Enhanced monopole and dipole transitions in medium-heavy nuclei induced by $\alpha$ clusters

M. Ito$^1$, M. Nakao$^1$, T. Okuno$^1$, S. Ebata$^2$

$^1$Department of Pure and Applied Physics, Kansai University

$^2$Lab. For Advanced Nuclear Energy, Institute of Innovative Research, TIT

1. **Background**: Isoscalar transitions in light nuclei ($^{16,18}$O, $^{12}$Be)

2. **Our subjects**: Monopole and dipole transitions in heavy nuclei

3. **Framework**: $\alpha$ potential model + complex boundary condition

4. **Results(1)**: IS0 and IS1 transitions in $^{44}$Ti = $\alpha + ^{40}$Ca

5. **Results(2)**: IS1 in $^{104\sim110}$Te = $\alpha + ^{100\sim106}$Sn (E1 in $^{135}$Cs = $\alpha + ^{131}$I)

5. **Discussion and future studies**: Improvement of calculations
**α clustering phenomena and monopole transition**

1. **α clustering phenomena in nuclei**

   α cluster structures are well known in excited states of lighter systems

   \[ ^{8}\text{Be} = 2\alpha, \; ^{12}\text{C} = 3\alpha, \; ^{16}\text{O} = \alpha + ^{12}\text{C}, \ldots \Rightarrow \text{Identified by } E_x(0_{\text{ex}}^+), \Gamma_{\alpha}, \text{etc} \]

2. **IS0 transition and α cluster structures**

   IS0 transition is strongly enhanced if α cluster structures are well developed

   \[ M(\text{IS0}) = \left\langle 0_f^+ \right| \sum_{i=1}^{A} r_i^2 \left| 0_1^+ \right\rangle \]

   Discrete IS0 strength at \( E_x < 15 \text{ MeV} \) is possible to be described by α cluster model

   **IS0 will be new (or modern) probe to assign α cluster structure**

   cf. \( E_x > 15 \text{ MeV} \) in M.F. model

   \( 2hw \sim 30 \text{ MeV} \)

3. **Investigations on IS0 transition**

   IS0 transitions are mainly investigated in lighter systems.

   \[ ^{12}\text{C} \Rightarrow 3\alpha, \; ^{16,18}\text{O} \Rightarrow \alpha + ^{12,14}\text{C}, \; ^{12}\text{Be} \Rightarrow \alpha + ^{8}\text{He} \text{ are compared with experiments} \]
Monopole transition in $^{16,18}$O with $\alpha$ cluster model

$^{16}$O: 4$\alpha$ model

$$M(ISO) = \langle 0_f^+ | \sum_{i=1}^{A} r_i^2 | 0_1^+ \rangle$$

$^{18}$O: $\alpha + ^{14}$C coupled-ch.

$$M(E0) = \langle 0_f^+ | \sum_{i=1}^{A} \tau_i(p) r_i^2 | 0_1^+ \rangle$$

Theory (0$^+_1 \Rightarrow 0^+_2$) = 1.1 W.U.

Experiment (0$^+_1 \Rightarrow 0^+_2$) = 1.5 W.U.

$^{16,18}$O $\Rightarrow \alpha + ^{12,14}$C excitation nicely describe discrete strength below $E_x < 15$ MeV
**ISO transitions in $^{12}$Be**

ISO strength is enhanced for $^{12}$Be $\rightarrow$ $\alpha + ^8$He excitation; M. Ito and K. Ikeda, RPP77 (2014)

Theoretical cal. of ISO transition

Exp. of $^{12}$C $+$ $^{12}$Be

Z. H. Yang et al.
PRL112 (2014)
Our subject: Extension of IS transition to heavier systems

1. Studies of IS transition (\(A < 50\))
   ① IS0 transition is effective probe to identify cluster structures
   \[ \hat{O}(IS0) = \sum_i r_i^2 \]
   ② IS1 transition is proposed as a new probe for asymmetric cluster with \(N=Z\)
   \[ \hat{O}(IS1) = \sum_i r_i^3 Y_{1,0}(\hat{r}_i) \]
   - Y. Chiba et al., PRC93, 034319 (2016)
   - \(\alpha + {}^{16}\text{O} \) in \(^{20}\text{Ne}\) and \(\alpha + {}^{40}\text{Ca}\) in \(^{44}\text{Ti}\)

