Pairing rotation and pairing energy density functional

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Conventional pairing observable

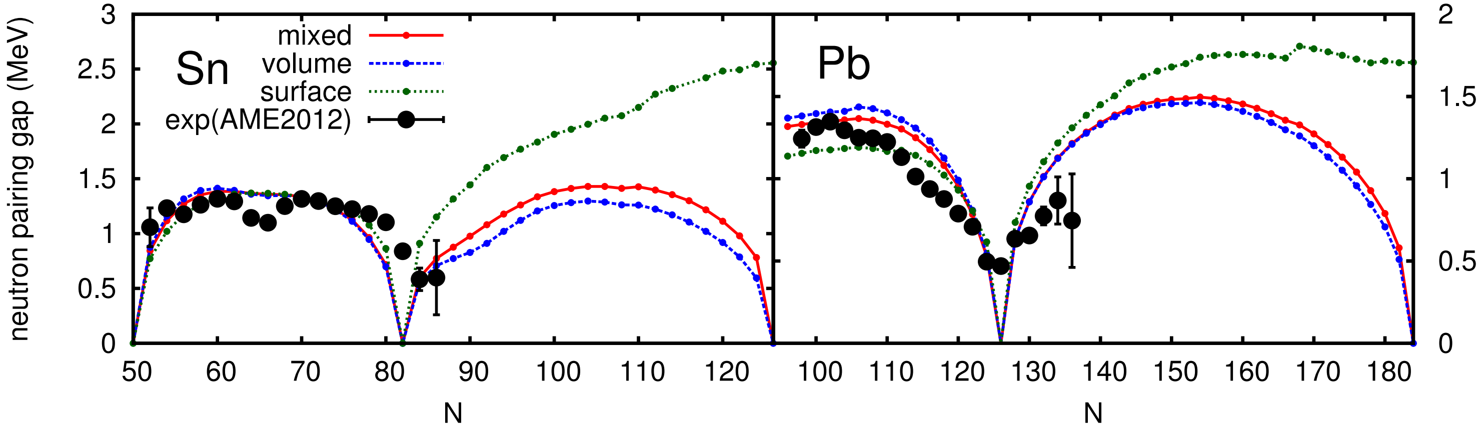
Odd-even staggering (OES)

$$\Delta_n^{(3)}(N,Z) = -\frac{(-1)^N}{2}[S_n(N,Z) - S_n(N+1,Z)]$$

theoretical pairing gap (average pairing gap)

pairing potential

$$\Delta_n(\rho) = \frac{\mathrm{Tr}\tilde{h}_n[\rho,\tilde{\rho}_n]\rho_n}{\mathrm{Tr}\rho_n} \qquad \Delta_n(\tilde{\rho}) = \frac{\mathrm{Tr}\tilde{h}_n[\rho,\tilde{\rho}_n]\tilde{\rho}_n}{\mathrm{Tr}\tilde{\rho}_n} \qquad \tilde{h}_n[\rho,\tilde{\rho}_n] = \frac{V_n}{2}\left(1-\eta\frac{\rho_0}{\rho_c}\right)\tilde{\rho}_n$$



Problems

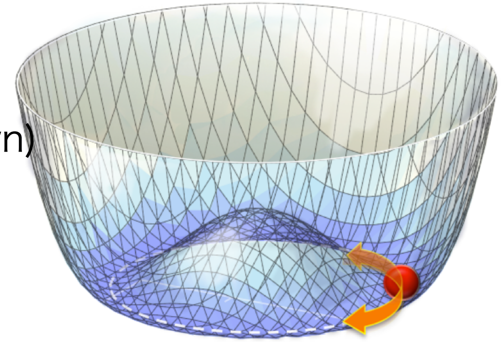
- pairing gap: not an experimental observable
- multiple definitions both in OES and pairing gaps
- time-reversal symmetry: broken in OES, pairing gap is from even-system
- contribution of the time-odd interaction is present only in OES

Pairing as symmetry breaking

- Pairing breaks the gauge symmetry.
- Pairing rotation appears as a zero-energy collective mode to restore the broken symmetry.
- Nambu-Goldstone (NG) mode appears in a solution of self-consistent QRPA when mean field (DFT) breaks continuous symmetries which the original EDF has

Three quantities describe the NG modes in QRPA

momentum P_{NG}: broken symmetry (known)
coordinate Q_{NG}: canonically conjugate coordinate operators (unknown)
mass M_{NG}: Thouless-Valatin inertia (mass)



Nambu-Goldstone modes in finite nuclei

Broken symmetry	Mean field	NG mode	P_{NG}	Q_{NG}	M_{NG}
translation	center of mass fixed	translation	center of mass momentum	C.M. coordinates	total mass (mA)
rotation	deformed	rotation	angular momentum	angle operator	moment of inertia
gauge symmetry	superconducting	pairing rotation	particle number	gauge angle operator	pairing rotational MOI

