Competition between long- range (collective) and short-range (pairing) correlations in two-neutron transfer reactions

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Outline

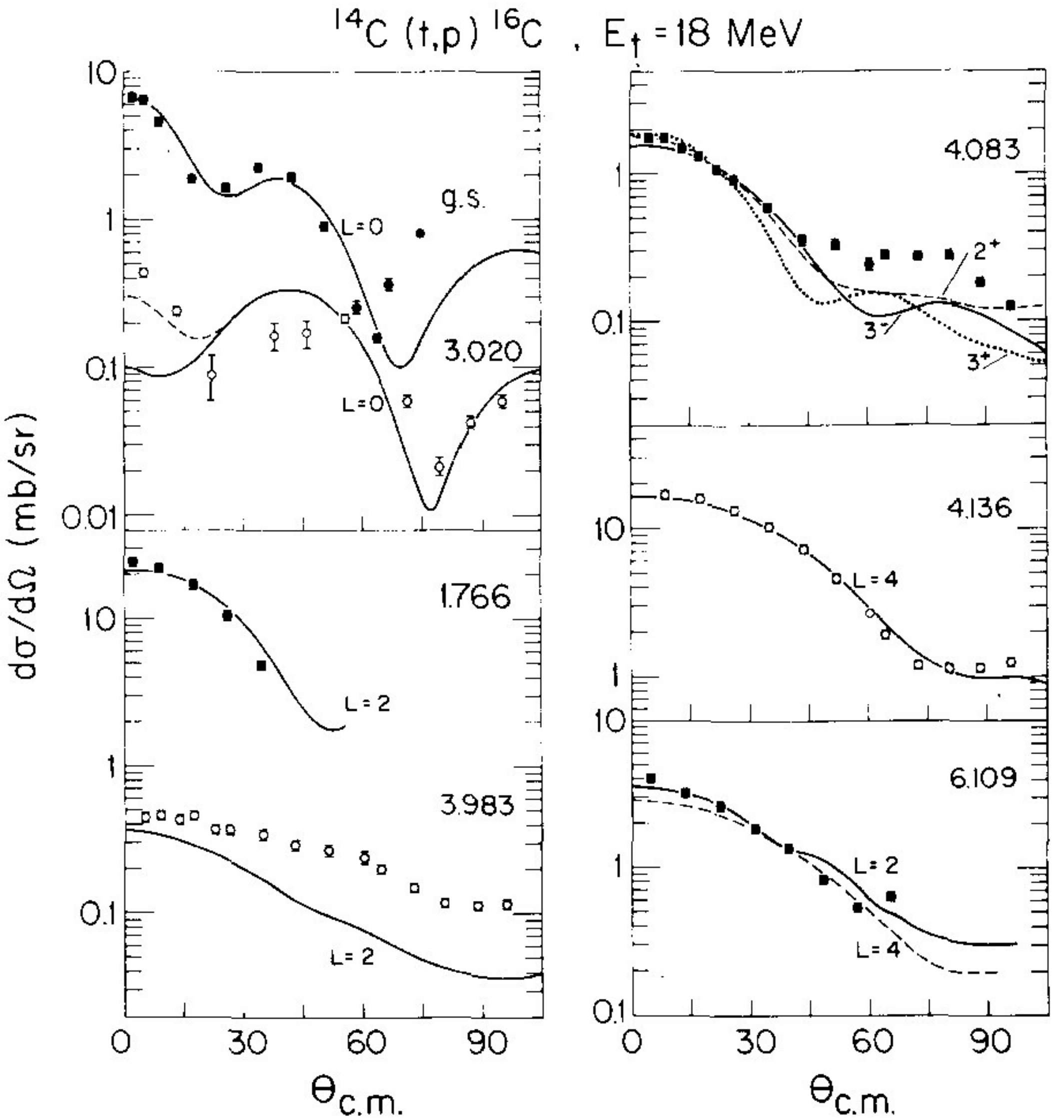
Nuclear spectroscopy via transfer reactions between heavy ions

- The (^{18}O,^{16}O) reaction

- Experimental results about ^{12,13}C(^{18}O,^{16}O)^{14,15}C, ^{16}O(^{18}O,^{16}O)^{18}O, ^{64}Ni(^{18}O,^{16}O)^{66}Ni and ^{28}Si(^{18}O,^{16}O)^{30}Si reactions @ 84 MeV incident energy

- CRC and two-step DWBA calculations
- Microscopic cluster calculations

Brief introduction. (t,p) reactions



Volume 70B, number 4 PHYSICS LETTERS 24 October 1977

SPECTROSCOPY OF ^{16}C ★

H.T. FORTUNE, R. MIDDLETON, M.E. COBERN,
G.E. MOORE, S. MORDECHAI, R.V. KOLLARITS
Physics Department, University of Pennsylvania, Philadelphia, Pennsylvania 19104, USA

and

H. NANN ‡, W. CHUNG and B.H. WILDENTHAL
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Received 30 June 1977

The $^{14}C(t,p)^{16}C$ reaction locates five new states in ^{16}C, at excitation energies of 3020 ± 15, 3983 ± 10, 4083 ± 10, 4136 ± 10 and 6109 ± 15 keV, in addition to the g.s. and 1.76 MeV states. The 3.02 and 3.98 MeV states appear to be the second 0^+ and 2^+ 2p-2h states, respectively. The 4.14 MeV state has $J^\pi = 4^+$ and the 6.11 MeV state has $J^\pi = 2^+$, 3^-, or 4^+.

E_x (I^π)	N
0.0 (0+)	300
1.766 (2+)	400
3.020 (0+)	300
3.983 (2+)	400
4.136 (4+)	360

Main reasons for these discrepancies:

- the use of oversimplified triton wave functions
- the use of the zero-range approximation
- the use of only simultaneous transfer.
- Numerical simplifications to solve six-dimension integrals to determine transition amplitudes???

The triton beam are not longer produced for safety reasons!

Nuclear spectroscopy via (^{18}O,^{16}O) reaction

The **(^{18}O,^{16}O)** reactions are good candidates to show the role of **pairing interaction** thanks to

- The presence of a **correlated pair** of neutrons in the $^{18}O_{g.s.}$ wave function
- The very low polarizability of the **^{16}O core**

^{14}C is a good benchmark for considerations on the reaction mechanism, **^{64}Ni and ^{28}Si** are good benchmark for studying long-range vs short-range correlations

Studies on both
$^{13}C(^{18}O,^{17}O)^{14}C$ **1n transfer** and $^{12}C(^{18}O,^{16}O)^{14}C$ **2n transfer**

- Presence of 2n correlations in the $^{14}C_{g.s.}$ wave function
- Strong selectivity in the populated states
- Absolute cross sections reproduced without any scaling factor

M. Cavallaro et al., PRC 88 (2013) 054601

Theoretical models and main ingredients

Exact finite range CRC
and
two-step CCBA calculations

- Sao Paulo Potential (**SPP**) used in the optical model

L.C. Chamon, et al., PRL 79 (1997) 5218

- **Wood-Saxon form factors** were used to generate single particle and cluster wave functions. Depth were adjusted to fit the exp. separation energies

- **Deformation parameters** for collective excitations

- **Spectroscopic Amplitudes** by shell-model in the $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$ model space (zbm interaction)

A.P. Zuker, et al., PRL 17 (1969) 983

Theoretical models and main ingredients

The CRC equations are in many cases of the form

$$\left[E_{\kappa pt} - T_{\kappa L}(R_\kappa) - U_\kappa(R_\kappa)\right] f_\alpha(R_\kappa) = \sum_{\alpha', \Gamma>0} i^{L'-L}\, V^{\Gamma}_{\alpha:\alpha'}(R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) + \sum_{\alpha', \kappa' \neq \kappa} i^{L'-L} \int_0^{R_m} V_{\alpha:\alpha'}((R_\kappa), R_{\kappa'}) f_{\alpha'}(R_{\kappa'}) dR_{\kappa'}$$

Single nucleon states are given by

$$\phi_{JM}(\xi_c, \mathbf{r}) = \sum_{\ell j I} A^{jIJ}_{\ell s j}\ \left[\phi_I(\xi_c)\varphi_{\ell s j}(\mathbf{r})\right]_{JM}$$

$$= \sum_{\ell j I, m\mu m_s m_\ell} A^{jIJ}_{\ell s j}\, \langle j m I \mu | J M\rangle\, \phi_{I\mu}(\xi_c)\, \langle \ell m_\ell s m_s | j m\rangle\, Y^{m_\ell}_\ell(\hat{\mathbf{r}}) \phi^{m_s}_s \frac{1}{r} u_{\ell s j I}(r)$$

and are the solution of

$$[T_\ell(r) + V(r) + \epsilon_I - E] u_{\ell s j I}(r) + \sum_{\ell' j' I',\ \Gamma>0} V^{\Gamma}_{\ell s j I:\ell' s j' I'}(r) u_{\ell' s j' I'}(r) = 0$$

Theoretical models and main ingredients

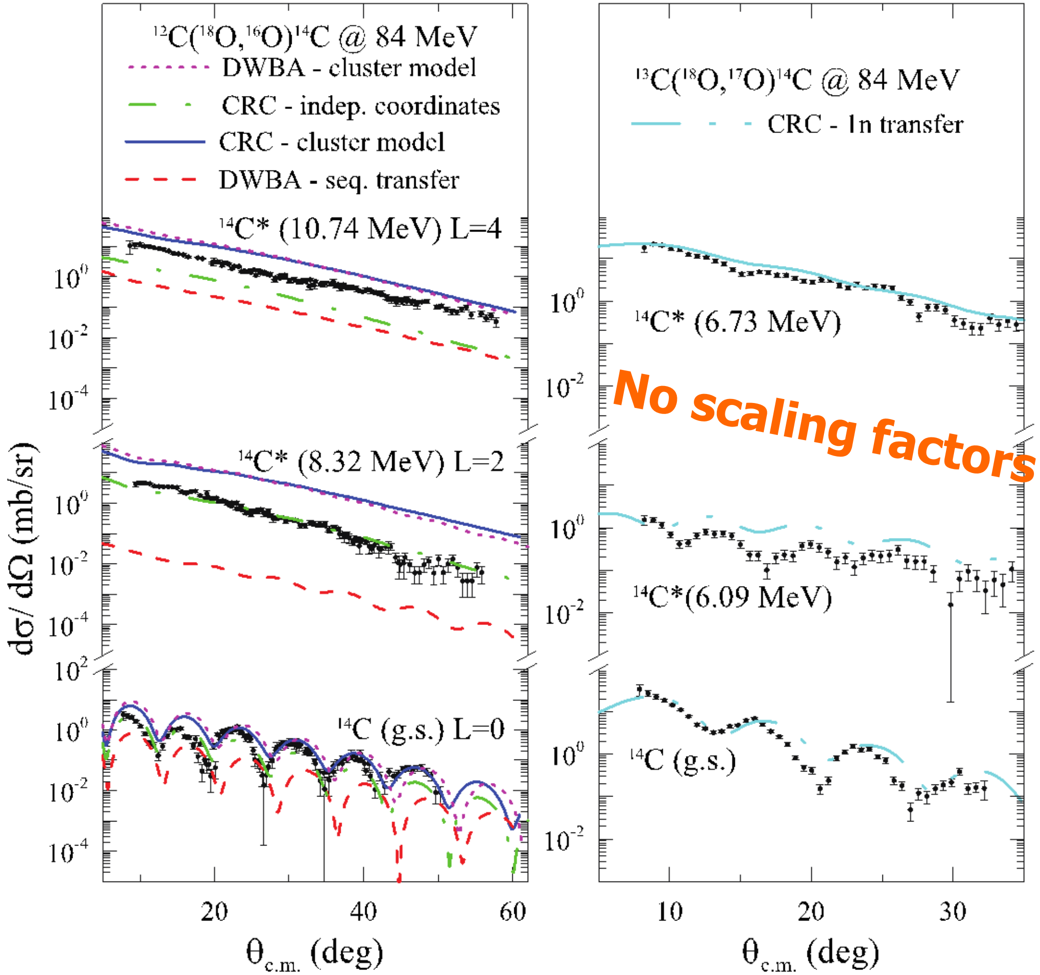
Independent coordinate model

$$
\begin{aligned}
\varphi_{12}(\mathbf{r}_1,\mathbf{r}_2) &= \sum_i c_i \ |(\ell_1(i),s_1)j_1(i),(\ell_2(i),s_2)j_2(i);\ \ J_{12}T\rangle \\
&\rightarrow \sum_u c_i \sum_{L\ell S j} |L,(\ell,(s_1 s_2)S)j;J_{12}T\rangle\, \phi^{J_{12}T,i}_{L(\ell S)j}(r,\rho)
\end{aligned}
$$

$$
\begin{aligned}
\phi^{J_{12}T,i}_{L(\ell S)j}(r,\rho) &= \langle L,(\ell,(s_1 s_2)S)j;J_{12}T|\,(\ell_1(i),s_1)j_1(i),(\ell_2(i),s_2)j_2(i);\ \ J_{12}T\rangle \\
&\quad \times\langle[Y_L(\hat{\mathbf{r}})Y_\ell(\hat{\rho})]_\lambda\,|\,[\varphi_{\ell_1 s_1 j_1}(\mathbf{r}_1)\varphi_{\ell_2 s_2 j_2}(\mathbf{r}_2)]_{J_{12}T}\rangle
\end{aligned}
$$

and the radial integral overlaps are derived from using Moshinsky harmonic oscillator expansion

Theoretical results for other channels



Presence of two-neutron pairing correlations in other ^{14}C states

M. Cavallaro et al., PRC 88 (2013) 054601

Extreme Cluster Model

(CRC)

❖ Relative motion of the 2n system frozen and separated by the c.m.

