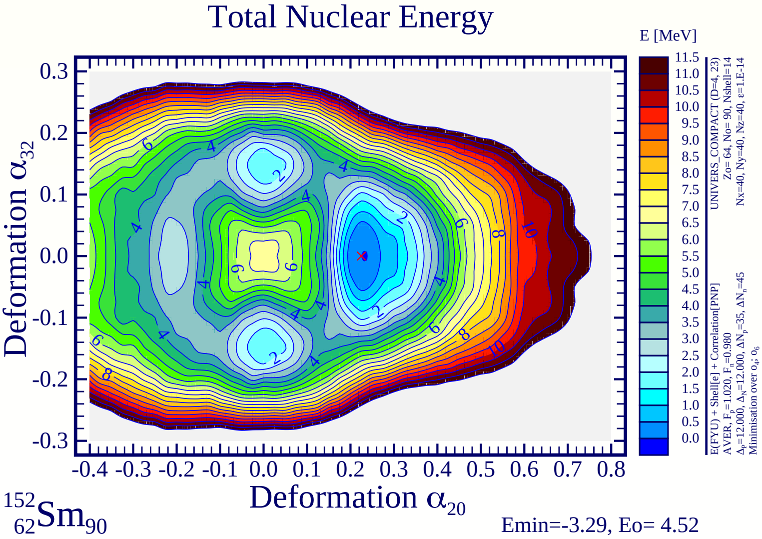
Consequently we should expect two independent parabolic structures

Suggestion:

Look for the experimental evidence
of the tetrahedral and octahedral symmetries
focussing on the $N = 90$ isotones
for which the best 'appropriate' experimental data exist

The Following Discussion Is Focussed on ^{152}Sm



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• Energy-wise – they are expected to form regular energy sequences

$$\boxed{E_I \propto \alpha_2 I^2 + \alpha_1 I + \alpha}$$

No Electromagnetic Transitions Expected ...

How do we start looking for rotational bands without rotational transitions?

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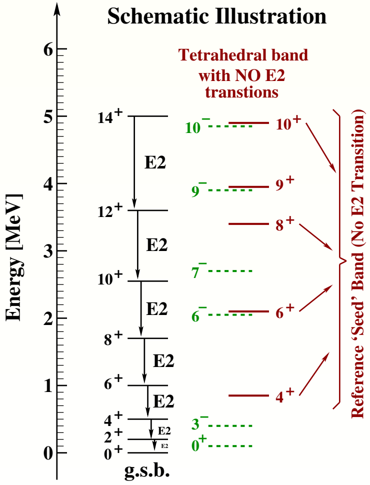
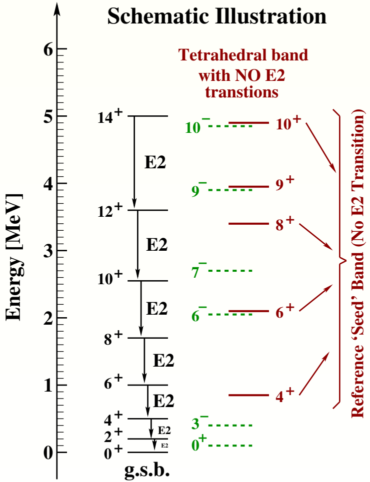
What To Start With?

Starting Point in Collecting the Experimental Evidence



We begin by looking for experimental candidates for the 'reference seed band'

We proceed like this:



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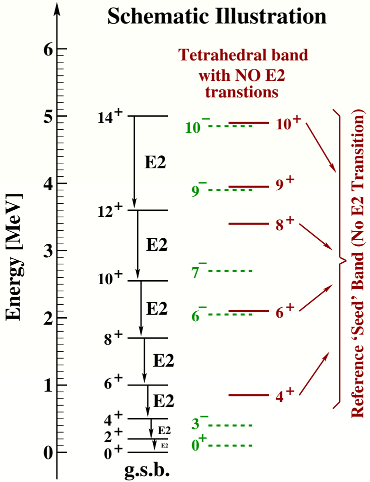
which is parabolic, no E2 transitions



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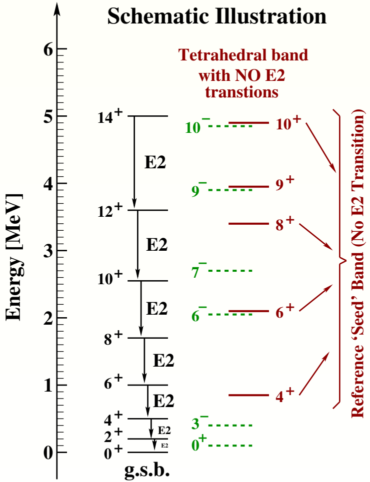