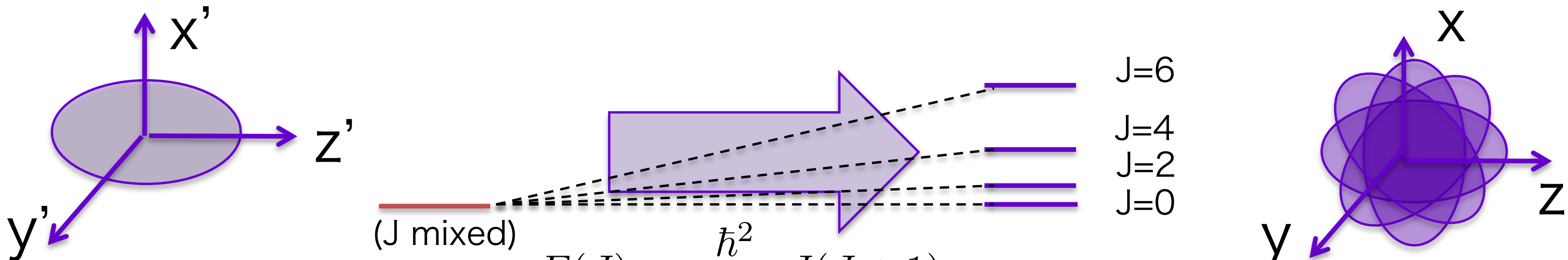
Analogy with rotational excitation

deformation: rotational symmetry breaking

intrinsic frame

NG mode excitation (rotation)
(angular momentum projection)

Laboratory frame



$$E(J) = \frac{\hbar^2}{2\mathcal{J}_{\rm rot}} J(J+1)$$

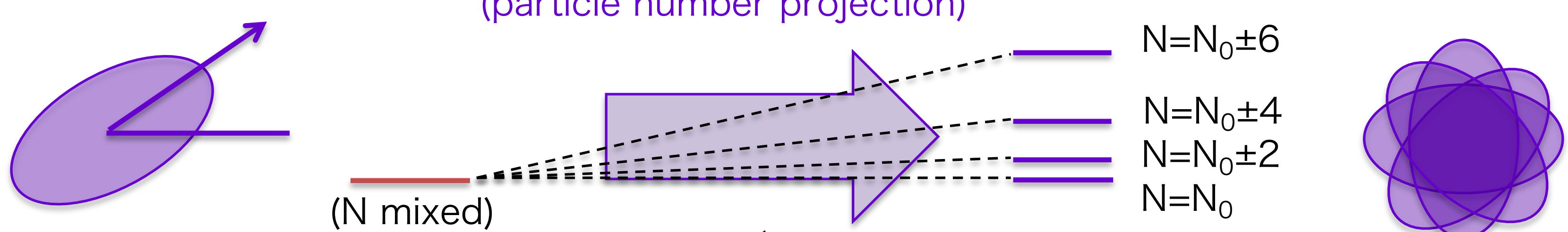
moment of inertia (or $E(2_1^+)$): magnitude of quadrupole collectivity

pair condensation(superconductivity): gauge symmetry breaking

superconducting state
in intrinsic frame

NG mode excitation (pairing rotation)
(particle number projection)

Laboratory frame



$$E(N) = \frac{1}{2\mathcal{J}_{\rm N}} (N - N_0)^2$$

pairing rotational moment of inertia: magnitude of pairing collectivity

How to observe pairing rotational band

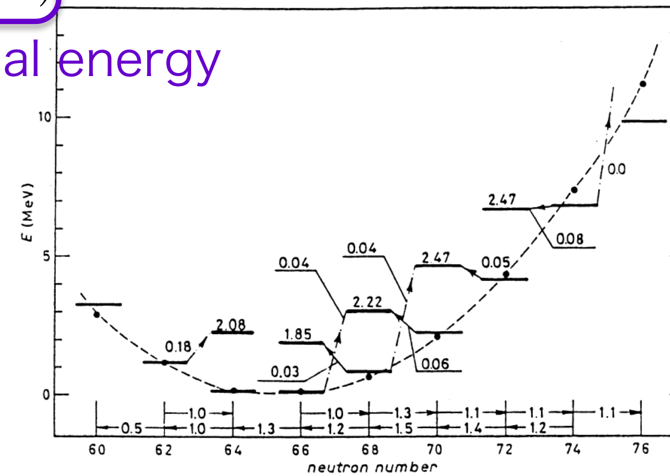
total energy of even-even ground state measured from a reference nucleus

$$E(N+\Delta N, Z) = E(N,Z) + \lambda_n(N,Z)\Delta N + \boxed{\frac{(\Delta N)^2}{2\mathcal{J}_{nn}(N,Z)}} + \mathcal{O}((\Delta N)^3)$$

pairing rotational energy

Brink and Broglia "Nuclear Superfluidity, Pairing in finite systems"
review: Broglia et al., Phys. Rep. **335**, 1(2000)

$$E_{\rm pair}(N) = -B_{\rm exp}(N) + 8.58N + 45.3({\rm MeV})$$



pairing rotational moment of inertia (from experimental data)

$$\mathcal{J}_{nn}(N,Z) = \frac{4}{E(N+2,Z) + E(N-2,Z) - 2E(N,Z)} = \frac{4}{\delta_{2n}(N,Z)}$$

$$\begin{aligned}\delta_{2n}(N,Z) =& S_{2n}(N,Z) - S_{2n}(N+2,Z) \\ =& -B(N-2,Z) + 2B(N,Z) - B(N+2,Z)\end{aligned}$$

