

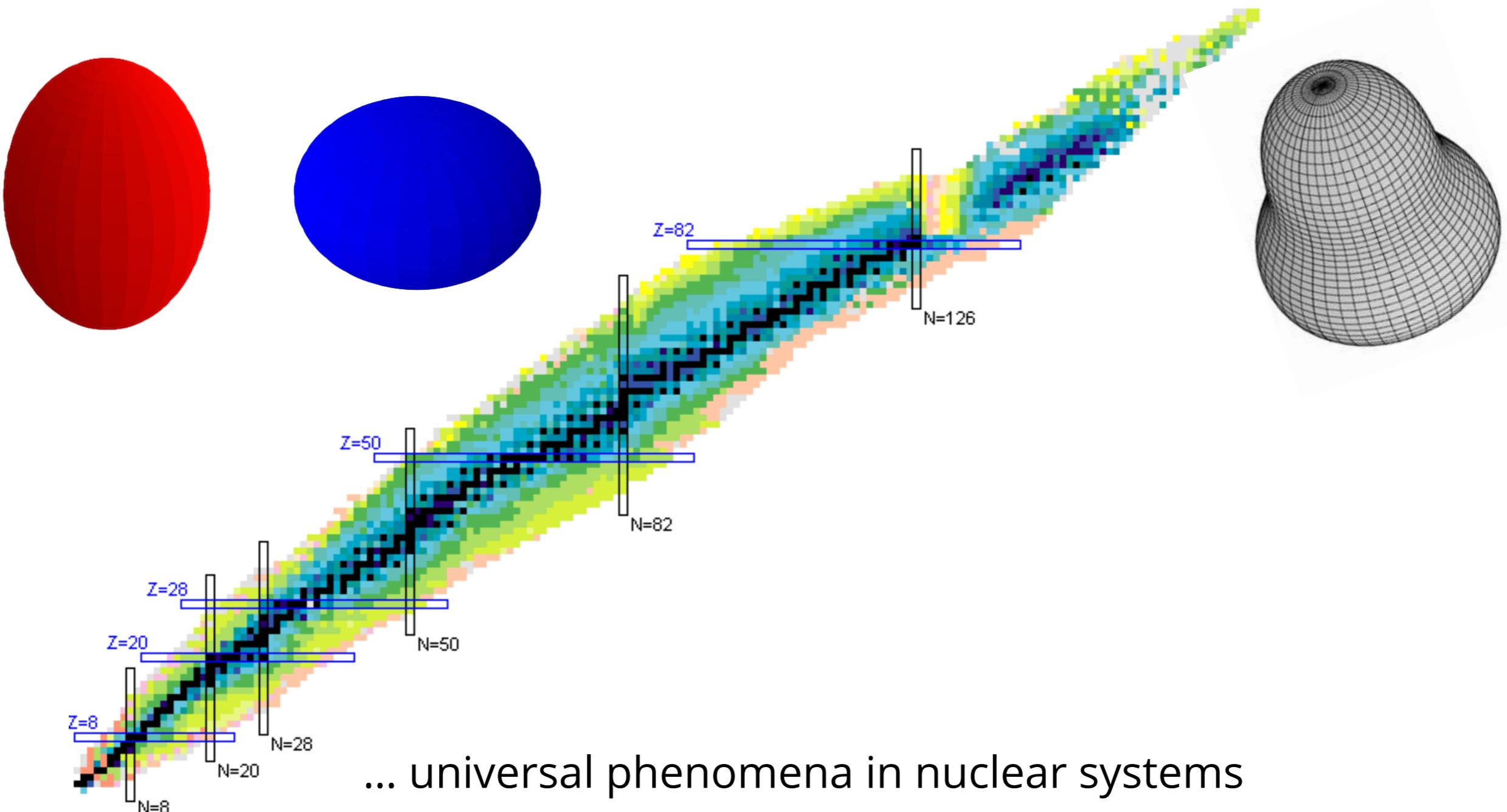
Venice, May 2019

# Coexistence and evolution of shapes: mean-field-based interacting boson model

*Kosuke Nomura - U. Zagreb*



# Variety of shapes and excitations

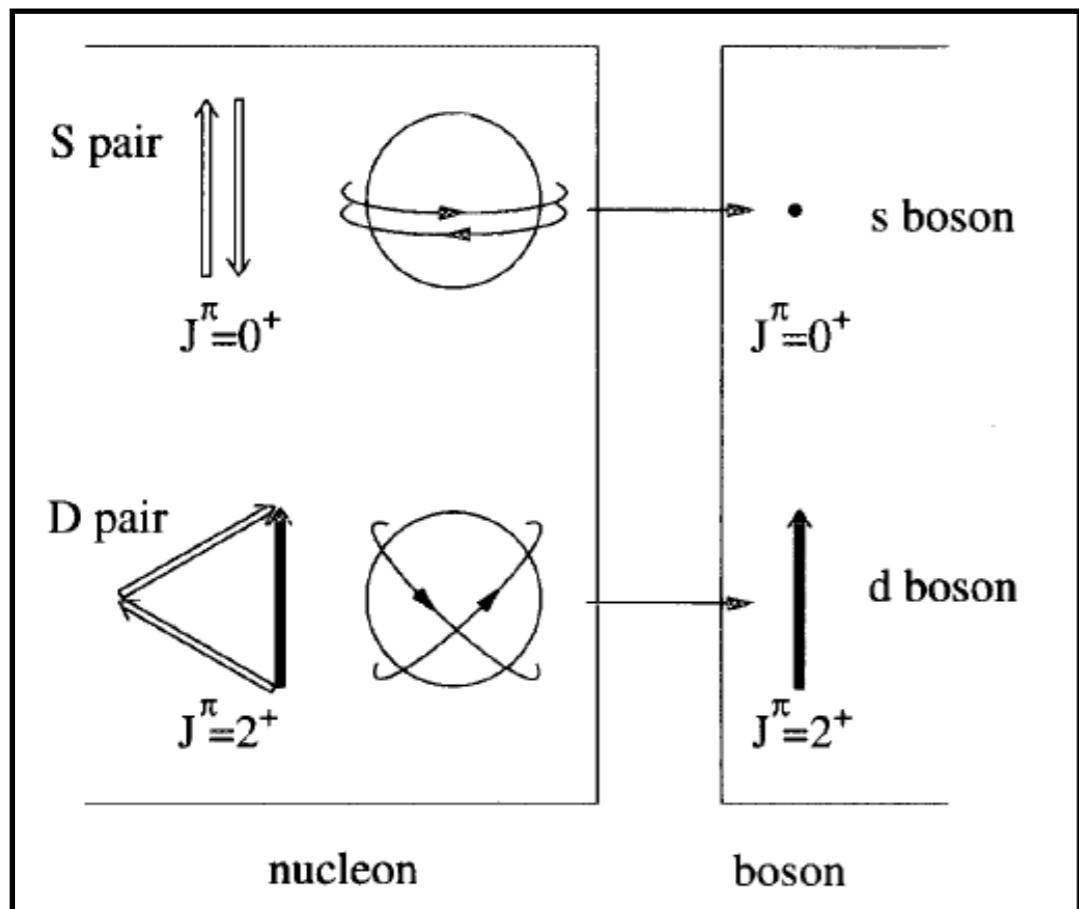


A unified framework: IBM from DFT

# Interacting boson model (IBM) - synopsis

A. Arima, F. Iachello (1974)

- Pair of valence nucleons represented by **boson**  
→ drastic reduction of Hilbert space
- Use of group theory (U(5), SU(3), SO(6))



## Dimension:

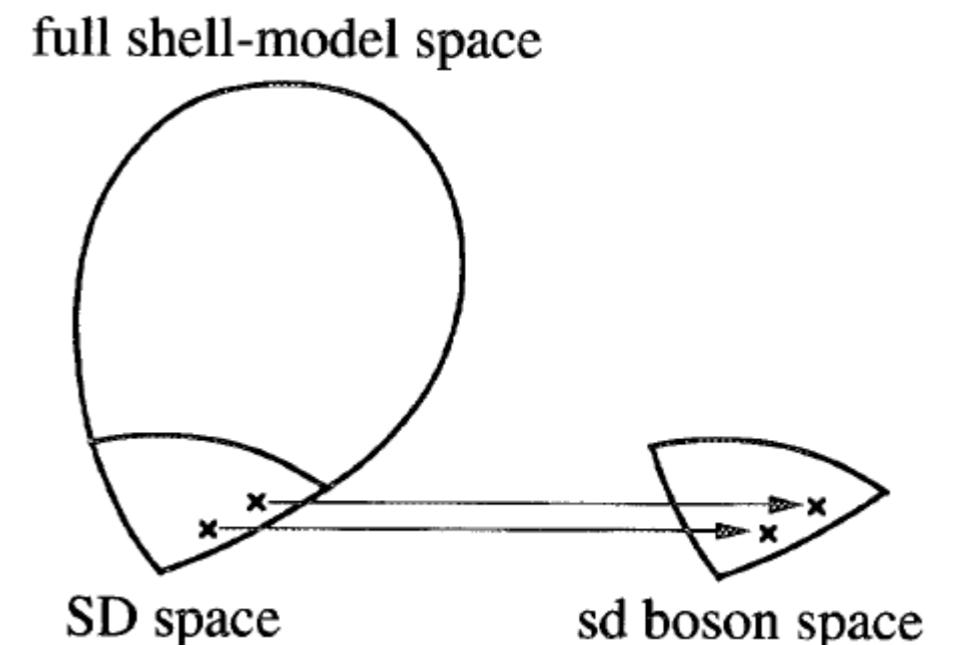
e.g.,  $^{154}\text{Sm}$ : 22 valence nucleons  
(12 protons + 10 neutrons)

- Shell model:  $O(10^{14})$  2+ states
- IBM:  $26$  2+ states

IBM used to be a purely phenomenological model!

