Intertwined Quantum Phase Transitions in the Zr Isotopes

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Quantum Phase Transitions (QPTs)

- Type I: Single configuration (shape-phase transitions)

$$\hat{H} = (1-\xi)\,\hat{H}_1 + \xi \hat{H}_2$$

neutron number 90 region: Nd-Sm-Gd

- Type II: Two configurations A , B (normal-intruder states)

$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi^A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi^B) \end{bmatrix}$$

nuclei near shell-closure: Cd, Pb-Hg

- Intertwined quantum phase transitions (IQPTs)

Type II QPT and Type I QPT coexist
configuration crossing accompanied by pronounced individual shape-evolutions

Microscopic approach: NRMF (Robledo), RMF (Vretenar), MCSM (Togashi, Otsuka 2016)
Algebraic approach: THIS TALK

Zr isotopes: EXP (Kremer 2016; Ansari 2017; Paul 2017; Witt 2018; Singh 2018)

Algebraic approach to shape coexistence near shell closure

- Multiparticle-multihole intruder excitations across shell gaps

- Interacting boson model with configuration mixing (IBM-CM) [Duval, Barrett, PLB 81]

0p-0h, 2p-2h, 4p-4h,… → [N] ⊕ [N+2] ⊕ [N+4]… normal ⊕ intruder states

- Hamiltonian

$$\hat{H} = \begin{bmatrix} \hat{H}_A(\xi^A) & \hat{W}(\omega) \\ \hat{W}(\omega) & \hat{H}_B(\xi^B) \end{bmatrix} = \hat{H}_A^{(N)} + \hat{H}_B^{(N+2)} + \hat{W}$$

$$\hat{H}_A = \epsilon_d^{(A)} \hat{n}_d + \kappa^{(A)} \hat{Q}_\chi \cdot \hat{Q}_\chi$$

$$\hat{H}_B = \epsilon_d^{(B)} \hat{n}_d + \kappa^{(B)} \hat{Q}_\chi \cdot \hat{Q}_\chi + \kappa'^{(B)} \hat{L} \cdot \hat{L} + \Delta_p^{(B)}$$

$$\hat{W} = \omega \left[(d^\dagger \times d^\dagger)^{(0)} + (s^\dagger)^2 + \text{H.c.} \right]$$

$$\hat{Q}_\chi = d^\dagger s + s^\dagger \tilde{d} + \chi (d^\dagger \times \tilde{d})^{(2)}$$

$$\hat{T}(E2) = e^{(A)} \hat{Q}_\chi^{(N)} + e^{(B)} \hat{Q}_\chi^{(N+2)}$$

- Geometry

$$E(\beta,\gamma) = \begin{pmatrix} E_N(\beta,\gamma) & W(\beta,\gamma) \\ W(\beta,\gamma) & E_{N+2}(\beta,\gamma) \end{pmatrix}$$

