Neutron Skin Effects in Mirror Energy Differences: The Case of $^{23}\text{Mg}-^{23}\text{Na}$

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Charge radii and nuclear skin

Charge radii are usually measured via electron scattering. These measurements are limited to stable nuclei. Laser spectroscopy allows to measure radial shifts along isotopic chains. This applies to ground states or isomeric states.

Neutron skin is still more difficult to measure.

Can we get any information on the evolution of radii in excited states and on the neutron skin?

We will see that mirror energy differences are sensitive to the nuclear radius.
Charge invariance and isospin

In fact the nuclear force is slightly asymmetric:

\[ a_{nn} = -18.8 \pm 0.3 \text{ fm} \]
\[ a_{pp} = -17.3 \pm 0.4 \text{ fm} \]

\[ \Rightarrow \text{the nn interaction is about 0.5\% more attractive.} \]

The nuclear force is also slightly charge dependent:

\[ a_{np} = -23.75 \pm 0.01 \text{ fm} \]

\[ \Rightarrow \text{the np interaction is about 2.5\% more attractive than the average of nn and pp} \]

• These will break symmetry slightly
• Coulomb force also breaks symmetry

BOTH can be though of as a perturbation – underlying symmetry will be retained.
Energy differences along isospin multiplets

Mirror nuclei
- Same A, interchanged Z and N
- Differences in Binding energies
  - **CDE** Coulomb Displacement Energies
  - ~ tens of **MeV** – charged sphere
- Excitation energies
  - Binding energies normalized to the g.s.
  - ~ tens of **keV**
  - Large Coulomb effects almost vanish
- Charge Symmetry $V_{pp} = V_{nn}$
  - Identical level scheme

Mirror Energy Differences

$$\text{MED}_J = \text{Ex}_{J, T_z = \frac{3}{2}^{-}} - \text{Ex}_{J, T_z = \frac{3}{2}^{+}} = -\Delta b_J$$

How do we calculate them?
Contributions to the MED

**Monopole Term**

- **Radius Variation with J**
  - Related to orbitals occupation numbers

**Multipole Term**

- **Nucleon Alignment** with increasing Angular Momentum J
  - W.f. overlap decreases with increasing J

- **Single Particle Energies**
  - Electromagnetic Spin Orbit
  - Thomas precession

Calculated in the harmonic oscillator basis
Calculation of Mirror Energy Differences

\[ MED_J^{\text{exp}} = E_J^*(Z)> - E_J^*(Z)> \]

\[ MED_J^{\text{theo}} = \Delta_M \langle V_{Cm} \rangle_J + \Delta_M \langle V_{CM} \rangle_J + \Delta_M \langle V_B \rangle_J \]

The multipole Coulomb contribution gives information on the nucleon alignment.

The monopole Coulomb contribution gives information on changes in the nuclear radius (deformation).

Important contribution from the “nuclear” ISB term, of the same order as the Coulomb contributions!!!!!!

Now, without changing the parametrization, see how the rest of the MED for nuclei along the f7/2 shell are described by the calculations…
Is the $V_B$ term needed also in other mass regions? The answer went much beyond the scope of the question.
The experiment: $^{16}\text{O} + ^{12}\text{C}$

- EXOGAM
- DIAMANT
- NEUTRON-WALL
Selection of the channel

**Channel selection**

Pulse Shape vs. Energy

**Efficiency estimate**
From $\gamma$ spectra

<table>
<thead>
<tr>
<th>$\gamma$ energy</th>
<th>$I_{op}/I_p$</th>
<th>$\epsilon_\alpha$</th>
<th>$I_{op}/I_\alpha$</th>
<th>$\epsilon_p$</th>
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<td>440</td>
<td>0.32</td>
<td>24 %</td>
<td>0.63</td>
<td>39 %</td>
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<tr>
<td>627</td>
<td>0.29</td>
<td>22 %</td>
<td>0.60</td>
<td>38 %</td>
</tr>
</tbody>
</table>

**Kinematical correction**

Trajectory reconstructed from the angle of the evaporated particles

**Resolution improvement:** 1.6
Selection of the channel

50 BC501A liquid scintillators

Neutron Wall was placed at forward angle to maximize the efficiency

Pulsed beam allows to discriminate between single $\gamma$ and neutron hits

In neutron efficiency estimated to be $\sim 21\%$

$n$-$\gamma$ discrimination

Channel selectivity
Mirror symmetry at work
Previously known level scheme confirmed and extended:

- 2 new levels
- 4 new transitions
Theory I: USD interaction

Multipole term
Calculated from Coulomb matrix elements in the harmonic oscillator basis