❖ Only the term with the 2n coupled to S = 0 participates to the transfer

Sequential transfer

(DWBA)

Introducing the $^{17}O + ^{13}C$ intermediate partition

Independent coord.

(CRC)

CRC **-1n transfer**

No arbitrary scaling

New works published in 2016-2017

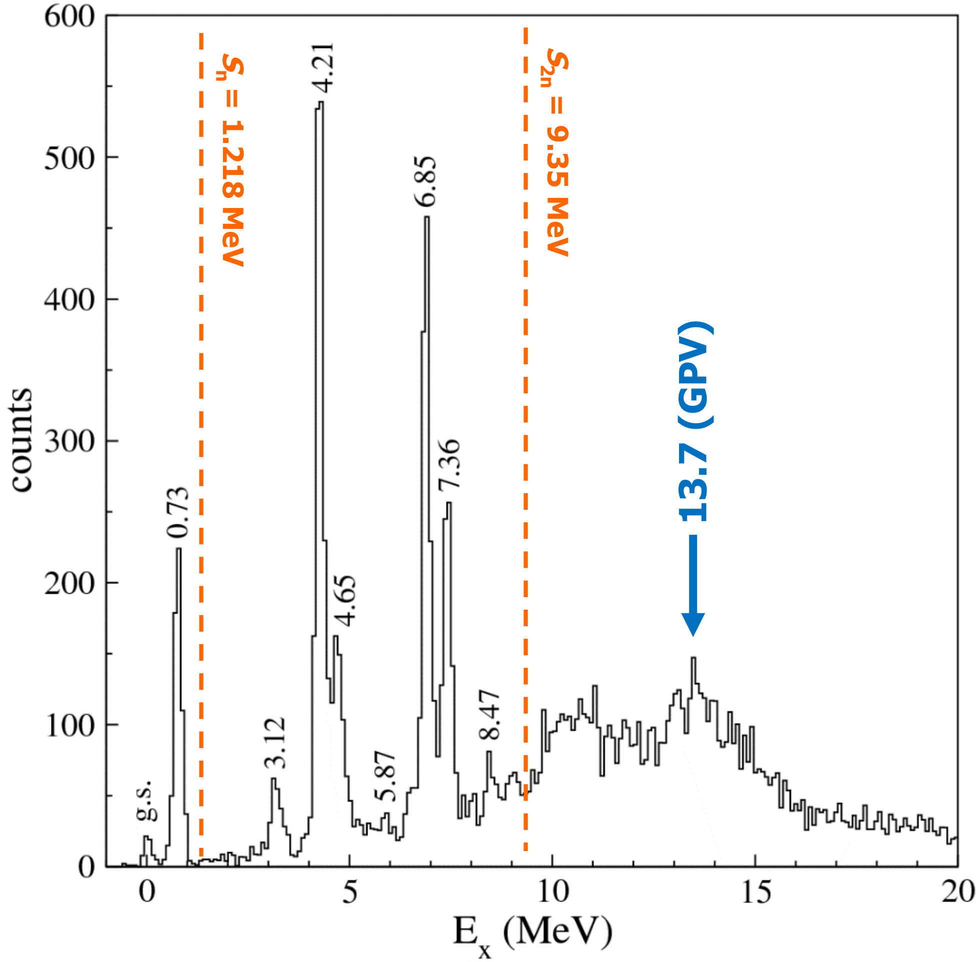
What happens if we add a neutron to the ^{14}C system?

Study of the $^{13}C(^{18}O,^{16}O)^{15}C$ reaction at 84 MeV incident energy

D. Carbone et al., PRC 95, 034603 (2017)

^{15}C energy spectrum

$^{13}C(^{18}O,^{16}O)^{15}C$



$9° < \theta_{lab} < 10°$

- Same states populated in the (t,p) reactions
- Strong population of states with $^{13}C + 2n$ configurations
- Population of the Giant Pairing Vibration above S_{2n}
  - **F. Cappuzzello et al., Nat. Commun. 6, 6743 (2015)**
  - **D. Carbone, EPJ Plus (2015) 130:143**

Energy resolution ~ 200 keV

nature COMMUNICATIONS

ARTICLE

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Signatures of the Giant Pairing Vibration in the ^{14}C and ^{15}C atomic nuclei

F. Cappuzzello[1,2], D. Carbone[2], M. Cavallaro[2], M. Bondì[1,2], C. Agodi[2], F. Azaiez[3], A. Bonaccorso[4], A. Cunsolo[2], L. Fortunato[5,6], A. Foti[1,7], S. Franchoo[3], E. Khan[3], R. Linares[8], J. Lubian[8], J.A. Scarpaci[9] & A. Vitturi[5,6]

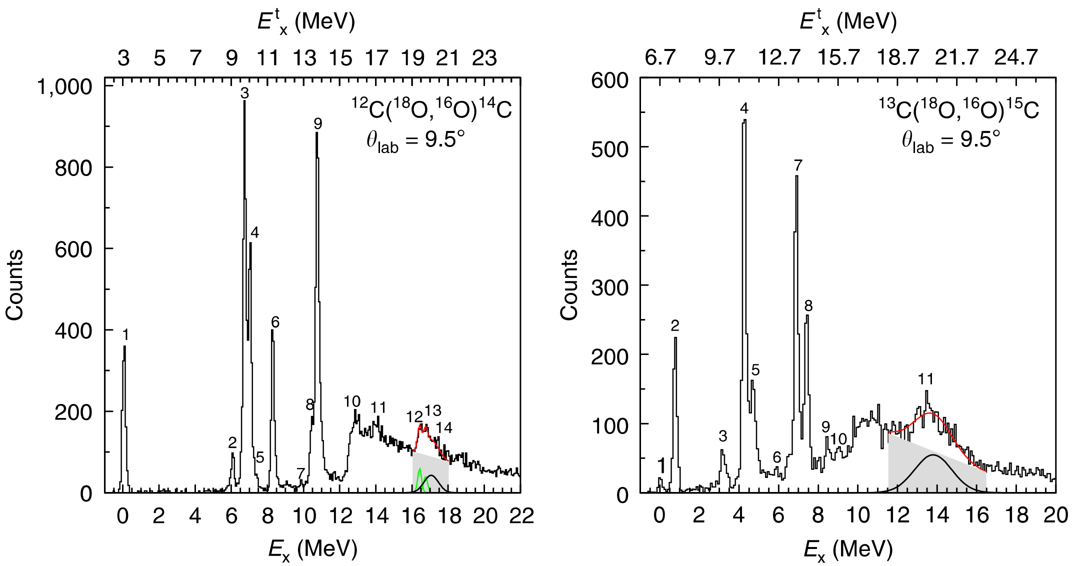
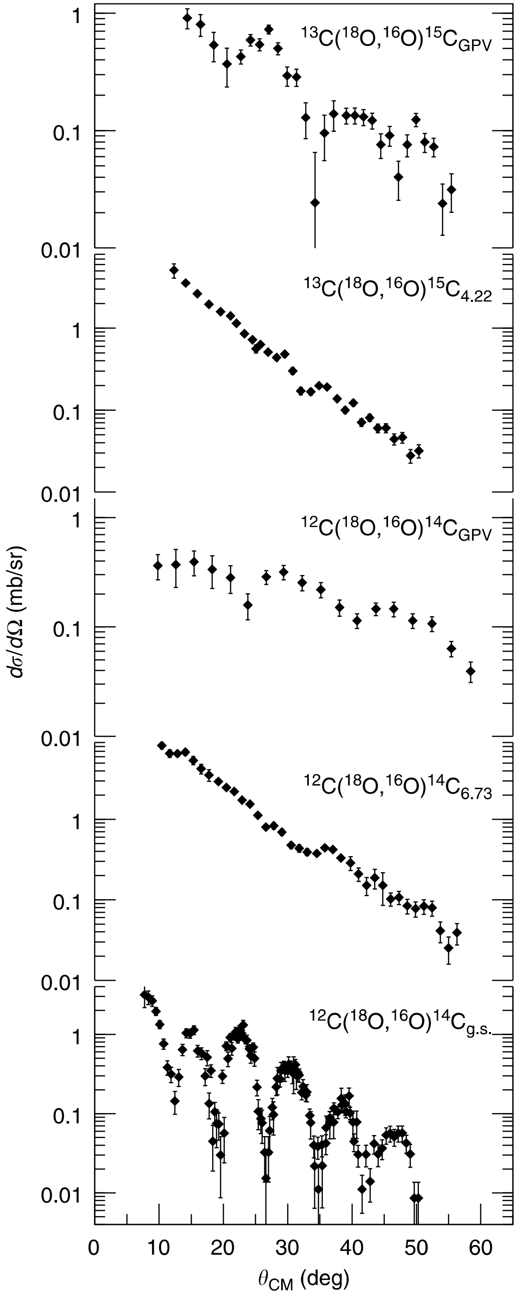


Table 1 | Main spectroscopic features of the populated states.

S.No.	Excitation energy (MeV) (present work)	Excitation energy (MeV) (values from ref. 38)	J^π (*)
^{15}C states			
1	0.00 ± 0.02	0	$1/2^+$
2	0.73 ± 0.02	0.7400	$5/2^+$
3	3.12 ± 0.02	3.103	$1/2^-$
4	4.21 ± 0.02	4.220	$5/2^-$
5	4.65 ± 0.02	4.657	$3/2^-$
6	5.87 ± 0.02	5.866	$1/2^-$
7	6.85 ± 0.02	6.841	$7/2^-$
8	7.36 ± 0.02	7.352	$9/2^-$
9	8.47 ± 0.02	8.47	$1/2^+$, $3/2^+$, $5/2^+$ (from ref. 39)
10	9.06 ± 0.02	9.00	
11	13.7 ± 0.1		$1/2^-$ (present work)
^{14}C states			
1	0.00 ± 0.02	0	0^+
2	6.10 ± 0.02	6.0938	1^-
3	6.71 ± 0.02	6.7282	3^-
4	7.00 ± 0.02	7.0120	2^+
5	7.36 ± 0.02	7.3414	2^-
6	8.33 ± 0.02	8.3179	2^+
7	9.81 ± 0.02	9.7460	0^+
8	10.43 ± 0.02	10.425, 10.498	2^+, 3^-
9	10.73 ± 0.02	10.736	4^+
10	12.88 ± 0.02	12.963	3^-
11	13.96 ± 0.02	14.05	
12	16.42 ± 0.02	16.43	6^+ (from ref. 40)
13	16.74 ± 0.02	16.715	6^- (from ref. 40)
14	16.9 ± 0.1		0^+ (present work)

*Values from ref. 38, except those explicitly indicated.



