Neutrinoless Double-Beta Decay and Realistic Shell Model

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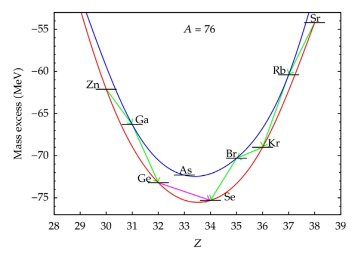
Nuclear Structure and Dynamics - NSD2019

Double β-decay (2νECEC) is the rarest process yet observed in nature.



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  $(A, Z) \rightarrow (A, Z+2) + e^- + e^- + \overline{\nu}_e + \overline{\nu}_e$
- Historically, G. Racah (1937) and W. Furry (1939) were the first ones, to suggest to test the neutrino as a Majorana particle, considering the process:
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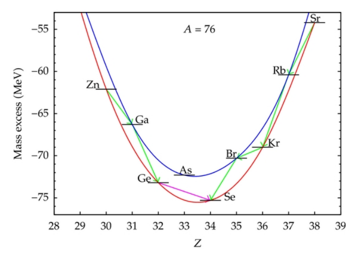

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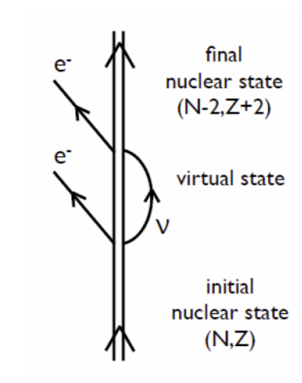

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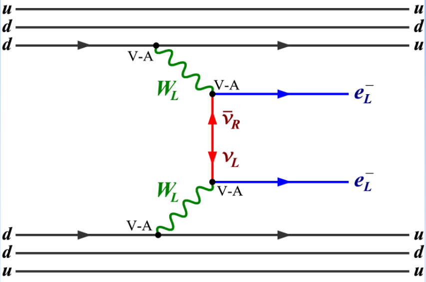
Neutrinoless double β-decay

The detection of the $0\nu\beta\beta$ decay is nowadays one of the main targets in many laboratories all around the world, triggered by the search of "new physics" beyond the Standard Model.

- Its detection
  - would correspond to a violation of the conservation of the leptonic number
  - may provide more informations on the nature of neutrinos (neutrino as a Majorana particle, determination of its effective mass, ..).

Neutrinoless double β-decay

The inverse of the $0\nu\beta\beta$-decay half-life is proportional to the squared nuclear matrix element (NME).
This evidences the relevance to calculate the NME



$$\left[T_{1/2}^{0\nu}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2$$

- $G^{0\nu}$ $\rightarrow$ phase-space factor
- $\langle m_{\beta\beta}\rangle = \mid \sum_k m_k U_{ek}^2 \mid$ effective mass of the Majorana neutrino, U_{ek} being the lepton mixing matrix

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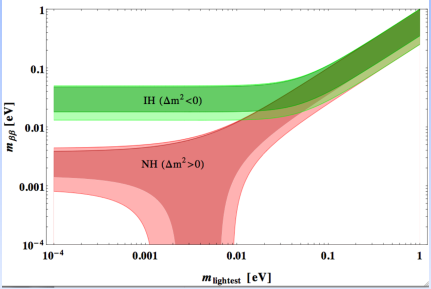

$$\left[T^{0\nu}_{1/2}\right]^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 \langle m_\nu \rangle^2$$

- constraints from oscillation data
- to exclude IH $\Rightarrow$ $m_{\beta\beta} = 8\text{meV}$

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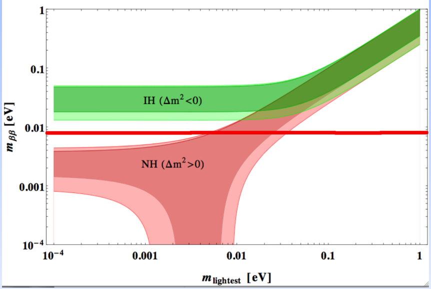

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- constraints from oscillation data
- to exclude IH $\Rightarrow$ $m_{\beta\beta} = 8\text{meV}$

