Structure and reactions of N=7 isotones: parity inversion and transfer.

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Spirit of the talk: Mean Field at the limits

Mean Field is probably the most genuine Nuclear Structure concept. Intimately linked to the collective behavior of nuclei, it is expected to emerge as a useful tool when the number of nucleons increases.

On the other hand, Nuclear Field Theory (NFT) is the systematic perturbative procedure for improving the zero order Mean Field description towards the exact description of nuclei, by incorporating higher and higher order many body effects.

In this talk we explore how things are going in the **region of very light, one valence neutron N=7 isotones (from Z=3 to Z=6)**, and find that very good description of low-energy spectra and associated reaction processes is attainable as far as the NFT procedure is pursued **up to third order terms.**

It will be shown that a consistent description of

O spectra,
O one neutron transfer absolute dσ/dEdΩ
O dipole transitions and
O isotopic rms charge radius change

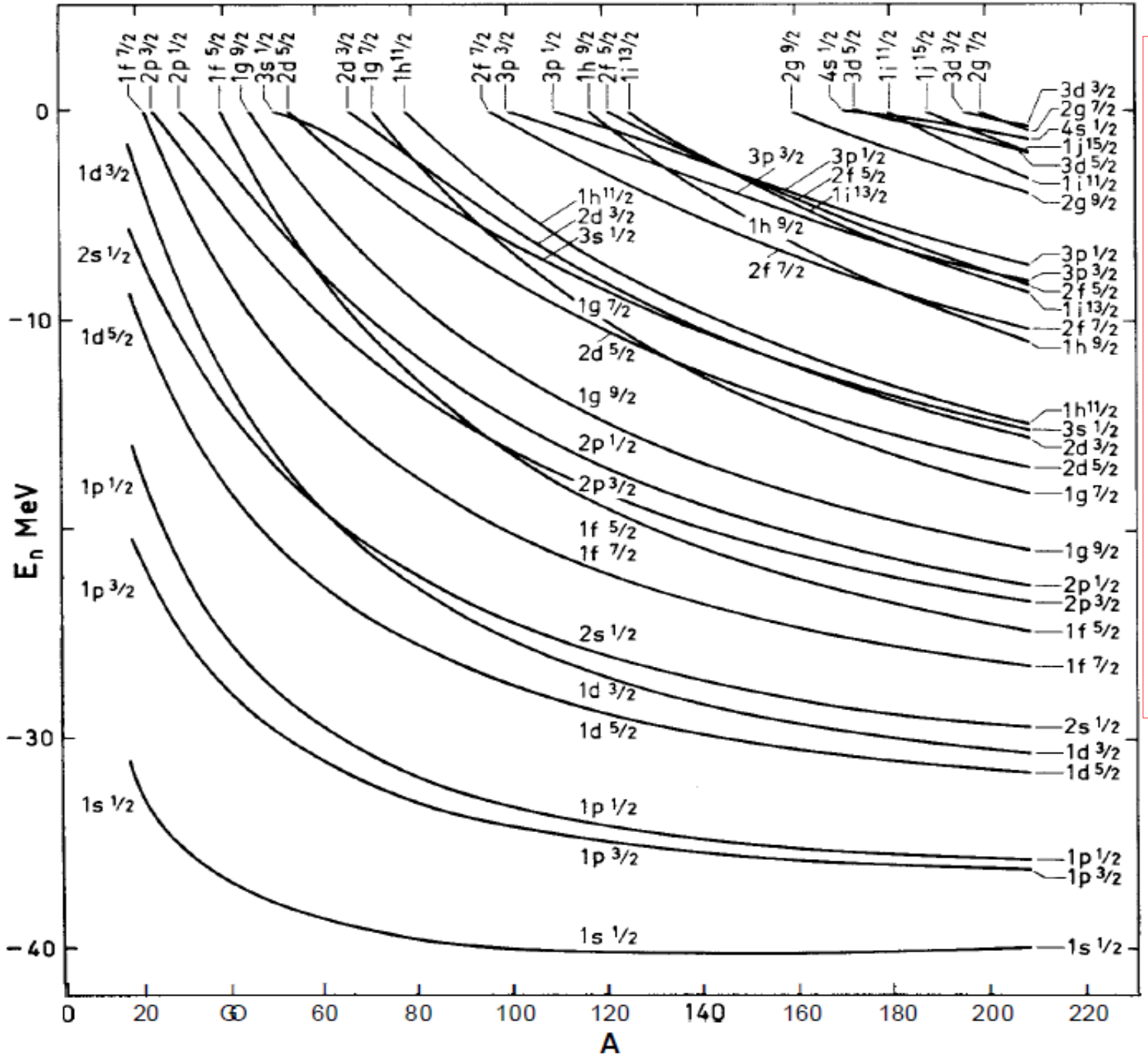
It is obtained by including

+ up to "**3p-2h"** configurations, **and**

+ coupling to the **single-particle continuum.**

It will be stressed that antisymmetrization (**Pauli principle**) between the valence neutron and core, plays a crucial role.

By incorporating coupling to the **single-particle continuum**, a **common framework for the study of structure and reactions**, based on the Nuclear Field Theory emerges.
A formulation in terms of **Generalized Coupled Channels allows for the proper inclusion of Pauli principle.**



(B&M *Nucl..Struct., Vol. I*)

WS potential

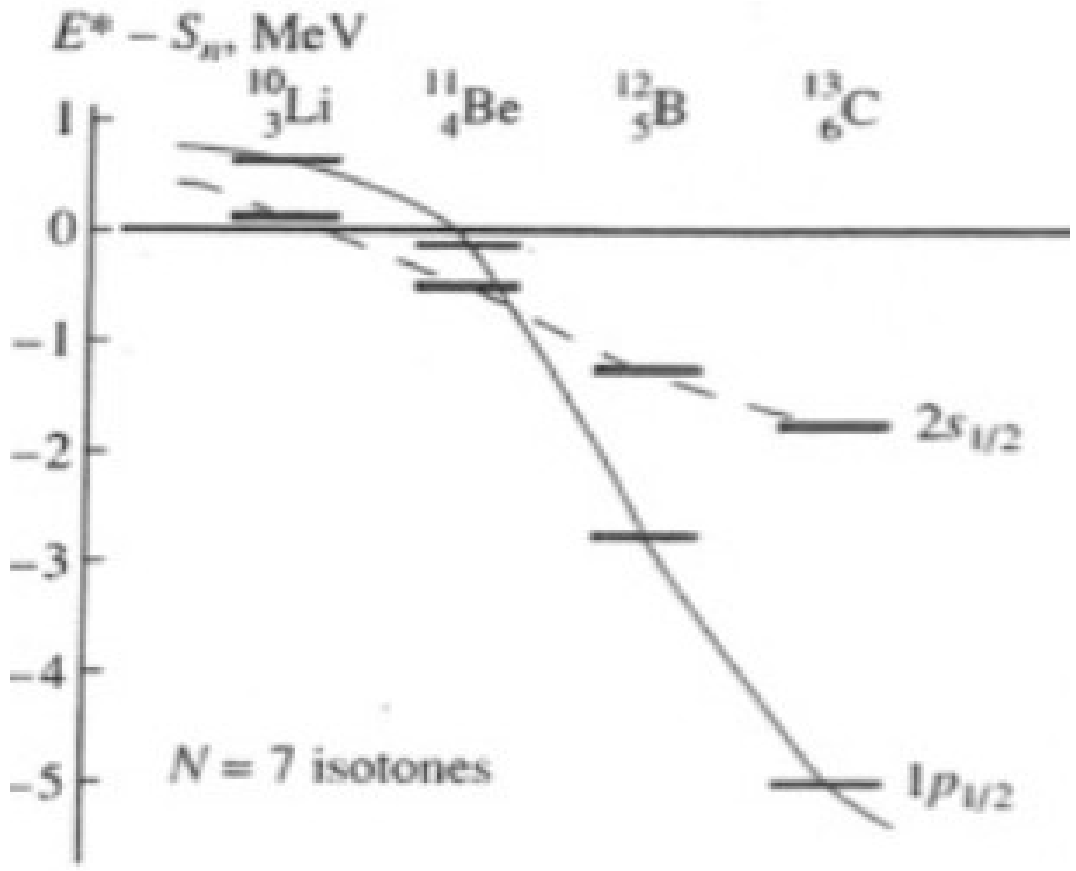
V=-51+-33(N-Z)/A (MeV)

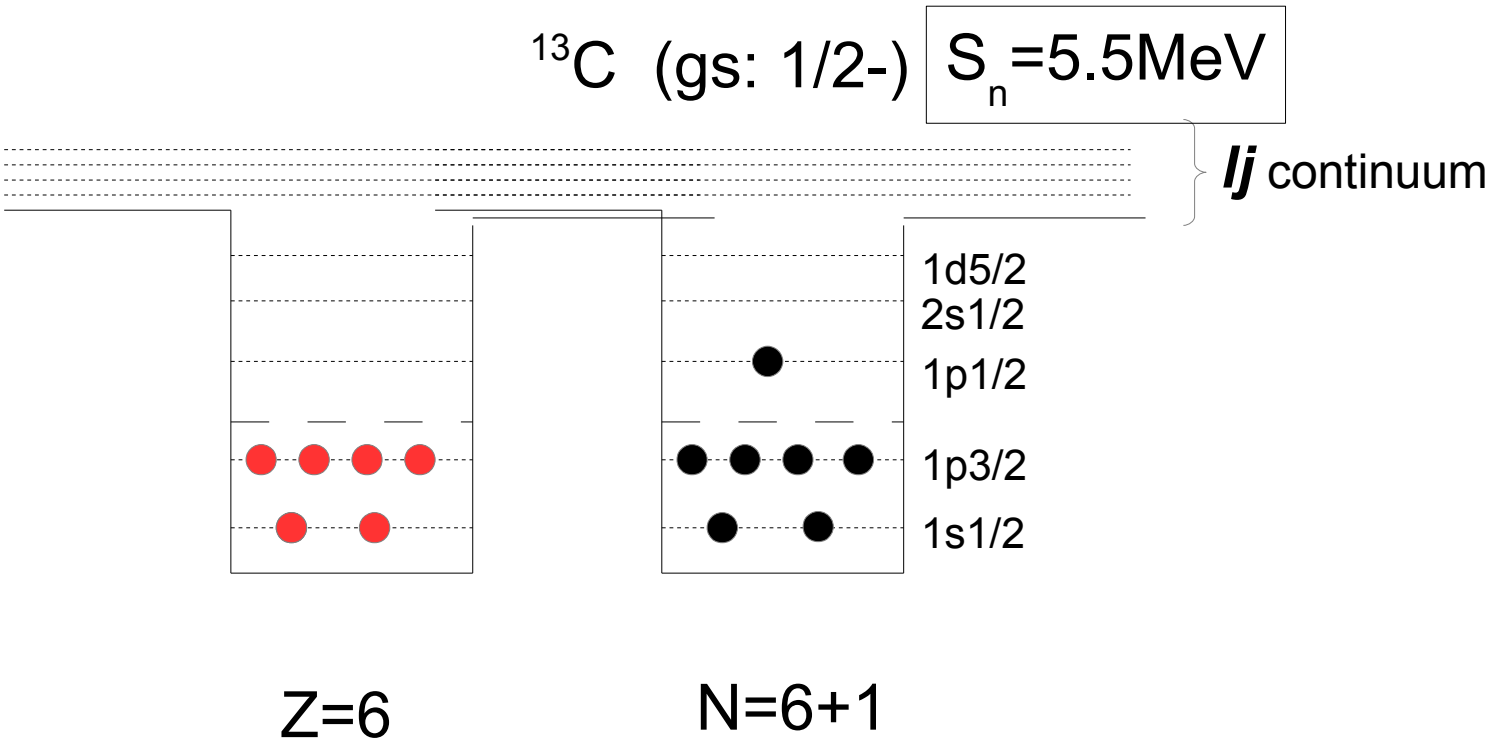
a=0.67fm

$R=1.27A^{1/3}$ fm

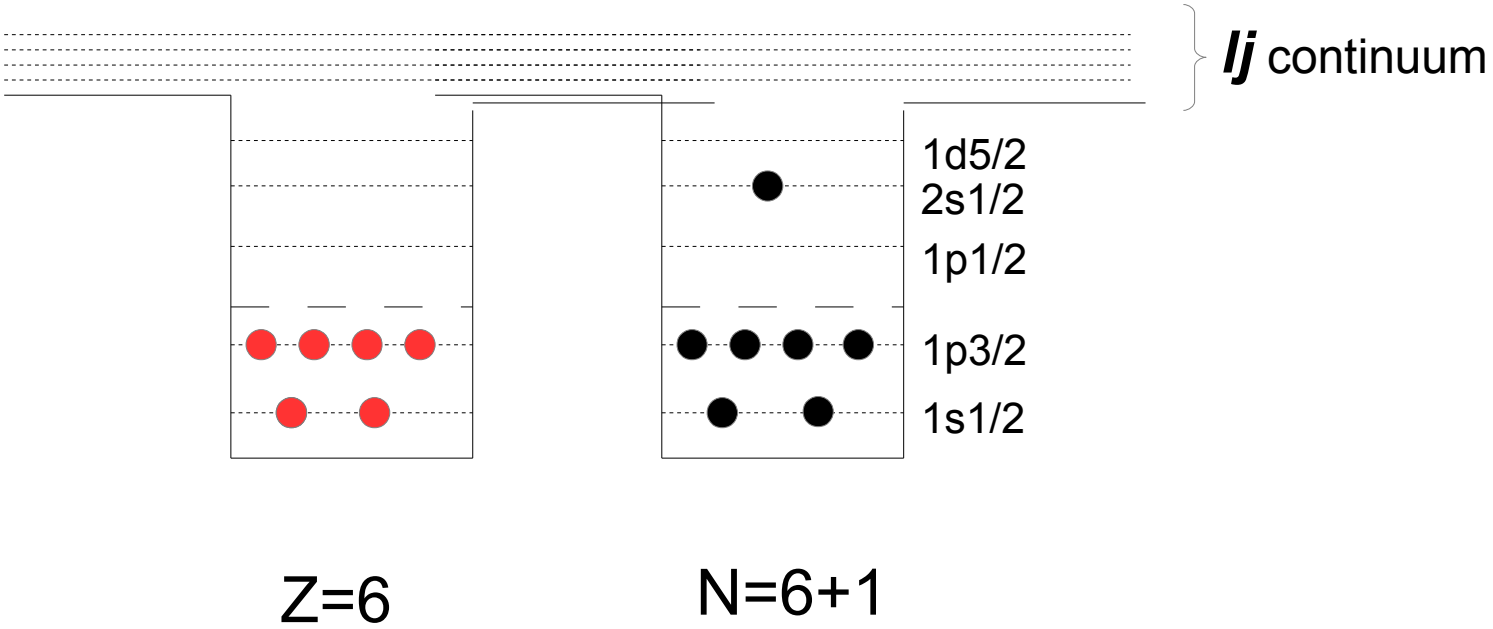
Vls=0.44V

N=7 isotnes, from stability valley to drip line

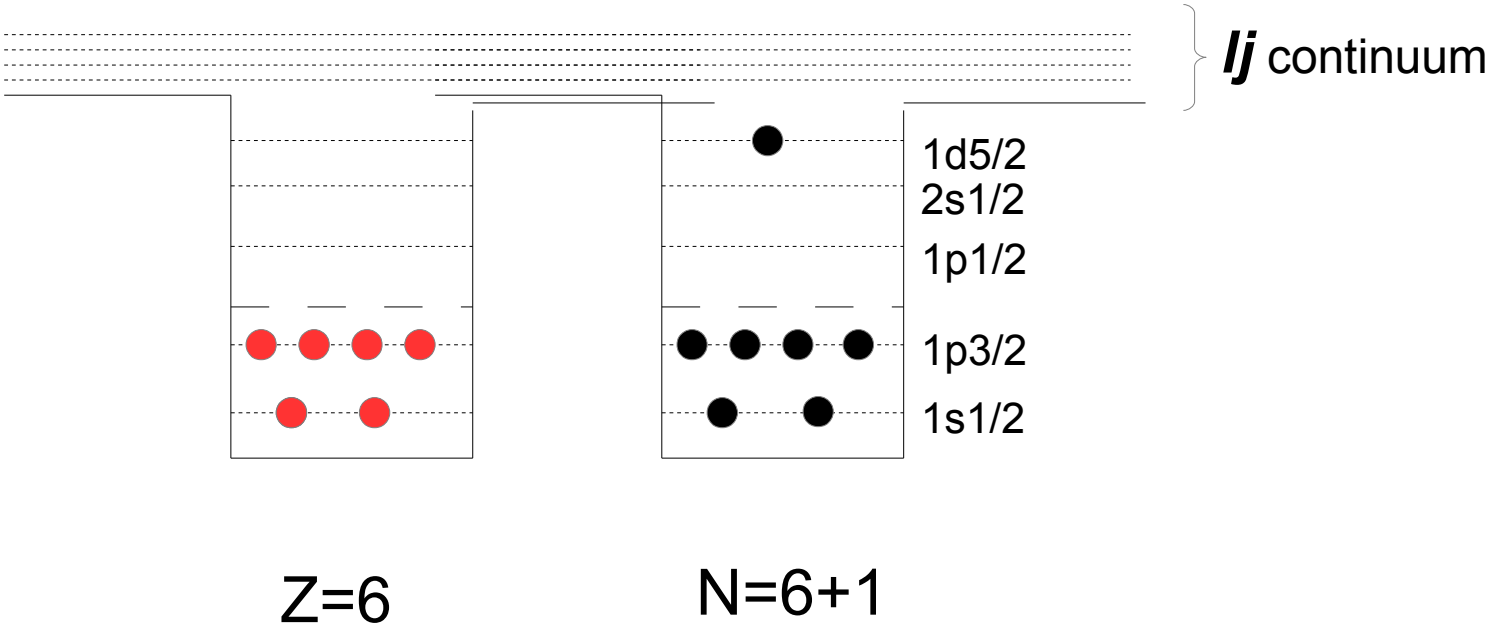


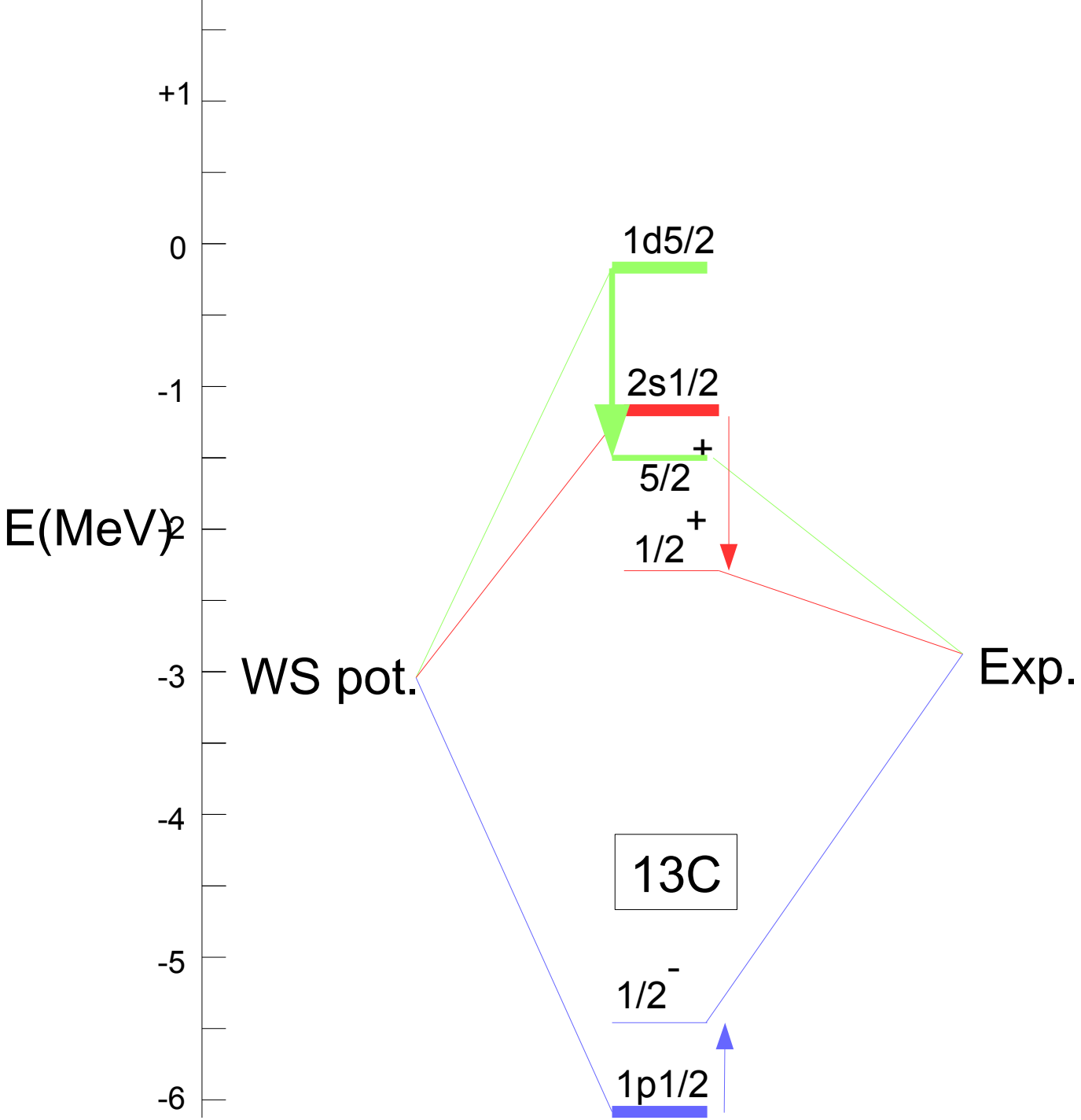

^{13}C (gs: 1/2-)
S_n=5.5MeV
lj continuum
1d5/2
2s1/2
1p1/2
1p3/2
1s1/2
Z=6
N=6+1

^{13}C (1^{st} exc.: 1/2+)

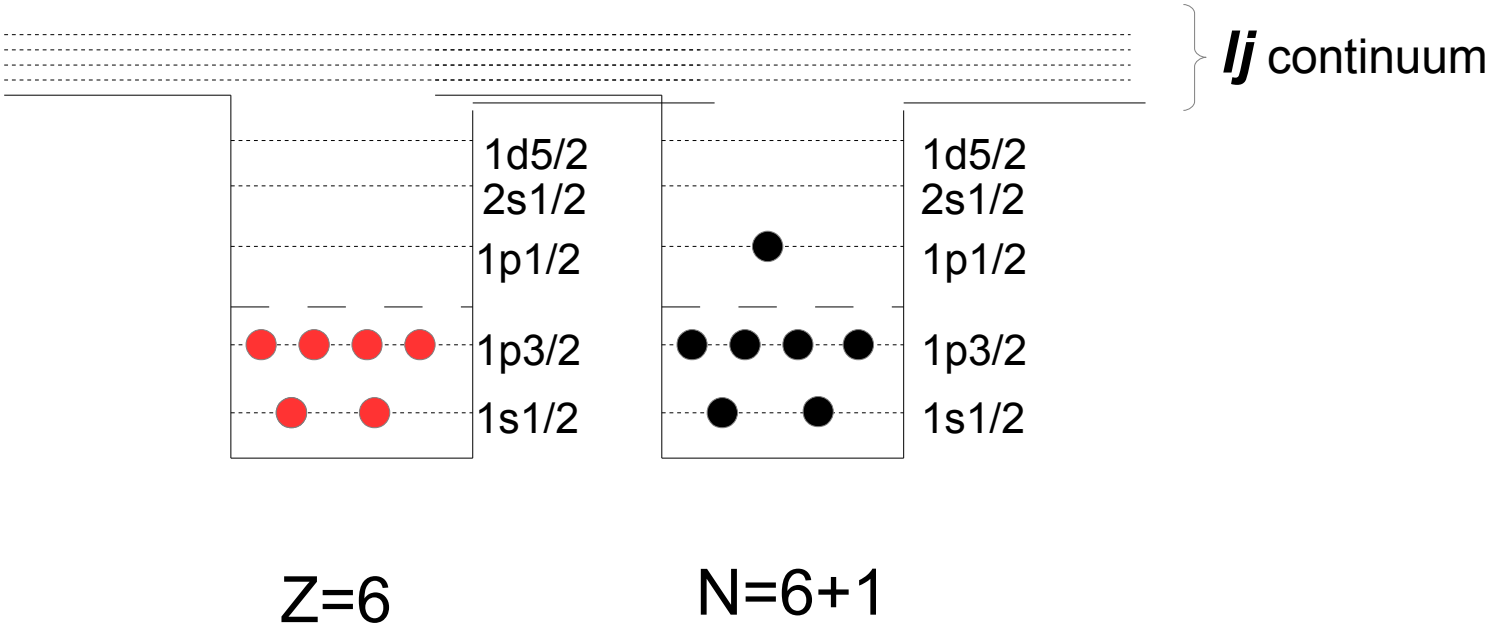


^{13}C (2^{nd} exc.: 5/2+)




+1
0
-1
-2
-3
-4
-5
-6
E(MeV)
WS pot.
Exp.
1d5/2
2s1/2
5/2+
1/2+
13C
1/2-
1p1/2

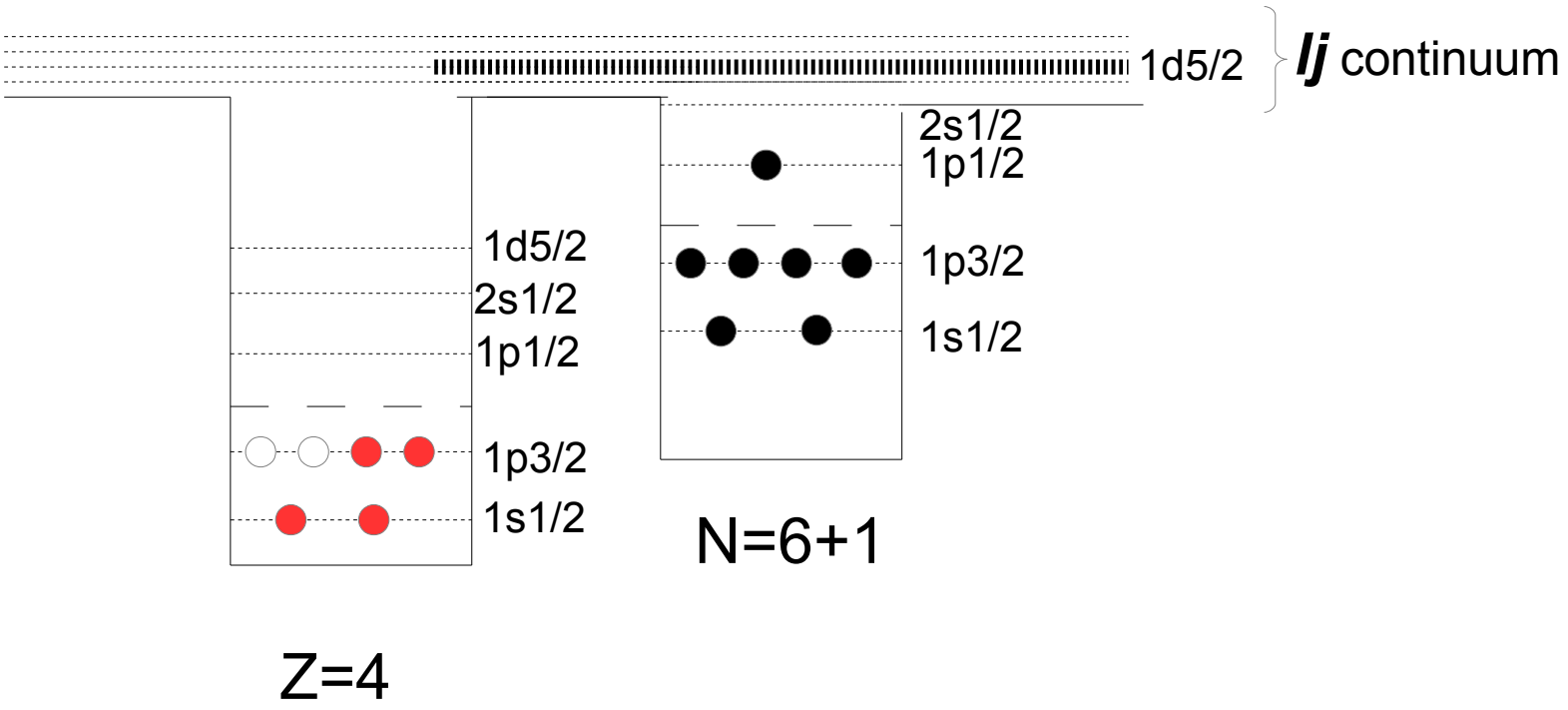
^{13}C (gs: 1/2-)



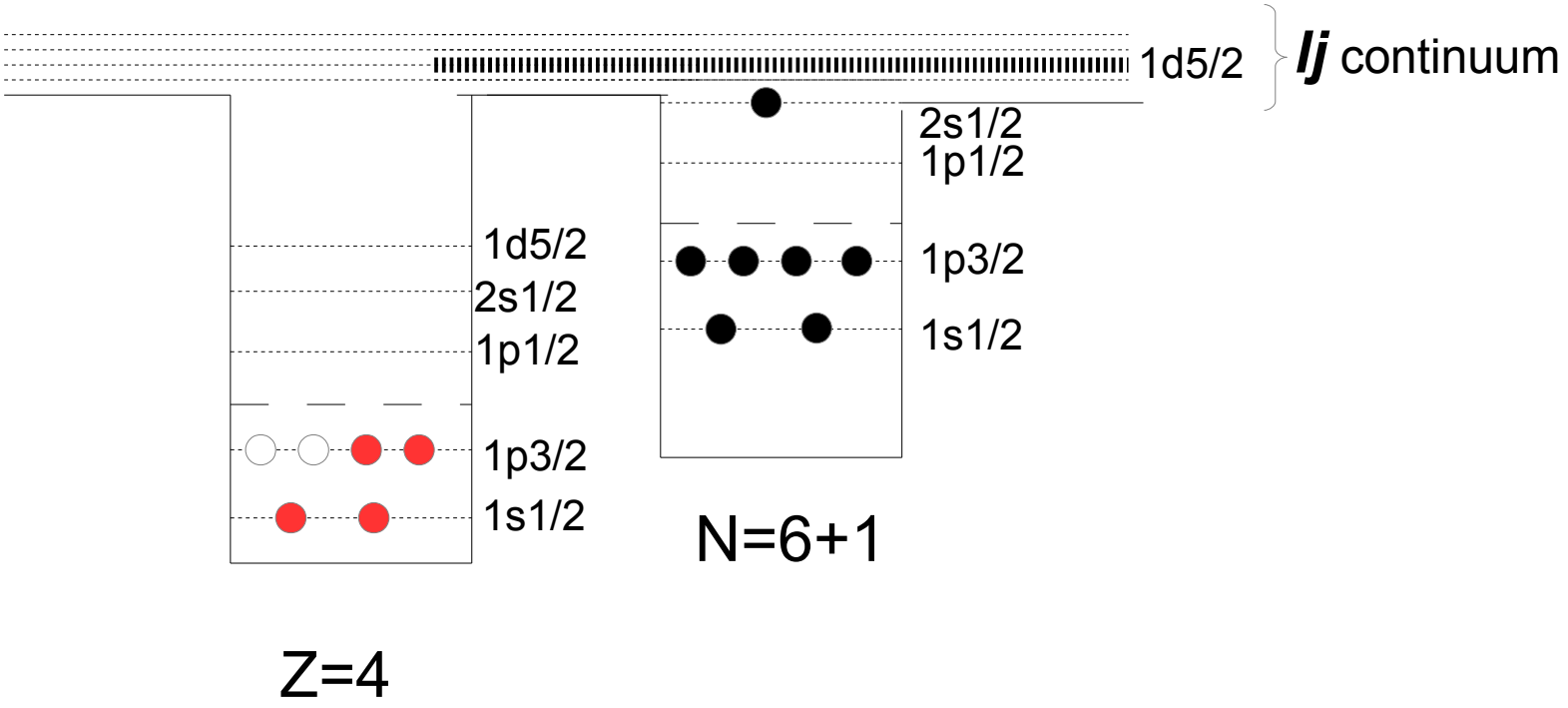
Z=6

N=6+1

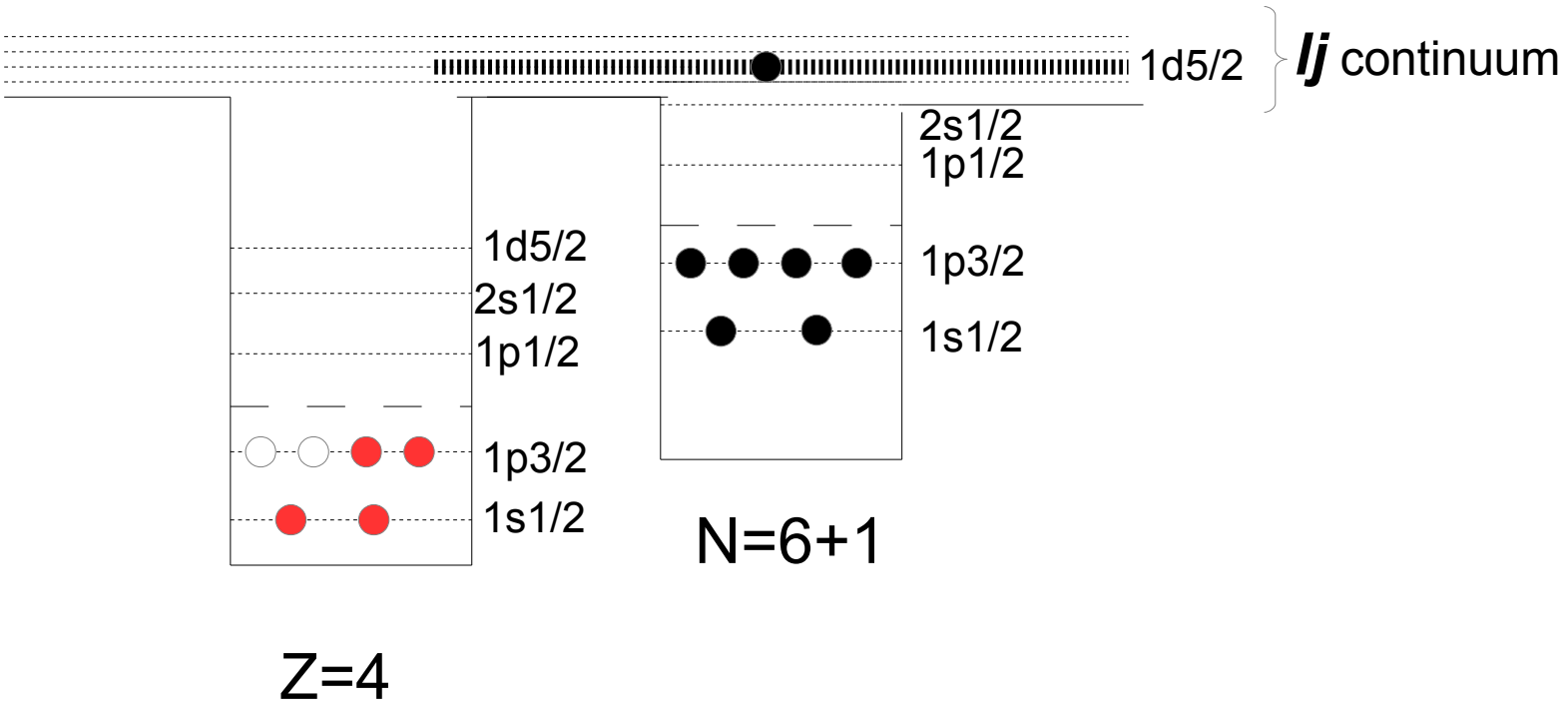
^{11}Be (gs: 1/2-)


1d5/2
lj continuum
2s1/2
1p1/2
1p3/2
1s1/2
N=6+1
1d5/2
2s1/2
1p1/2
1p3/2
1s1/2
Z=4

^{11}Be (1st exc.: 1/2+)

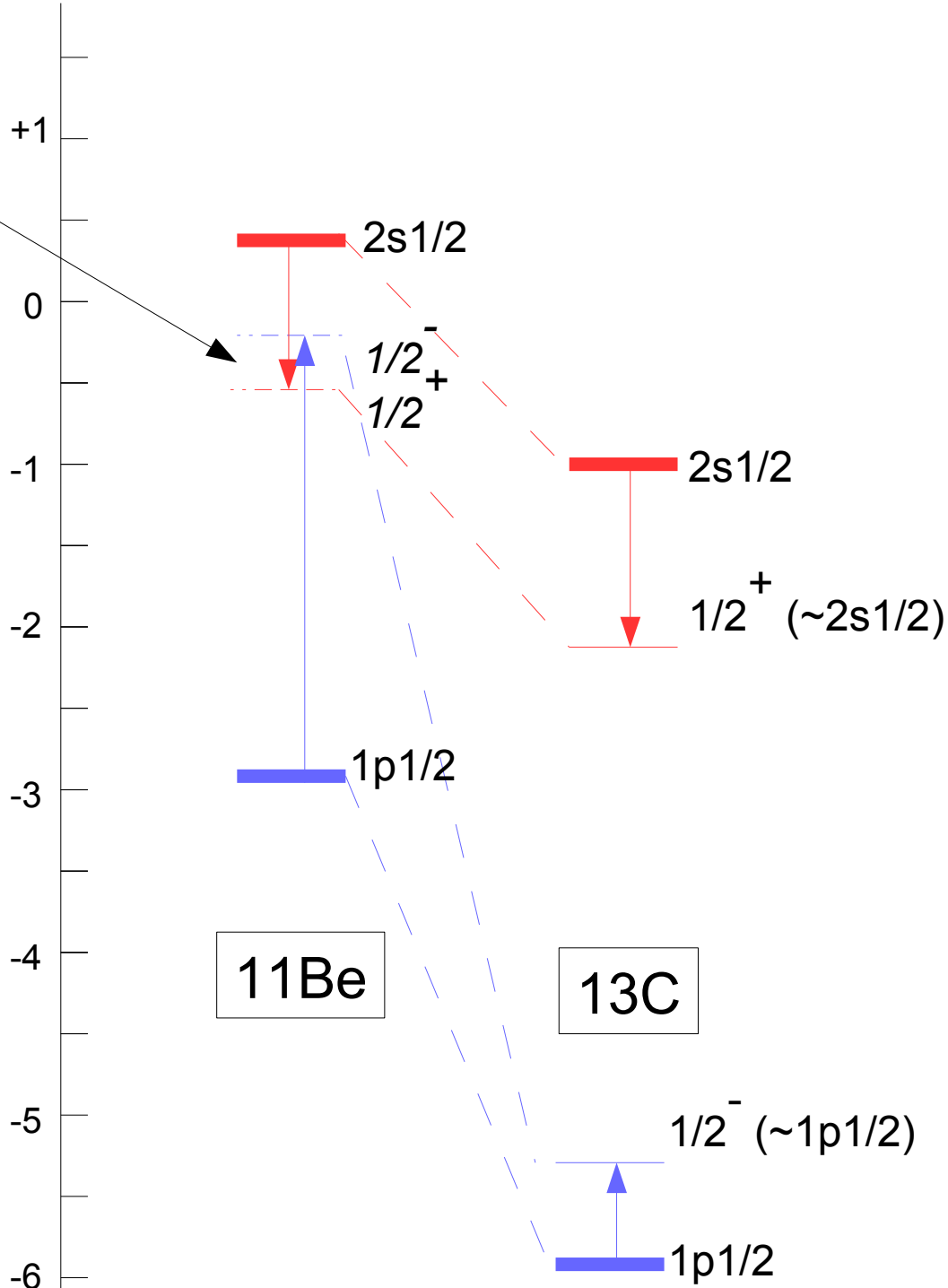


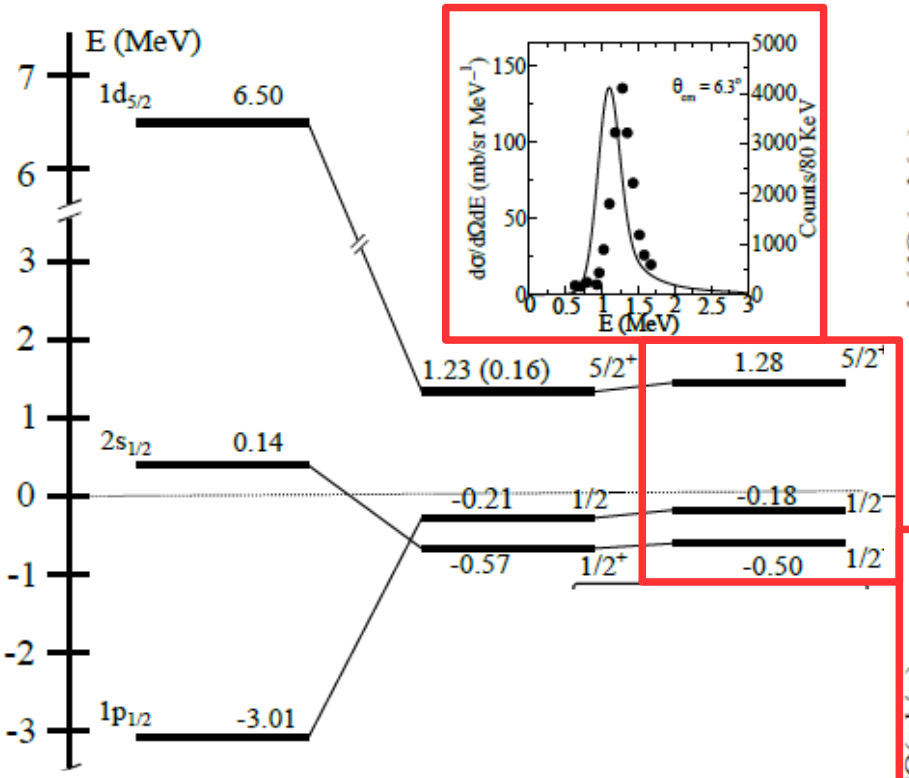
^{11}Be (2nd exc.: 5/2+)

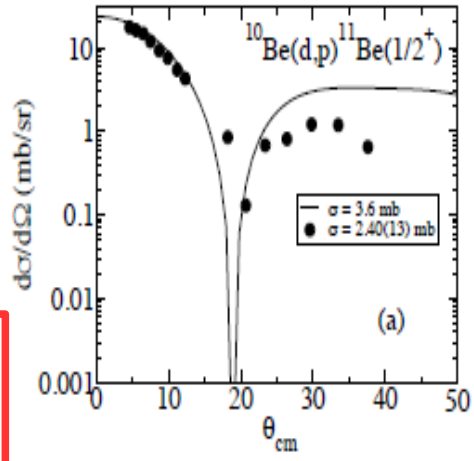


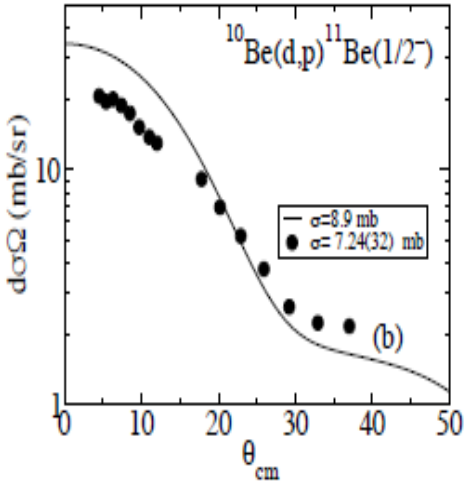
Parity inversion.