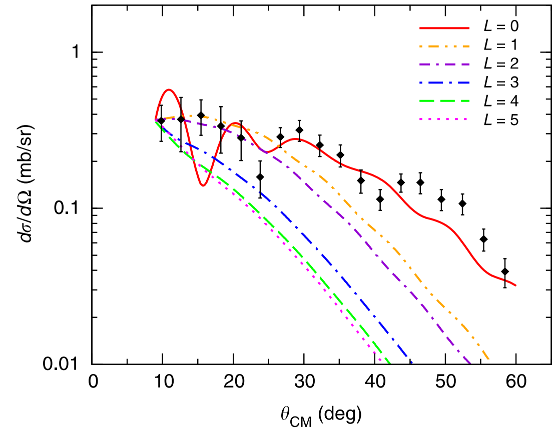
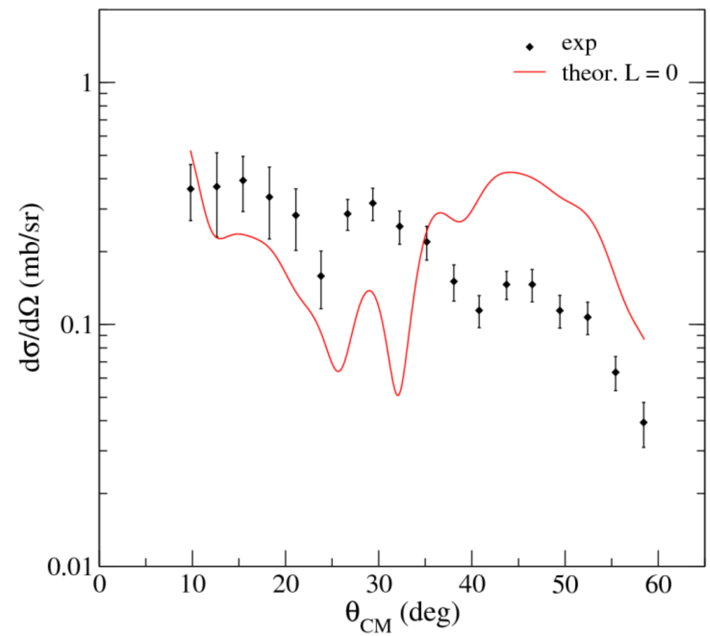
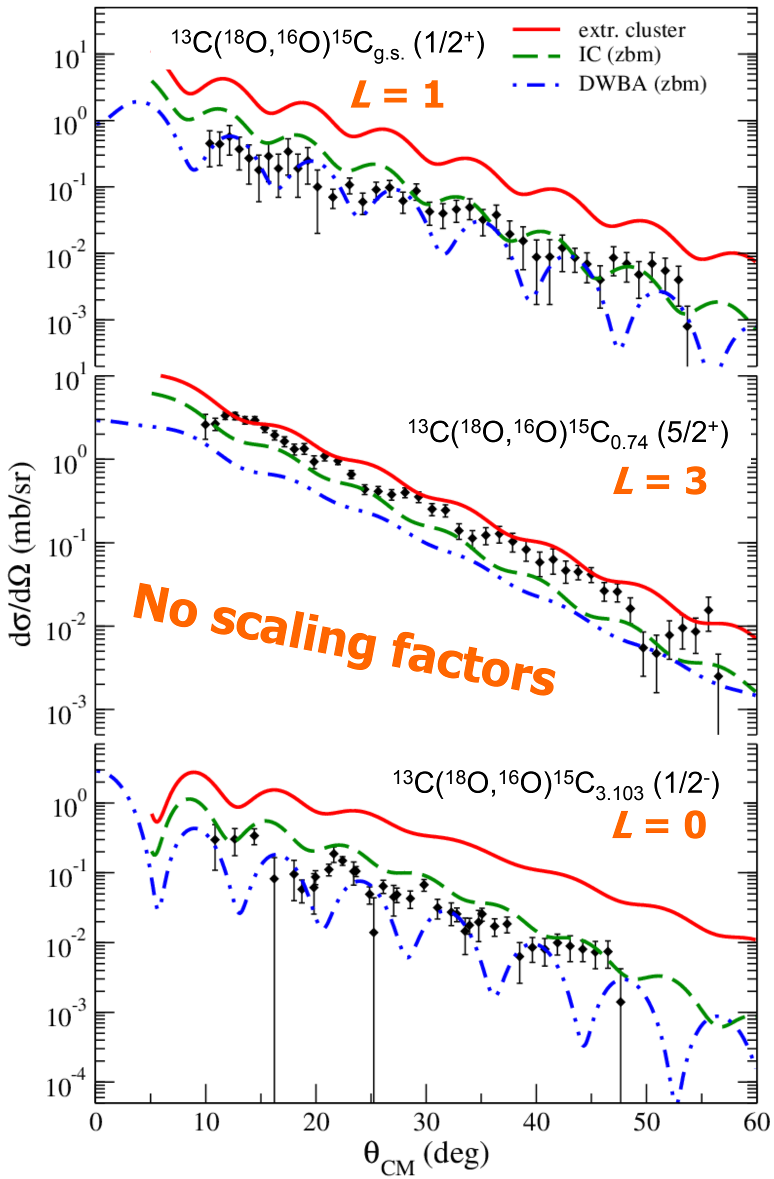


Figure 3 | Comparison between the CRC calculations and the measured cross section for the ^{14}C resonance at 16.9 MeV. The calculations are performed at 12 MeV excitation energy for $L = 0$ to 5 and are normalized to coincide at $\theta_{CM} = 9°$. The error bars correspond to the combination of uncertainties coming from the solid angle determination, the statistical error and the background subtraction (see text).



Supplementary Figure 7 – Comparison with calculations. Discretized continuum scheme calculations for the $L = 0$ case (red line) and experimental cross section angular distribution for the ^{14}C resonance at 16.9 ± 0.1 MeV. No scaling factors are used.

CRC and DWBA calculations



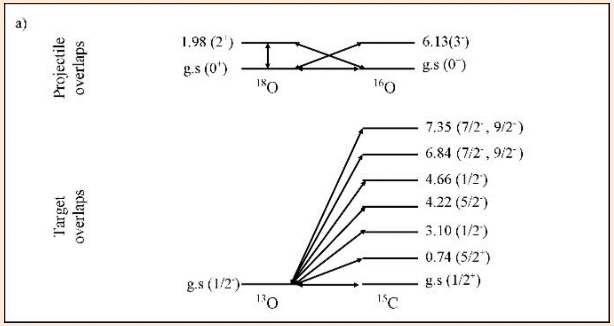
Extreme cluster model

- Relative motion of the 2n frozen and separated by the c.m.
- Only the term with the 2n coupled to S = 0 participates to the transfer
- S.A. = 1 for all configurations

Independent coordinate model

- The transfer is described taking into account spectroscopic information obtained by shell model calculations

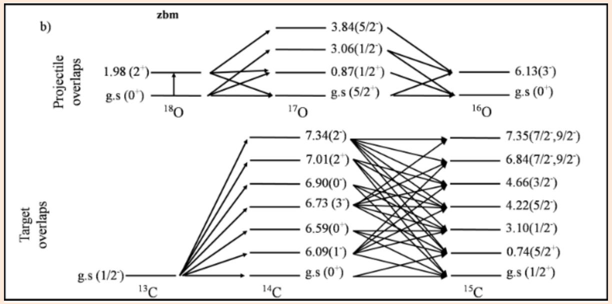
Coupling scheme



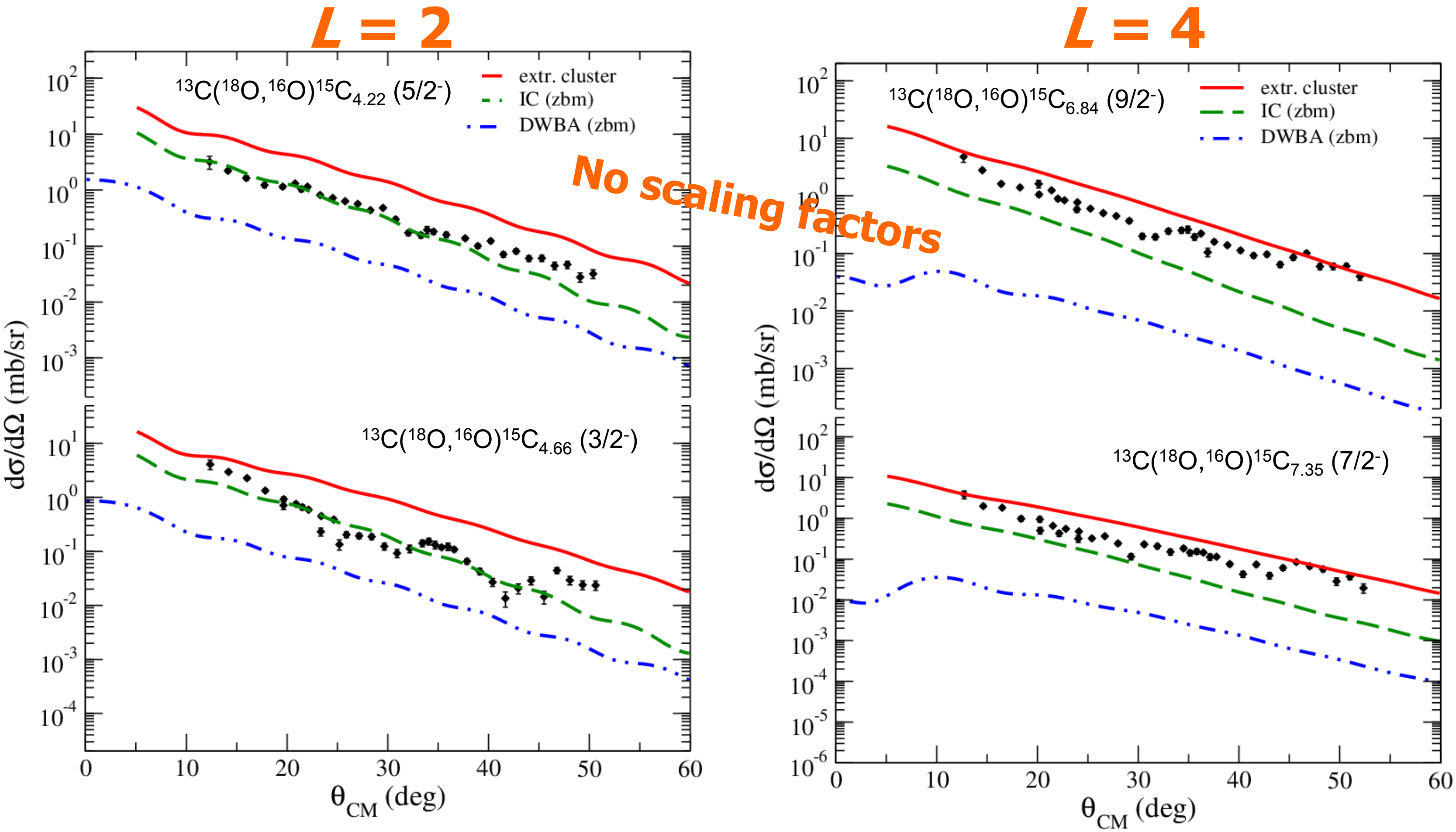
Sequential transfer (DWBA)

- Introducing the $^{17}O + {}^{14}C$ intermediate partition

Coupling scheme



CRC and DWBA calculations



Extreme cluster model overestimate the cross section (S.A. = 1)
Independent coordinate model describes quite well the cross section
Sequential transfer (DWBA) underestimate the cross section

Microscopic cluster calculations

Wave functions for two particles in an harmonic oscillator common potential (*j-j* coupling)

wave functions in terms of the relative and centre of mass coordinates of the two particles (*LS* coupling)

(n,l) internal cluster state
(N,L) cluster motion relative to the core

$$S_{\alpha J\beta J'}\left[(nl)(NL)\Lambda S;J\right]=\sum_{n_1l_1n_2l_2}\sum_{j_1j_2}\hat{S}\hat{L}\hat{j}_1\hat{j}_2\begin{Bmatrix} l_1 & 1/2 & j_1 \\ l_2 & 1/2 & j_2 \\ \Lambda & S & J \end{Bmatrix} C^L\left(n_1l_1n_2l_2;nlNL\right)S_{\alpha J\beta J'}\left(n_1l_1j_1n_2l_2j_2;J\right)$$

$$\hat{a}=\sqrt{2a+1}$$

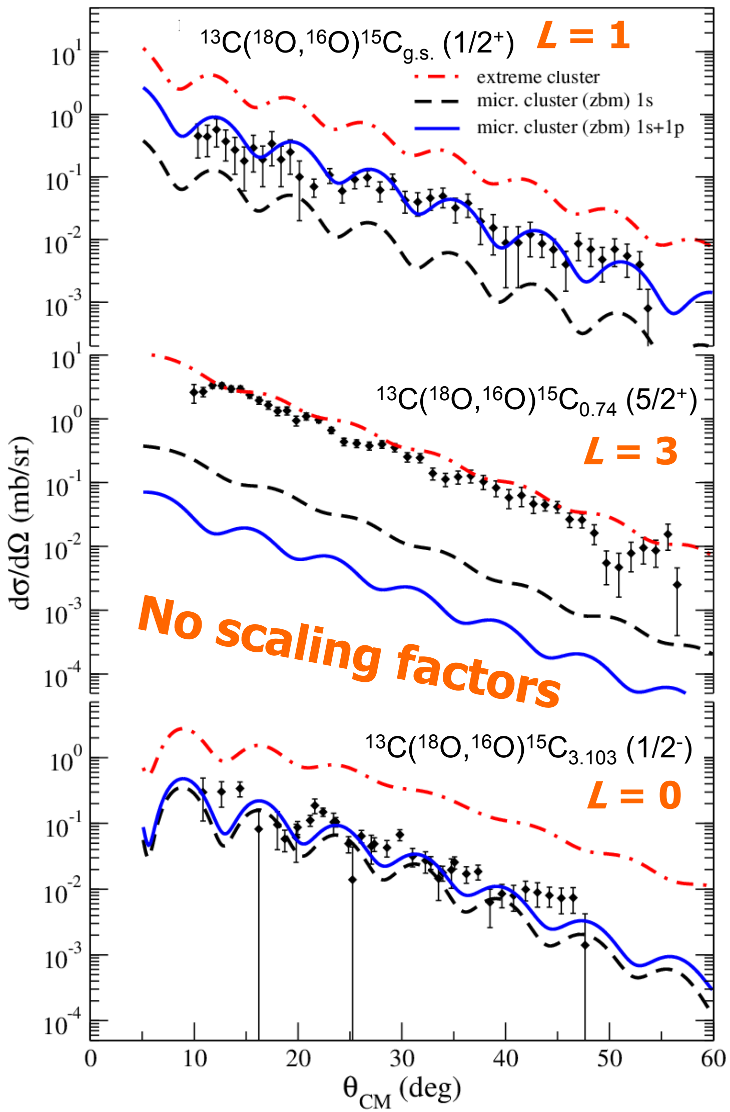
Moshinsky coefficients

S.A. in j-j coupling

Two neutron amplitudes – zbm interaction

Initial state	j_1j_2	J_{12}	Final state	Spectr. Amp.	n	l	N	L	Λ	S	Spec. Amp. (c.m.)
$^{13}C_{g.s.}(1/2^-)$	$(p_{1/2}s_{1/2})$	0	$^{15}C_{g.s.}(1/2^+)$	-0.641	1	0	2	1	1	1	-0.292
					1	1	1	2	1	1	0.338
					1	1	2	0	1	1	-0.075
	$(p_{1/2}s_{1/2})$	1		-1.110	1	0	2	1	1	0	0.292
					1	1	1	2	1	0	-0.338
					1	1	2	0	1	0	0.075
					1	0	2	1	1	1	-0.413
					1	1	1	2	1	1	0.477
					1	1	2	0	1	1	-0.107