The calculation of the NME

The nuclear matrix element (NME) is expressed as

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_F + M^{0\nu}_T ,$$

where

$$M^{0\nu}_{GT} = < 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_{GT}(r_{mn}) \vec{\sigma}_m \cdot \vec{\sigma}_n \mid 0^+_i >$$

$$M^{0\nu}_F = < 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_F(r_{mn}) \mid 0^+_i >$$

$$M^{0\nu}_T = < 0^+_f \mid \sum_{m,n} \tau^-_m \tau^-_n H_T(r_{mn}) \left[3 \left(\vec{\sigma}_m \cdot \hat{r}_{mn}\right) \left(\vec{\sigma}_n \cdot \hat{r}_{mn}\right) - \vec{\sigma}_m \cdot \vec{\sigma}_n\right] \mid 0^+_i >$$

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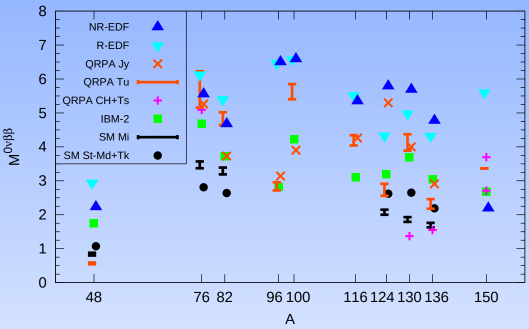

Engel & Menendez Reports on Progress in Physics 80, 046301 (2017)

- The spread of nuclear structure calculations evidences inconsistencies among results obtained with different models

Realistic Shell-Model Calculations

Shell model ⇒ well-established approach to obtain a microscopic description of both collective and single-particle properties of nuclei

Napoli-Caserta group

- L. Coraggio (INFN-NA)
- L. De Angelis (INFN-NA)
- T. Fukui (INFN-NA)
- A. Gargano (INFN-NA)
- N. I. (Università ”Vanvitelli” and INFN-NA)
- F. Nowacki (IPHC-CNRS Strasbourg)

Realistic Shell-Model Calculations

Focus on ^{76}Ge, ^{82}Se, ^{130}Te, and ^{136}Xe.

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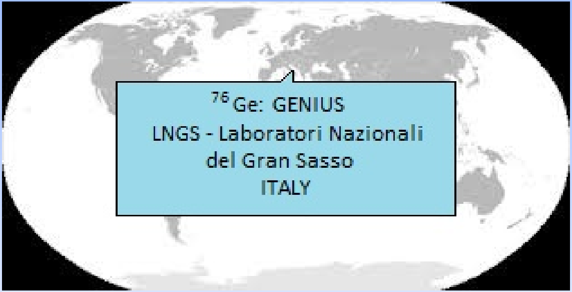

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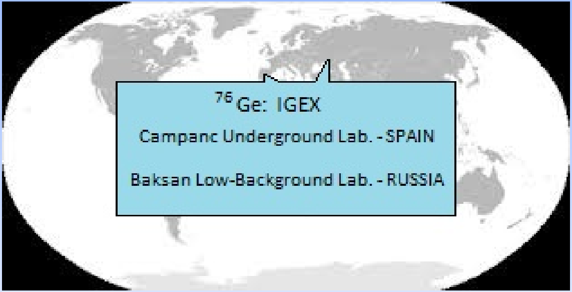

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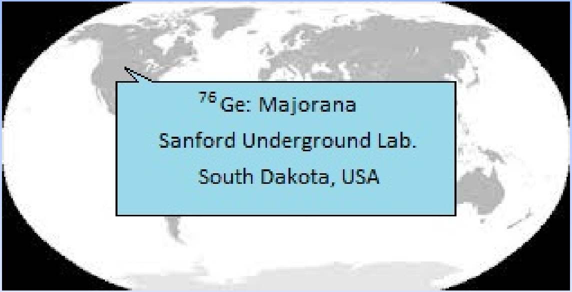

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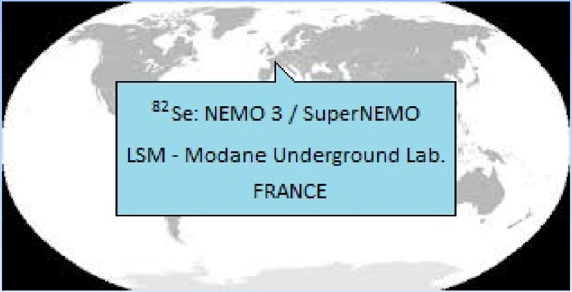

