Microscopic optical potential from chiral forces

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In collaboration with
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Motivation
Experimental matter densities

The aim of the EXL experiment is to study the structure of unstable exotic nuclei in light-ion scattering experiments at intermediate energies. 

see Nuclear matter distribution of Ni56 measured with EXL, M. Von Schmid (2015)

Antiproton physics

Proton elastic scattering from tin isotopes at 295 MeV and systematic change of neutron density distributions, PRC77 (2008) 024317


Theoretical point of view

It is important to constrain and to test the most recent chiral potentials

• convergence
• accuracy
• predictive power
Method
Nuclear reaction theory relies on reducing the many-body problem to a problem with few degrees of freedom: optical potentials.

\[ \frac{d\sigma}{d\Omega} \] 

Optical potentials:

\[ V_{NN} = \begin{cases} G_{NN} \\ t_{NN} \end{cases} \]

\[ U_{opt}(k', k, E) \]

\[ \frac{d\sigma}{d\Omega} \]
Model
The general goal when solving the scattering problem of a nucleon from a nucleus is to solve the corresponding **Lippmann-Schwinger equation** for the many-body transition amplitude $T$

$$T = V + VG_0(E)T$$

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**Spectator expansion**

Two nucleon interaction dominates the scattering process

$$T_{0i} = v_{0i} + v_{0i}G_0(E)T,$$

$$T_{0i} = v_{0i} + v_{0i}G_0(E)\sum_j T_{0j}$$

$$= v_{0i} + v_{0i}G_0(E)T_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j}$$

$$(1 - v_{0i}G_0(E))T_{0i} = v_{0i} + v_{0i}G_0(E)\sum_{j\neq i} T_{0j}$$

$$T_{0i} = t_{0i} + t_{0i}G_0(E)\sum_{j\neq i} T_{0j}.$$

---

**Watson multiple scattering**

First-order optical potential

Kerman, McManus and Thaler, Ann. Phys. 8 (1959) 551 and many others

\[ \hat{U}(k', k; \omega) = (A - 1) \langle k', \Phi_A | t(\omega) | k, \Phi_A \rangle \]

\[ q \equiv k' - k, \quad K \equiv \frac{1}{2}(k' + k) \]

\[ \hat{U}(q, K; \omega) = \frac{A - 1}{A} \eta(q, K) \]

\[ \times \sum_{N=n,p} t_{pN} \left[ q, \frac{A + 1}{A} K; \omega \right] \rho_N(q) \]

\[ \eta(q, K) = \]

**Møller factor**

\[ \left[ \frac{E_{\text{proj}}(k')}{E_{\text{proj}}(-k')} \frac{E_{\text{proj}}(-k)}{E_{\text{proj}}(k)} \frac{E_{\text{proj}}(-\kappa)}{E_{\text{proj}}(\kappa)} \right]^{\frac{1}{2}} \]

Optimum factorization factor

First-order optical potential

\[
\hat{U}(q, K; \omega) = \hat{U}^c(q, K; \omega) + \frac{i}{2} \sigma \cdot q \times K \hat{U}^{ls}(q, K; \omega)
\]

Central component

\[
\hat{U}^c(q, K; \omega) = \frac{A - 1}{A} \eta(q, K)
\]

\[
\times \sum_{N=n,p} t^c_{pN} \left[ q, \frac{A + 1}{A} K; \omega \right] \rho_N(q)
\]

Spin-orbit component

\[
\hat{U}^{ls}(q, K; \omega) = \frac{A - 1}{A} \eta(q, K) \left( \frac{A + 1}{2A} \right)
\]

\[
\times \sum_{N=n,p} t^{ls}_{pN} \left[ q, \frac{A + 1}{A} K; \omega \right] \rho_N(q)
\]
Matter densities

\[ \rho_{\text{ch}} (e/\text{fm}^3) \]

\[ r (\text{fm}) \]

\[ ^{16}\text{O} \]

\[ \text{expt.} \]

\[ \mathcal{L} = \bar{\psi} \left[ \gamma^\mu \left( i \partial_\mu - \gamma_\omega A_\mu^{(\omega)} - \gamma_\nu \frac{\tau}{2} \cdot A_\mu^{(\nu)} - e \frac{1 + \tau_3}{2} A_\mu^{(\gamma)} \right) - (\rho - \Gamma_\phi \phi) \right] \psi 
+ \frac{1}{2} \left[ \partial_\mu \phi \partial^\mu \phi - m_\phi^2 \phi^2 - \frac{1}{2} F^{(\omega)}_{\mu\nu} F^{(\omega)\mu\nu} + m_\omega^2 A_\mu^{(\omega)} A^{(\omega)\mu} 
- \frac{1}{2} F_{\mu\nu}^{(\gamma)} F^{(\gamma)\mu\nu} \right] \]