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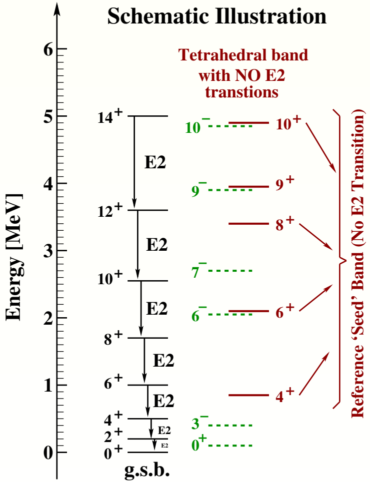
• Once this parabola is known – we select other experimental candidate states - close to reference seed-band



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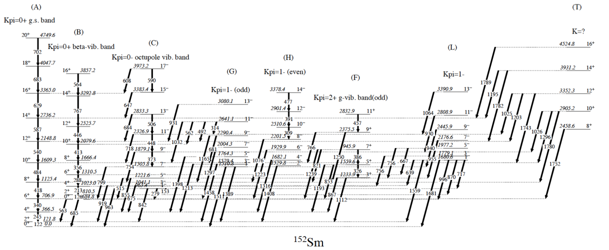

Experimental spectrum of ^{152}Sm from NNDC data base

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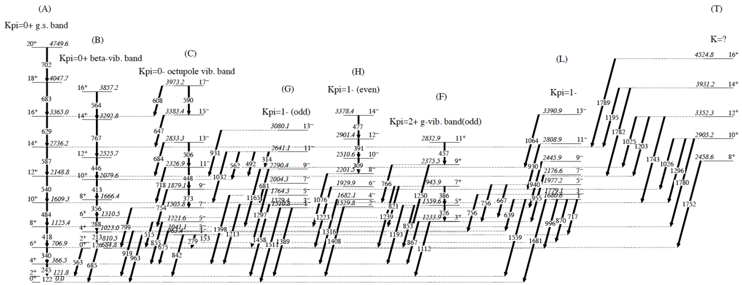

^{152}Sm

Experimental spectrum of ^{152}Sm from NNDC data base:

Notice the fantasist nomenclature of the bands

... invented long ago by an NNDC data base evaluator(s)

“OUR BAND” is called ... Band (T) like ...

I could NOT stop laughing seeing it for the first time

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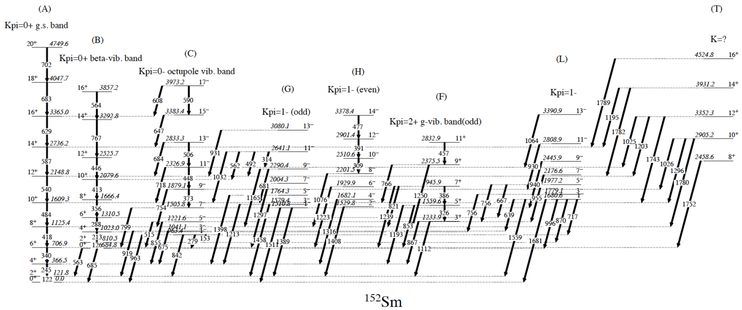

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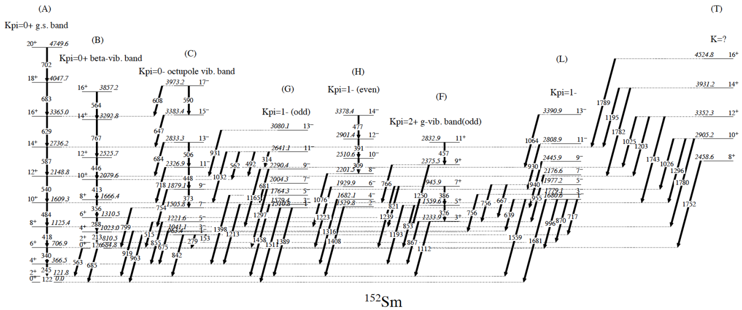

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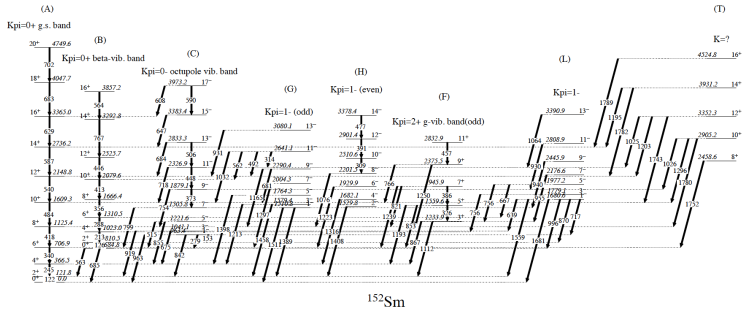

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... or (T)etrahedral ... or ...