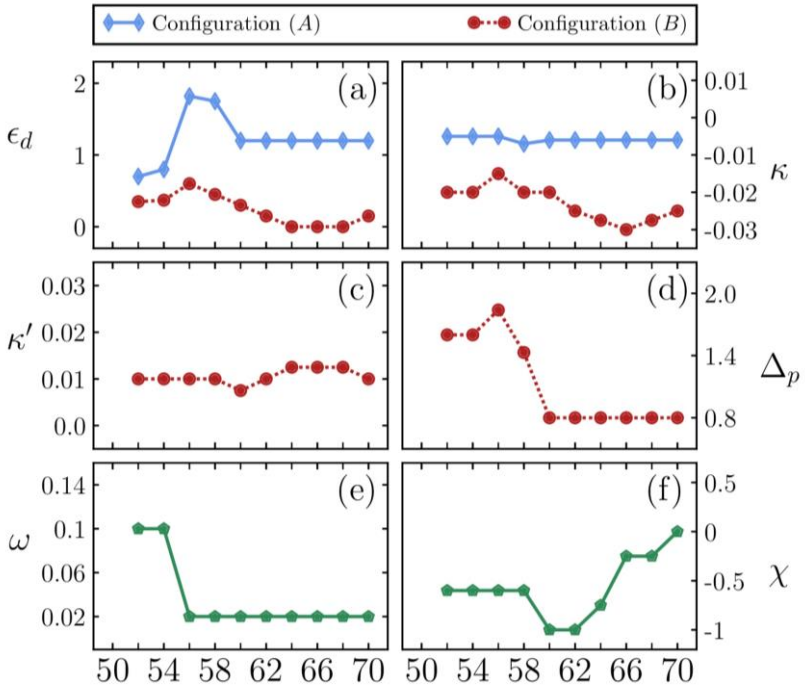
Matrix coherent states

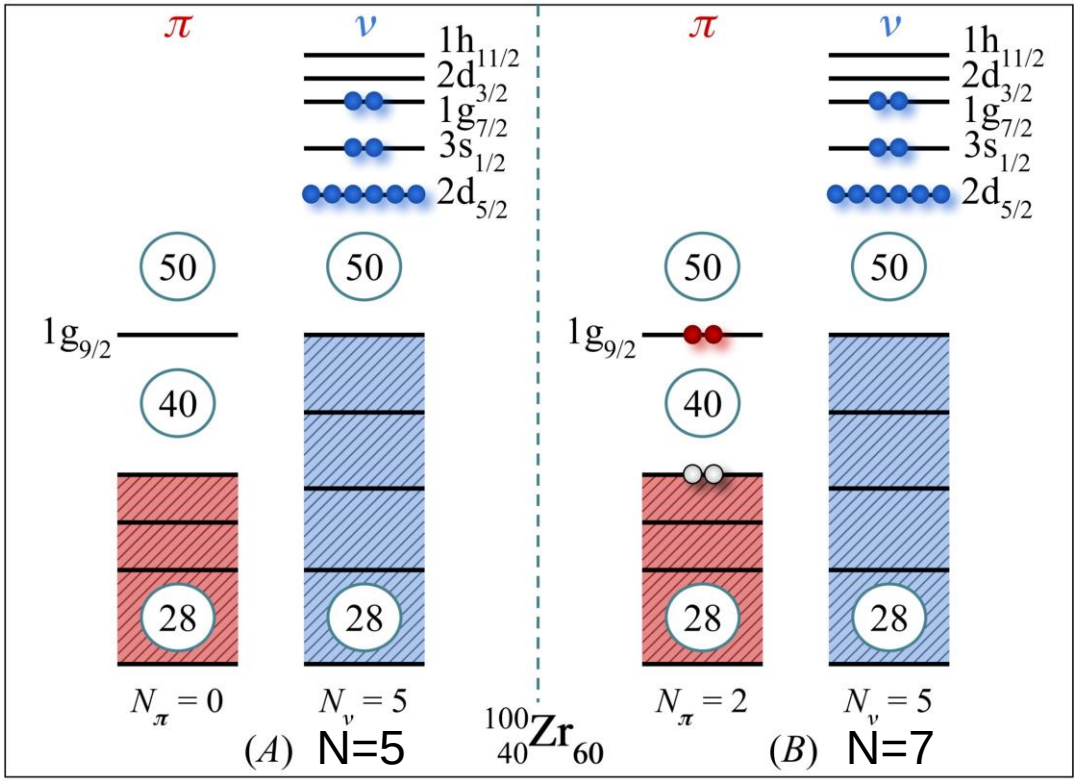
$E_\pm(\beta,\gamma)$ Eigenpotentials

[Frank, Van Isacker, Vargas, PRC 2004;
Frank, Van isacker, Iachello, PRC 2006]

Applications: Mo, Po, Hg, Pb.. [Sambataro, Molnar, Garcia-Ramos, Heyde, Van Isacker, Nomura,…]

IBM-CM in the Zr chain (z = 40, n = 52-70)





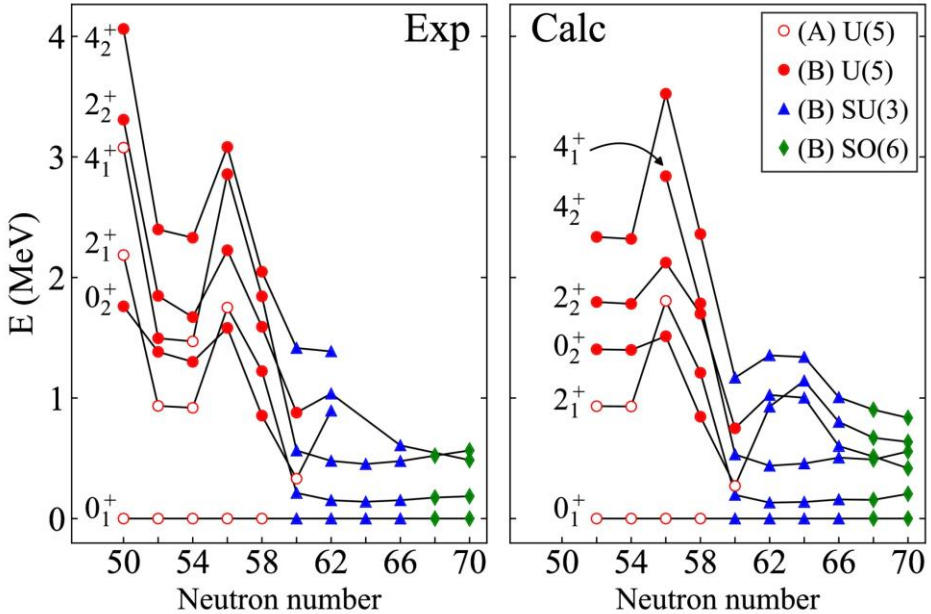
$$\hat{H} = \hat{H}_A^{(N)} + \hat{H}_B^{(N+2)} + \hat{W}$$

