

Recent applications of the subtracted second random-phase approximation



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Microscopic description of the Giant Resonances

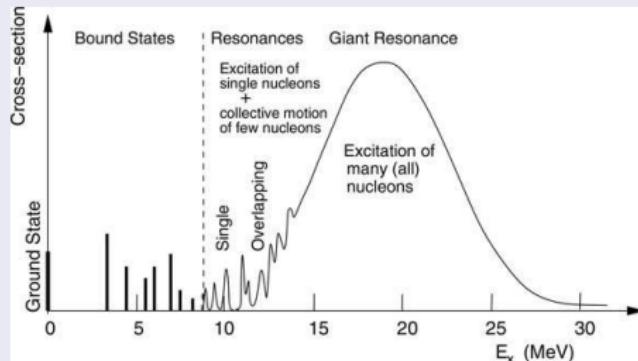
- The Random Phase Approximation (RPA)
- Beyond the RPA: The Second RPA
- The Subtraction method: Why and How

Recent Applications

- Monopole and Quadrupole response for ^{16}O
- Dipole response in ^{48}Ca : Low-Lying states (PDR) and GDR
- ISGQR systematic calculations for spherical nuclei

Nuclear response

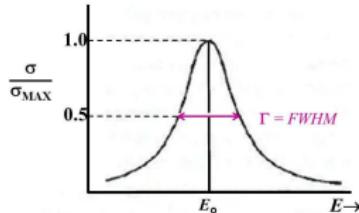
Total Strength



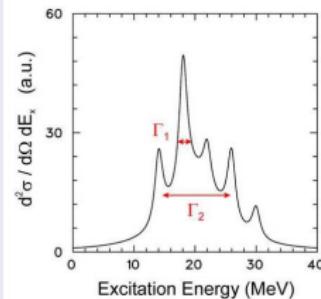
Centroid (E_0), Width (Γ) and Fine structure

Breit-Wigner Resonance Curve

$$\sigma(E) = \sigma_{\max} \frac{\Gamma^2 / 4}{(E - E_0)^2 + \Gamma^2 / 4}$$



Centroid (E_0), Width (Γ)

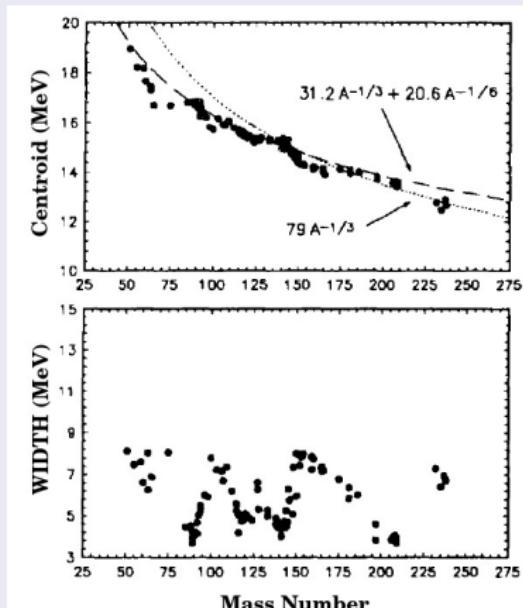


High precision studies



The Giant Dipole Resonance (GDR)

Centroid Energy and Width

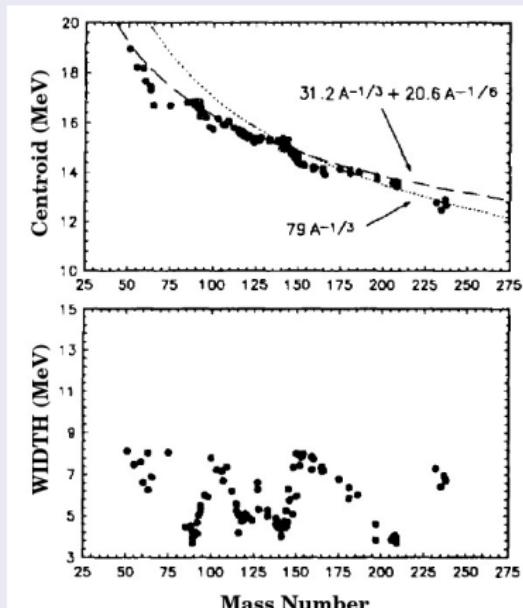


From: Ph. Chomaz, N. Frascaria, Physics Reports 252 (1995)

Centroid Energy: Very smooth dependence on A
Width: Strongly dependent on A

The Giant Dipole Resonance (GDR)

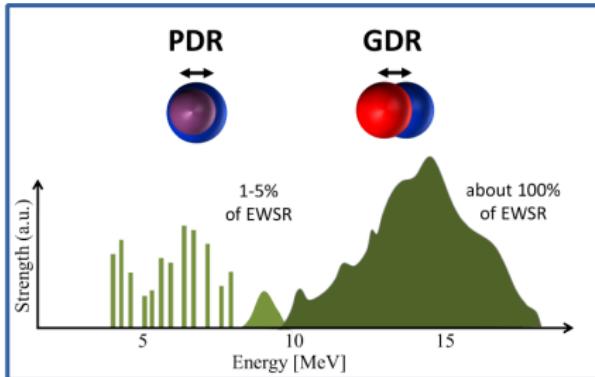
Centroid Energy and Width



From: Ph. Chomaz, N. Frascaria, Physics Reports 252 (1995)

Centroid Energy: Very smooth dependence on A
Width: Strongly dependent on A **Very Challenging !**

Future Investigations of the PDR and GDR @ ELI-NP



GDR: collective motion of the neutrons against the protons.

