

Chiral three-body force and monopole properties of shell-model Hamiltonian

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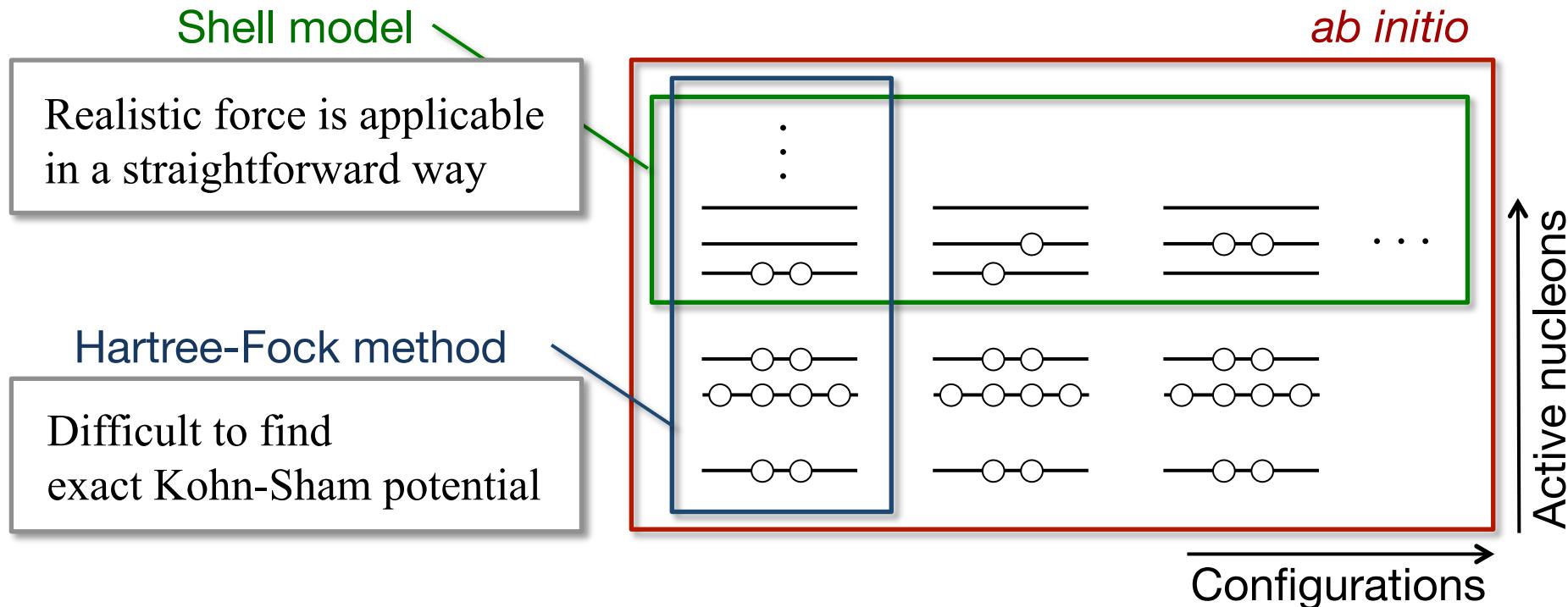
³*Dipartimento di Matematica e Fisica,
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T. Fukui *et al.*, Phys. Rev. C **98**, 04430 (2018).
Y. Z. Ma *et al.*, arXiv:1812.03284.

15/March/2019

Shell model and density functional theory



Example of realistic shell model (RSM)

- ⌚ Shell model with chiral effective field theory
→ **Many-body force** on an equal footing

S. Weinberg, Phys. A **96**, 327 (1979).
R. Machleidt and D. Entem, Phys. Rep. **503**, 1 (2011).

RSM and 3NF | RSM Hamiltonian

3NF contribution to RSM

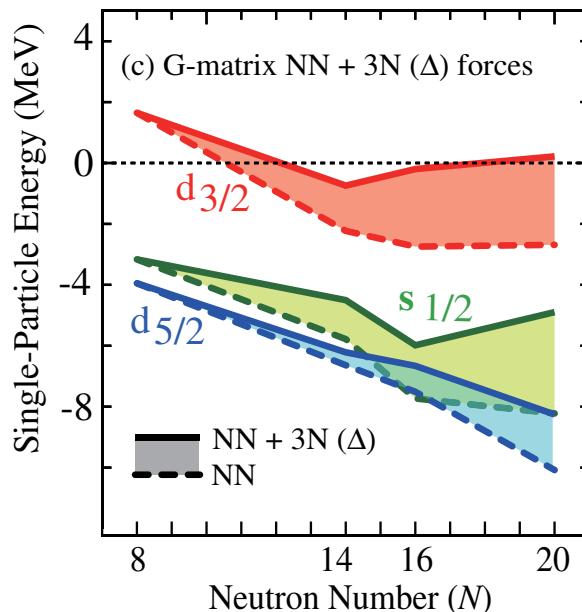
- ④ RSM Hamiltonian needs to be revised due to **3-nucleon force (3NF)**. Specifically → **Monopole Hamiltonian**

A. P. Zuker, Phys. Rev. Lett. **90**, 042502 (2003).

cf.) Oxygen dripline and 3NF

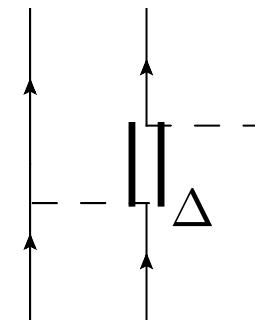
T. Otsuka *et al.*, Phys. Rev. Lett., **105**, 032501 (2010).

- ④ The 3NF accounts for the oxygen dripline (^{24}O).