Density-dependent couplings

DDME1 parametrization
T. Nikšić, D. Vretenar, P. Finelli and P. Ring
Scattering observables

\[ M(k_0, \theta) = A(k_0, \theta) + \sigma \cdot \hat{N} C(k_0, \theta) \]

\[ A(\theta) = \frac{1}{2\pi^2} \sum_{L=0}^{\infty} [(L + 1)F_L^+(k_0) + LF_L^-(k_0)] P_L(\cos \theta) \]

\[ F_{LJ}(k_0) = -\frac{A}{A-1} 4\pi^2 \mu(k_0) \langle \hat{T}_{LJ} | k_0, k_0; E \rangle \]

\[ C(\theta) = \frac{i}{2\pi^2} \sum_{L=1}^{\infty} [F_L^+(k_0) - F_L^-(k_0)] P_L^{1}(\cos \theta) \]

Differential cross section

\[ \frac{d\sigma}{d\Omega}(\theta) = |A(\theta)|^2 + |C(\theta)|^2 \]

Analyzing power

\[ A_y(\theta) = \frac{2 \text{Re}[A^*(\theta) C(\theta)]}{|A(\theta)|^2 + |C(\theta)|^2} \]

\[ U(r) + L \cdot S \neq 0 \]

\[ U(r) + L \cdot S > 0 \]
NN potential
## Chiral potentials: why?

<table>
<thead>
<tr>
<th>Chiral potentials</th>
<th>Phenomen. potentials</th>
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</thead>
<tbody>
<tr>
<td>1. QCD symmetries are consistently respected</td>
<td>1. QCD symmetries are not respected</td>
</tr>
<tr>
<td>2. Systematic expansion (order by order you know exactly the terms to be included)</td>
<td>2. Expansion determined by phenomenology (add whatever you need). A lot of freedom</td>
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<tr>
<td>3. Theoretical errors</td>
<td>3. Errors can’t be estimated</td>
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<tr>
<td>4. Two- and three- body forces belong to the same framework</td>
<td>4. Two- and three- body forces are not related one to each other</td>
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Nucleon-Nucleon Chiral potentials

Entem, Machleidt, Nosyk (N4LO)

\[
V(k, k') \rightarrow V(k, k') f^\Lambda(k, k')
\]

\[
f^\Lambda = \exp\left(-\left(\frac{k'}{\Lambda}\right)^{2n} - \left(\frac{k}{\Lambda}\right)^{2n}\right)
\]

Epelbaum, Krebs, Meissner (N4LO)

\[
V_{\text{long-range}}(r) = V_{\text{long-range}}(r) f\left(\frac{r}{R}\right)
\]

\[
f\left(\frac{r}{R}\right) = \left(1 - \exp\left(-\frac{r^2}{R^2}\right)\right)^n
\]

spectral function subtraction

\[
V(k, k') \rightarrow V(k, k') f^\Lambda(k, k')
\]

\[
f^\Lambda = \exp\left(-\left(\frac{k'}{\Lambda}\right)^{2n} - \left(\frac{k}{\Lambda}\right)^{2n}\right)
\]
Chiral potentials - Phase shifts


New renormalisation technique in the coordinate space with the cutoff $R$ being chosen in the range of $R = 0.8 \ldots 1.2$ fm. For contact interactions, they use a non-local Gaussian regulator in momentum space with the cutoff $\Lambda = 2R^{-1}$

$$f \left( \frac{r}{R} \right) = \left[ 1 - \exp \left( -\frac{r^2}{R^2} \right) \right]^6$$
Chiral potentials - Phase shifts

Antinucleon-nucleon interaction at next-to-next-to-next-to-leading order in chiral effective field theory
Ling-Yun Dai, Johann Haidenbauer and Ulf-G. Meißner JHEP07(2017)078

Real and imaginary parts of various $\bar{N}N$ phase shifts at $N^3$LO for cutoffs $R = 0.7–1.2$ fm
Results
NN amplitudes - 200 MeV

\[ M(\kappa', \kappa, \omega) = \langle \kappa' | M(\omega) | \kappa \rangle = -4\pi^2 \mu \langle \kappa' | t(\omega) | \kappa \rangle \]

\[ a_{pN} = \frac{1}{f_{pN}\pi^2} \sum_{L=0}^{\infty} P_L(\cos \phi) \left[ (2L + 1) M_{LL}^{L,S=0} + (2L + 1) M_{LL}^{L,S=1} + (2L - 1) M_{LL}^{L-1,S=1} \right] \]

\[ c_{pN} = \frac{i}{f_{pN}\pi^2} \sum_{L=1}^{\infty} P_L^1(\cos \phi) \left[ \left( \frac{2L + 3}{L+1} \right) M_{LL}^{L+1,S=1} - \left( \frac{2L - 1}{L} \right) M_{LL}^{L-1,S=1} \right] \]
Oxygen 16

![Graphs showing angular distributions of different optical potentials for Oxygen 16](22)

- **Graph (a)**: Angular distribution of differential cross section $d\sigma/d\Omega$ in units of [mb/sr].
- **Graph (b)**: Alternating current $A_y$.
- **Graph (c)**: Quadrupole moment $Q$.

These graphs illustrate the comparison between $N^2$LO, $N^3$LO, and $N^4$LO optical potentials, with data points shown as symbols.