PDR: motion of neutrons skin against proton-neutron core



ELI-NP: high-intensity, mono-chromatic and linear-polarized gamma ray beam facility:

- Separate measure of E1 and M1:** no need of model-dependent determination of M1 strength
- Complementary studies:** strength below (NRF) and above (ELI-GANT) the neutron threshold
- Mono-chromatic beam:** fine structure of the response
- Model independent results:** pure electromagnetic excitation process

The Random Phase Approximation (RPA)

- The RPA is a widely used approximation for the description of GRs
- Very successful especially within the Energy Density Functional framework (interactions à la Skyrme or Gogny, or Covariant EDF)
- It provides global properties of the GRs

However, extensions of the RPA are also required for:

- Spreading Width
- Fine Structure
- Low Lying excitations in closed shell nuclei
- Double excitations and Anharmonicities
- ...

The Second RPA (SRPA): more general excitation operators are introduced

Phonon Operators: RPA vs SRPA

Random Phase Approximation (RPA)

$$Q_\nu^\dagger = \underbrace{\sum_{ph} X_{ph}^{(\nu)} \underbrace{a_p^\dagger a_h}_{1p-1h} - \sum_{ph} Y_{ph}^{(\nu)} \underbrace{a_h^\dagger a_p}_{1h-1p}}_{\text{Only Landau Damping, Centroid Energy and Total Strength of GRs}}$$

Second Random Phase Approximation (SRPA)

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p) + \underbrace{\sum_{p_1 < p_2, h_1 < h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{p_1}^\dagger a_{h_1}^\dagger a_{p_2} a_{h_2}}_{2p-2h} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} \underbrace{a_{h_1}^\dagger a_{p_1}^\dagger a_{h_2} a_{p_2}}_{2h-2p})}_{\text{Spreading Width, Fragmentation, Double GRs and Anharmonicites, Low-Lying States}}$$

RPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} X_{ph}^{(\nu)} a_p^\dagger a_h - \sum_{ph} Y_{ph}^{(\nu)} a_h^\dagger a_p$$

RPA Equations of Motion ($1 \leftrightarrow 1p1h$)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{B}_{11} \\ -\mathcal{B}_{11}^* & -\mathcal{A}_{11}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{Y}_1^\nu \end{pmatrix}$$

SRPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p) + \sum_{p_1 < p_2, h_1 < h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

SRPA Equations of Motion ($1 \leftrightarrow 1p1h$, $2 \leftrightarrow 2p2h$)

$$\begin{pmatrix} \mathcal{A}_{11} & \mathcal{A}_{12} & \mathcal{B}_{11} & \mathcal{B}_{12} \\ \mathcal{A}_{21} & \mathcal{A}_{22} & \mathcal{B}_{21} & \mathcal{B}_{22} \\ -\mathcal{B}_{11}^* & -\mathcal{B}_{12}^* & -\mathcal{A}_{11}^* & -\mathcal{A}_{12}^* \\ -\mathcal{B}_{21}^* & -\mathcal{B}_{22}^* & -\mathcal{A}_{21}^* & -\mathcal{A}_{22}^* \end{pmatrix} \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix} = \omega_\nu \begin{pmatrix} \mathcal{X}_1^\nu \\ \mathcal{X}_2^\nu \\ \mathcal{Y}_1^\nu \\ \mathcal{Y}_2^\nu \end{pmatrix}$$

SRPA Phonon Operators

$$Q_\nu^\dagger = \sum_{ph} (X_{ph}^{(\nu)} a_p^\dagger a_h - Y_{ph}^{(\nu)} a_h^\dagger a_p) + \sum_{p_1 < p_2, h_1 < h_2} (X_{p_1 h_1 p_2 h_2}^{(\nu)} a_{p_1}^\dagger a_{h_1} a_{p_2}^\dagger a_{h_2} - Y_{p_1 h_1 p_2 h_2}^{(\nu)} a_{h_1}^\dagger a_{p_1} a_{h_2}^\dagger a_{p_2})$$

Second RPA calculations

- Computationally very demanding
- Realistic studies were done using strong truncations in the s.p space and/or approximations in the evaluation of the SRPA matrices
- Only recently full SRPA calculations have been performed: ^a

^aP. Papakonstantinou and R. Roth PLB 671, 356 (2009) ; D. G. et al. PRC 81, 054312 (2010)

Large scale SRPA calculations have shown that:

- The SRPA strength distribution is systematically shifted towards lower energies compared to the RPA one
- This shift is very strong ($\simeq 5$ MeV), RPA description often spoiled

Origins and Causes:

- ① Quasi Boson Approximation and stability problems in SRPA
- ② Use of effective interactions in beyond-mean field methods

The Subtraction procedure (I. Tselyaev Phys. Rev. C 75, 024306 (2007))

- Designed for beyond RPA approaches
- It restores the Thouless theorem, e.g. instabilities are removed
- Static ($\omega = 0$) limit of the SRPA imposed to be equal to the RPA one

From SRPA to an Energy dependent RPA-like problem

- The SRPA problem as an energy-dependent RPA problem

$$A_{1,1'} \mapsto \tilde{A}_{1,1'}(\omega) = A_{1,1'}^{RPA} + \sum_{2,2'} A_{1,2}(\omega + i\eta - A_{2,2'})^{-1} A_{2',1'} = A_{1,1'}^{RPA} + A_{1,1'}^{Cor}(\omega)$$

From SRPA to an Energy dependent RPA-like problem

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The Subtraction procedure is SRPA (SSRPA)

