Proton-neutron pairing and alpha-like quartetting in nuclei

Nicolae Sandulescu

National Institute of Physics and Nuclear Engineering, Bucharest

J. Dukelsky (CSIC-Madrid)
D. Gambacurta (ELI-NP, Bucharest)
C. W. Johnson (SDSU -San Diego)
D. Negrea (NIPNE-Bucharest)
M. Sambataro (INFN-Catania)

why alpha-like quartet correlations ?

alpha-like quartet = two neutrons and two protons coupled to isospin $T=0$

SOVIET PHYSICS JETP VOLUME 11, NUMBER 3 SEPTEMBER, 1960

SUPERFLUIDITY OF LIGHT NUCLEI

V. B. BELYAEV, B. N. ZAKHAR'EV, and V. G. SOLOV'EV

Joint Institute of Nuclear Research

Submitted to JETP editor October 12, 1959

J. Exptl. Theoret. Phys. (U.S.S.R.) **38**, 952-954 (March, 1960)

"we must take into consideration the quadruple correlation of alpha-particle-like nucleons [...]; these new correlations evidently play a very important role and somewhat mask the effect of pair correlations"

EFFECT OF QUADRUPLE CORRELATIONS IN LIGHT NUCLEI

V G SOLOVIEV
Joint Institute of Nuclear Research, Dubna, USSR

Received 25 December 1959

Fingerprints of alpha-like (quadruple) correlations

1) Extracting a pn pair from a even-even N=Z nucleus costs more energy than adding to it a pn pair

$$^{22}\mathrm{Na} \xleftarrow[22.1\ \mathrm{MeV}]{} {}^{24}\mathrm{Mg} \xrightarrow[13.6\ \mathrm{Mev}]{} {}^{26}\mathrm{Al} \xleftarrow[24.4\ \mathrm{MeV}]{} {}^{28}\mathrm{Si}$$

2) Extracting one neutron from a even-even N=Z nucleus costs more energy than from neighbouring nuclei

$$B(^{24}\mathrm{Mg})-B(^{23}\mathrm{Mg}) = 16.6\ \mathrm{MeV}$$

$$B(^{25}\mathrm{Mg}) -B(^{24}\mathrm{Mg}) = 7.3\ \mathrm{Mev}$$

$$B(^{26}\mathrm{Mg})-B(^{25}\mathrm{Mg}) = 11.3\ \mathrm{Mev}$$

to brake a quadruple (quartet) in pairs takes about 4-5 MeV

conclusion: 4-body alpha-like correlations are important in N=Z nuclei

Volume 34B, number 1 PHYSICS LETTERS 18 January 1971

EVIDENCE FOR QUARTET STRUCTURE IN MEDIUM AND HEAVY NUCLEI

M. DANOS‡ and V. GILLET
Service de Physique Théorique, Centre d Etudes Nucléaires de Saclay, B.P. no 2-91-Gif-sur-Yvette, France

Received 1 November 1970

The second differences of the nuclear masses keeping T constant are discussed for even-even nuclei throughout the mass table. They are shown to be consistent with the quartet picture of weakly interacting tight two-proton two-neutron structures

VOLUME 57, NUMBER 2 PHYSICAL REVIEW LETTERS 14 JULY 1986

Quartet Effects in Rare-Earth Nuclei

H. J. Daley, M. A. Nagarajan, and N. Rowley
Science and Engineering Research Council Daresbury Laboratory, Daresbury, Warrington WA4 4AD, England

D. Morrison
University of Liverpool, Liverpool, England

and

A. D. May
University of Sheffield, Sheffield, England
(Received 5 March 1986)

Quartet effects in deformed rare-earth nuclei are confronted from a phenomenological point of view. Some very simple systematic trends are evident in the experimental data when plotted as a function of a quartet number. The interacting-boson model has been modified to include quartet effects explicitly and it is able to reproduce accurately the experimental trends with fixed parameters.

PACS numbers: 21.10.Re, 21.60.Fw, 23.20.Lv

Theoretical studies on pn pairing & alpha correlations

V. G. Soloviev NP18 (1960)

B. H. Flowers and M. Vijicic,NPA49(1963)

B. Bremond and J. G. Valatin NP41(1963)

J. Eichler and M. Yamamura, NPA182(1972)

R. Chasman, PLB577(2003)

J. Dobes and S. Pittel PRC57(1998)

R. A. Senkov and V. Zelevinski (2011)

....