$$|\Psi; L\rangle = a\,|\Psi_A; [N], L\rangle + b\,|\Psi_B; [N+2], L\rangle$$

- $N = N_\pi + N_\nu$; Beyond mid-shell (n=66), bosons counted as holes
- Parameters smooth function of n, or constant, similar to Mo (Sambataro 82) and Ge (Duval 83, Padilla 06)
- Some fluctuations due to sub-shell closure (n=56; $2d_{5/2}$ Talmi 65)
- Descending cross shell-gap excitations and onset of deformation
  (n>56; V_{pn} between protons and neutrons occupying $1g_{9/2}$ and $1g_{7/2}$ Federman Pittel 79)

Normal (A) configuration
Intruder (B) configuration

$^{90\text{-}100}$Zr



n=50-56: config. (A) spherical (seniority-like) $R^{(A)}_{4/2} \sim 1.6$
config. (B) weakly-deformed $R^{(B)}_{4/2} \sim 2.3$

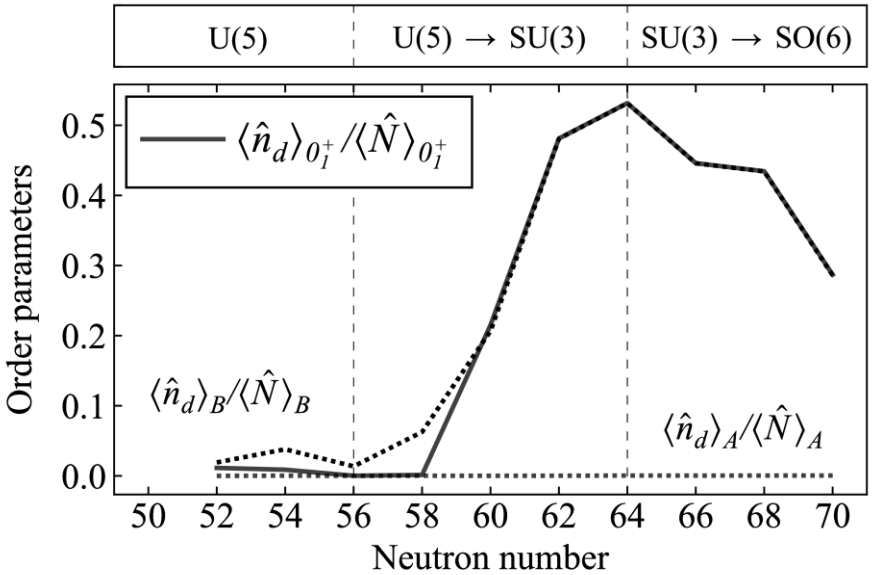
From n=58: pronounced drop in energy for states of config. (B)

n=60: two configurations exchange role $\Rightarrow$ Type II QPT
config. (B) at critical point of U(5)-SU(3) Type I QPT
first excited $L=0^+_B$ low ($E(0^+_3)$ in ^{100}Zr), similar to Sm-Gd

n>60: intruder config. (B) strongly deformed [SU(3)]
^{104}Zr: $E(2_1) = 139.3$ keV , $R^{(B)}_{4/2} = 3.24$

n=66: g.s. becomes γ-unstable (or triaxial) SU(3) $\rightarrow$ SO(6) crossover
^{106}Zr : $E(2_2) = 607$, $E(4_1) = 476.5$ keV
^{110}Zr : $E(4_1) = 565$, $E(2_2) = 485$ keV (Paul PRL 2017) SO(6) signature

Order parameters



Config. (A): spherical for all n

Config. (B): weakly-deformed for n=52-58

clear jump between n=58 & 60 from (A) to (B)
$\Rightarrow$ 1st order Type II QPT

further increase at n=60-64
$\Rightarrow$ U(5)-SU(3) Type I QPT

decrease at n=66
$\Rightarrow$ SU(3) to SO(6) crossover & shift of particle to hole bosons at mid-shell

$$|\Psi; L = 0_1^+\rangle = a\,|\Psi_A; [N], L = 0_1^+\rangle + b\,|\Psi_B; [N+2], L = 0_1^+\rangle$$

$$\langle \hat{N} \rangle_A = N \quad , \quad \langle \hat{N} \rangle_B = N + 2$$