Microscopic cluster calculations



Extreme cluster model

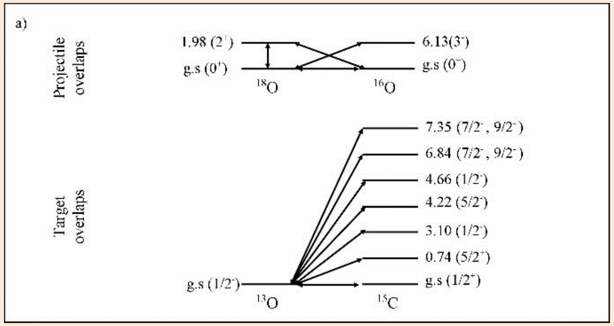
Microscopic cluster 1s

- Taking into account configurations with n = 1 l = 0

Microscopic cluster 1s + 1p

- Taking into account configuration with n = 1 l = 0, 1

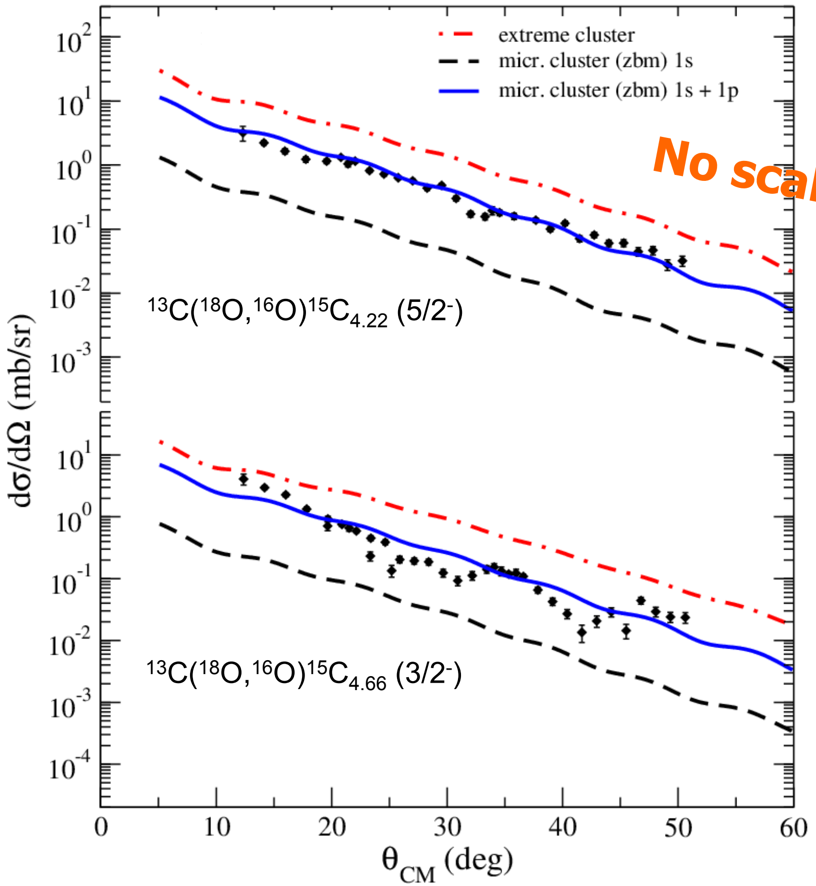
Coupling scheme



- Transitions to ground and 3.103 MeV states **reproduced rather well** with 1s + 1p waves
- Transition to 0.74 MeV state probably needs more configurations

Microscopic cluster calculations

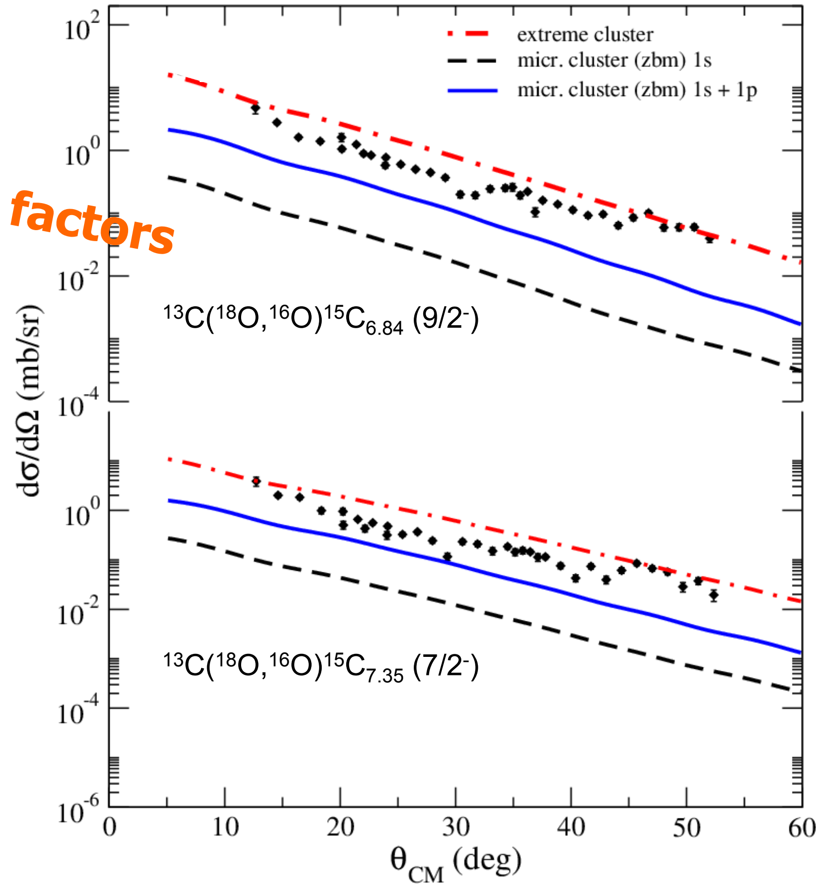
L = 2



Cross section **reproduced quite well** with 1s + 1p waves

No scaling factors

L = 4



Cross section **underestimated** with 1s + 1p waves

Relevant $d_{3/2}$ contributions expected, excluded in our model space

New works published in 2016-2017

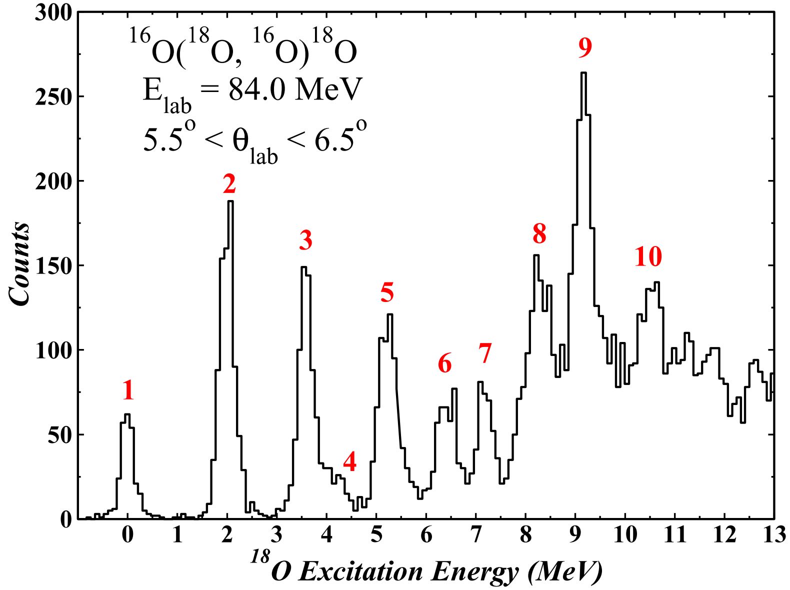
Test of model space for the <^{18}O|^{16}O> projectile overlaps

Study of the $^{18}O(^{16}O,^{18}O)^{16}O$ reaction at 84 MeV incident energy zbm vs psdmod interactions

Model space	valence orbitals
zbm (^{12}C-core)	$1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$
psdmod (^{4}He core)	$1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$

Experimental results

S_n= 8.045 MeV; S_{2n}= 12.189 MeV



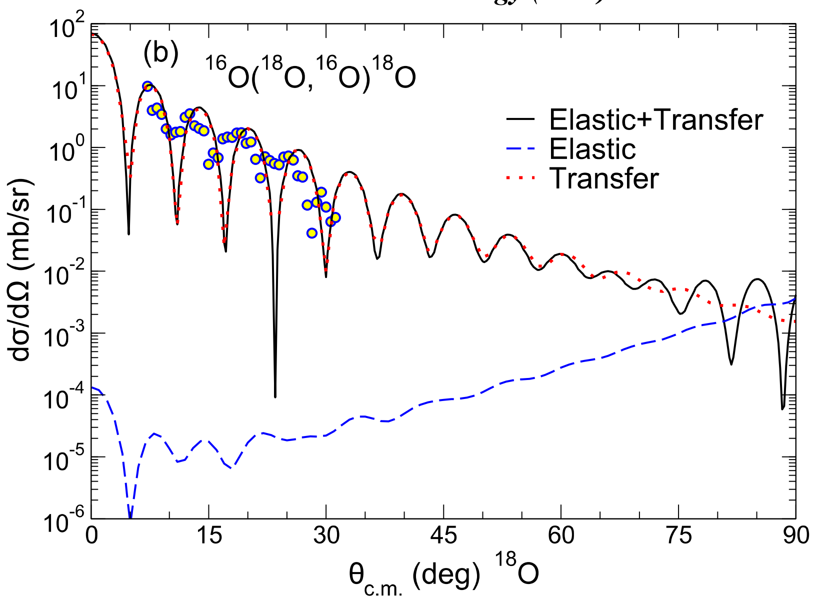


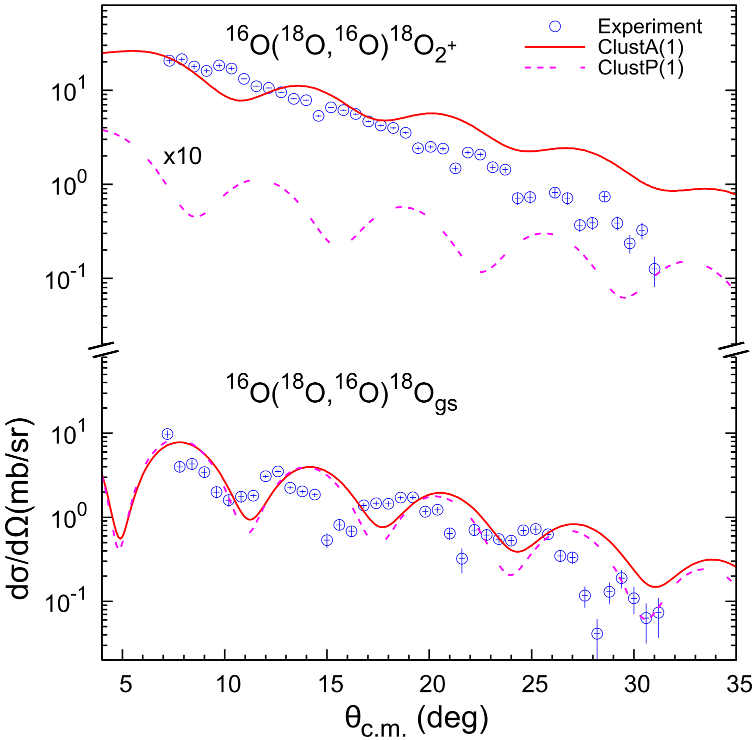
TABLE I. States populated in the $^{16}O(^{18}O,^{16}O)^{18}O$ reaction at 84 MeV (see Fig. 1).