Fujita-Miyazawa 3NF

Repulsive contribution to monopole Hamiltonian



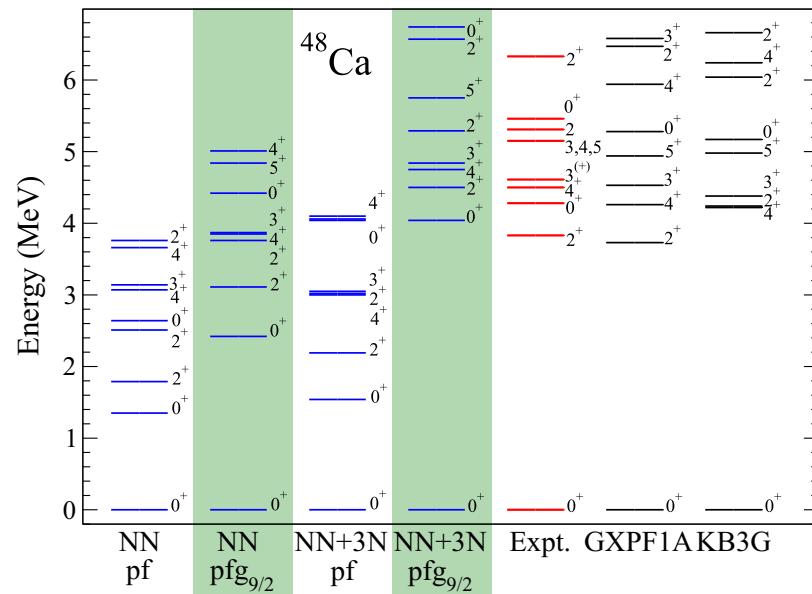
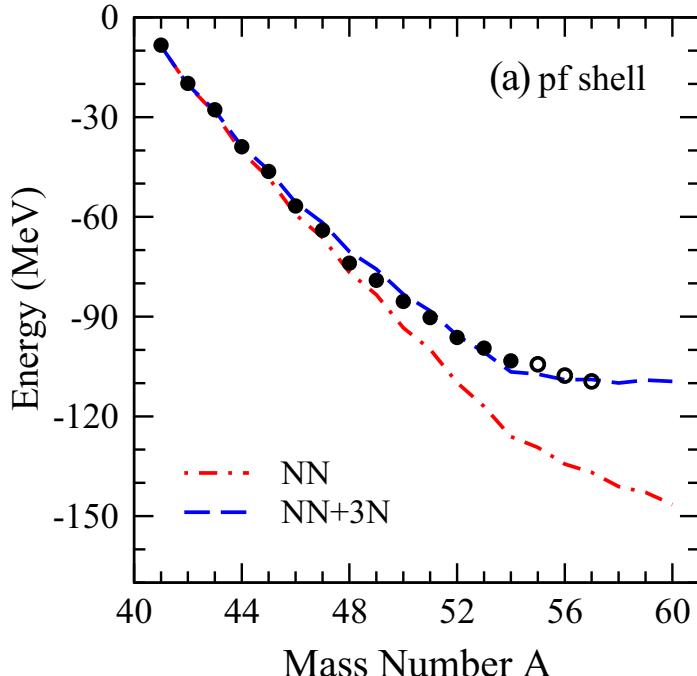
J. Fujita and H. Miyazawa,
Prog. Theor. Phys. **17**, 360 (1957).

Motivation | RSM and 3NF

Shell evolution on *fp*-shell and RSM

J. D. Holt *et al.*, Phys. Rev. **90**, 024312 (2014).

- Ⓐ A crucial role played by 3NF (Chiral N²LO) for Ca isotopes.



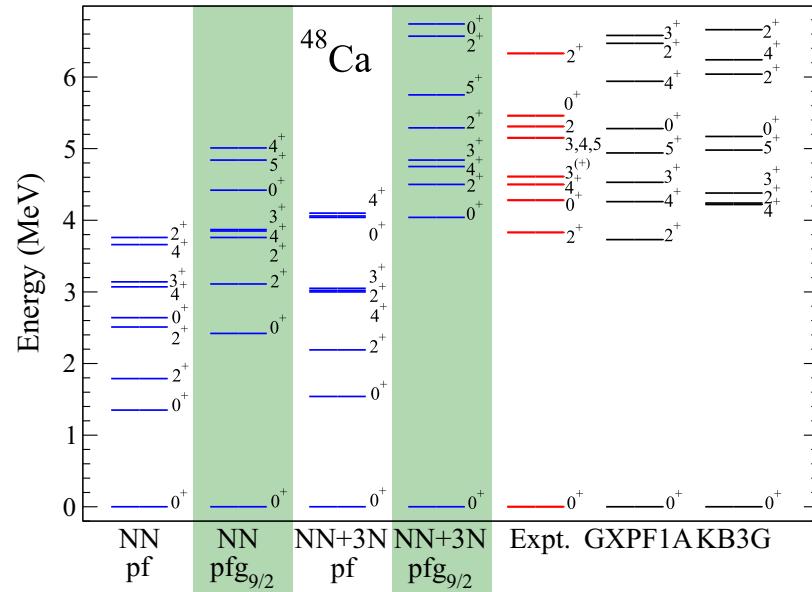
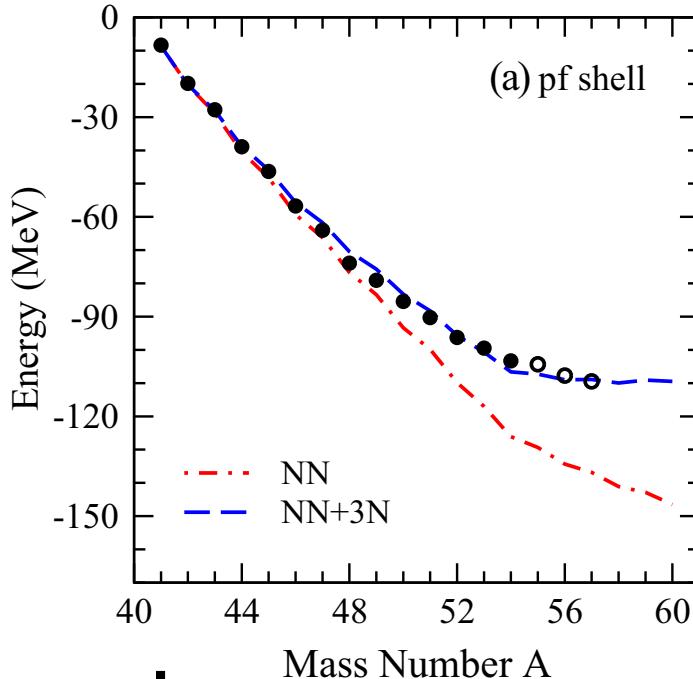
3NF contribution to monopole Hamiltonian?