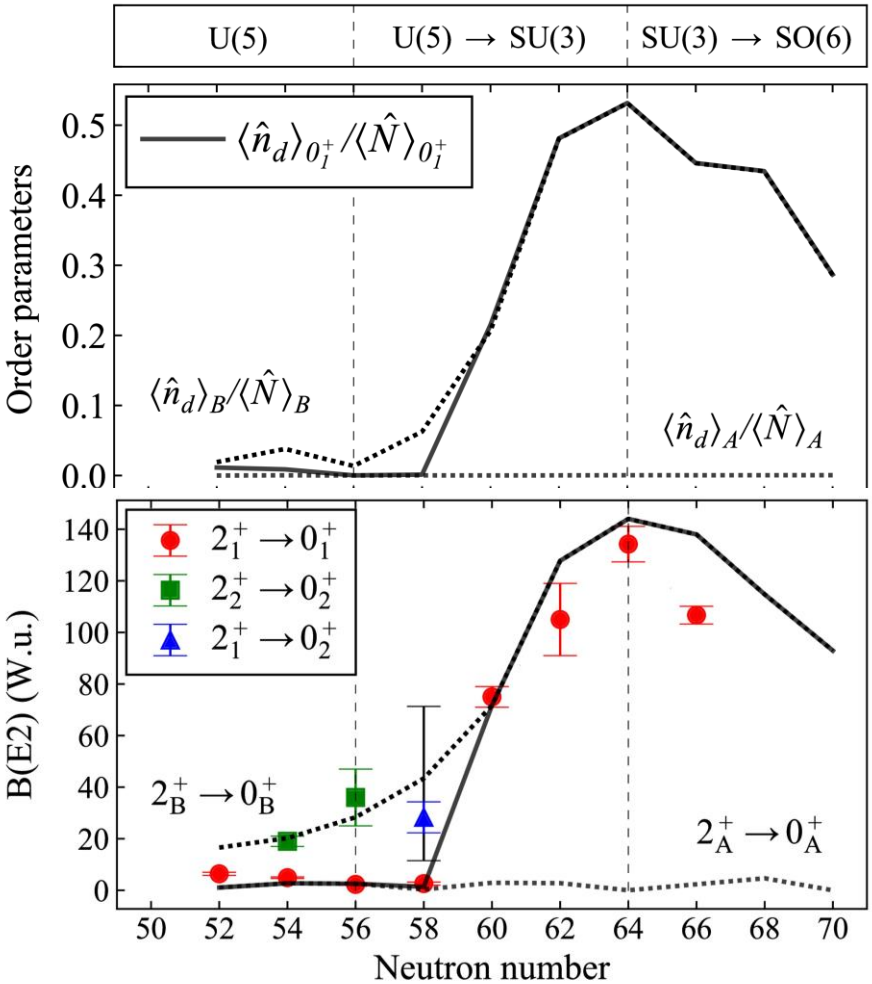
$$\langle \hat{N} \rangle_{0_1^+} = a^2\, N + b^2\,(N+2)$$

$\langle \hat{n}_d \rangle_A \quad , \quad \langle \hat{n}_d \rangle_B$ shapes of configurations A, B

$\langle \hat{n}_d \rangle_{0_1^+} = a^2\ \langle \hat{n}_d \rangle_A + b^2\ \langle \hat{n}_d \rangle_B$ normal-intruder mixing

Coexisting Type I QPT (shape-evolution) and Type II QPT (configuration crossing)

Order parameters and B(E2)'s



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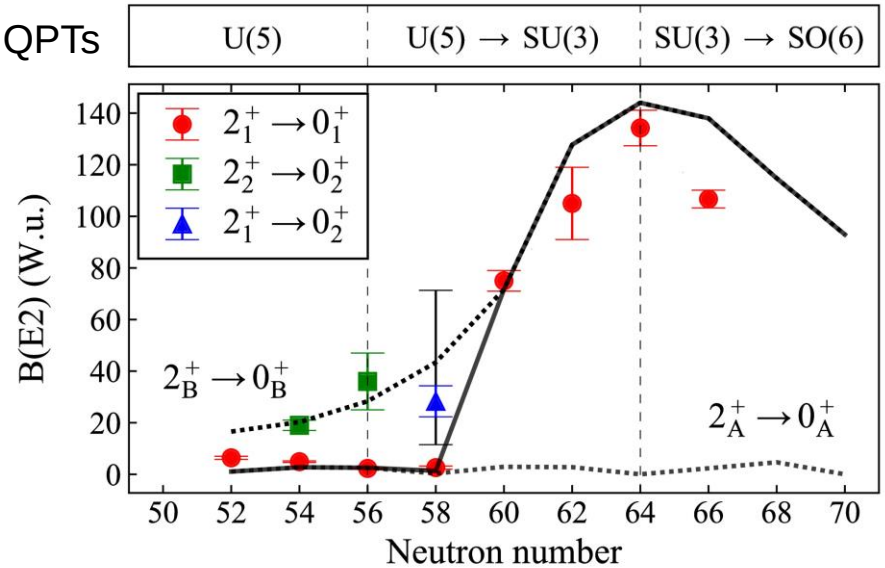
Coexisting Type I QPT and Type II QPT ⇒ IQPTs