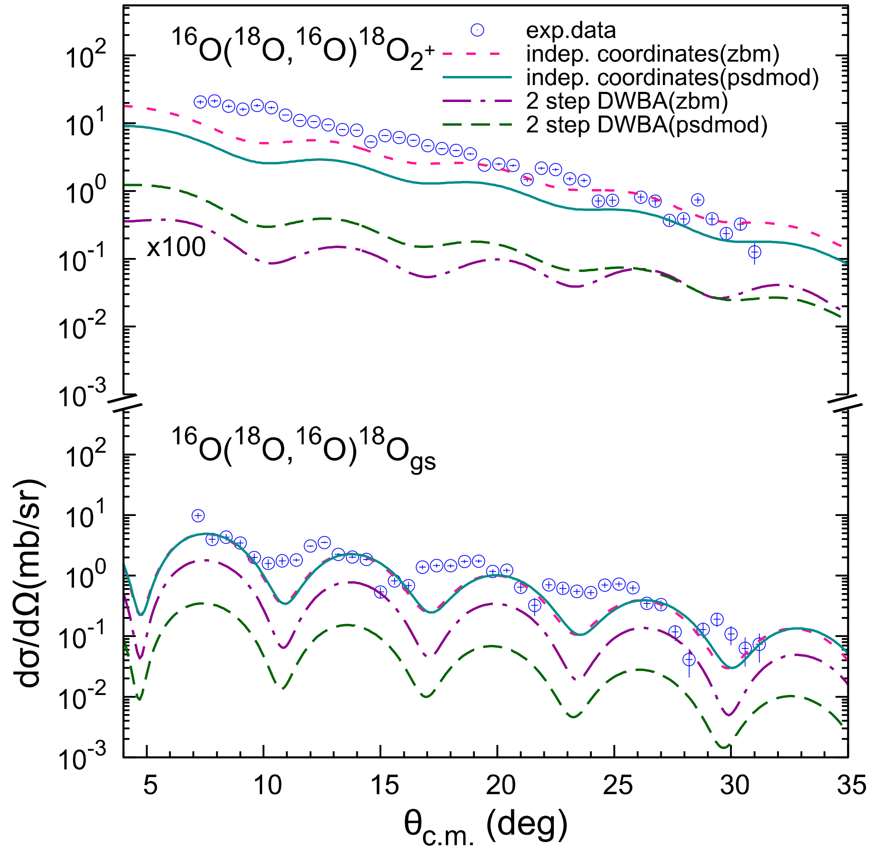
Label	Energy (MeV)	J^π
1	0.00	0^+
2	1.98	2^+
3	3.55, 3.63, 3.92	
4	4.46	1^-
5	5.10, 5.26, 5.34, 5.38, 5.53	
6	6.20, 6.35, 6.40, 6.88	
7	7.12	4^+
8	8.28[a]	3^-
9	9.03[a]	
10	10.40[a], 10.61[a]	

[a]The most intense states according to (t,p) reaction [37] were considered.

M. J. Ermamatov et al., PRC 94 (2016) 024610

Results of theoretical calculations





g.s. only S=0 (A)
2+ S=0 (A) or (P)

Extreme cluster model works

M. J. Ermamatov et al., PRC 94 (2016) 024610

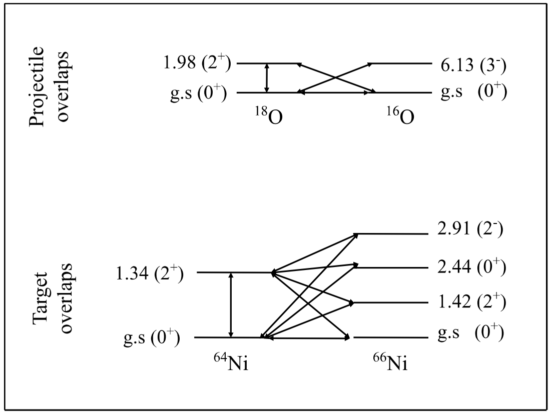
For the lower states of projectile overlaps the zbm model- space is enough.

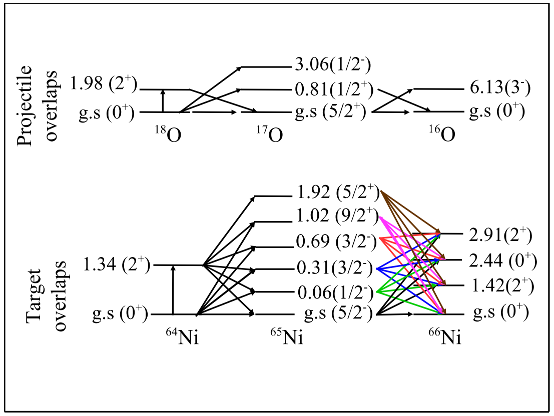
The study of the higher excited states is in progress

New works in progress (some results)

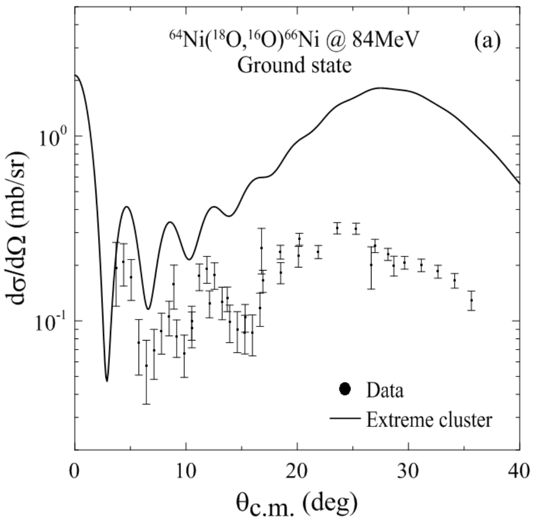
Study of the $^{18}O(^{64}Ni,^{66}Ni)^{16}O$ reaction at 84 MeV incident energy

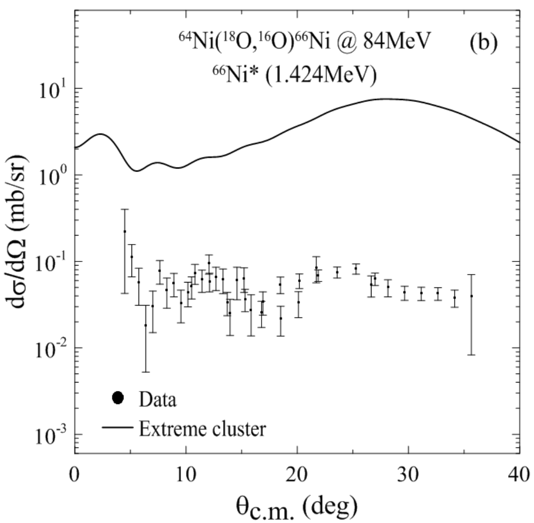
Model space	valence orbitals
protons	$1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$
neutrons	$1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2,\,1}$, $1g_{7/2}$

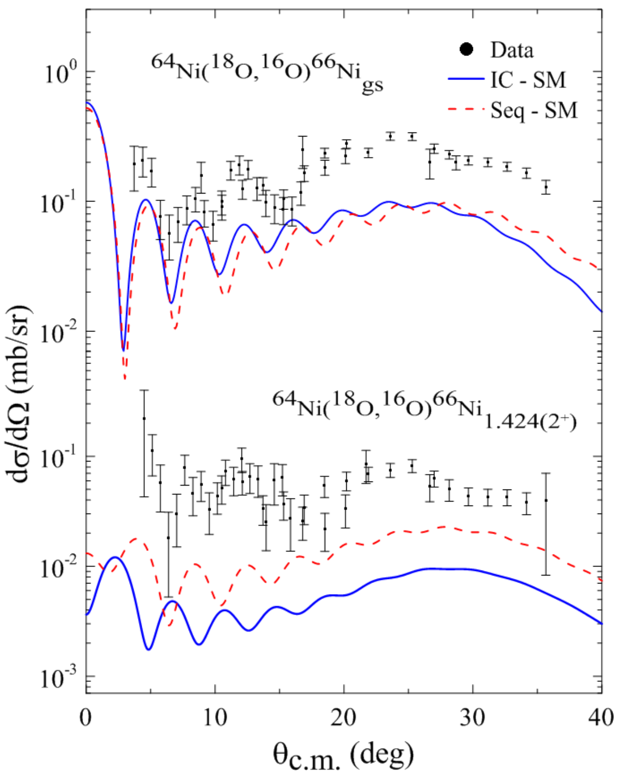




Results of theoretical calculations







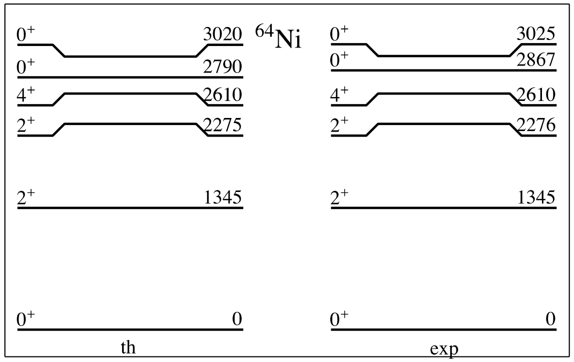
Microscopic results:

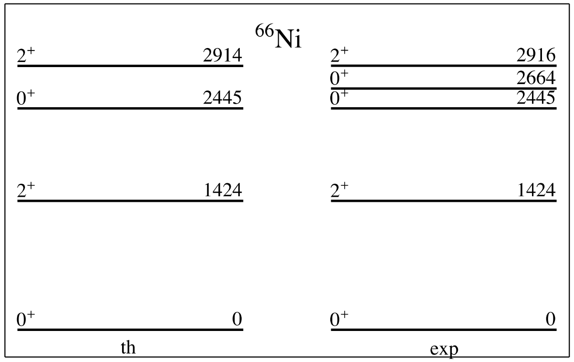
g.s.: IC results are better, specially in the bell-shaped region. Same order: one and two step.

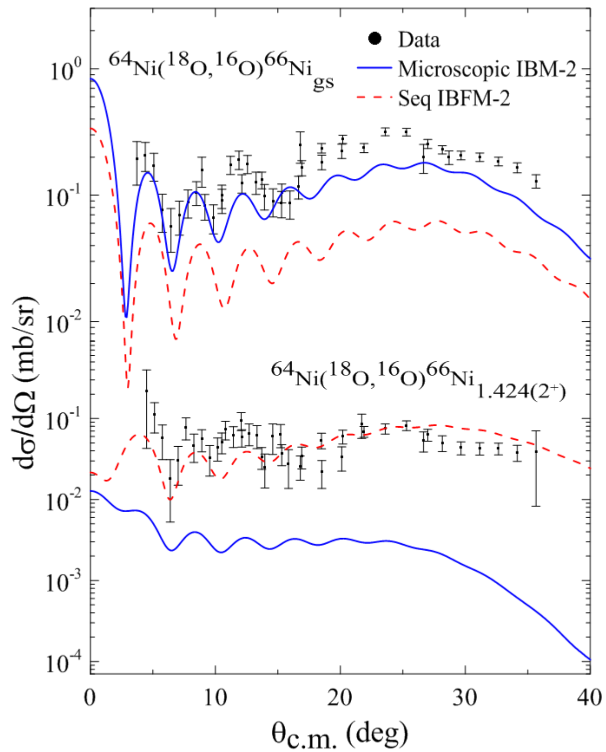
2^+ : Long-range correl. (coll.) dominates over the short-range (pairing)

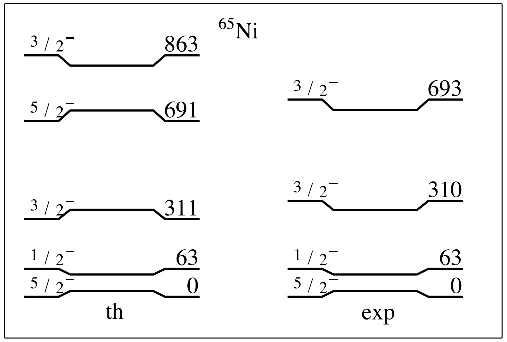
Cluster model is not good for ^{64,66}Ni

Results of theoretical calculations









Microscopic results:
g.s.: IC results are better, specially in the bell-shaped region.
2^+ : Long-range correl. (coll.) dominates over the short-range (pairing)