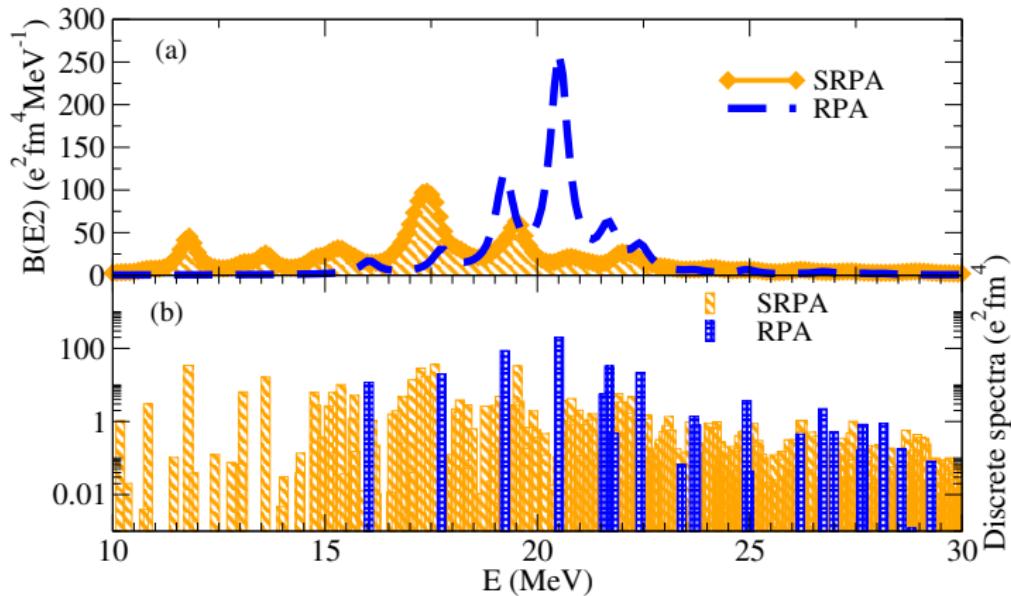
- Subtraction of the zero-frequency limit of the SRPA correction

$$A_{1,1'}^{Cor} \mapsto \tilde{A}_{1,1'}^{Cor}(\omega) = A_{1,1'}(\omega) - A_{1,1'}(\omega = 0) \Rightarrow$$

$$\tilde{A}_{1,1'}(\omega = 0) = A_{1,1'}^{RPA}$$

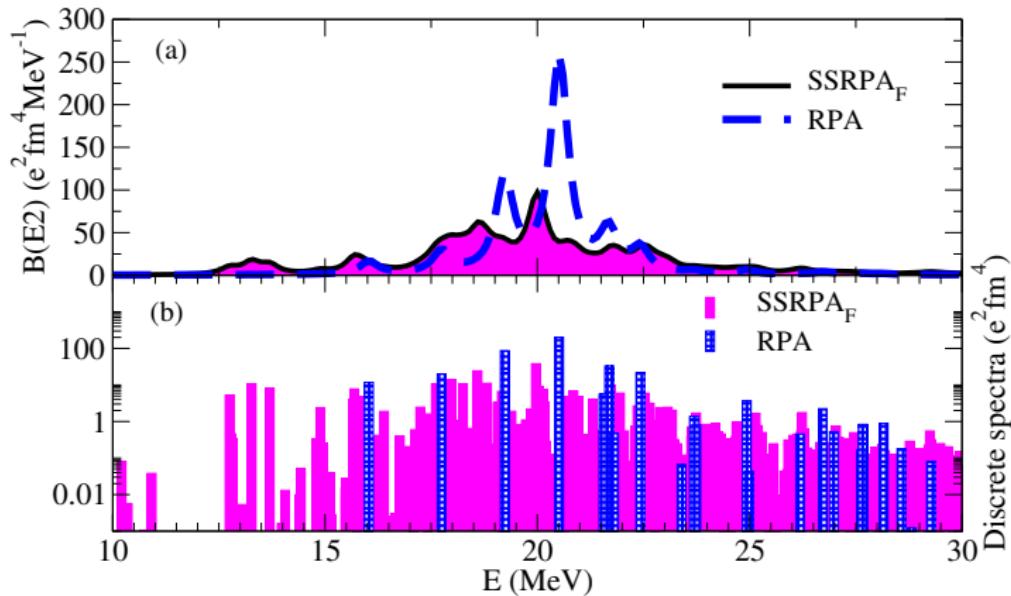
$$\Rightarrow \Pi^{SSRPA}(\omega = 0) = \Pi^{RPA}$$

Quadrupole Strength Distribution in ^{16}O : RPA, SRPA and SSRPA



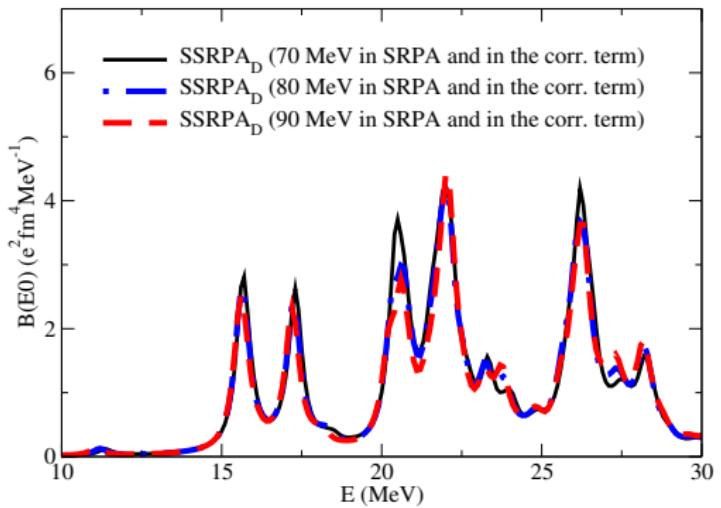
D. G., M. Grasso and J. Engel, Phys. Rev. C 92 , 034303 (2015)