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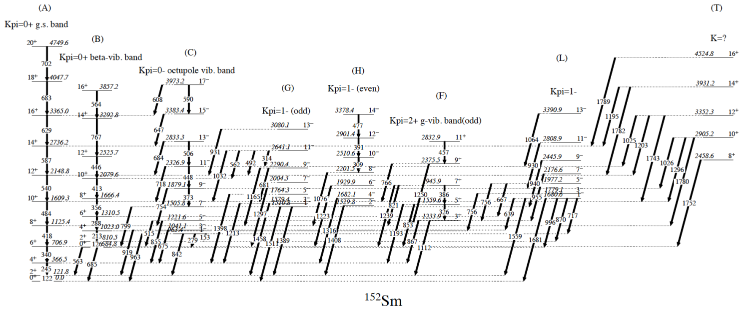


Experimental spectrum of ^{152}Sm

By the way, band (T) was NOT retained in the final analysis

We Proceed Looking for the Other Candidate States

Criterion no. 1:
Accepted states must neither be populated nor depopulated by any strong E1 or E2 transitions, preferably populated by nuclear reaction

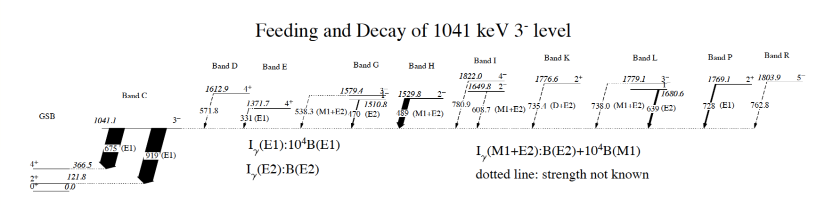
Criterion No. 2:
Their energies should be 'reasonably' close to the reference parabola

Observation:
Since they do not decay via a single strong transition it is instructive verifying that they decay into several states – with weak intensities

Next Steps in the Procedure: Part II

A typical diagram among a hundred in this analysis

Feedig the tetrahedral $I^\pi = 3^-$ candidate (among five others)



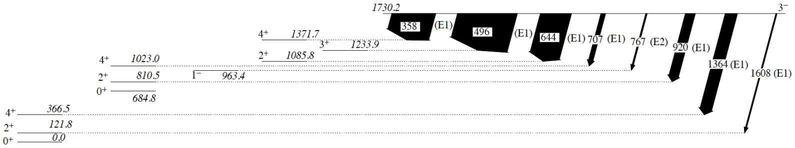
Let us note that 3^- does not decay to the 0^+ ground-states (suggesting that it is not an octuple vibrational state built on the other) and that there are numerous states populating it suggesting that its structure is exotic from our point of view.

[By the way, this state was not retained at the final steps]

Next Steps in the Procedure: Part II

A typical diagram among a hundred in this analysis

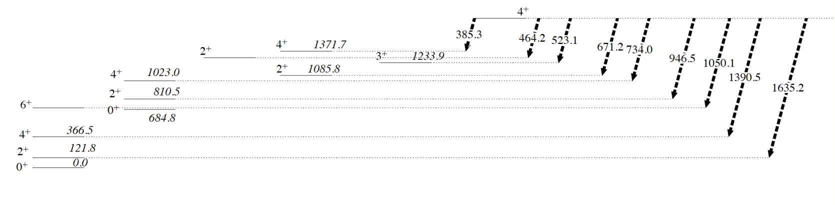
Decay from the tetrahedral $I^\pi = 3^-$ candidate (among five others)



Let us observe that this state decays to many others suggesting its 'exotic' structure of interest in our context

A typical diagram among a hundred in this analysis

Decay from the tetrahedral $I^\pi = 4^+$ candidate level



Let us observe that this state decays to many others via very weak transitions suggesting no resemblance to quadrupole-deformed rotational states...

and many, many other states analysed within this project...

Tetrahedral Band: Our First Candidate

Proposed experimental energy levels candidates as members of **the** tetrahedral band in ^{152}Sm after analysing numerous hypotheses. Columns 3 and 4 give the numbers of decay-out transitions and feeding transitions, respectively.

Spin	**E[keV]**	No. D-out	No. Feed	Reaction
$\mathbf{3^-}$	**1579.4**	10	none	CE & α
$\mathbf{4^+}$	**1757.0**	9	1+(1)	CE & α
$\mathbf{6^-}$	**1929.9**	2	(1)	CE & α
$\mathbf{6^+}$	**2040.1**	7	none	CE & α
$\mathbf{7^-}$	**2057.5**	6	2+(1)	CE & α
$\mathbf{8^+}$	**2391.7**	3	1	CE & α
$\mathbf{9^-}$	**2388.8**	4	3	CE & α
$\mathbf{9^+}$	**2588**	2	1	α
$\mathbf{10^-}$	**2590.7**	4	1	α
$\mathbf{(10^+)}$	**2810**	2	none	α
$\mathbf{11^-}$	**2808.9**	2	none	CE