Motivation | RSM and 3NF

Shell evolution on *fp*-shell and RSM

J. D. Holt *et al.*, Phys. Rev. **90**, 024312 (2014).

- ⌚ A crucial role played by 3NF (Chiral N²LO) for Ca isotopes.



This work

- ⌚ We focus on the **monopole contribution** induced by the chiral N²LO 3NF.
→ Shell evolution in *fp*-shell nuclei.
- ⌚ We develop **our own code** for 3-body matrix elements (MEs) for RSM inputs.
→ Benchmark test for *p*-shell nuclei.

Chiral 3-body ME | New approach for 2π term

3-body ME with nonlocal regulator

Pioneering works

- ⌚ Center-of-mass separation
A. Nogga *et al.*, Phys. Rev. C **73**, 064002 (2006).
- ⌚ Antisymmetrization
P. Navrátil *et al.*, Phys. Rev. C **61**, 044001 (2000).
- ⌚ c_D and c_E terms
P. Navrátil, Few-Body Syst. **41**, 117 (2007).

Our new formalism

- ⌚ **Triple-fold multipole expansion** for 2π term.

T. Fukui *et al.*, Phys. Rev. C **98**, 04430 (2018).

Chiral N²LO 3NF

$$\begin{aligned}
 V_{3N} = & \quad \text{---} \bullet \text{---} \quad \textbf{2}\pi (c_1, c_3, c_4) \\
 + & \quad \text{---} \diagup \diagdown \bullet \text{---} \quad \textbf{1}\pi + \textbf{contact} (c_D) \\
 + & \quad \diagup \diagdown \bullet \text{---} \quad \textbf{Contact} (c_E)
 \end{aligned}$$

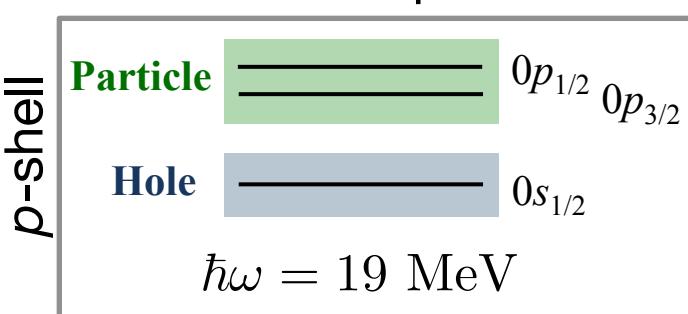
S. Weinberg, Phys. Lett. B **295**, 114 (1992).
U. van Kolck, Phys. Rev. C **49**, 2932 (1994).

Low-energy constants

2NF: N³LO D. R. Entem and R. Machleidt, Phys. Rev. C **68**, 041001(R) (2003).

3NF: N²LO P. Navrátil *et al.*, Phys. Rev. Lett. **99**, 042501 (2007).

Model space



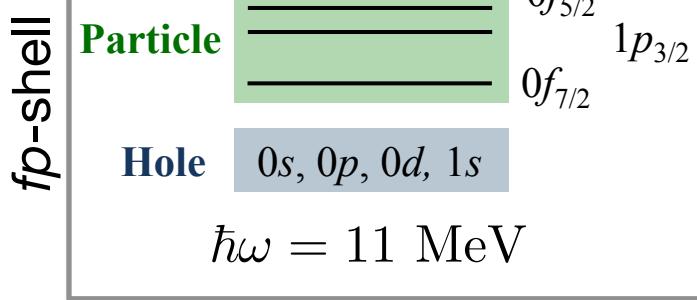
Many-body perturbation theory

$$H \approx H_{\text{eff}}$$

2NF: Up to 3rd-order folded-diagram expansion

3NF: Up to 1st-order (normal-order approx.)