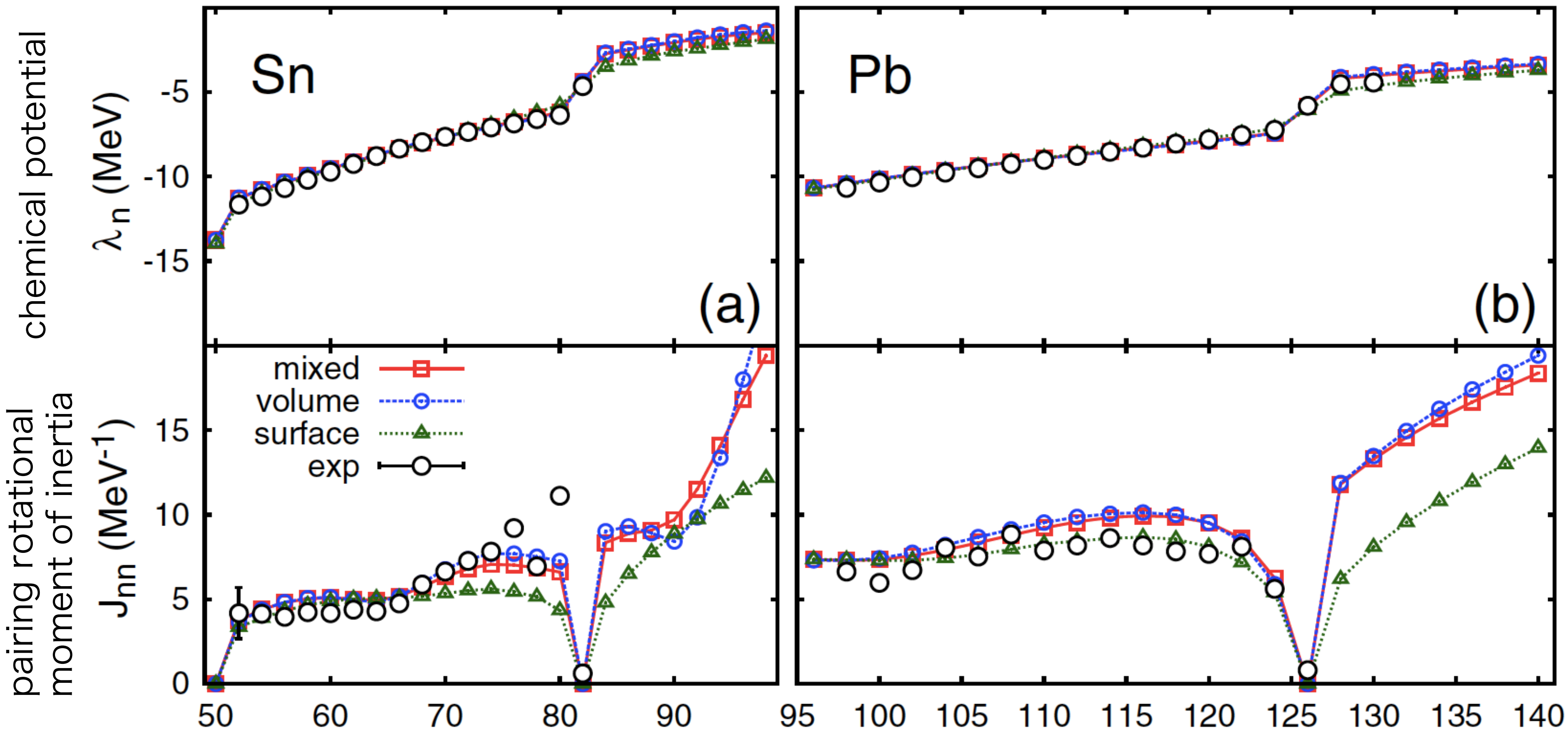
- inverse of double binding-energy differences (shell gap indicator)
- quantity from gauge symmetry breaking
- time-even quantity
- Thouless-Valatin inertia with neutron particle number operator (QRPA)

Pairing rotation in single-closed shell nuclei

broken symmetry: neutron U(1), $\Delta n \neq 0$

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)