Alpha-like quartetting

for

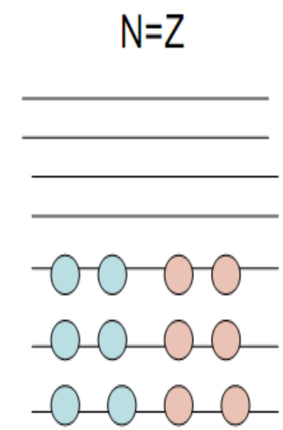
Isovector (T=1) pairing

Isovector pairing in terms of quartets

$$H = \sum_i \varepsilon_i (N_i^{(\nu)} + N_i^{(\pi)}) + g \sum_{ij,\tau} P_{i,\tau}^+ P_{j,\tau}$$

$$P_{i1}^+ \propto \nu_i^+ \nu_{\bar{i}}^+ \qquad P_{i-1}^+ \propto \pi_i^+ \pi_{\bar{i}}^+ \qquad P_{i0}^+ \propto \nu_i^+ \pi_{\bar{i}}^+ + \pi_i^+ \nu_{\bar{i}}^+$$

non-collective quartets



$$Q_{ij}^+ = [P_{i\tau}^+ P_{j\tau'}^+]^{T=0} \propto P_{\nu\nu,i}^+ P_{\pi\pi,j}^+ + P_{\pi\pi,i}^+ P_{\nu\nu,j}^+ - P_{\nu\pi,i}^+ P_{\nu\pi,j}^+$$

collective quartet

$$Q^+ = \sum_{ij} x_{ij} [P_{i\tau}^+ P_{j\tau'}^+]^{T=0}$$

quartet condensate

$$|QCM> = Q^{+n_q} |->$$ (has T=0, J=0)

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Quartets in terms of Cooper pairs

$$Q^{+} = \sum_{ij} x_{ij} [P^{+}_{i\tau} P^{+}_{j\tau'}]^{T=0} \qquad |QCM>= Q^{+n_q} |->$$

$$Q^{+} = 2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi} \qquad \Gamma^{+}_{\tau} = \sum_{i} x_i P^{+}_{i,\tau}$$ collective pairs

$$\boxed{|QCM>= (2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+}_{\nu\pi}\Gamma^{+}_{\nu\pi})^{n_q} |->}$$

'coherent' mixing of condenstates formed by nn, pp and pn pairs

PBCS solutions: $(\Gamma^{+}_{\nu\upsilon}\Gamma^{+}_{\pi\pi})^{n_q}$ $\quad (\Gamma^{+2}_{\nu\pi})^{n_q}$

calculations

$$\delta_x < QCM | H | QCM >= 0$$

(14 non-linear coupled equations solved iteratively)

Accuracy of quartet condensation versus pair condensation

$$H = \sum_i \varepsilon_i \left(N_i^{(\nu)} + N_i^{\pi} \right) - g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$

Skyrme

g=-24/A

	SM	QCM $(Q^+)^{n_q}$	PBCS1 $(\Gamma^+_{\nu\nu}\Gamma^+_{\pi\pi})^{n_q}$	PBCS0 $(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{20}Ne	6.55	6.539 (0.168%)	5.752 (12.183%)	4.781 (27.008%)
^{24}Mg	8.423	8.388 (0.415%)	7.668 (8.963%)	6.829 (18.924%)
^{28}Si	9.661	9.634 (0.279%)	9.051 (6.314%)	8.384 (13.218%)
^{32}S	10.263	10.251 (0.117%)	9.854 (3.985%)	9.372 (18.682%)
^{44}Ti	3.147	3.142 (0.159%)	2.750 (12.615%)	2.259 (28.217%)
^{48}Cr	4.248	4.227 (0.494%)	3.854 (9.275%)	3.423 (19.421%)
^{52}Fe	5.453	5.426 (0.495%)	5.033 (7.702%)	4.582 (15.973%)
^{104}Te	1.084	1.082 (0.184%)	0.964 (11.070%)	0.832 (23.247%)
^{108}Xe	1.870	1.863 (0.374%)	1.697 (9.264%)	1.514 (19.037%)
^{112}Ba	2.704	2.688 (0.592%)	2.532 (6.361%)	2.184 (19.230%)

Conclusion: T=1 pairing is accurately described by quartets, not by pairs

Note: $|PBCS(N,T)\rangle = \hat{P}_T \hat{P}_N |BCS\rangle$ gives errors much larger than QCM

N. S, D. Negrea, J. Dukelsky, C.W. Johnson, PRC85, 061303(R) (2012)

Isovector pairing with distinct quartets

$$H = \sum_i \varepsilon_i \left(N_i^{(\nu)} + N_i^{\pi}\right) - g \sum_{ij,\tau} P_{i,\tau}^{+} P_{j,\tau}$$

$$|QM\rangle = Q_1^+ Q_2^+ .. Q_{n_q}^+ |-\rangle \qquad |QCM\rangle = Q^{+n_q} |-\rangle$$

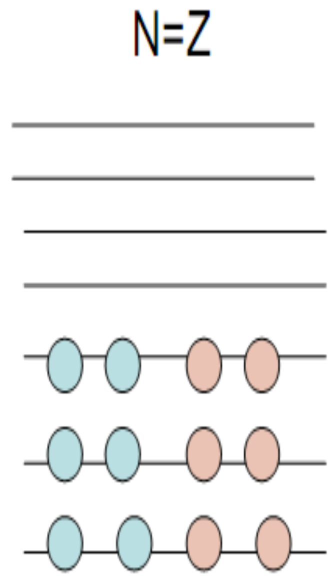
	exact	QM	QCM
^{20}Ne	-6.5505	-6.5505	-6.539 (0.18%)
^{24}Mg	-8.4227	-8.4227	-8.388 (0.41%)
^{28}Si	-9.6610	-9.6610	-9.634 (0.28%)
^{32}S	-10.2629	-10.2629	-10.251 (0.12%)
^{44}Ti	-3.1466	-3.1466	-3.142 (0.15%)
^{48}Cr	-4.2484	-4.2484	-4.227 (0.50%)
^{52}Fe	-5.4532	-5.4531	-5.426 (0.50%)
^{104}Te	-1.0837	-1.0837	-1.082 (0.16%)
^{108}Xe	-1.8696	-1.8696	-1.863 (0.35%)
^{112}Ba	-2.7035	-2.7034	-2.688 (0.57%)

QM reproduces the exact results up to the 4th digit !!