$$\hat{T}(E2) = e^{(A)}\hat{Q}_\chi^{(N)} + e^{(B)}\hat{Q}_\chi^{(N+2)}$$

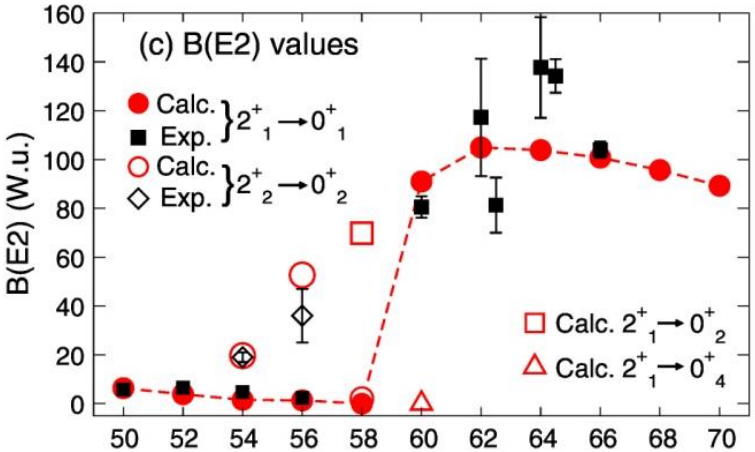
$$\hat{Q}_\chi = d^\dagger s + s^\dagger \tilde{d} + \chi(d^\dagger \times \tilde{d})^{(2)}$$

B(E2)'s in the Zr chain

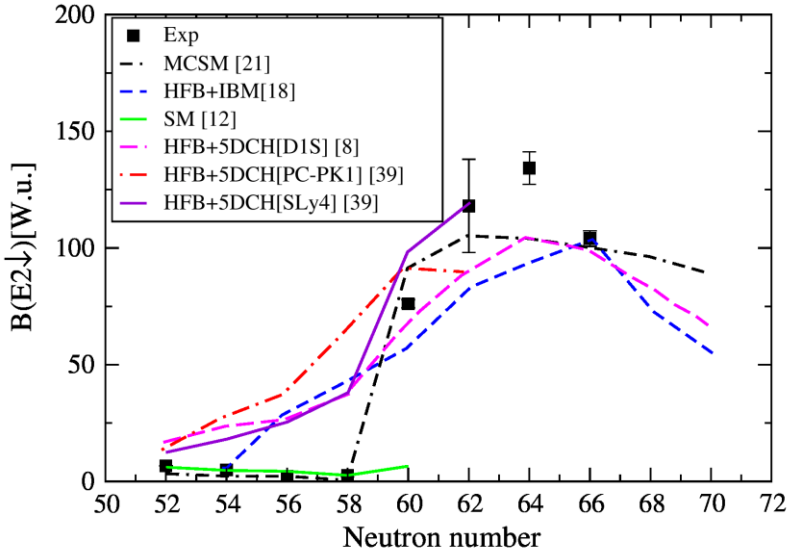
IQPTs



MCSM (Togashi et al. PRL 2016)



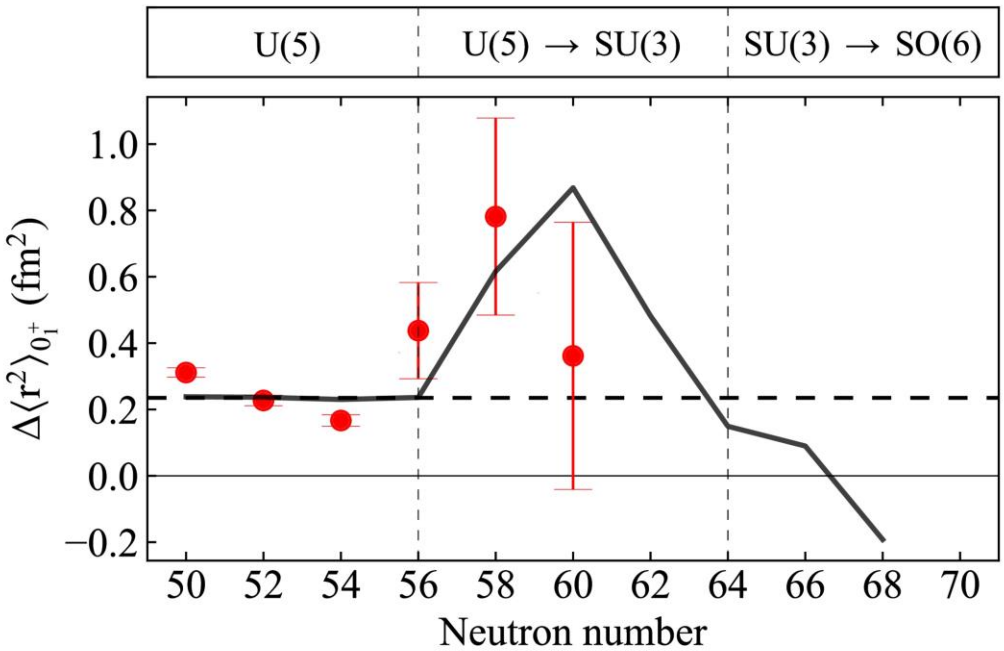
(Singh et al. PRL 2016)