L. Coraggio *et al.*, Ann. Phys. **327**, 2125 (2012).

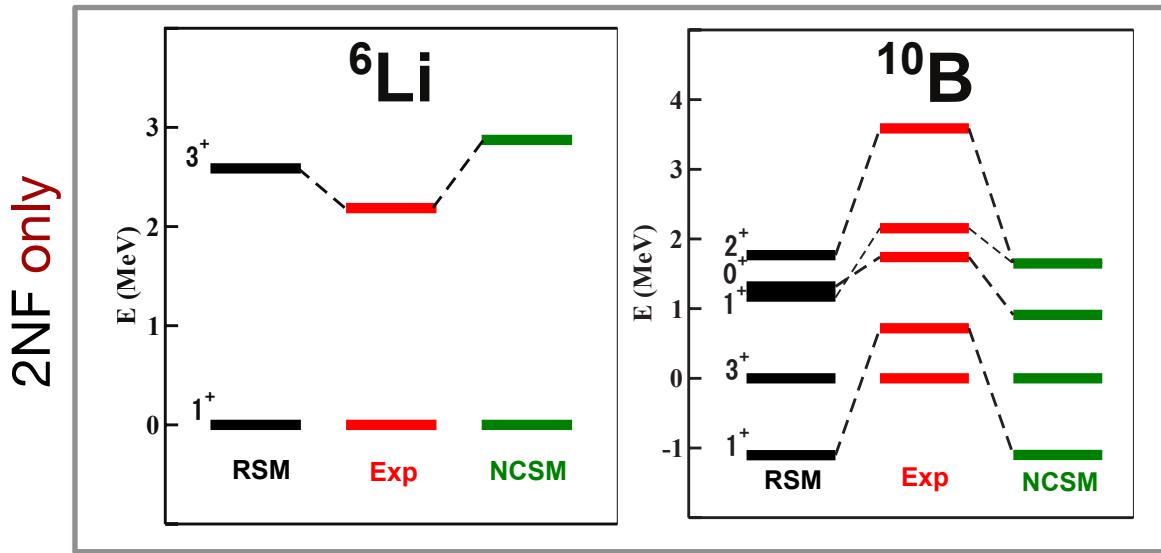


- (1) Effective Hamiltonian H_{eff} involving ***Q*-space effect**.
 (2) **Theoretical single-particle energies** and **2-body MEs**.

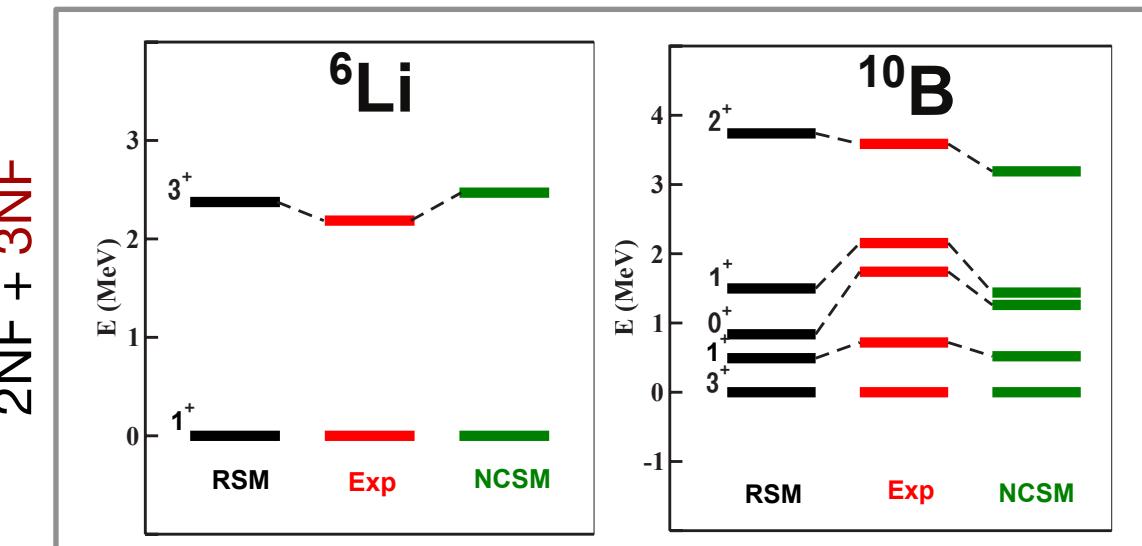
No empirical input

Benchmark calculations | p -shell nuclei

Comparison with *ab initio* no-core shell model (NCSM)



- Ⓐ RSM and NCSM agree with each other for **low-lying states**.
- Ⓑ Significant 3NF effect can be seen.