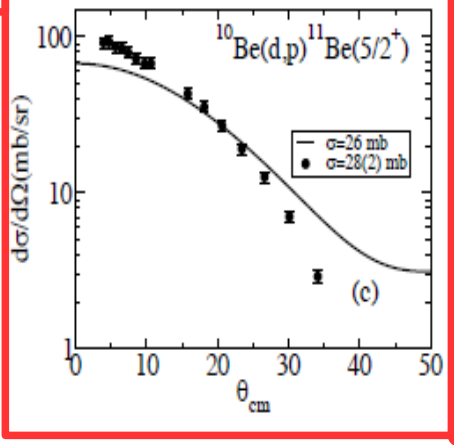
No spherical mean field can reproduce it

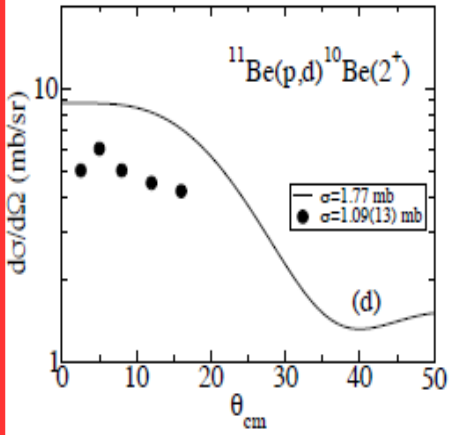


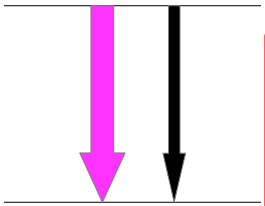












$B_{exp}(E1)=0.10\ e^2\ fm^2$

$B_{bare}(E1)=0.29\ e^2\ fm^2$

10Be Quadrupole moment much smaller than rigid body (about 30%).

$\Delta\langle r^2\rangle^{1/2}_{11Be}$ =0.11 fm / 0.27 fm

HOW TO IMPROVE SIMPLE (SPHERICAL STATIC) ZERO ORDER MEAN FIELD DESCRIPTION?

Include quadrupole deformed mean field potential

+*Static/rigid and strong coupling limt (A.Macchiavelli)*

+*Static/rigid and weak coupling (F.Nunes; A. Moro)*

+*Dynamic and weak/medium coupling (This work,Esbensen&Sagawa)*

HOW TO IMPROVE SIMPLE ZERO ORDER MEAN FIELD DESCRIPTION?

Nuclear Fierld Theory (NFT)

shows how to construct an ***EXACT*** solution of the many body problem, for a given two body force, in terms of a **rapidly convergent series expansion** for each observable, similar to the Feynman's QED diagrams expansion, and **based on the use of realistic (coherent-collective) core excitations** rather than independent 1p-1h, 2p-2h,... configurations, that is in the **dynamical mean field response rather than in the static (zero order) one.**

1. Fermions: Neutron moving in a HF mean field
2. Bosons: Core realistic excited states; RPA Vibrational like
3. Fermion-Boson vertex: Microscopically evaluated from RPA and HF
4. Expansion parameter: inverse of effective degeneracy, Ω.

Empirical NFT: fix Fermions, Vibrations and Vertex from experiment in order to give a unified description of “ALL(II)” data of “ALL” nuclei

PVC: Vibrational Core (even-even) + One particle (neutron)

$$H = H_{sp} + H_{coll} + H_{PVC}$$

?

$$H_{sp} = -d/d\vec{r}\,\hbar^2/2\mu(r)\,d/d\vec{r} + V(r) + V_{ls}(r)$$

$$H_{coll} = \sum_{\lambda\mu\nu} \hbar\omega_{\lambda,\nu}\left[\Gamma^{+.}_{\lambda\mu,\nu}\Gamma_{\lambda\mu,\nu} + 1/2\right]$$

Dynamic deformed nucleus

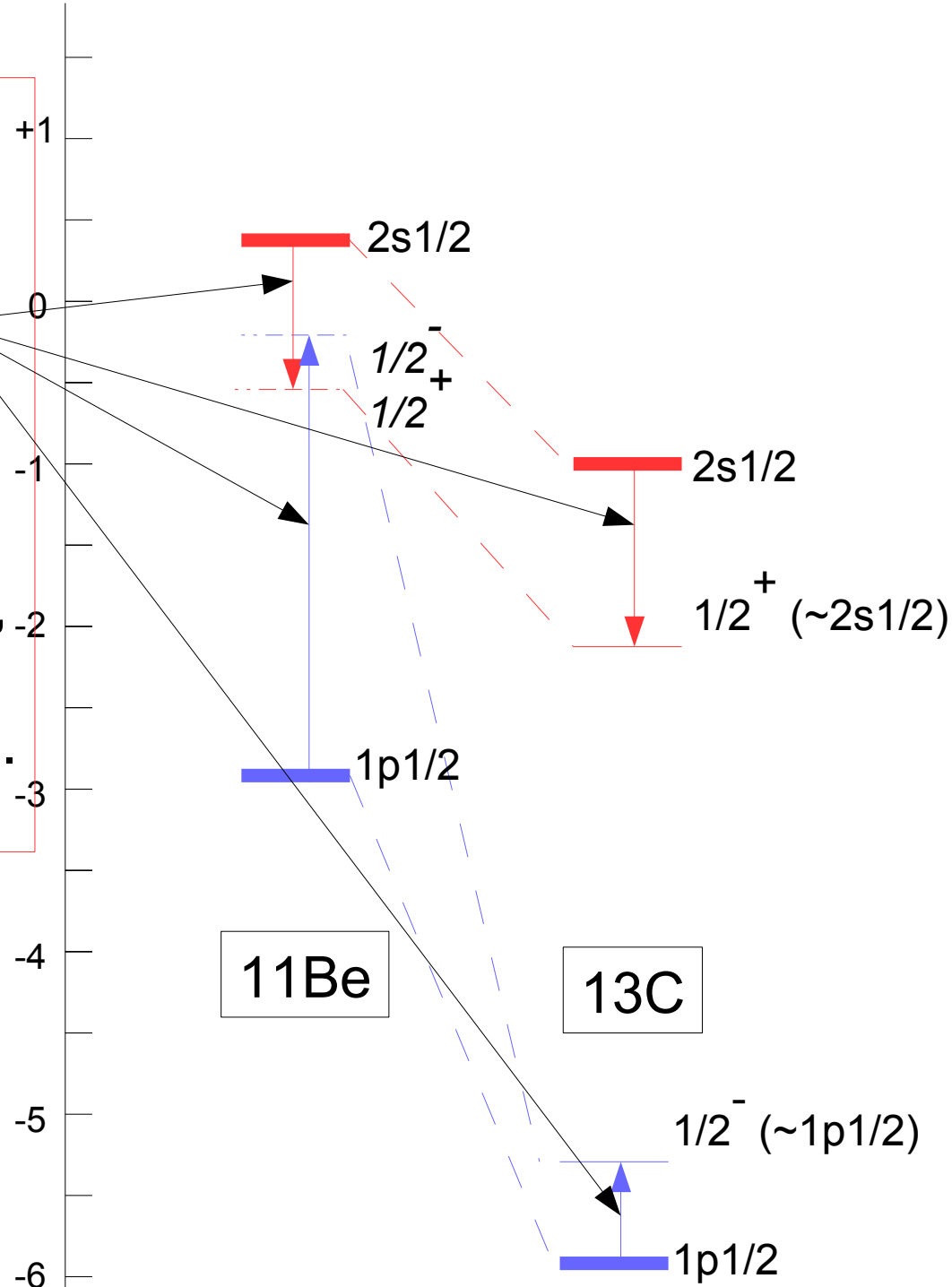
$$H_{PVC} = \sum_{\lambda\mu\nu} \delta V_{\lambda\nu}(r)\, Y_{\lambda\mu}(\hat{r})\left[\Gamma^{+.}_{\lambda\mu,\nu} + (-1)^{\mu}\Gamma_{\lambda\mu,\nu}\right]$$

$$where\ \delta V_{\lambda\nu}(r) \approx -r\,dV/dr\,\beta_{\lambda,\nu}$$

$$\mu(r) = \mu_0 - \frac{0.2\mathrm{M}}{1+\exp(r-R)/a}\,;\ \mu_0 = \frac{A}{(A+1)}M$$

Where λμ is the excited core angular momentum and z-projection, v labels the different states with the same λμ. For the collective states the form factor in PVC is the semiclassic one.

The idea is to determine a new Mean Field consistent with the PVC effects that correctly describe the data:
Single-part. Energies,
Spectroscopic Factors,
E-M transitions,
Exc. states population..



Coupled channels equations.

$$\Psi_{j_a m_a} = \left[\psi^{x}_{j_a m_a} + \left[\psi^{C}_{jb} \cdot \Gamma^{+.}_{\lambda} \right]_{j_a m_a} \right] \Phi_{GS}$$

particle x phonon (vibrational quanta) configuration.

but if

$$\Psi_{GS} = \left[1 + \varepsilon \left[\psi^{-1}_{ja,occ} \left[\psi^{x}_{jb} \cdot \Gamma^{+.}_{\lambda} \right]_{ja} \right]_{J=0} \right] \Phi^{HF}_{GS}$$

Creation of a neutron in an "occupied" level jc levels + phonon anihilation may/will give a non nule contribution

a new term can / must be added

$$\Psi_{j_a m_a} = \left[\psi^{x}_{j_a m_a} + \left[\psi^{C}_{b} \cdot \Gamma^{+.}_{\lambda} \right]_{j_a m_a} - \psi^{y}_{j_a m_a} - \left[\psi^{D}_{c} \cdot \Gamma_{\lambda} \right]_{j_a m_a} + \ldots \right] \Phi_{GS}$$

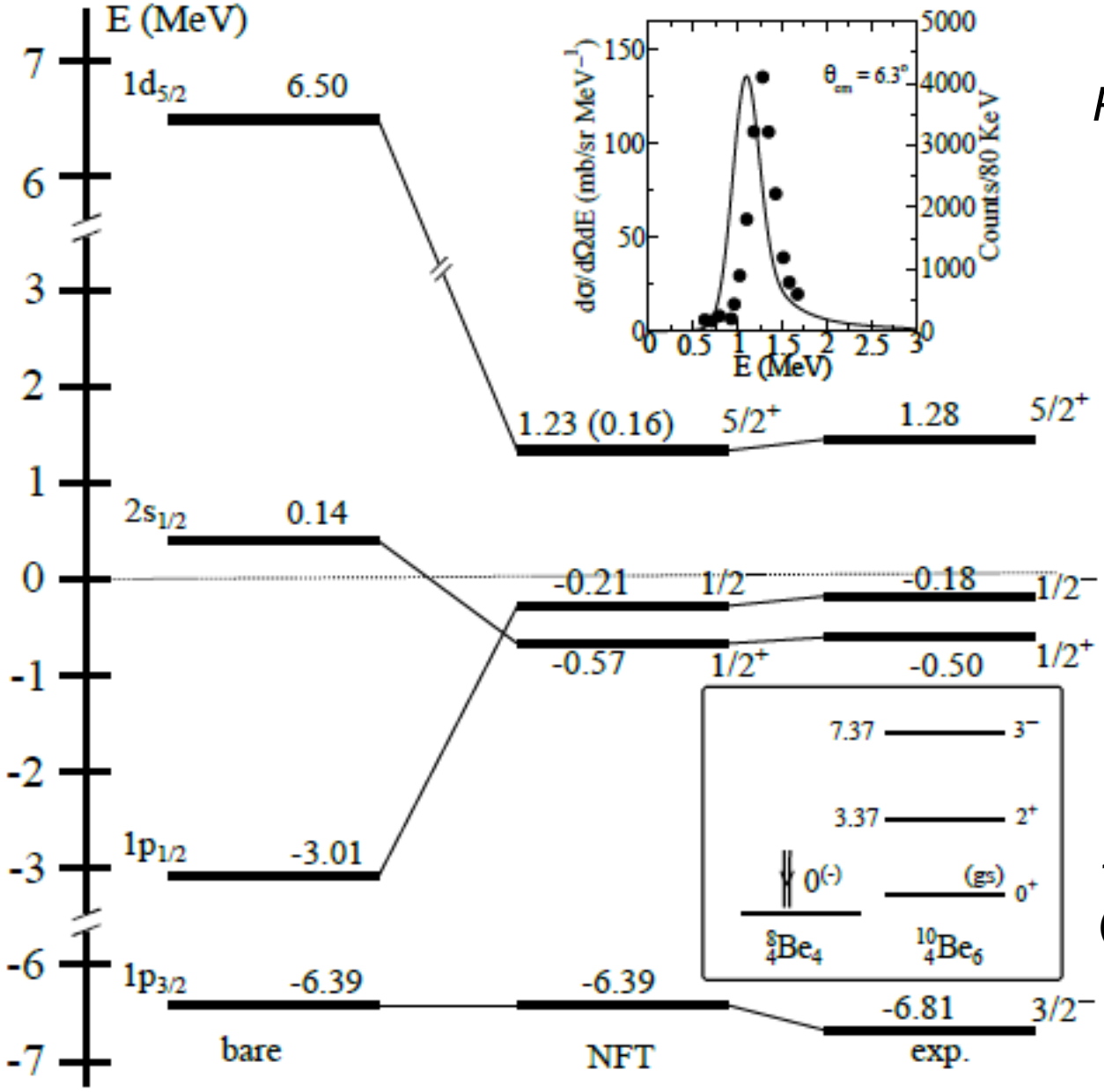
Creation of a neutron in an "occupied" level ja level may/will give a non nule contribution

GSC lead to auxiliary Coupled Channels

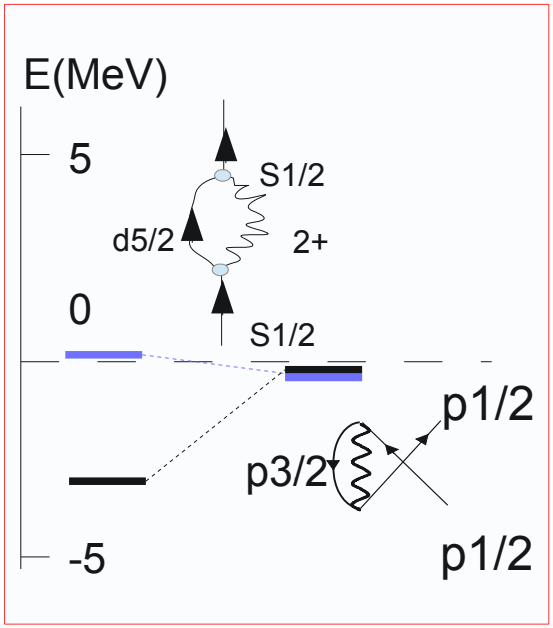
$$u^{x}_{a}(r) = \sum_i x_{ai} R^{WS}_{ai}(r); e_{ai} > e_F$$
$$u^{C}_{b}(r) = \sum_i C_{bi} R^{WS}_{bi}(r); e_{bi} > e_F$$
$$v^{D}_{c}(r) = \sum_i D_{ci} R^{WS}_{ci}(r); e_{ci} < e_F$$
$$v^{y}_{a}(r) = \sum_i y_{ai} R^{WS}_{ai}(r); e_{ai} < e_F$$

e_{cut}=100MeV

$$\begin{pmatrix} H_p - e_F & \Xi_{a,b\lambda} f(r) & 0 & \Xi_{a,c\lambda} f(r) \\ \Xi_{a,b\lambda} f(r) & H_p - e_F + \hbar\omega & \Xi_{a,b\lambda} f(r) & 0 \\ 0 & \Xi_{a,b\lambda} f(r) & (H_p - e_F) & -\Xi_{a,c\lambda} f(r) \\ \Xi_{a,c\lambda} f(r) & 0 & -\Xi_{a,c\lambda} f(r) & (H_p - e_F) - \hbar\omega \end{pmatrix} \begin{pmatrix} u^{x}_{a} \\ u^{C}_{b} \\ -v^{y}_{a} \\ -v^{D}_{c} \end{pmatrix} = \tilde{E} \begin{pmatrix} u^{x}_{a} \\ u^{C}_{b} \\ -v^{y}_{a} \\ -v^{D}_{c} \end{pmatrix}$$



*PRL**119**,082501(2017)*

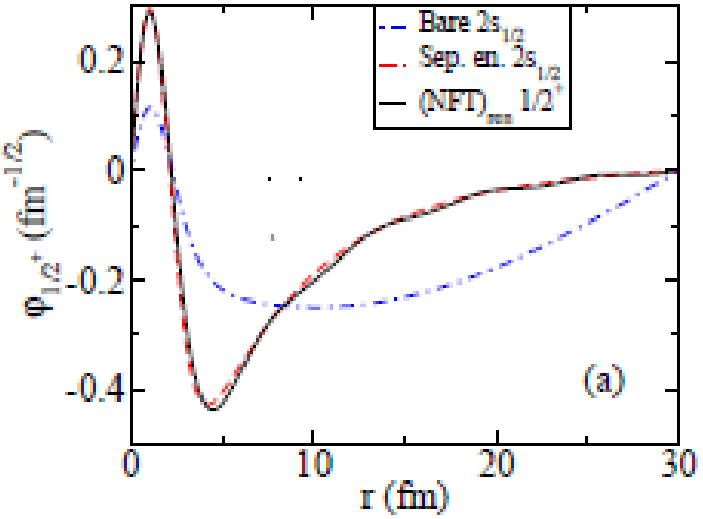


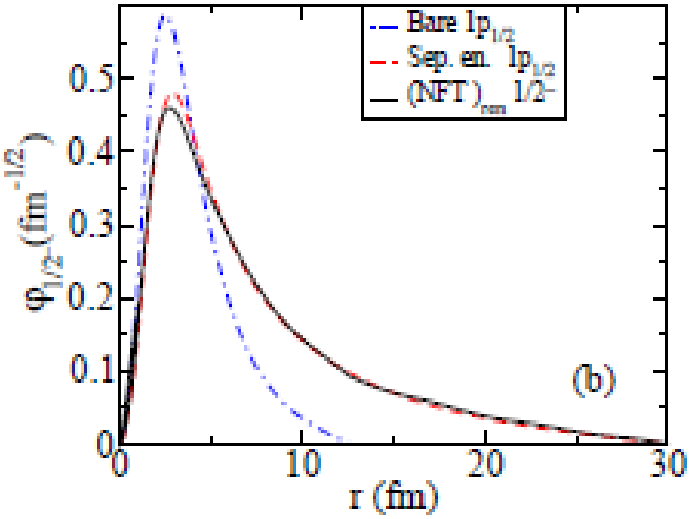
10Be Quadrupole moment (to be done)

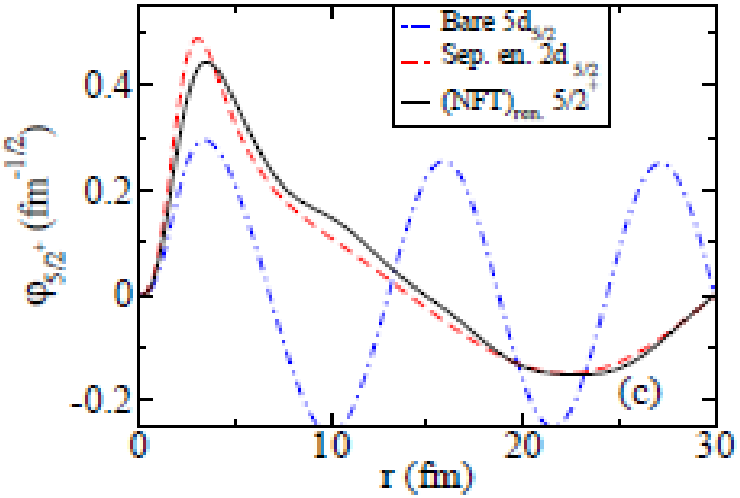
V0=-72MeV ; R=2.14fm ; a= 0.72

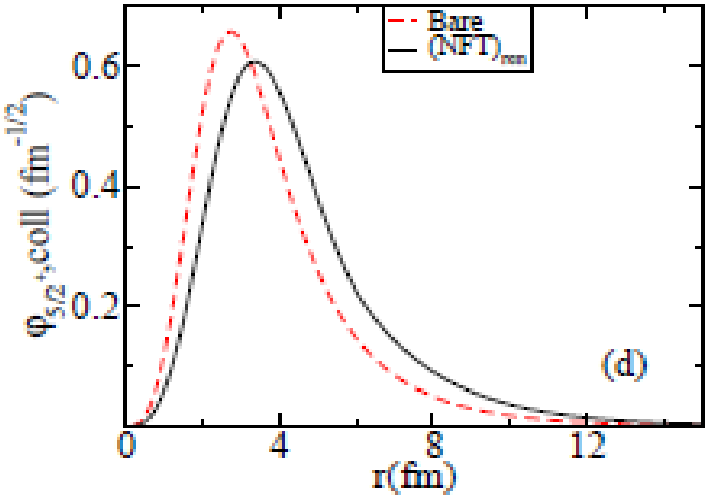
Waves/Transfer form factors

(see also PRC89,014333(2016),Lay et al.)









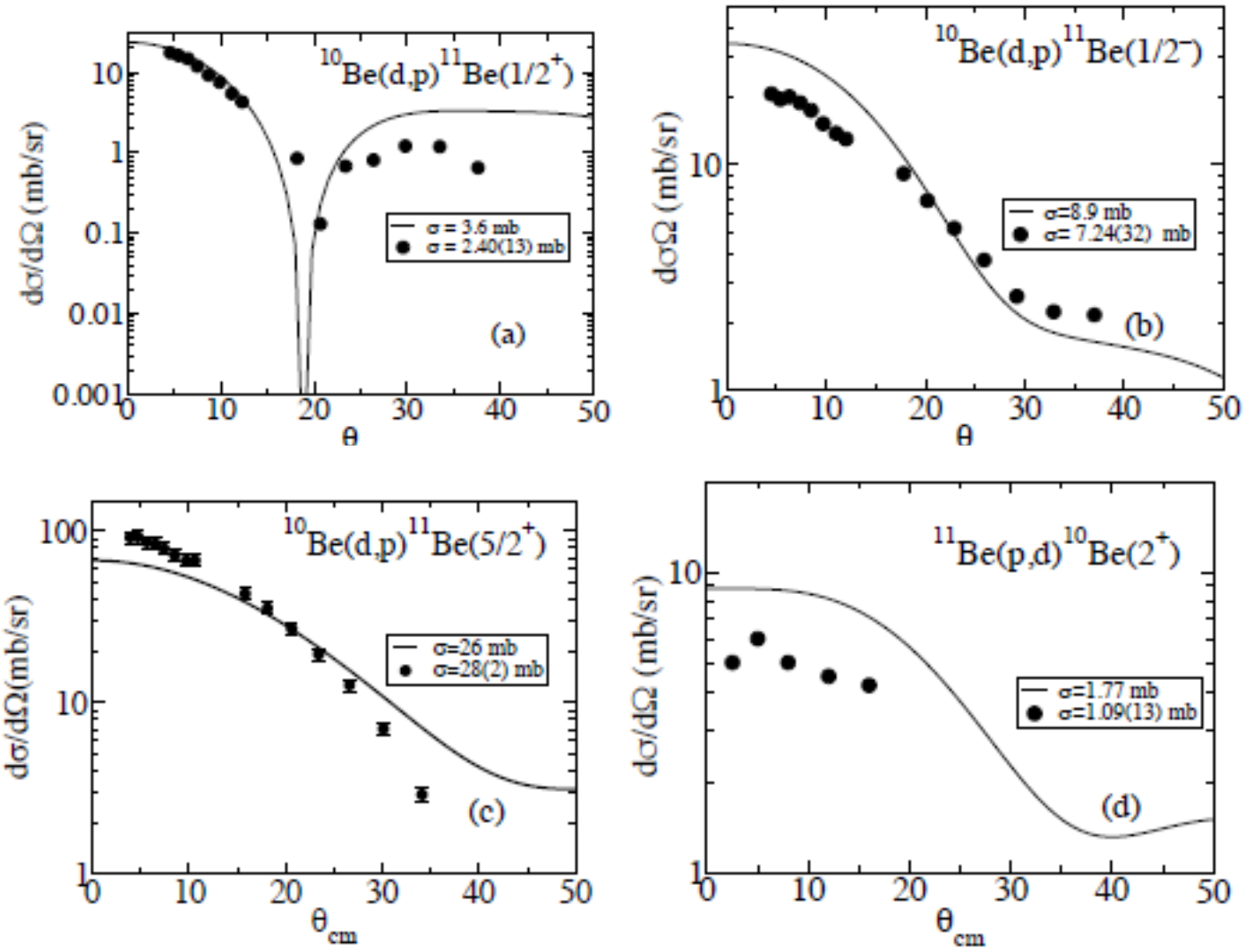
Charge Radii

$\Delta\langle r^2\rangle^{1/2}_{^{11}\mathrm{Be}-^{10}\mathrm{Be}} = 0.11$ fm --> *0.28 fm (exp. 0.27fm)*

Dipole transition

$B_{bare}(E1) = 0.29\ e^2\ \mathrm{fm}^2$ ---> *0.11 e^2 fm^2* (exp.0.10 e^2 fm^2)

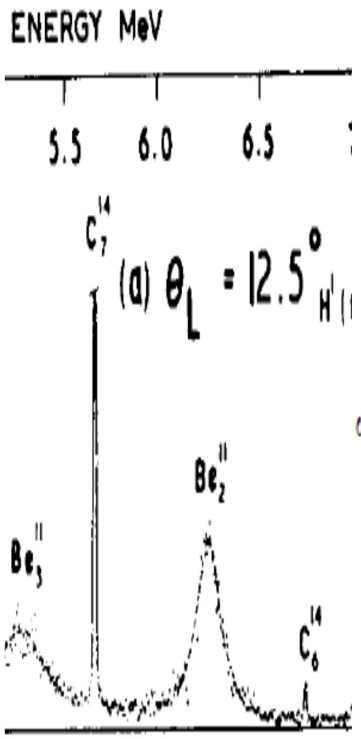
Theor.: *PRL**119**,082501(2017)* 10Be(d,p)11Be

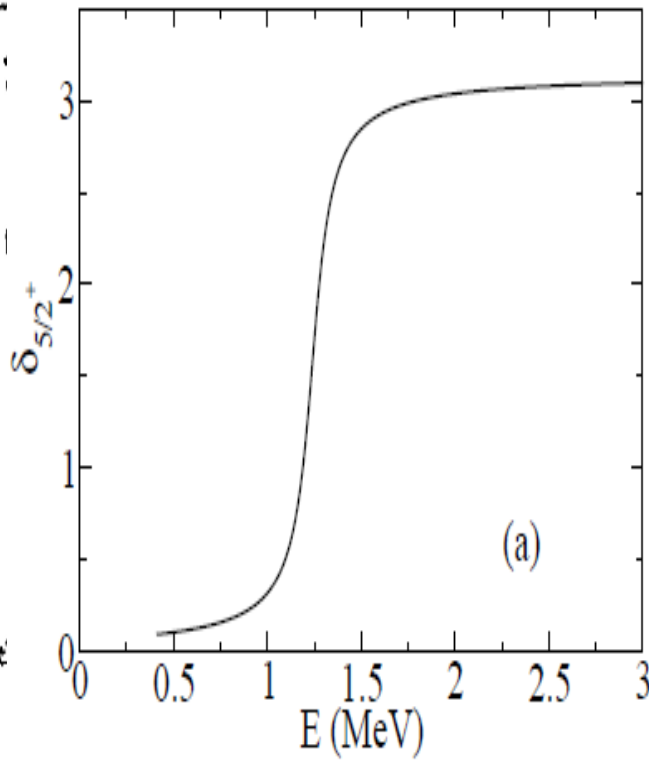


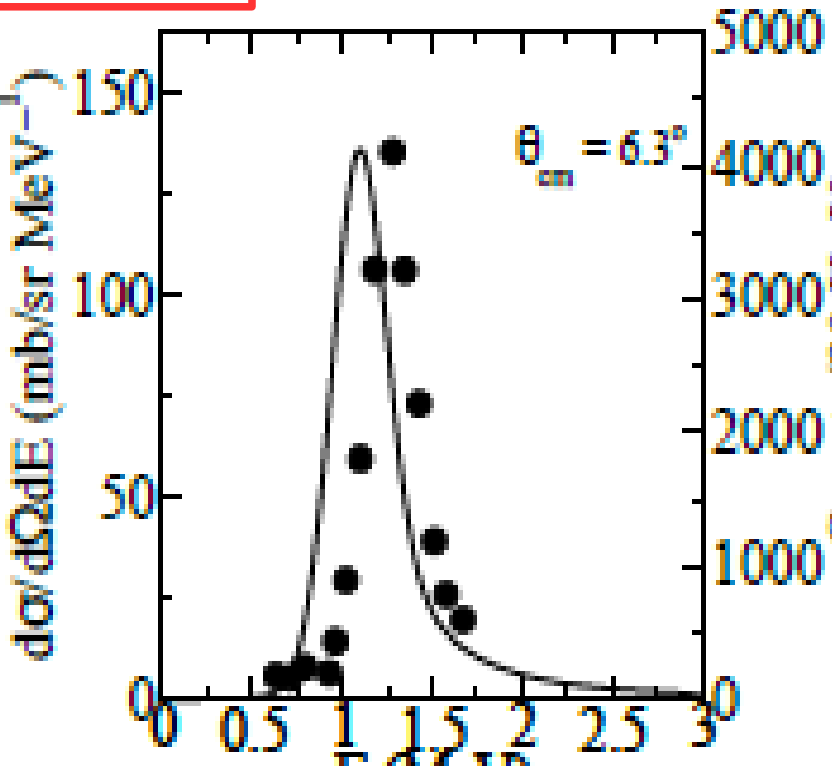
Data: Schmitt, PRC88,064612(2013); Winfield,NPA683,48(2001)

DWBA; Optical potential: Koening-Delaroche/Han et al.

Three FWHM's (keV)





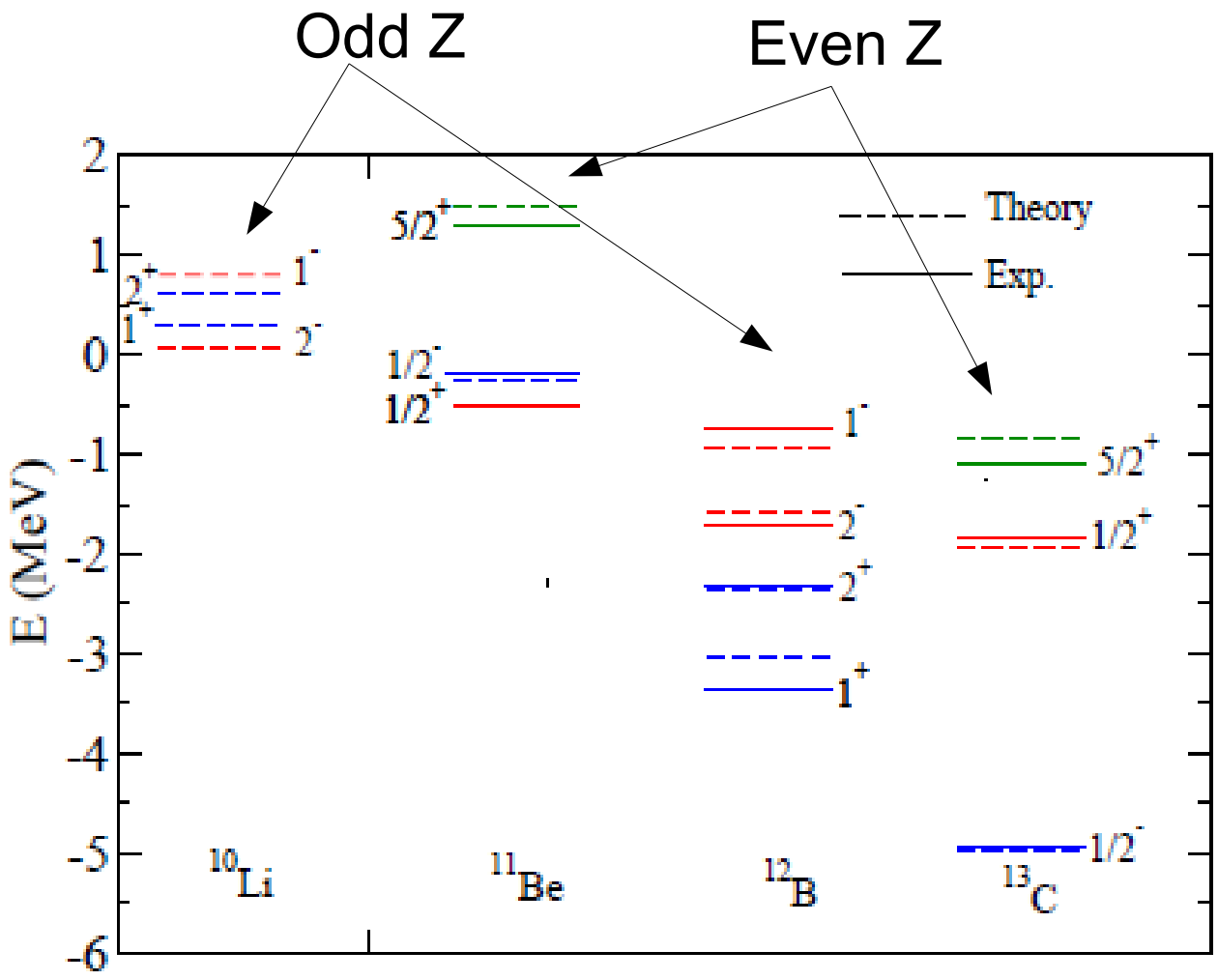


110+-20	///	160	///	300

9Be(t,p)11Be:
NP36(1962)1;
PRC42(1990)167

10Be(d,p)11Be:
points:PRC88(2013)04612

N=7 isotones results



Bare mean field potential for N=7 isotones

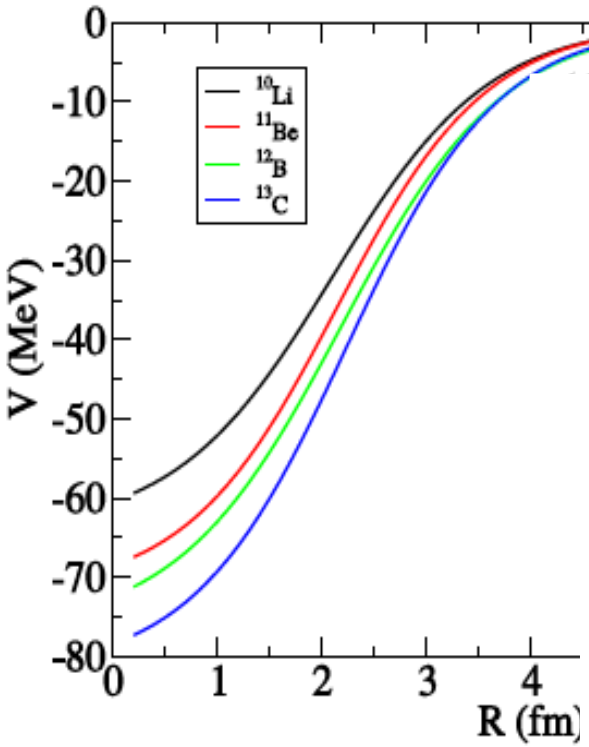
	$\hbar\omega_{2^+}$ (MeV)	β_2^n	V_{WS} (MeV)	V_{ls}	a_{WS} (fm)	R_{WS} (fm)
^{10}Li	3.37	0.68	64.0	0.50	0.75	2.10
^{11}Be	3.37	0.71	72.0 (73.8)	0.58 (0.54)	0.72 (0.78)	2.14
^{12}B	3.80	0.57	76.8	0.65	0.78 (0.76)	2.18
^{13}C	4.4	0.46(0.43)	82(83.0)	0.76(0.7)	0.73 (0.72)	2.23 (2.25)

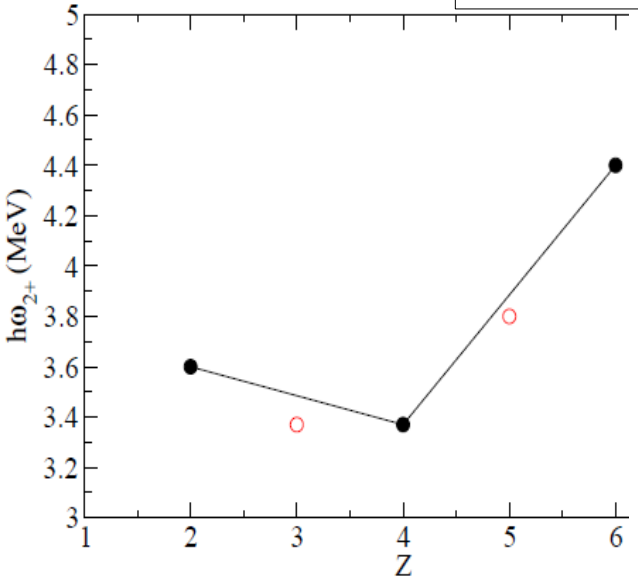
New simple parametrization:

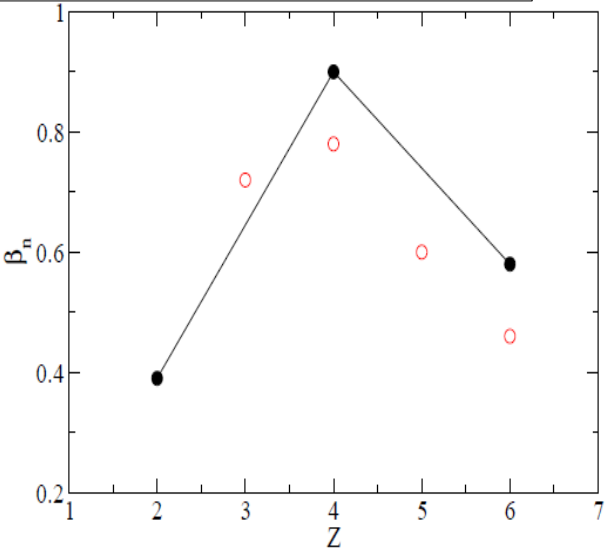
V_{WS}=-82+-54(N-Z)/A *MeV*

a = 0.75 *fm;* R_{WS}=0.99$A^{1/3}$ *fm*

V_{LS}=0.0082V_{WS}

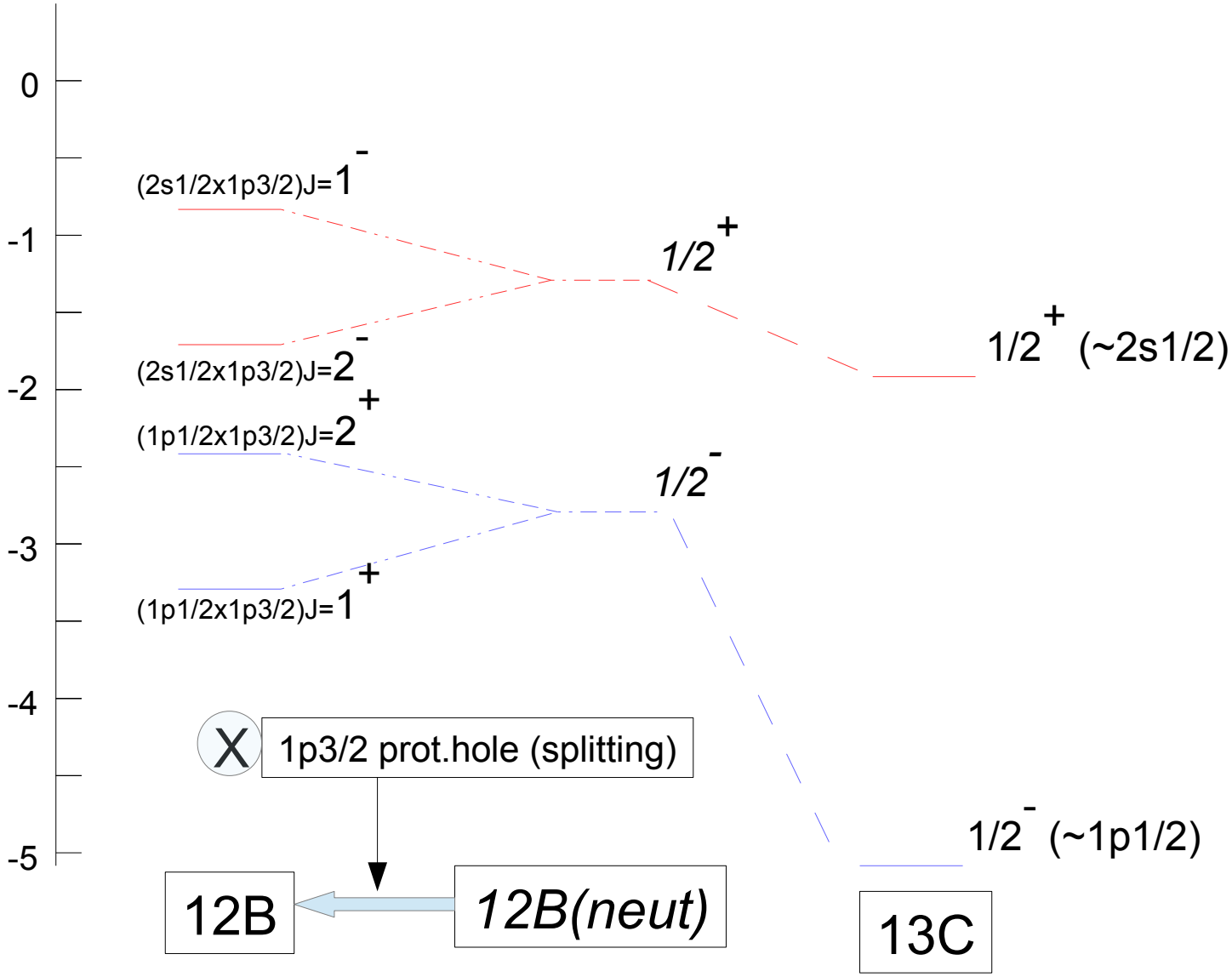






Odd Z nuclei

Effect of coupling to 1p3/2 proton hole in 12B



(K. Heyde, *The Nuclear Shell Model*)

The neutron proton interaction

$$V_{np}(r_n,r_p) = F_0\ \delta(r_n - r_p)\ (1-a+a\ \sigma_n .\ \sigma_p)$$

Proton frozen on the $1p_{3/2}$ orbital

No neutron mixing with other lj_n's

An additional $J_{np}(lj)_n$ dependent neutron potential

$$\delta V_{J\_np(lj)\_n}(r_n) = F_0\ |\Phi_{1p3/2}(r_p)|^2\ (1-a+a\ \sigma_n .\ \sigma_p)_{J\_np(lj)\_n}$$

to be added to the bare potential;

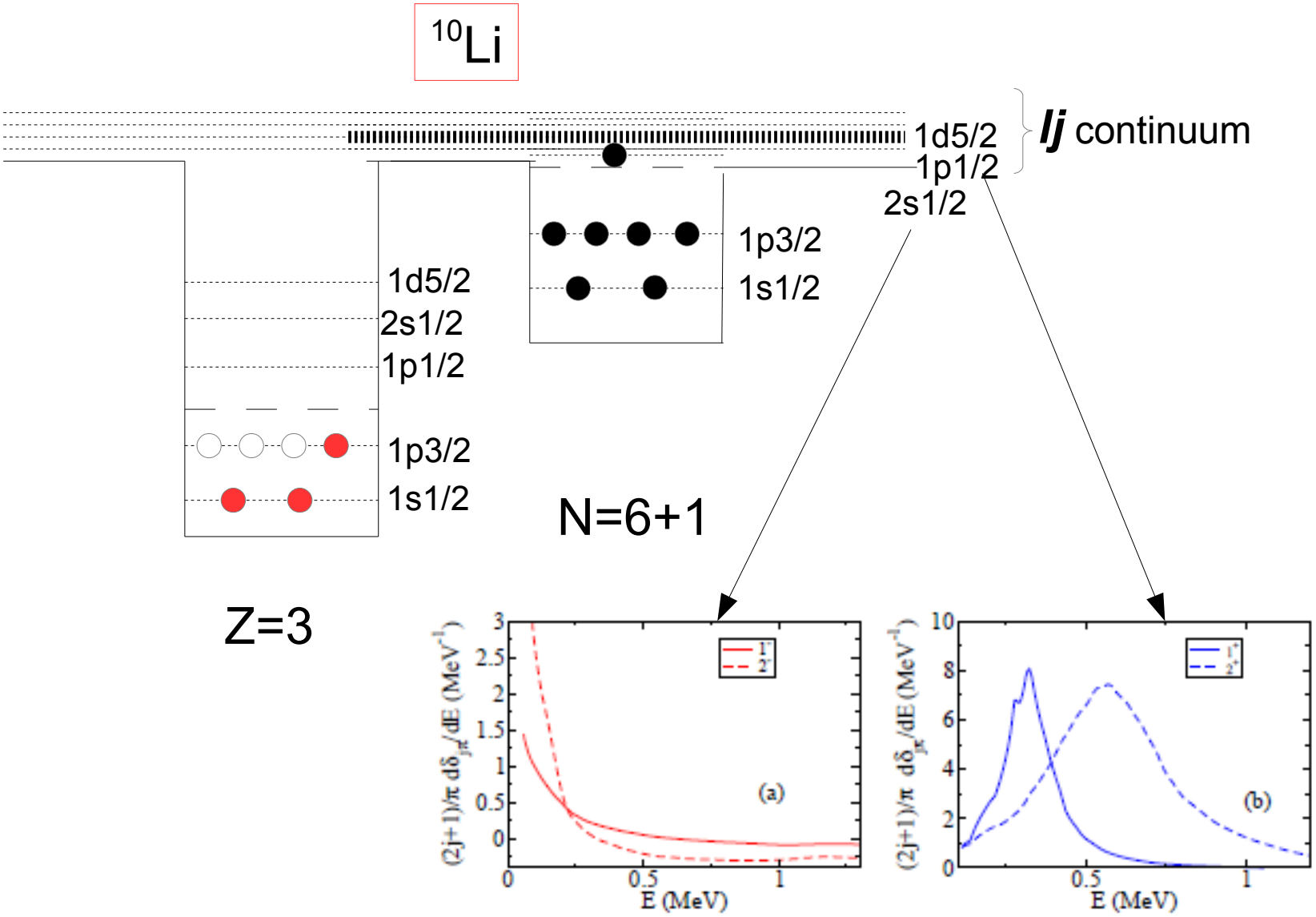
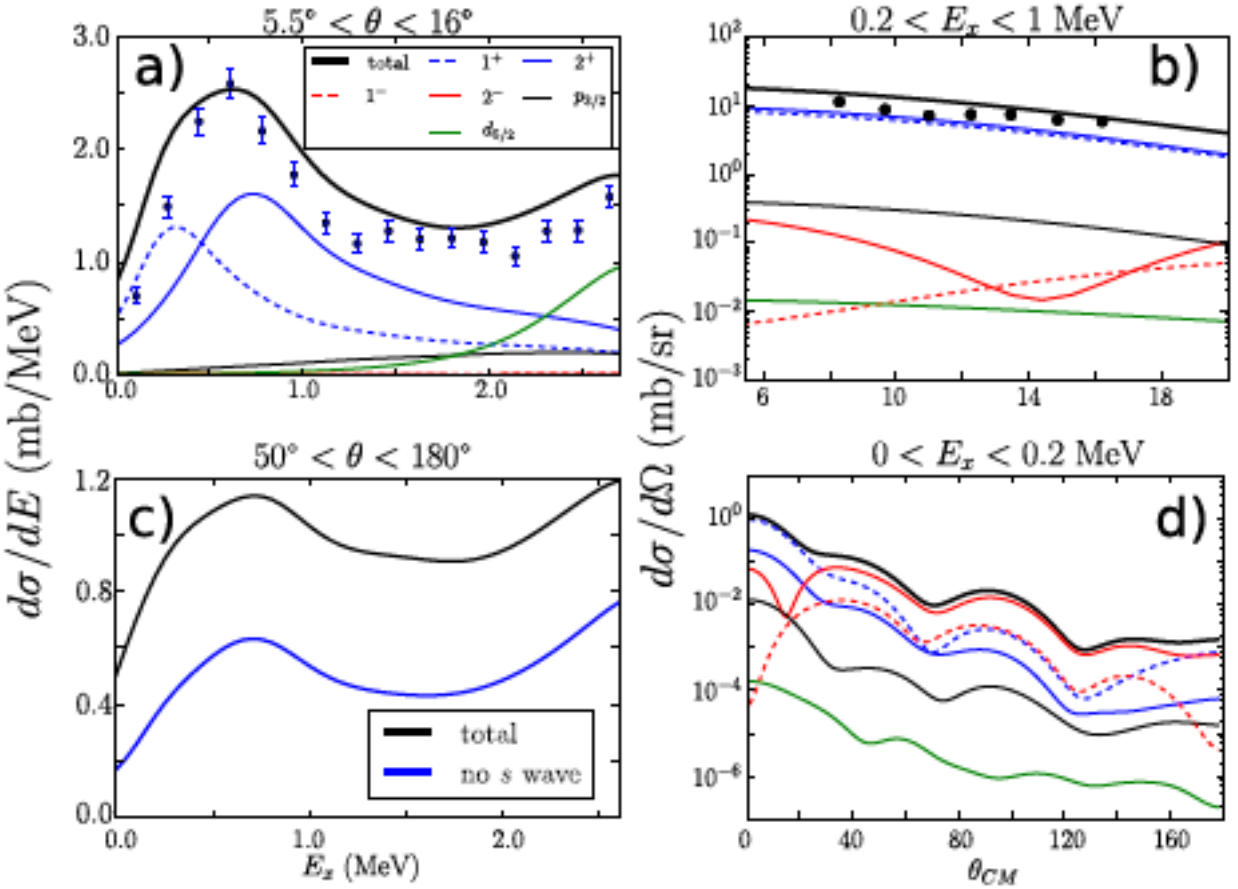


FIG. 2. Spectral functions of the low-lying negative (a) and positive (b) parity states in ^{10}Li.

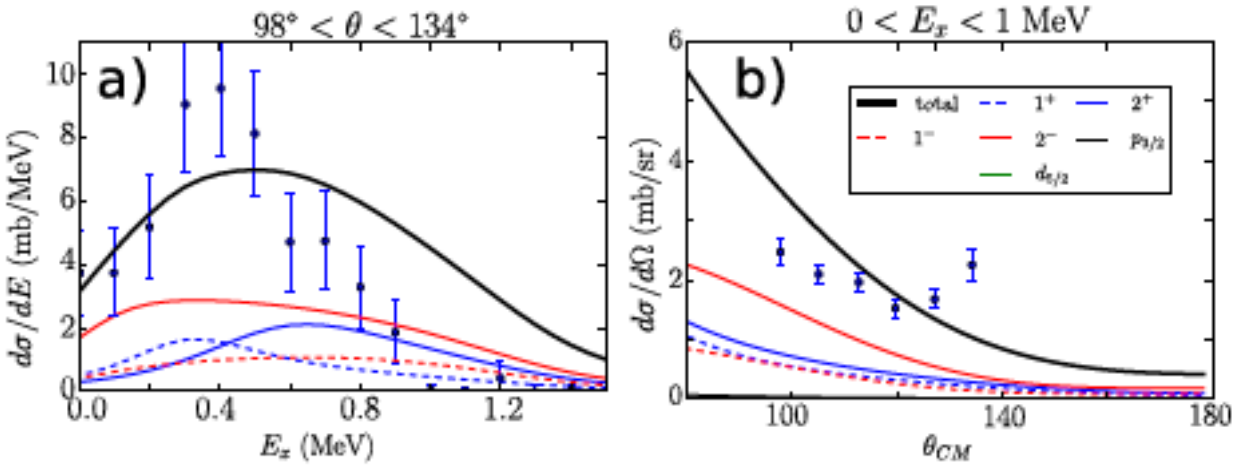
$d(^{9}Li,p)^{10}Li$ @ E_{lab} 100MeV ; θ_{CM} [5.5º,16º]



Theor.: *ArXiV1812.01761; see also A.Moro PLB793,13(2019)*

Data: Cavallaro et al. PRL118:012701, 2017

$d(^{9}Li,p)^{10}Li$ @ E_{lab} 21MeV ; θ_{CM} [98º,134º]



Theor.: *ArXiV1812.01761;*
see also A.Moro PLB793,13(2019),S.Orrigo PLB633,469(2006)

Data: Jeppesen et al. PLB 642:449, 2006

We plan to produce an universal bare mean field
to be used together with quadrupole (eventually also octupole)
coupling (as generalisation of the classical B&M static one)
that can be applied to the whole mass table from A about 10
and from proton to neutron dripline in the unified study of
low lying spectroscopic observables.

Results in the Sn isotopes region already exist.

A QRPA based protocol for calculating realistic
quadrupole properties exists.

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Thank you!

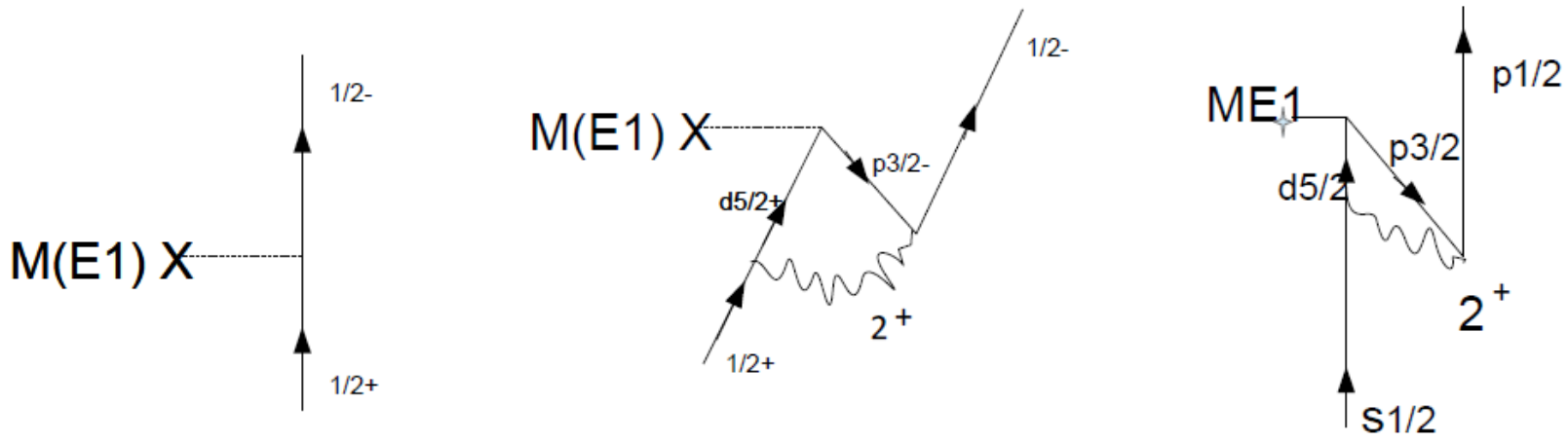


FIG. 5: Main processes contributing to the dipole transition between the first excited state and the ground state of ^{11}Be. The vertex correction associated with the low-lying 1^- strength (incipient GDPR) is estimated to be small.

One has to consider, however, that one expects other important contributions from many-body processes. The two time orderings associated with the most important ones are shown in the right part of Figs. 5. They interfere in a destructive way with the leading contribution, leading to

$$B(E1; I_i \to I_f) = \frac{e^2_{eff}}{2I_i+1} a^2_{1/2^+} a^2_{1/2^-} |M^0 + M^{1(a)} + M^{1(b)}|^2 = \frac{16}{121}\frac{1}{2} 0.66 |\sqrt{\frac{1}{2\pi}} \times 5 - 0.25 - 0.19|^2 \quad e^2fm^2$$

$B_{exp}(E1)=0.10\ e^2\ fm^2$

$$= 0.11 \quad e^2fm^2. \qquad (16)$$

$B_{bare}(E1)=0.29\ e^2\ fm^2$

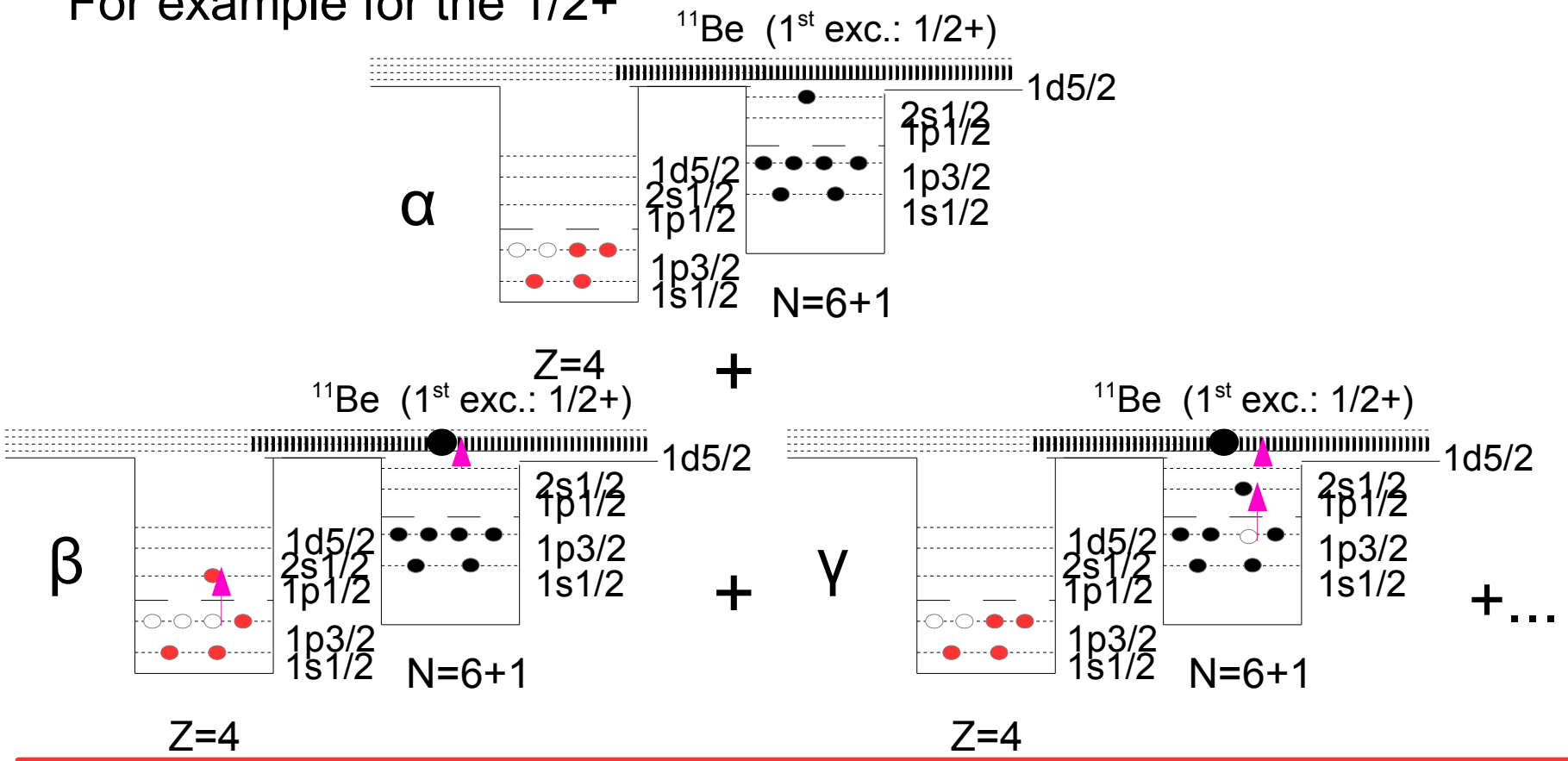
Charge Radii

$$\langle r^2\rangle_{11\mathrm{Be}} = \left(\langle r^2\rangle_{10\mathrm{Be}} + \left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2\right)\times S^2 + (1-S^2)\times\left(\langle r^2\rangle_{10\mathrm{Be}}\left(1+\frac{2}{4\pi}\beta_\pi^2\right) + \left(\frac{\langle r^2\rangle^{1/2}_{d5/2\,coll}}{11}\right)^2\right) = .$$

$$\langle r^2\rangle_{10\mathrm{Be}} + \left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2\times S^2 + (1-S^2)\times\left(\left(\frac{\langle r^2\rangle^{1/2}_{d5/2\,coll}}{11}\right)^2 + \langle r^2\rangle_{10\mathrm{Be}}\frac{2}{4\pi}\beta_\pi^2\right)$$

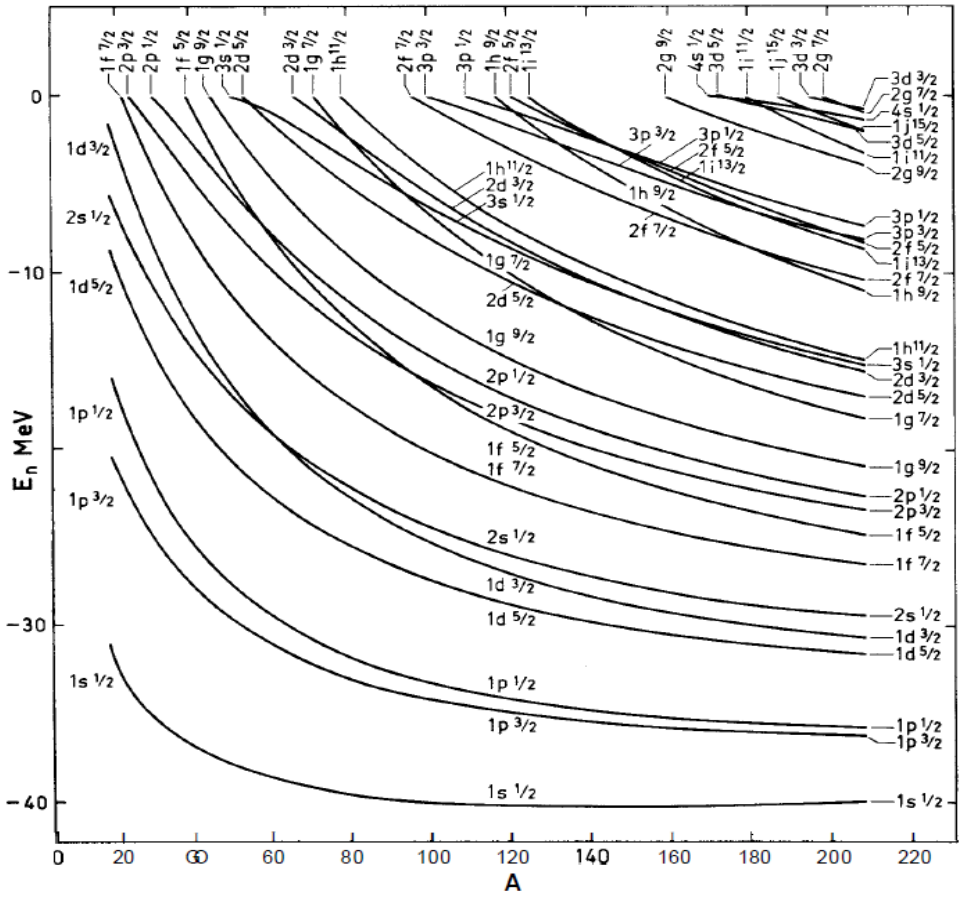
$\Delta\langle r^2\rangle^{1/2}_{11\mathrm{Be}}$ = 0.11 fm / 0.27 fm

Another standard strategy is mixing more complex configurations, including 1p-1h, 2p-2h,... core excitations.
For example for the 1/2+



$$\alpha\,|2s1/2 \times \mathrm{Core(gs)}\rangle + \beta\,|1d5/2 \times \mathrm{Core(exc.1)}\rangle + \gamma\,|1d5/2 \times \mathrm{Core(exc.2)}\rangle + \ldots$$

The amplitudes are obtained diagonalizing the nuclear many body hamiltonian , in a more or less extended basis. But this method is rather slowly convergent.



B&M I

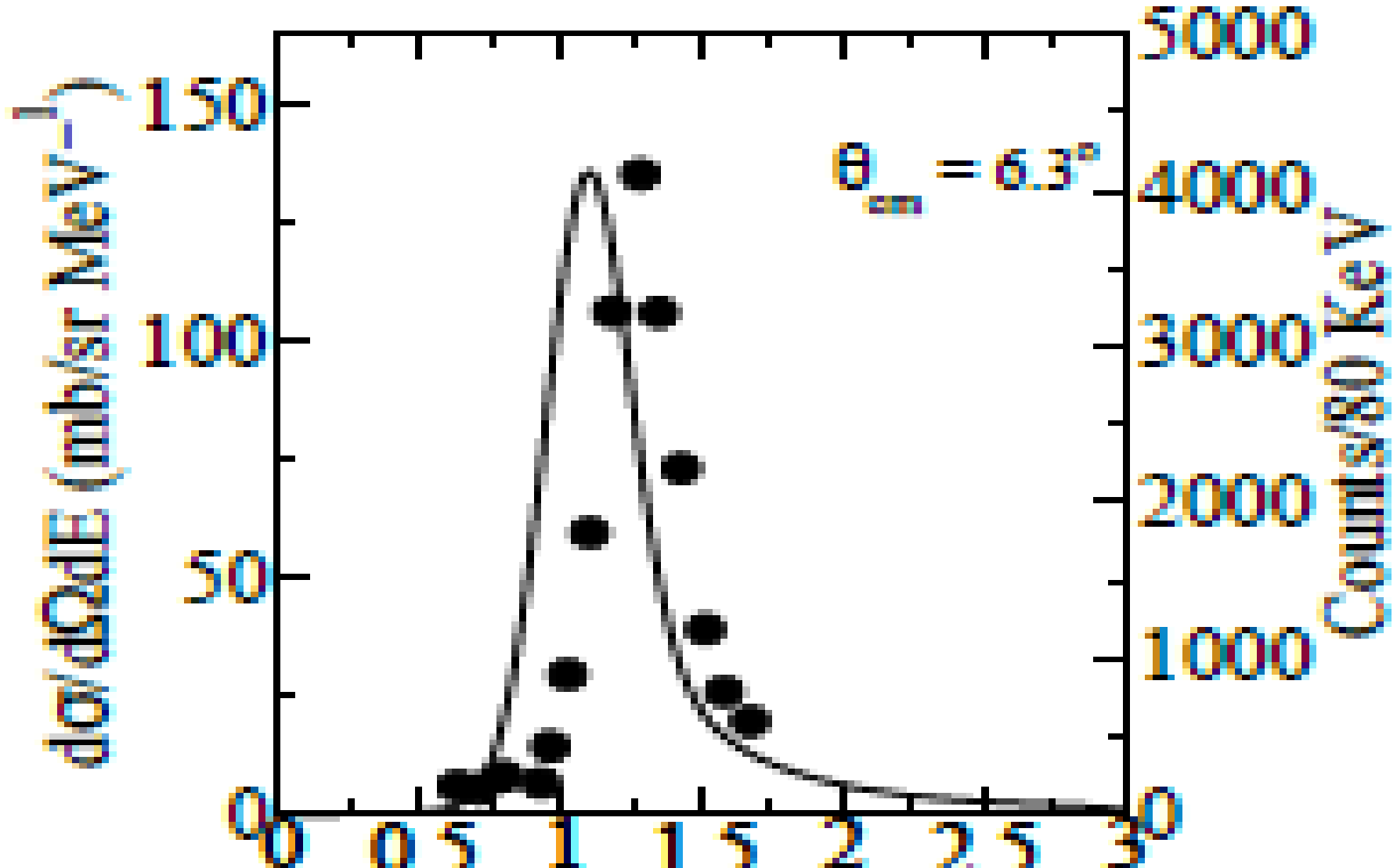
surface thickness parameter a is taken to be A independent.

$$R = r_0 A^{1/3} \qquad r_0 = 1.27 \text{ fm}$$
$$a = 0.67 \text{ fm} \tag{2-181}$$

The potential strengths include a term depending on the neutron excess, in order to describe approximately the potential acting on a single neutron,

$$V = \left(-51 + 33\,\frac{N-Z}{A}\right) \text{ MeV}$$
$$V_{ls} = -0.44V = \left(22 - 14\,\frac{N-Z}{A}\right) \text{ MeV} \tag{2-182}$$

and the neutron excess for each A has been chosen to correspond to the


dσ/dΩdE (mb/sr MeV⁻¹)
Counts/80 KeV
θ_cm = 6.3°
0
50
100
150
1000
2000
3000
4000
5000
0
0.5
1
1.5
2
2.5
3

PVC: Vibrational Core (even-even) + One particle (neutron)

$$H\,\Psi_a = \tilde{E}\,\Psi_a$$

partial waves analisys for J= j_a (1/2+ for example)

$$\Psi_a = [\,\psi^x_a + [\psi^C_b \cdot \Gamma^{+}_{\lambda}]_{j_a} + \ldots]\,\Phi^A_{GS}$$

$$\psi^x_a = \left(R^x_a(r)/r\right)\Theta_{j_a m_a}$$

$$[\psi^C_b \cdot \Phi_\lambda]_{j_a} = \left(R^C_b(r)/r\right)[\Theta_{j_b} \cdot \Gamma^{+}_{\lambda}]_{j_a}$$

$$\Phi^A_{GS}: core\ ground\ state\ (J^\pi = 0^+)$$

particle x phonon (vibrational quanta) configuration.

Radial wave functions and energy to be determined

Standard Coupled Channels Equations

$$\begin{bmatrix} [-\hbar^2/2\mu\ d^2/dr^2 + V_a(r)] & \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) & [-\hbar^2/2\mu\ d^2/dr^2 + V_b(r) + \hbar\omega] \end{bmatrix} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix}$$

$$with \quad \Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} \sum_\mu Y_{\lambda\mu} [\Gamma^{+}_{\lambda\mu} + (-1)^\mu \Gamma_{\lambda\mu}] [\Theta_{j_b} \cdot \Phi_\lambda]_{j_a m_a} \rangle \sim \langle j_b, 1/2\,; \lambda\, 0 | j_a, 1/2 \rangle$$

Core GS is not simply the HF (zero order) lower levels filled state

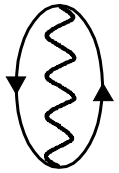
$$\sum_{\lambda} 1/2\,\hbar\omega_{\lambda}$$

This is the zero point motion associated to the normal modes in the harmonic approximation. Its theoretical value depends on the used model for calculating the frequencies w_L. In fact the GSC energy can be calculated as (c.f. *Ring&Schuck*)

$$E_{corr} = \sum_{\lambda,\nu} 1/2\,\hbar\omega^{RPA}_{\lambda,\nu} - 1/2\,\hbar\omega^{HF}_{\lambda,\nu}$$

This is a negative energy indicating that the RPA Ground State is more bound than the HF one,

What in turn means that in the core GS time by time fluctuations happen around the HF (zero order core GS) (c.f. Bohr&Mottelson Vol.II)



p-h + phonon “virtual” excitations with respect to the HF GS.

Heyde, Shell Model book

The neutron proton interaction

$$V_{np}(r_n,r_p) = F_0\ \delta(r_n - r_p)\ (1-a+a\ \sigma_n .\ \sigma_p)$$

Proton frozen on the $1p_{3/2}$ orbital

No neutron mixing with other lj_n's

An additional $J_{np}(lj)_n$ dependent neutron potential

$$\delta V_{J\_np(lj)\_n}(r_n) = F_0\ |\Phi_{1p3/2}(r_p)|^2\ (1-a+a\ \sigma_n .\ \sigma_p)_{J\_np(lj)\_n}$$

to be added to the bare potential;

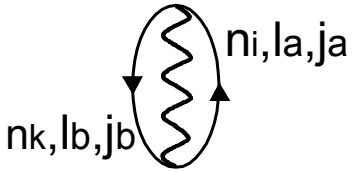
(finally for simplicity the form factor is taken the same that for the bare potential)

$$\delta V_{J\_np(lj)\_n}(r_n) = F'_{WS0}\ WSff(r_n)(1-a+a\ \sigma_n .\ \sigma_p)_{J\_np(lj)\_n}$$

$$a=0.5 \text{ and } F'_{WS0}\ (1-a+a\ \sigma_n .\ \sigma_p)_{J\_np=1(p1/2)\_n} = -2\text{MeV}$$

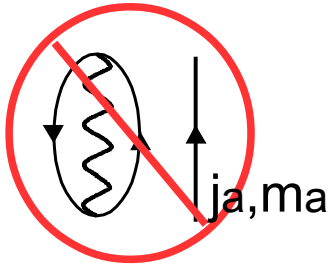
Single-particle energy correction due to GSC Pauli rearrangement

The contribution of a given p-h configuration to the GS Correlation Energy is (*B&MII*)



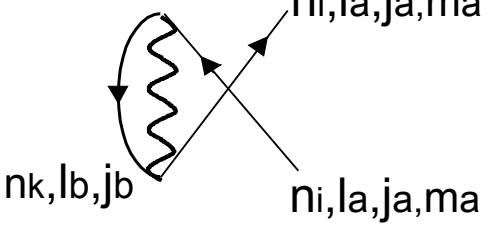
$$\delta E = \frac{\left(-h_{ai,bk,\lambda}\sqrt{(2j_a+1)}\right)^2}{0-\left(E_{ai}+E_{bk}+\hbar\omega_\lambda\right)} < 0$$

The presence of a new neutron (scattering- or bound-like) inhibits some of these correlations, producing an energy modification of the core state...



$$-\frac{\delta E}{2j_a+1} > 0$$

This is the meaning/value of the NFT self energy diagram (*B&MII, eq.6.225*)



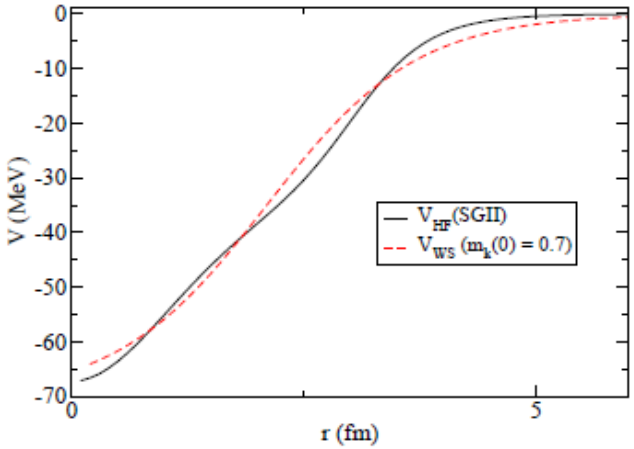
$$(-1)\frac{\left(-h_{ai,bk,\lambda}\sqrt{2j_a+1}\right)^2\langle((j_{a1},j_{a2})J=0,j_{a3};j_a|(j_{a1},j_{a3})J=0,j_{a2};j_a)\rangle}{E_{ai}-\left(2E_{ai}+E_{bk}+\hbar\omega_\lambda\right)}$$

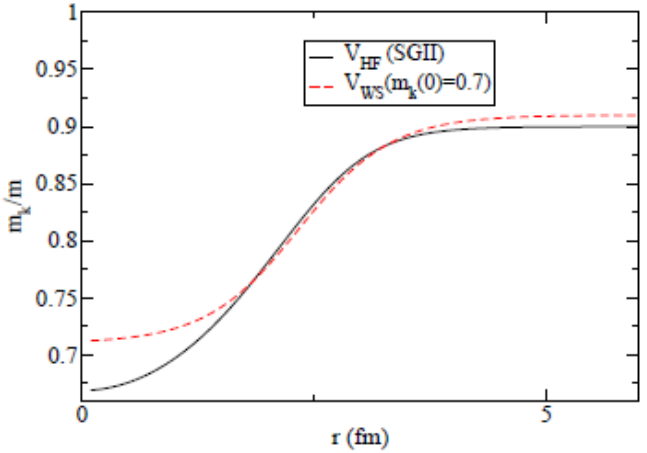
$$.=\frac{\left(h_{ai,bk,\lambda}\right)^2}{\left(E_{ai}+E_{bk}+\hbar\omega_\lambda\right)}$$

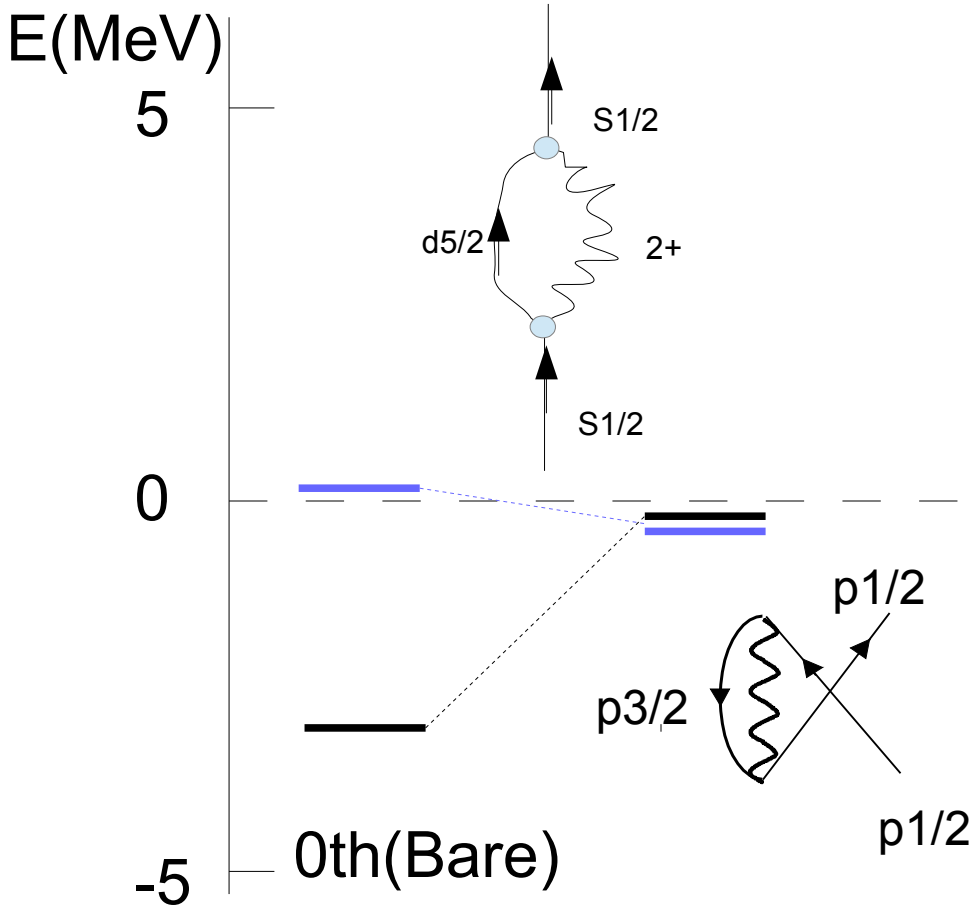
V_0=70MeV

a=0.81fm

R=2.1fm







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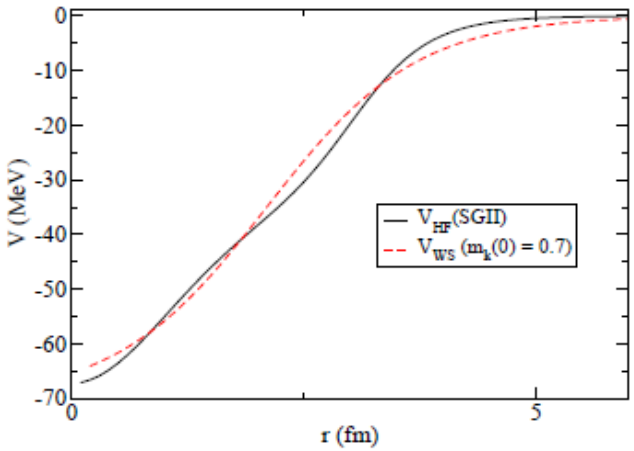
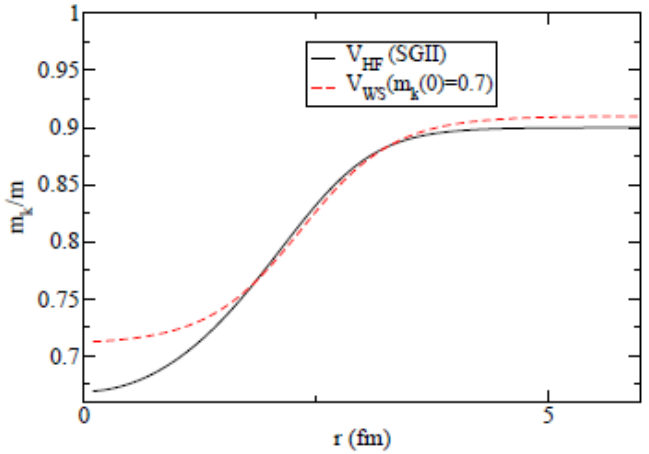
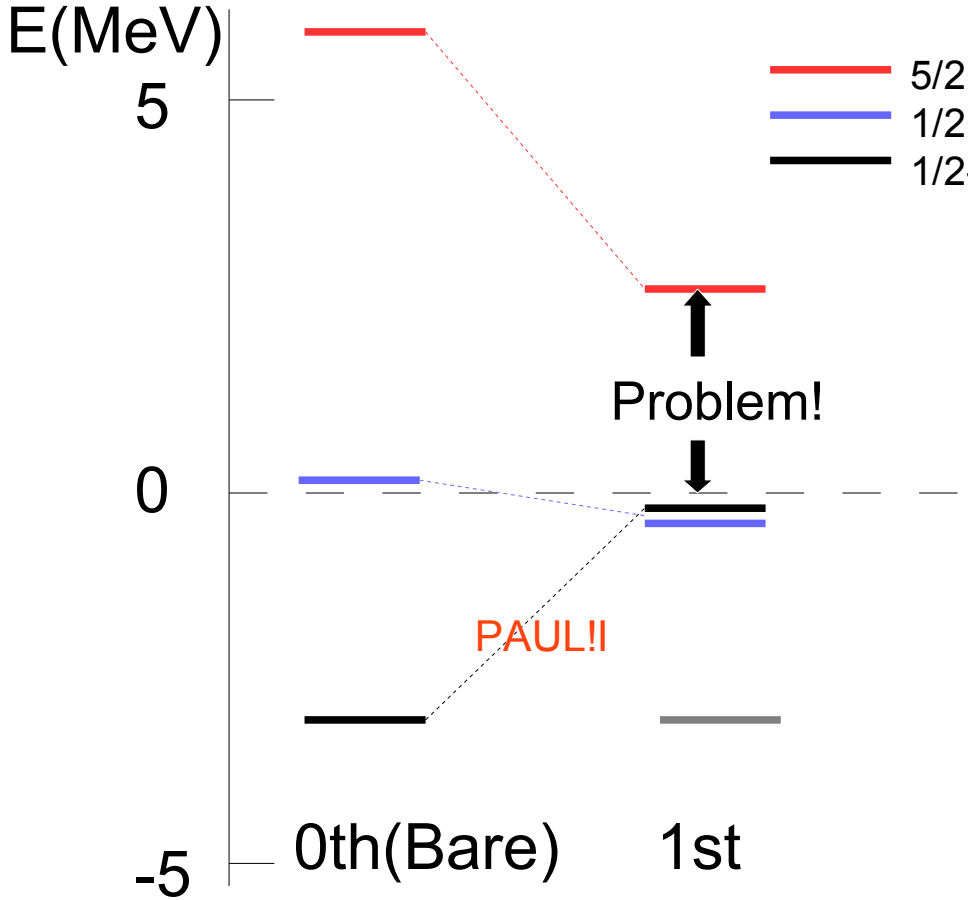