B. Paes et al PRC 96.044612 (2017)

IBM2 for ^{64,66}Ni and IBFM for ^{65}Ni

Results of theoretical calculations

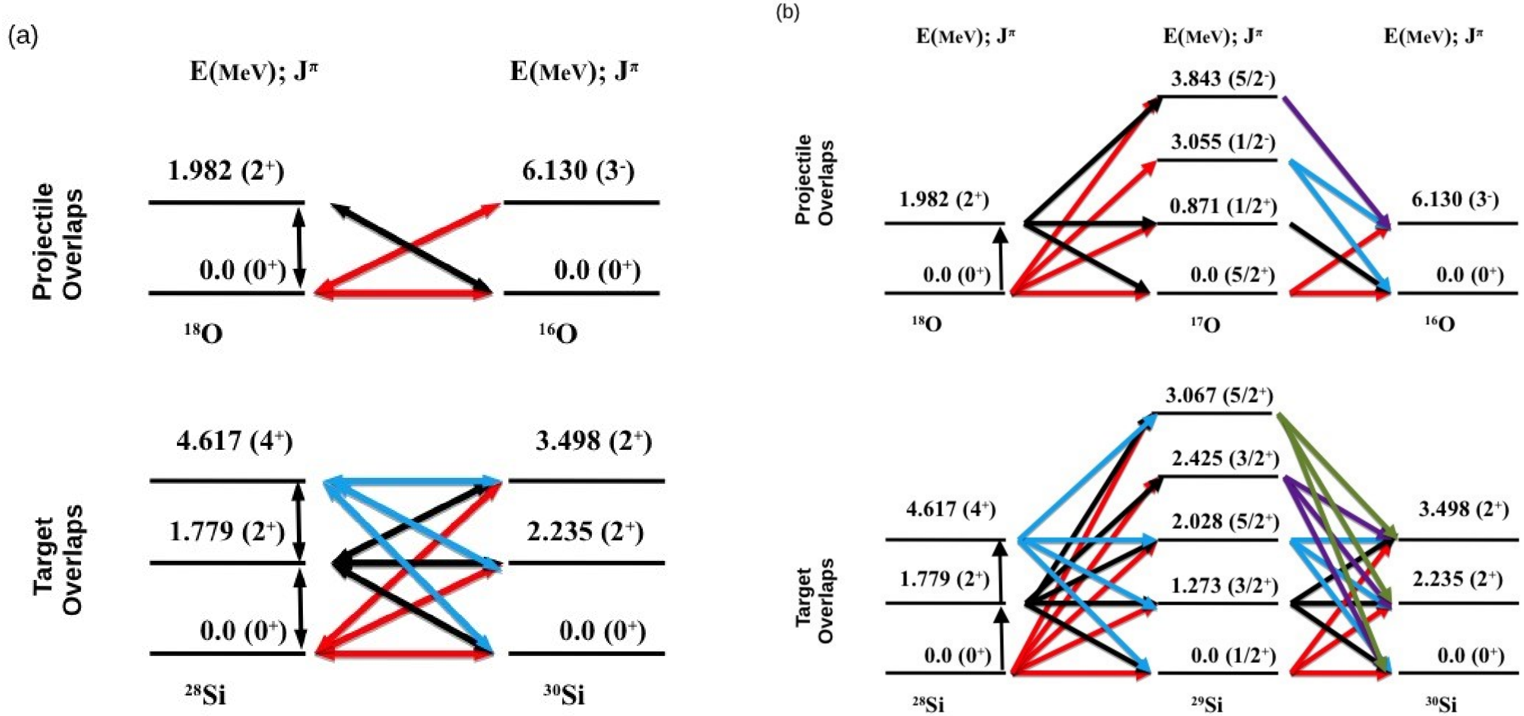
Nucleus	B(E2); 0^+ ! 2^+ (e^2b^2)
^{14}C	0.0018
^{18}O	0.0045
^{28}Mg	0.035
^{66}Ni	0.060
^{76}Ge	0.270

Small for ^{14}C ^{18}C
Big for ^{28}Mg ^{66}Ni ^{76}Ge

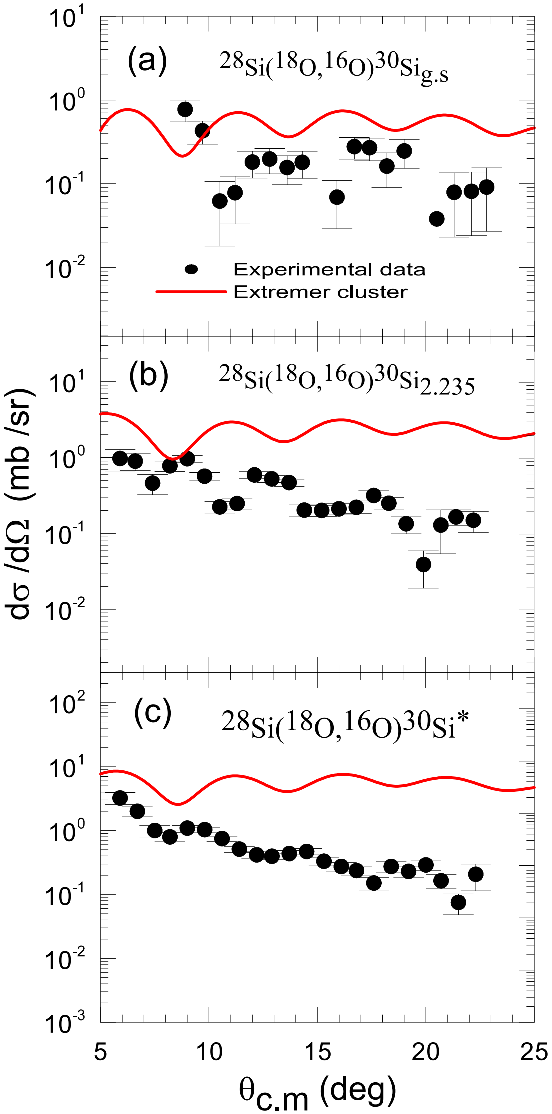
More recent results

Study of the $^{18}O(^{28}Si,^{30}Si)^{16}O$ reaction at 84 MeV incident energy

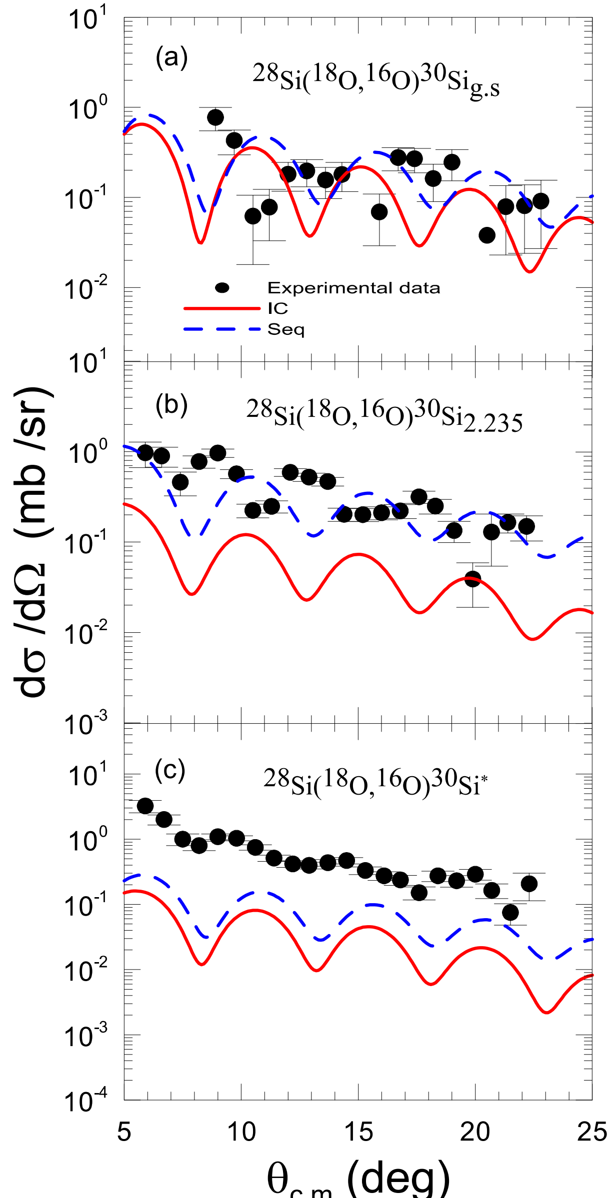
Model space (^{4}He core)	valence orbitals (similar to Ni)
Protons	$1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$
neutrons	$1p_{3/2}$, $1p_{1/2}$, $1d_{5/2}$, $2s_{1/2}$, $1d_{3/2}$



Results of theoretical calculations



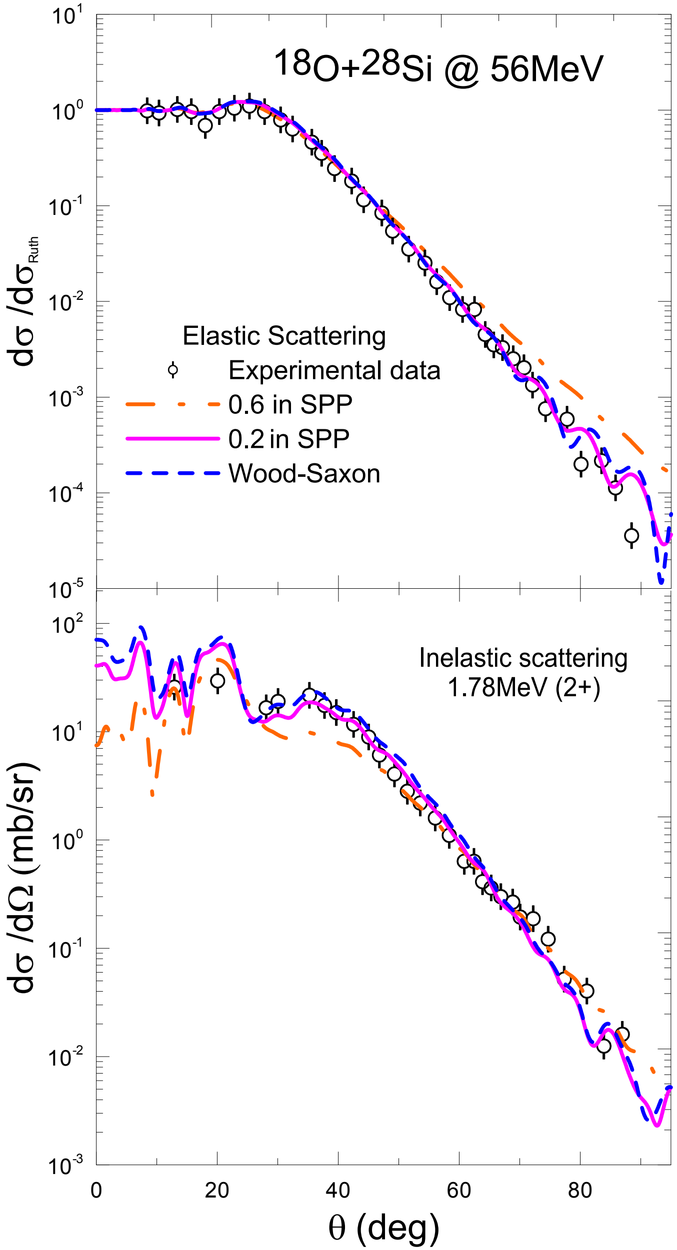
Cluster model is not good for ^{28,30}Si



Microscopic results:
g.s.: Two-step DWBA results are better. Same order: one and two step.
2^+ : Long-range correl. (coll.) dominates over the short-range (pairing)
Si* the same results as the 2^+ state

Nucleus	B(E2); $0^+ \to 2^+$ (e^2b^2)
^{14}C	0.0018
^{18}O	0.0045
^{28}Mg	0.035
^{30}Si	0.022
^{66}Ni	0.060
^{76}Ge	0.270

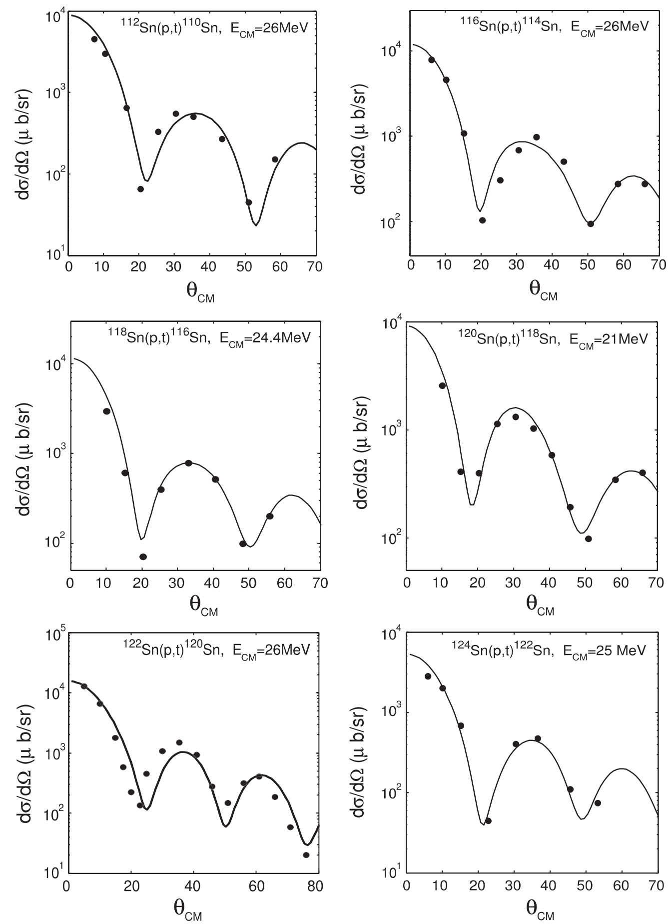
Results of theoretical calculations



Does our theoretical calculations describe other observables?