Pairing rotational moment of inertia as a function of a particle number



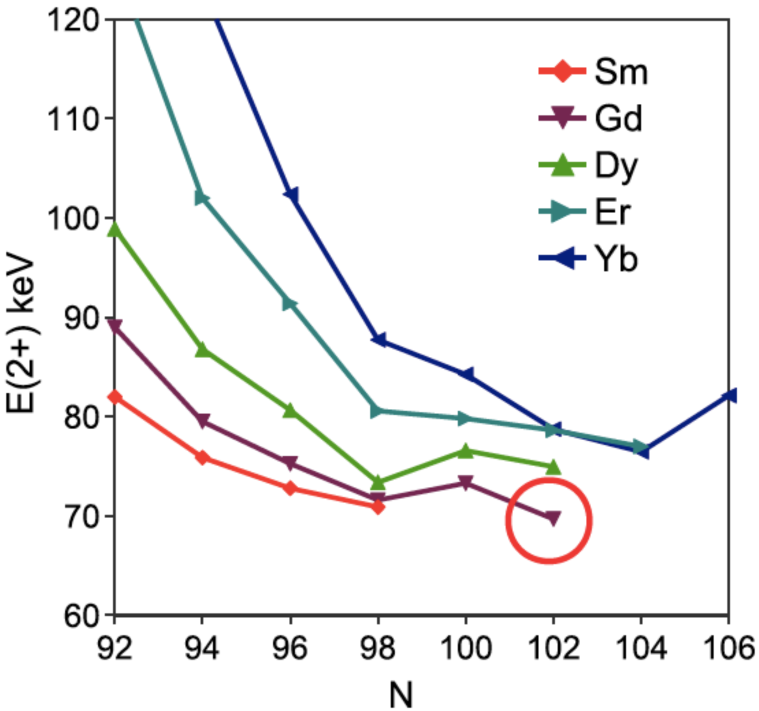
- three pairing EDFs (strength fitted to ^{120}Sn neutron gap)
- QRPA calculations (finite-amplitude method for NG mode)

- chemical potential (S_{2n}): small pairing EDF dependence (agrees well)
- pairing rotational moment of inertia: pairing EDF dependence visible
- experimental value collapses at the normal phase(N=82,126)

Pairing rotation in single-closed shell nuclei

quadrupole collectivity

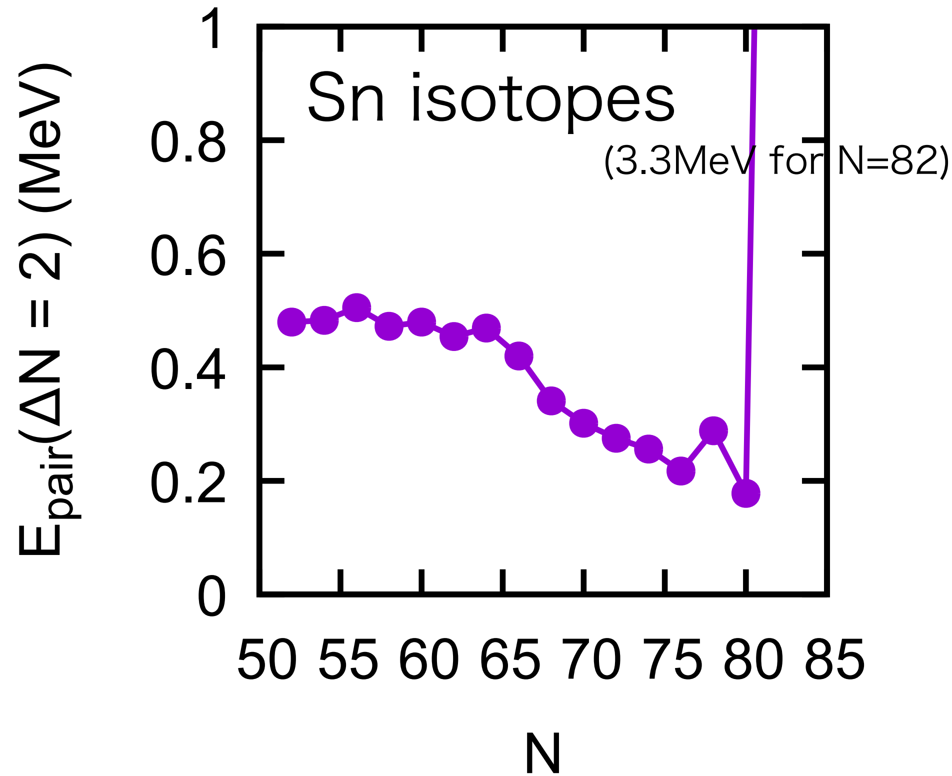
2⁺ energy



Patel et al., Phys. Rev. Lett. **113**, 262502 (2014)

pairing collectivity

ΔN=2 energy



$$E_{\rm pair}(\Delta N = 2) = \frac{2}{\mathcal{J}_{nn}(N,Z)} = \frac{\delta_{2n}(N,Z)}{2} = \frac{S_{2n}(N,Z) - S_{2n}(N+2,Z)}{2}$$

mass measurement of even-even isotope will clarify the pairing collectivity

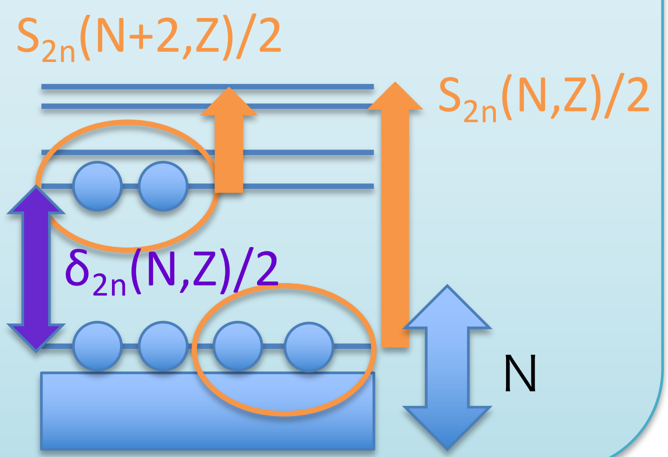
Pairing observables

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)