Both MCSM & IQPTs: large jump in B(E2; $2_1 \to 0_1$) between ^{98}Zr and ^{100}Zr $\Rightarrow$ 1st order QPT

Mean-field based calculations: smooth out the QPT behavior, show no jump at the critical point

Isotope shift in the Zr chain

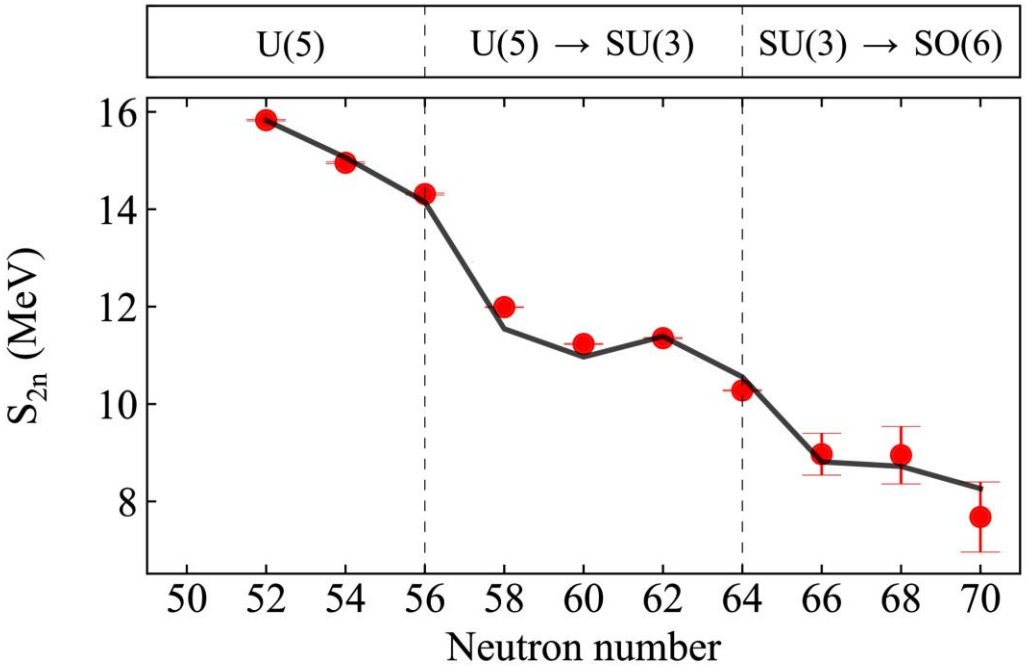


$$\Delta\,\langle\hat{r}^2\rangle_{0_1^+} = \langle\hat{r}^2\rangle_{0_1^+;A+2} - \langle\hat{r}^2\rangle_{0_1^+;A}$$

$$\langle\hat{r}^2\rangle = r_c^2 + \alpha\, N_v + \eta\left[\,\langle\hat{n}_d^{(N)}\rangle + \langle\hat{n}_d^{(N+2)}\rangle\,\right]$$

$\Delta\langle r^2\rangle$ increases at the transition point and decreases

Two-neutron separation energies in the Zr chain



$$S_{2n} = -\tilde{A} - \tilde{B}N_v \pm S_{2n}^{\mathrm{def}} - \Delta_n$$

S_{2n}^{def} $\langle$ H $\rangle$ in g.s.