Tetrahedral Symmetry Hypothesis: One Parabolic Branch

$$\underbrace{\mathrm{A}_1: \quad 0^+, 3^-, 4^+, \underbrace{(6^+, 6^-)}_{\text{doublet}}, 7^-, 8^+, \underbrace{(9^+, 9^-)}_{\text{doublet}}, \underbrace{(10^+, 10^-)}_{\text{doublet}}, 11^-, \underbrace{2\times 12^+, 12^-}_{\text{triplet}}, \cdots}_{\textbf{Forming a common parabola}}$$

• We performed the test of the tetrahedral A_1-type hypothesis by fitting the parameters of the parabola to the energies in the Table. The obtained root-mean-square deviation:

$$\boxed{T_d:\; A_1 \;\to\; r.m.s. \approx 80.5\,\mathrm{keV} \quad \leftrightarrow \quad 11 \text{ levels } I^\pi = I^\pm}$$

For comparison:

$$\boxed{\mathrm{G.s.b.} \;\to\; r.m.s. \approx 52.4\,\mathrm{keV} \quad \leftrightarrow \quad 7 \text{ levels } I^\pi = I^+}$$

Parabolic Relations: R.M.S.-Deviation Analysis (II)

Octahedral Symmetry Hypothesis: Two Parabolic Branches

$$\underbrace{A_{1g}:\; 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \;\ldots,\; I^\pi = I^+}_{\text{Forming a common parabola}}$$

$$\underbrace{A_{2u}:\; 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots,\; I^\pi = I^-}_{\text{Forming another (common) parabola}}$$

• We performed the test of the octahedral A_{1g}-A_{2u} hypothesis by fitting the parameters of the parabolas to the energies in the Table. The obtained root-mean-square deviations:

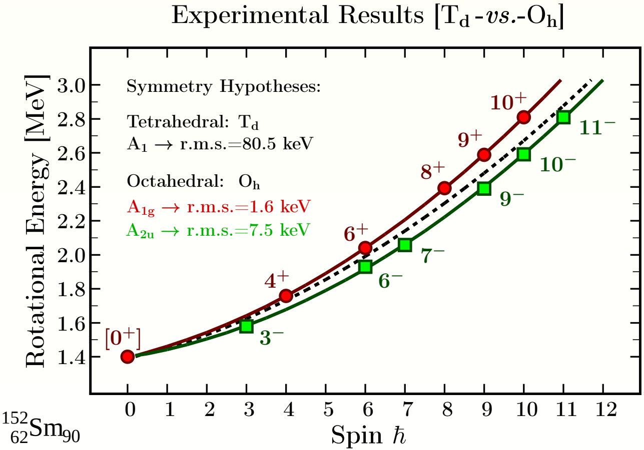
$$\boxed{O_h:\; A_{1g} \;\to\; r.m.s. \approx 1.6\,\mathrm{keV} \;\leftrightarrow\; 5 \text{ levels } I^\pi = I^+}$$

$$\boxed{O_h:\; A_{2u} \;\to\; r.m.s. \approx 7.5\,\mathrm{keV} \;\leftrightarrow\; 6 \text{ levels } I^\pi = I^-}$$

For comparison:

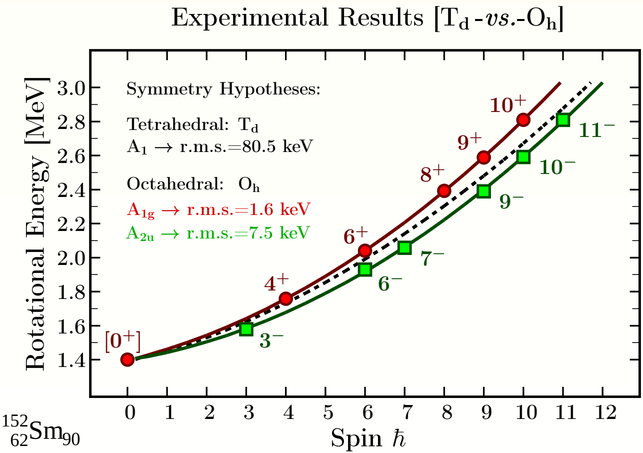
$$\boxed{T_d:\; A_1 \;\to\; r.m.s. \approx 80.5\,\mathrm{keV} \;\leftrightarrow\; 11 \text{ levels } I^\pi = I^\pm}$$

Dominating Octahedral-Symmetry Hypothesis



Graphical representation of the experimental data from the summary Table. Curves represent the fit and are *not* meant 'to guide the eye'. Markedly, point $[I^{\pi} = 0^{+}]$, is a prediction by extrapolation - not an experimental datum.

A Comment About Extrapolation to $\mathbf{I}^\pi \to \mathbf{0}^+$



Notice: The negative parity sequence lies entirely below the positive parity one. Extrapolating the parabolas to zero-spin we find $E^-_{I=0} = 1.396\,8$ MeV compared to $E^+_{I=0} = 1.396\,1$ MeV, the difference of 0.7 keV at the level 1.4 MeV excitation!