M. Sambataro and N.S, PRC88 (2013)

Isoscalar and isovector pairing in N=Z nuclei

$$P^{+}_{i,T_z} = [a_i^{+} a_i^{+}]^{T=1,J=0}_{T_z}$$



$$D^{+}_{ij,J_z} = [a_i^{+} a_j^{+}]^{J=1,T=0}_{J_z}$$

Quartetting for isovector (J=0) and isoscalar (J=1) pairing

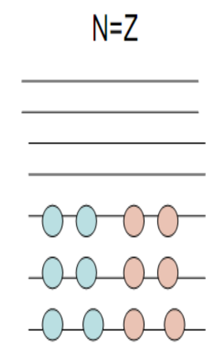
$$H = \sum \varepsilon_i N_i + \sum_{ij} V^{T=1}_{J=0}(i,j) \sum_{\tau} P^+_{i\tau} P_{j\tau} + \sum_{ij} V^{T=0}_{J=1}(ij,kl) \sum_{\sigma} D^+_{ij\sigma} D_{kl\sigma}$$

isovector

$$P^+_{i,T_z} = [a^+_i a^+_i]^{T=1,J=0}_{T_z}$$

isoscalar

$$D^+_{ij,J_z} = [a^+_i a^+_j]^{J=1,T=0}_{J_z}$$



collective quartets

$$Q^+_{iv} = \sum_{i,j} x_{ij} [P^+_i P^+_j]^{T=0}$$

$$Q^+_{is} = \sum_{i,j} y_{ij,kl} [D^+_{ij} D^+_{kl}]^{J=0}$$

generalised quartet

$$Q^+ = Q^+_{iv} + Q^+_{is}$$

ground state

$$|QCM> = Q^{+n_q} |->$$

superposition of T=0 and T=1 quartets

M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Quartet condensation versus pair condensation for isovector & isoscalar pairing

$$H = \sum \varepsilon_i N_i + \sum_{ij} V_{J=0}^{T=1}(i,j) \sum_{\tau} P_{i\tau}^{+} P_{j\tau} + \sum_{ij} V_{J=1}^{T=0}(i,j) \sum_{\sigma} D_{i\sigma}^{+} D_{j\sigma}$$

$$(Q^{+})^{n_q} |-> \qquad (\Gamma_{\nu\nu}^{+}\Gamma_{\pi\pi}^{+})^{n_q} |-> \qquad (\Gamma_{\nu\pi}^{+})^{2n_q} |-> \qquad (\Delta_0^{+})^{2n_q} |0\rangle$$

	QCM	PBC1	$\mathrm{PBCS0}_{iv}$	$\mathrm{PBCS0}_{is}$
^{20}Ne	15.985 (-)	14.011 (12.35%)	13.664 (14.52%)	13.909 (12.99%)
^{24}Mg	28.595 (0.24%)	21.993 (23.35%)	20.516 (28.50%)	23.179 (19.22%)
^{28}Si	35.288 (0.57%)	27.206 (23.58%)	25.293 (28.95%)	27.740 (22.19%)
^{44}Ti	7.019 (-)	5.712 (18.62%)	5.036 (28.25%)	4.196 (40.22%)
^{48}Cr	11.614 (0.21%)	9.686 (16.85%)	8.624 (25.97%)	6.196 (46.81%)
^{52}Fe	13.799 (0.42%)	11.774 (15.21%)	10.591 (23.73%)	6.673 (51.95%)
^{104}Te	3.147 (-)	2.814 (10.58%)	2.544 (19.16%)	1.473 (53.19%)
^{108}Xe	5.489 (0.20%)	4.866 (11.61%)	4.432 (19.49%)	2.432 (55.82%)
^{112}Ba	7.017 (0.34%)	6.154 (12.82%)	5.635 (20.17%)	3.026 (57.13%)

- quartet condensation wins over Cooper pair condensates
- T=1 and T=0 pairing correlations <u>always</u> coexist in quartets

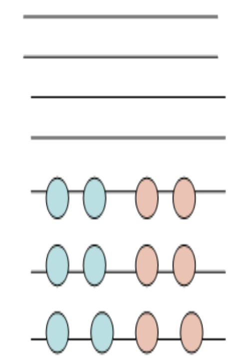
M. Sambataro and N.S, Phys. Rev C93, 054320 (2016)