# Microscopic study of IBM from nucleons

- from shell model
  - OAI mapping (1978)
- applied to vibrational and  $\gamma$ -unstable nuclei
- general cases (strongly-deformed nuclei) ?
- from DFT mean field (2008):

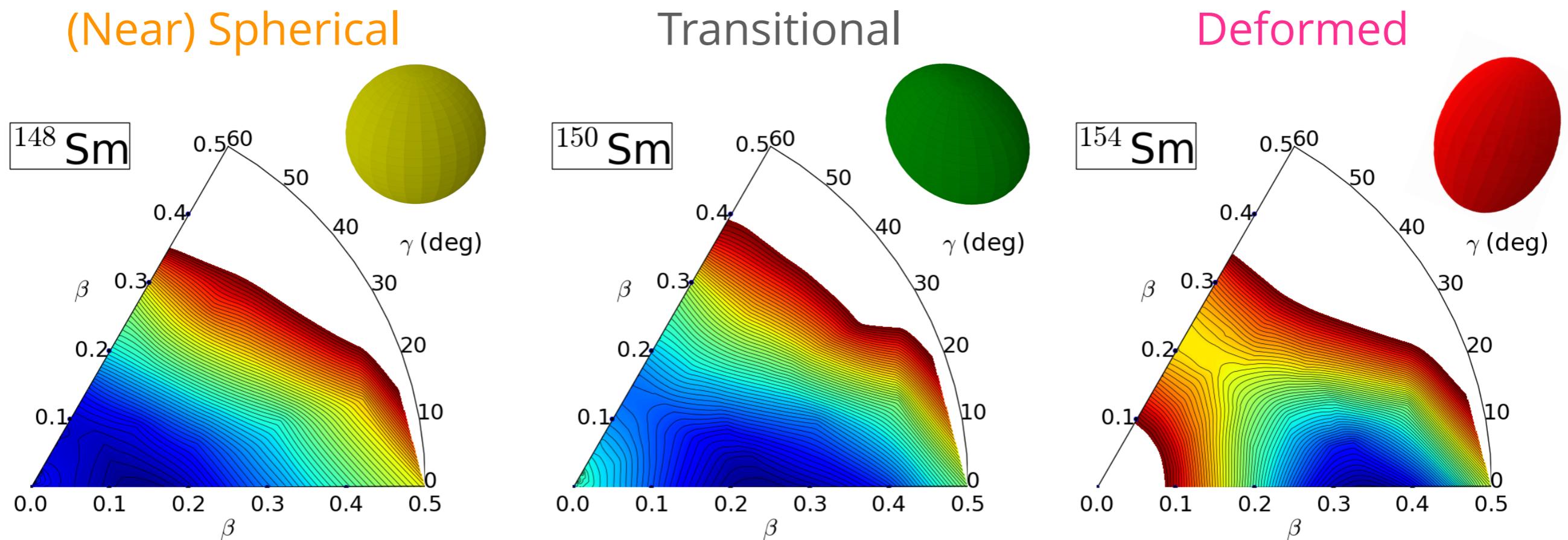


## Mean-Field Derivation of the Interacting Boson Model Hamiltonian and Exotic Nuclei

Kosuke Nomura,<sup>1</sup> Noritaka Shimizu,<sup>1</sup> and Takaharu Otsuka<sup>1,2,3,4</sup>

# Density functional theory: self-consistent mean field

- ... universal microscopic description of intrinsic properties in arbitrary nuclei
- ... calculation of spectra within DFT is demanding...

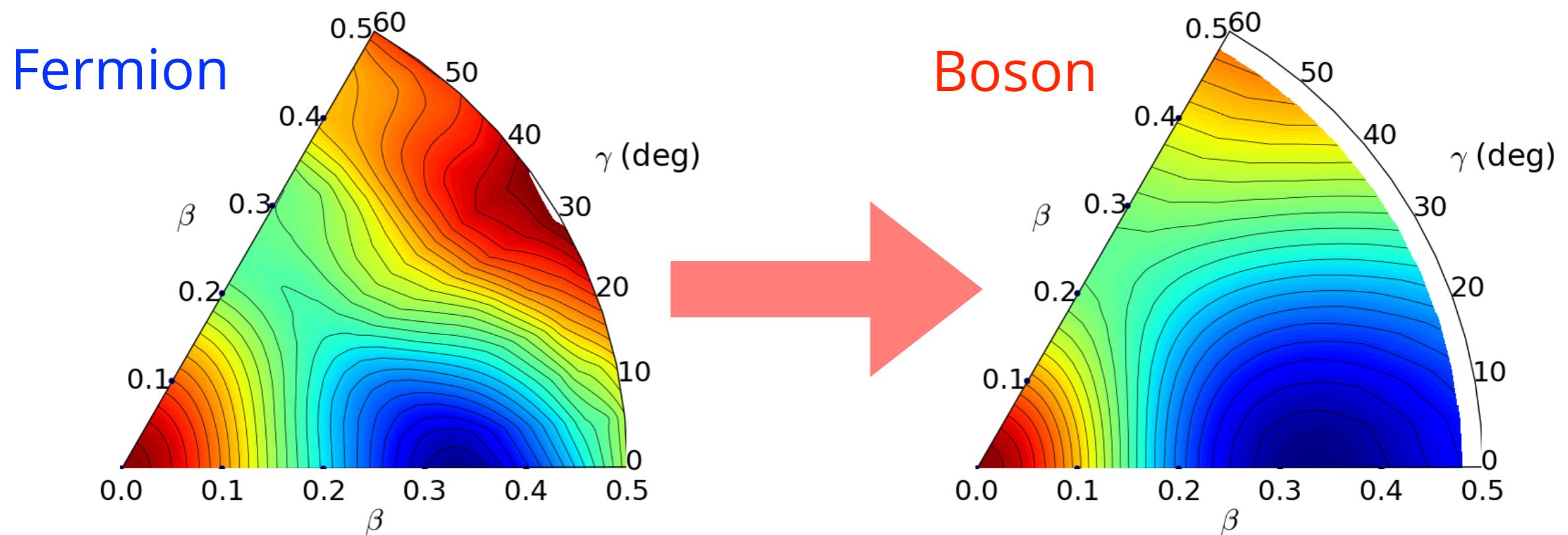


*from HFBCS calc. with Skyrme SkM\**  
*EDF +  $\delta$ -type pairing force*

# SCMF-to-IBM mapping

1. SCMF potential energy surface

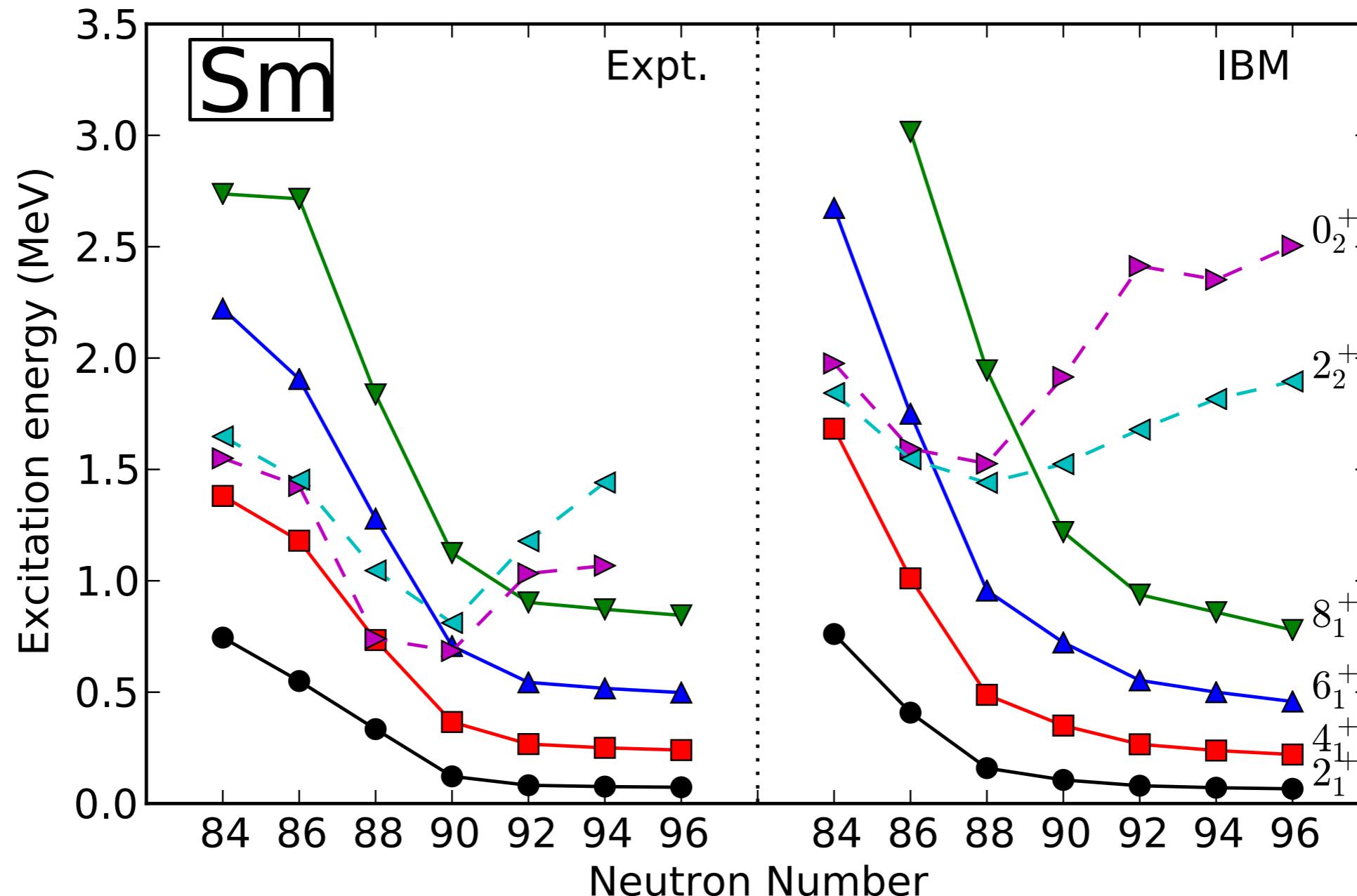
2. ... mapped onto the expectation value of the IBM Hamiltonian in the boson condensate state



3. Diagonalise the mapped IBM Hamiltonian in lab. frame  
→ Observables (energies & electromagnetic transitions)

# Calculated spectra

... unified description of vibrational and rotational spectra  
... no phenomenological adjustment to experiment



# A summary of major developments and applications

- Microscopic formulation of IBM for rotational nuclei  
*K.N., T. Otsuka, N. Shimizu, & L. Guo, PRC83, 041302R (2011)*
- Microscopic realisation of y-softness and higher-order interactions in IBM  
*K.N., N. Shimizu, D. Vretenar, T. Nikšić, & T. Otsuka, PRL108, 132501 (2012)*
- Systematic study of shape evolution in neutron-rich nuclei with A~100  
*K.N., N. Shimizu, & T. Otsuka, PRC81, 044307 (2010); M. Albers, K.N., et al., PRL108, 062701 (2012); K.N., R. Rodríguez-Guzmán, & L. M. Robledo, PRC94, 044314 (2016), etc.*
- Implement intruder configurations for shape coexistence  
*K.N., R. Rodríguez-Guzmán, & L. M. Robledo, PRC86, 034322 (2012); [Review] K.N., T. Otsuka, & P. Van Isacker, JPG43, 024008 (2016); K.N., & J. Jolie, PRC98, 024303 (2018), etc.*
- Microscopic description of octupole shapes in actinides and rare-earth nuclei  
*K.N., D. Vretenar, T. Nikšić, & B.-N. Lu, PRC89, 024312 (2014), etc.*
- Particle-core coupling for odd nuclei  
*K.N., T. Nikšić, & D. Vretenar, PRC93, 054305 (2016); PRC97, 024317 (2018); K.N., R. Rodríguez-Guzmán, & L. M. Robledo, PRC99, 034308 (2019), etc.*
- Two-nucleon transfers as a signature of shape phase transition  
*K.N., & Y. Zhang, PRC99, 024324 (2019)*

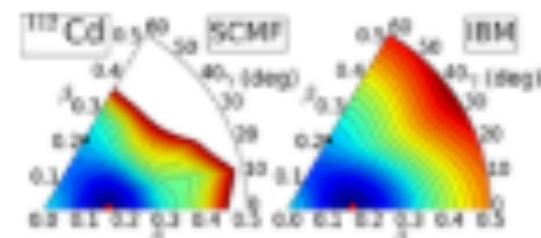
# Intruder v.s. vibrational excitations in Cd

Editors' Suggestion

## Structure of even-even cadmium isotopes from the beyond-mean-field interacting boson model

K. Nomura and J. Jolie

Phys. Rev. C **98**, 024303 (2018) – Published 1 August 2018



The cadmium isotopes have been a classic testing ground for vibrational models with various multiphonon states identified.