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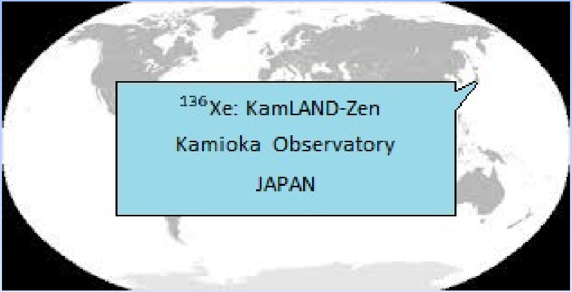

Effective shell-model hamiltonian

The shell-model hamiltonian has to take into account in an effective way all the degrees of freedom not explicitly considered

Two alternative approaches

- phenomenological
- microscopic

$$V_{NN} \ (+V_{NNN}) \Rightarrow \text{many-body theory} \Rightarrow H_{\text{eff}}$$

Definition

The eigenvalues of H_{eff} belong to the set of eigenvalues of the full nuclear hamiltonian

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Workflow for a realistic shell-model calculation

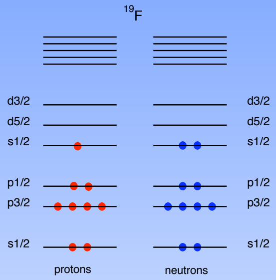
1. Choose a realistic *NN* potential (*NNN*)
2. Determine the model space better tailored to study the system under investigation
3. Derive the effective shell-model hamiltonian and operators by way of a many-body theory
4. Calculate the physical observables (energies, e.m. transition probabilities, ...)

The shell-model effective hamiltonian

A-nucleon system Schrödinger equation

$$H|\Psi_\nu\rangle = E_\nu|\Psi_\nu\rangle$$

$$H = H_0 + H_1 = \sum_{i=1}^{A}(T_i + U_i) + \sum_{i<j}(V_{ij}^{NN} - U_i)$$



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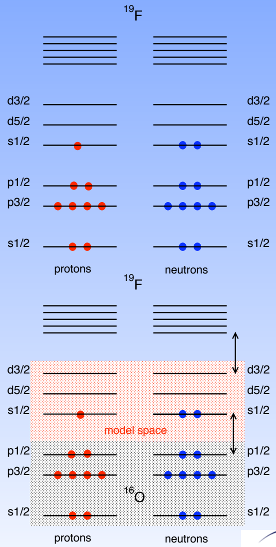
Model space

$$|\Phi_i\rangle = [a_1^\dagger a_2^\dagger \ldots a_n^\dagger]_i|c\rangle \Rightarrow P = \sum_{i=1}^{d}|\Phi_i\rangle\langle\Phi_i|$$

Model-space eigenvalue problem

$$H_{\mathrm{eff}}P|\Psi_\alpha\rangle = E_\alpha P|\Psi_\alpha\rangle$$



The perturbative approach to the shell-model H^{eff}

$$H_{\mathrm{eff}} = \hat{Q} - \hat{Q}' \int \hat{Q} + \hat{Q}' \int \hat{Q} \int \hat{Q} - \hat{Q}' \int \hat{Q} \int \hat{Q} \int \hat{Q} \cdots ,$$

$$\hat{Q} \text{ box} \Rightarrow \hat{Q}(\epsilon) = PH_1P + PH_1Q \frac{1}{\epsilon - QHQ} QH_1P$$

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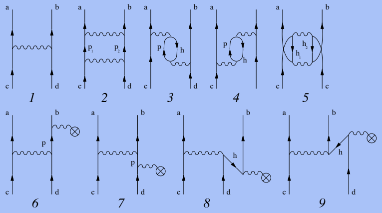
Perturbative expansion

$$\frac{1}{\epsilon - QHQ} = \sum_{n=0}^{\infty} \frac{(QH_1Q)^n}{(\epsilon - QH_0Q)^{n+1}}$$



Effective operators

Φ_α = eigenvectors obtained diagonalizing $H_{\rm eff}$ in the reduced model space $\Rightarrow |\Phi_\alpha\rangle = P|\Psi_\alpha\rangle$

$$\langle\Phi_\alpha|\hat{\Theta}|\Phi_\beta\rangle \neq \langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle$$