- Elastic scattering
- Inelastic scattering

2n transfer. Heavier target, other authors



G. Potel et al.PRL 107, 092501 (2011)

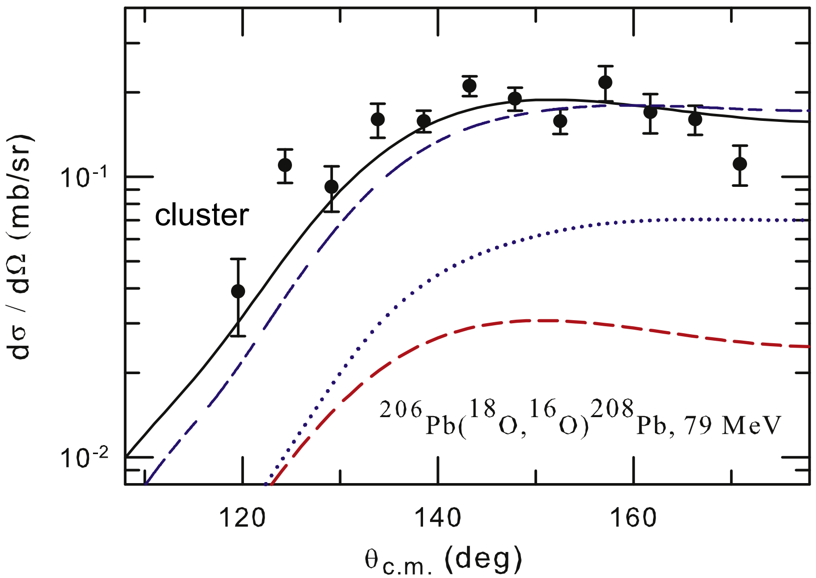
	σ (μb)			
	5.11 MeV	6.1 MeV	10.07 MeV	15.04 MeV
Total	1.29×10^{-17}	3.77×10^{-8}	39.02	750.2
Successive	9.48×10^{-20}	1.14×10^{-8}	44.44	863.8
Simultaneous	1.18×10^{-18}	8.07×10^{-9}	10.9	156.7
Nonorthogonal	2.17×10^{-17}	7.17×10^{-8}	22.68	233.5
Nonorthogonal + simultaneous	1.31×10^{-17}	3.34×10^{-8}	3.18	17.4
Pairing	1.01×10^{-19}	6.86×10^{-10}	0.97	14.04

$^{132}\mathrm{Sn}(p,t)^{130}\mathrm{Sn}(g.s.)$

integrating

$0° \leq \theta_{\mathrm{c.m.}} \leq 80°$

2n transfer. Heavier target, other authors

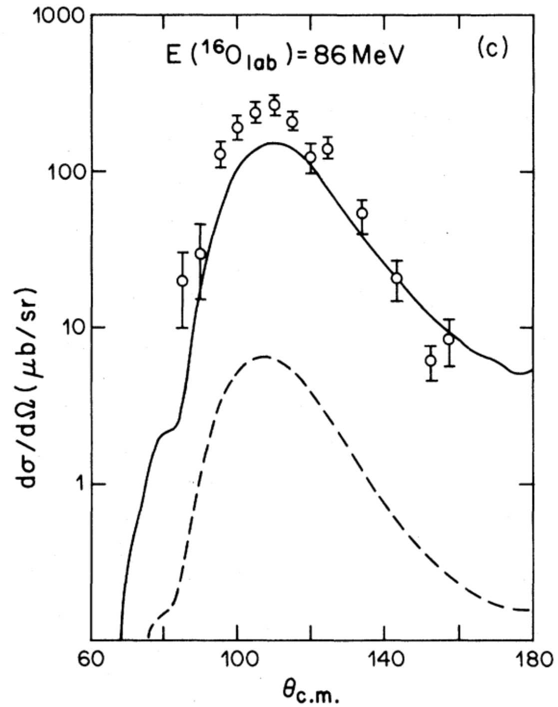


Coherent sum

Sequential (successive)

direct

A. Parmar et al. / Nuclear Physics A 940 (2015) 167



direct

B.M. Bayman & Jongsheng Chen, PRC26, 1509 (82)

Conclusions and outlooks

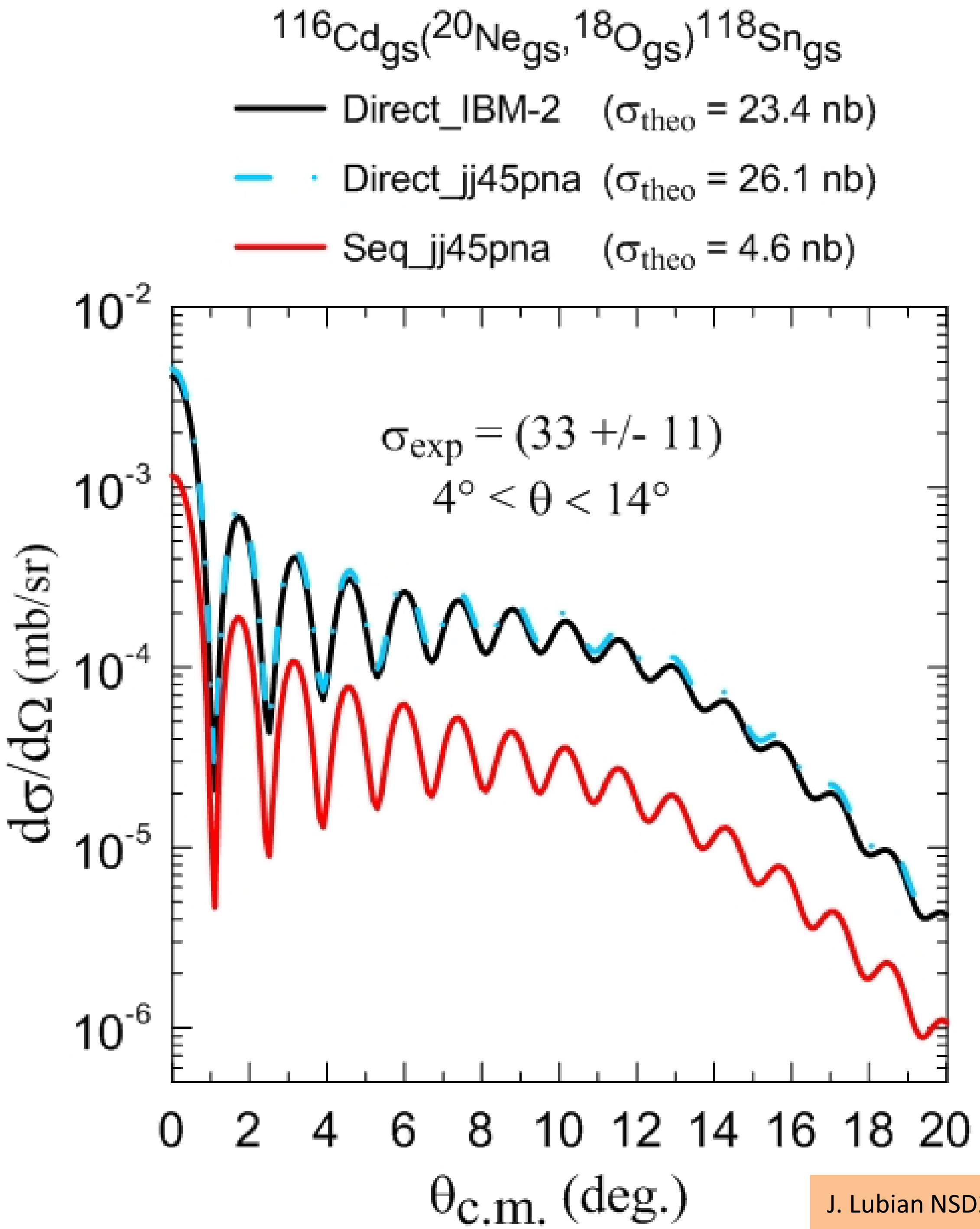
- $^{12,13}C(^{18}O,^{16}O)^{15}C$, $^{16}O(^{18}O,^{16}O)^{18}O$, $^{64}Ni(^{18}O,^{16}O)^{66}Ni$, $^{28}Si(^{18}O,^{16}O)^{30}Si$, at 84 MeV incident energy
- Four models were used to calculate the cross section:
  - ✓ Extreme cluster
  - ✓ Independent coordinate
  - ✓ DWBA
  - ✓ Microscopic cluster (only for ^{13}C)
- no need for any "unhappiness" factor to reproduce the absolute cross sections

- In ^{13}C importance of a two-neutron correlation in the nuclear wave function, the extra neutron does not destroy the correlations observed in the ^{14}C case
- Dominance of the 1s and 1p waves in the two-neutron cluster internal wave function
- Adequacy of zbm interaction for low-lying overlaps of the projectile were established for the projectile.
- Dominance of long-range correlations for the excited 2^+ state of ^{66}Ni over the short-range pairing correlations. The opposite for the g.s.
- Dominance of long-range correlations in all states of ^{30}Si.

Conclusions and outlooks

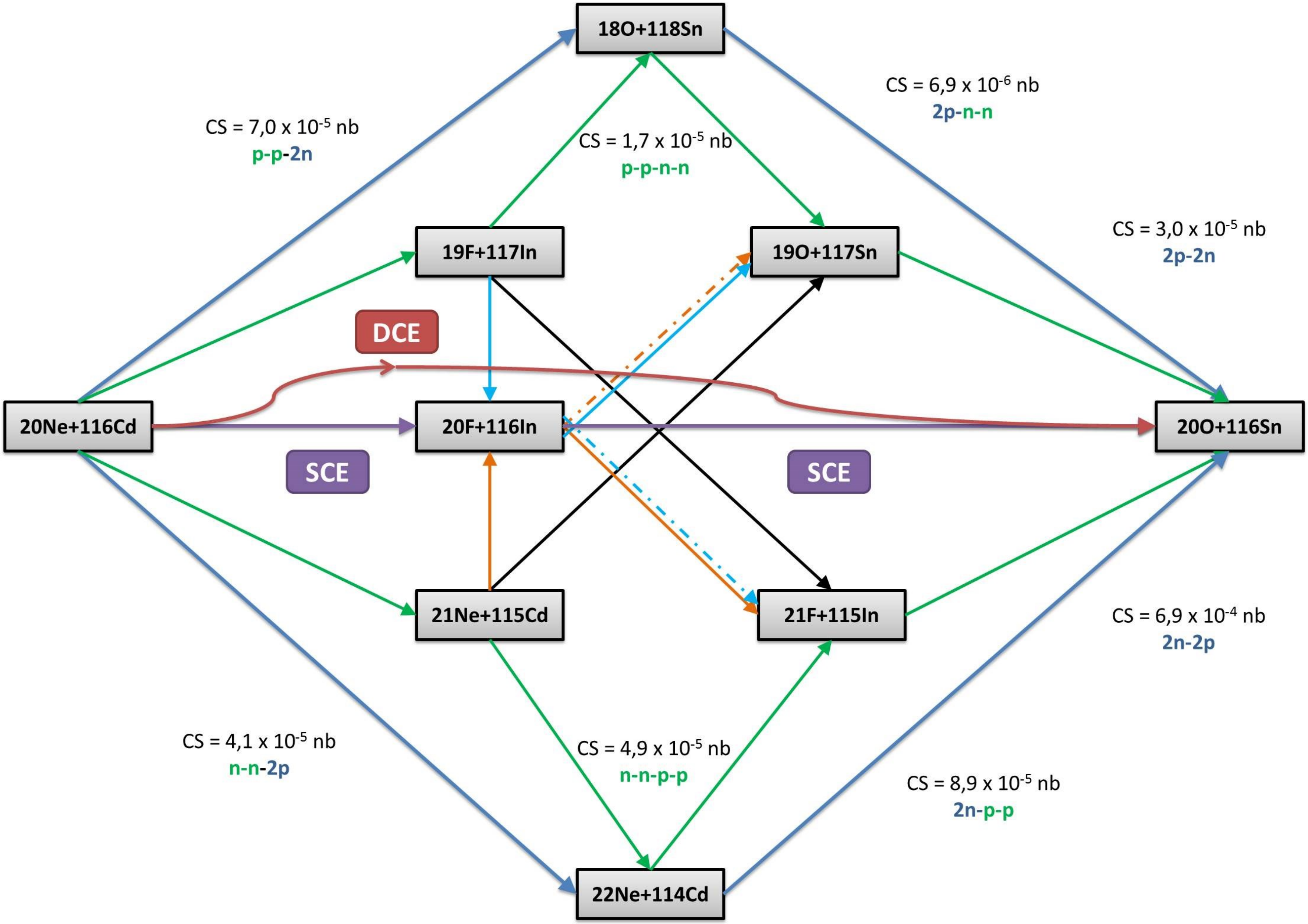
Outlooks:

- Include other waves in the microscopic cluster calculations
- Enlarge the model space for higher energy transitions ($d_{3/2}$)
- Describe high excited states of the projectile.
- Study the 2p and np transfers to study the pairing correlations.