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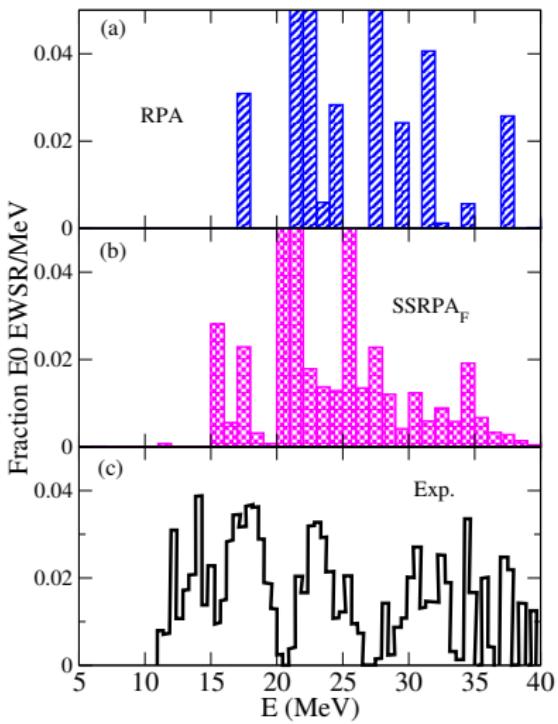
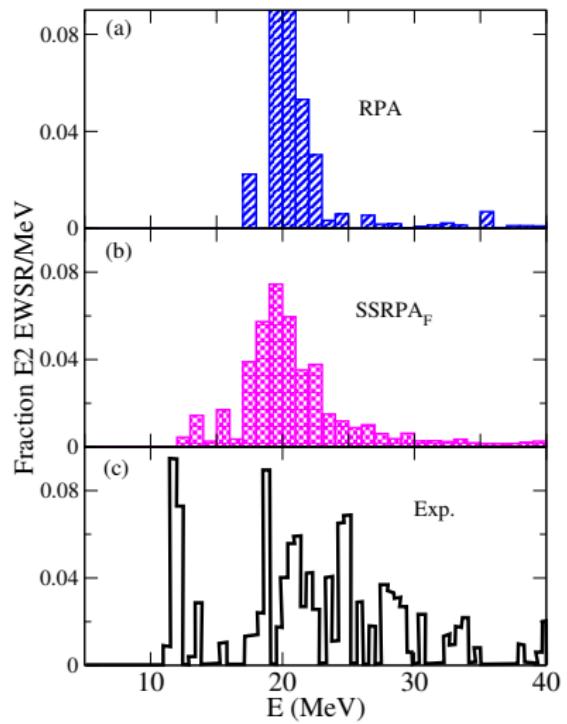


D. G., M. Grasso and J. Engel, Phys. Rev. C 92 , 034303 (2015)

Monopole Strength Distribution ^{16}O : cutoff dependence



Comparison with experimental data for ^{16}O



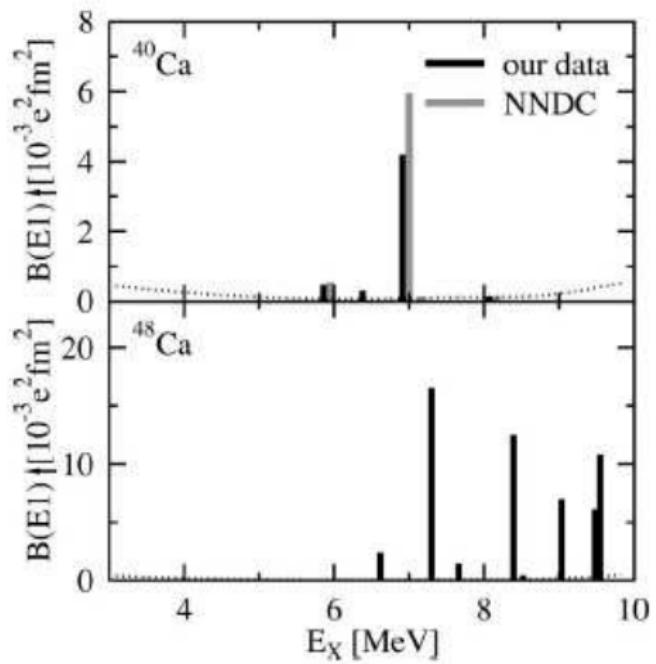
Low-lying dipole response in ^{48}Ca : Motivation

- Experimental low-lying dipole (from 5 to 10 MeV) response in ^{48}Ca
- Pygmy Dipole Resonance (PDR) type?
- Not described in relativistic and non-relativistic RPA models
- What happens in SRPA ^a ?
- and in the SSRPA ^b ?