SUMMARISING
THIS PART of DISCUSSION

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• The two branches characteristic for octahedral symmetry are very close to the single parabola predicted for the tetrahedral symmetry

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• The negative parity branch lies entirely below the positive parity branch: Can positions of rotational band members be 'accidental'?

• What is the probability that "due to enormous complexity of the nuclear interactions" the discussed energies are positioned in reality at random and the discussed structures incidentally form parabolas?

$$\boxed{\boldsymbol{T_{\mathrm{d}} - incidental:} \quad \boldsymbol{P_{11\ \mathrm{levels}}^{\sqrt{\sigma^2}=80\ keV} \approx 1.1 \cdot 10^{-14}}}$$

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- Emphasise: None of the geometrical nuclear symmetries can be considered exact because of the zero-point motion (Bohr model) and various polarisation mechanism, e.g. by nucleons outside shells

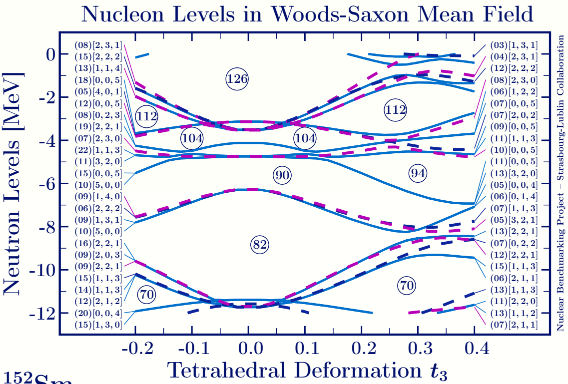
- Consequently relatively weak electromagnetic transitions are to be expected and this mechanism can/should be used to obtain a more complete information about electromagnetic decay, transition details

Part 4

Highly Deformed Tetrahedral and/or Octahedral Configurations

Based On the Deformation-Driving Orbitals

Deformation-Driving Particle-Hole Configurations

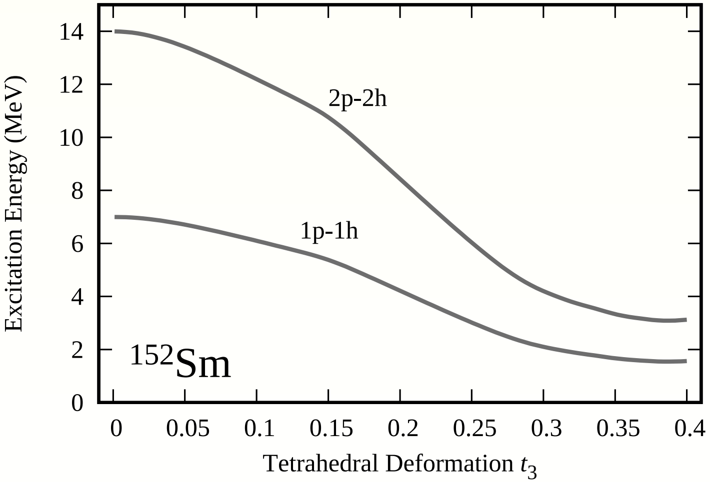


$^{152}_{62}\mathrm{Sm}_{90}$

- Exciting down-sloping orbitals from up-sloping orbitals we construct:

– Strongly deformation-driving configurations
and
– Gain several MeV in terms of excitation energy

How Powerful This Mechanism Is: Illustration 2



- Constructing particle-hole excited configurations with down-, and up-sloping orbitals we gain several MeV: Deformations increase significantly!

Improving Degeneracies with Increasing Deformation

SHINGO TAGAMI, YOSHIFUMI R. SHIMIZU, AND JERZY DUDEK   PHYSICAL REVIEW C **87**, 054306 (2013)

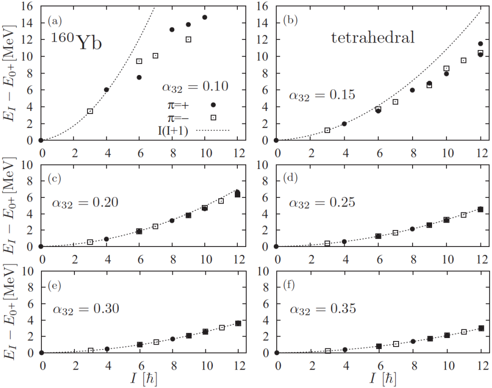
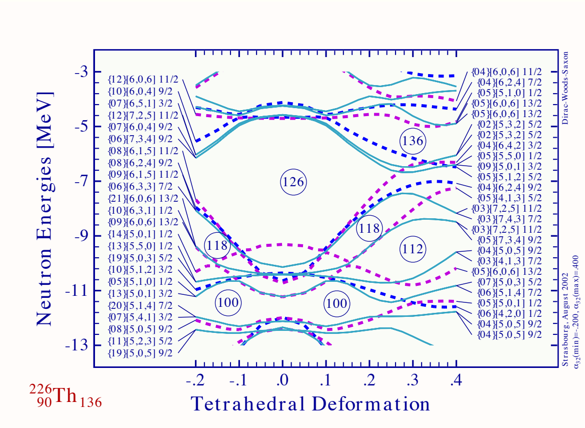