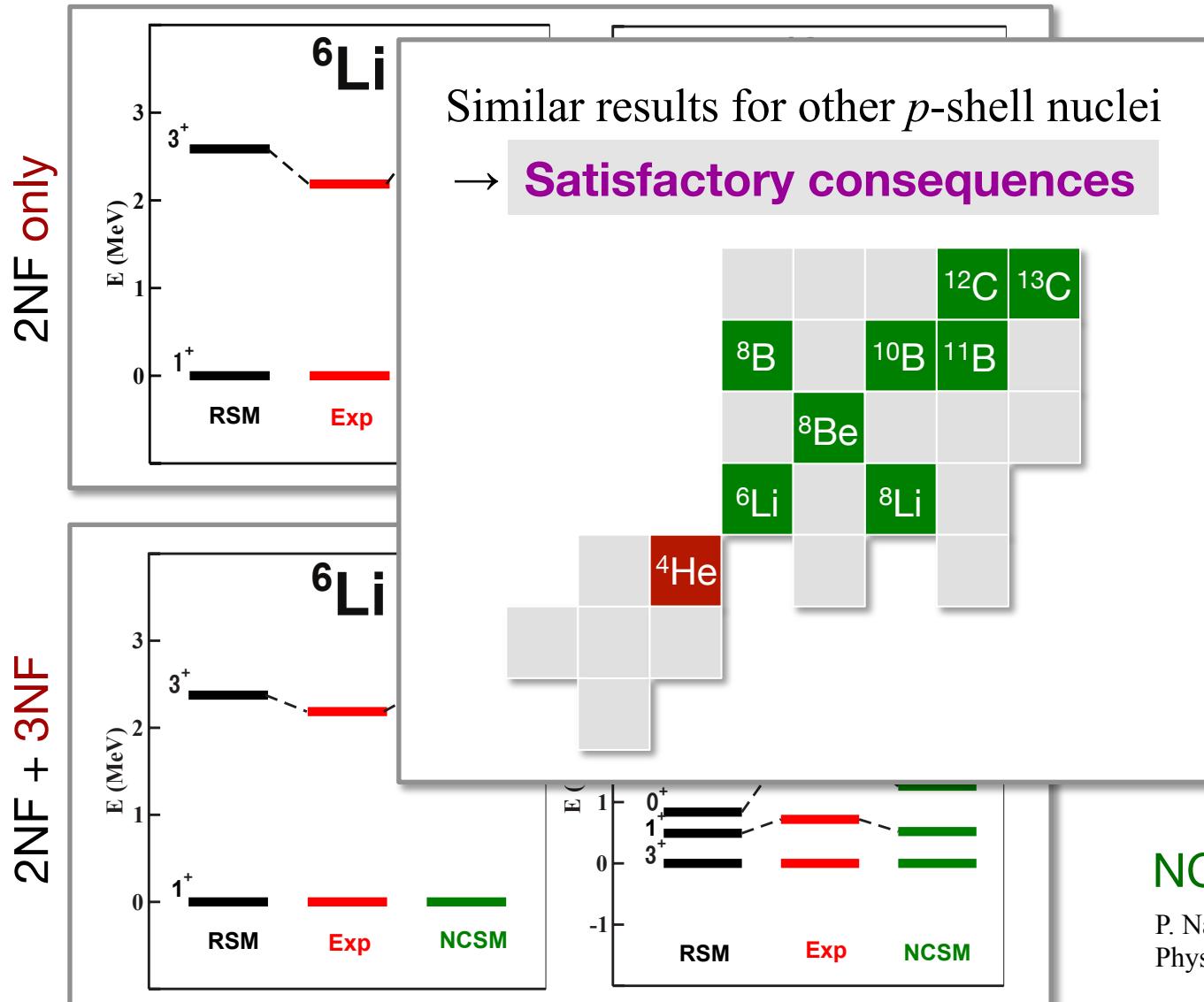


NCSM

P. Navrátil *et al.*,
Phys. Rev. Lett. **99**, 042501 (2007).

Benchmark calculations | p -shell nuclei

Comparison with *ab initio* no-core shell model (NCSM)



Similar results for other p -shell nuclei
→ **Satisfactory consequences**

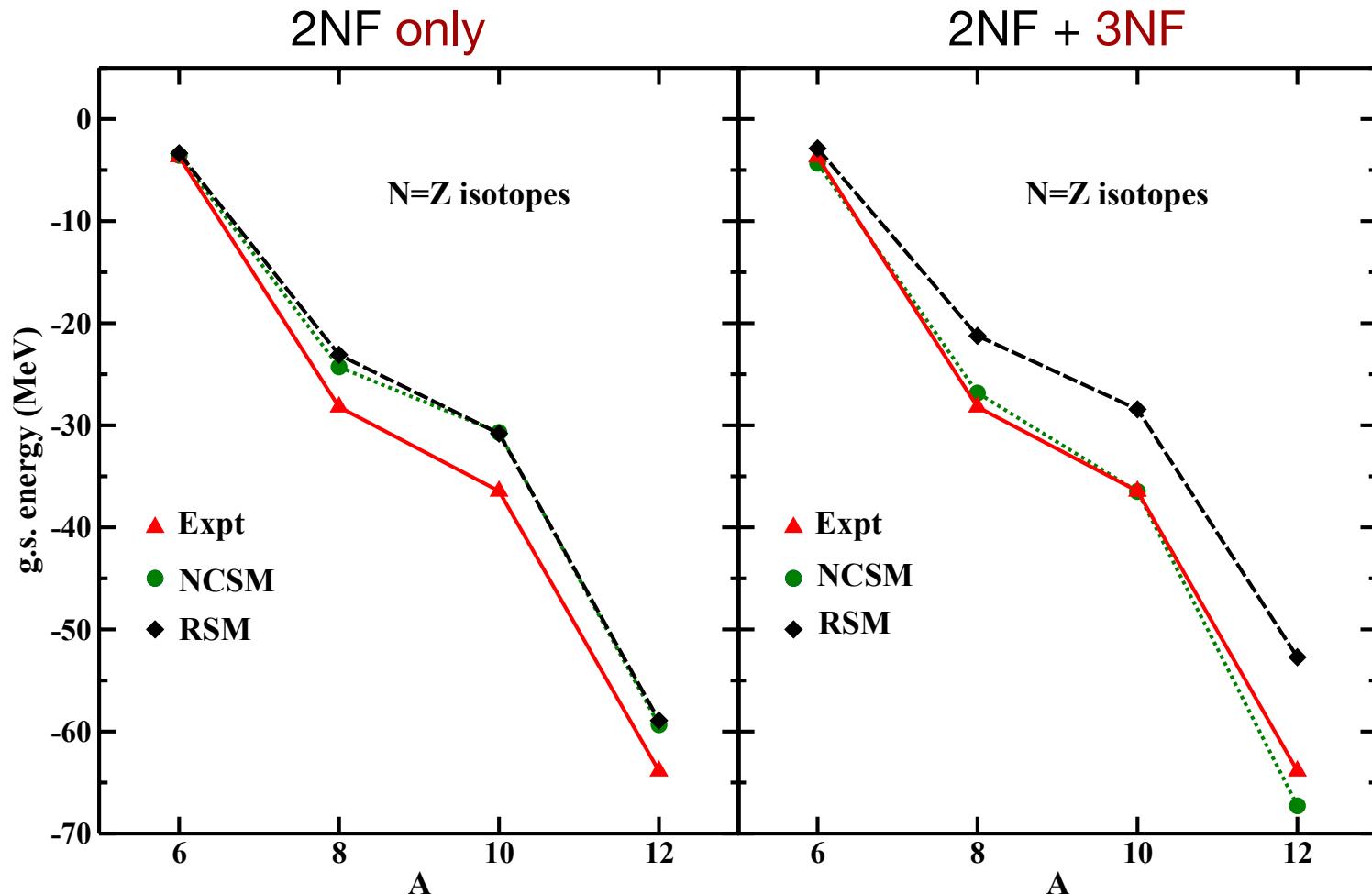
M and NCSM agree
with each other
low-lying states.