^aD. G., M. Grasso, and F. Catara, Phys. Rev. C 84, 034301 (2011)

^bD. G., M. Grasso and O. Vasseur, Physics Letters B 777 (2018) 163168

Experimental low-lying dipole strength in $^{40,48}\text{Ca}$. (Photon Scattering)



$$\sum B(E1) = 5.1 \pm 0.8 (10^{-3} \text{ e}^2 \text{ fm}^2),$$

$$\sum B(E1) = 68.7 \pm 7.5 (10^{-3} \text{ e}^2 \text{ fm}^2),$$

From T. Hartmann *et al.*, PRC 65, 034301, (2002)

Relativistic RPA results

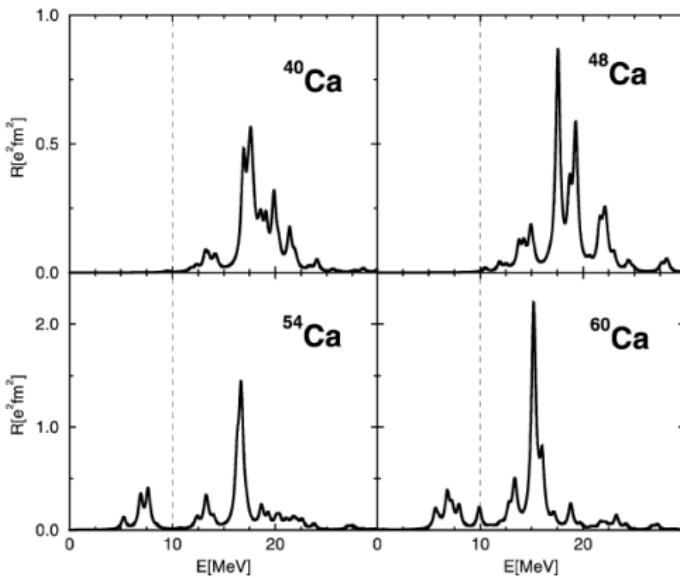
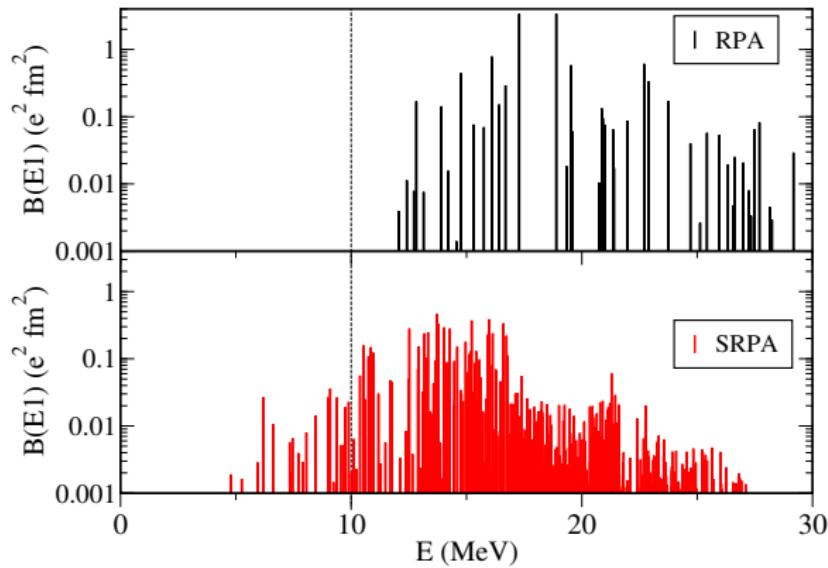


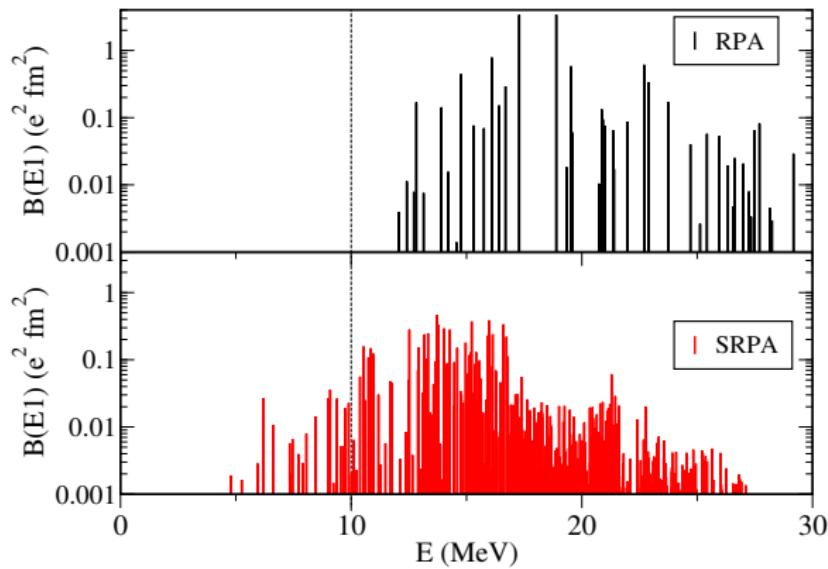
Fig. 5. RRPA isovector dipole strength distributions in Ca isotopes. The thin dashed line tentatively separates the region of giant resonances from the low-energy region below 10 MeV.

From D. Vretenar *et al.*, Nucl. Phys. A 692, 496 (2001)