The present calculation follows a well-defined prescription in which the potential energy surface (PES) from a microscopic calculation is input to an interacting boson model calculation by matching that PES. This reduces the number of parameters and allows one to distinguish the vibrational states from cross-shell intruder states.

# Cadmium isotopes: a long-standing problem

$^{112}\text{Cd}$

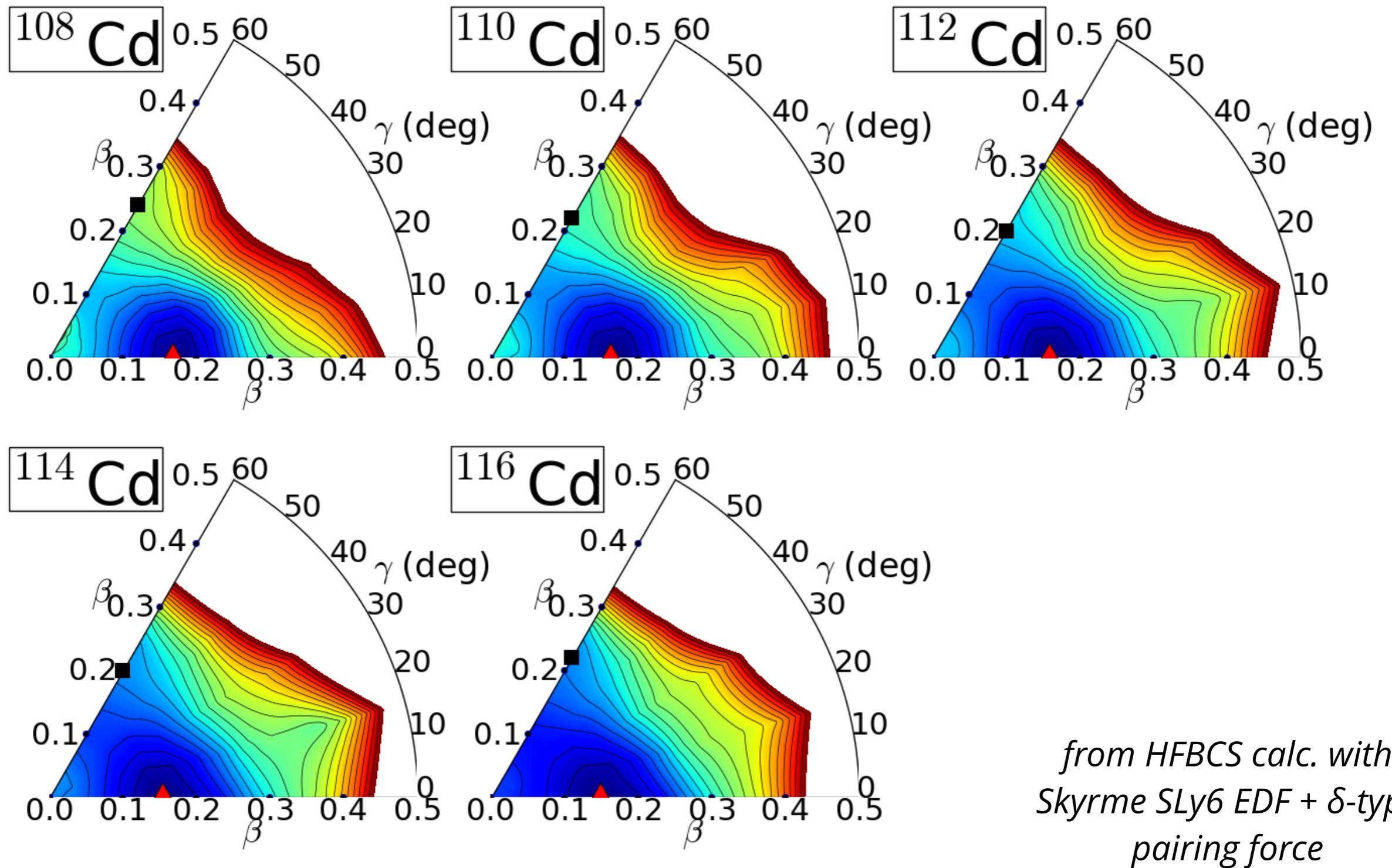
Additional low-spin  
states observed at low  
energy: intruder or  
vibrational?

$0^+ \underline{\quad 1433 \quad} \quad 4^+ \underline{\quad 1416 \quad} \quad 2^+ \underline{\quad 1468 \quad}$   
 $\underline{\quad 0^+ \quad 1224 \quad} \quad \underline{\quad 2^+ \quad 1312 \quad}$