5/2+ : a |d5/2> + b |s1/2x2+> + c|d5/2x2+>

1/2+ : A |s1/2> + B |d5/2x2+>

1/2- α |p1/2> + β |p3/2x2+>

Expansion parameter

$1/\Omega_{eff}$

Ω_{eff}=3-4

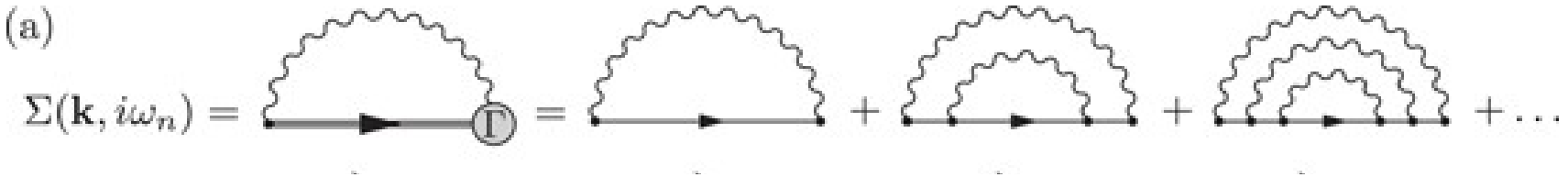
Ω_{eff}=6 (^{120}Sn)

Including higher orders

This is the non-crossing phonon aproximation to the many phonons full solution.
It excludes the vertex correction diagrams, that should be incorporated independently

Rainbow 1st 2nd 3rd

(a)



$$(H_p(r)-e_F)u^x_{ai}(r)+\int dr'\left[\sum_{bk}\frac{\Xi^2_{a,b\lambda}u^x_{bk}(r')u^x_{bk}(r)f(r')f(r)}{\tilde{E}_{ai}-(\tilde{E}_{bk}+\hbar\omega_\lambda)}+\sum_{ck}\frac{\Xi^2_{a,b\lambda}v^x_{ck}(r')v^x_{ck}(r)f(r')f(r)}{\tilde{E}_{ai}+(\tilde{E}_{ck}+\hbar\omega_\lambda)}\right]u^x_{ai}(r')$$

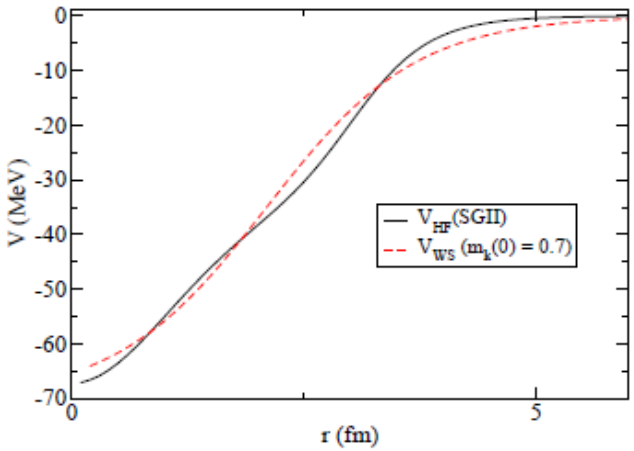
$$=\tilde{E}_{ai}u^x_{ai}(r)$$

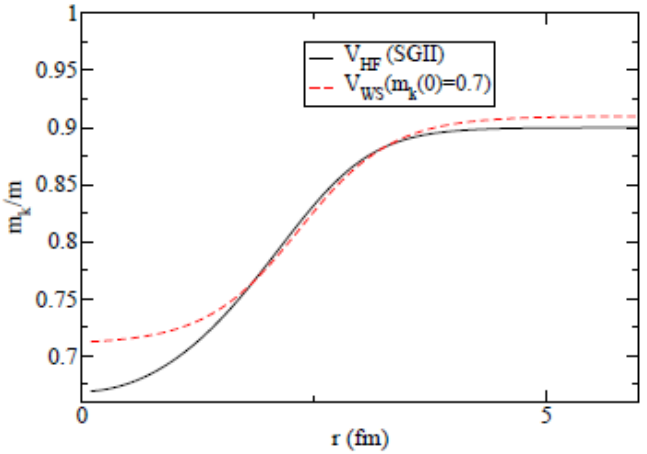
These equations must be solved iteratively

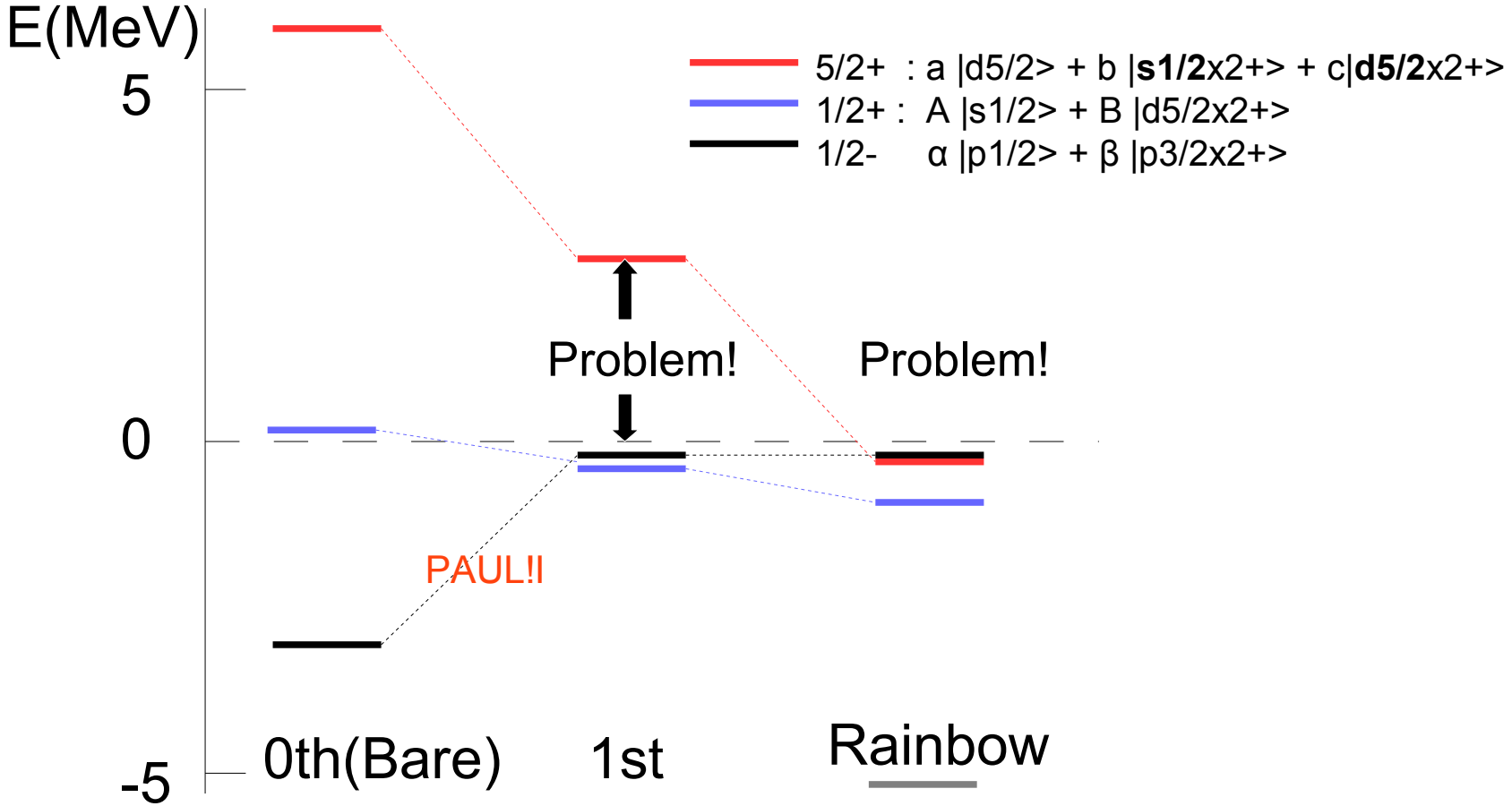
V_0=70MeV

a=0.81fm

R=2.1fm



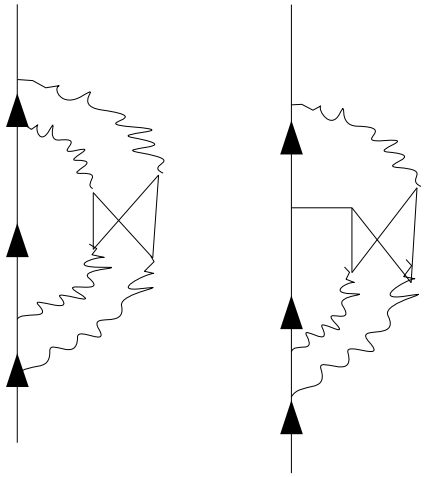


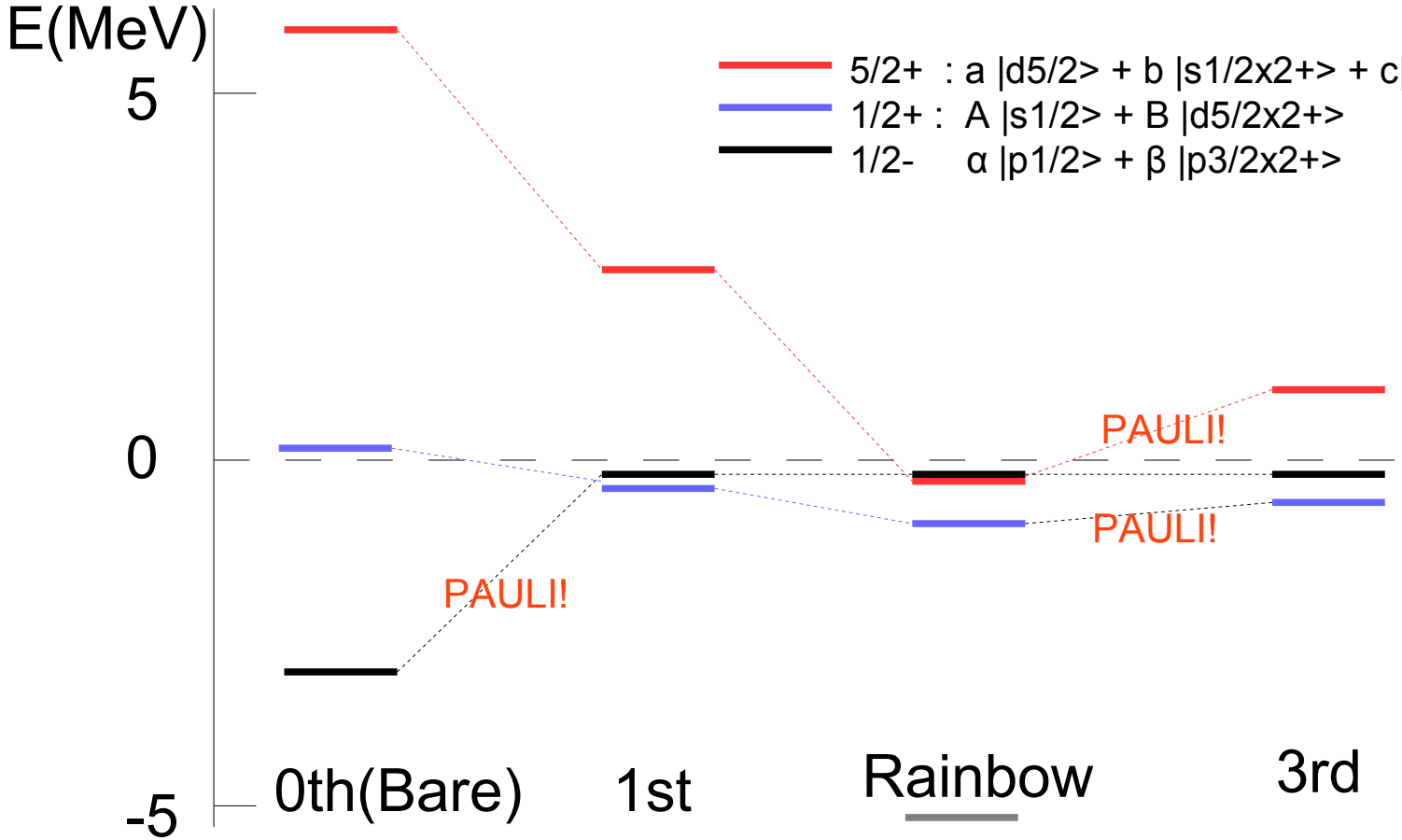


V_0=70MeV
a=0.81fm
R=2.1fm

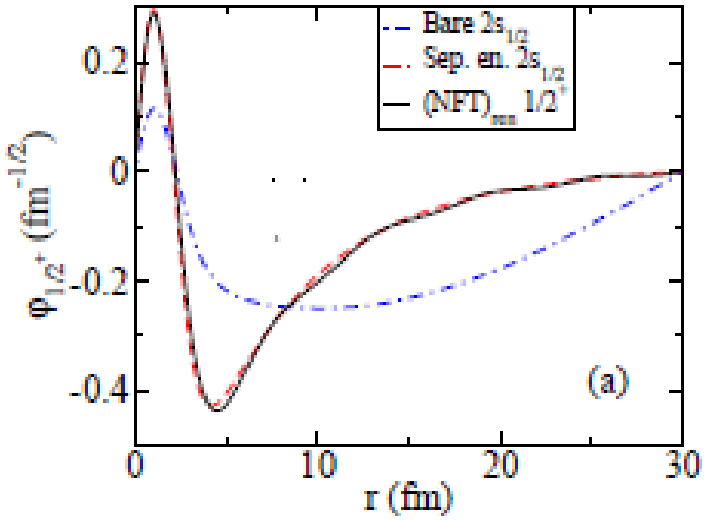
Two Phonon Anharmonicities

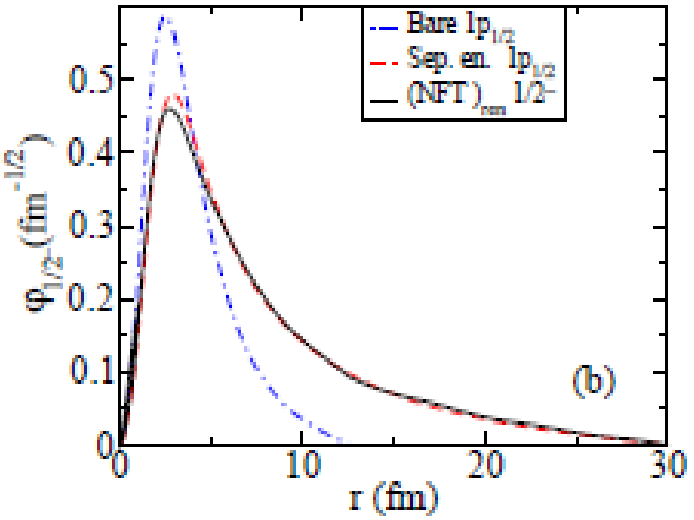
Butterfly Diagrams

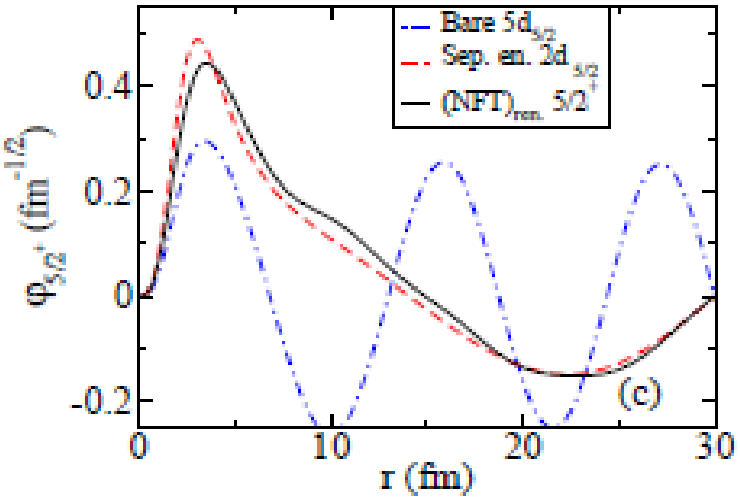


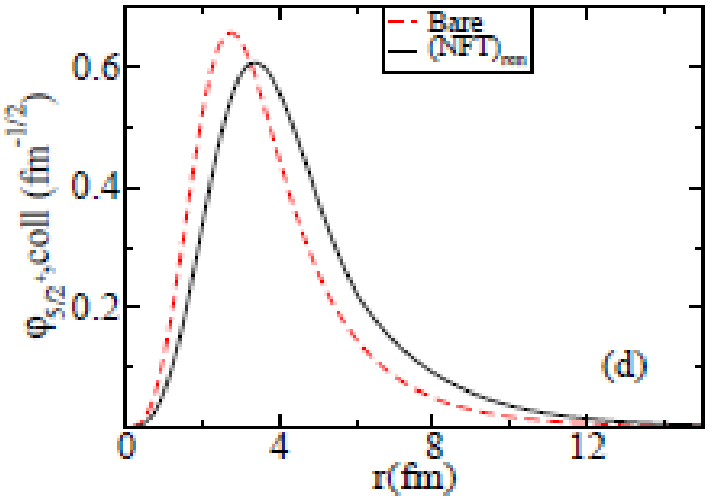


Transfer form factors









$$\widetilde{|1/2^+\rangle} = \sqrt{0.83}\ |s_{1/2}\rangle + \sqrt{0.17}|(d_{5/2}\otimes 2^+)_{1/2^+}\rangle + \sqrt{0.01}|(s^2_{1/2}(0)\otimes 0^{(-)})_{0^+}s_{1/2}\rangle \quad (1)$$

$$\widetilde{|1/2^-\rangle} = \sqrt{0.81}\ |p_{1/2}\rangle + \sqrt{0.15}|(\ p_{1/2},1p_{3/2})^{-1}_{2^+}\otimes 2^+)_{0^+}\ p_{1/2}\rangle + \sqrt{0.02}|(d_{5/2}\otimes 3^-)_{1/2^-}\rangle$$

$$+\sqrt{0.04}|(p^2_{1/2}(0)\otimes 0^{(-)})_{0^+}p_{1/2}\rangle \quad (2)$$

$$|0^{(-)}\rangle = |^8\mathrm{Be}(\mathrm{gs},0^+)\rangle \quad \text{monopole pair removal mode of the closed shell system } {}^{10}_{4}\mathrm{Be}_6 \quad (3)$$

$$\widetilde{|5/2^+\rangle} = \sqrt{0.34}|d_{5/2}\rangle + \sqrt{0.32}|(s_{1/2}\otimes 2^+)_{5/2^+}\rangle + \sqrt{0.34}|(d_{5/2}\otimes 2^+)_{5/2^+}\rangle \quad (4)$$

$$+\sqrt{0.003}|(d_{5/2}(0)^2\otimes 0^{(-)})_{0^+}d_{5/2}\rangle$$

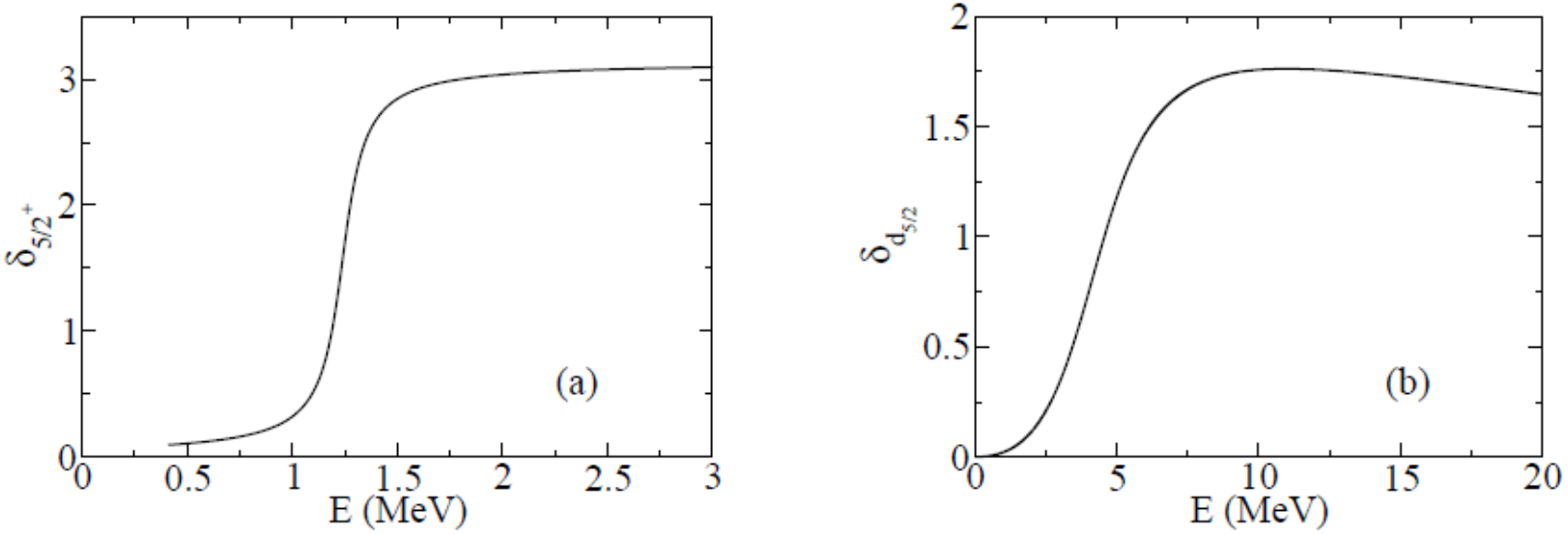
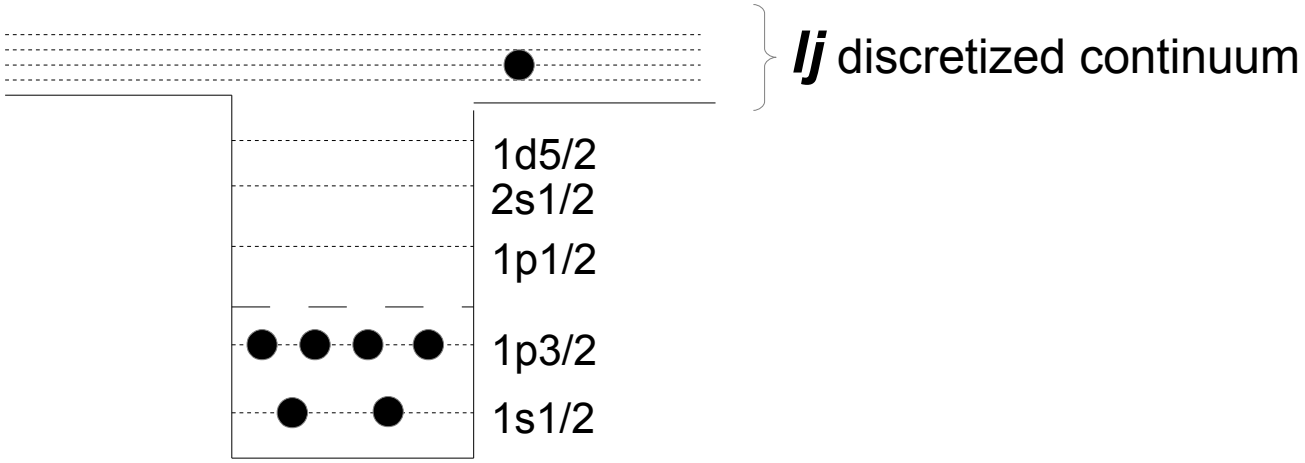


FIG. 2: (a) $5/2^+$ $(\mathrm{NFT})_{ren}$ phase shifts as a function of energy. (b) $d_{5/2}$ phase shifts calculated with the bare potential.

Taking care of the Pauli principle.1st part



Project out the occupied waves in HF GS

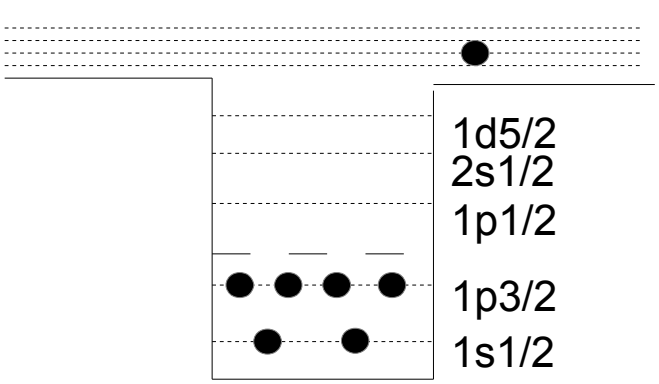
$$R_a^x(r)=\int dr'\left[\delta(r-r')-\sum_{k\in occ.} R_{ak}(r)\,R_{ak}(r')\right]R_a^x(r')=\int dr'\left[\delta(r-r')-P(r,r')\right]R_a^x(r')$$

Similarly for R_b^C

The CC equation becomes non-local (integro-differential): must be solved in R^2

$$\begin{bmatrix} \left[-\hbar^2/2\mu\ \ d^2/dr^2+V_a(r)\right] & \Xi_{a,b\lambda}\left(-\beta_\lambda r dV/dr\right) \\ \Xi_{a,b\lambda}\left(-\beta_\lambda r dV/dr\right) & \left[-\hbar^2/2\mu\ \ d^2/dr^2+V_b(r)+\hbar\omega\right] \end{bmatrix}\begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix}=\tilde{E}\begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix}$$

Taking care of the Pauli principle.1st part



lj discretized continuum

Alternative method: Expand using only ***non-occupied*** waves

$$R_a^x(r)=\sum_{i=1}^{N} x_{ai} R_{ai}^{WS}(r) \qquad i\in non-occ. \qquad\qquad R_b^C(r)=\sum_{i=1}^{N} C_{bi} R_{bi}^{WS}(r) \qquad i\in non-occ.$$

$$\begin{bmatrix} [-\hbar^2/2\mu\ d^2/dr^2+V_a(r)] & \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) & [-\hbar^2/2\mu\ d^2/dr^2+V_b(r)+\hbar\omega] \end{bmatrix} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R_a^x(r) \\ R_b^C(r) \end{bmatrix}$$

$$\left(\begin{array}{cc|cc} e_{a1} & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots \\ 0 & e_{a2}\ldots & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots \\ \hline h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}+\hbar\omega & 0\ldots \\ h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}+\hbar\omega\ldots \end{array}\right) \left(\begin{array}{c} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{array}\right) = \tilde{E} \left(\begin{array}{c} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{array}\right)$$

FULL HAMILTONIAN MATRIX (Block-**Arrowhead**, **energy independent**, matrix), where

$$h_{ai,bj\lambda}=\Xi_{a,b\lambda}\beta_\lambda \int R_{ai}^{WS}(r)(-r dV/dr) R_{bj}^{WS}(r)\, dr$$

Energy dependent Self-energy

$$R_a^x(r)=\sum_i x_{ai} R_{ai}^{WS}(r) \qquad i\in non-occ.$$
$$R_b^C(r)=\sum_i C_{bi} R_{bi}^{WS}(r) \qquad i\in non-occ.$$

$$\begin{pmatrix} e_{a1} & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots \\ 0 & e_{a2}\ldots & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots \\ h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}+\hbar\omega & 0\ldots \\ h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}+\hbar\omega\ldots \end{pmatrix} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix} = \tilde{E} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2}\ldots \end{pmatrix}$$

Projecting onto the single-particle basis

$$\begin{pmatrix} e_{a1}+\Sigma_{11}(\tilde{E}) & \Sigma_{12}(\tilde{E})\ldots \\ \Sigma_{21}(\tilde{E}) & e_{a2}+\Sigma_{22}(\tilde{E})\ldots \end{pmatrix} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix} = \tilde{E} \begin{pmatrix} x_{a1} \\ x_{a2}\ldots \end{pmatrix}$$

SINGLE-PARTICLE HAMILTONIAN MATRIX (energy **dependent**), where

$$\Sigma_{ij}(\tilde{E})=\sum_{bk} \frac{h_{ai,bk,\lambda} h_{aj,bk,\lambda}}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}$$

Where the so called basic vertex is

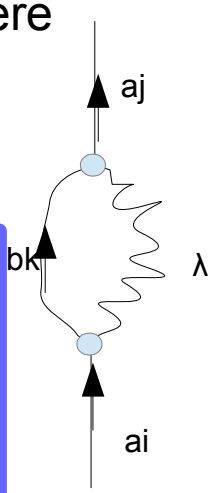
$$h_{ai,bk\lambda}=\Xi_{a,b\lambda}\beta_\lambda \int R_{ai}^{WS}(r)(-rdV/dr)R_{bk}^{WS}(r)\,dr$$

Often, for bound states, a 1 s-p state approximation is used

$$e_{a1}+\Sigma_{11}(\tilde{E})=\tilde{E}$$

In this approximation the radial dependence is blocked. Only the amplitudes are determined

To be noted that this Feynman Diagram contains all the richness of the CC equation, and in fact it is completely equivalent to it, eventual open inelastic channels included.



Optical Potential as the r,r'-E-dependent Self-energy

..obtained by means of the standard algebraic manipulations:

$$C_{bk}=\frac{\sum_{ai} x_{ai}\, h(a_i,b_k\lambda)}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}=\frac{\Xi_{a,b\lambda}\int dr'\,R^{WS}_{bk}(r')\,\delta\rho(r')\,R^{x}_{ai}(r')}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}$$

$$h_{ai,bk\lambda}=\Xi_{a,b\lambda}\beta_\lambda\int R^{WS}_{ai}(r)(-rdV/dr)R^{WS}_{bk}(r)\,dr$$

$$R^{x}_{a}(r)=\sum_i x_{ai}\,R^{WS}_{ai}(r);\; E_{ai}>0$$

$$R^{C}_{b}(r)=\sum_k C_{bk}\,R^{WS}_{bk}(r)=\sum_k R^{WS}_{bk}(r)\frac{\Xi_{a,b\lambda}\int dr'\,R^{WS}_{bk}(r')\,\delta\rho(r')\,R^{x}_{ai}(r')}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)};\; E_{bk}>0$$

$$\begin{pmatrix} H_p-e_F & \Xi_{a,b\lambda}f(r) \\ \Xi_{a,b\lambda}f(r) & H_p-e_F+\hbar\omega \end{pmatrix}\begin{pmatrix} R^{x}_{a} \\ R^{C}_{b} \end{pmatrix}=\tilde{E}\begin{pmatrix} R^{x}_{a} \\ R^{C}_{b} \end{pmatrix}$$

$$(H_p(r)-e_F)\,R^{x}_{ai}(r)+\int dr'\sum_{bk>0}\frac{\Xi^2_{a,b\lambda}\,R^{WS}_{bk}(r')\,R^{WS}_{bk}(r)\,\delta\rho(r')\,\delta\rho(r)}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda-i\eta)}R^{x}_{ai}(r')=\tilde{E}\,R^{x}_{ai}(r)$$

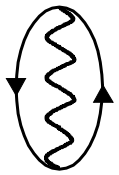
Microscopic optical potentials for high enough E's

$$\sum_{\lambda} 1/2\,\hbar\omega_{\lambda}$$

This is the zero point motion associated to the normal modes in the harmonic approximation. Its theoretical value depends on the used model for calculating the frequencies w_L. In fact the GSC energy can be calculated as (c.f. *Ring&Schuck*)

$$E_{corr} = \sum_{\lambda,\nu} 1/2\,\hbar\omega^{RPA}_{\lambda,\nu} - 1/2\,\hbar\omega^{HF}_{\lambda,\nu}$$

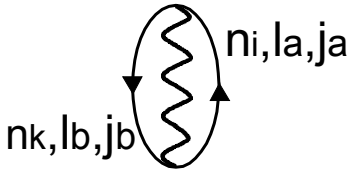
This is a negative energy indicating that the RPA Ground State is more bound than the HF one, what in turn is due to the Correlations Included in the RPA (c.f. Bohr&Mottelson Vol.II)



p-h + phonon “virtual” excitations with respect to the HF GS.

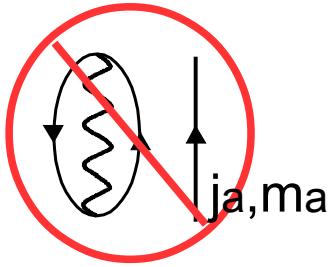
Matrix elements due to GSC Pauli rearrangement

The contribution of a given p-h configuration to the GS Correlation Energy is (*B&MII*)



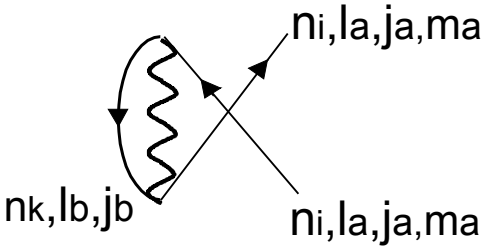
$$\delta E = \frac{\left(-h_{ai,bk,\lambda}\sqrt{(2j_a+1)}\right)^2}{0-\left(E_{ai}+E_{bk}+\hbar\omega_\lambda\right)} < 0$$

The presence of a new neutron (scattering- or bound-like) inhibits some of these correlations, producing an energy modification of the core state...



$$-\frac{\delta E}{2j_a+1} > 0$$

This is the meaning/value of the NFT self energy diagram (*B&MII, eq.6.225*)



$$(-1)\frac{\left(-h_{ai,bk,\lambda}\sqrt{2j_a+1}\right)^2\langle((j_{a1},j_{a2})J=0,j_{a3};j_a|(j_{a1},j_{a3})J=0,j_{a2};j_a)\rangle}{E_{ai}-\left(2E_{ai}+E_{bk}+\hbar\omega_\lambda\right)}$$

$$.=\frac{\left(h_{ai,bk,\lambda}\right)^2}{\left(E_{ai}+E_{bk}+\hbar\omega_\lambda\right)}$$

Meaning of the auxiliary GSCPR channel.

$$u_a^x(r)=\sum_i x_{ai} R_{ai}^{WS}(r); e_{ai}>e_F$$
$$u_b^C(r)=\sum_i C_{bi} R_{bi}^{WS}(r); e_{bi}>e_F$$
$$v_c^D(r)=\sum_i D_{ci} R_{ci}^{WS}(r); e_{ci}<e_F$$

$$\Psi_{jama}=[\psi^x_{jama}+[\psi^C_{jb}\cdot\Gamma^{+.}_{\lambda}]_{jama}]\Phi_{GS}$$

but if

$$\Psi_{GS}=[1+\varepsilon[\psi^x_{ja}\cdot[\psi^{-1}_{jc}\cdot\Gamma^{+.}_{\lambda}]_{ja}]]_{J=0}\Phi^{HF}_{GS}$$

a new term can / must be added

$$\Psi_a=[\psi^x_a+[\psi^C_b\cdot\Gamma^{+.}_{\lambda}]_{j_a}-[\psi^D_c\cdot\Gamma_{\lambda}]_{j_a}+\ldots]\Phi_{GS}$$

with the hole − phonon anihilator

$$[\psi^D_c\cdot\Gamma_{\lambda}]_{j_a}=(v^D_c(r)/r)[\Theta_{j_c}\cdot\Phi^{-1}_{\lambda}]_{j_a},$$

$$v_c^D(r)=\sum_i D_{ci} R_{ci}^{WS}(r); e_{ci}<e_F$$

p-h + phonon
"virtual"excitations
with respect to the HF GS.

Creation of a neutron in an "occupied" level jc levels + phonon anihilation may/will give a non nule contribution

GSC Pauli rearr.: An auxiliary Coupled Channel

!

$$\begin{pmatrix} H_p-e_F & \Xi_{a,b\lambda}f(r) & \Xi_{a,c\lambda}f(r) \\ \Xi_{a,b\lambda}f(r) & H_p-e_F+\hbar\omega & 0 \\ \Xi_{a,c\lambda}f(r) & 0 & (H_p-e_F)-\hbar\omega \end{pmatrix}\begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix}=\tilde{E}\begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix}$$

PVC: Vibrational Core (even-even) + One particle (neutron)

$$H = H_{coll} + H_{sp} + H_{PVC}$$

$$H_{coll} = \sum_{\lambda\mu\nu} \hbar\omega_{\lambda,\nu} [\Gamma^{+.}_{\lambda\mu,\nu}\Gamma_{\lambda\mu,\nu} + 1/2]$$

$$H_{sp} = -\hbar^2/2\mu\, d^2/d\vec{r}^2 + V(r) + V_{ls}(r)$$

$$H_{PVC} = \sum_{\lambda\mu\nu} \delta V_{\lambda\nu}(r) Y_{\lambda\mu}(\hat{r}) [\Gamma^{+.}_{\lambda\mu,\nu} + (-1)^{\mu}\Gamma_{\lambda\mu,\nu}]$$

$$\delta V_{\lambda\nu}(r) \approx -r dV/dr\, \beta_{\lambda,\nu}$$

$$H\Psi_a = \tilde{E}\Psi_a$$

partial.waves analisys

$$\Psi_a = [\psi^x_a + [\psi^C_b \cdot \Gamma^{+.}_{\lambda}]_{j_a} + ...]\Phi^A_{fbGS}$$

$$\psi^x_a = (R^x_a(r)/r)\Theta_{j_a m_a}$$

$$[\psi^C_b \cdot \Phi_\lambda]_{j_a} = (R^C_b(r)/r)[\Theta_{j_b} \cdot \Gamma^{+.}_{\lambda}]_{j_a}$$

$$\Phi^A_{fbGS} : fermion \cdot boson\ GS's$$

Standard Coupled Channels Equations

$$\begin{bmatrix} [-\hbar^2/2\mu\ d^2/dr^2 + V_a(r)] & \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) & [-\hbar^2/2\mu\ d^2/dr^2 + V_b(r) + \hbar\omega] \end{bmatrix} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix}$$

with $\Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} \sum_\mu Y_{\lambda\mu} [\Gamma^{+.}_{\lambda\mu} + (-1)^{\mu}\Gamma_{\lambda\mu}][\Theta_{j_b} \cdot \Phi_\lambda]_{j_a m_a} \rangle \sim \langle j_b, 1/2; \lambda 0 | j_a, 1/2 \rangle$

1. Here the zero point energy $\sum_\lambda 1/2\,\hbar\omega_\lambda$ is assigned to the GS. Later we will discuss a more dynamical role of this term

2. In H_{PVC} only linear terms are/must be included. Higher orders will be discussed later.

3. The core is supposed to be closed shell. Open shell (superfluid) cores will be discussed later.

4. (n,d) and (n,t) channels will be discussed later

RESULTADOS poner lo del PRL

Treatment of the neutron-proton interaction.

simple but quantitative treatment of the effects related to the presence of the odd proton in 12 B, that can be directly extrapolated to case of 10 Li.