FIG. 4. Calculated spectra of tetrahedral states in ^{160}Yb with $\alpha_{32} = 0.10, 0.15, 0.20, 0.25, 0.30$, and 0.35, respectively, for (a), (b), (c), (d), (e), and (f). The dotted line in each panel denotes an ideal $I(I+1)$ sequence going through the first excited 3^- state. Note that almost exact degeneracies for $I = (6^+, 6^-), (9^+, 9^-), (10^+, 10^-), (2 \times 12^+, 12^-)$ states are obtained for $\alpha_{32} \geqslant 0.25$ demonstrating the nearly perfect rotor character of the rotational excitation of the system.

Observe the sequences containing both parities; note also degeneracies

$$I^\pi = 6^\pm,\ I^\pi = 9^\pm,\ I^\pi = 10^\pm,\ \ldots$$

Technical Construction Problem: Level Crossings



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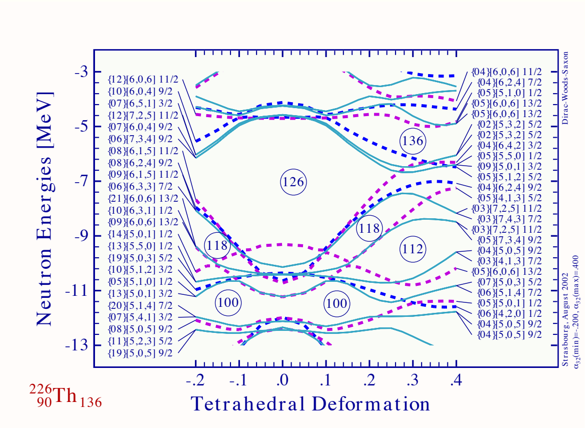

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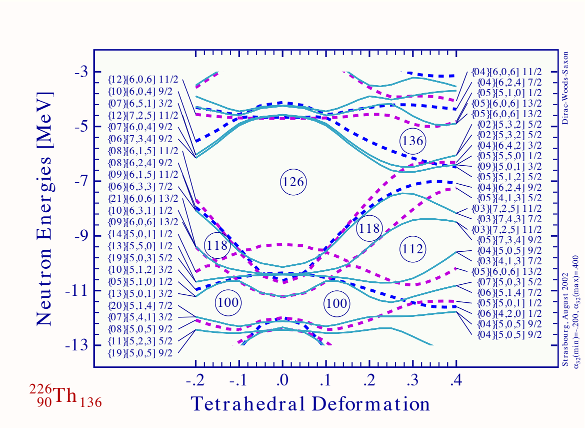
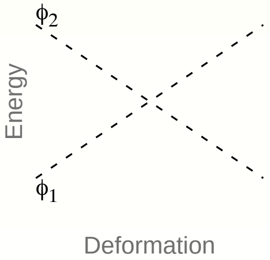


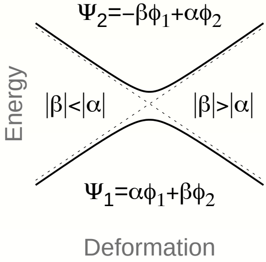
- We will need to construct the maps of the potential energy surfaces for **n-particle n-hole excited states**
- However: In general, there exist numerous level crossings as above
- We will need an automatic algorithm of the level crossing removal!

Heuristic Considerations Based on Two-Level Model

• Consider Hamiltonian H with two eigenstates. The matrix representation of solutions within a basis say, ϕ_1 and ϕ_2, can be given as:

$$\Psi_1 = +\alpha\phi_1 + \beta\phi_2$$
$$\Psi_2 = -\beta\phi_1 + \alpha\phi_2$$





• Schematic illustration of the two discussed variants of the level crossing:

– Left: No mixing and no repulsion of levels, levels cross

– Right, full lines: Mixing, the levels repel each other

Level Crossing 'Removal': Position of the Problem

- How to construct an automatic algorithm, which connects the levels of the same structure before the crossing and after the crossing, rather than connecting the levels according to the energy order:

  "The first level connected always with the first level",
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> *"The second level connected always with the second".*

• Consider two successive deformation points, one to the left of the crossing point, "L" and one to the right of the crossing point, "R"

• We say that a single particle state ${}_R\langle k|$ at R is of a similar structure compared to $|m\rangle_L$ at L, if and only if the absolute value of the scalar product of corresponding eigenvectors at L and R is close to 1, *i.e.*:

$$\boxed{|{}_L\langle \boldsymbol{k}|\boldsymbol{m}\rangle_R| \approx \mathbf{1}}$$

and dissimilar otherwise

SUMMARISING
THIS PART of DISCUSSION

The Mechanism of Increasing Symmetry Effects

- By activating down-sloping and deactivating up-sloping orbitals we gain huge amounts of excitation energy in terms of several MeV