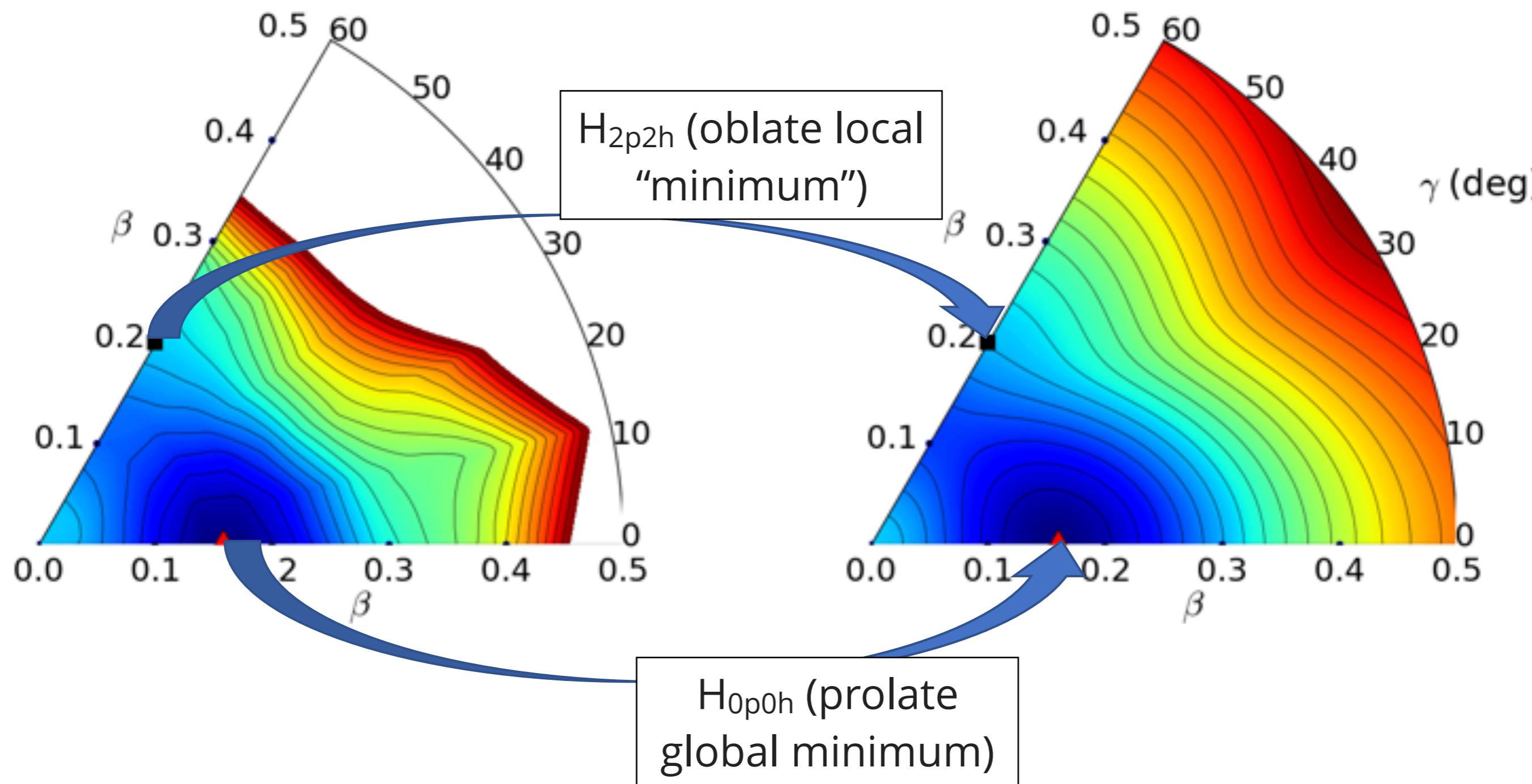
$2^+ \underline{\quad 618 \quad}$

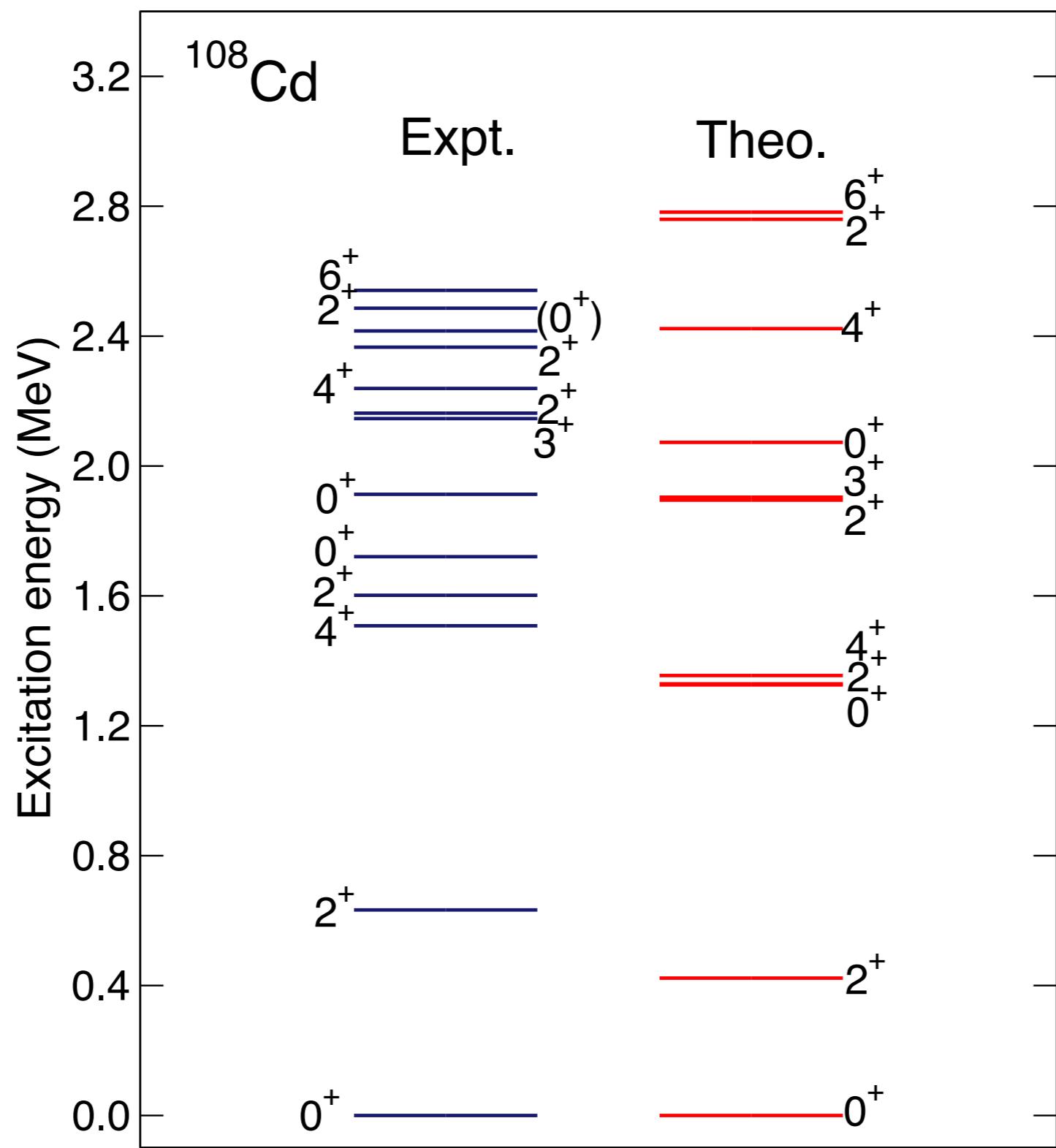
$0^+ \underline{\quad 0 \quad}$

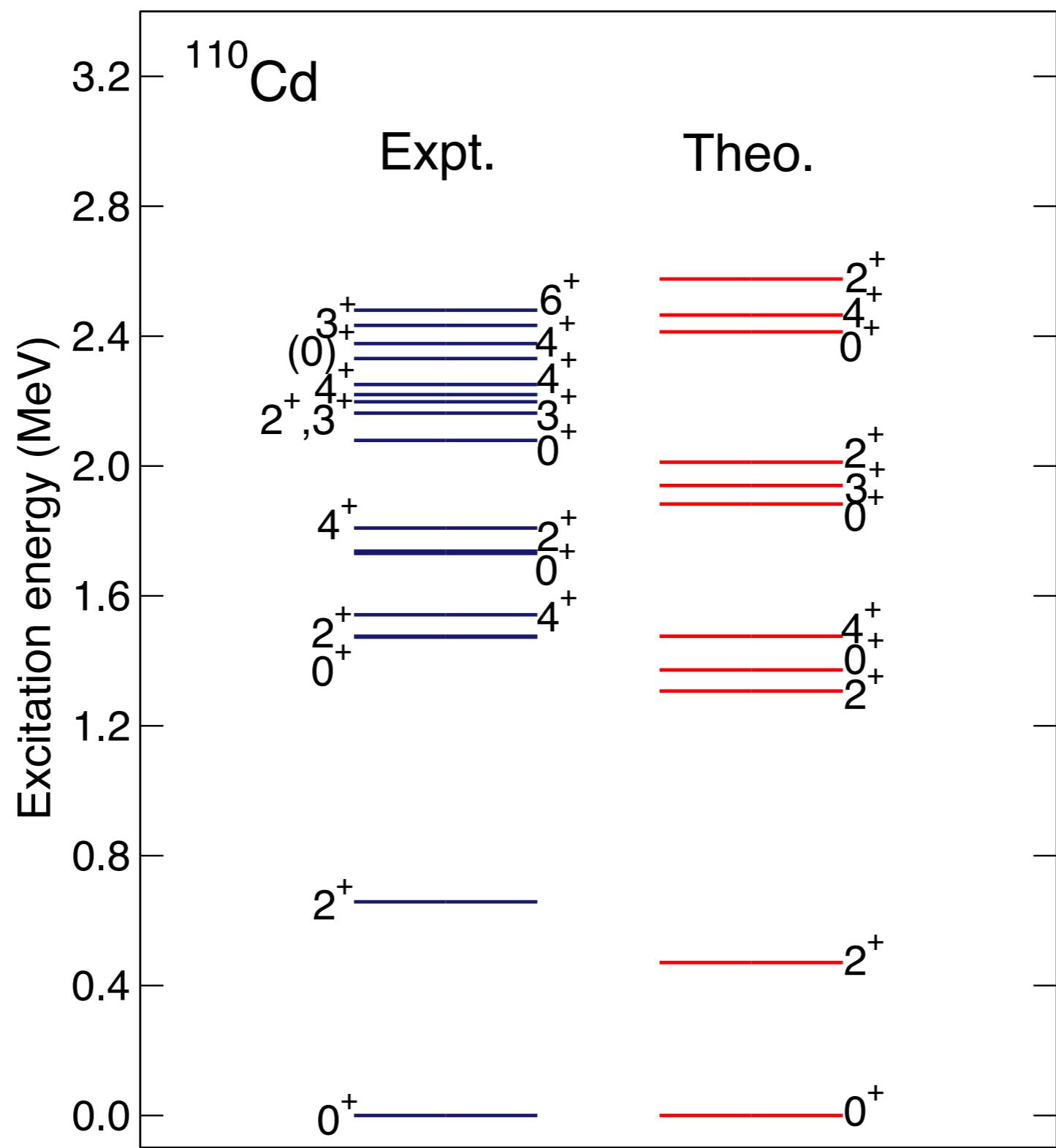
# Self-consistent mean-field energy surfaces