The lowest states observed in the energy spectrum of the odd-odd nucleus 12B are two doublets with angular momentum and parity (1+; 2+) and (1-; 2-) which can be naturally interpreted as arising from the coupling of the odd proton lying on the 1p3=2 orbital with the odd neutron lying on the 1p1=2 (1p3=2 1p1=2) or on the 2s1=2 (1p3=2 2s1=2) weakly bound orbitals. The doublets display a similar energy splitting equal to approximately 0.95 MeV. The 1+ and the 2- states lie lowest in the two multiplets. The relative position of the centroids of the (1+; 2+) (Ecent = 0:59 MeV) and (1-; 2-) doublets (Ecent = 2:02) in 12B indicates a dierence in the position of the 1p1=2 or on the 2s1=2 neutron states equal to ps = -1.43 MeV, to be compared with with the value ps = -3.09 MeV associated with the relative position of the States 1/2- (g.s.) and 1/2+ (3.09 MeV) in 13C.

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Parity inversion and breakdown of shell closure in Be isotopes

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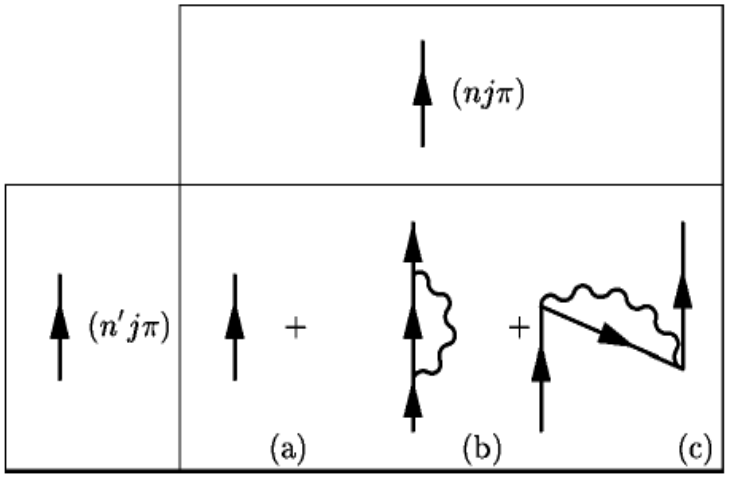

			Theory	
		Expt.	Particle vibration	Mean field
$^{11}_{4}Be_7$	$E_{s_{1/2}}$	−0.504 MeV	−0.48 MeV	~0.14 MeV
	$E_{p_{1/2}}$	−0.18 MeV	−0.27 MeV	−3.12 MeV
	$E_{d_{5/2}}$	1.28 MeV	~0 MeV	~2.4 MeV
	$S[1/2^+]$	0.65–0.80 [19]	0.87	1
		0.73±0.06 [20]		
		0.77 [21]		
	$S[1/2^-]$	0.63±0.15 [20]	0.96	1
		0.96 [21]		1
	$S[5/2^+]$		0.72	1

Why? What for?

NFT has been mostly used in the study of the structure of well bound nuclei:

Single-particle and multiplet states: Hamamoto Phys.Rep.(73),
Mahaux,Bortignon, Broglia and Dasso, Phys.Rep.(85)
Density distribution: F.B. and Broglia, PLB(85)
Width of the Giant Resonances: Colo', Bortignon and Broglia, NPA(87)
Superfluidity properties: Idini et al.,PRC(2012,2015)

Studies of the **structure** of nuclei close to the **neutron drip line** like **11-Li, 11-Be and 12-Be** led us naturally to the need of **including the continuum** in the model basis for describing **ground state** and bound excited states properties (*Barranco et a.,EPJ(2001*) (nicely confirmed in 11-Li by the Tanihata et al. (p,t) data) (*Potel et al. PRL(2010*))

The study of **resonant states** is now under development, the formal **connection with scattering and reaction** processes has appeared naturally.

These formal advances find simple expression in the **Coupled Channels** formalism. (*Parallel developments have been pursuid by **Mizuyama et. al**. and by **Blanchon et al.** in terms of microspical optical potentials; connections will be established*)

Using NFT as the (exact!) starting point allows in principle the exact treatment of some interesting aspects of the reaction side, as the **antisymmetrization in n-N reactions.**

Summary/Message

Small nuclei mean field/PVC based description must consider several levels of correlations, but **still a useful tool,** in order to give a **global** description of low energy spectroscopy data.

Nice laboratories for many body physics.

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Particle motion in a deformed potential using a transformed oscillator basis

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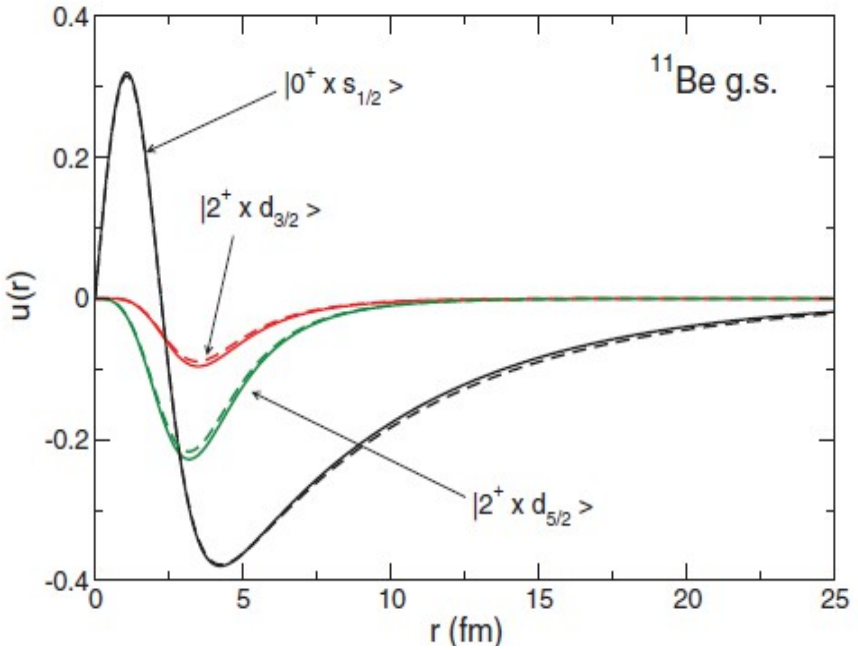

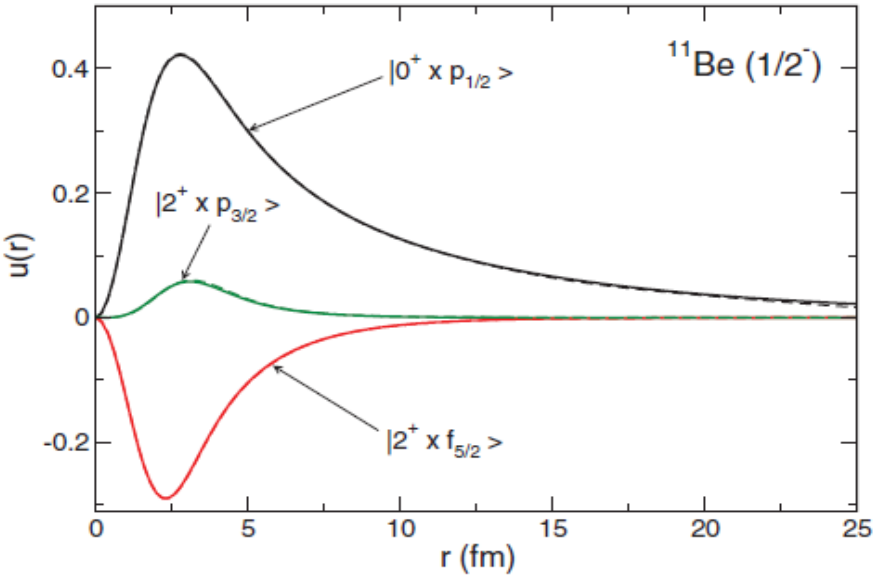

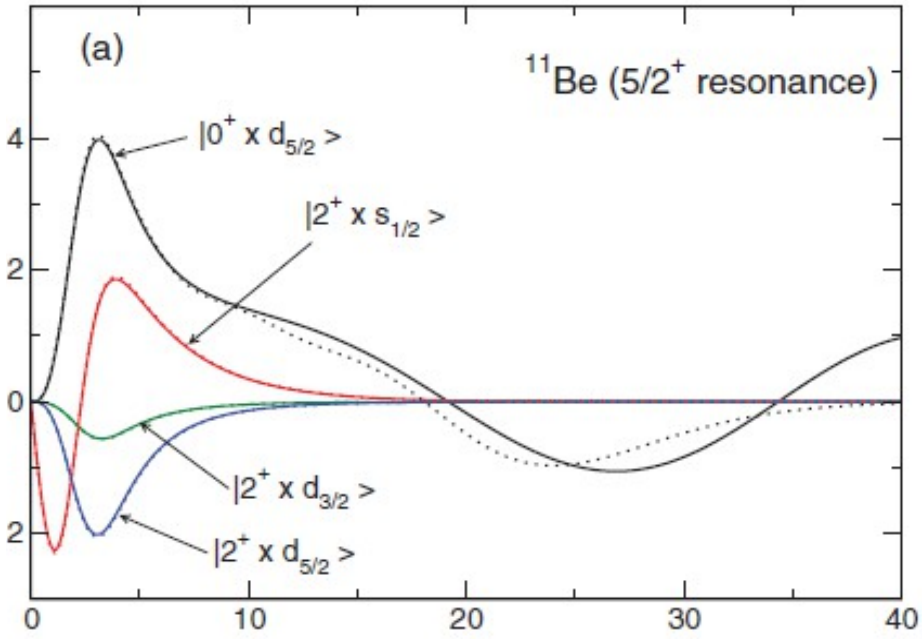

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Single-neutron transfer from $^{11}Be_{gs}$ via the (p, d) reaction with a radioactive beam

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Abstract

The ^{11}Be(p, d)^{10}Be reaction has been performed in inverse kinematics with a radioactive ^{11}Be beam of $E/A = 35.3$ MeV. Angular distributions for the 0^+ ground state, the 2^+, 3.37 MeV state and the multiplet of states around 6 MeV in ^{10}Be were measured at angles up to 16°_{cm} by detecting the ^{10}Be in a dispersion-matched spectrometer and the coincident deuterons in a silicon array. Distorted wave and coupled-channels calculations have been performed to investigate the amount of 2^+ core excitation in $^{11}Be_{gs}$. The use of "realistic" ^{11}Be wave functions is emphasised and bound-state form factors have been obtained by solving the particle-vibration coupling equations. This calculation gives a dominant $2s$ component in the $^{11}Be_{gs}$ wave function with a 16% $[2^+ \otimes 1d]$ core excitation admixture. Cross sections calculated with these form factors are in good agreement with the present data. The Separation Energy prescription for the bound-state wave function also gives satisfactory fits to the data, but leads to a significantly larger $[2^+ \otimes 1d]$ component in $^{11}Be_{gs}$.

Appendix A. Transfer form factors in a vibrational coupling approach

It has long been established that in the presence of core-polarisation admixtures created by an interaction Hamiltonian $H_{\rm int}$, radial form factors of one-nucleon transfer reactions should no more be approximated by the product of a spectroscopic amplitude and a single-particle wave function $U_{lj}^{\rm SE}$ deduced from the standard separation-energy procedure, but have to be determined by solving the Pinkston–Satchler coupled equations [71]. In the present analysis of the ^{11}Be(p, d)^{10}Be reaction, these coupled equations were solved in the framework of the particle-vibration coupling model [72,73], with the program CCVIB [55]. The interaction Hamiltonian was chosen as:

$$H_{\rm int} = -R\frac{{\rm d}V}{{\rm d}r}\sum_{LM}\frac{{\rm i}^{-L}\beta_L}{\sqrt{2L+1}}\left(c_{LM}^{\dagger}+(-1)^{L+M}c_{L-M}\right)Y_{LM}^{*}(\theta,\phi), \qquad \text{(A.1)}$$

where $c_{LM}^{\dagger}$ and c_{L-M} are operators for phonon creation and annihilation and β_L is the deformation parameter. Here the term ${\rm d}V/{\rm d}r$ corresponds to the deformation of the central part $V(r)$ of the potential, with no action on the spin–orbit part. Coupled-channel calculations were performed for both $1/2^+$ ground state and $1/2^-$ first excited state at 0.32 MeV, with the additional constraint that the same well depth V_0 reproduced the experimental separation energies relative to the ground state of ^{10}Be, thus ensuring the right asymptotic behaviour of the wave functions. This reproduction of the separation energies could be obtained by adjusting either the value of the β_2 parameter or the strength of the spin–orbit part of the potential. The configuration space was truncated to the $0^+_{\rm gs}$ and 2^+_1 core states in ^{10}Be (first-order vibrational coupling), coupled with one neutron in the $1p_{3/2}$ or $1p_{1/2}$, and $2s_{1/2}$ or $1d_{5/2}$ orbitals, for the $1/2^-$ and $1/2^+$ states, respectively.

Renormalization processes acting on s-, p- and d-waves
Coupling to pair vibrations included

Parameters are varied around a standard parameterization of the neutron-core potential (Bohr-Mottelson)

$$V_0 = -51 + \frac{33(N-Z)}{A}\ \mathrm{MeV} \quad ; \quad a = 0.67\ \mathrm{fm}; \quad ; \quad R = 1.27A^{1/3},$$

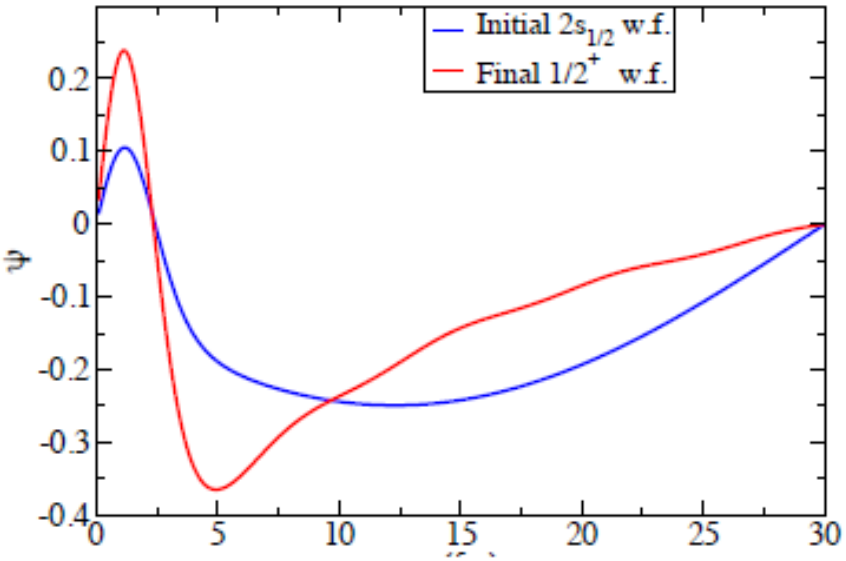
$(\beta_2)_0^{\ nuc} = 0.67$ $(\beta_2)_0^{\ Coul} = 1.15$

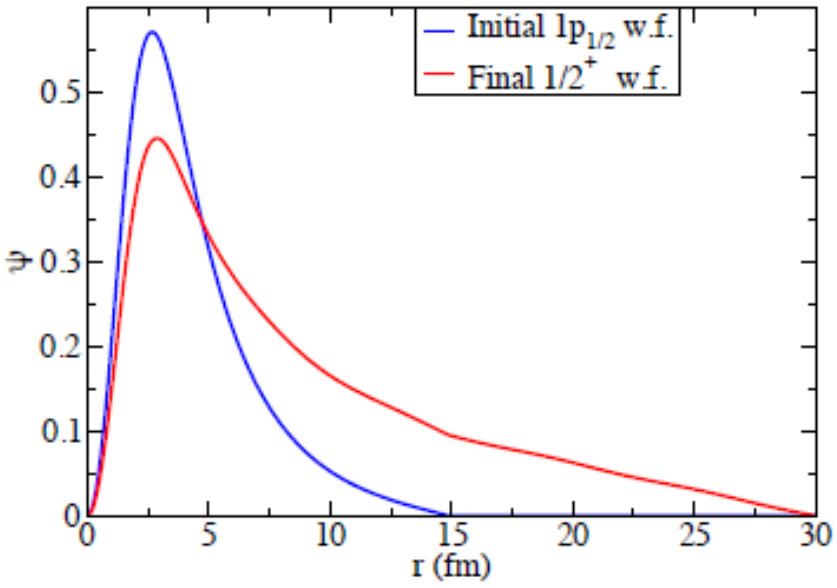
$E(p_{1/2})$	$\tilde{E}(1/2^-)$	$A(1/2^-)$	$E(s_{1/2})$	$\tilde{E}(1/2^+)$	$A(1/2^+)$	$E_{res}(d_{5/2})$	$\tilde{E}(5/2^+)$
-2.22	-0.19	0.15	0.18	-0.18	0.07	3.5	1.21

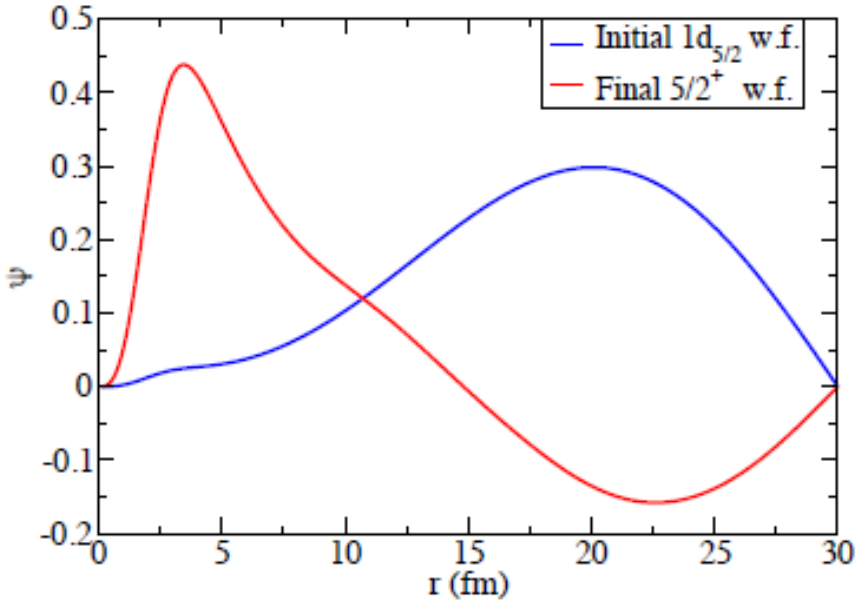
$\beta_2/(\beta_2)_0$	$\beta_3/(\beta_3)_0$	$V_{WS}/(V_{WS})_0$	$V_{ls}/(V_{ls})_0$	a	$m_k(0)/m$
1.3	0.9	1.0	1.0	0.74	0.9

Initial single-particle wavefunctions
and
Final many-body wavefunctions

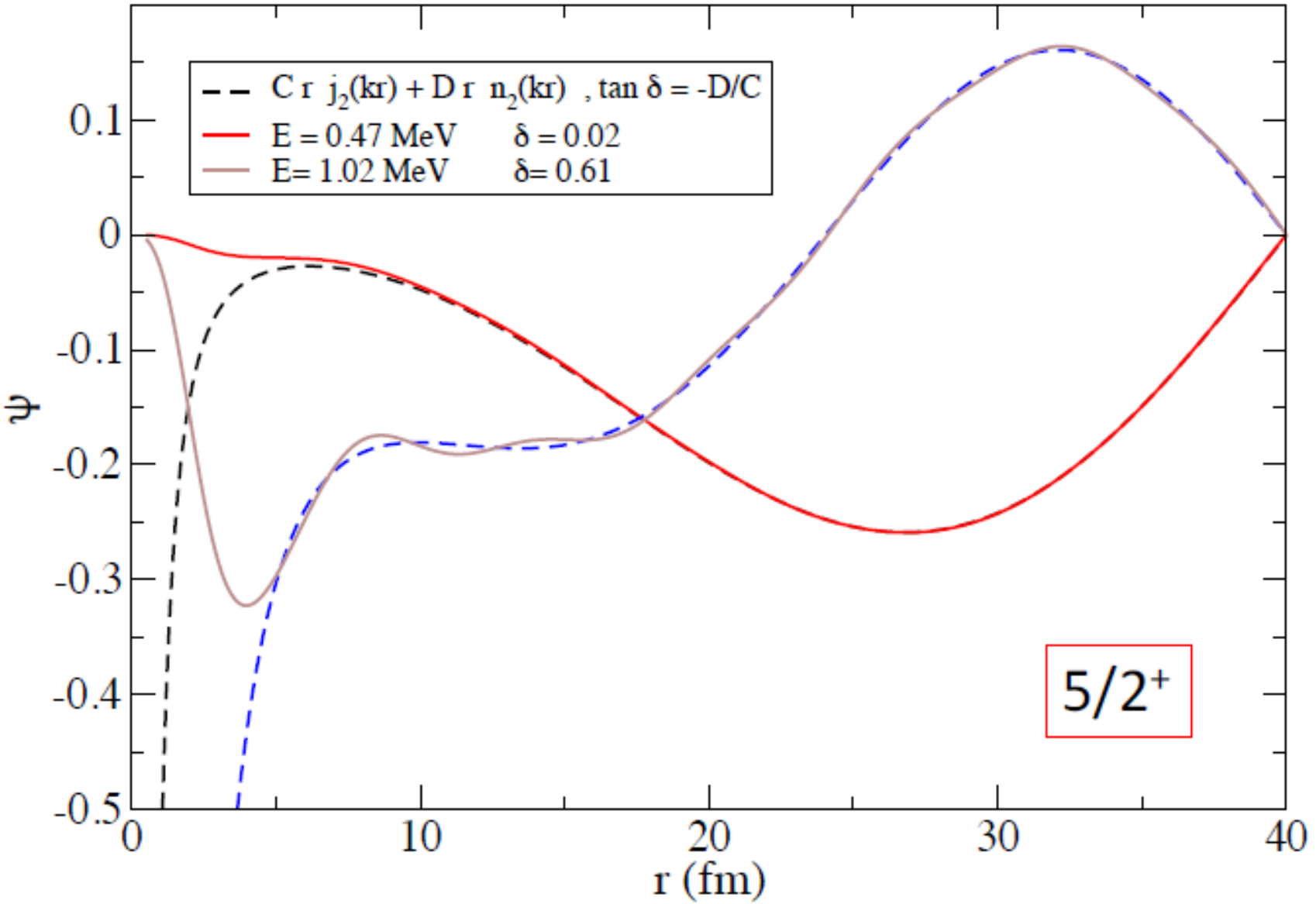
R_{box} = 30 fm







Determination of phase shifts, R_{box} = 40 fm



PHYSICAL REVIEW C 85, 054618 (2012)

Particle motion in a deformed potential using a transformed oscillator basis

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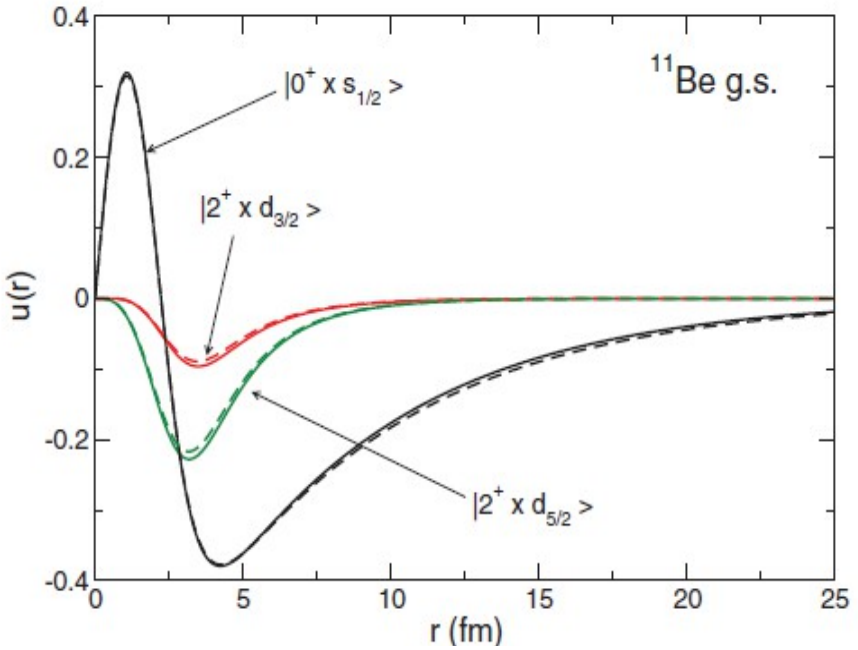
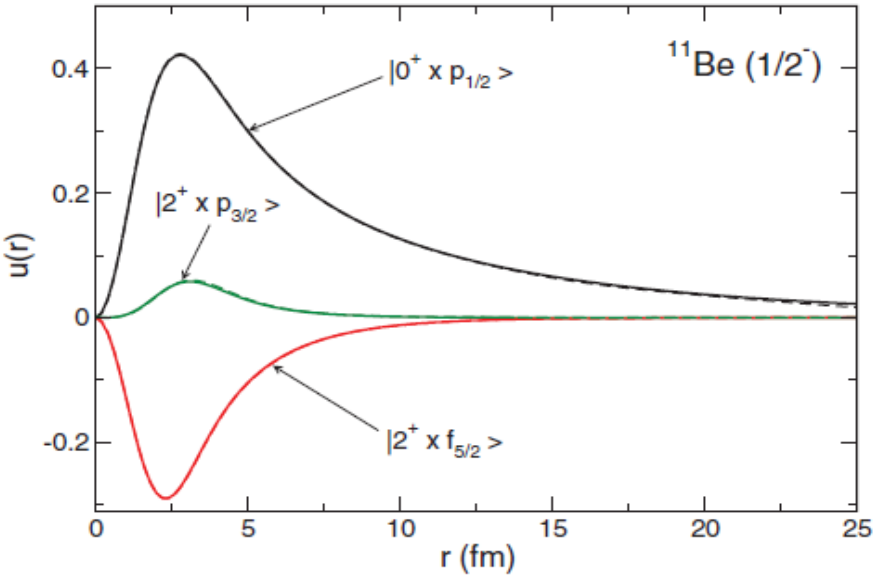
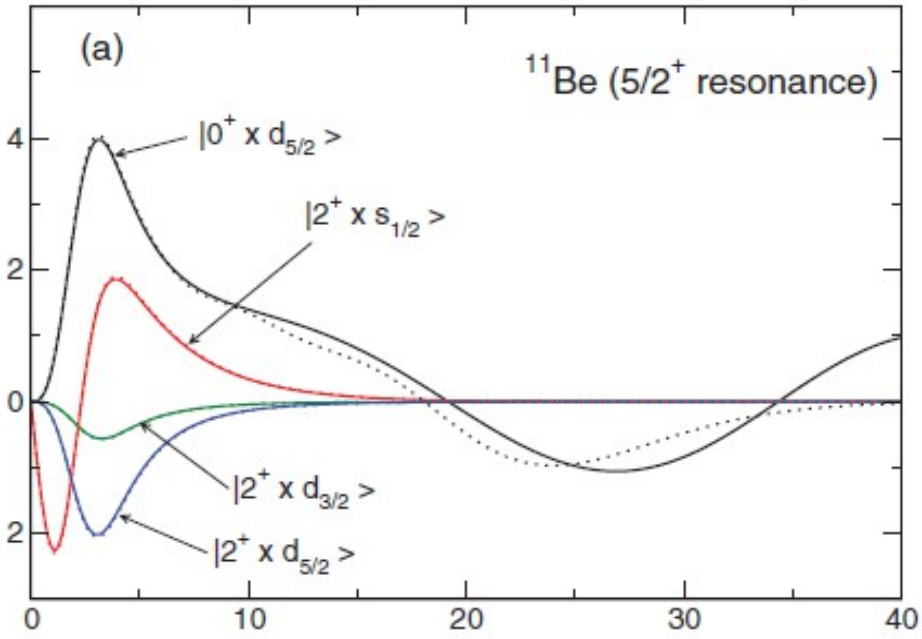
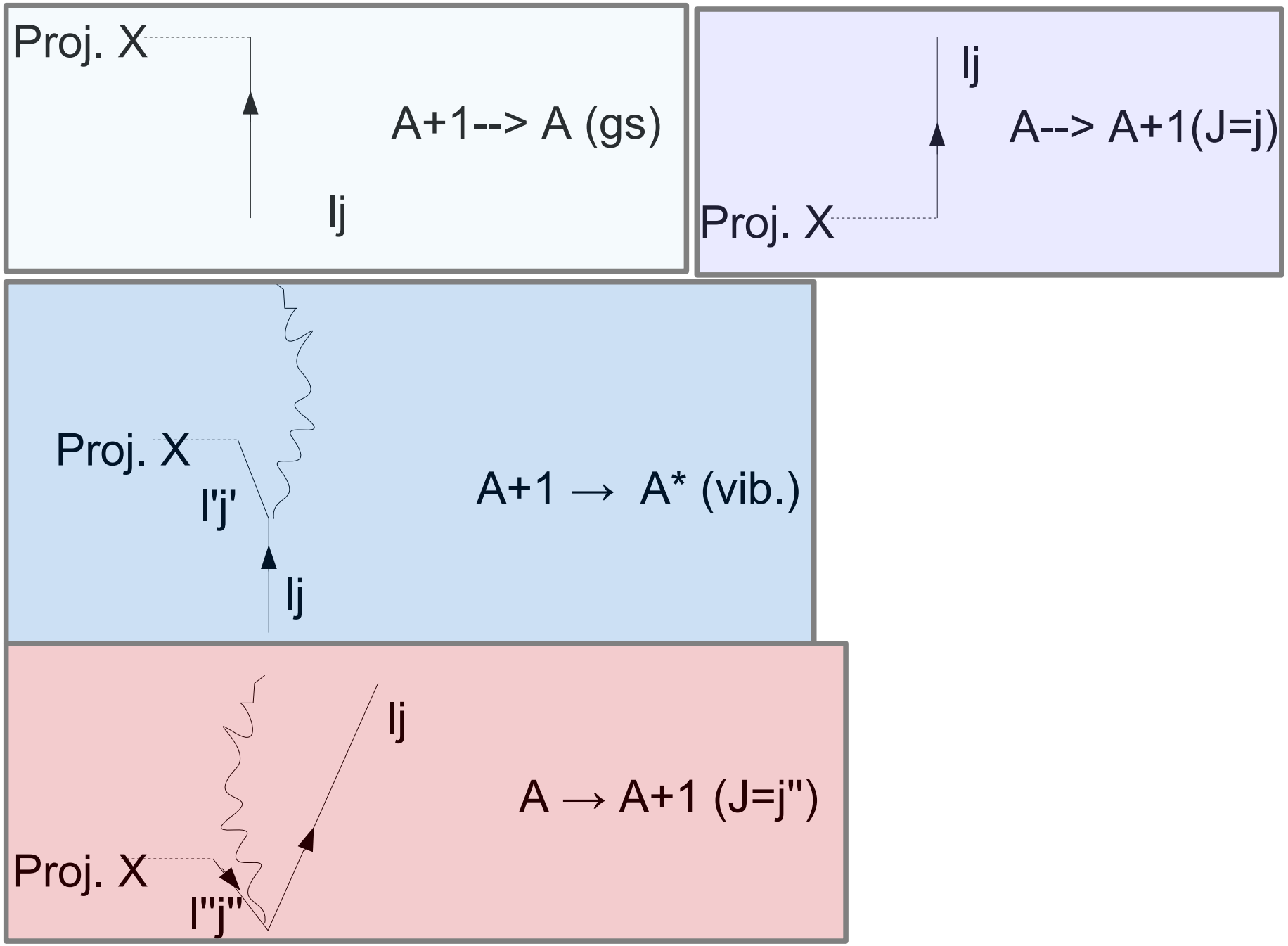
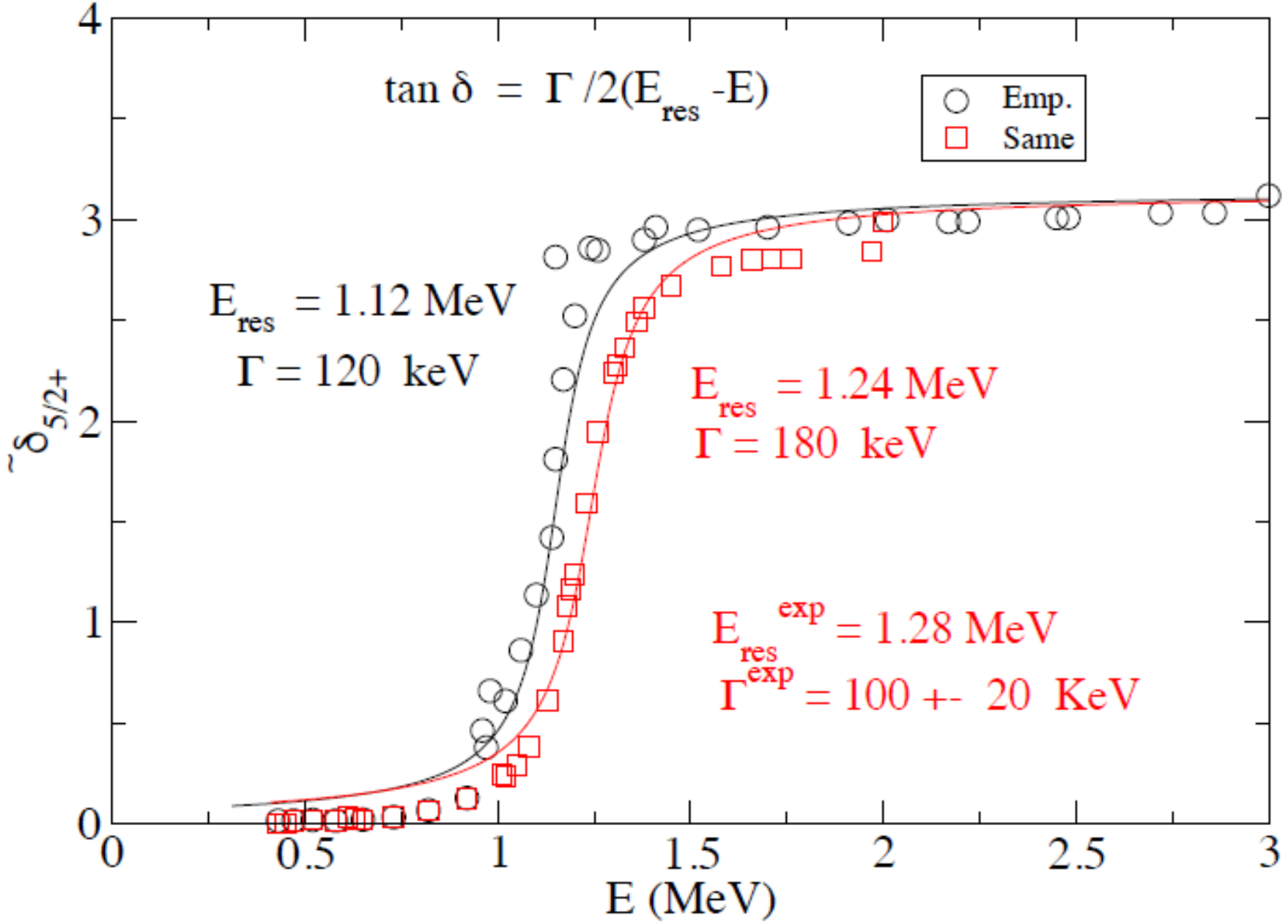




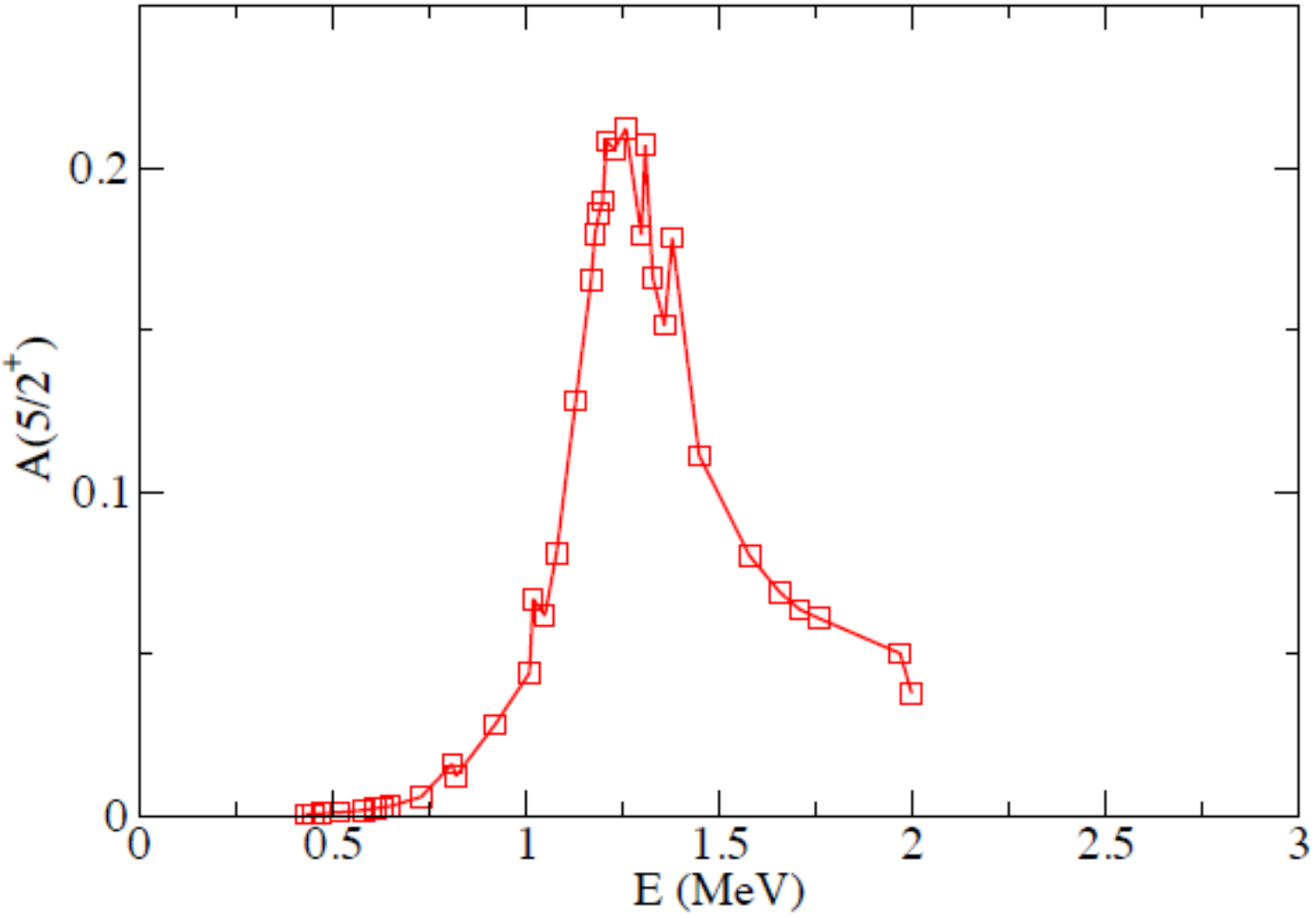


TRANSFER FORM FACTORS

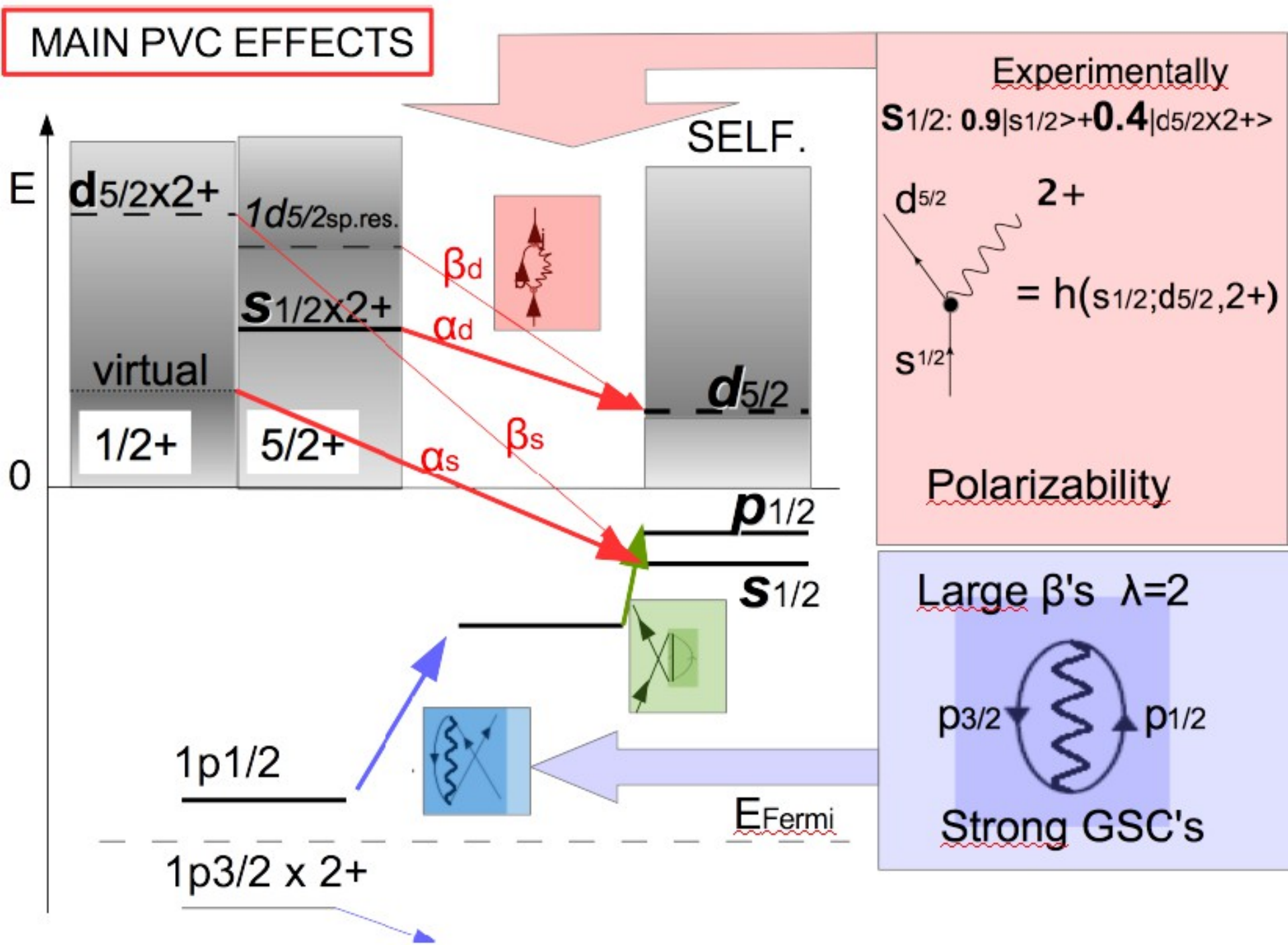



tan δ = Γ /2(E_res -E)
Emp.
Same
E_res = 1.12 MeV
Γ = 120 keV
E_res = 1.24 MeV
Γ = 180 keV
E_res^exp = 1.28 MeV
Γ^exp = 100 +- 20 KeV
~δ_5/2+
E (MeV)
0
0.5
1
1.5
2
2.5
3
4