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Effective operator $\hat{\Theta}_{\rm eff}$: definition

$$\hat{\Theta}_{\rm eff} = \sum_{\alpha\beta} |\Phi_\alpha\rangle\langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle\langle\Phi_\beta|$$

$$\langle\Phi_\alpha|\hat{\Theta}_{\rm eff}|\Phi_\beta\rangle = \langle\Psi_\alpha|\hat{\Theta}|\Psi_\beta\rangle$$

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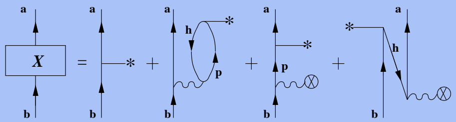
$\hat\Theta_{\rm eff}$ can be derived consistently in the MBPT framework

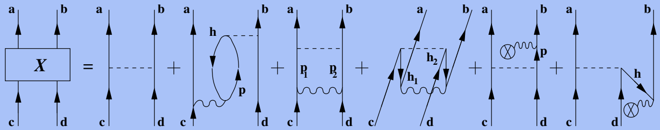
K. Suzuki and R. Okamoto, Prog. Theor. Phys. **93** *, 905 (1995)*

The shell-model effective operators

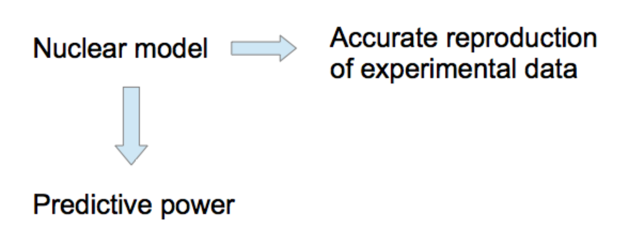
One-body operator



Two-body operator



Nuclear models and predictive power



Realistic shell-model calculations for
^{130}Te, ^{136}Xe, ^{76}Ge and ^{82}Se

$\Downarrow$

Test our approach calculating observables related to the GT strengths and $2\nu\beta\beta$ decay and comparing the results with data.

Realistic Shell-Model Calculations

- ^{76}Ge,^{82}Se: four proton and neutron orbitals outside double-closed ^{56}Ni $\longrightarrow 0f_{5/2}, 1p_{3/2}, 1p_{1/2}, 0g_{9/2}$
- ^{130}Te,^{136}Xe: five proton and neutron orbitals outside double-closed ^{100}Sn $\longrightarrow 0g_{7/2}, 1d_{5/2}, 1d_{3/2}, 2s_{1/2}, 0h_{11/2}$

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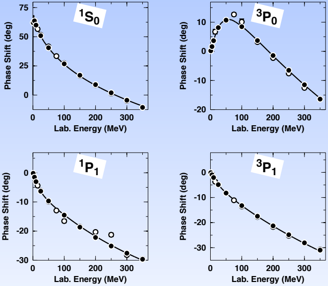
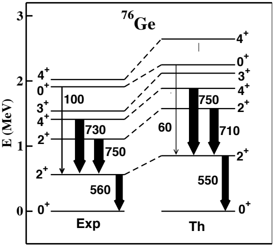
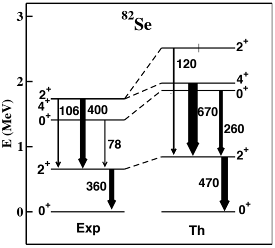
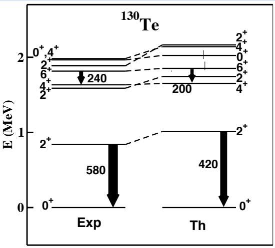
- Input V_{NN}: $V_{\rm low-k}$ derived from the high-precision *NN* CD-Bonn potential with a cutoff: $\Lambda = 2.6$ fm^{-1}.
- $H_{\rm eff}$ obtained calculating the Q-box up to the 3rd order in $V_{\rm low-k}$
- Effective operators are consistently derived by way of the MBPT

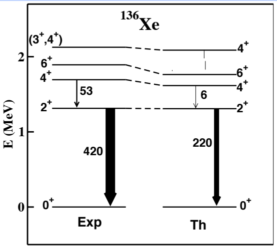


Spectroscopic properties (B(E2)s in e^2fm^4)