Significant 3NF effect
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NCSM

P. Navrátil *et al.*,
Phys. Rev. Lett. **99**, 042501 (2007).

Ground-state energies



⌚ At maximum
 ~ 1 MeV discrepancy

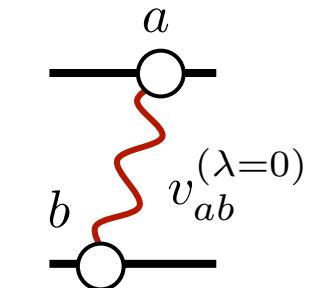
⌚ Higher-order 3NF contribution
may be necessary.

Monopole interaction

Monopole interaction $v_{ab}^{(\lambda=0)}$

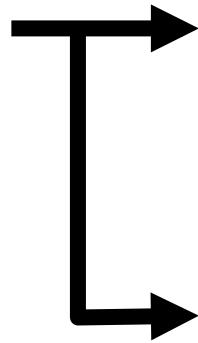
= J -averaged 2-body interaction

$$v_{ab}^{(\lambda=0)} = \frac{\sum_J (2J+1) \langle ab; J | V | ab; J \rangle}{\sum_J (2J+1)}$$



Core

$v_{ab}^{(\lambda=0)}$



Effective single-particle energy (ESPE) ε'_a

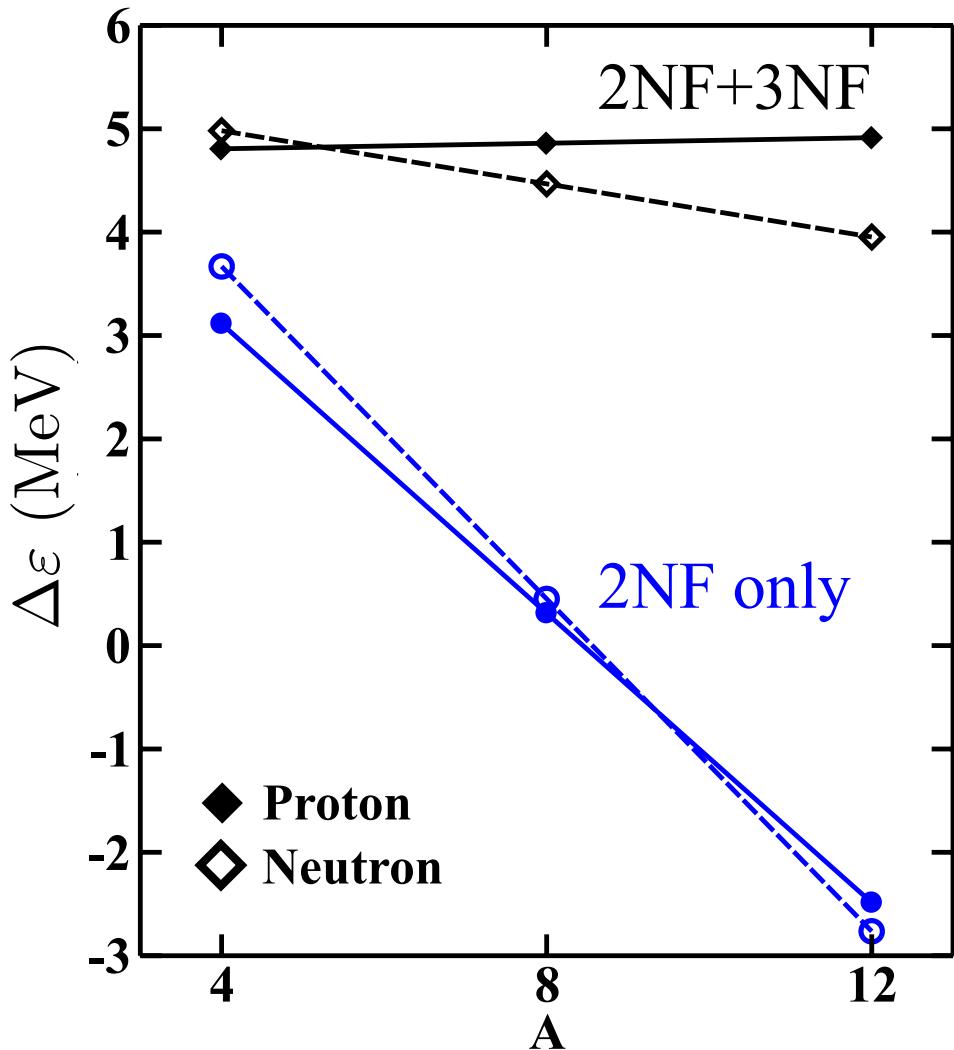
= Evolved single-particle energy $\varepsilon_a \xrightarrow{v_{ab}^{(\lambda=0)}} \varepsilon'_a$

Monopole-Hamiltonian ME

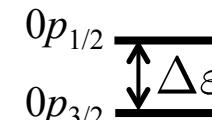
$$= \varepsilon_a + \varepsilon_b + v_{ab}^{(\lambda=0)}$$

Spherical mean field

ESPEs



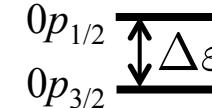
2NF + 3NF



- ⌚ $\Delta\epsilon$ is almost constant.