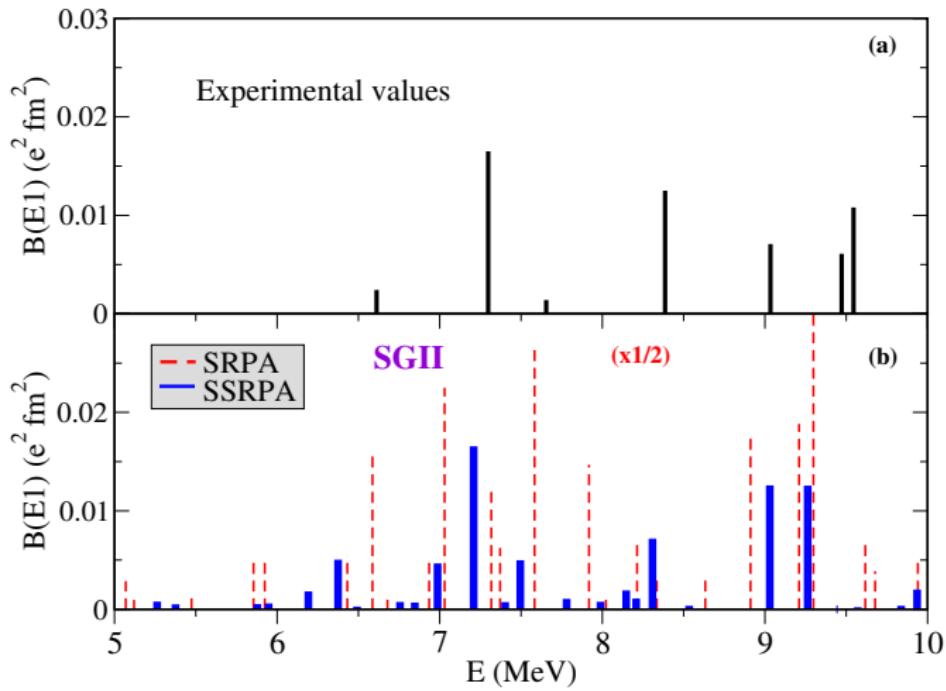
Dipole Strength ^{48}Ca



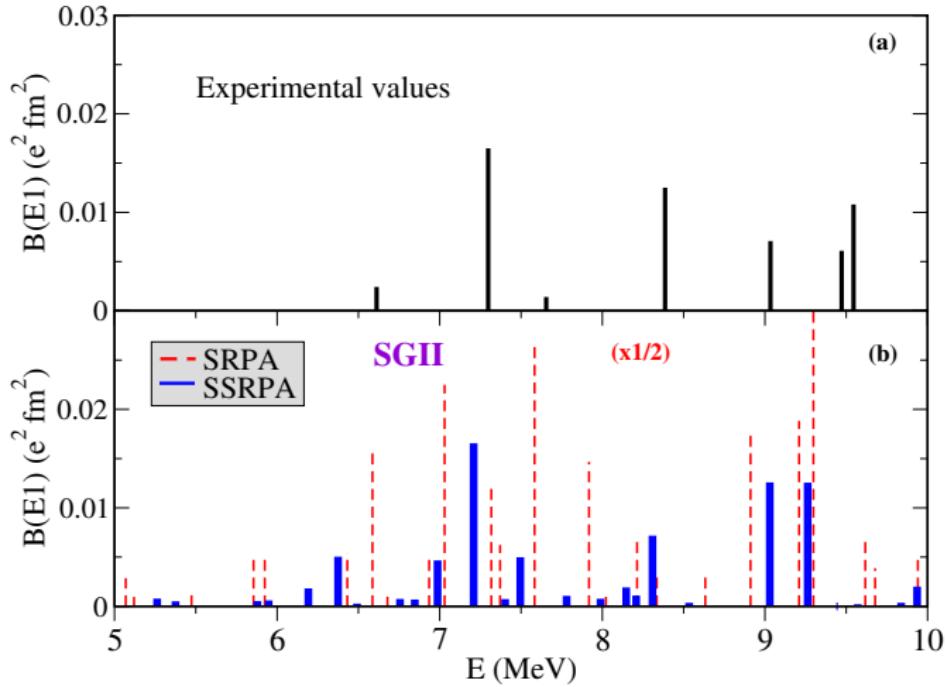
Dipole Strength ^{48}Ca



SRPA provides the strength below 10 MeV, but total strength is overestimated.



D. Gambacurta , M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163168



D. Gambacurta , M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163168
Interaction is the only input, e.g. no parameters are adjusted.

Total $B(E1)$ and EWSRs (From 5 to 10 MeV)

	Exp	SRPA SGII	SSRPA SGII	SRPA SLy4	SSRPA SLy4
$\sum B(E1)$	0.068 ± 0.008	0.563	0.078	1.012	0.126
$\sum_i E_i B_i(E1)$	0.570 ± 0.062	4.618	0.621	8.795	1.062

Experimental and theoretical $\sum B(E1)$ in ($e^2 \text{ fm}^2$) and $\sum_i E_i B_i(E1)$ in ($\text{MeV } e^2 \text{ fm}^2$) summed between 5 and 10 MeV.

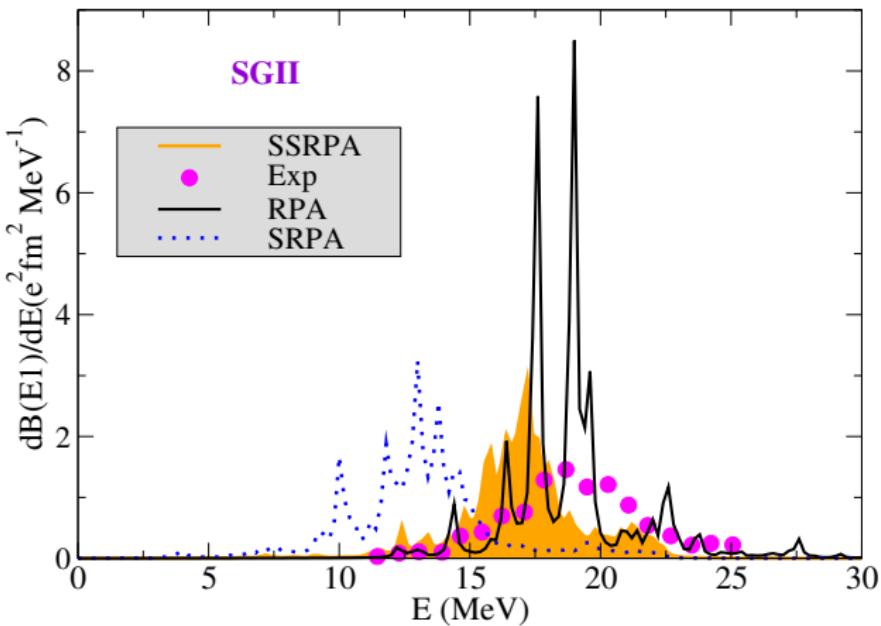
From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163168