GT^- strength distribution

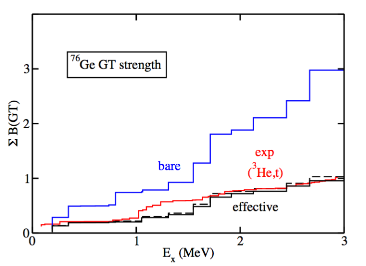
Charge-exchange experiments

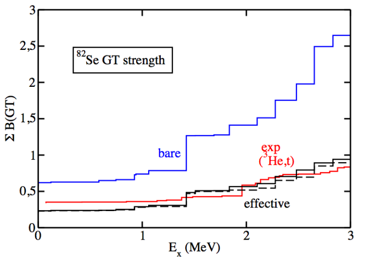
$$\left[\frac{d\sigma}{d\Omega}(q=0)\right] = \hat{\sigma} B_{exp}(GT)$$

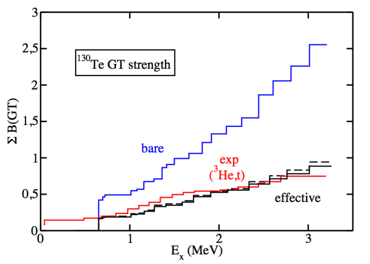
Theory

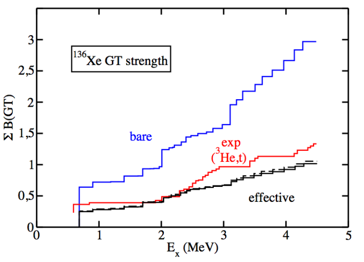
$$B_{th}(\mathrm{GT}) = \frac{\left|\langle\Phi_f| \sum_j \vec{\sigma}_j \vec{\tau}_j |\Phi_i\rangle\right|^2}{2J_i + 1}$$

GT⁻ strength distribution



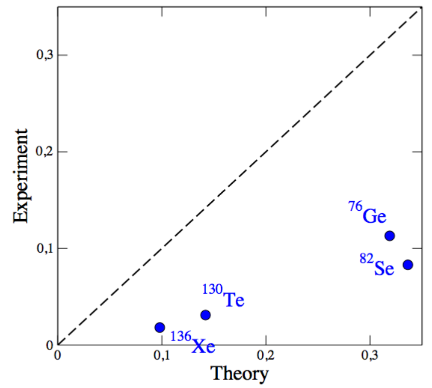






$2\nu\beta\beta$ nuclear matrix elements

$$M_{2\nu}^{\mathrm{GT}} = \sum_n \frac{\langle 0_f^+ || \vec{\sigma}\tau^- || 1_n^+ \rangle \langle 1_n^+ || \vec{\sigma}\tau^- || 0_i^+ \rangle}{E_n + E_0}$$



Blue dots: bare GT operator

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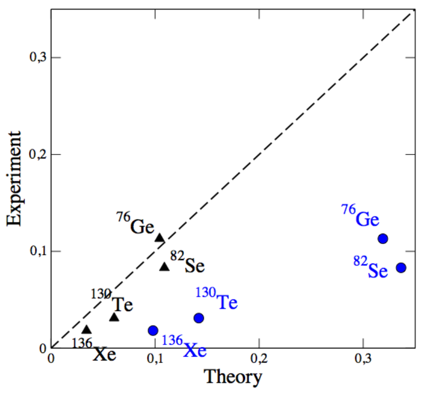

Blue dots: bare GT operator
Black triangles: effective GT operator

$2\nu\beta\beta$ nuclear matrix elements perturbative properties

- RSM calculations provide a satisfactory description of observed GT-strength distributions and $2\nu 2\beta$ NME
- what about perturbative properties ?