→ Better closure properties

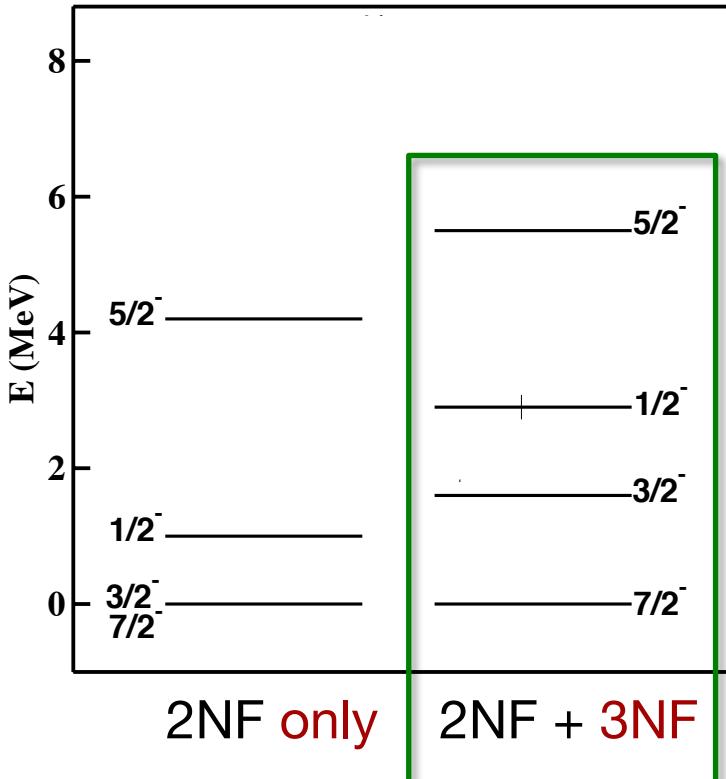
2NF only



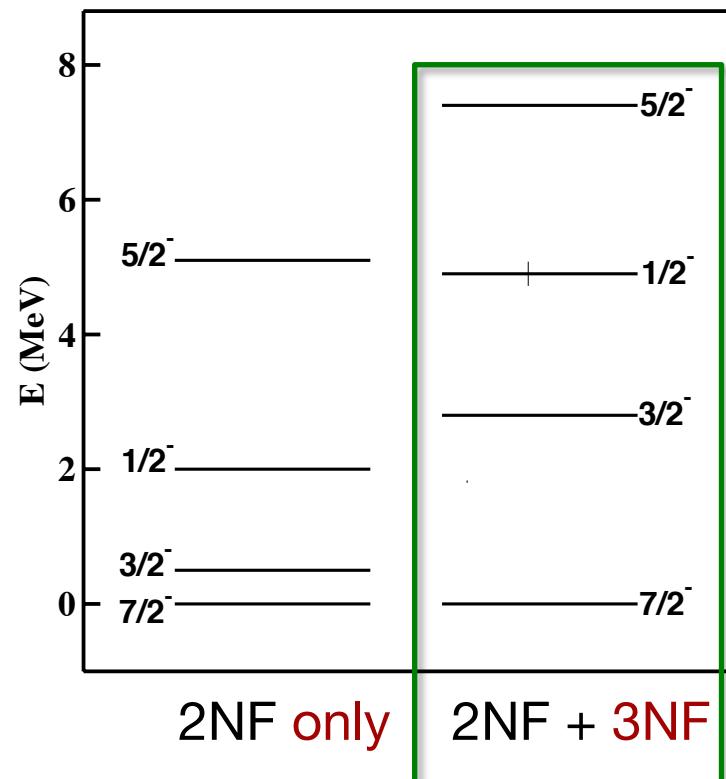
- ⌚ $\Delta\epsilon$ decreases with A , and $p_{3/2}$ - $p_{1/2}$ orbits become inverted.

fp-shell nuclei | Calculated SPEs

Proton (^{41}Sc)



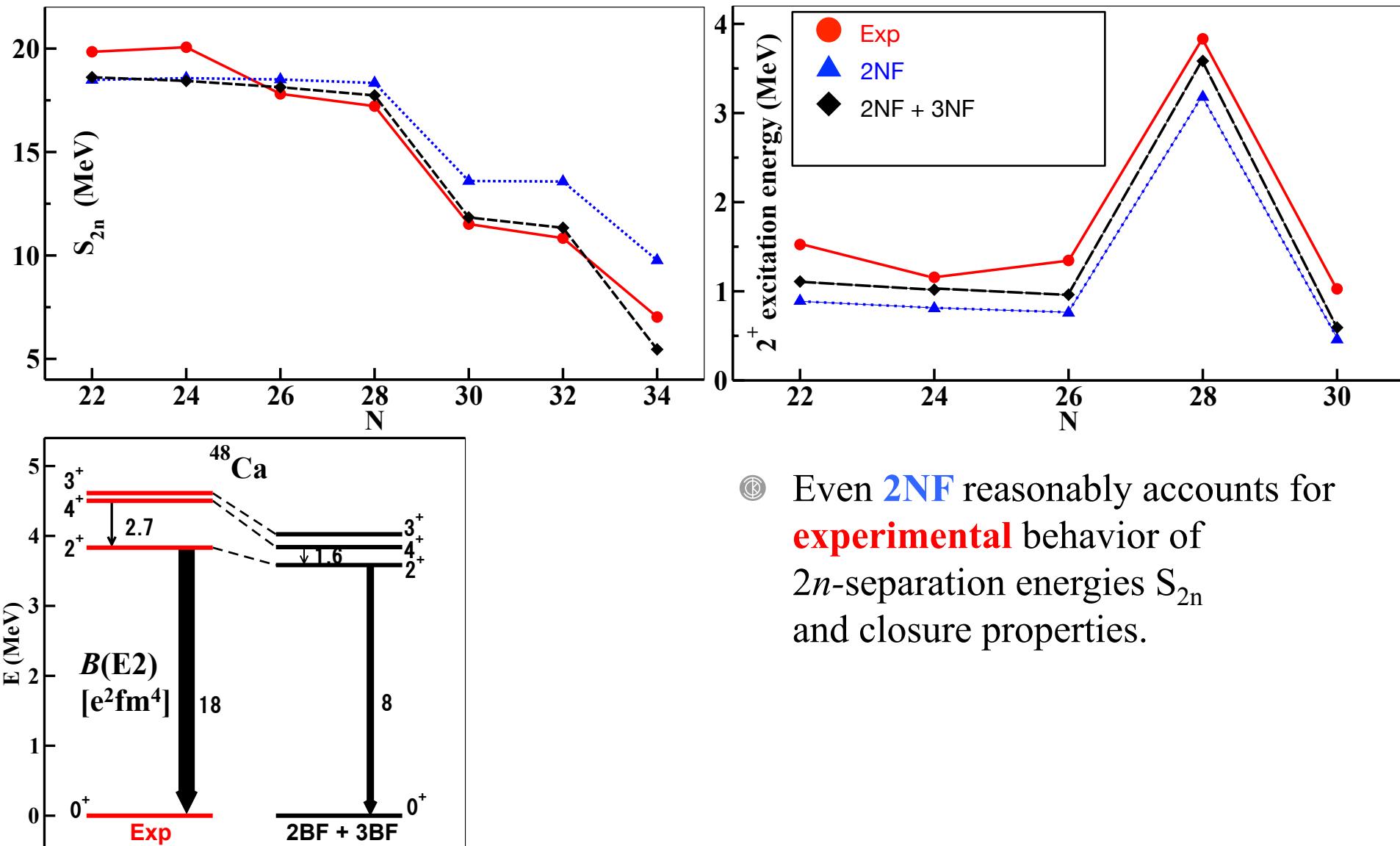
Neutron (^{41}Ca)