2. Studies of cluster structure in heavy systems
   \(^{44}\text{Ti}\) with \(\alpha + {}^{40}\text{Ca}\) is deeply studied by microscopic and potential models
   Heavier systems (Actinide, rare-earth, ...) are extensively studied by Potential model
   Intensive contributions from B. Buck et al. \(\rightarrow\) Ground and negative rot. band structure

Our subject: Studies on IS0 and IS1 transitions in heavier systems

3. Today’s report
   We discuss continuum strength of isoscalar transitions by potential model for
   IS0 and IS1: \(^{44}\text{Ti}\) with \(\alpha + {}^{40}\text{Ca}\) model
   IS1: \(^{104}\text{Te}\) with \(\alpha + {}^{100}\text{Sn}\) and Te isotopes
   \(^{104}\text{Te}\) Exp. K. Auren et al., PRL121 (2018)
Framework (1): Potential and boundary condition

1. Double folding model

\[ U_{\text{DF}} = \int \int \rho_\alpha(r_\alpha) \rho_{\text{Ca}}(r_{\text{Ca}}) \nu_{\text{DDM3Y}}^{\text{NN}}(s, \rho) \, dr_\alpha \, dr_{\text{Ca}} \]

Exp. Charge F.F. M.F. model (by S. Ebata)

2. Schrödinger equation

\[ (T + V(r) - E) \chi(r) = 0 \]

Nuclear interaction

\[ V(r) = N_r \cdot U_{\text{DF}}(r) + V_C(r) + V^{\text{PF}} \]

Double Folding pot. (\( N_r \) is optimized)

Coulomb pot. (Uniform charge)

Pseudo pot. (Pauli’s principle of \( \alpha + ^{40}\text{Ca} \))

3. Boundary condition for continuum strength

We apply absorbing boundary condition (ABC) or complex scaling method (CSM)

ABC: M. Iwasaki et al., PTEP2015

Framework (2) : Calculation of Strength function

\[
S_\lambda(E) = \sum_\nu \left| \langle \Psi_\nu | \tilde{O}_\lambda | \Psi_0 \rangle \right|^2 \delta(E - E_\nu)
\]

Extended Completeness Relation of ABC or CSM solutions

\[
\sum_m |\Psi_m\rangle \langle \tilde{\Psi}_m| = 1
\]

ABC or CSM Solution

\[
S_\lambda(E) = -\frac{1}{\pi} \sum m \text{Im} \left[ \frac{\langle \Psi_0 | \tilde{O}_\lambda^\dagger | \Psi_m \rangle \langle \tilde{\Psi}_m | \tilde{O}_\lambda | \Psi_0 \rangle}{E - E_m} \right]
\]

Complex Energy

\text{Smooth continuum strength function is possible to calculate}
Result of $\alpha + ^{40}\text{Ca} :$ ISO strength function

※ Naïve M.F. $E_x > 2\hbar\omega = 30$ MeV

This strength appears below 15 MeV which is lower than M.F. model.

$$\text{Excitation energy [MeV]}$$

$$\text{ISO transition } S(E) [\text{fm}^4/\text{MeV}]$$
Result of $\alpha + ^{40}\text{Ca} : \text{IS1} \text{ stt} \text{rength function}$

This strength appears around 5MeV which is lower than M.F. model

※ Naïve M.F. $E_x > 1\hbar\omega = 15 \text{ MeV}$

Fraction of EWSR

$\int_0^E \varepsilon S_{\lambda=1}(\varepsilon) d\varepsilon / M_{\text{sum}}^{\lambda=1}$
Result of $\alpha + \text{Sn: IS1 strength function}$

Peak at $E_x \sim 5 \text{ MeV}$ appear in even Te isotopes.

Peak Energy is lower than $1\hbar\omega = 9 \text{ MeV}$ for nucleon excitation.

Strength in $^{104}\text{Te}$ is the most enhanced of all other systems.
Summary

1. Features of clustering phenomena
   ① Spatial extension is induced by the development of cluster structure
   ② Cluster structure appears in low excited region of Ex < 15 MeV

2. Importance of IS0 and IS1 transition
   IS0 and IS1 are naturally enhanced when cluster structures are developed
   ⇒ Important probe to identify the cluster deg. of freedom

   Important subject: Extension of analyses on IS0 and IS1 to heavier systems

Results

Enhanced IS transition appears at low energy region with EWSR fraction of 25 % for IS0 and 6 % for IS1 (α + 40Ca, α + 100Sn)
⇒ These strengths are a little difficult to explain in a simple M.F. picture

Low-lying and discrete IS strength due to clusters will be valid in heavy systems

Improvement of theory: Ground state correlation, Spreading width for continuum, etc...