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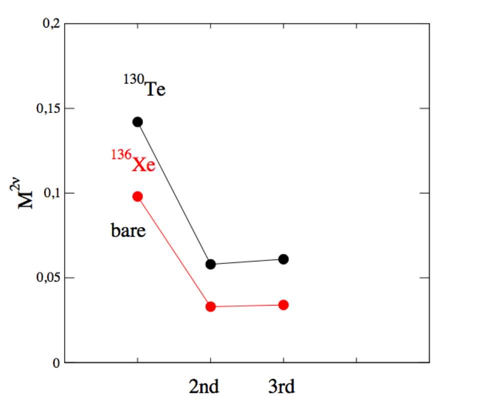

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The calculation of the $0\nu\beta\beta$ NME

The NME is given by

$$M^{0\nu} = M^{0\nu}_{GT} - \left(\frac{g_V}{g_A}\right)^2 M^{0\nu}_F + M^{0\nu}_T \quad ,$$

The matrix elements $M^{0\nu}_\alpha$ are defined, for a SM calculation, as follows:

$$M^{0\nu}_\alpha = \sum_{j_p j_{p'} j_n j_{n'} J_\pi} TBTD\,(j_p j_{p'}, j_n j_{n'}; J_\pi)\,\langle j_p j_{p'}; J^\pi T \mid \tau_1^- \tau_2^- O^\alpha_{12} \mid j_n j_{n'}; J^\pi T\rangle$$

$$\text{with } \alpha = (GT,\ F,\ T)$$

The *TBTD* are the two-body transition-density matrix elements, and the Gamow-Teller (*GT*), Fermi (*F*), and tensor (*T*) operators:

$$O^{GT}_{12} = \vec{\sigma}_1 \cdot \vec{\sigma}_2 H_{GT}(r)$$
$$O^F_{12} = H_F(r)$$
$$O^T_{12} = [3\,(\vec{\sigma}_1 \cdot \hat{r})\,(\vec{\sigma}_1 \cdot \hat{r}) - \vec{\sigma}_1 \cdot \vec{\sigma}_2]\,H_T(r)$$

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Light Majorana neutrino exchange

The neutrino potentials H_α are defined using the closure approximation

$$H_\alpha(r) = \frac{2R}{\pi} \int_0^\infty f_\alpha(qr) \frac{h_\alpha(q^2)}{q + \langle E \rangle} qdq$$

where $f_{F,GT}(qr) = j_0(qr)$ and $f_T(qr) = j_2(qr)$, $\langle E \rangle$ is the average energy used in the closure approximation.

- closure approximation
- higher order corrections (HOC)
- finite nucleon size corrections (FNS)

Short-range correlations

Empirical approach

$$\psi_{nl} \to [1 + f(r)]\, \psi_{nl} \qquad f(r) = -ce^{-ar^2}(1 - br^2)$$

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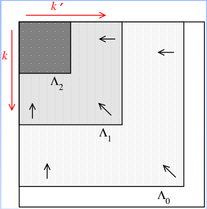
$V_{\rm low-k}$: the configurations of $V_{NN}(k,k')$ are restricted to those with $k, k' < k_{\rm cutoff} = \Lambda$



$$V_{NN}(k,k') \to V_{\rm low-k}(k,k') = \Omega^{-1} V_{NN}(k,k')\Omega$$

Short-range correlations

Empirical approach

$$\psi_{nl} \rightarrow [1 + f(r)]\, \psi_{nl} \qquad f(r) = -ce^{-ar^2}(1 - br^2)$$

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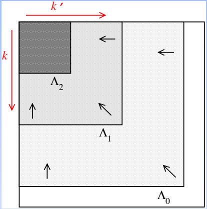
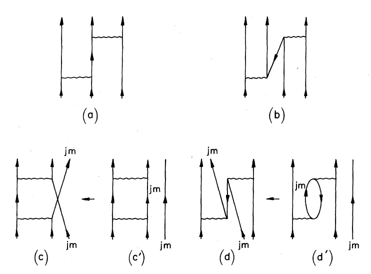


$$V_{NN}(k,k') \rightarrow V_{\mathrm{low-k}}(k,k') = \Omega^{-1} V_{NN}(k,k') \Omega$$

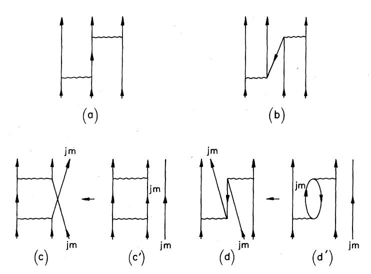
Consistently, we transform the $0\nu\beta\beta$ operator by way of the same similarity transformation Ω

$$\Theta(k,k') \rightarrow \Theta_{\mathrm{low-k}}(k,k') = \Omega^{-1} \Theta(k,k') \Omega$$