J=0  J=2  J=4  J=6

Single particle energies
Electromagnetic spin orbit effect

Radial Term
From variation of occupation number of $s_{1/2}$ shell with $J$

$V_C \sim \Delta n_{s_{1/2}}(J)$

Nuclear ISB term
Non-null matrix element: $V_B(d_{5/2}^2, J=0)$
Theory I: USD interaction

- Calculations performed as in the classical in $f_{7/2}$ shell
  - **Important multipole term** (alignment)
  - Improved by radial term correction
  - $V_B$ needed: ~ 30 keV

→ **What is the origin?** From NN interaction?
New Approach for fit to the charge radii: Duflo-Zuker formula

\[ \sqrt{\langle r_{\pi}^2 \rangle} = \rho_{\pi} = A^{1/3} \left[ \rho_0 - \frac{\zeta}{2} \frac{t}{A^{4/3}} - \frac{\nu}{2} \left( \frac{t}{A} \right)^2 \right] e^{g/A} + \lambda \left[ \frac{z(D_{\pi} - z)}{D_{\pi}^2} \times \frac{n(D_{\nu} - n)}{D_{\nu}^2} \right] A^{-1/3} \]

- Fit yield very good results for A<60 N>Z nuclei
- The quality of the DZ fit does not depend on \( \zeta \) = Isovector monopole polarizability – thickness of nuclear skin

How to understand Z>N radii?

- Mirror nuclei seen as core+p or core+n
- charged sphere + particle yield severe underestimation of MDE \( \Rightarrow \) Nolen-Schiffer Anomaly (NSA)
- MDE depends strongly on the radii
- Let's play with the radii to reproduce MDE, solving NSA

Fit yield very good results for A<60 N>Z nuclei

The quality of the DZ fit does not depend on \( \zeta \) = Isovector monopole polarizability – thickness of nuclear skin

\[ t = 0, 1, 2 \]
\[ \text{rmsd}_{Z \leq 30} = 42 \text{ fm} \]

\[ \text{rmsd}_{Z \leq 30} = 18 \text{ fm} \]
Isovector monopole polarizability

Toy model

The extra particle polarizes the system by inducing particle-hole jumps from the core.

\[ \hat{H} = \hat{h} + \hat{H}_0 + \hat{H}_1 \]

- \( \hat{h} \): unperturbed system
- \( \hat{H}_0 \): isoscalar: overall increase
- \( \hat{H}_1 \): isovector: differential contraction-dilation of the fluids

Approximate solution:

The NSA disappears as the proton and neutron radii tends to equalize.

\[ \hbar \omega_\nu > \hbar \omega_\pi \]

No-core shell model with potentials from chiral N3LO interaction that incorporates all isospin-breaking effects.
Results for binding energies

Single-particle/hole states built on $^{16}\text{O}$ and $^{40}\text{Ca}$

$s_{1/2}$ and $p_{3/2,1/2}$ orbits in the $^{17}\text{O}$ and $^{41}\text{Ca}$ respectively are huge

Bonnard, Lenzi, Zuker, PRL 113, 212501 (2016)
Can we relate the neutron skin with the difference of occupation numbers of protons and neutrons in low-L orbitals?

**How to get $\hbar \omega_{\nu, \pi}$ for both nuclei??**

- $\hbar \omega_{\pi, \nu} \propto \frac{1}{\langle \pi_{\pi, \nu}^2 \rangle}$
- 4 radii to determine
- IS: $r_{\pi}(N > Z) = r_{\nu}(N < Z)$
- $r_{\pi}(N > Z)$ is **measured**
- $r_{\nu}(N > Z)$ from MED?

**MED decrease** when skin increases

**Linear** dependence

For **each state** we can determine the value of the skin

From MED we can determine the neutron skin value as a function of $J$ (not measurable)
Neutron skin vs $s_{1/2}$ occupation numbers

- Clear correlation between the difference in occupation numbers in the $s_{1/2}$ orbit of protons and neutrons and the neutron skin
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Conclusions

- Experimental Mirror Energy Differences in mirror nuclei $A=23$ extended up to $J = 15/2^+$

- Mirror Energy differences interpreted via the **USD interaction** and the procedure successfully adopted in the $f_{7/2}$ mirror nuclei

- Realistic chiral N3LO potential with different potential wells for $\pi$ and $\nu$ adopted to reproduce MED

- Proved the possibility to determine the neutron skin for each excited state from the measured MED

- **Strong correlation between neutron skin and difference in occupation numbers of the $s_{1/2}$ orbit**