Microscopic vs. Phenomenological


Microscopic vs. Phenomenological


Figure 2. (Color online) The same as in Fig. 1 for $^{40}\text{Ca}$, $^{42}\text{Ca}$, $^{44}\text{Ca}$, $^{48}\text{Ca}$ isotopes at 200 MeV. Experimental data from Refs. [45, 46].

Figure 3. (Color online) The same as in Fig. 1 for Ni isotopes: $^{58}\text{Ni}$ at $E = 192$ and 295 MeV, $^{60}\text{Ni}$ at $E = 178$ MeV, and $^{62}\text{Ni}$ at $E = 156$ MeV. Experimental data from Refs. [45, 46].

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Microscopic vs. Phenomenological

Figure 4. (Color online) The same as in Fig. 1 for Sn isotopes: $^{116}\text{Sn}$ at $E = 295$ MeV and $^{118},^{116},^{120},^{122},^{124}\text{Sn}$ at $E = 295$ MeV. Experimental data from Refs. [45, 46].


Microscopic vs. Phenomenological


Figure 5. (Color online) The same as in Fig. 1 for Pb isotopes: $^{208}$Pb at $E = 200$ MeV and $^{204},^{206},^{208}$Pb at $E = 295$ MeV. Experimental data from Refs. [45, 46].
Microscopic vs. Phenomenological


Figure 6. (Color online) The same as in Fig. 1 for $^{16}$O and $^{40}$Ca, $^{42}$Ca, $^{44}$Ca, $^{48}$Ca at $E = 318$ MeV and $^{58}$Ni at $E = 333$ MeV. Experimental data from Refs. [45, 46].
Microscopic vs. Phenomenological

Figure 7. (Color online) The same as in Fig. 1 for $^{56}\text{Ni}$ at $E = 400\text{ MeV}$. 

Figure 8. (Color online) The same as in Fig. 2 but for the analyzing power $A_y$. Experimental data from Refs. [45, 46].

Figure 10. (Color online) Analyzing power $A_y$ as a function of the angle $\theta$ for elastic proton scattering on $^{16}\text{O}$, and $^{208}\text{Pb}$ at $E = 200\text{ MeV}$, $^{58}\text{Ni}$ at $E = 192\text{ MeV}$, and $^{60}\text{Ni}$ at $E = 178\text{ MeV}$. 


Nuclear matter distribution of $^{56}$Ni measured with EXL

Kerndichteverteilung von $^{56}$Ni gemessen mit EXL

Vom Fachbereich Physik der Technischen Universität Darmstadt zur Erlangung des Grades eines Doktors der Naturwissenschaften (Dr. rer. nat.) genehmigte Dissertation von Mirko von Schmid M.Sc. aus Fulda

2015 — Darmstadt — D 17
Antiproton

\[ U(\alpha, q, K; E) = \sum_{N=n,p} \int d^3P \, \eta(P, q, K) \, t_{\alpha N} \left[ q, \frac{1}{2} \left( \frac{A+1}{A} K - P \right) ; E \right] \]

Projectiles
\[ \alpha = (p, n, \bar{p}) \]
\[ \times \rho_N \left( P - \frac{A-1}{2A} q, P + \frac{A-1}{2A} q \right) \]
\[ q = k' - k \]
\[ K = \frac{1}{2} (k' + k) \]

Basic ingredients

- (Anti)nucleon-nucleon scattering matrix \( t_{\alpha N} \)
- Non-local nuclear densities

\[ \rho_{op} = \sum_{i=1}^{A} \delta(r - r_i) \delta(r' - r_i') \]

The matrix elements between a general initial and final state are obtained from the NCSM PRC 97, 034619 (2018)

NCSM

\[ |\Psi_A^{J^\pi T}\rangle = \sum_{N=0}^{N_{\text{max}}} \sum_i c_i^{J^\pi T} |AN_iJ^\pi T\rangle \]

In collaboration with P. Navratil (TRIUMF), to appear soon

- **Target density**
  - NN - N^4LO500  Entem, Machleidt, Nosyk, PRC 96 024004 (2017)
  - + 3N - N^2LO  Navratil, Few-Body Syst. 41 117 (2007)

- **Scattering matrix**

  ➢ Local regulator for the long range part: \( R = 0.9 \text{ fm} \)

  ➢ Non-local regulator for the contact terms: \( \Lambda = 2 R^{-1} \)

  ➢ The interaction is connected to the NN one through the G-parity in an unambiguous way
Antiproton

Experimental data are from the LEAR collaboration at CERN

Local: factorized optical potential

\[ U(q, K; E) \sim \sum_{\alpha=n,p} t_{p\alpha} \left[ q, \frac{A+1}{2A} K; E \right] \rho_\alpha(q) \]

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Factorized optical potential

\[ U(q, K; E) \sim \sum_{\alpha=n,p} t_{p\alpha} \left[ q, \frac{A + 1}{2A} K; E \right] \rho_{\alpha}(q) \]

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Future
Include three-body forces and medium effects

- Contribution to the potential through a density-dependent two-body forces
- To be consistent, also medium corrections to the propagator should be included
- Not consistent with the perturbative order of the two-body sector (N4LO)

In collaboration with R. Machleidt (Idaho), work in progress
Thank you very much