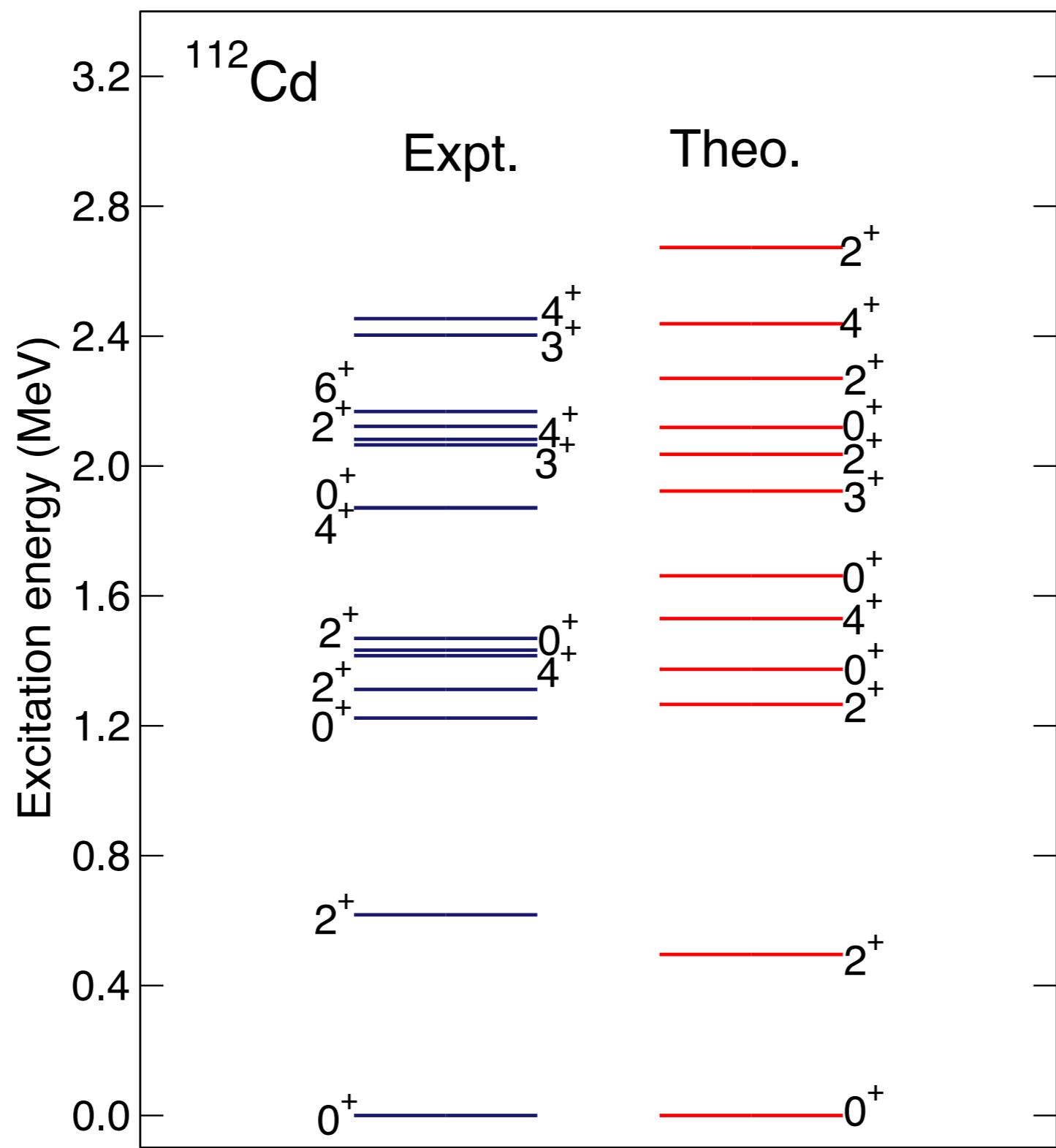


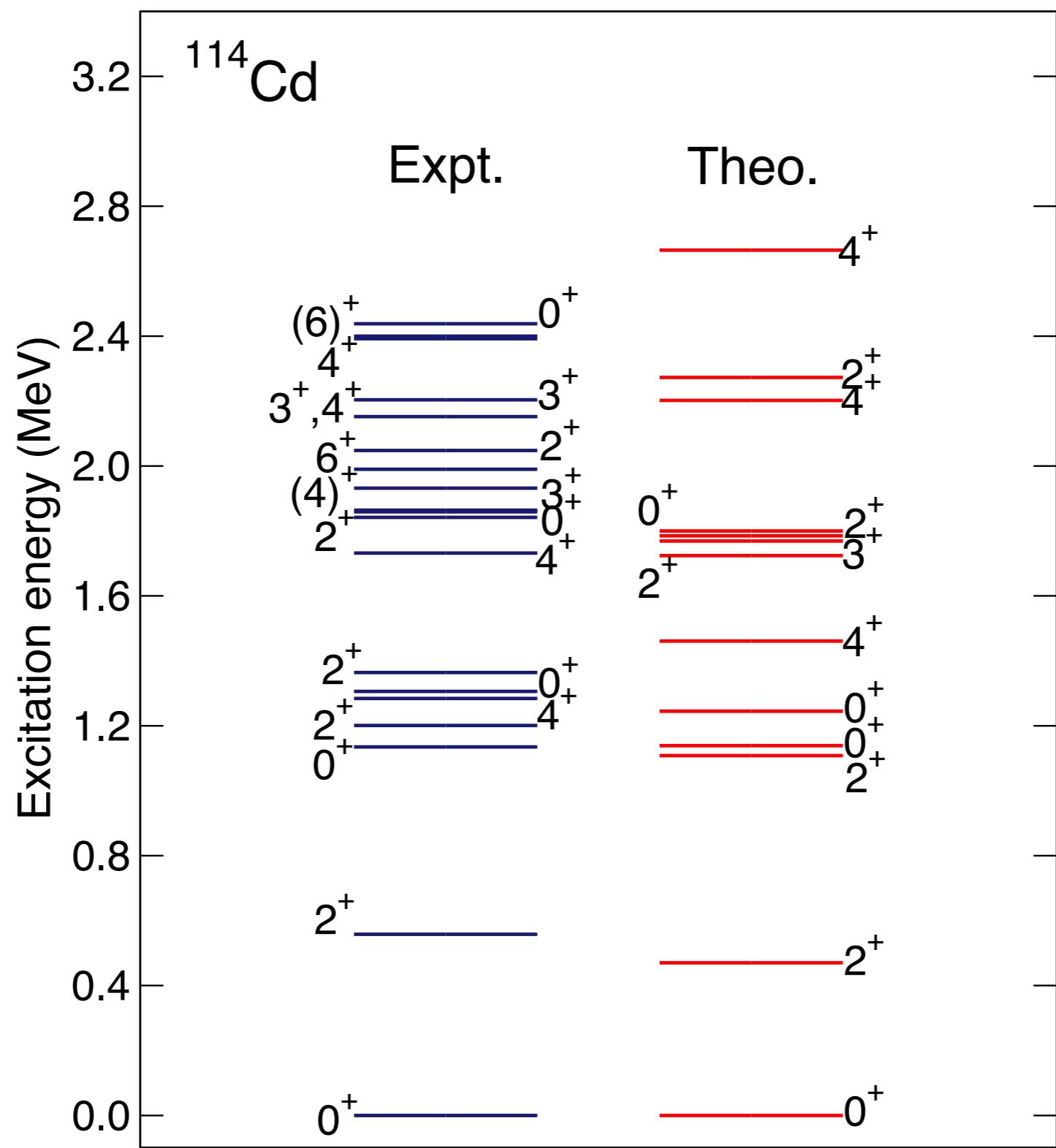
# IBM with configuration mixing











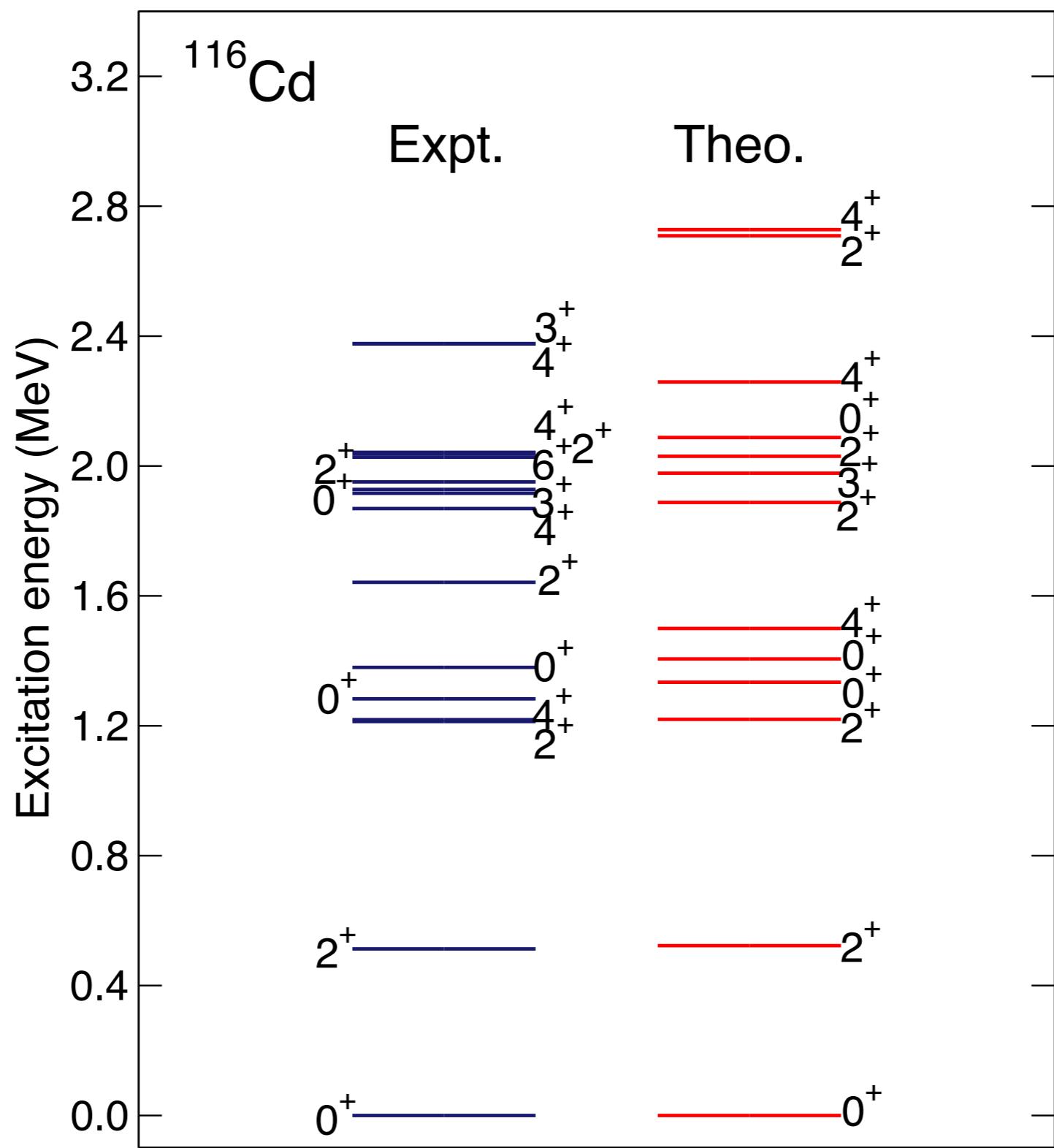


TABLE II: Comparison between experimental and theoretical  $B(E2; J_i^+ \rightarrow J_f^+)$  values in Weisskopf units.

$J_i^+$	$J_f^+$	$^{108}\text{Cd}$		$^{110}\text{Cd}$		$^{112}\text{Cd}$		$^{114}\text{Cd}$		$^{116}\text{Cd}$	
		Expt	Theo	Expt	Theo	Expt	Theo	Expt	Theo	Expt	Theo
2 <sub>1</sub>	0 <sub>1</sub>	26.6(3)	29	27.0(8)	33	30.3(2)	39	31.1(19)	46	33.5(12)	36
0 <sub>2</sub>	2 <sub>1</sub>	-	1.4	<40	2.8	51(14)	4.5	27.4(17)	2.9	0.79(22)	9.5
2 <sub>2</sub>	0 <sub>1</sub>	1.8(3)	1.1	0.68(14)	1.7	0.65(11)	2.4	0.48(6)	3.2	1.11(18)	1.9
2 <sub>2</sub>	2 <sub>1</sub>	17(5)	6	19(4) or 30(5)	11	39(7)	18	22(6)	21	25(10)	27
2 <sub>2</sub>	0 <sub>2</sub>	-	1.7	1.35(20)	1.2	-	2.6	3.4(7)	11	-	1.7
4 <sub>1</sub>	2 <sub>1</sub>	41(6)	39	42(9)	47	63(8)	55	62(4)	65	56(14)	51
0 <sub>3</sub>	2 <sub>1</sub>	-	0.003	<7.9	0.10	0.0121(17)	0.82	0.0026(4)	4.4	30(6)	1.6
0 <sub>3</sub>	2 <sub>2</sub>	-	13	<1680	29	99(16)	42	127(16)	39	-	96
2 <sub>3</sub>	0 <sub>1</sub>	-	0.02	0.28(4)	0.051	0.88(17)	0.085	0.33(4)	0.072	1.11(18)	0.25
2 <sub>3</sub>	2 <sub>1</sub>	-	0.02	0.7 <sup>+3</sup> <sub>-4</sub>	0.068	0.12(7)	0.14	<0.045	0.17	6.2 <sup>+22</sup> <sub>-26</sub>	0.0083
2 <sub>3</sub>	0 <sub>2</sub>	-	16	29(5)	20	120(50)	25	65(9)	32	-	2.8
2 <sub>3</sub>	2 <sub>2</sub>	-	0.17	<8	0.46	-	0.79	-	0.22	-	7.8
2 <sub>3</sub>	0 <sub>3</sub>	-	0.56	-	0.43	-	0.98	-	1.9	86 <sup>+24</sup> <sub>-30</sub>	76
3 <sub>1</sub>	2 <sub>1</sub>	-	1.5	0.85(25)	2.5	1.8(5)	3.3	-	4.2	2.6(7)	2.0
3 <sub>1</sub>	2 <sub>2</sub>	-	30	22.7(69)	38	64(18)	47	-	55	61(17)	39
3 <sub>1</sub>	4 <sub>1</sub>	-	3.9	2.4 <sup>+9</sup> <sub>-8</sub>	6.8	25(8)	10	-	12	18(10)	11
3 <sub>1</sub>	2 <sub>3</sub>	-	2.3	<5	1.9	-	1.6	-	1.9	-	3.6
4 <sub>2</sub>	2 <sub>1</sub>	-	0.035	0.14(6)	0.083	0.9(3)	0.14	0.50(5)	0.32	3.0(7)	0.22
4 <sub>2</sub>	2 <sub>2</sub>	-	15	22(10)	23	58(17)	31	32(4)	45	230(130)	44
4 <sub>2</sub>	4 <sub>1</sub>	-	4.8	10.7 <sup>+49</sup> <sub>-48</sub>	8.6	24(8)	13	17(6)	16	150(90)	18
4 <sub>2</sub>	2 <sub>3</sub>	-	1.4	<0.5	0.98	59(20)	0.79	119(12)	5.9	-	31
6 <sub>1</sub>	4 <sub>1</sub>	-	39	62(18)	49	-	59	119(15)	72	110(46)	58
8 <sub>1</sub>	6 <sub>1</sub>	-	34	80(22)	45	-	58	86(28)	73	-	61

TABLE IV. Comparison between experimental and theoretical  $\rho^2(E0; J_i^+ \rightarrow J_f^+)$  values. The experimental  $\rho^2(E0)$  values are not known for  $^{108}\text{Cd}$  and  $^{116}\text{Cd}$ .