The Pauli blocking effect



The Pauli blocking effect



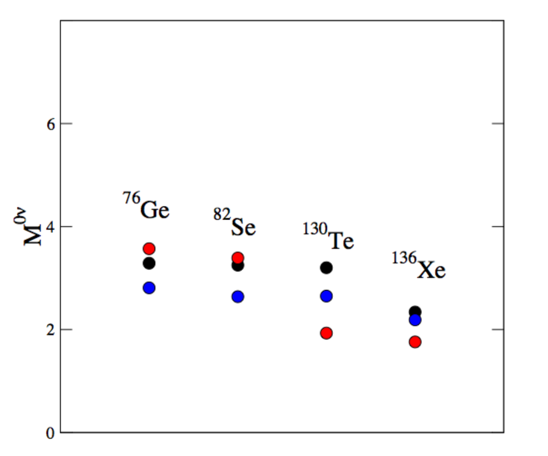
Two-body operator ⇒ Three-body operator

We calculate three-body diagrams and sum over one of the incoming/outcoming nucleons

⇓

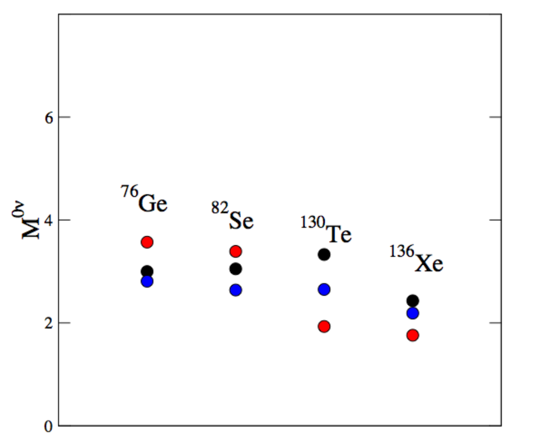
nucleus-dependent effective operator

Shell model calculations of $M^{0\nu}$



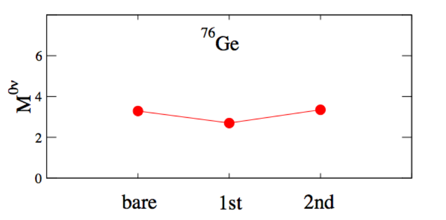
- Blue dots: Madrid-Strasbourg group, $0\nu\beta\beta$ operator
- Red dots: Horoi *et al.*, $0\nu\beta\beta$ operator
- Black dots: RSM, bare $0\nu\beta\beta$ operator

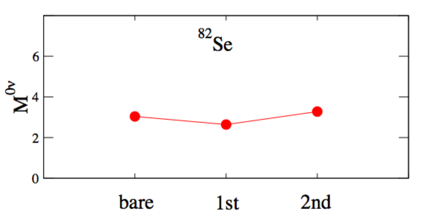
Shell model calculations of $M^{0\nu}$

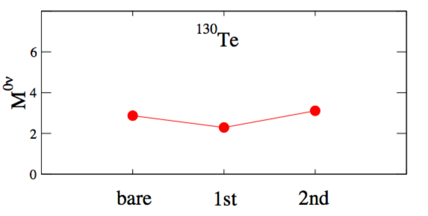


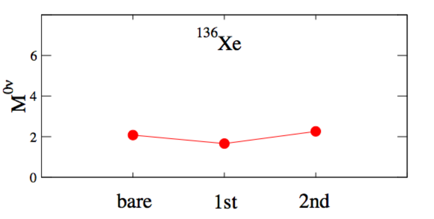
- Blue dots: Madrid-Strasbourg group, $0\nu\beta\beta$ operator
- Red dots: Horoi *et al.*, $0\nu\beta\beta$ operator
- Black dots: RSM, effective $0\nu\beta\beta$ operator