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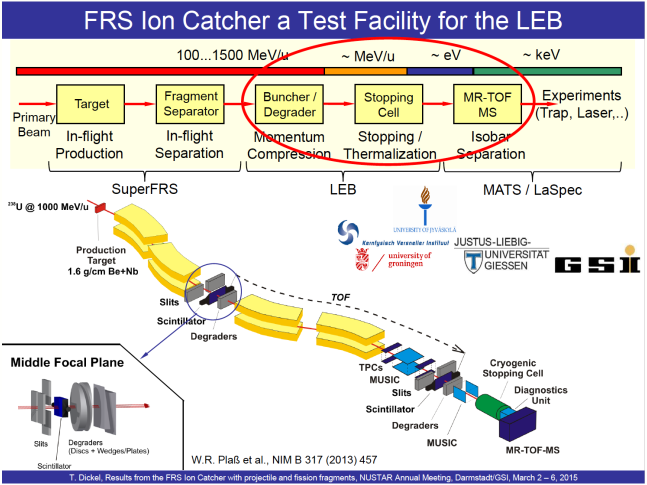
- All these arguments bring us to the question of the optimisation of the population of the highly excited states via nuclear processes!
- The vanishing E1 and E2 transitions at the exact symmetry limit emphasises the detection via modern mass spectrometry methods

Part 5

Recent Experimental Evolution: Collaboration GSI Strasbourg

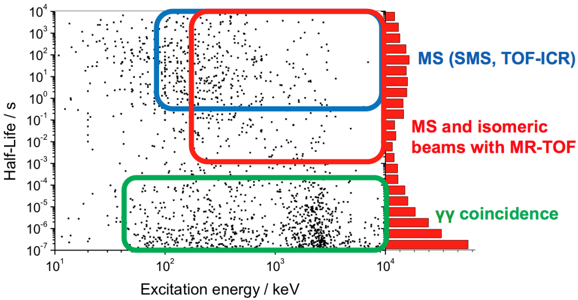
Specifically Designed Facilities at the GSI

- **FRS [FRagment Separator];** • **LEB [Low-Energy Branch]**
- **MR-TOF-MS [Multiple-Reflection Time-of-Flight Mass Spectrometer]**



Specifically Designed Facilities at the GSI

• Scatter plot of all known isomers [from the NUBASE 2016 database]. In the ms region there are fewer isomers known. This coincides with the region relatively difficult to access for decay spectroscopy and 'traditional' mass spectrometry techniques.



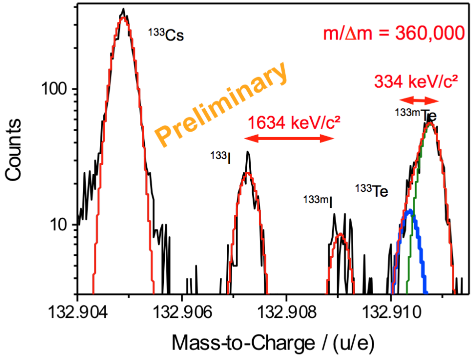
• GSI system MR-TOF-MS can measure about 2 orders of magnitude faster then the existing high resolution mass spectrometry techniques: Accessing region in red!

• TOF-ICR [Time of Flight Ion Cyclotron Resonance]; • MS [Mass Spectrometry]; • SMS [Schottky Mass Spectrometry]; • MR-TOF Multiple Reflection Time of Flight • Other MR-TOF facilities: ISOLDE, RIKEN, TRIUMF

Examples of Isomer Measurements

Uranium Fission Fragments

- Mass measurement of uranium fission products produced at 1000 MeV/u
- MR-TOF-MS will enable efficient search of new isotopes and isomers



S. Ayet et al.

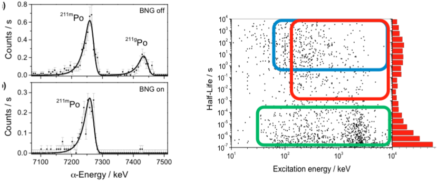
T. Dickel, Isomer Studies with the FRS Ion Catcher, Super-FRS Experiement Collaboration Meeting, Walldorf, Germany, May 2 – 4, 2018

- **MR-TOF-MS [Multiple-Reflection Time-of-Flight Mass Spectrometer]**
  - **Approximate FWHM of the peaks is of the order of 300 keV.**

Perspectives of New Measurements

The FRS Ion Catcher

- Powerful tool for the measurement of isomers:
  Identification, excitation energies, isomeric ratios
- Isomer-clean beams



Experiment proposal for next G-PAC focusing on experiments for long-lived isomers:

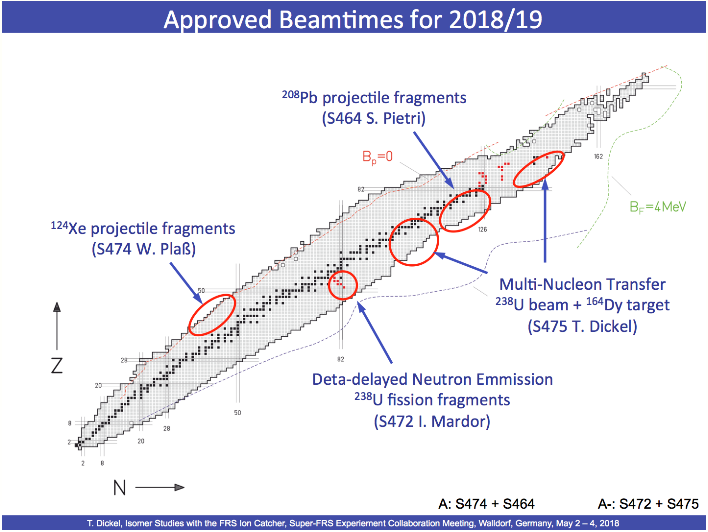
- Spectroscopy of isomer-clean beams
- Search for tetrahedral isomeric states
- New isomer search
- Isomeric ratios for reaction studies
- Half-life and decay modes
- ...

J. Dudek, A. Jain,
S. Pietri, P. Walker,
Z. Patyk, ⋯
YOU?

one day workshop on ideas for “isomer” proposal planed

T. Dickel, Isomer Studies with the FRS Ion Catcher, Super-FRS Experiement Collaboration Meeting, Walldorf, Germany, May 2 – 4, 2018

Approved Beam - Times



- **Our interests are numerous but in order of priority: ^{152}Sm, ^{160}Yb and ^{80}Zr**

Part 6

Summary and Prospects

$\mathrm{T_d}$ and $\mathrm{O_h}$ Symmetries in ^{152}Sm – Summary

• **We believe having demonstrated signs of simultaneous presence of tetrahedral and octahedral symmetries in existing experimental data of other authors – following the strict group representation criteria**

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- **Octahedral symmetry hypothesis leads to two parabolic branches**

$$\underbrace{A_{1g}:\; 0^+, 4^+, 6^+, 8^+, 9^+, 10^+, \;\ldots,\; I^\pi = I^+ \;\leftrightarrow\; \textbf{r.m.s.} \approx \textbf{1.6 keV},\; \textbf{5 states}}_{\text{Forming a common parabola}}$$

$$\underbrace{A_{2u}:\; 3^-, 6^-, 7^-, 9^-, 10^-, 11^-, \ldots,\; I^\pi = I^- \;\leftrightarrow\; \textbf{r.m.s.} \approx \textbf{7.5 keV},\; \textbf{6 states}}_{\text{Forming another (common) parabola}}$$

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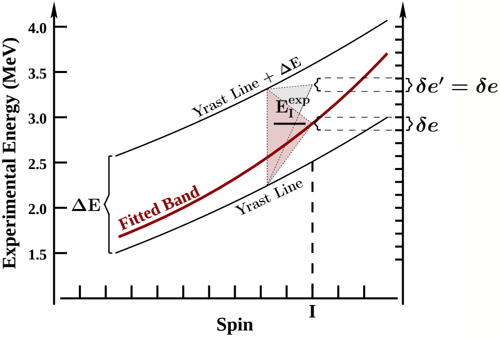
$$\underbrace{\mathrm{A}_1:\quad 0^+,\ 3^-,\ 4^+,\ \underbrace{(6^+, 6^-)}_{\text{doublet}},\ 7^-,\ 8^+,\ \underbrace{(9^+, 9^-)}_{\text{doublet}},\ \underbrace{(10^+, 10^-)}_{\text{doublet}},\ 11^-,\ \underbrace{2\times 12^+,\ 12^-}_{\text{triplet}},\ \cdots}_{\text{Forming a common parabola with r.m.s.=80 keV over 11 states}}$$

T_d and O_h Symmetries in ^{152}Sm – Summary

- **Estimated probability of obtaining discussed correlations at the discovered precision level and within the discussed excitation energy range out of random numbers is:**

$$\boxed{T_d - \text{incidental}: \; P^{\sqrt{\sigma^2}=80\,\text{keV}}_{11\;\text{levels}} \approx 1.1 \cdot 10^{-14}}$$



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– Ally: Some electromagnetic transitions, even weak, can be present
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• By populating excited (np-nh) tetrahedral symmetry states we open the way towards the unique symmetry identification conditions!

Publicity Section

New Informal Nuclear Structure Physics Group

Presentation of the members:

1. A Baran, 2. N Benhamouda, 3. D Curien, 4. I Dedes,
5. A Gaamouci, 6. H-Y Meng, 7. D Rouvel, 8. K Starosta,
9. H-L Wang, 10. M Warda, 11. J Yang
and 12. J Dudek – spokesperson

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Theoretical Determination of Stability of Exotic Nuclei
with Estimates of Modelling Uncertainties

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- Shape coexistence and exotic shape isomers.

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