new pairing observable: two-neutron/proton separation energy

$$E_{\text{pair}}(\Delta N = 2) = \frac{2}{\mathcal{J}_{nn}(N,Z)} = \frac{\delta_{2n}(N,Z)}{2} = \frac{S_{2n}(N,Z) - S_{2n}(N+2,Z)}{2}$$

shell-model picture
without gauge symmetry breaking

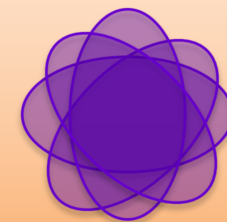


Lunney et al. Rev. Mod. Phys. **75**, 1021 (2003)

size of the magic shell gap

collective pairing picture
with gauge symmetry breaking

$$\delta_{2n}(N,Z) = \frac{4}{\mathcal{J}_{nn}(N,Z)}$$



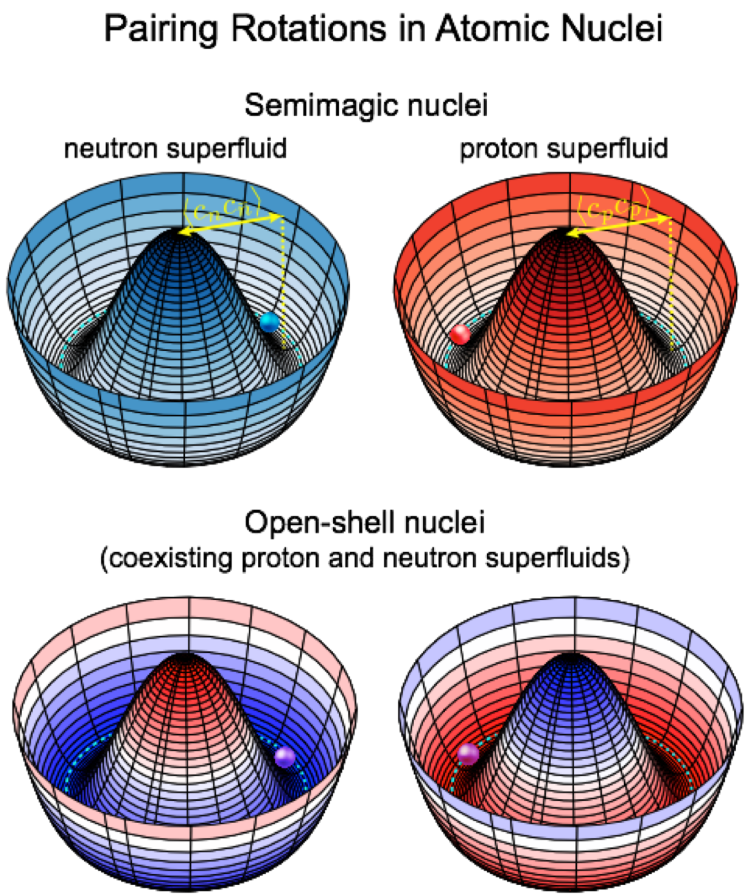
$$E_{\text{pair}} = \frac{(\Delta N)^2}{2\mathcal{J}_{nn}}$$

gauge symmetry breaking
pairing rotational MOI

$\delta_{2n,2p}$: size of the magic shell gap / pairing rotational MOI

Pairing rotation in doubly open-shell nuclei

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)



broken symmetry:
neutron and proton U(1), Δn≠0 and Δp≠0

$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{n}}] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_{\mathrm{p}}] \neq 0$$

$$\hat{N}_1 = \hat{N}_{\mathrm{n}} \cos\theta + \alpha \hat{N}_{\mathrm{p}} \sin\theta$$
$$\hat{N}_2 = -\hat{N}_{\mathrm{n}} \sin\theta + \alpha \hat{N}_{\mathrm{p}} \cos\theta$$

$$[\hat{H}_{\mathrm{HFB}}, \hat{N}_1] \neq 0 \qquad [\hat{H}_{\mathrm{HFB}}, \hat{N}_2] \neq 0$$