Isoscalar and isovector proton-neutron pairing in time-reversed states

$$\hat{H}=\sum_{i,\tau=\pm 1/2}\varepsilon_{i\tau}N_{i\tau}+\sum_{i,j}V^{T=1}(i,j)\sum_{t=-1,0,1}P^{+}_{i,t}P_{j,t}+\sum_{i,j}V^{T=0}(i,j)D^{+}_{i,0}D_{j,0}$$

isovector isoscalar

N=Z



$$P^{+}_{i,0} = (\nu^{+}_{i}\pi^{+}_{\bar{i}} + \pi^{+}_{i}\nu^{+}_{\bar{i}})/\sqrt{2}. \qquad D^{+}_{i,0} = (\nu^{+}_{i}\pi^{+}_{\bar{i}} - \pi^{+}_{i}\nu^{+}_{\bar{i}})/\sqrt{2}$$

$$P^{+}_{i1}=\nu^{+}_{i}\nu^{+}_{\bar{i}} \qquad P^{+}_{i-1}=\pi^{+}_{i}\pi^{+}_{\bar{i}}$$

$$Q^{+}_{T=1}=\sum_{ij}x_{i}x_{j}[P^{+}_{i\tau}P^{+}_{j\tau'}]^{T=0} \qquad \Delta^{+}_{0}=\sum y_{i}D^{+}_{i,0}$$

ansatz for ground state

$$|\Psi>=(Q^{+}_{T=1}+\Delta^{+2}_{0})^{n_q}\,|->$$

superposition of T=1 quartet condensates and T=0 pair condensates

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B751 (2015) 348

Competition between isovector and isoscalar pairing

pairing on top of deformed Skyrme-HF

$$V_{paring}^{T=\{0,1\}} = \mathrm{v}_0^{T=\{0,1\}}\delta(r_1 - r_2)\hat{P}_{S=\{0,1\}} \qquad \mathrm{v}_0^{T=0} = 1.5\ \mathrm{v}_0^{T=1}$$

$(Q_{T=1}^{+} + \Delta_0^{+2})^{n_q}$ $\quad (Q_{T=1}^{+})^{n_q}$ $\quad (\Delta_0^{+2})^{n_q}$

	exact	$\mid \Psi\rangle$	$\mid iv\rangle$	$\mid is\rangle$	$\langle iv \mid is\rangle$
^{20}Ne	11.38	11.38 (0.00%)	11.31 (0.62%)	10.92 (4.00%)	0.976
^{24}Mg	19.32	19.31 (0.03%)	19.18 (0.74%)	18.93 (2.00%)	0.980
^{28}Si	18.74	18.74 (0.01%)	18.71 (0.14%)	18.54 (1.07%)	0.992
^{44}Ti	7.095	7.094 (0.02%)	7.08 (0.18%)	6.30 (10.78%)	0.928
^{48}Cr	12.78	12.76 (0.1%)	12.69 (0.67%)	12.22 (4.37%)	0.936
^{52}Fe	16.39	16.34 (0.26%)	16.19 (1.17%)	15.62 (4.65%)	0.946
^{104}Te	4.53	4.52 (0.06%)	4.49 (0.82%)	4.02 (11.26%)	0.955
^{108}Xe	8.08	8.03 (0.61%)	7.96 (1.45%)	6.75 (16.47%)	0.814
^{112}Ba	9.36	9.27 (0.93%)	9.22 (1.43 %)	7.50 (19.81%)	0.784

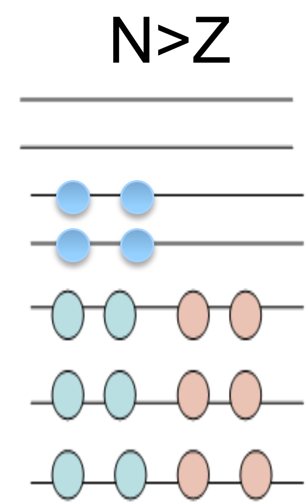
isovector and isoscalar pairing **always coexist** together

large overlaps between |iv> and |is>

N.S, D.Negrea, D. Gambacurta, Phys. Lett. B (2015)

Isovector-isoscalar pairing and quartetting for N>Z nuclei

nuclei with N-Z=2n_N



ansatz

- all protons are correlated in alpha-like quartets
- neutrons in excess form a pair condensate

$$|QCM\rangle = (\tilde{\Gamma}^{+}_{\nu\nu})^{n_N} (Q^{+}_{T=1} + \Delta^{+2}_{0})^{n_q} |-\rangle$$

$$Q^{+}_{T=1} = 2\Gamma^{+}_{\nu\nu}\Gamma^{+}_{\pi\pi} - \Gamma^{+2}_{\nu\pi} \qquad \Delta^{+}_{0} = \sum y_i D^{+}_{i,0}$$