	$J_i^+$	$J_f^+$	$\rho^2(E0) \times 10^3$	
			Exp	Theory
$^{110}\text{Cd}$	$0_2$	$0_1$	$<31(5)^{\text{a}}$	37
	$0_3$	$0_1$	$<11^{\text{b}}$	1.1
	$2_2$	$2_1$	$20(15)^{\text{c}}$	1.1
	$2_3$	$2_1$	$9(8)^{\text{a}}$	26
	$4_3$	$4_1$	$106_{-91}^{+98\text{a}}$	0.44
$^{112}\text{Cd}$	$0_2$	$0_1$	$34(9)^{\text{d}}$	36
	$0_3$	$0_1$	$0.87(5)^{\text{d}}$	8.6
	$0_3$	$0_2$	$10.7(6)^{\text{d}}$	12
	$2_3$	$2_1$	$31(20)^{\text{c}}$	27
$^{114}\text{Cd}$	$0_2$	$0_1$	$19(2)^{\text{d}}$	12
	$0_3$	$0_1$	$1.83(13)^{\text{d}}$	44
	$0_3$	$0_2$	$0.65(5)^{\text{d}}$	100
	$0_4$	$0_1$	$0.9(4)^{\text{d}}$	8.8
	$2_2$	$2_1$	$<28^{\text{c}}$	0.25
	$2_3$	$2_1$	$38(5)^{\text{e}}$	22
	$2_3$	$2_2$	$22(6)^{\text{e}}$	1.1
	$2_4$	$2_2$	$<20^{\text{e}}$	57
	$3_2$	$3_1$	$<130^{\text{e}}$	35
	$4_2$	$4_1$	$67(10)^{\text{e}}$	0.38

# Particle-core coupling for odd-A nuclei

Interacting boson-fermion model (IBFM) Hamiltonian from DFT

$$\hat{H} = \hat{H}_B + \hat{H}_F + \hat{H}_{BF}$$

Input from DFT:

- (i) Potential energy surface for even-even core
- (ii) Spherical single-particle energies and occupation probabilities of odd particle

# IBFM with octupole degree of freedom

PHYSICAL REVIEW C **97**, 024317 (2018)

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## Signatures of octupole correlations in neutron-rich odd-mass barium isotopes

K. Nomura, T. Nikšić, and D. Vretenar

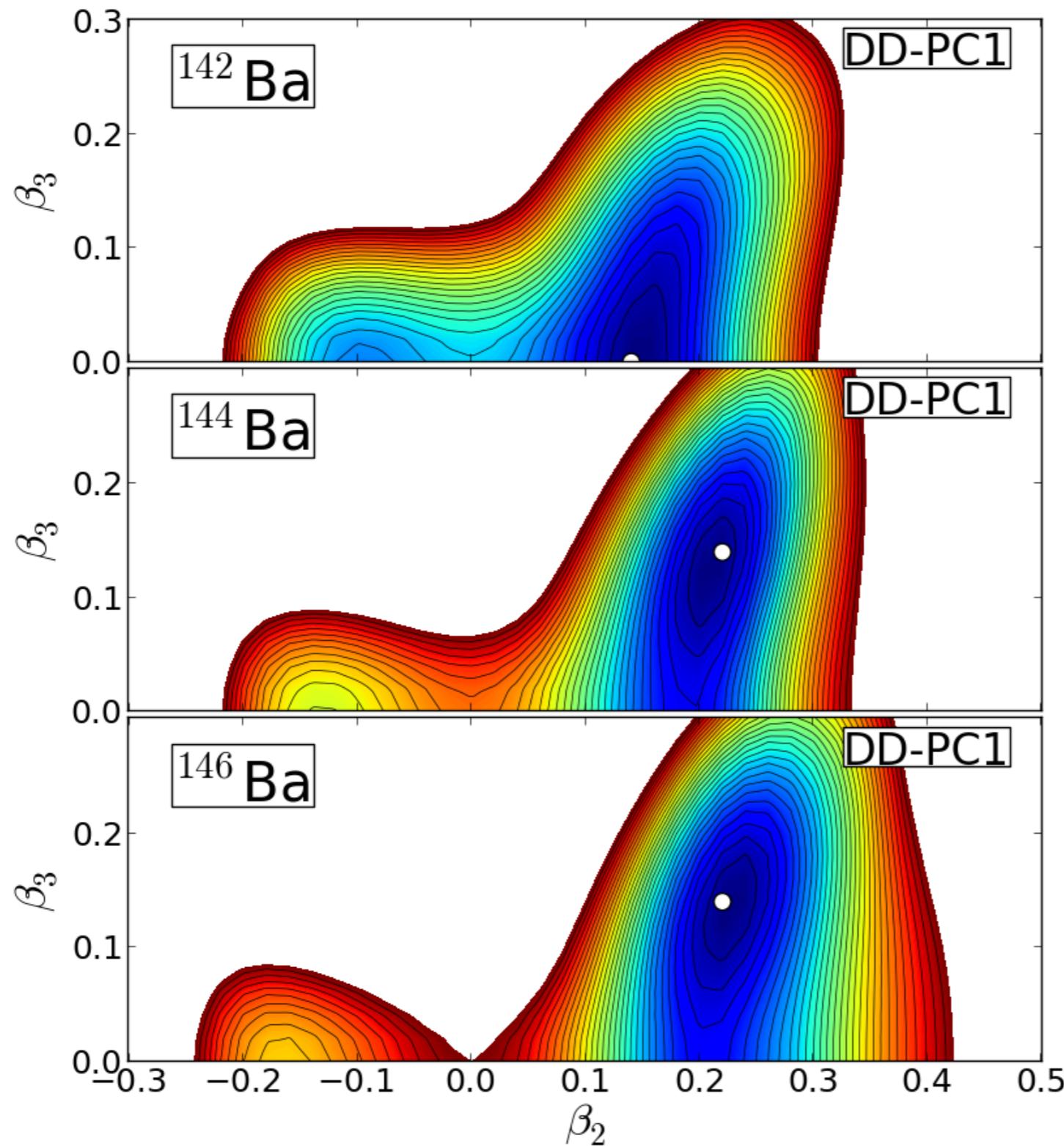
*Physics Department, Faculty of Science, University of Zagreb, 10000 Zagreb, Croatia*



(Received 27 November 2017; revised manuscript received 18 January 2018; published 16 February 2018)