mixing due to residual pn interactions

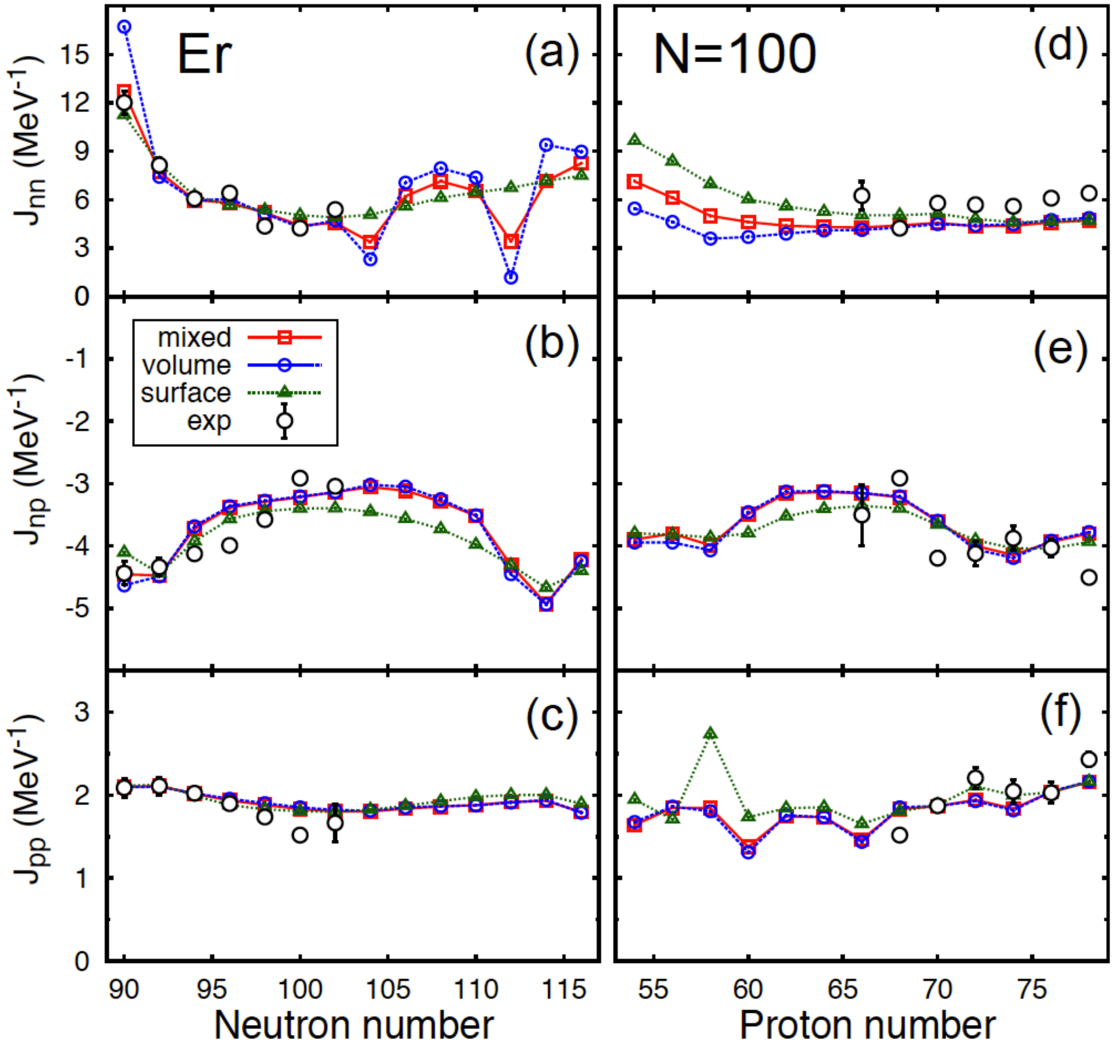
$$E_{\mathrm{pairrot}}(N+\Delta N, Z+\Delta Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{nn}(N,Z)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{np}(N,Z)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{pp}(N,Z)}$$

- two pairing rotations about tilted axes in neutron-proton space
- np component in the pairing rotational moment of inertia

Pairing rotation in doubly open-shell nuclei

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)

three pairing rotational MOIs along deformed Er isotope/N=100 isotone



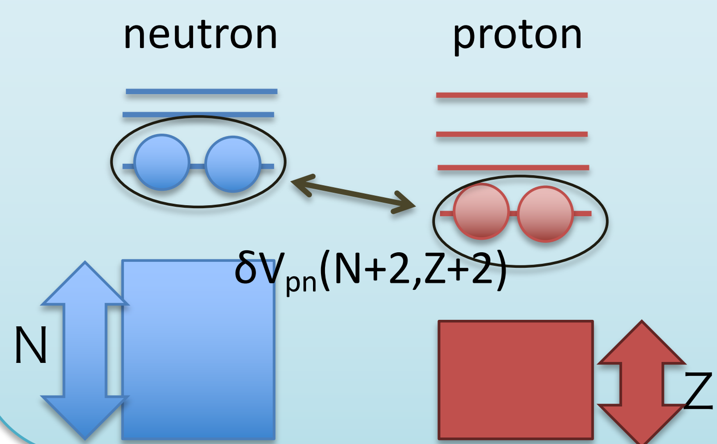
simultaneous agreement of three inertia: mixing of neutron and protons in NG modes

δVpn and pairing rotational moment of inertia

Experimental (off-diagonal) pairing rotational moment of inertia

$$\mathcal{J}_{np}^{-1}(N,Z) = \frac{E(N+2,Z+2) - E(N+2,Z) - E(N,Z+2) + E(N,Z)}{4} = -\delta V_{pn}(N+2,Z+2)$$

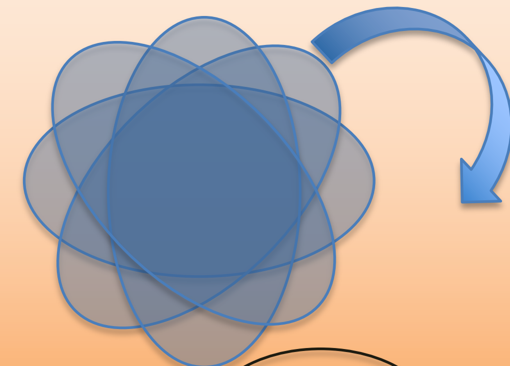
shell model picture
(before symmetry breaking)