Shell model calculations of $M^{0\nu}$

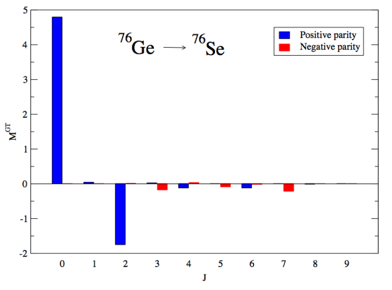


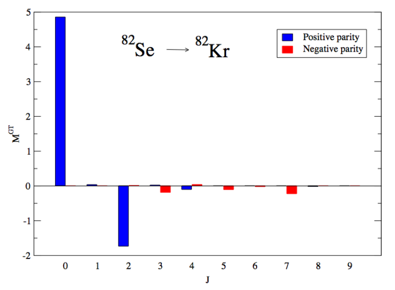


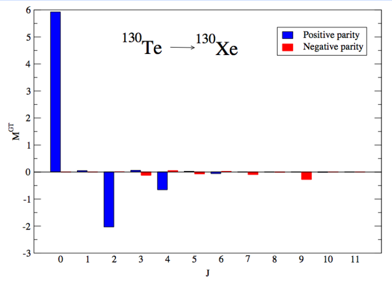


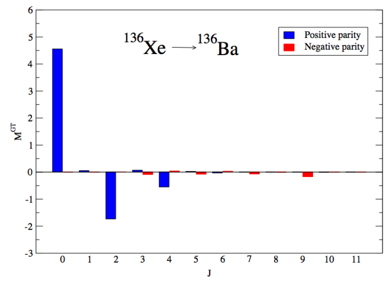


J-pair decomposition

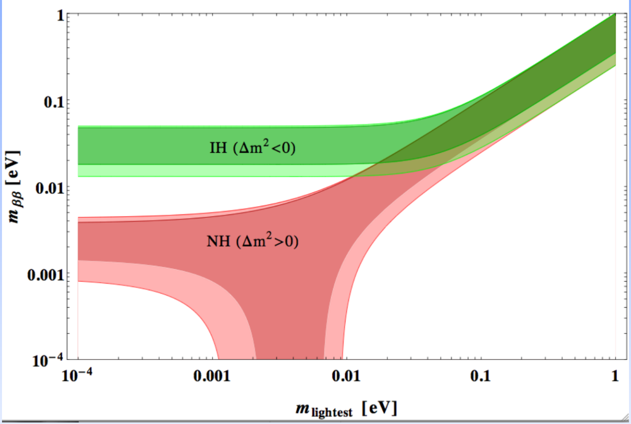




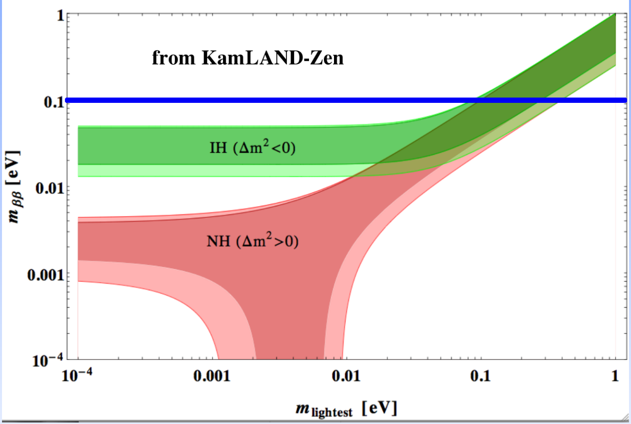




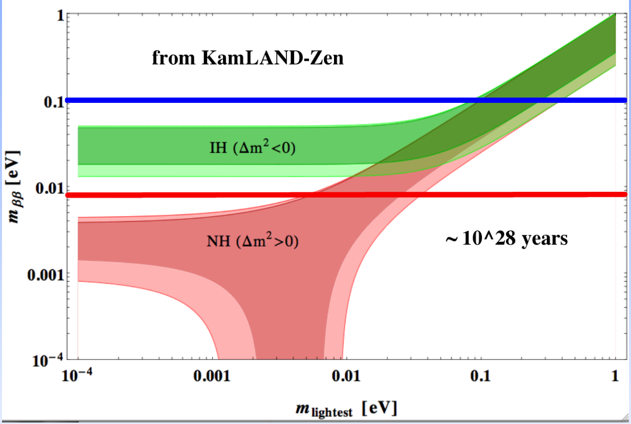
Experimental upper bounds & Sensitivity



Experimental upper bounds & Sensitivity



Experimental upper bounds & Sensitivity



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Perspectives

- H_{eff} derived from chiral two- and three-body potentials: effects of chiral two-body currents (for both $2\nu\beta\beta$ and $0\nu\beta\beta$ decays)
- Beyond closure approximation
- Blocking effect at higher order

Neutrinoless Double-Beta Decay and Realistic Shell Model

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