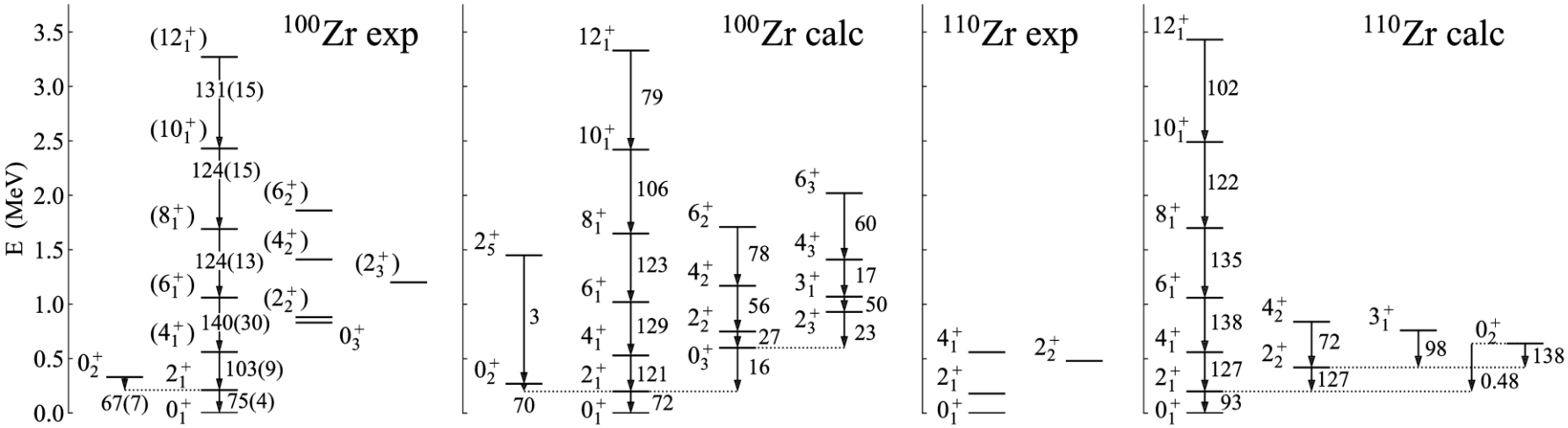
n=52-56: S_{2n} straight line, g.s. spherical (seniority-like)

After 56: S_{2n} first goes down (sub-shell closure) then flattens $\Rightarrow$ 1st order QPT (as in Sm)

After 62: S_{2n} goes down (increasing deformation)

n=66-70: S_{2n} flattens (SU(3)$\rightarrow$ SO(6) crossover)

Quantum analysis



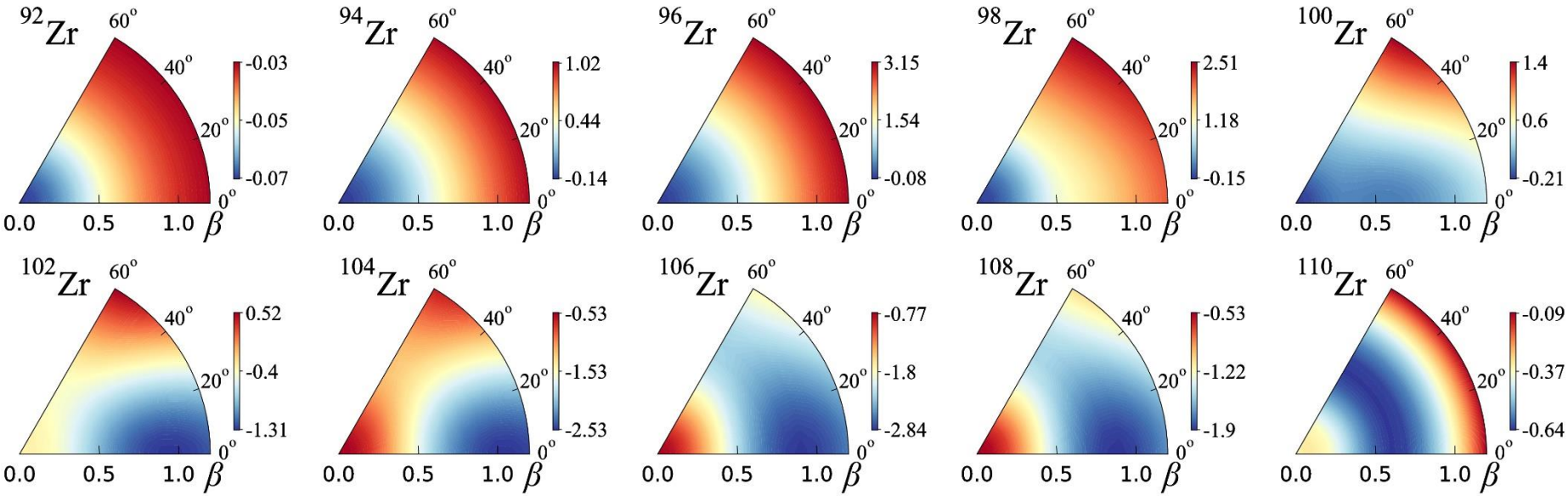
^{100}Zr: near the critical point of both Type I and Type II QPT

g.s. band, config. (B): X(5) symmetry

spherical config. (A): excited band 0_2

^{110}Zr: SO(6) symmetry

Classical analysis



$^{92\text{-}98}$Zr: spherical

^{100}Zr: flat-bottomed potential

$^{102\text{-}104}$Zr: axially deformed

$^{106\text{-}110}$Zr: γ-unstable

Concluding remarks

- Intertwined Quantum Phase Transitions (IQPTs):