Zhang et al., Phys. Lett. B **227**, 1 (1989)

interaction energy of valence 2n and 2p (proton-neutron interaction energy)

collective pairing picture
(after simultaneous symmetry breaking)



$$E_{\mathrm{pairrot}}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\mathrm{nn}}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\mathrm{np}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\mathrm{pp}}(N_0,Z_0)}$$

measure of simultaneous gauge symmetry breaking of neutron and proton
(off-diagonal pairing rotational MOI)

Pairing picture of binding energy differences

NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)

pairing collectivity

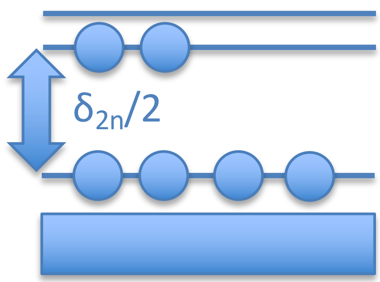
shell gap indicator

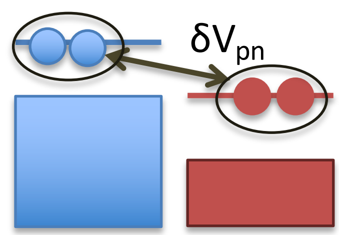
δ_{2n}, δ_{2p}

proton-neutron interaction energy

δV_{pn}

magic nuclei

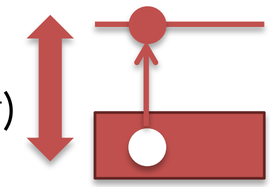




$E(2_1^+)$

doubly magic nuclei

J^π=2+ ph excitation



quadrupole collectivity

superconducting nuclei

excitation of two NG modes

$$E_{\text{pairrot}}(N,Z) = \frac{(\Delta N)^2}{2\mathcal{J}_{\text{nn}}(N_0,Z_0)} + \frac{2(\Delta N)(\Delta Z)}{2\mathcal{J}_{\text{np}}(N_0,Z_0)} + \frac{(\Delta Z)^2}{2\mathcal{J}_{\text{pp}}(N_0,Z_0)}$$

pairing energy/MOI

$\mathcal{J}_{nn}, \mathcal{J}_{pp}$ $\mathcal{J}_{np}$

magnitude of pairing collectivity
(gauge symmetry breaking)

deformed nuclei

excitation of NG mode

$$E(2_1^+) \sim \frac{3}{\mathcal{J}_{\text{rot}}}$$

magnitude of quadrupole collectivity
(deformation)

Extending pairing functional

NH, J. Phys. G **45**, 024004 (2018)

conventional pairing functional in Skyrme EDF: simple density-dependent delta int.

$$\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^{\rho}(\rho_0)|\tilde{\rho}_t|^2 \qquad \tilde{C}_t^{\rho}[\rho_0] = \frac{1}{4}V_t\left(1 - \eta_t \frac{\rho_0(\boldsymbol{r})}{\rho_c}\right)$$

simple pairing EDF because of lack of observables.

pairing rotational MOI may be useful to determine detailed pairing EDF

general pair-density-bilinear form of the isovector pairing functional

Perlinska et al., Phys. Rev. C **69**, 014316 (2004)

$$\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^{\rho}(\rho_0)|\tilde{\rho}_t|^2 + \tilde{C}_t^{\Delta\rho}\mathrm{Re}(\tilde{\rho}_t^*\Delta\tilde{\rho}_t) + \tilde{C}_t^{\tau}\mathrm{Re}(\tilde{\rho}_t^*\tilde{\tau}_t) + \tilde{C}_t^{J0}|\tilde{J}_t|^2 + \tilde{C}_t^{J1}|\tilde{\boldsymbol{J}}_t|^2 + \tilde{C}_t^{J2}|\underline{\tilde{\mathsf{J}}}_t|^2 + \tilde{C}_t^{\nabla J}\mathrm{Re}(\tilde{\rho}_t^*\boldsymbol{\nabla}\cdot\tilde{\boldsymbol{J}}_t)$$

conventional momentum-dependent tensor spin-orbit

- local gauge invariance related $C^{\Delta\rho}$ and C^{τ}, and $C^{\nabla J}=0$
- omitting the tensor-pairing terms for simplicity