Admixture of $5/2^+$ state with 2^+ phonon



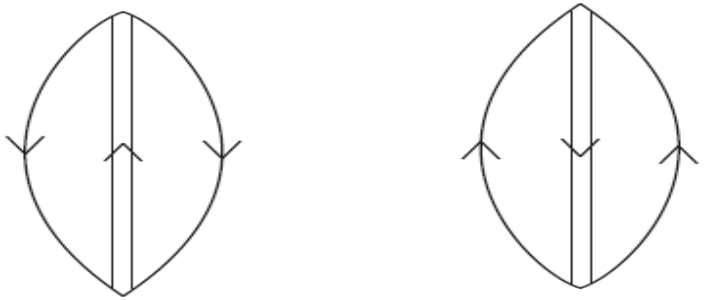
MAIN PVC EFFECTS



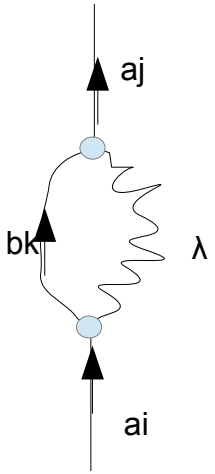
(n,nn) channel: Coupling to pair modes

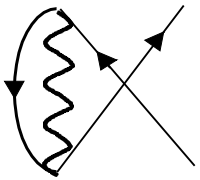
There are Ground State Correlations apart from
a particle-hole plus a surface vibration (a correlated p-h)

In fact: 2 holes plus a (correlated) 2 particles-state (pair addition)
and 2 particles plus a (correlated) 2 holes-state (pair removal)



(n,nn) channel: Coupling to pair modes

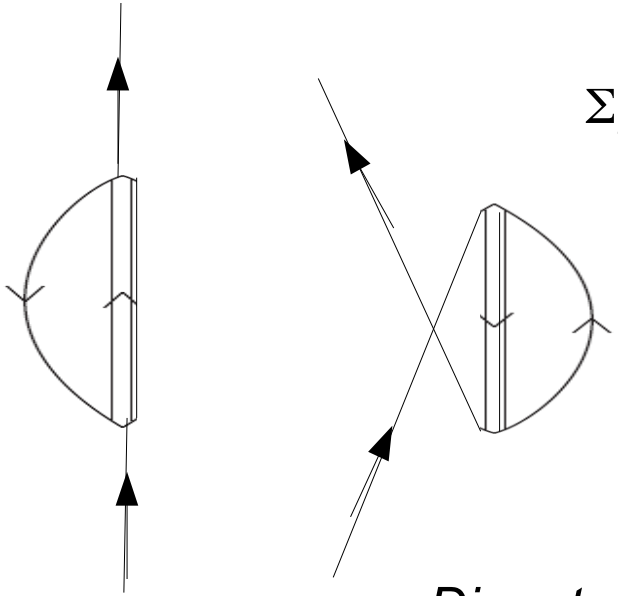




$$\Sigma_{ij}(\tilde{E}) = \sum_{bk}^{Non\,occ.} \frac{h_{ai,bk,\lambda} h_{aj,bk,\lambda}}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \sum_{ck}^{Occ.} \frac{h_{ai,ck,\lambda} h_{aj,ck,\lambda}}{\tilde{E} + (E_{ck} + \hbar\omega_\lambda)}$$

$$C_{bk} = \frac{\Xi_{a,b\lambda} \int dr'\, R^{WS}_{bk}(r') f(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{bk} + \hbar\omega_\lambda)}$$

$$D_{ck} = \frac{-\Xi_{a,b\lambda} \int dr'\, R^{WS}_{ck}(r') f(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{ck} + \hbar\omega_\lambda)}$$



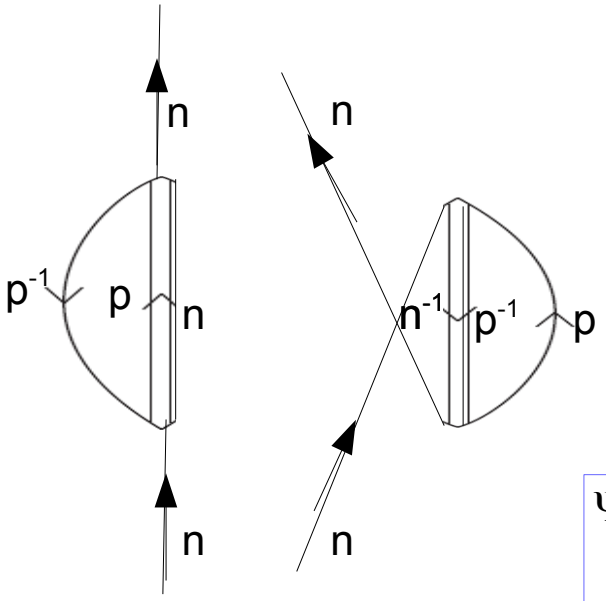
$$\Sigma_{ij}(\tilde{E}) = \sum_{ck}^{Occ.} \frac{g_{ai,ck,\lambda} g_{aj,ck,\lambda}}{\tilde{E} - (E_{ck} + E_{2n}(\lambda))} + \sum_{bk}^{Nonocc.} \frac{g_{ai,bk,\lambda} g_{aj,bk,\lambda}}{\tilde{E} + (E_{bk} + E_{-2n}(\lambda))}$$

$$C_{ck} = \frac{\Xi_{a,b\lambda} \int dr'\, R^{WS}_{ck}(r') g(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{ck} + E_{2n}(\lambda))}$$

$$D_{bk} = \frac{-\Xi_{a,b\lambda} \int dr'\, R^{WS}_{bk}(r') g(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{bk} + E_{-2n}(\lambda))}$$

Dineutron is not stable, but...

...(n,np) channel: Coupling to T=0 pairs



$$C_{ck} = \frac{\Xi_{a,b\lambda} \int dr' R^{WS}_{ck\pi}(r') g^{T=0}_{\lambda}(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{ck\pi} + E_{np}(\lambda))}$$

$$D_{bk} = \frac{-\Xi_{a,b\lambda} \int dr' R^{WS}_{bk\pi}(r') g^{T=0}_{\lambda}(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{bk\pi} + E_{-np}(\lambda))}$$

$$\Psi_a = [\psi^{x}_{a} + \sum_{\lambda} [\psi^{C^{-1}}_{c\pi} \cdot \Phi_{np\lambda}]_{j_a} - [\psi^{D}_{b} \cdot \Phi_{\lambda}]_{j_a} + \ldots] \Phi_{GS}$$

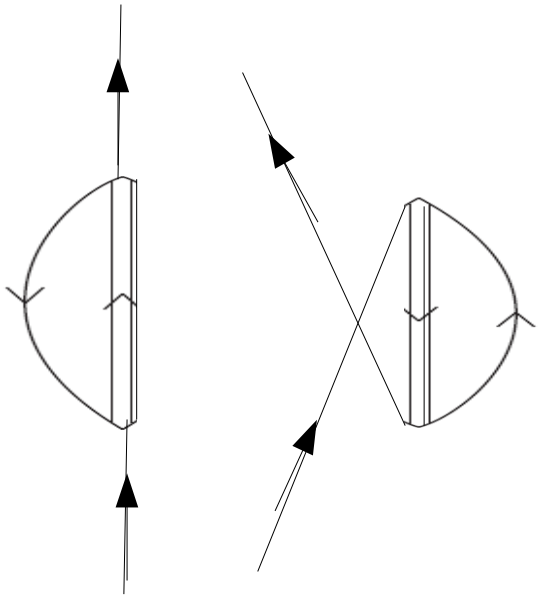
with

$$[\psi^{C^{-1}}_{c\pi} \cdot \Phi_{np\lambda}]_{j_a} = (v^{C}_{c}(r)/r)[\Theta_{j_b} \cdot \Phi_{np\lambda}]_{j_a},$$

$$v^{C}_{c}(r) = \sum_{i} C_{ci} R^{WS}_{ci\pi}(r); e_{ci\pi} < e_F$$

Φ_{npL} *is the deuteron-mode wave with angular momentum L (up to 8 @ 1MeV),*
scattering ones included!!...(n,d) can be calculated

...(n,np) channel: Coupling to T=0 pairs



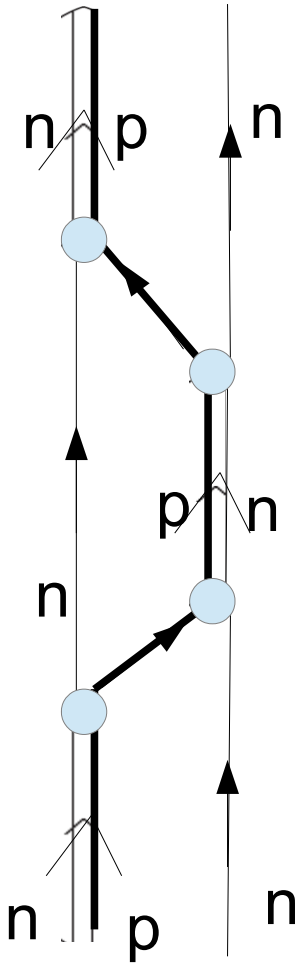
Note: The T=0 pair correlations continue to be a Nuclear Structure challenge.

The direct connection to reaction processes opens new windows for experimentally studying it.

One notable difference with T=1 pairing is the low energy GSC viability, which for T=1 is always perfect, while for T=0, due to the asymmetry between proton and neutron mean field, is much more dificult.

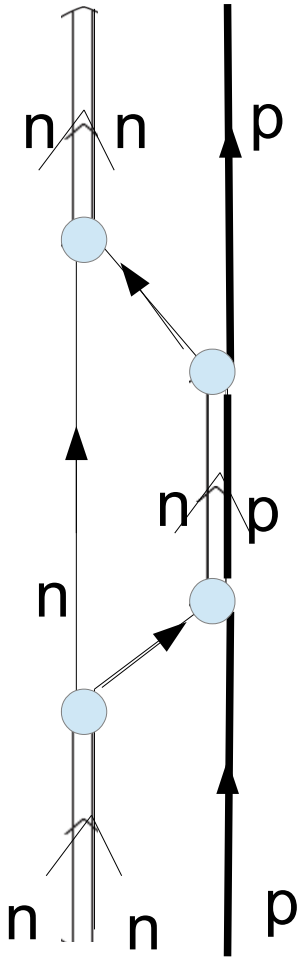
At higher energies instead the strongest n-p interaction favours the T=0 channel

(n,t) channel



The interaction between the T=0 pair and the incoming neutron will lead to bound structures (tritium) whose scattering waves can be analyzed to obtain the transfer amplitudes

(p,t) channel



The interaction between the T=1 pair and the incoming proton will lead to bound structures (tritium) whose scattering waves can be analyzed to obtain the transfer amplitudes

(p,t) using 2nd order DWBA

Phys.Scripta (2016)

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Phys. Scr. 00 (2016) 000000 (37pp)

Invited Comment

Unified description of structure and reactions: implementing the nuclear field theory program

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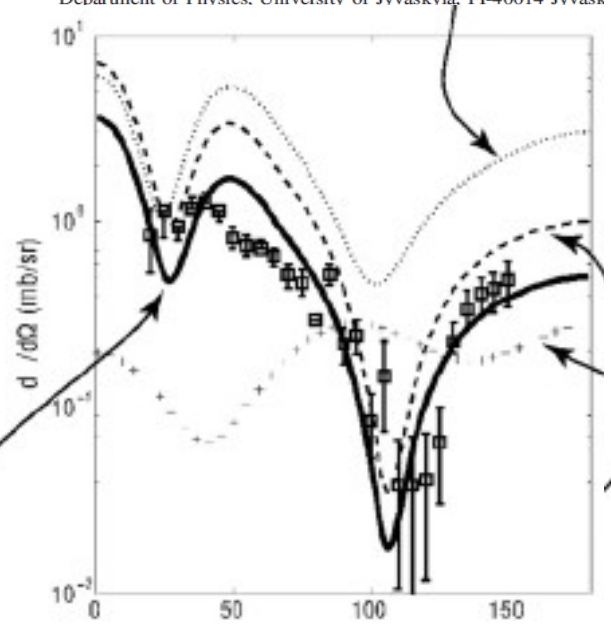
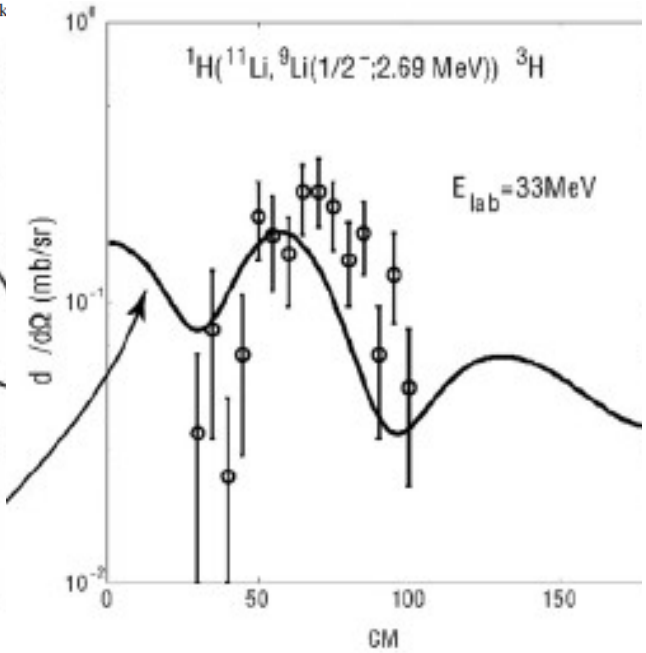
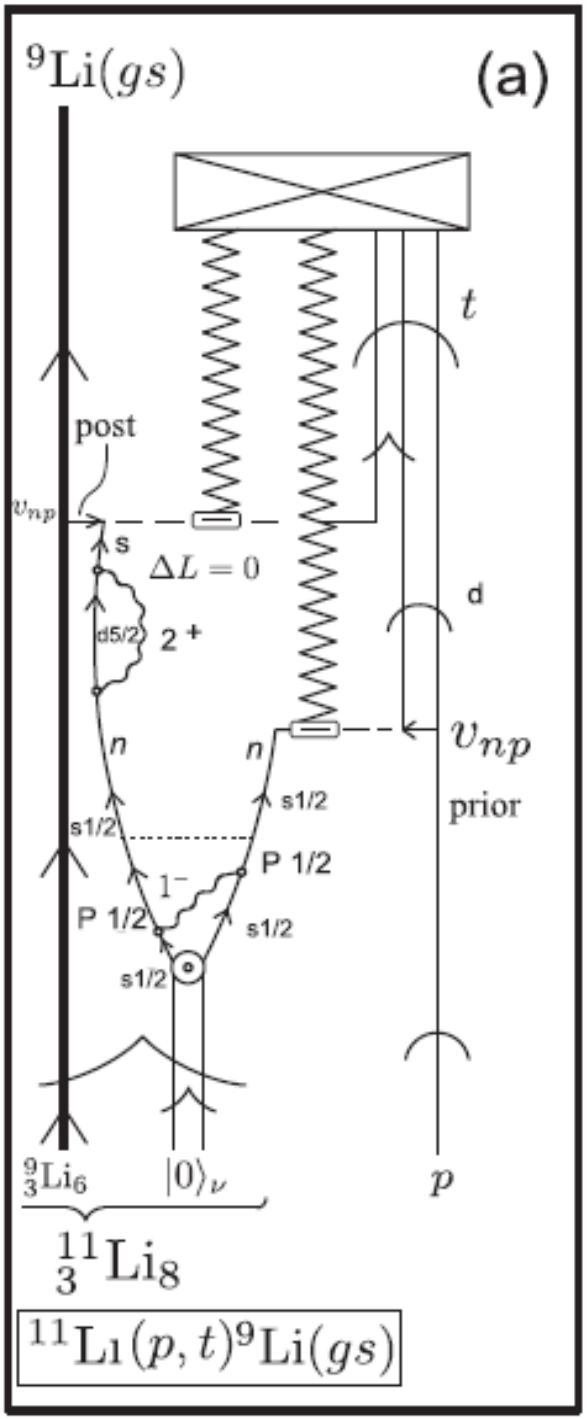
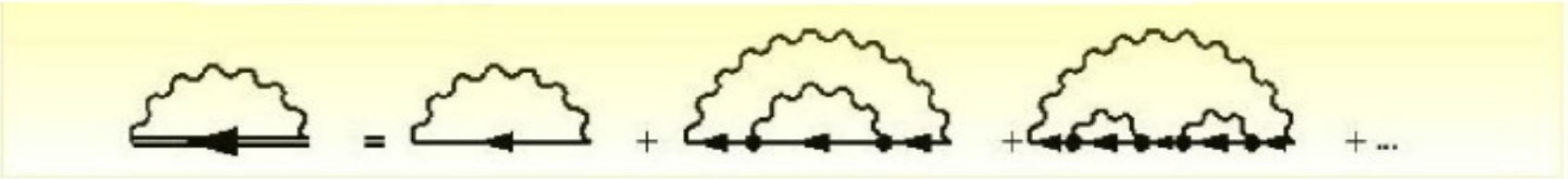
[5] Department of Physics, University of Jyvaskyla, FI-40014 Jyvask







THANK YOU



In the previous calculation, many-phonon contributions have been neglected

Configuration of the dressed $5/2^+$ state E ~1.4 MeV :

$0.9\ 1d_{5/2} + 0.4\ 2s_{1/2} \times 2+$, where the $2s_{1/2}$ state is not bound

However, our dressed $1/2^+$ configuration is bound, with
$1/2^+ = 0.95\ s_{1/2} + 0.3\ d_{5/2} \times 2^+$

By iterating, in the calculation of the dressed $5/2^+$ state we take into account the 2-phonon contribution and the fact that the $1/2^+$ state is localized.

Empirical renormalization : instead of iterating, take empirical states as intermediate states, and check the consistency of the final solution

The energy of the 5/2+ state can be raised reducing the depth of the potential for l=2 waves

Add the admixture

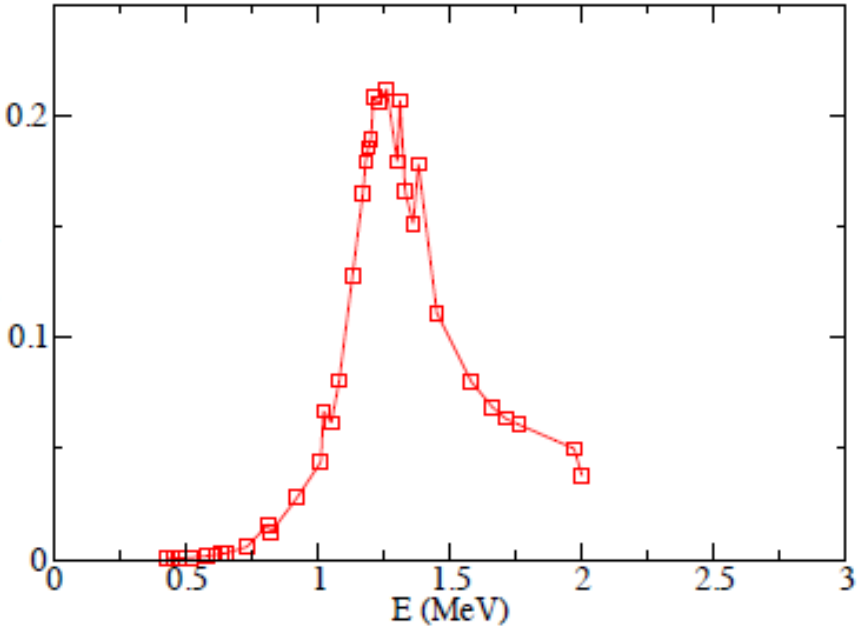
$V(d_{5/2})/(V_{WS})_0$	$E(p_{1/2})$	$\tilde{E}(p_{1/2})$	$A(p_{1/2})$	$E(s_{1/2})$	$\tilde{E}(s_{1/2})$	$A(s_{1/2})$	$E_{res}(d_{5/2})$	$\tilde{E}(d_{5/2})$
0.5	-1.97	-0.21	0.15	0.18	-0.51	0.16	-	1.65
0.6	-1.97	-0.21	0.15	0.18	-0.51	0.16	-	1.42
0.7	-1.97	-0.21	0.15	0.18	-0.51	0.16	-	1.13
0.8	-2.22	-0.35	0.12	0.18	-0.31	0.13	-	1.15
0.9	-1.98	-0.15	0.12	0.22	-0.44	0.12	6.5	0.66
1.0	-1.98	-0.02	0.12	0.22	-0.18	0.13	4.2	0.09
1.1	-1.98	-0.02	0.13	0.22	-0.17	0.12	2.6	-0.71

Admixture of $5/2^+$ state with 2^+ phonon doubles with empirical renormalization

$V(d5/2) = (V(d5/2))_0$

No confined intermediate $s1/2 \times 2^+$ state

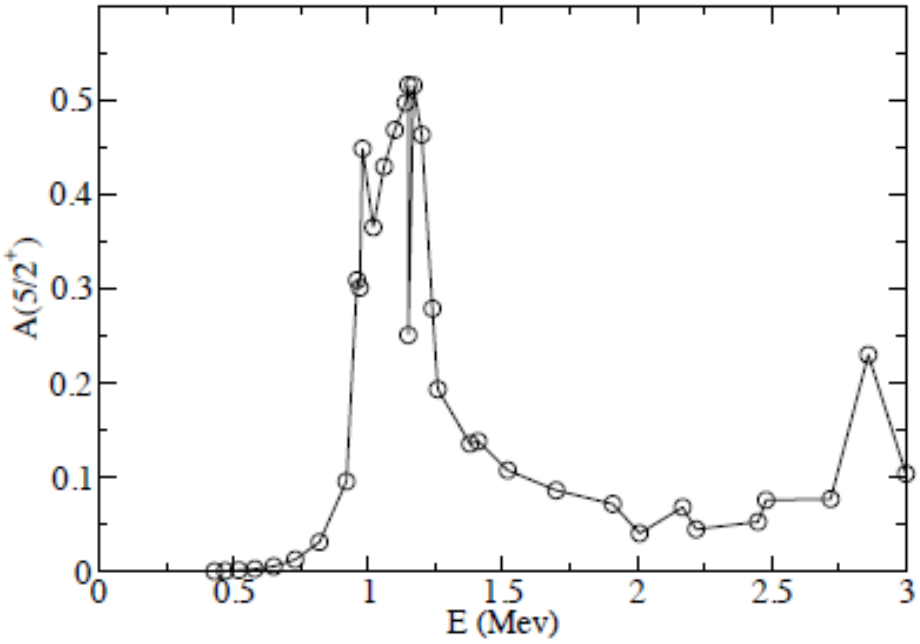
Single particle resonance at 3.5 MeV

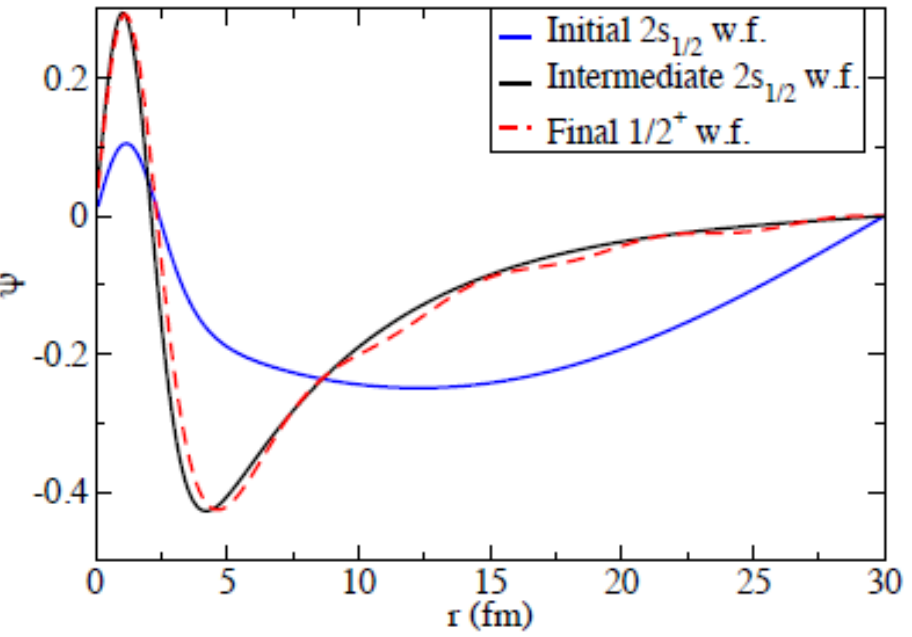


$V(d5/2) = 0.7\ (V(d5/2))_0$

Confined intermediate $s1/2 \times 2+$ state at 3.3 - 0.5 = 2.8 MeV

No single particle resonance



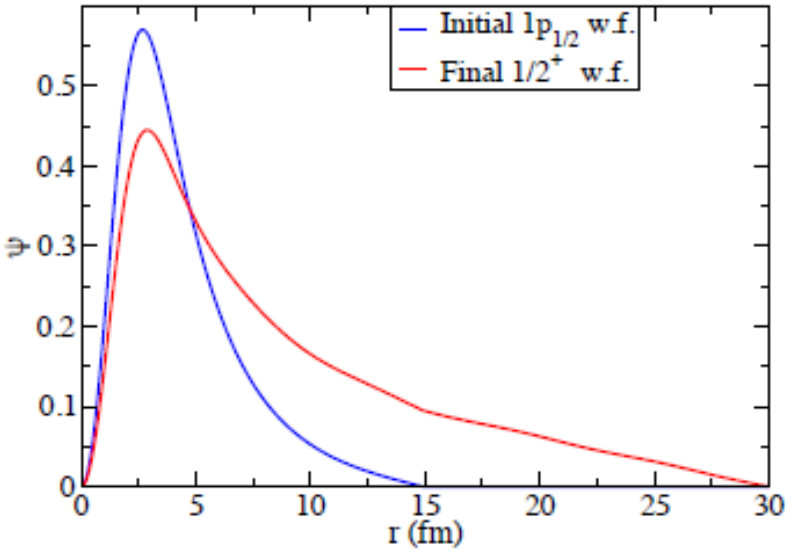


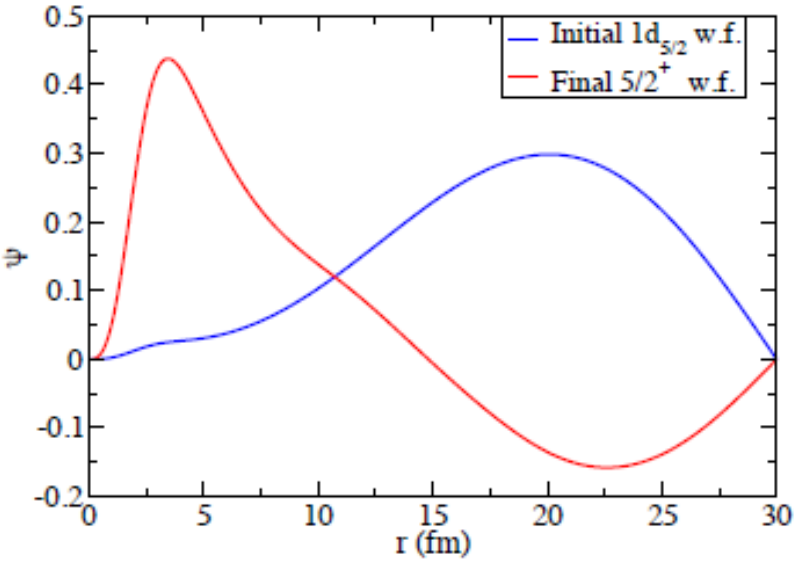
Initial single-particle wavefunctions

Intermediate single-particle wavefunctions

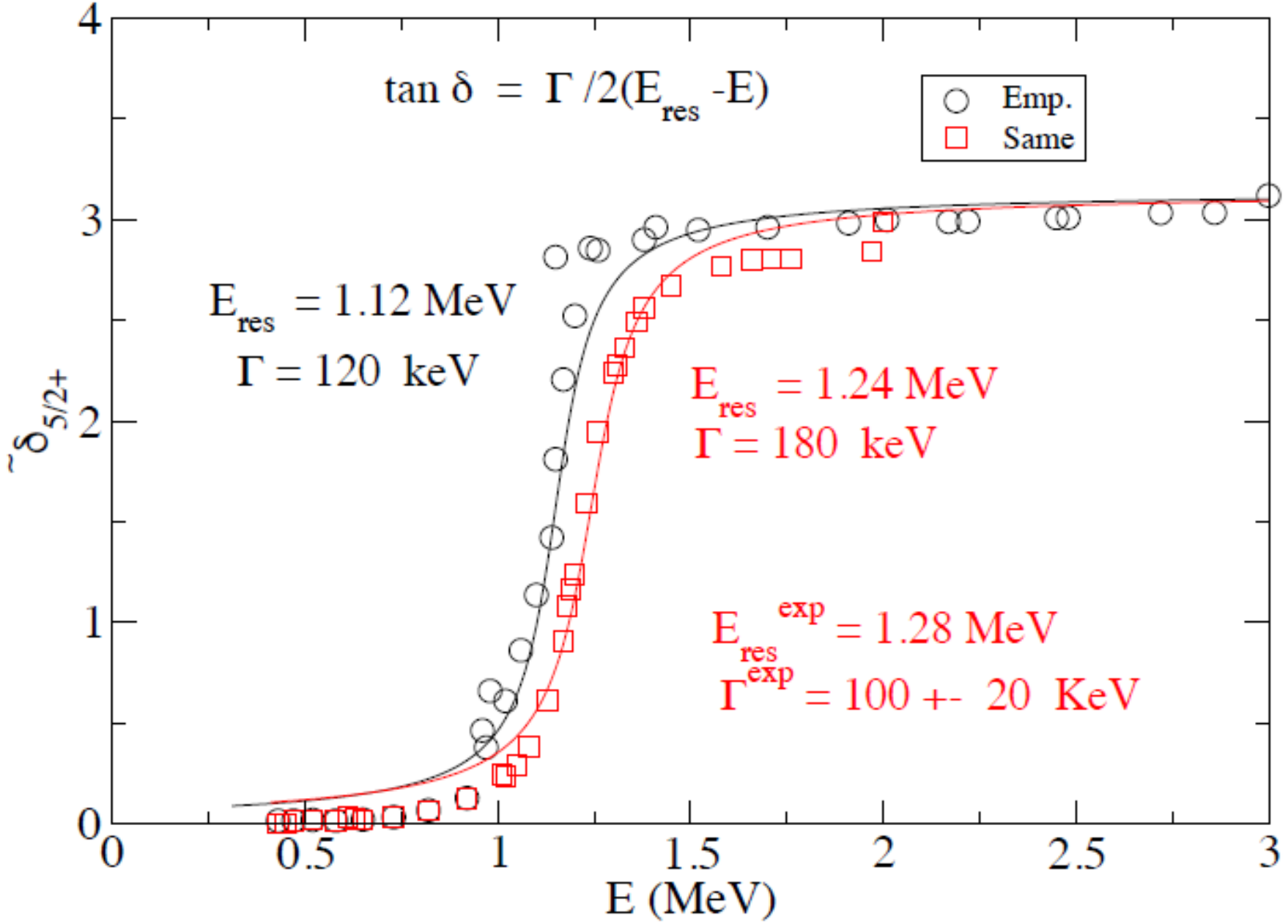
Final many-body wavefunctions

R_{box} = 30 fm





Lay's Waves...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E_res = 1.12 MeV
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Γ = 180 keV
E_res^exp = 1.28 MeV
Γ^exp = 100 +- 20 KeV
~δ_5/2+
E (MeV)
0
0.5
1
1.5
2
2.5
3
4

Optical Potential as the r,r'-E-dependentSelf-energy (*c.f. Idini and Mizuyama*)

$$u_a^x(r)=\sum_i x_{ai} R_{ai}^{WS}(r);\, E_{ai}>0$$
$$u_b^C(r)=\sum_i C_{bi} R_{bi}^{WS}(r);\, E_{bi}>0$$
$$v_c^D(r)=\sum_i D_{ci} R_{ci}^{WS}(r);\, E_{ci}<0$$

$$\begin{pmatrix} H_p-e_F & \Xi_{a,b\lambda} f(r) & \Xi_{a,c\lambda} f(r) \\ \Xi_{a,b\lambda} f(r) & H_p-e_F+\hbar\omega & 0 \\ \Xi_{a,c\lambda} f(r) & 0 & (H_p-e_F)-\hbar\omega \end{pmatrix} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix}$$

$$C_{bk}=\frac{\Xi_{a,b\lambda}\int dr'\, R_{bk}^{WS}(r') f(r') u_{ai}^x(r')}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}$$

$$D_{ck}=\frac{-\Xi_{a,b\lambda}\int dr'\, R_{ck}^{WS}(r') f(r') u_{ai}^x(r')}{\tilde{E}+(E_{ck}+\hbar\omega_\lambda)}$$

$$(H_p(r)-e_F)u_{ai}^x(r)+\int dr' \left[\sum_{bk>0}\frac{\Xi_{a,b\lambda}^2 R_{bk}^{WS}(r') R_{bk}^{WS}(r) f(r') f(r)}{\tilde{E}-(E_{bk}+\hbar\omega_\lambda)}+\sum_{ck<0}\frac{\Xi_{a,b\lambda}^2 R_{ck}^{WS}(r') R_{ck}^{WS}(r) f(r') f(r)}{\tilde{E}+(E_{ck}+\hbar\omega_\lambda)}\right] u_{ai}^x(r') = \tilde{E}\, u_{ai}^x(r)$$

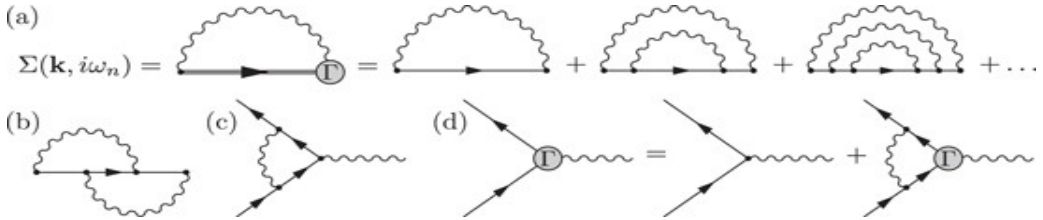
- *Microscopic optical potentials*

In HPVC only linear terms are/must be included.
Higher orders will be discussed here.

$$C_{bk}=\frac{\Xi_{a,b\lambda}\int dr'\,R^{WS}_{bk}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}-(E_{bk}+\hbar\omega_{\lambda})} \quad\rightarrow\quad \frac{\Xi_{a,b\lambda}\int dr'\,u^{x}_{bk}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}-(\tilde{E}_{bk}+\hbar\omega_{\lambda})} \qquad \left(\tilde{E}_{bk}(part.)>0\right)$$

$$D_{ck}=\frac{-\Xi_{a,b\lambda}\int dr'\,R^{WS}_{ck}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}+(E_{ck}+\hbar\omega_{\lambda})} \quad\rightarrow\quad -\frac{\Xi_{a,c\lambda}\int dr'\,v^{x}_{ck}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}+(\tilde{E}_{ck}+\hbar\omega_{\lambda})} \qquad \left(\tilde{E}_{ck}(hole)>0\right)$$

This is the non-crossing phonon aproximation to the many phonons full solution.
It excludes the vertex correction diagrams, that should be incorporated independently

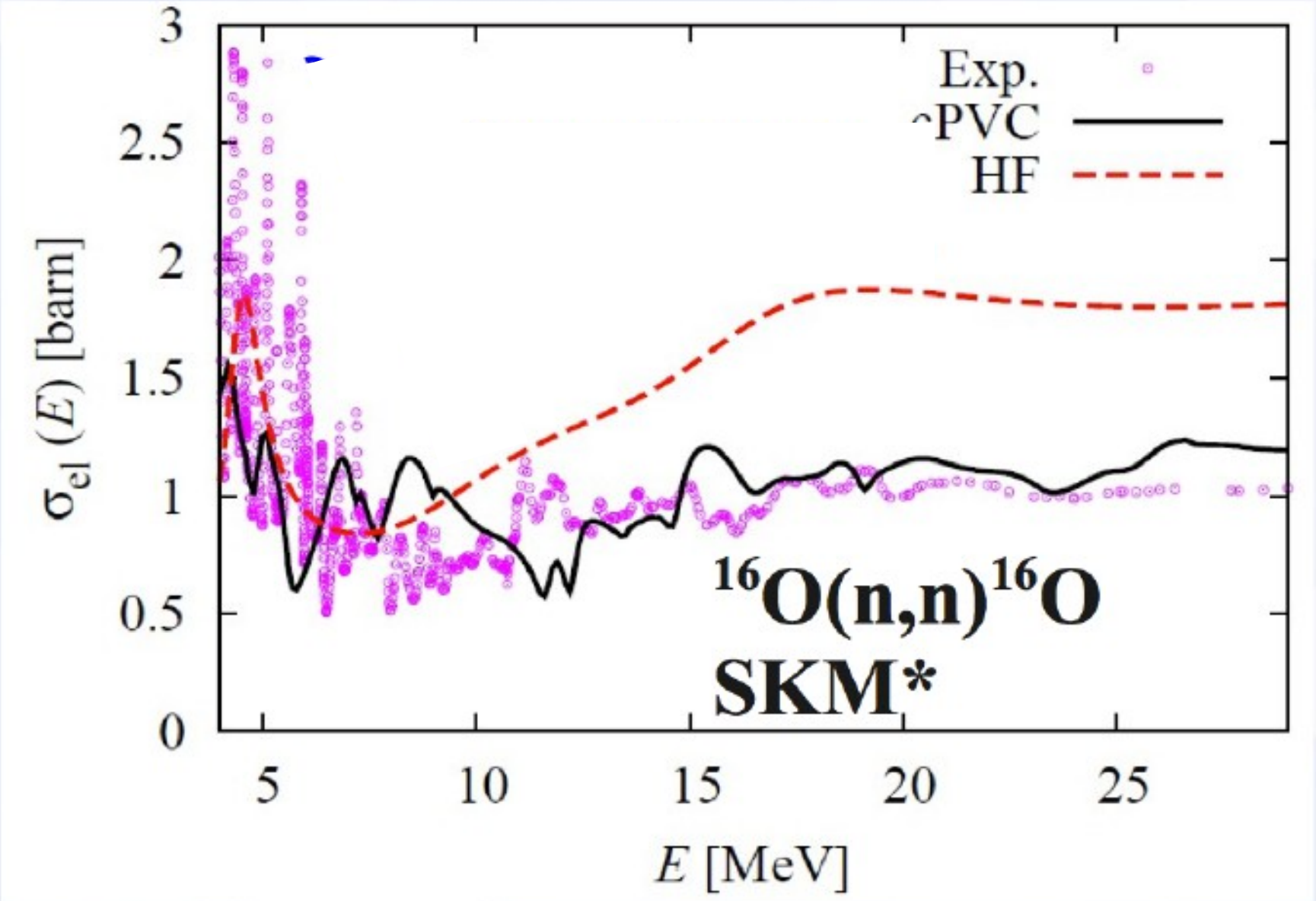


$$(H_p(r)-e_F)u^{x}_{ai}(r)+\int dr'\left[\sum_{bk}\frac{\Xi^2_{a,b\lambda}u^{x}_{bk}(r')u^{x}_{bk}(r)f(r')f(r)}{\tilde{E}_{ai}-(\tilde{E}_{bk}+\hbar\omega_{\lambda})}+\sum_{ck}\frac{\Xi^2_{a,b\lambda}v^{x}_{ck}(r')v^{x}_{ck}(r)f(r')f(r)}{\tilde{E}_{ai}+(\tilde{E}_{ck}+\hbar\omega_{\lambda})}\right]u^{x}_{ai}(r') = \tilde{E}_{ai}u^{x}_{ai}(r)$$

These equations must be solved iteratively or
*Making a good ansatz for the final solution->**experimental info**:*

Empirical Renormalization

No free parameter !