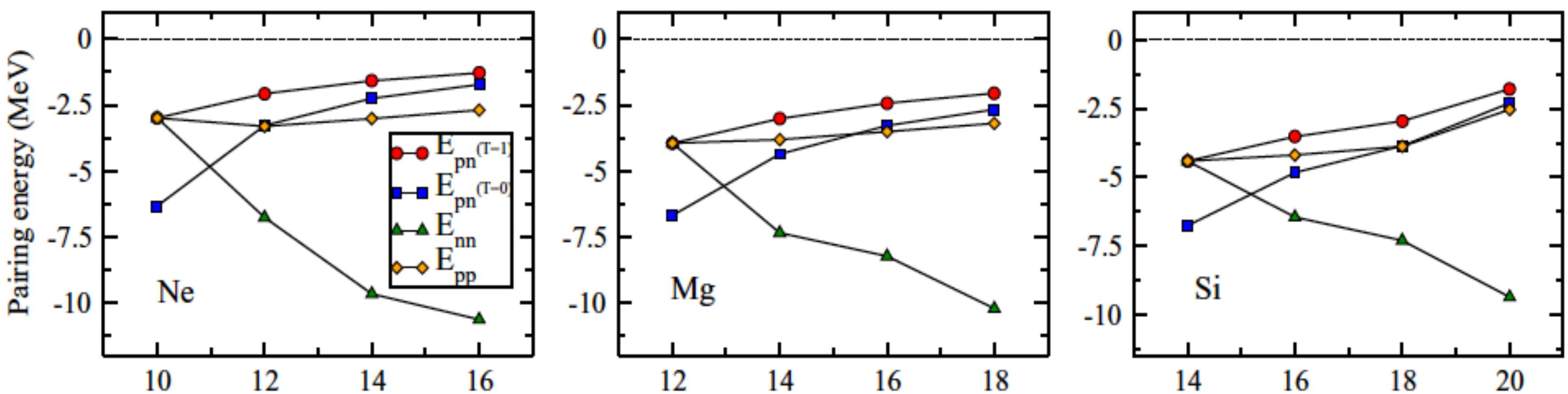
how fast are suppressed the pn correlations away of N=Z ?

D. Negrea, P. Buganu, D. Gambacurta, N. S., PRC98 (2018)

Isovector-isoscalar pairing and quartetting for N>Z nuclei

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P^+_{i\tau} P_{j\tau} + g_{T=0} \sum_{ij} D^+_{i0} D_{j0} \qquad g_{T=0} = 1.5\, g_{T=1}$$

$$|QCM\rangle = (\tilde{\Gamma}^+_{\nu\nu})^{n_N} (Q^+_{T=1} + \Delta_0^{+2})^{n_q} |-\rangle \qquad \Delta_0^+ = \sum y_i D^+_{i,0}$$



pn pairing and quartet correlations survive in N > Z nuclei !

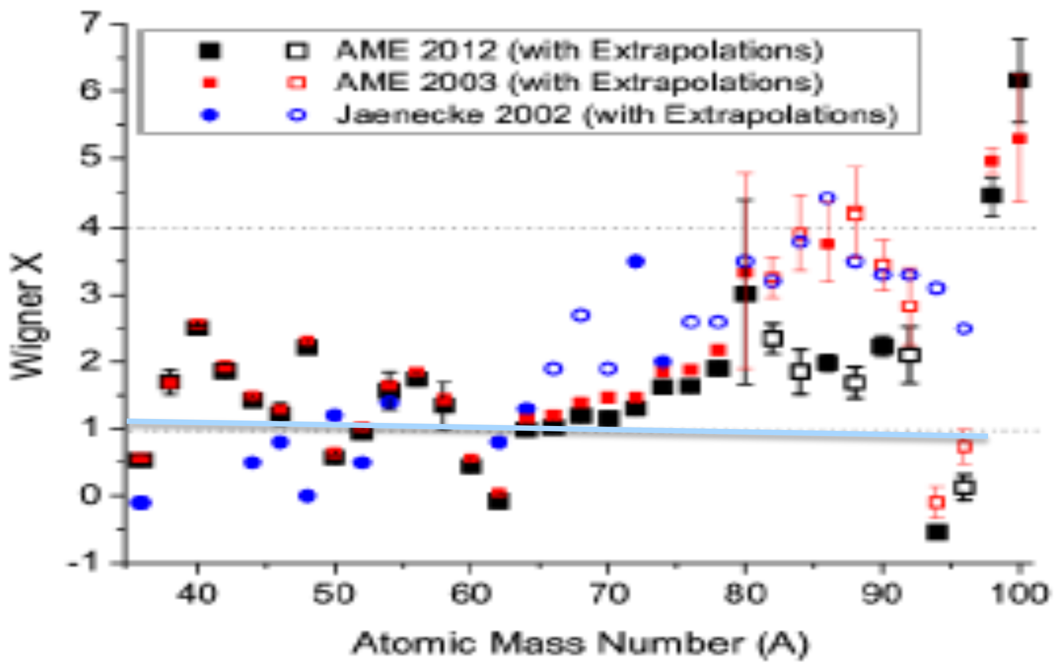
D. Negrea, P. Buganu, D. Gambacurta, N. S., PRC98 (2018)