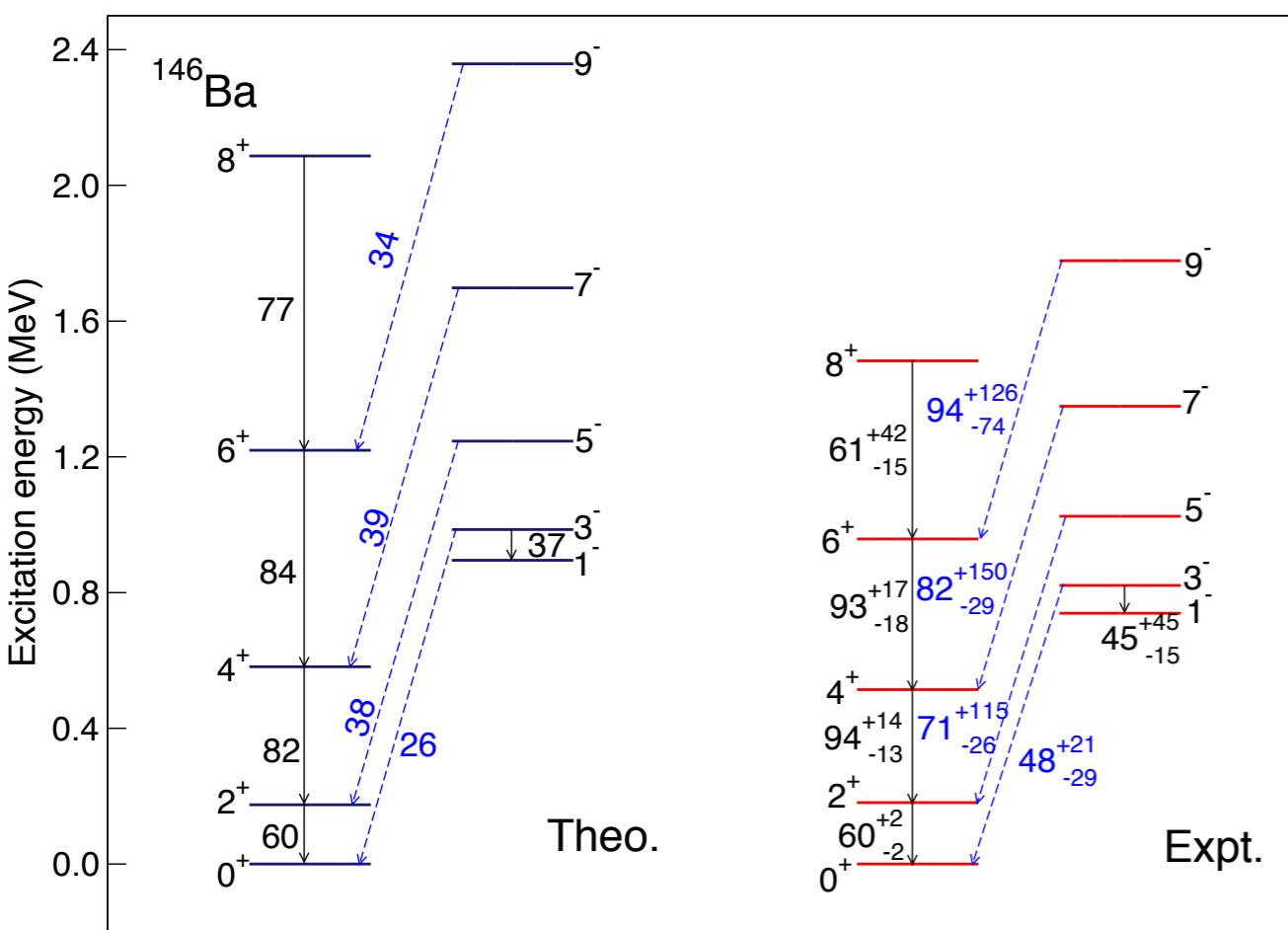
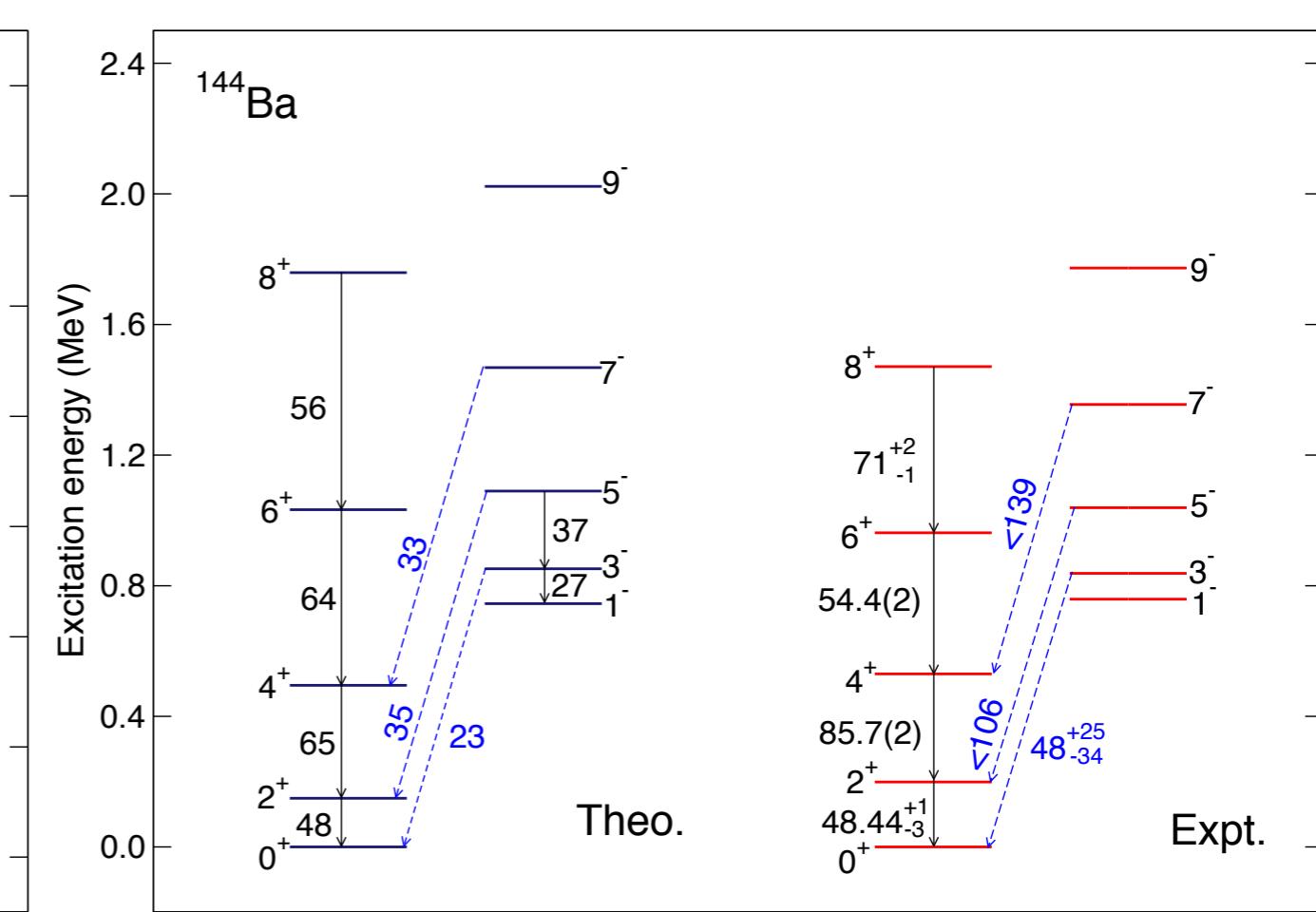
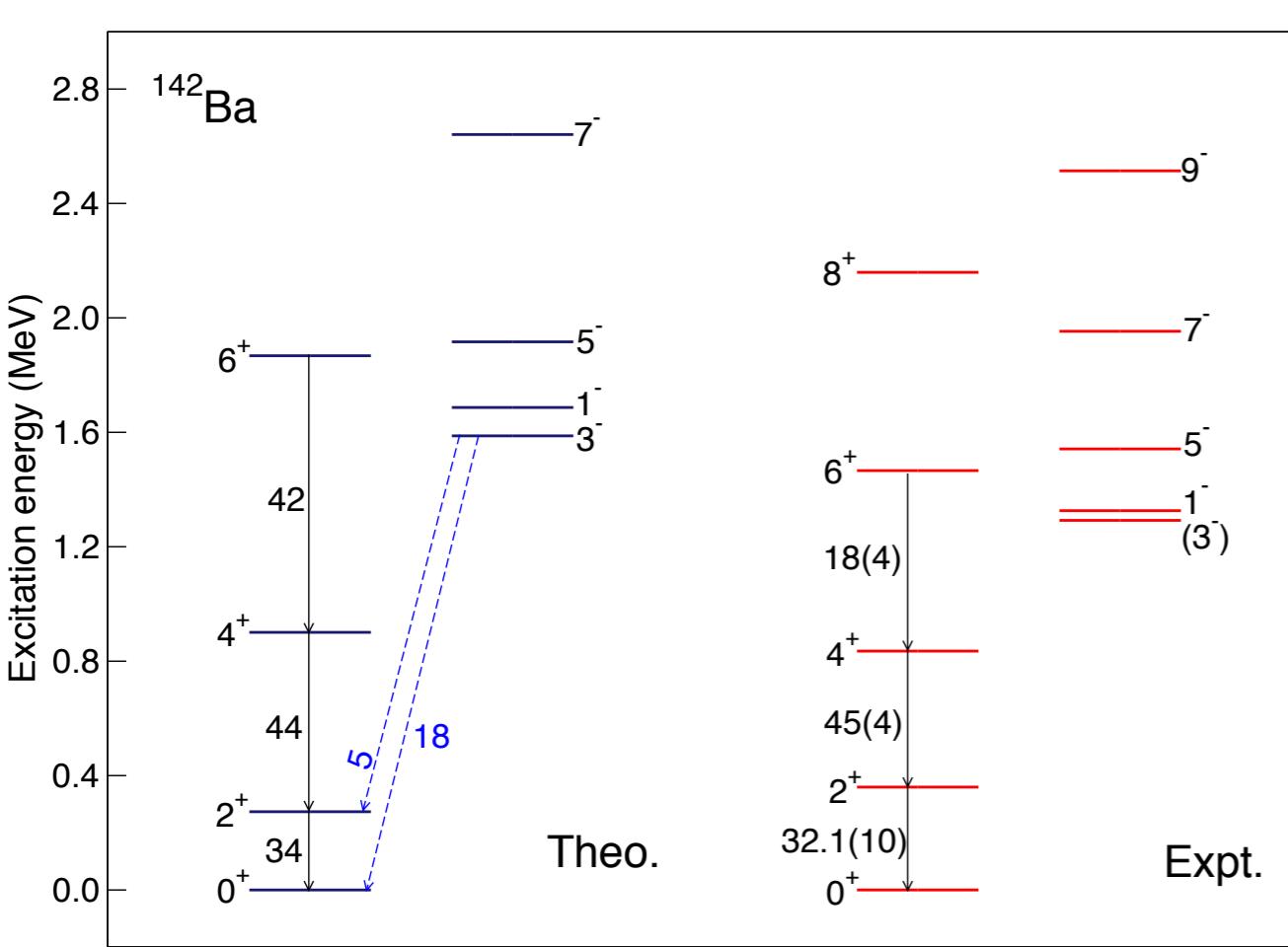
Octupole deformation and the relevant spectroscopic properties of neutron-rich odd-mass barium isotopes are investigated in a theoretical framework based on nuclear density functional theory and the particle-core coupling scheme. The interacting-boson Hamiltonian that describes the octupole-deformed even-even core nucleus, as well as the single-particle energies and occupation probabilities of an unpaired nucleon, are completely determined by microscopic axially symmetric  $(\beta_2, \beta_3)$ -deformation constrained self-consistent mean-field calculations for a specific choice of the energy density functional and pairing interaction. A boson-fermion interaction that involves both quadrupole and octupole degrees of freedom is introduced, and their strength parameters are determined to reproduce selected spectroscopic data for the odd-mass nuclei. The model reproduces recent experimental results for both even-even and odd-mass Ba isotopes. In particular, for  $^{145,147}\text{Ba}$  our results indicate, in agreement with recent data, that octupole deformation does not determine the structure of the lowest states in the vicinity of the ground state, and only becomes relevant at higher excitation energies.