Mizuyama and Ogata, PRC86,041603(2012)

Mizuyama, Colo' and Vigezzi PRC86,034318(2012)

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THE ROYAL SOCIETY

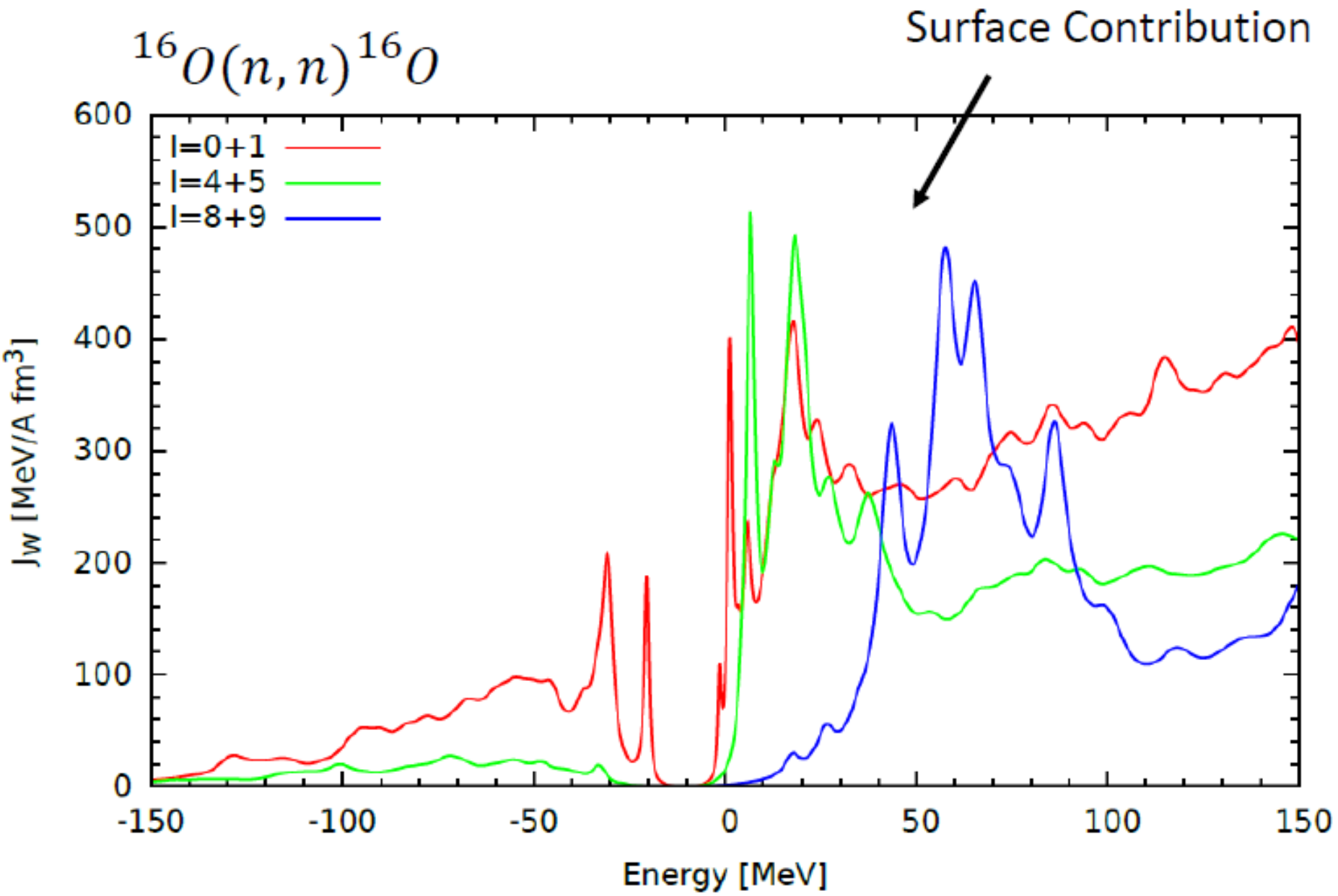
Structure and scattering from (Gorkov-type) self-energies

Andrea Idini

Towards consistent approaches for nuclear structure and reactions

09 Jun 2016

Partial Waves Contribution



PHYSICS REPORTS (Review Section of Physics Letters) 120, Nos. 1–4 (1985) 1–274. North-Holland, Amsterdam

DYNAMICS OF THE SHELL MODEL

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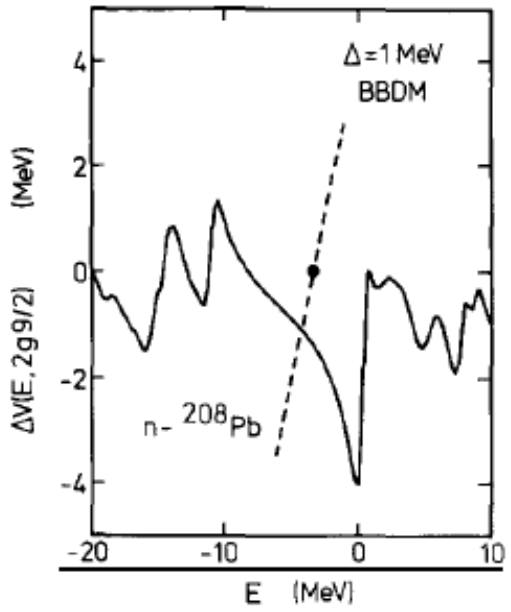


Fig. 4.45a. Energy dependence of the quantity $\Delta V_{2g9/2}(E)$ defined by eq. (3.18a), for the energy-averaging interval $\Delta = 1$ MeV. The straight dashed line represents $E - \varepsilon^{(0)}_{2g9/2}$, where $\varepsilon^{(0)}_{2g9/2} = -3.3$ MeV is derived from the Skyrme III–Hartree–Fock approximation. The approximation scheme is that of ref. [283].

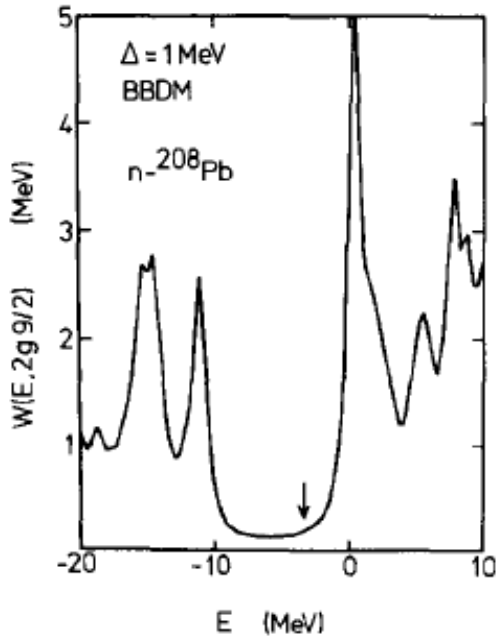
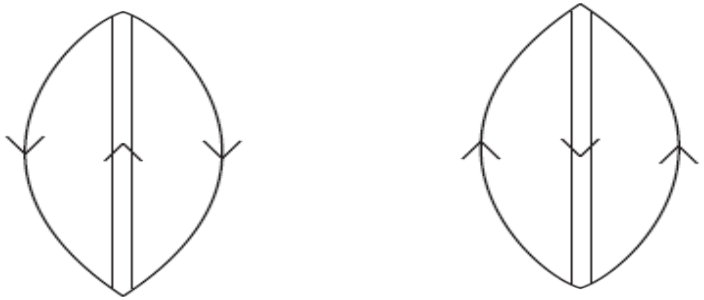


Fig. 4.45b. Energy dependence of the quantity $W_{2g9/2}(E)$ defined by eq. (3.18b), for $\Delta = 1$ MeV. The location of the Skyrme III–Hartree–Fock approximation to the single-particle energy is indicated by an arrow. The large peak of $W_{2g9/2}(E)$ at $E \approx 0.5$ MeV is due to the coupling to the $\{1j_{15/2})_* 3^-\}$ configuration.

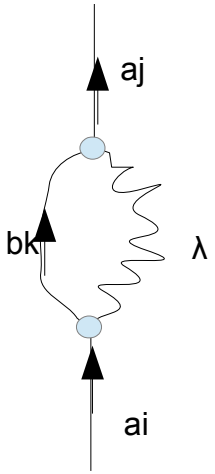
(n,nn) channel: Coupling to pair modes

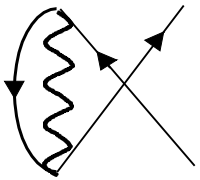
There are Ground State Correlations apart from
a particle-hole plus a surface vibration (a correlated p-h)

In fact: 2 holes plus a (correlated) 2 particles-state (pair addition)
and 2 particles plus a (correlated) 2 holes-state (pair removal)



(n,nn) channel: Coupling to pair modes

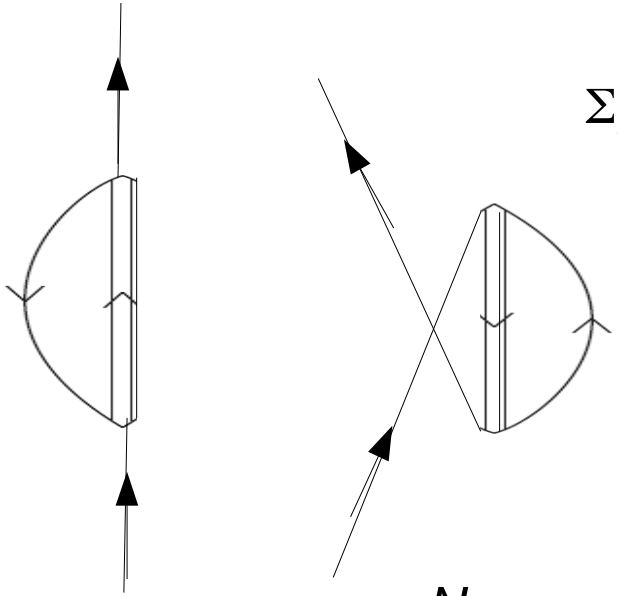




$$\Sigma_{ij}(\tilde{E}) = \sum_{bk}^{Non\,occ.} \frac{h_{ai,bk,\lambda} h_{aj,bk,\lambda}}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \sum_{ck}^{Occ.} \frac{h_{ai,ck,\lambda} h_{aj,ck,\lambda}}{\tilde{E} + (E_{ck} + \hbar\omega_\lambda)}$$

$$C_{bk} = \frac{\Xi_{a,b\lambda} \int dr'\, R^{WS}_{bk}(r') f(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{bk} + \hbar\omega_\lambda)}$$

$$D_{ck} = \frac{-\Xi_{a,b\lambda} \int dr'\, R^{WS}_{ck}(r') f(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{ck} + \hbar\omega_\lambda)}$$



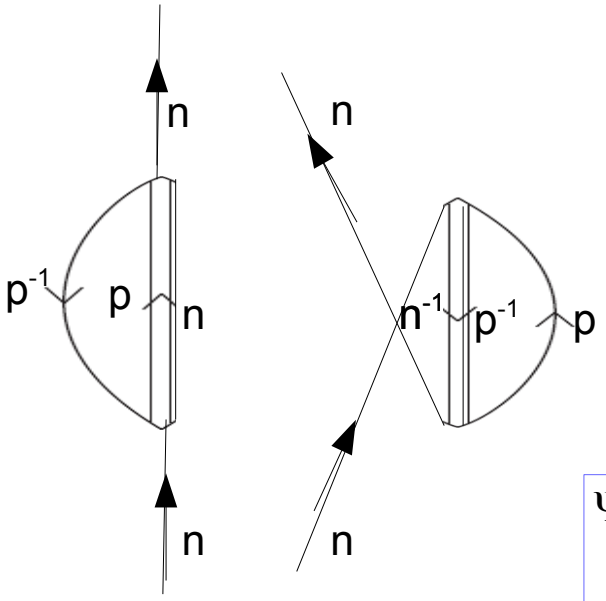
$$\Sigma_{ij}(\tilde{E}) = \sum_{ck}^{Occ.} \frac{g_{ai,ck,\lambda} g_{aj,ck,\lambda}}{\tilde{E} - (E_{ck} + E_{2n}(\lambda))} + \sum_{bk}^{Nonocc.} \frac{g_{ai,bk,\lambda} g_{aj,bk,\lambda}}{\tilde{E} + (E_{bk} + E_{-2n}(\lambda))}$$

$$C_{ck} = \frac{\Xi_{a,b\lambda} \int dr'\, R^{WS}_{ck}(r') g(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{ck} + E_{2n}(\lambda))}$$

$$D_{bk} = \frac{-\Xi_{a,b\lambda} \int dr'\, R^{WS}_{bk}(r') g(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{bk} + E_{-2n}(\lambda))}$$

No open channels since dineutron is not stable, but...

...(n,np) channel: Coupling to T=0 pairs



$$C_{ck} = \frac{\Xi_{a,b\lambda} \int dr' R^{WS}_{ck\pi}(r') g^{T=0}_{\lambda}(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} - (E_{ck\pi} + E_{np}(\lambda))}$$

$$D_{bk} = \frac{-\Xi_{a,b\lambda} \int dr' R^{WS}_{bk\pi}(r') g^{T=0}_{\lambda}(r') u^{x}_{ai}(r')}{\tilde{E}_{ai} + (E_{bk\pi} + E_{-np}(\lambda))}$$

$$\Psi_a = [\psi^{x}_{a} + \sum_{\lambda} [\psi^{C^{-1}}_{c\pi} \cdot \Phi_{np\lambda}]_{j_a} - [\psi^{D}_{b} \cdot \Phi_{\lambda}]_{j_a} + \ldots] \Phi_{GS}$$

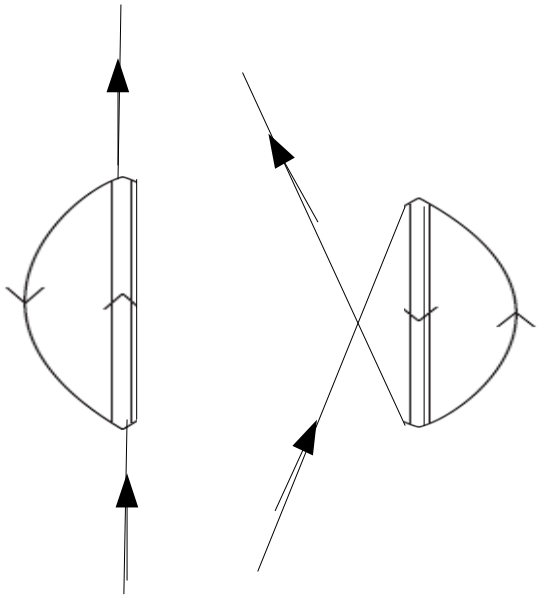
with

$$[\psi^{C^{-1}}_{c\pi} \cdot \Phi_{np\lambda}]_{j_a} = (v^{C}_{c}(r)/r)[\Theta_{j_b} \cdot \Phi_{np\lambda}]_{j_a},$$

$$v^{C}_{c}(r) = \sum_{i} C_{ci} R^{WS}_{ci\pi}(r); e_{ci\pi} < e_F$$

Φ_{npL} *is the deuteron-mode wave with angular momentum L (up to 8 @ 1MeV),*
scattering ones included!!

...(n,np) channel: Coupling to T=0 pairs



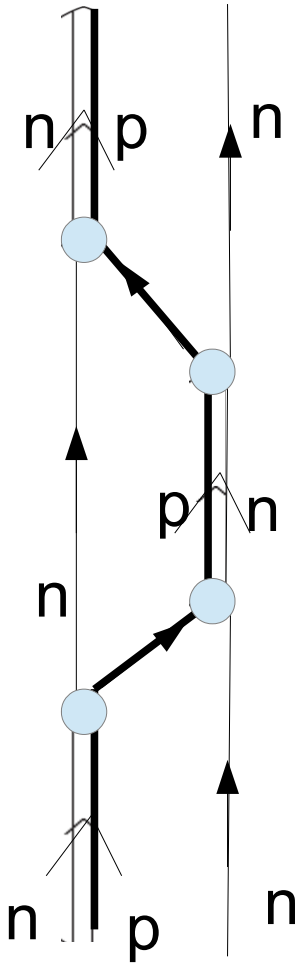
Note: The T=0 pair correlations continue to be a Nuclear Structure challenge.

The direct connection to reaction processes opens new windows for experimentally studying it.

One notable difference with T=1 pairing is the low energy GSC viability, which for T=1 is always perfect, while for T=0, due to the asymmetry between proton and neutron mean field, is much more dificult.

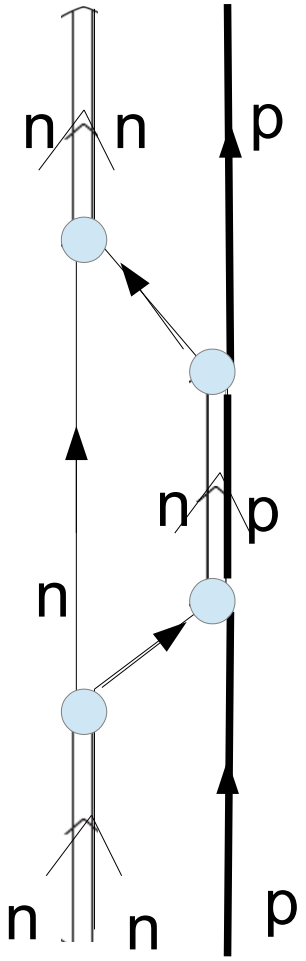
At higher energies instead the strongest n-p interaction favours the T=0 channel

(n,t) channel



The interaction between the T=0 pair and the incoming neutron will lead to bound structures (tritium) whose scattering waves can be analyzed to obtain the transfer amplitudes

(p,t) channel



The interaction between the T=1 pair and the incoming proton will lead to bound structures (tritium) whose scattering waves can be analyzed to obtain the transfer amplitudes

(p,t) using 2nd order DWBA

Phys.Scripta (2016)

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Phys. Scr. 00 (2016) 000000 (37pp)

Invited Comment

Unified description of structure and reactions: implementing the nuclear field theory program

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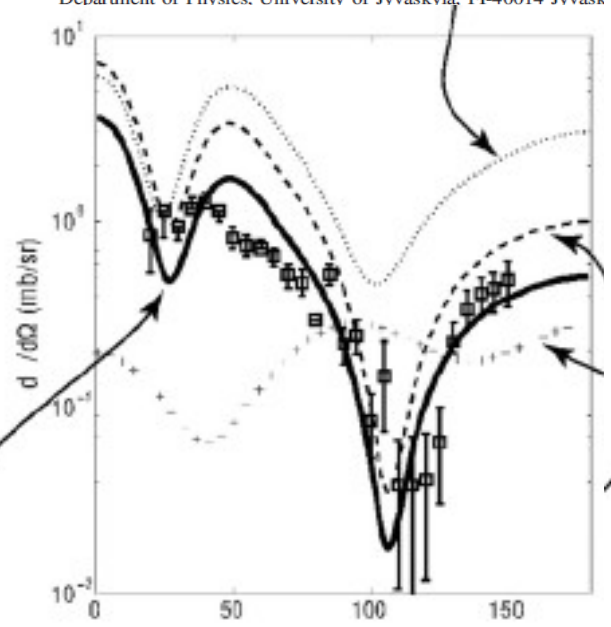

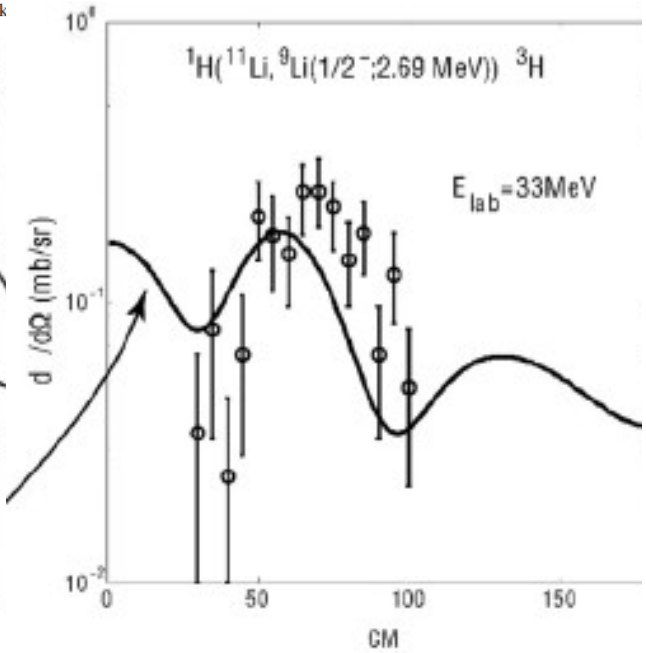

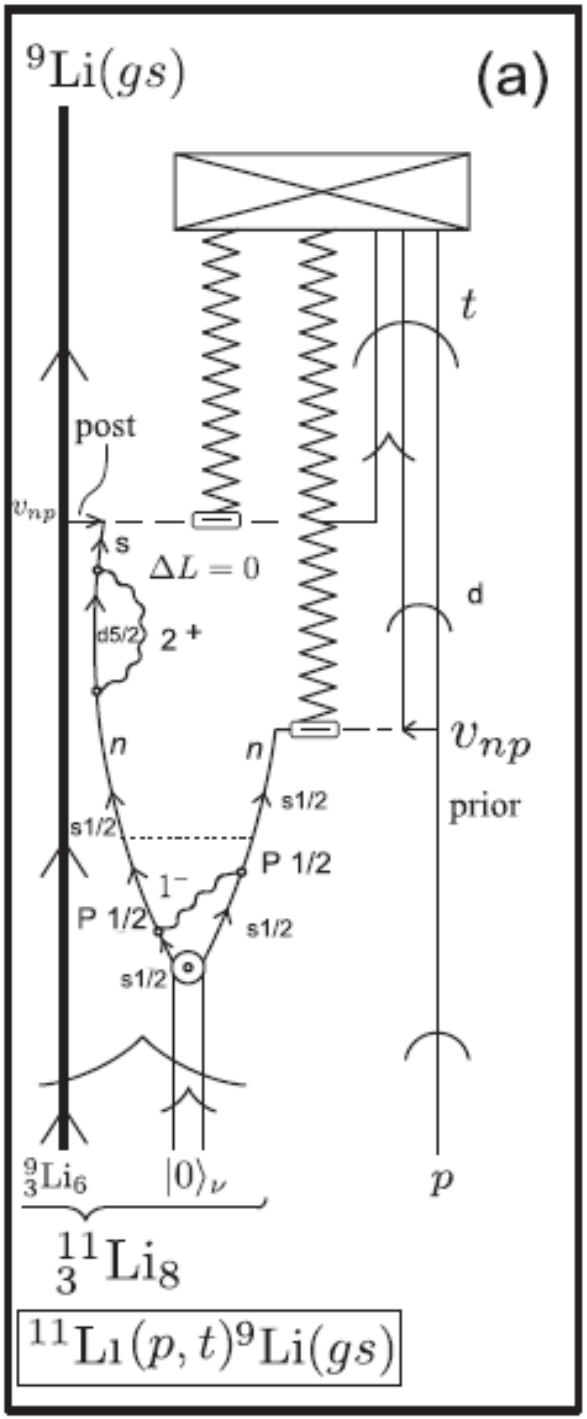

...and beyond

Totally especulative:

Heavy ions collisions.... TDHF... TDNFT

Open shell (superfluid) nucleii.1 Spinor CC equations

$$\begin{pmatrix} H_p - e_F & \Xi_{a,b\lambda} f(r) & \Xi_{a,c\lambda} f(r) \\ \Xi_{a,b\lambda} f(r) & H_p - e_F + \hbar\omega & 0 \\ \Xi_{a,c\lambda} f(r) & 0 & (H_p - e_F) - \hbar\omega \end{pmatrix} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x \\ u_b^C \\ -v_c^D \end{pmatrix}$$

⟸ Particles

$$\begin{pmatrix} -(H_p - e_F) & -\Xi_{a,b\lambda} f(r) & \Xi_{a,c\lambda} f(r) \\ -\Xi_{a,b\lambda} f(r) & -(H_p - e_F) + \hbar\omega & 0 \\ \Xi_{a,c\lambda} f(r) & 0 & -(H_p - e_F) - \hbar\omega \end{pmatrix} \begin{pmatrix} v_a^x \\ v_b^C \\ -u_c^D \end{pmatrix} = \tilde{E} \begin{pmatrix} v_a^x \\ v_b^C \\ -u_c^D \end{pmatrix}$$

⟸ Holes

Using the Pair-Spin notation, these six equation can be written as

$$\begin{pmatrix} H_p - e_F & 0 \\ 0 & -(H_p - e_F) \end{pmatrix} \begin{pmatrix} u_a^x \\ v_a^x \end{pmatrix} + \Xi_{a,b\lambda} f(r) \sum_k \begin{pmatrix} C_{bk} & 0 \\ 0 & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk} \\ v_{bk} \end{pmatrix} + \Xi_{a,c\lambda} f(r) \sum_k \begin{pmatrix} 0 & -D_{ck} \\ -D_{ck} & 0 \end{pmatrix} \begin{pmatrix} u_{ck} \\ v_{ck} \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x \\ v_a^x \end{pmatrix}$$

$$\Xi_{a,b\lambda} f(r) \begin{pmatrix} u_a^x \\ v_a^x \end{pmatrix} + \sum_k (E_{bk} + \hbar\omega) \begin{pmatrix} C_{bk} & 0 \\ 0 & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk} \\ v_{bk} \end{pmatrix} = \tilde{E} \sum_k \begin{pmatrix} C_{bk} & 0 \\ 0 & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk} \\ v_{bk} \end{pmatrix}$$

$$\Xi_{a,b\lambda} f(r) \begin{pmatrix} u_a^x \\ v_a^x \end{pmatrix} + \sum_k (E_{ck} + \hbar\omega) \begin{pmatrix} 0 & -D_{ck} \\ -D_{ck} & 0 \end{pmatrix} \begin{pmatrix} u_{ck} \\ v_{ck} \end{pmatrix} = \tilde{E} \sum_k \begin{pmatrix} 0 & -D_{ck} \\ -D_{ck} & 0 \end{pmatrix} \begin{pmatrix} u_{ck} \\ v_{ck} \end{pmatrix}$$

Open shell (superfluid) cores 2. Adding pairing field

$$\begin{pmatrix} H_p - e_F & \Delta(r) \\ \Delta(r) & -(H_p - e_F) \end{pmatrix} \begin{pmatrix} u_a^x(r) \\ v_a^x(r) \end{pmatrix} + \sum_b \Xi_{a,b\lambda} f(r) \sum_k \begin{pmatrix} C_{bk} & -D_{bk} \\ -D_{bk} & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk}(r) \\ v_{bk}(r) \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x(r) \\ v_a^x(r) \end{pmatrix}$$

$$\Xi_{a,b\lambda} f(r) \begin{pmatrix} u_a^x(r) \\ v_a^x(r) \end{pmatrix} + \sum_k (E_{bk} + \hbar\omega) \begin{pmatrix} C_{bk} & -D_{bk} \\ -D_{bk} & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk}(r) \\ v_{bk}(r) \end{pmatrix} = \tilde{E} \sum_k \begin{pmatrix} C_{bk} & -D_{bk} \\ -D_{bk} & -C_{bk} \end{pmatrix} \begin{pmatrix} u_{bk}(r) \\ v_{bk}(r) \end{pmatrix}$$

$$C_{bk} = \frac{\Xi_{a,b\lambda}}{\tilde{E}_{ai} - (E_{bk} + \hbar\omega_\lambda)} \langle (u_{bk}(r), -v_{bk}(r)) f(r) \begin{pmatrix} u_{ai}^x(r) \\ v_{ai}^x(r) \end{pmatrix} \rangle$$

$$D_{bk} = \frac{-\Xi_{a,b\lambda}}{\tilde{E}_{ai} + (E_{bk} + \hbar\omega_\lambda)} \langle (v_{bk}(r), u_{bk}(r)) f(r) \begin{pmatrix} u_{ai}^x(r) \\ v_{ai}^x(r) \end{pmatrix} \rangle$$

Open shell (superfluid) cores 3. Self-energy

$$\int dr' \left(\begin{pmatrix} H_p(r,r') - e_F & \Delta(r,r') \\ \Delta(r,r') & -(H_p(r,r') - e_F) \end{pmatrix} + \begin{pmatrix} \Sigma_a^{11}(r,r';\tilde{E}) & \Sigma_a^{12}(r,r';\tilde{E}) \\ \Sigma_a^{21}(r,r';\tilde{E}) & \Sigma_a^{22}(r,r';\tilde{E}) \end{pmatrix} \right) \begin{pmatrix} u_a^x(r') \\ v_a^x(r') \end{pmatrix} = \tilde{E} \begin{pmatrix} u_a^x(r) \\ v_a^x(r) \end{pmatrix}$$

$$\Sigma_a^{11}(r,r';\tilde{E}) = \sum_{bk} \Xi^2_{a,b\lambda} f(r) f(r') \left[\frac{u_{bk}(r) u_{bk}(r')}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \frac{v_{bk}(r) v_{bk}(r')}{\tilde{E} + (E_{bk} + \hbar\omega_\lambda)} \right]$$

$$\Sigma_a^{12}(r,r';\tilde{E}) = \sum_{bk} \Xi^2_{a,b\lambda} f(r) f(r') \left[\frac{-u_{bk}(r) v_{bk}(r')}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \frac{v_{bk}(r) u_{bk}(r')}{\tilde{E} + (E_{bk} + \hbar\omega_\lambda)} \right]$$

$$\Sigma_a^{21}(r,r';\tilde{E}) = \sum_{bk} \Xi^2_{a,b\lambda} f(r) f(r') \left[\frac{-v_{bk}(r) u_{bk}(r')}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \frac{u_{bk}(r) v_{bk}(r')}{\tilde{E} + (E_{bk} + \hbar\omega_\lambda)} \right]$$

$$\Sigma_a^{22}(r,r';\tilde{E}) = \sum_{bk} \Xi^2_{a,b\lambda} f(r) f(r') \left[\frac{v_{bk}(r) v_{bk}(r')}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \frac{u_{bk}(r) u_{bk}(r')}{\tilde{E} + (E_{bk} + \hbar\omega_\lambda)} \right]$$

c.f. Van der Sluys et al. NPA551.

Open shell (superfluid) cores 4. Self-energy

Nuclear Physics A551 (1993) 210–240
North-Holland

NUCLEAR PHYSICS A

Fragmentation of single-particle strength in spherical open-shell nuclei: Application to the spectral functions in ^{142}Nd

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Received 24 March 1992
(Revised 1 July 1992)

$$\left[\begin{pmatrix} E_a & 0 \\ 0 & -E_a \end{pmatrix} + \begin{pmatrix} \Sigma_{11}(E) & \Sigma_{12}(E) \\ \Sigma_{12}(E) & \Sigma_{22}(E) \end{pmatrix}\right]\begin{pmatrix} x_0 \\ y_0 \end{pmatrix} = E\begin{pmatrix} x_0 \\ y_0 \end{pmatrix}. \tag{24}$$

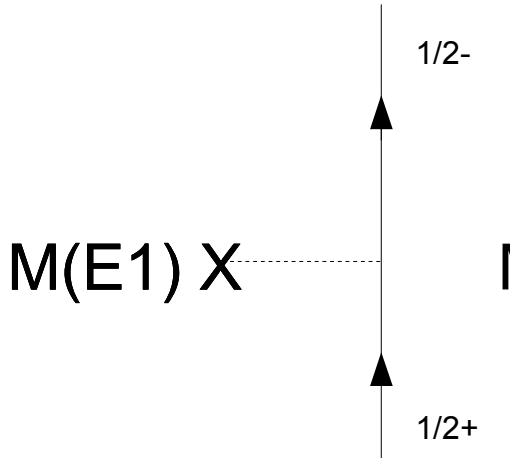
The energy-dependent self-energy matrix elements $\Sigma_{ij}(E)$ stand for:

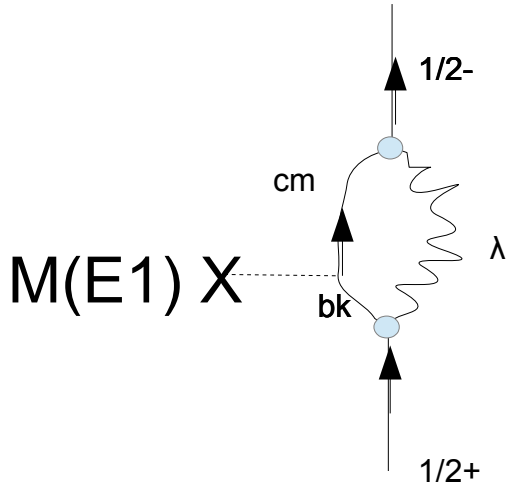
$$\Sigma_{11}(E) = \sum_{bJ\nu}\left(\frac{|V(abJ\nu)|^2}{E-(E_b+E_{J\nu})} + \frac{|W(abJ\nu)|^2}{E+(E_b+E_{J\nu})}\right),$$

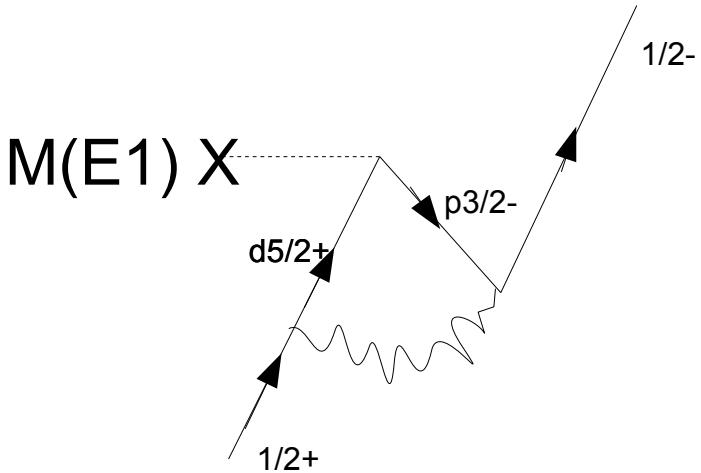
$$\Sigma_{22}(E) = \sum_{bJ\nu}\left(\frac{|W(abJ\nu)|^2}{E-(E_b+E_{J\nu})} + \frac{|V(abJ\nu)|^2}{E+(E_b+E_{J\nu})}\right),$$

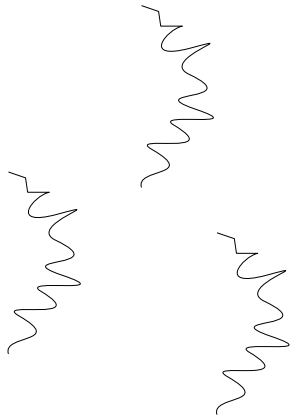
$$\Sigma_{12}(E) = -\sum_{bJ\nu} V(abJ\nu)\,W(abJ\nu)\left(\frac{1}{E-(E_b+E_{J\nu})} - \frac{1}{E+(E_b+E_{J\nu})}\right). \tag{25}$$

B(E1)

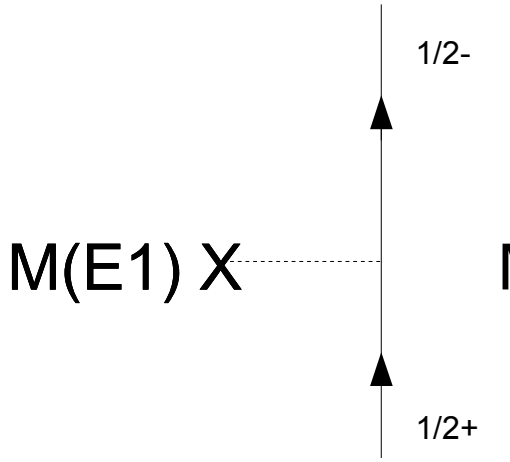


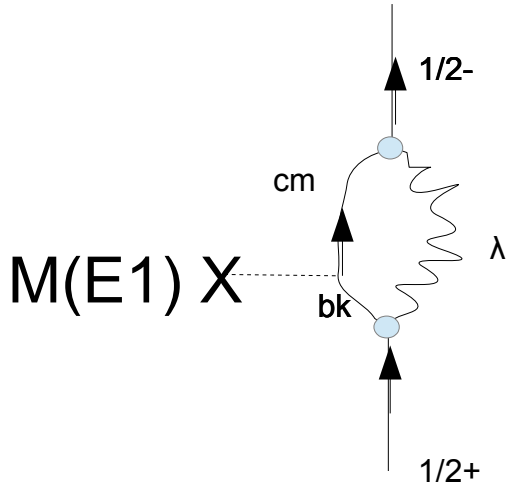


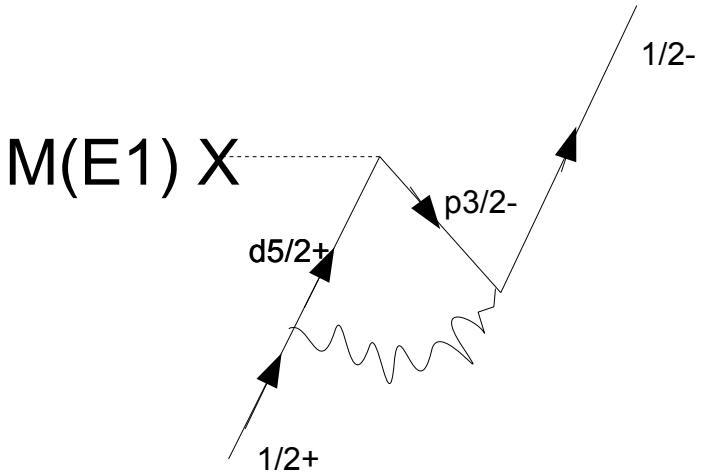


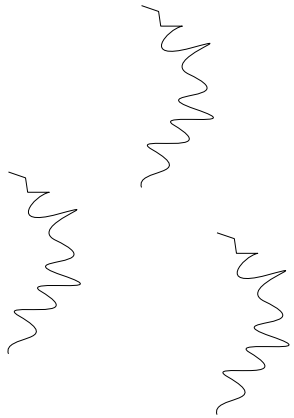


B(E1)

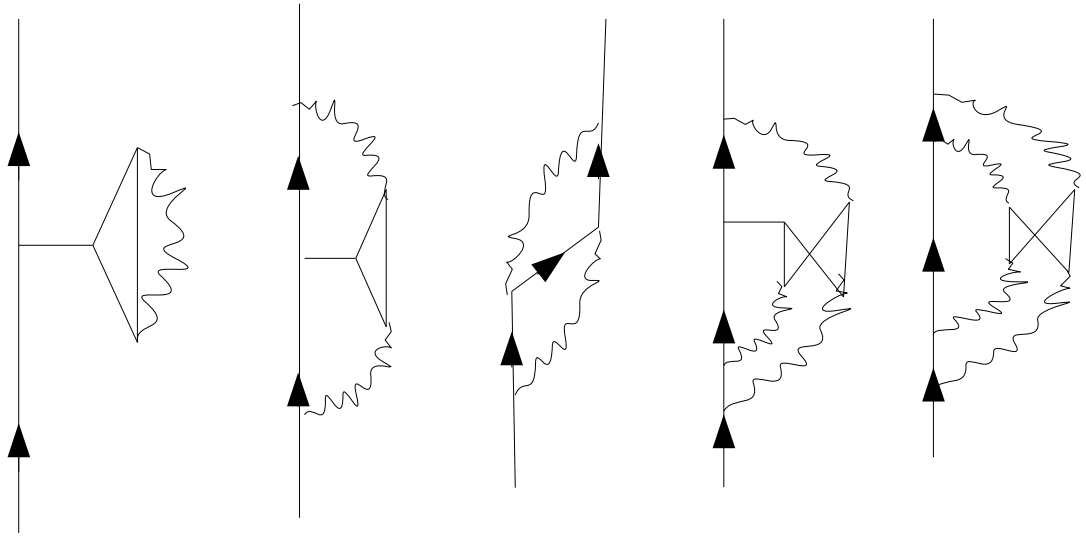




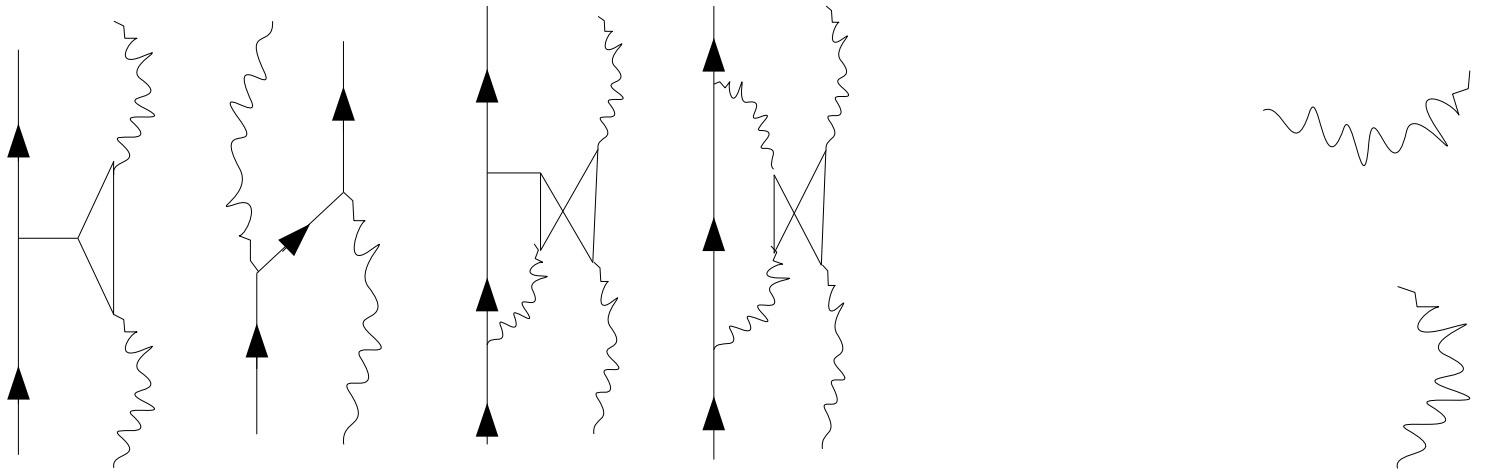




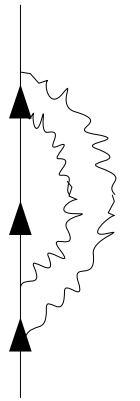
Tadpole, Triangle, Multiplet-C and Butterfly Diagrams