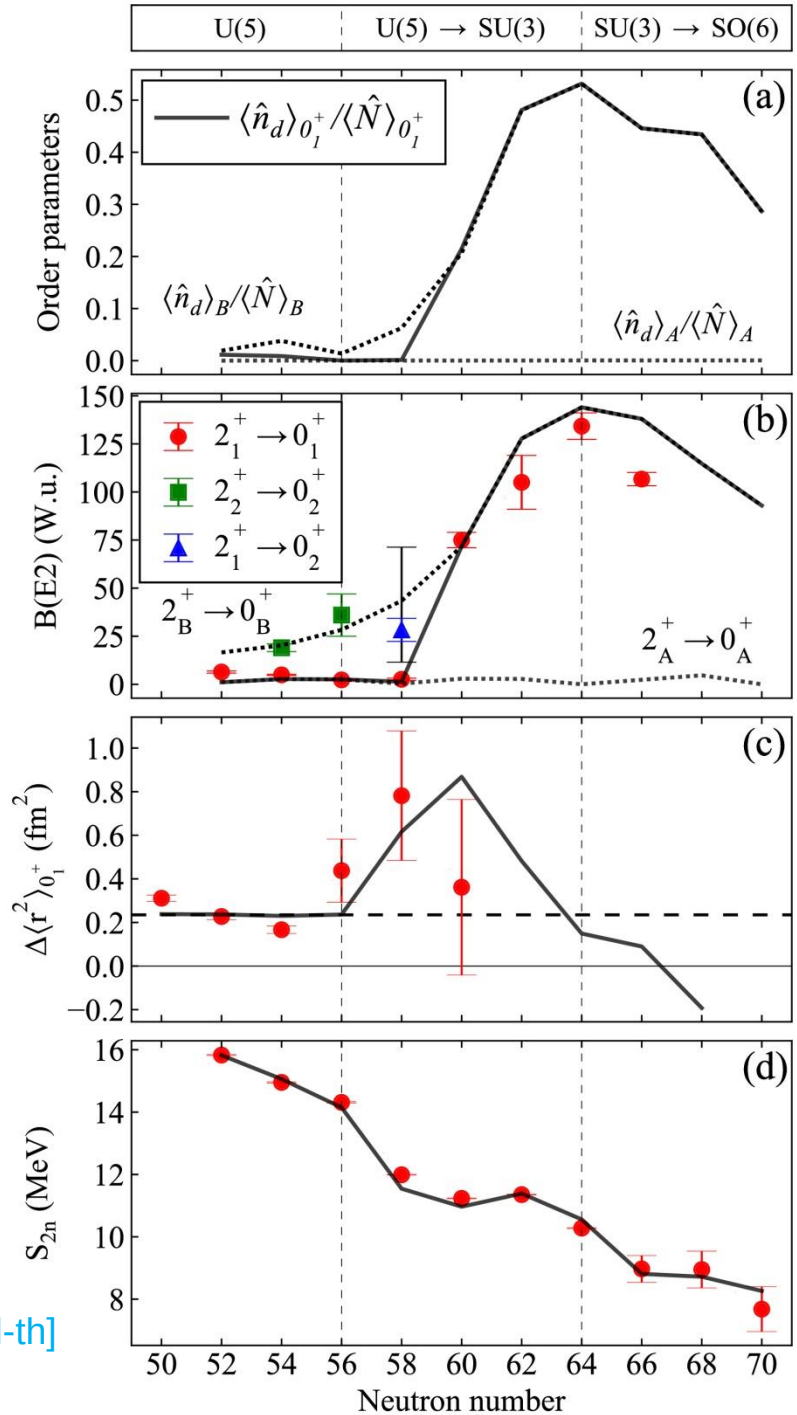
  QPT involving a crossing of two configurations (Type II) each of which is in itself undergoing a QPT (Type I)

- Comprehensive analysis in the IBM-CM framework of spectra and other observables (B(E2), $\Delta\langle r^2\rangle$, S_{2n}) in the Zr chain (n=52-70)

- Two configurations, one spherical (A) and the other (B) undergoing first a QPT U(5)-SU(3) and then a crossover SU(3)-SO(6), coexisting with a normal-intruder configuration- crossing

IQPTs appear to be manifested empirically in the Zr chain

- Initial indications for similar features in the Sr chain



N. Gavrielov, A. Leviatan and F. Iachello, arXiv:1904.09919 [nucl-th]

Thank you