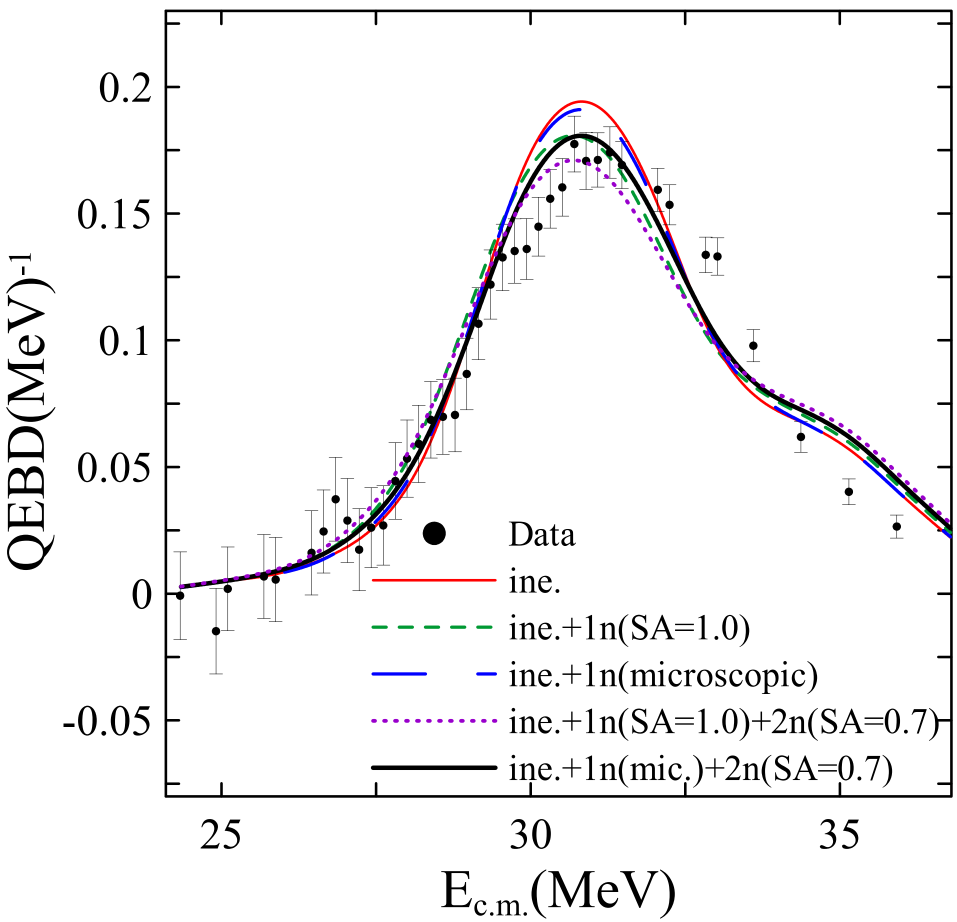
$^{116}Cd_{gs}(^{20}Ne_{gs}, ^{18}O_{gs})^{118}Sn_{gs}$
Direct_IBM-2 (σ_{theo} = 23.4 nb)
Direct_jj45pna (σ_{theo} = 26.1 nb)
Seq_jj45pna (σ_{theo} = 4.6 nb)
σ_{exp} = (33 +/- 11)
$4° < \theta < 14°$
$d\sigma/d\Omega$ (mb/sr)
10^{-2}
10^{-3}
10^{-4}
10^{-5}
10^{-6}
0 2 4 6 8 10 12 14 16 18 20
$\theta_{c.m.}$ (deg.)

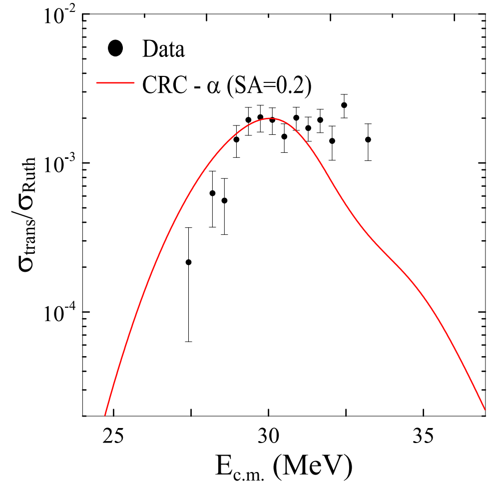
REACTIONS SCHEME CONCERNING THE $^{116}Cd(^{20}Ne, ^{20}O)\,^{116}Sn$

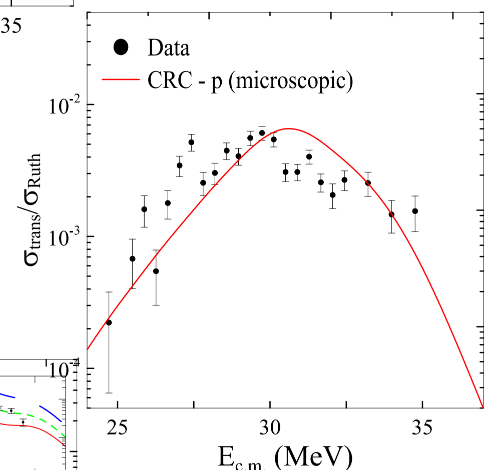


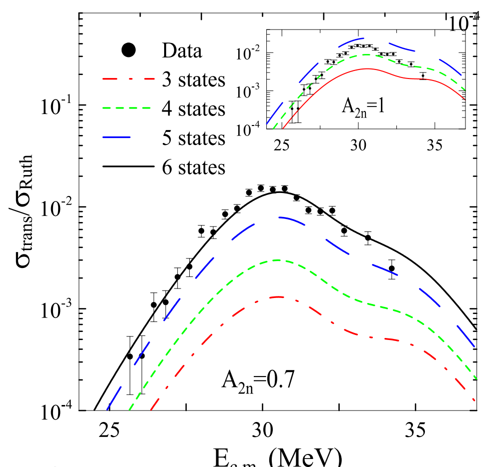
Very recent results

$^{18}O + ^{63}Cu$ @ 161^0









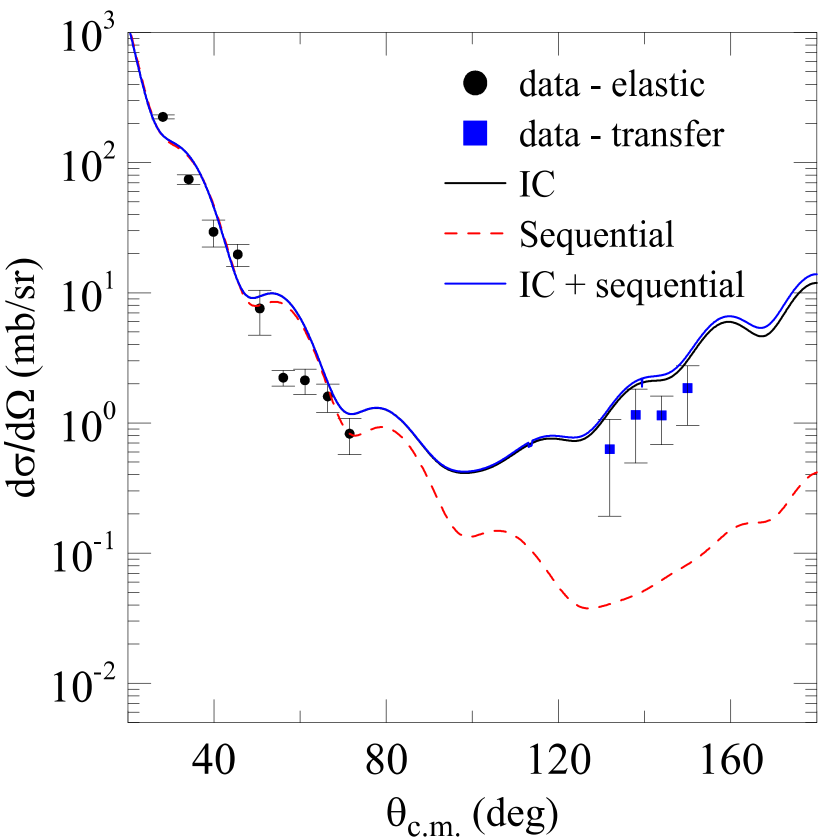
B(E2) ^{65}Cu: 185 e^2fm^4

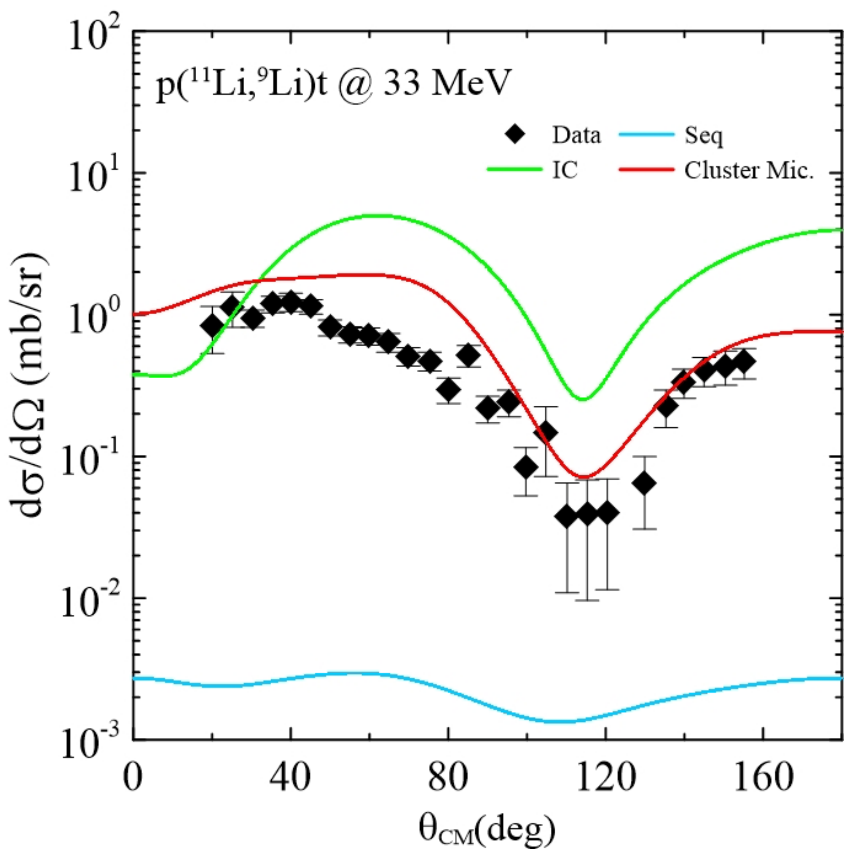
B(E2) ^{66}Ni: 600 e^2fm^4

E. Crema et al. PRC98.046614 (2018)

Very recent results

$^{7}Be + {}^{9}Be$ @ 23.1 MeV





2n pairing correlation is not an exclusive property of the inert-core + 2n configuration

Working group

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Thank you

Nuclear mean field and residual interaction

original Hamiltonian $H = T^{(1)} + V^{(2)} + V^{(3)}$

add and subtract a 1-body "mean field potential" $V^{(1)}$
(derive from Hartree-Fock theory)

$$H = T^{(1)} + V^{(1)} + V^{(2)} + V^{(3)} - V^{(1)}$$

regroup and separate terms

$H = H_{mf} + H_{res}$ complete many-body Hamiltonian

$H_{mf} = T^{(1)} + V^{(1)}$ mean field Hamiltonian, 1-body central field

$H_{res} = V^{(2)} + V^{(3)} - V^{(1)}$ residual interaction, "small"