Proton-neutron pairing & Wigner energy

Reminder on Wigner energy

$$E(N,Z) = E(N=Z) + a_s \frac{(N-Z)^2}{A} + a_W \frac{|N-Z|}{A} + \delta E_{shell} + \delta E_P$$ (no Coulomb)

$$E(N,Z) = E(N=Z) + \frac{T_z(T_z+X)}{2\Theta} \qquad T_z = 0,2,4$$



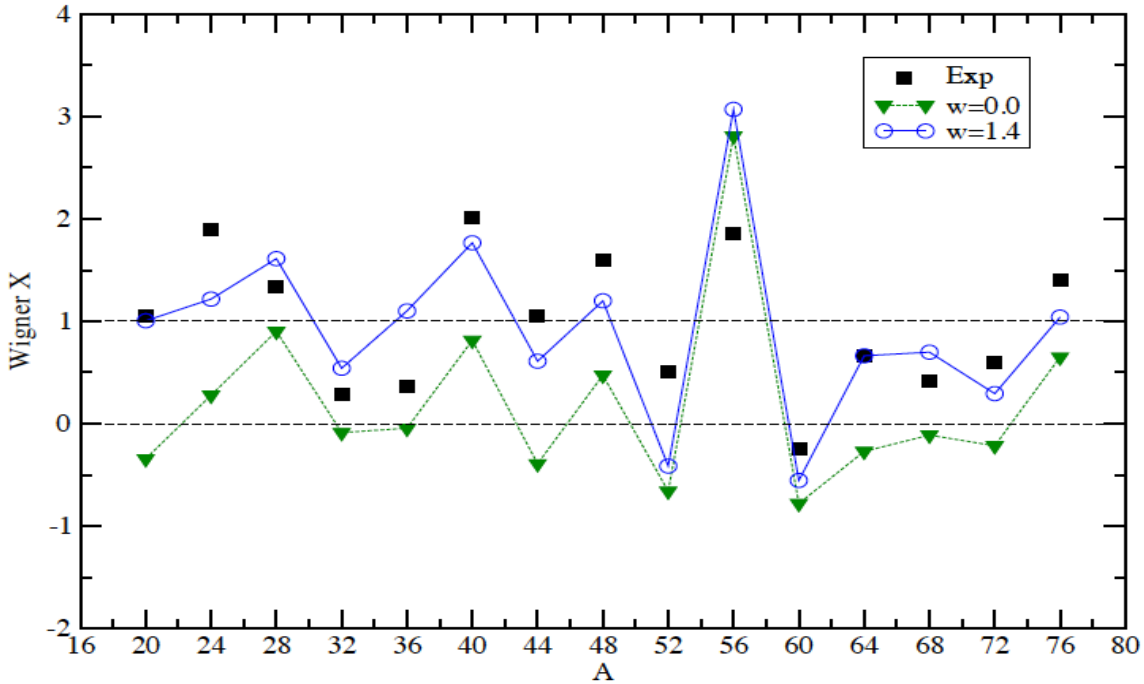
I. Bentley & S. Frauendorf, PRC(2013)

Effect of proton-neutron pairing on Wigner energy

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P^+_{i\tau} P_{j\tau} + g_{T=0} \sum_{ij} D^+_{i0} D_{j0}$$

Skyrme $\quad g_1 = -24/A \quad g_0 = w\, g_1$

preliminary Skyrme-HF+QCM results*



- it is suggested the need of T=0 pairing;
- however, the effects of T=0 and T=1 pairing are difficult to disentangle;
- the effects of T=0 and T=1 pairing in odd-odd N=Z nuclei ?

* N. S et all, in preparation

The effect of pn pairing on Wigner energy: example

$$H = \sum \varepsilon_i N_i + g_{T=1} \sum_{ij,\tau} P^+_{i\tau} P_{j\tau} + g_{T=0} \sum_{ij} D^+_{i0} D_{j0}$$