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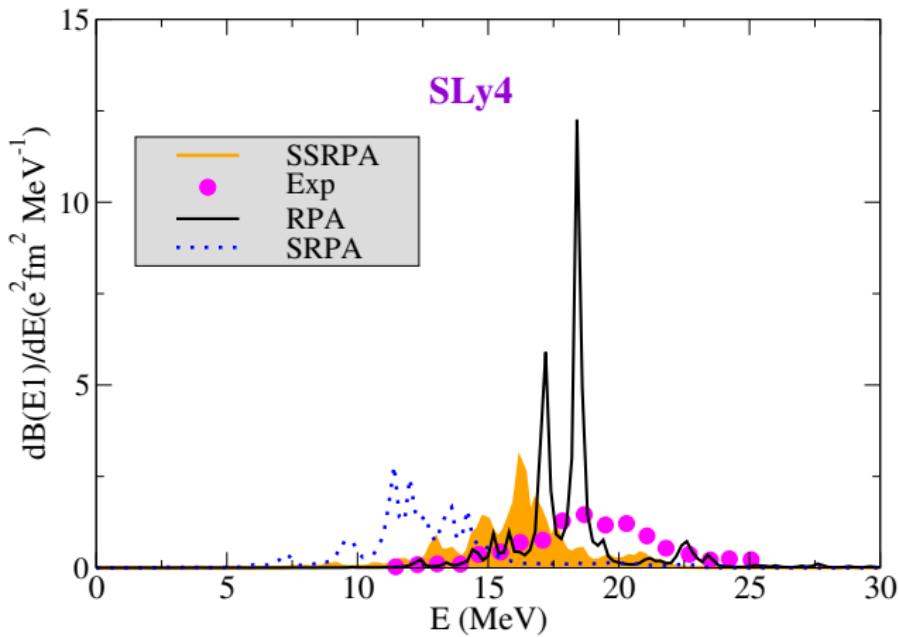
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From D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163168

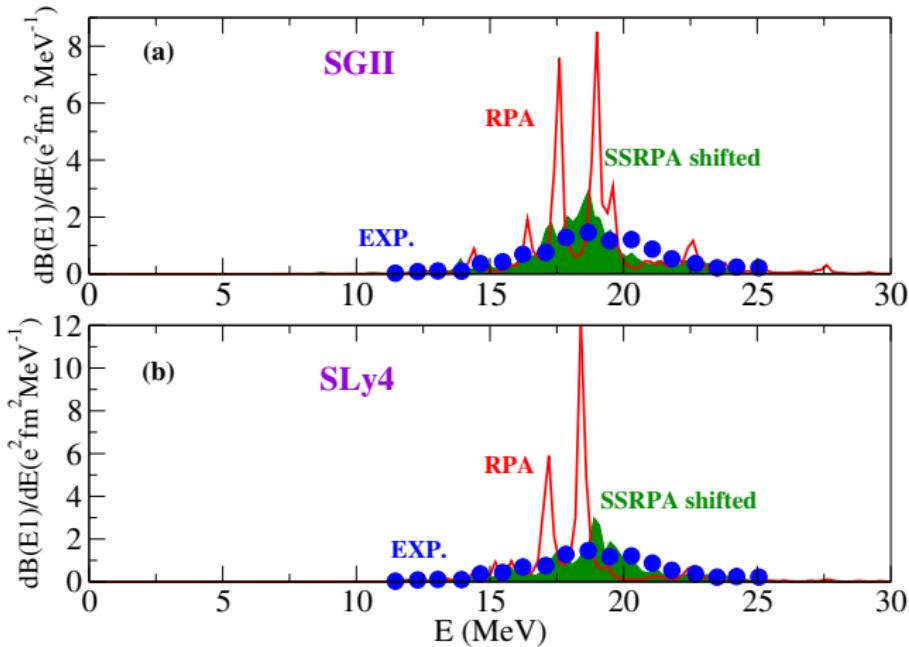


Data From J. Birkhan *et al.*, Phys. Rev. Lett. 118, 252501 (2017);
Theoretical results folded with a Lorentzian having a width of 0.25 MeV
D. G., M. Grasso , O. Vasseur, Physics Letters B 777 (2018) 163168



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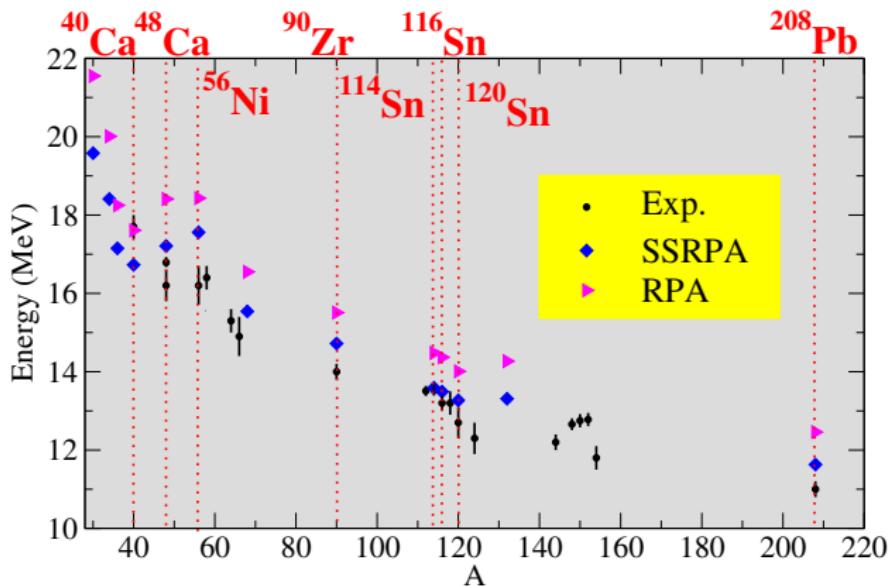
SSRPA vs Data, GDR case