# Quadrupole-octupole energy surfaces in even-even Ba



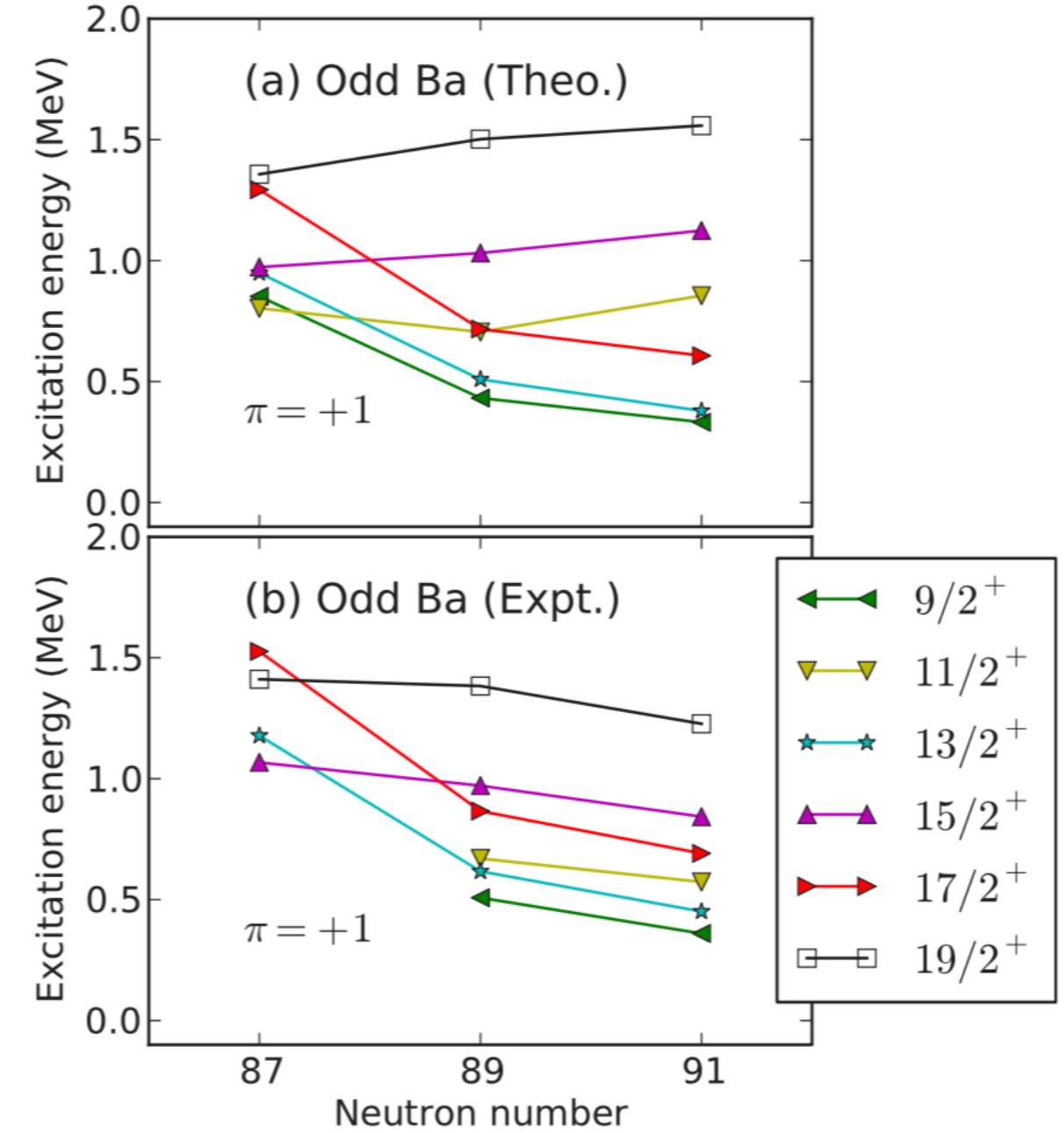
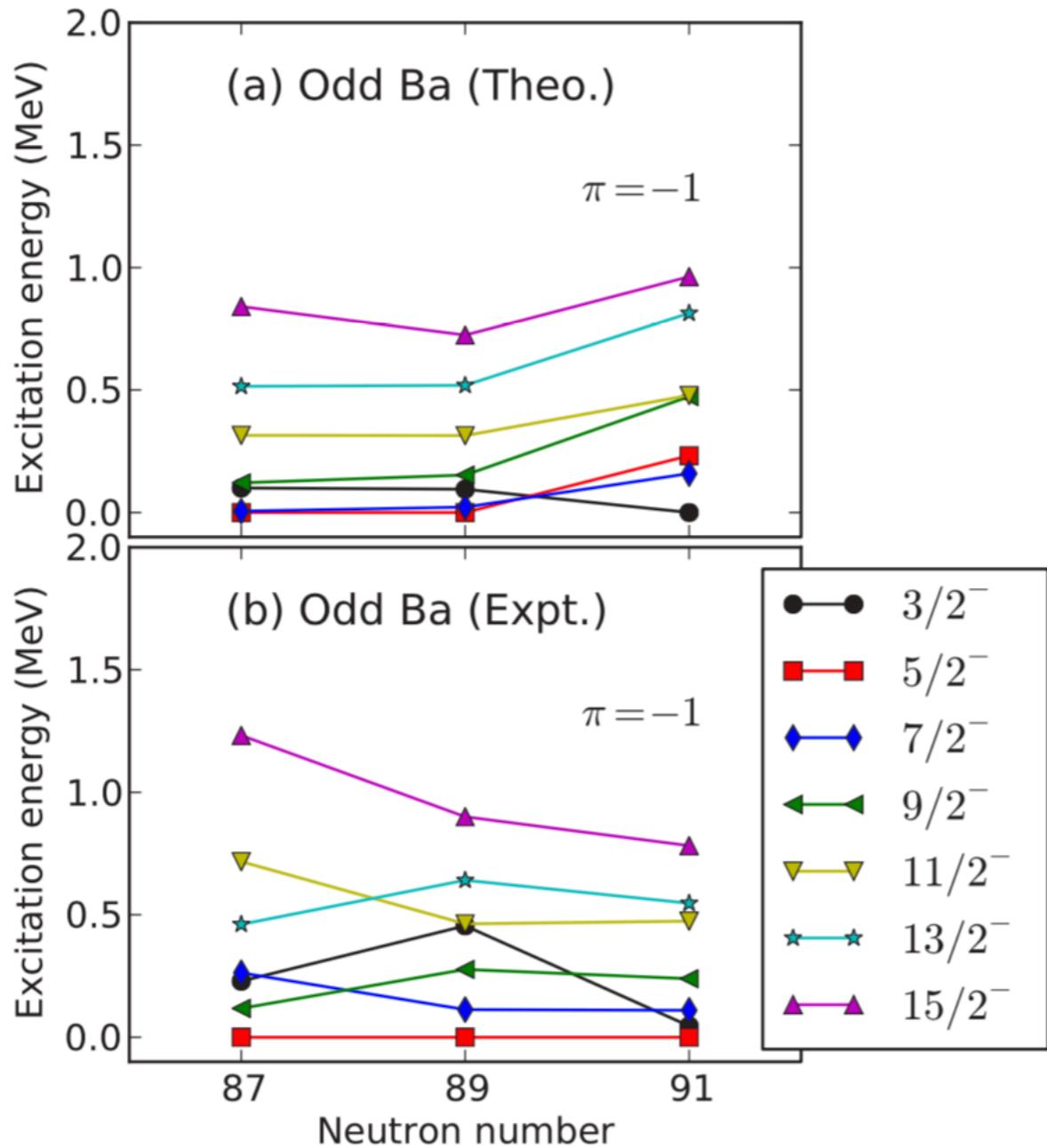
*from Relativistic Hartree-Bogoliubov calc. with separable pairing force and DD-PC1 EDF*

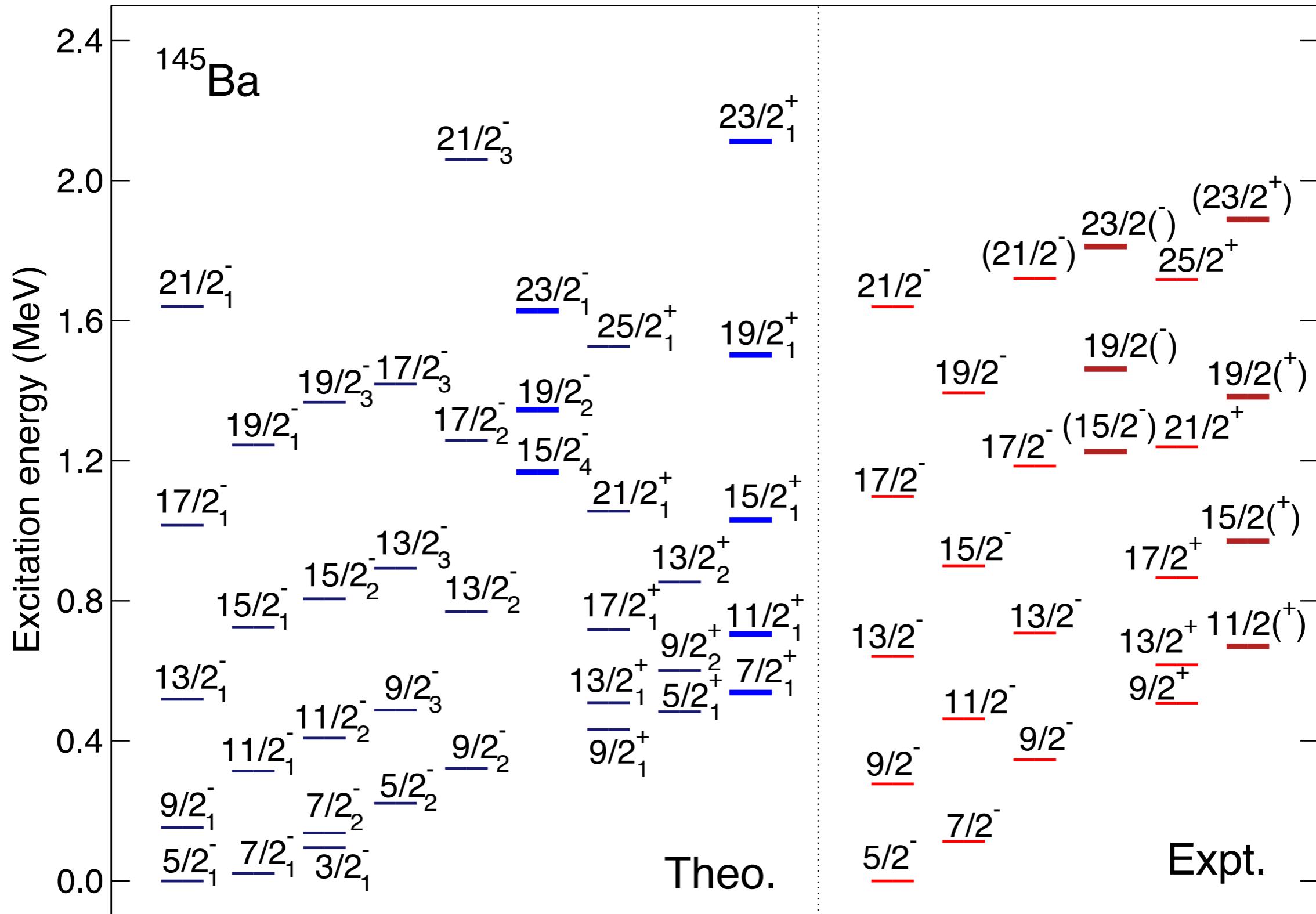
... mapped onto the IBM Hamiltonian that includes f ( $J=3^-$ ) boson



$B(E3)_{\text{th}} \approx 20\text{-}40 \text{ W.u.}$

# Excitation spectra in odd-A Ba





... predicts octupole (one-f boson) bands at medium spin & energy  
 ... large E3 transitions to g.s. (should be confirmed experimentally)

# DFT-based IBM - summary

- IBM is derived for general cases, and attains predictive power;
- allows for an accurate, systematic, and computationally feasible prediction of exotic nuclear shapes and spectroscopy (octupole, shape coexistence, odd nuclei, etc.)
- IBM and DFT mean-field can be “friends” (as an alternative DFT-based approach to nuclear spectroscopy)



# IBM with configuration mixing

Duval-Barrett (1981)

Normal ( $\pi(0p-0h)$ ) configuration

Intruder ( $\pi(2p-2h)$ ) configuration

$$\hat{H} = \hat{\mathcal{P}}_{0p0h} \hat{H}_{0p0h} \hat{\mathcal{P}}_{0p0h} + \hat{\mathcal{P}}_{2p2h} (\hat{H}_{2p2h} + \Delta) \hat{\mathcal{P}}_{2p2h} + \hat{H}_{\text{mix}}$$

energy needed to promote  
protons across Z=50

$$\hat{H}_i = \epsilon_d (\hat{n}_{d\nu} + \hat{n}_{d\pi}) + \kappa \hat{Q}_\nu \cdot \hat{Q}_\pi + \kappa' \hat{L} \cdot \hat{L},$$

$$\hat{H}_{\text{mix}} = \omega (s_\pi^\dagger s_\pi^\dagger + d_\pi^\dagger d_\pi^\dagger)^{(0)} + H.c.$$