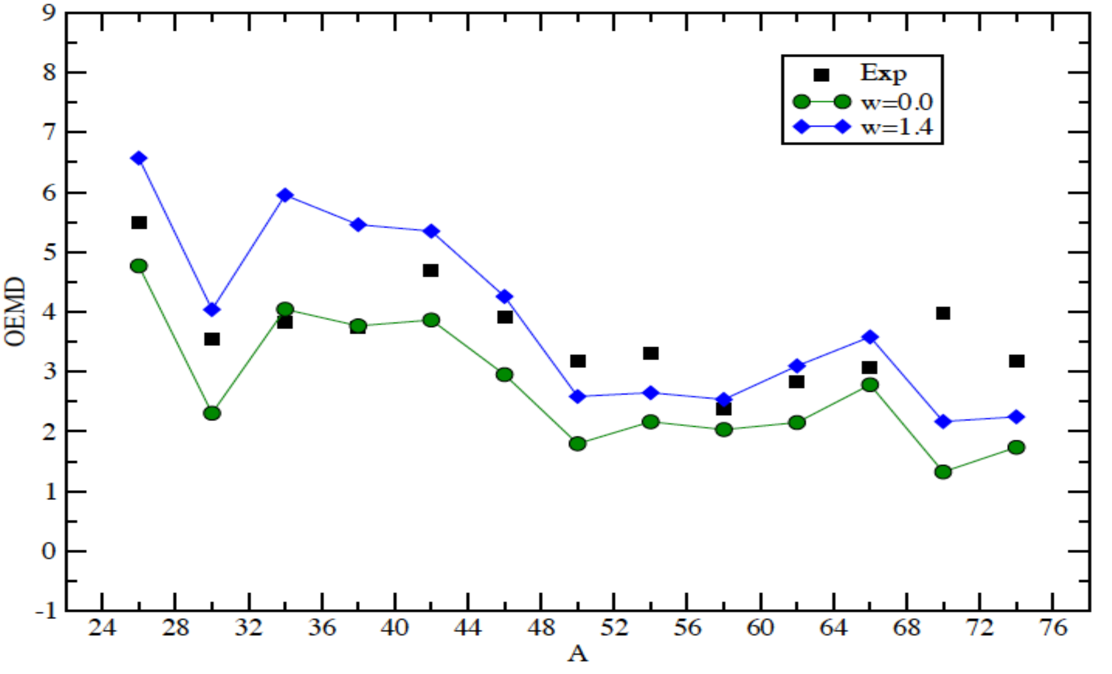
$g_1 = -24/A$ $g_0 = w\, g_1$

	Exp	QCM(w)	QCM(w=0)	HF+BCS	HF
24Mg (3q)	**-198.257**	**-195.420 (0.4)** **-195.905 (0.8)** **-196.500 (1.0)** **-197.577 (1.2)** **-199.609 (1.4)** **-203.514 (1.6)**	**-195.362**	**-190.333**	**-189.992**

Odd-even mass difference along N=Z line

$$2\Delta(N,Z) \quad = \frac{B(N-1,Z-1)-2B(N,Z)+B(N+1,Z+1)}{2}$$

preliminary Skyrme-HF+QCM results



odd-odd nuclei: calculations with blocking !

Summary and Conclusions

Main message: *T=1 and T=0 pairing forces are accurately described by alpha-like quartets, not by Cooper pairs*

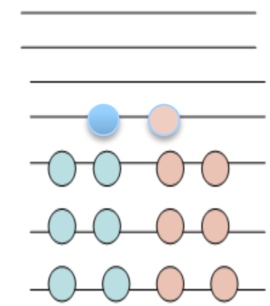
- *T=1 and T=0 pn pairing <u>always coexist</u> in quartet-type correlations*
- *T=0 pn pairing and quartetting* persist in N>Z nuclei
- *proton-neutron pairing has a significant effect on Wigner energy*

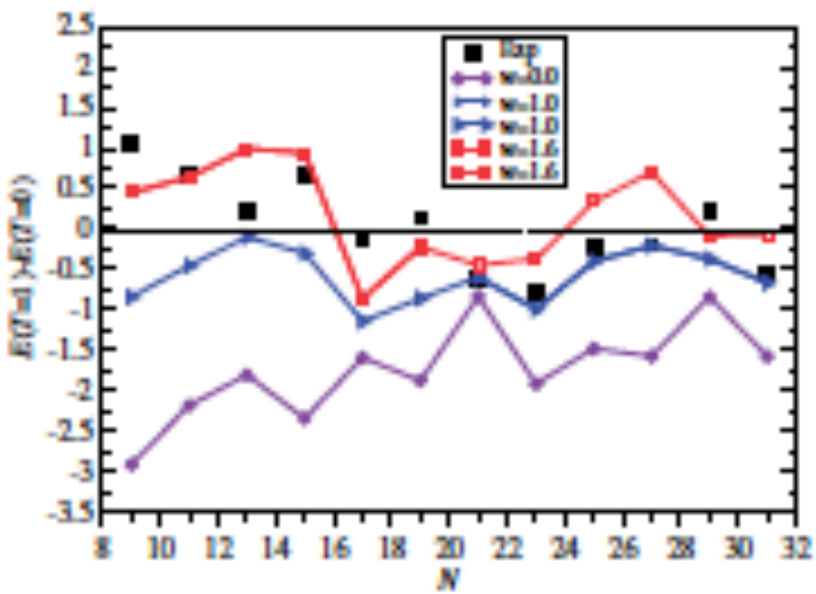
Isovector and isoscalar pairing in odd-odd N=Z

$$\hat{H} = \sum_{i,\tau=\pm 1/2} \varepsilon_{i\tau} N_{i\tau} + \sum_{i,j} V^{T=1}(i,j) \sum_{t=-1,0,1} P^{+}_{i,t} P_{j,t} + \sum_{i,j} V^{T=0}(i,j) D^{+}_{i,0} D_{j,0}$$

T=1 state $\quad | iv; QCM \rangle = \tilde{\Gamma}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} |-\rangle$

T=0 state $\quad | is; QCM \rangle = \tilde{\Delta}^{+}_{\nu\pi} (Q^{+}_{T=1} + \Delta^{+2}_{\nu\pi})^{n_q} |-\rangle$





$$v = \frac{V_0^{T=0}}{V_0^{T=1}} \qquad V_{paring}^{T=\{0,1\}} = V_0^{T=\{0,1\}} \delta(r_1 - r_2) \hat{P}_{S=\{0,1\}}$$

calculations on top of Skyrme-HF spectrum

effects of time-odd terms in Skyrme functional ?