Data From J. Birkhan *et al.*, Phys. Rev. Lett. 118, 252501 (2017);
Theoretical results folded with a Lorentzian having a width of 0.25 MeV

Systematic calculations for Isoscalar GQRs: from ^{30}Si to ^{208}Pb

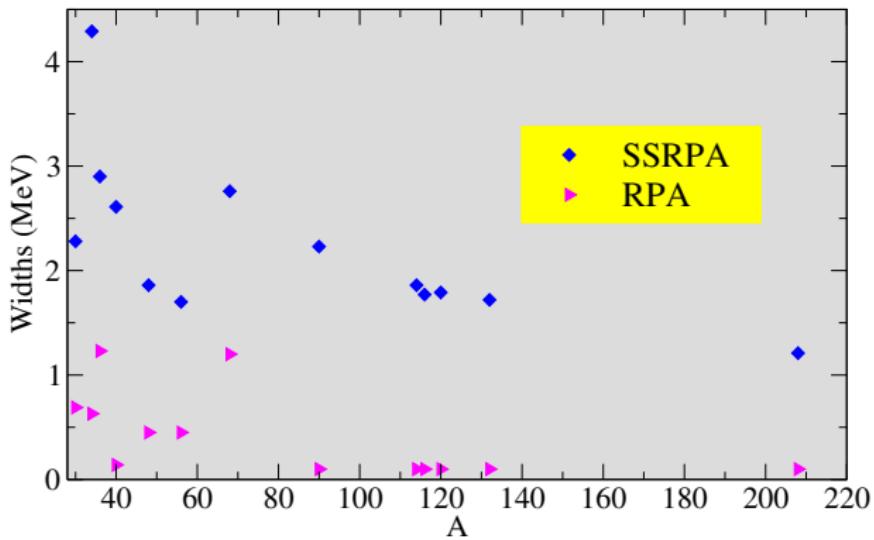
Centroid energy



Globally: better agreement with the experimental data compared to RPA
Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

Systematic calculations for Isoscalar GQRs: from ^{30}Si to ^{208}Pb

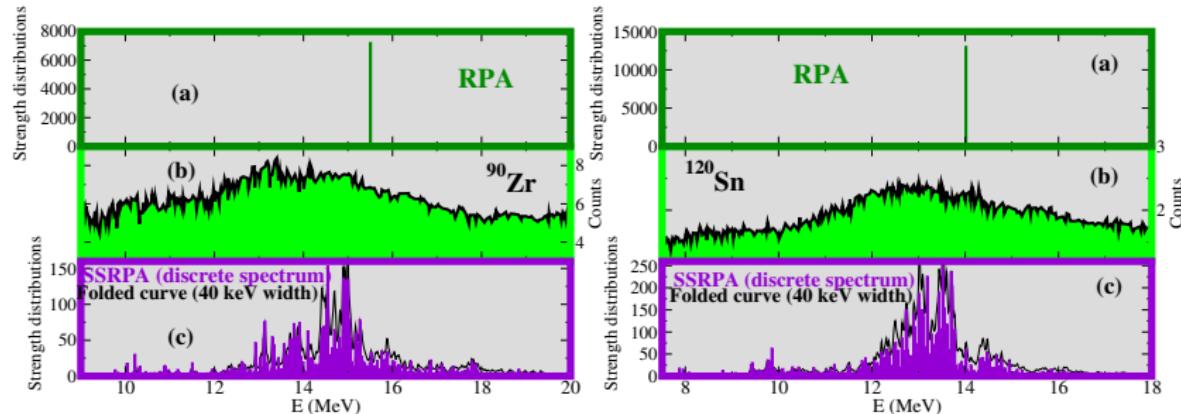
Width



General trend, found both in RPA and in SSRPA: the width is systematically reduced going from lighter to heavier nuclei (Landau damping)
Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

Systematic calculations for Isoscalar GQRs: from ^{30}Si to ^{208}Pb

Fine structure



Exp. data: Shevchenko et al, PRL 93, 122501 (2004)

From : Vasseur, Gambacurta, Grasso, PRC 98, 044313 (2018)

Conclusions:

Microscopic description on the nuclear response

- Microscopic description based on the RPA and SRPA
- SRPA: coupling between $1p - 1h$ and $2p - 2h$ is fully taken into account
- Subtraction procedure in SRPA (SSRPA) cures SRPA issues

SSRPA Applications and Results

- Monopole and Quadrupole response for ^{16}O : improvement with respect to the RPA (especially the fragmentation)
- Dipole response in ^{48}Ca :
 - i) Good description of PDR states and GDR width (Not in RPA),
 - ii) Centroid of the GDR is underestimated in SSRPA
- ISGQR systematic calculations for spherical nuclei: overall improvement of the centroids and fine structure (missing in RPA)