Part.-Phon.-Int.: Triangle, Multiplet-C and Butterfly Diagrams



Tadpole, Triangle, Multiplet-C and Butterfly Diagrams



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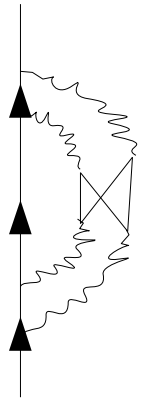
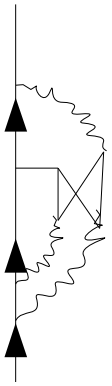
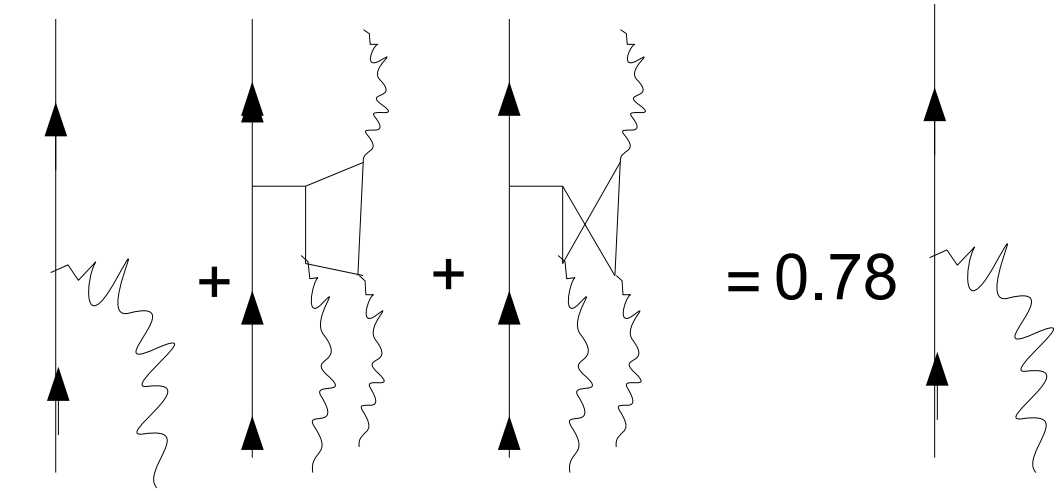
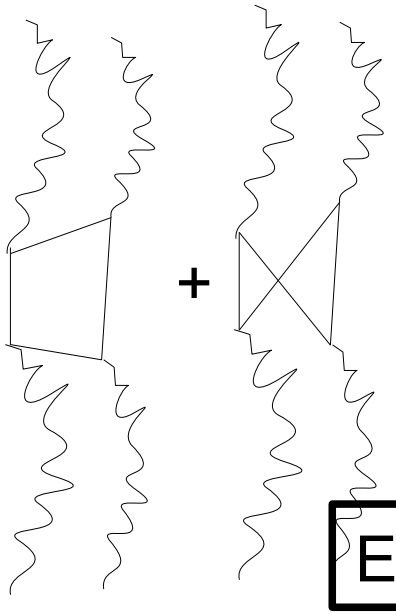




Part.-Phon.-Vertex and Two-Phonon Energy Butterfly Diagrams
QRPA estimate (using $u_{p1/2}=v_{p1/2}=1/\sqrt{2}$; $v_{p3/2}=\sqrt{3}u_{p3/2}=\sqrt{3/4}$)





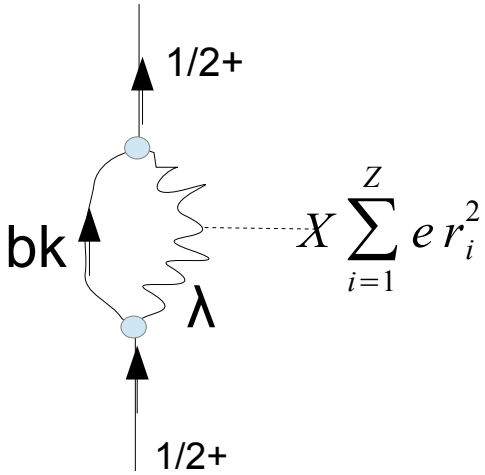
$\beta_{BUTT}=(-0.15-0.7)\beta_0=-0.22\beta_0$

$\beta_{BUTT}=(-0.19-0.9)\beta_0=-0.28\beta_0$ (without square)

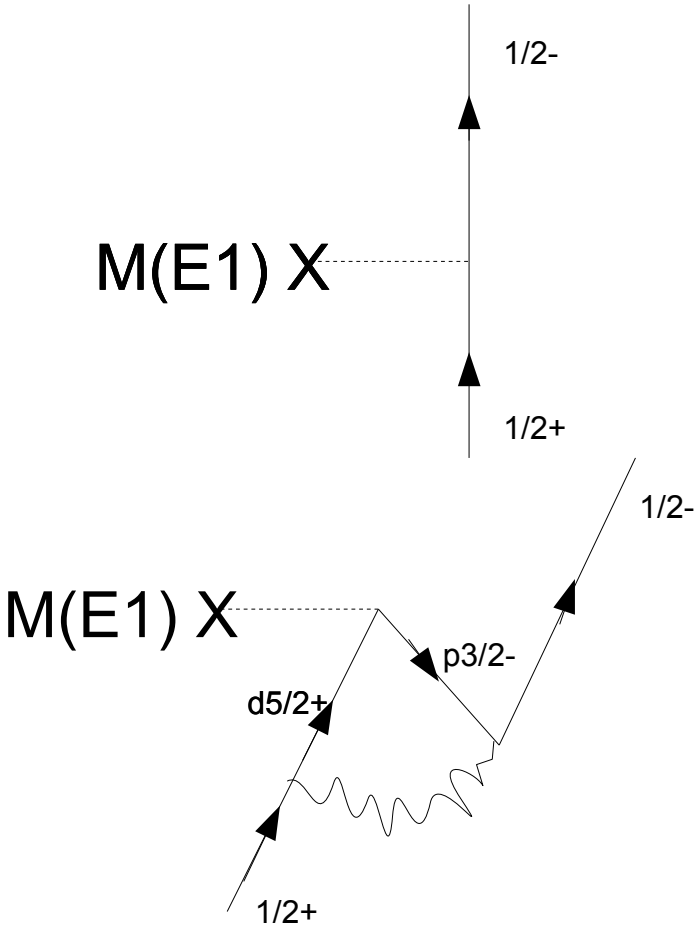
E_{BUTT}= 2 x (0.9MeV + 0.4MeV) = 2.6MeV

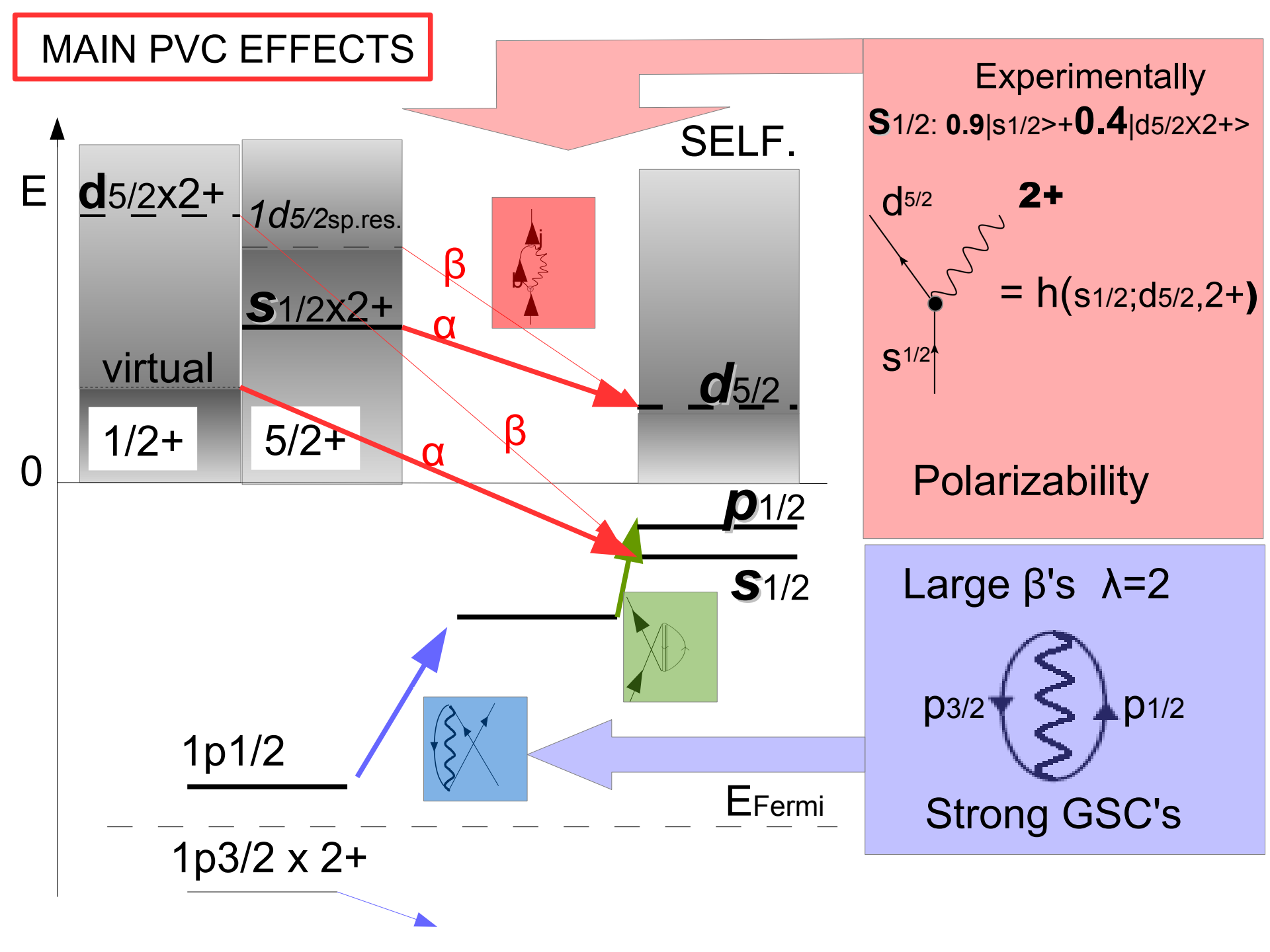
E_{BUTT}= 2 x (1.1MeV + 0.5MeV) = 3.2MeV (without square)

radius



B(E1)




MAIN PVC EFFECTS
E
0
d5/2x2+
1d5/2sp.res.
s1/2x2+
virtual
1/2+
5/2+
SELF.
d5/2
β
α
β
α
p1/2
s1/2
1p1/2
EFermi
1p3/2 x 2+
Experimentally
s1/2: 0.9|s1/2>+0.4|d5/2X2+>
d5/2
2+
= h(s1/2;d5/2,2+)
s1/2
Polarizability
Large β's λ=2
p3/2
p1/2
Strong GSC's

MAIN PVC EFFECTS

PVC/NFT: Vibrational Core (even-even) + One particle (neutron)

$$H = H_c + H_p + H_{PVC}$$

$$H_c = \sum_{\lambda\mu\nu} \hbar\omega_{\lambda,\nu} [\Gamma^{+.}_{\lambda\mu,\nu}\Gamma_{\lambda\mu,\nu} + 1/2]$$

$$H_p = -\hbar^2/2\mu\, d^2/d\vec{r}^2 + V(r) + V_{ls}(r)$$

$$H_{PVC} = -r dV/dr \sum_{\lambda\mu\nu} Y_{\lambda\mu}(\hat{r})\beta_{\lambda,\nu}[\Gamma^{+.}_{\lambda\mu,\nu} + (-1)^{\mu}\Gamma_{\lambda\mu,\nu}]$$

$$H\Psi_a = \tilde{E}\Psi_a$$

partial.waves analisys

$$\Psi_a = [\psi^x_a + [\psi^C_b \cdot \Phi_\lambda]_{j_a} + \ldots]\Phi^A_{GS}$$

$$\psi^x_a = (R^x_a(r)/r)\Theta_{j_a m_a}$$

$$[\psi^C_b \cdot \Phi_\lambda]_{j_a} = (R^C_b(r)/r)[\Theta_{j_b} \cdot \Phi_\lambda]_{j_a}$$

Standard Coupled Channels Equations

$$\begin{bmatrix} [-\hbar^2/2\mu\ d^2/dr^2 + V_a(r)] & \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) \\ \Xi_{a,b\lambda}(-\beta_\lambda r dV/dr) & [-\hbar^2/2\mu\ d^2/dr^2 + V_b(r) + \hbar\omega] \end{bmatrix} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix} = \tilde{E} \begin{bmatrix} R^x_a(r) \\ R^C_b(r) \end{bmatrix}$$

with $\Xi_{a,b\lambda} = \langle \Theta_{j_a m_a} \sum_{\lambda\mu} Y_{\lambda\mu}[\Gamma^{+.}_{\lambda\mu} + (-1)^{\mu}\Gamma_{\lambda\mu}][\Theta_{j_b} \cdot \Phi_\lambda]_{j_a m_a} \rangle \sim \langle j_b, 1/2\,; \lambda\, 0 | j_a, 1/2 \rangle$

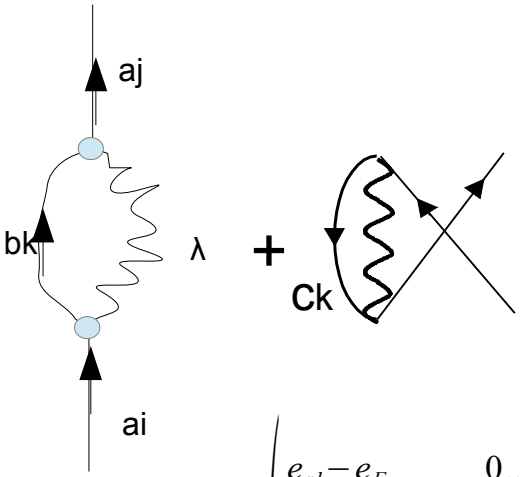
Other comments:

1- For r>10fm we have Bessel functions for positive E's → Phase shifts (S-matrix) may be extracted and thus cross sections (differential or integrated)

2.The theory input are as usual: V(r), Vls(r), the beta's and the hw's.

3. We aim to use a unique V(r) and Vls(r) functions: independent of lj, of E, local (non locality will be discussed)

Matrix elements due to GSC Pauli rearrangement. 2



$$\Sigma_{ij}(\tilde{E}) = \sum_{bk}^{Non\ occ.} \frac{h_{ai,bk,\lambda} h_{aj,bk,\lambda}}{\tilde{E} - (E_{bk} + \hbar\omega_\lambda)} + \sum_{ck}^{Occ.} \frac{h_{ai,ck,\lambda} h_{aj,ck,\lambda}}{\tilde{E} + (E_{ck} + \hbar\omega_\lambda)}$$

$$\left(\begin{array}{cc|cc|cc}
e_{a1}-e_F & 0\ldots & h_{a1,b1\lambda} & h_{a1,b2\lambda}\ldots & -h_{a1,c1\lambda} & -h_{a1,c2\lambda}\ldots \\
0 & e_{a2}-e_{F\ldots} & h_{a2,b1\lambda} & h_{a2,b2\lambda}\ldots & -h_{a2,c1\lambda} & -h_{a2,c2\lambda}\ldots \\
\hline
h_{a1,b1\lambda} & h_{a2,b1\lambda}\ldots & e_{b1}-e_F+\hbar\omega & 0\ldots & 0 & 0\ldots \\
h_{a1,b2\lambda} & h_{a2,b2\lambda}\ldots & 0 & e_{b2}-e_F+\hbar\omega\ldots & 0 & 0\ldots \\
\hline
-h_{a1,c1\lambda} & -h_{a2,c1\lambda}\ldots & 0 & 0 & e_{c1}-e_F-\hbar\omega & 0\ldots \\
-h_{a1,c2\lambda} & -h_{a2,c2\lambda}\ldots\ldots & 0 & 0 & 0 & e_{c2}-e_F-\hbar\omega\ldots
\end{array}\right)
\left(\begin{array}{c} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2\ldots} \\ D_{c1} \\ D_{c2}\ldots \end{array}\right)
= \tilde{E}
\left(\begin{array}{c} x_{a1} \\ x_{a2}\ldots \\ C_{b1} \\ C_{b2\ldots} \\ D_{c1} \\ D_{c2}\ldots \end{array}\right)$$

$$u_a^x(r) = \sum_i x_{ai} R_{ai}^{WS}(r);\ e_{ai} > e_F$$
$$u_b^C(r) = \sum_i C_{bi} R_{bi}^{WS}(r);\ e_{bi} > e_F$$
$$v_c^D(r) = \sum_i D_{ci} R_{ci}^{WS}(r);\ e_{ci} < e_F$$

(*)E<0 solutions belong to the A-1 system

A similar matrix appears for hole-like states

GSC Pauli rearr.: An additional Coupled Channel

!

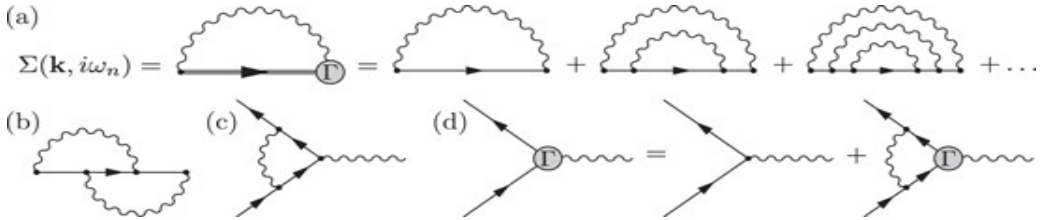
$$\left(\begin{array}{ccc}
H_p - e_F & \Xi_{a,b\lambda} f(r) & \Xi_{a,c\lambda} f(r) \\
\Xi_{a,b\lambda} f(r) & H_p - e_F + \hbar\omega & 0 \\
\Xi_{a,c\lambda} f(r) & 0 & (H_p - e_F) - \hbar\omega
\end{array}\right)
\left(\begin{array}{c} u_a^x \\ u_b^C \\ -v_c^D \end{array}\right)
= \tilde{E}
\left(\begin{array}{c} u_a^x \\ u_b^C \\ -v_c^D \end{array}\right)$$

In HPVC only linear terms are/must be included.
Higher orders will be discussed here.

$$C_{bk}=\frac{\Xi_{a,b\lambda}\int dr'\,R^{WS}_{bk}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}-(E_{bk}+\hbar\omega_{\lambda})} \quad\rightarrow\quad \frac{\Xi_{a,b\lambda}\int dr'\,u^{x}_{bk}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}-(\tilde{E}_{bk}+\hbar\omega_{\lambda})} \qquad \left(\tilde{E}_{bk}(part.)>0\right)$$

$$D_{ck}=\frac{-\Xi_{a,b\lambda}\int dr'\,R^{WS}_{ck}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}+(E_{ck}+\hbar\omega_{\lambda})} \quad\rightarrow\quad -\frac{\Xi_{a,c\lambda}\int dr'\,v^{x}_{ck}(r')\,f(r')\,u^{x}_{ai}(r')}{\tilde{E}_{ai}+(\tilde{E}_{ck}+\hbar\omega_{\lambda})} \qquad \left(\tilde{E}_{ck}(hole)>0\right)$$

This is the non-crossing phonon aproximation to the many phonons full solution.
It excludes the vertex correction diagrams, that should be incorporated independently



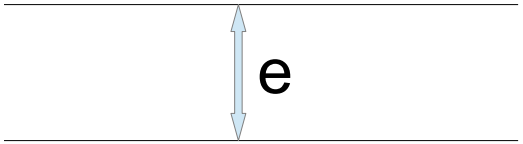
$$(H_p(r)-e_F)u^{x}_{ai}(r)+\int dr'\left[\sum_{bk}\frac{\Xi^2_{a,b\lambda}u^{x}_{bk}(r')u^{x}_{bk}(r)f(r')f(r)}{\tilde{E}_{ai}-(\tilde{E}_{bk}+\hbar\omega_{\lambda})}+\sum_{ck}\frac{\Xi^2_{a,b\lambda}v^{x}_{ck}(r')v^{x}_{ck}(r)f(r')f(r)}{\tilde{E}_{ai}+(\tilde{E}_{ck}+\hbar\omega_{\lambda})}\right]u^{x}_{ai}(r') = \tilde{E}_{ai}u^{x}_{ai}(r)$$

These equations must be solved iteratively or
*Making a good ansatz for the final solution->**experimental info**:*

Empirical Renormalization

The phonon's calculation will be discused here

The butterfly diagram in the TLM (two level model), after NPA260,1(76)

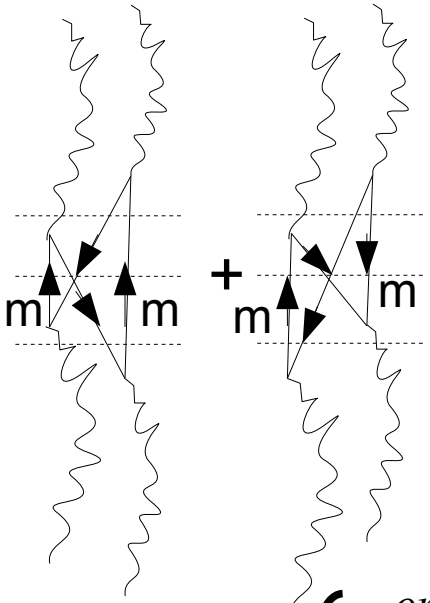


m=-j,...,j; 2Ω=2j+1

m=-j,...,j;2Ω=2j+1

TDA:
$$\begin{cases} \omega = \varepsilon - 2V\,\Omega \\ X = \dfrac{\Lambda}{\varepsilon - \hbar\omega} = \dfrac{1}{\sqrt{2\Omega}} \\ \Lambda = V\sqrt{2\Omega} \end{cases}$$

$$\sum_{m=1}^{2\Omega}$$ ·



+ p-h conjug. = $\dfrac{-2\times 2\times 2\Omega\times\Lambda^4}{(2\omega-\varepsilon-\omega)^2(2\omega-2\varepsilon)} = +2V$

c.f. NPA260 eqs-(28a&35)

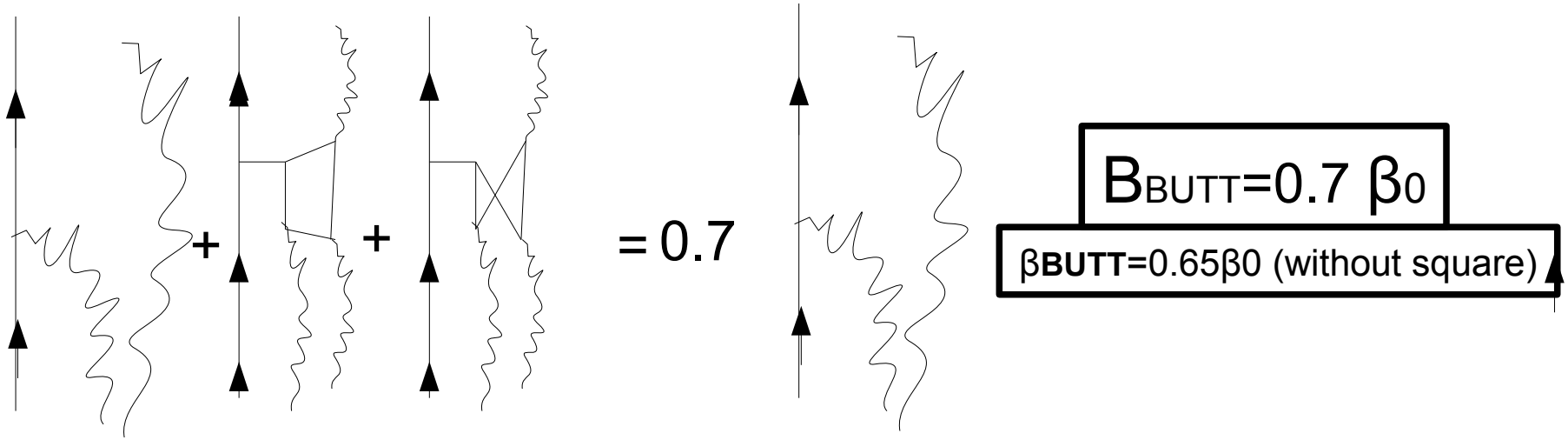
Blocked TDA:

one less jump , occupied by the other phonon: $\delta 2\Omega = -1 \quad \rightarrow \quad \delta\Omega = -1/2$

change for the one phonon energy: $\delta\omega = \dfrac{d\omega}{d\Omega}\times\delta\Omega = -2V\,\delta\Omega = +V$

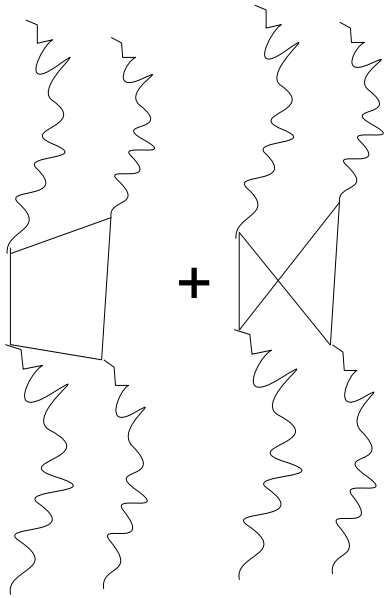
finally the change for the two phonon state: $2\times\delta\omega = +2V$

Part.-Phon.-Vertex and Two-Phonon Energy Butterfly Diagrams
QRPA estimate (using $u_{p1/2}=v_{p1/2}=1/\sqrt{2}$; $v_{p3/2}=\sqrt{3}u_{p3/2}=\sqrt{3/4}$)



$B_{BUTT}=0.7\ \beta_0$

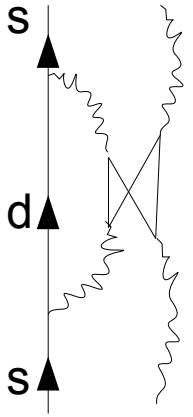
$\beta_{BUTT}=0.65\beta_0$ (without square)



$E_{BUTT}= 2 \times (1.35MeV) = 2.7MeV$

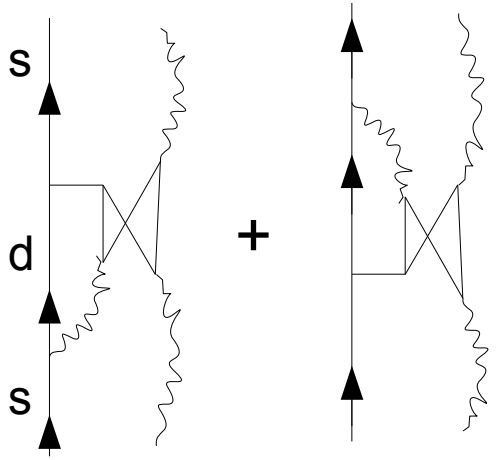
EBUTT= 2 x (1.50MeV) = 3.00MeV (without square)

Part.-Phon.-Int.: Butterfly Diagrams; sx2+



= (ADMIXs)(E_{BUTT}) = 0.2 * (2.7MeV) = +540keV

+1.20MeV



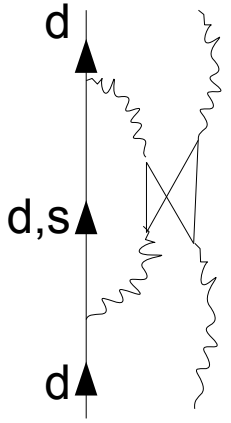
= -2 * (0.3 $\beta_{0*}V_{sd*}\beta_{0*}V_{sd}$) /(Deno)=

-0.6*ADMIXs*Deno=-0.56 ADMIXs (e_s-e_d -hw)

= 636keV

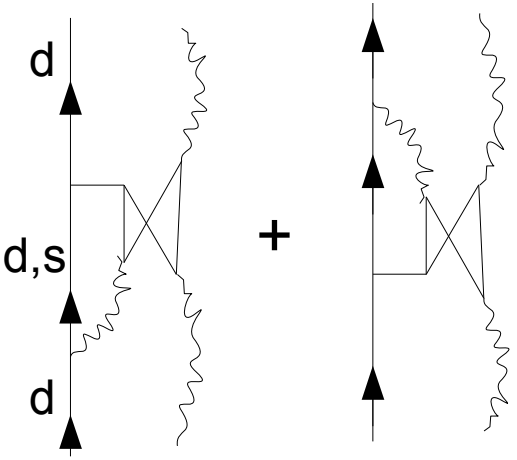
The factor of 2 is for the two internal affected verteces

Part.-Phon.-Int.: Butterfly Diagrams: dx2+



= (ADMIXd)(E_{BUTT}) = 0.5 * (2.70MeV) = +1350keV

+2.13MeV



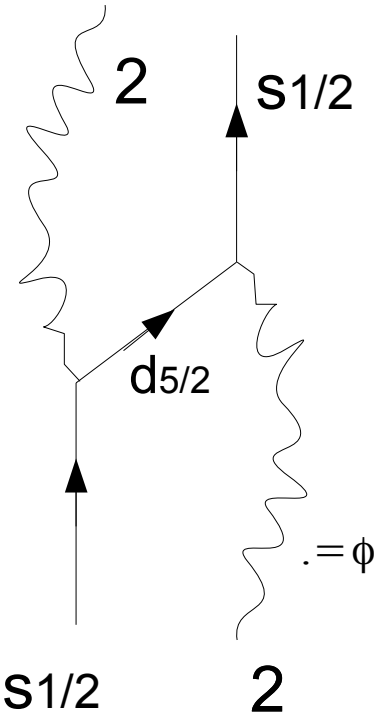
= $-2 * (0.3\ \beta_{0*}V_{sd*}\beta_{0*}V_{sd}) / (Deno) =$

-0.6*ADMIXd*Deno=-0.56 ADMIXd (e_d-e_ds -hw)

= 780keV

The factor of 2 is for the two internal affected verteces

The C-diagram



$$.=\phi_1\frac{\left(V\left(s;2\mathrm{d}\right)\times V\left(s;d2\right)\right)}{\left(e_s+\hbar\omega-e_d\ 2\hbar\omega\right)}\times\left\langle\left(2\ 5/2\right)1/2\ 2;5/2|2\left(5/2\ 2\right)1/2;5/2\right\rangle$$

with

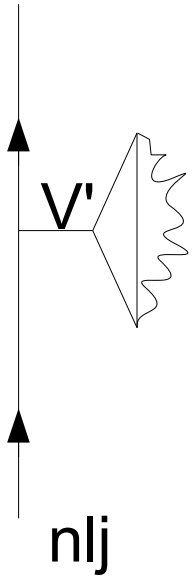
$$\phi_1=\left(-1\right)^{1/2+2-5/2}=+1;\quad V\left(s;2\mathrm{d}\right)\times V\left(s;d2\right)=V\left(s;2\mathrm{d}\right)^2\left(-1\right)^{1/2+2-5/2}=+V\left(s;2\mathrm{d}\right)^2$$

$$\left\langle\left(2\ 5/2\right)1/2\ 2;5/2|2\left(5/2\ 2\right)1/2;5/2\right\rangle=\sqrt{\left(2x1/2+1\right)\left(2x1/2+1\right)}\left(-1\right)^{2+5/2+2+5/2}\begin{Bmatrix}2 & 5/2 & 1/2\\ 2 & 5/2 & 1/2\end{Bmatrix}=2/30$$

$$\left(e_s+\hbar\omega-e_d-2\hbar\omega\right)\approx-5.2\mathrm{MeV};\qquad since\qquad\frac{V^2\left(s;2\mathrm{d}\right)}{5.2^2}\approx0.17\ \rightarrow\frac{V^2\left(s;2\mathrm{d}\right)}{-5.2\mathrm{MeV}}\approx-1.0\mathrm{MeV}$$

$$thus\ the\ diagram\ is-2/30\mathrm{MeV}=-0.066\mathrm{MeV}$$

Tadpole



This is an energy independent diagram. Its effect can be incorpored in static mean fields. The monopole part of V' is totally democratic and thus leads to a democratic (state independent correction), which is included in our common empirical mean filed.

Being the phonon of Quadrupole type (and thus of p1/2,p3/2^-1 nature), the quadrupole component of V' acts only in the nlj= 1p1/2, and thus with large Pauli inhibition.

In any case the tadpole is a small part of....

Charge Radii

$$\langle r^2\rangle_{11\mathrm{Be}} = \left(\langle r^2\rangle_{10\mathrm{Be}} + \left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2\right)\times S^2 + (1-S^2)\times\left(\langle r^2\rangle_{10\mathrm{Be}}\left(1+\frac{2}{4\pi}\beta_\pi^2\right) + \left(\frac{\langle r^2\rangle^{1/2}_{d5/2\,coll}}{11}\right)^2\right) = .$$

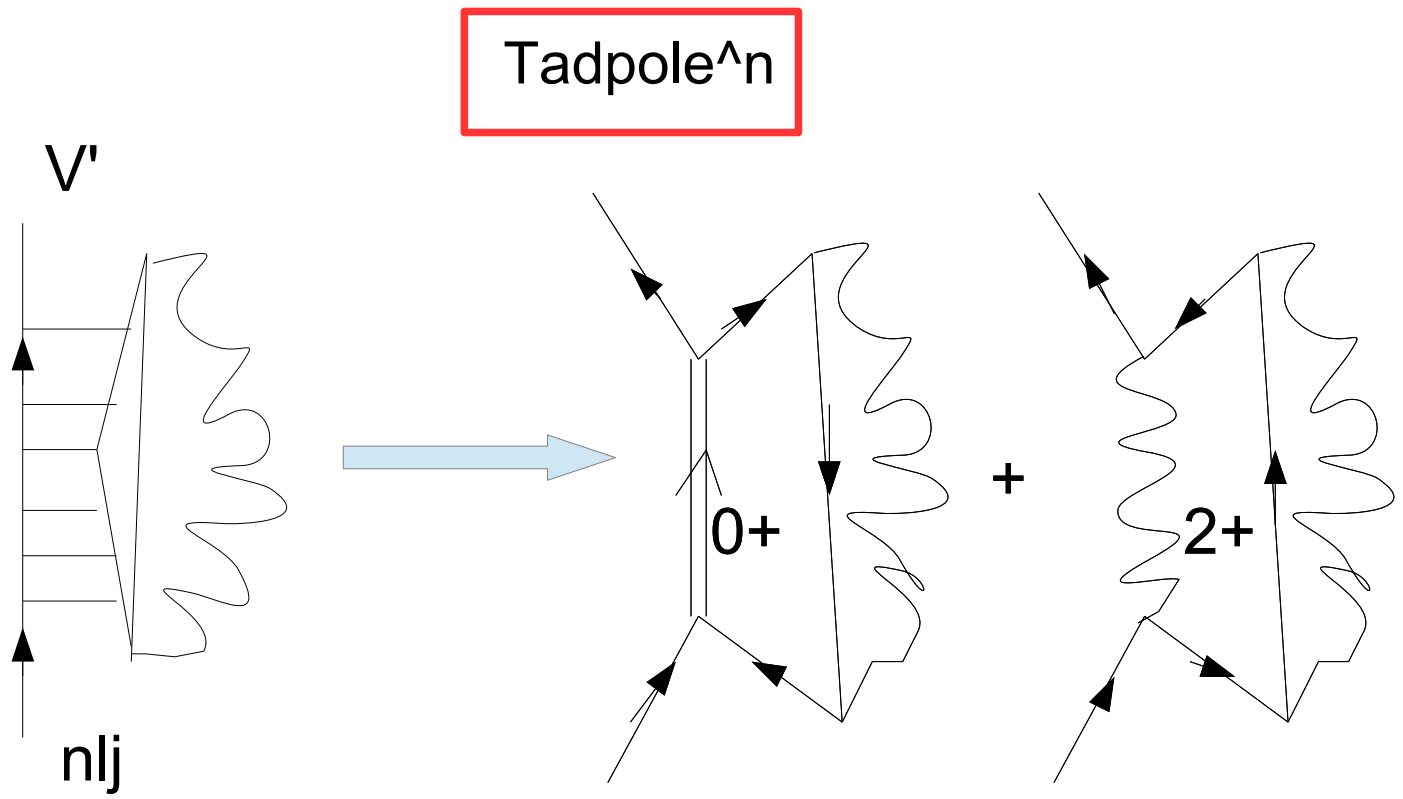
$$\langle r^2\rangle_{10\mathrm{Be}} + \left(\frac{\langle r^2\rangle^{1/2}_{1s1/2}}{11}\right)^2\times S^2 + (1-S^2)\times\left(\left(\frac{\langle r^2\rangle^{1/2}_{d5/2\,coll}}{11}\right)^2 + \langle r^2\rangle_{10\mathrm{Be}}\frac{2}{4\pi}\beta_\pi^2\right)$$

$$(2.47)^2 = (2.36)^2 + (7.10/11)^2\times S^2 + (1-S^2)\times(2.36^2\,2/4\pi\beta^2 + (3/11)^2)$$

$$6.10 = 5.57 + 0.42\,S^2 + (1-S^2)(0.89\beta^2 + 0.07)$$

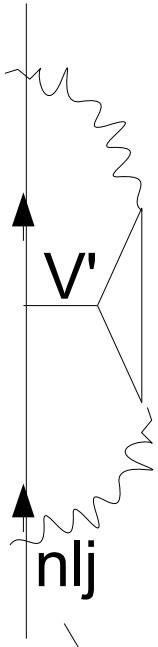
$$\textit{with}\quad \beta = 1.2 \qquad 6.10 = 5.57 + 0.42\,S^2 + (1-S^2)1.35 \rightarrow S^2 = 0.88$$

$$\textit{with}\quad S^2 = 0.83 \qquad 6.10 = 5.57 + 0.35 + 0.17\times(0.89\beta^2 + 0.07) \rightarrow \beta = 1.06$$

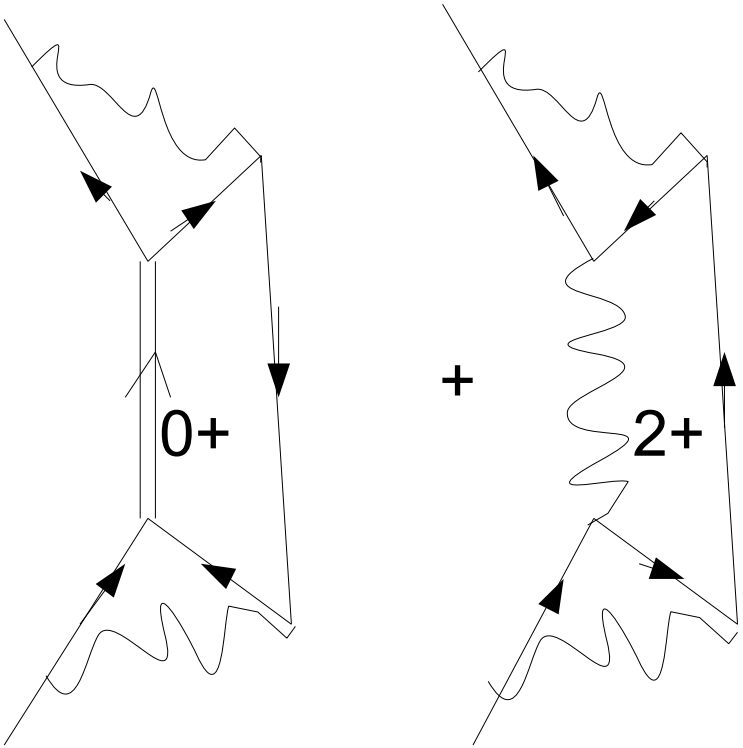


These are diagrams belonging to the Rainbow series, effectively included in the diagonalization.

Triangle



... the triangle is a small part of....



...which are vertex corrections, included in our effective vertices