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

The structure of lowest T=0 and T=1 states

T=0 ground state

		Exact	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Delta}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$(\Delta^{+}_{\nu\pi})^{2n_q+1}$	$\tilde{\Delta}^{+}_{\nu\pi}(\Gamma^{+2}_{\nu\pi})^{n_q}$
^{30}P	T=0	12.66	12.60 (0.44%)	12.55 (0.86%)	11.96 (5.86%)	11.94 (5.95%)

T=1 ground state

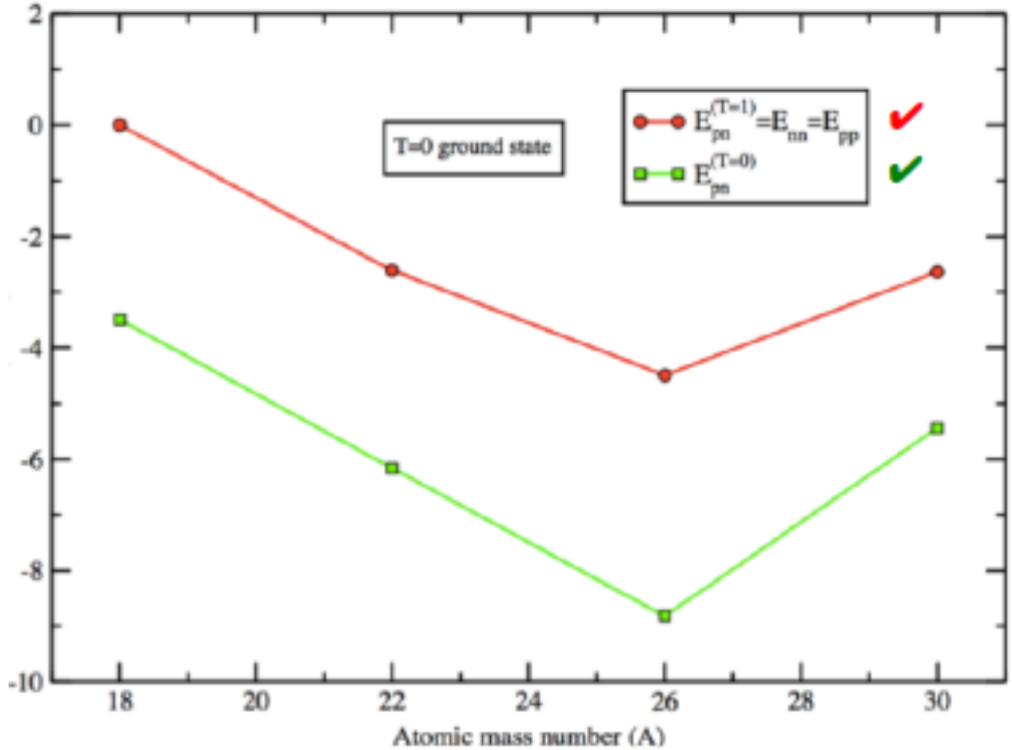
		Exact	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1}+\Delta^{+2}_{\nu\pi})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(Q^{+}_{T=1})^{n_q}$	$\tilde{\Gamma}^{+}_{\nu\pi}(\Delta^{+2}_{\nu\pi})^{n_q}$	$(\Gamma^{+}_{\nu\pi})^{2n_q+1}$
^{54}Co	T=1	16.14	16.12 (0.14%)	16.09 (0.28%)	15.67 (3.01%)	15.86 (1.78%)

conclusion

isovector correlations are stronger in both T=0 and T=1 low-lying states

D. Negrea, N.S. and D. Gambacurta, Prog. Theor. Exp. Phys. 073D05 (2017)

Average value of pairing interactions



Quartet correlations for general two-body forces ?

Quartet correlations for general two-body forces

$$H = \sum_i \varepsilon_i (N_i^{(n)} + N_i^{(p)}) + \sum_{ii',jj',J',T'} V_{JT}(ii';jj')[A^+_{ii'J'T'} A_{jj'J'T'}]^{J=0,T=0}$$

$$|QCM> = Q^{+n_q}|->$$

$$Q^+ = \sum_{ii',jj',JT} x_{ii',jj'} [A^+_{ii'JT} A^+_{jj'JT}]^{0,0}$$

	$E_{corr}(SM)$	$E_{corr}(QCM)$	$E_{corr}(QM)$	$\langle SM\|QCM\rangle$
^{20}Ne	24.77	24.77	24.77	1
^{24}Mg	55.70	53.04 (4.77%)	53.24 (4.41%)	0.85
^{28}Si	88.75	86.52 (2.52%)	87.12 (1.84%)	0.86
^{32}S	122.51	122.02 (0.40%)	122.29 (0.18%)	0.98

$$E(n_q) = n_q \times E(1) + \frac{n_q(n_q-1)}{2} \times V(n_q),$$

the interaction between the quartets is small compared to their binding energies

quartets acts as weakly interacting building blocks

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