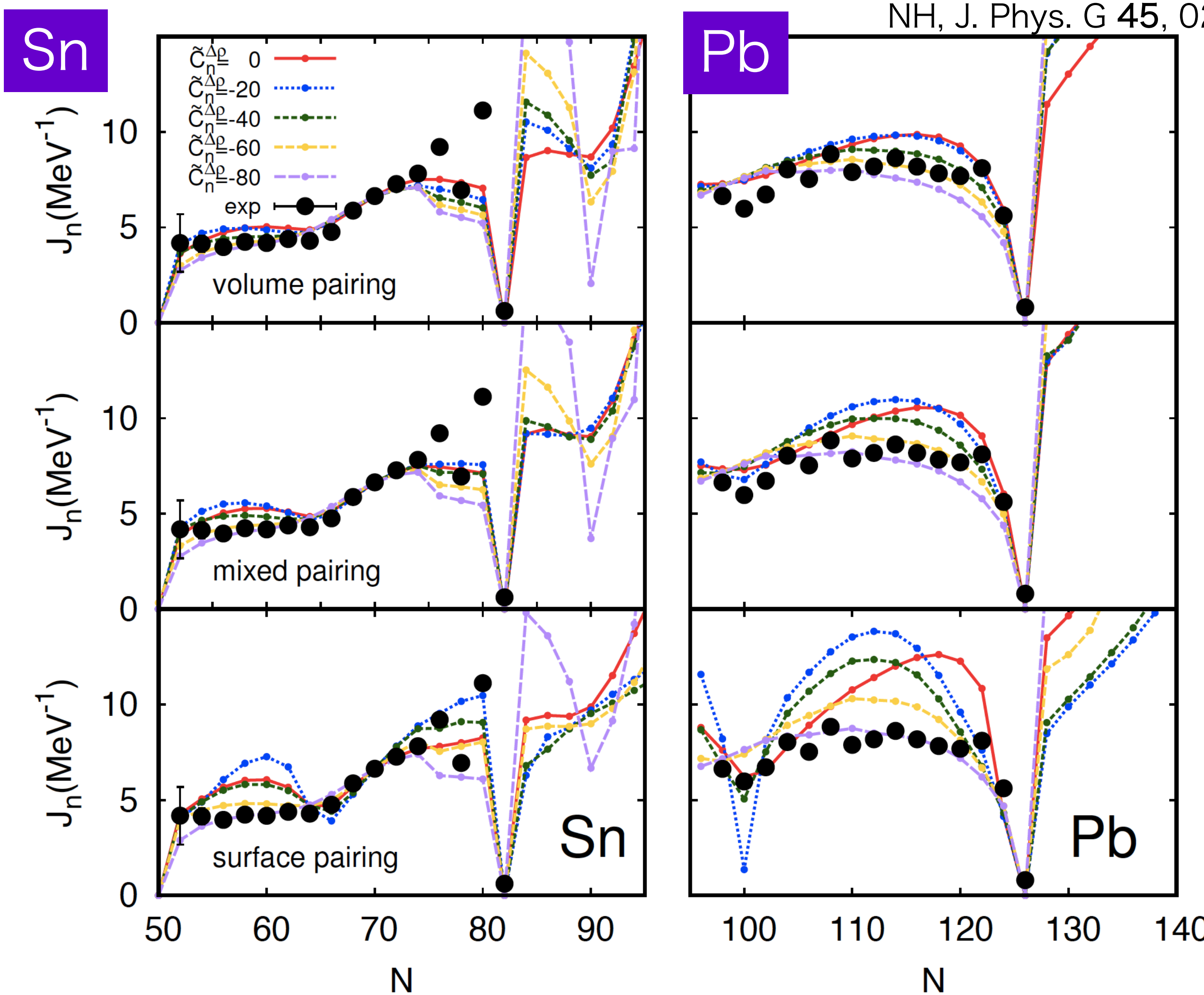
one new coupling constant

$$\tilde{\chi}_t(\boldsymbol{r}) = \tilde{C}_t^{\rho}(\rho_0)|\tilde{\rho}_t|^2 + \boxed{\tilde{C}_t^{\Delta\rho}}[\mathrm{Re}(\tilde{\rho}_t^*\Delta\tilde{\rho}_t) - 4\mathrm{Re}(\tilde{\rho}_t^*\tilde{\tau}_t)]$$

$$\tilde{\rho}_t(\boldsymbol{r}) = \tilde{\rho}_t(\boldsymbol{r},\boldsymbol{r}) \qquad \tilde{\tau}_t(\boldsymbol{r}) = (\boldsymbol{\nabla}\cdot\boldsymbol{\nabla}')\tilde{\rho}_t(\boldsymbol{r},\boldsymbol{r}')\Big|_{\boldsymbol{r}=\boldsymbol{r}'}$$

Extending pairing functional

NH, J. Phys. G **45**, 024004 (2018)



- Coupling constant (C^{ρ}) fitted to pairing rotational MOI in ^{120}Sn
- new terms improve the agreement in light Sn and whole Pb region

Summary

Pairing rotational MOI as a pairing indicator of finite nuclei

- pairing rotational MOI: double binding-energy differences

$$E_{\mathrm{pair}}(\Delta N = 2) = \frac{2}{\mathcal{J}_{nn}(N,Z)} = \frac{\delta_{2n}(N,Z)}{2} = \frac{S_{2n}(N,Z) - S_{2n}(N+2,Z)}{2}$$

- shell gap indicator δ_{2n} and proton-neutron interaction energy δV_{pn}

Extending pairing energy-density functional

- pair-density derivative term and kinetic-pair density term
- (terms from momentum-dependent force)
- improve the systematic values of pairing rotational MOI

References: NH and Nazarewicz, Phys. Rev. Lett. **116**, 152502 (2016)
(theory) NH, Phys. Rev. C **92**, 034321 (2015)
NH, J. Phys. G **45**, 024004 (2018)