- ⌚ 2NF only must not work.
→ Always we adopt the 2NF + 3NF SPEs.

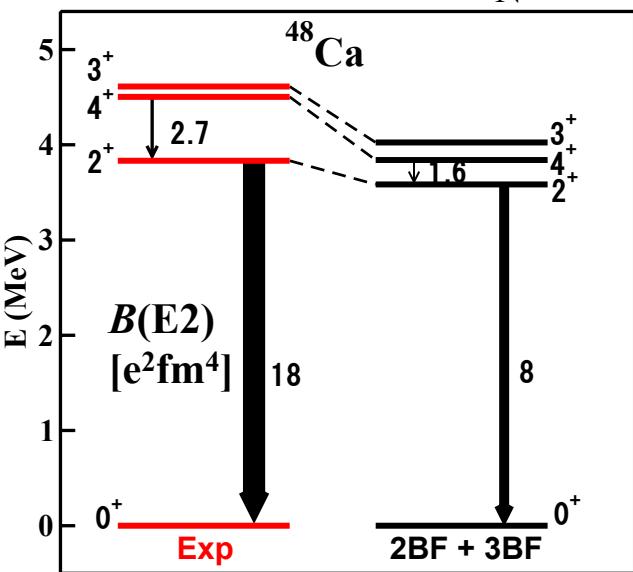
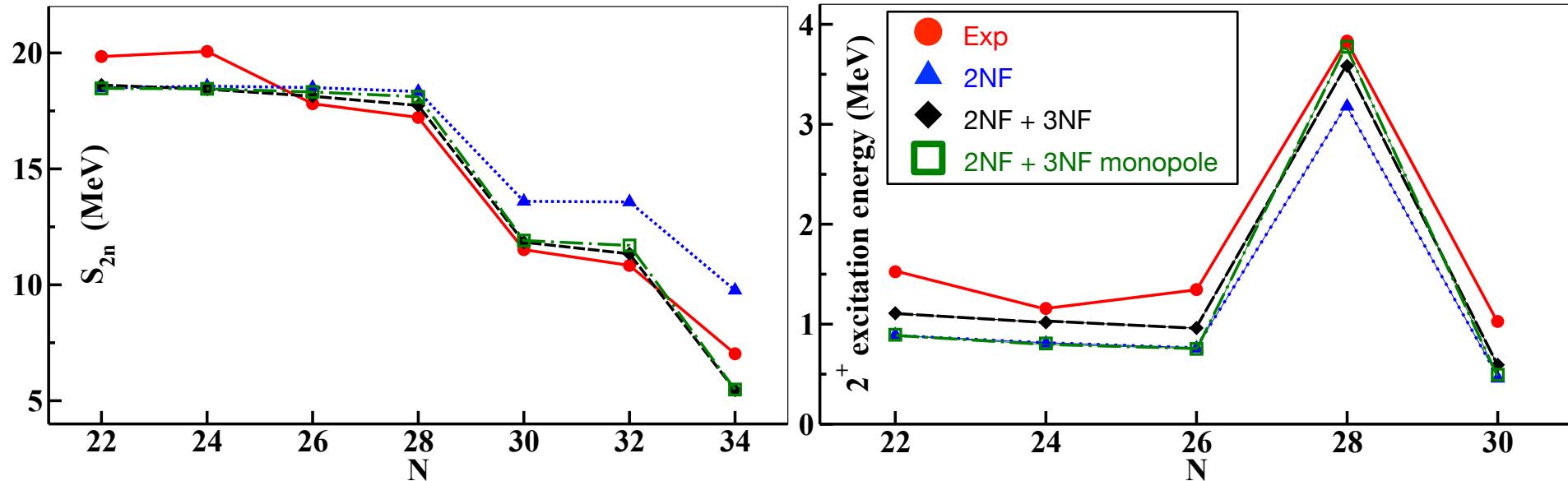
Focus only on monopole component

Ca isotopes



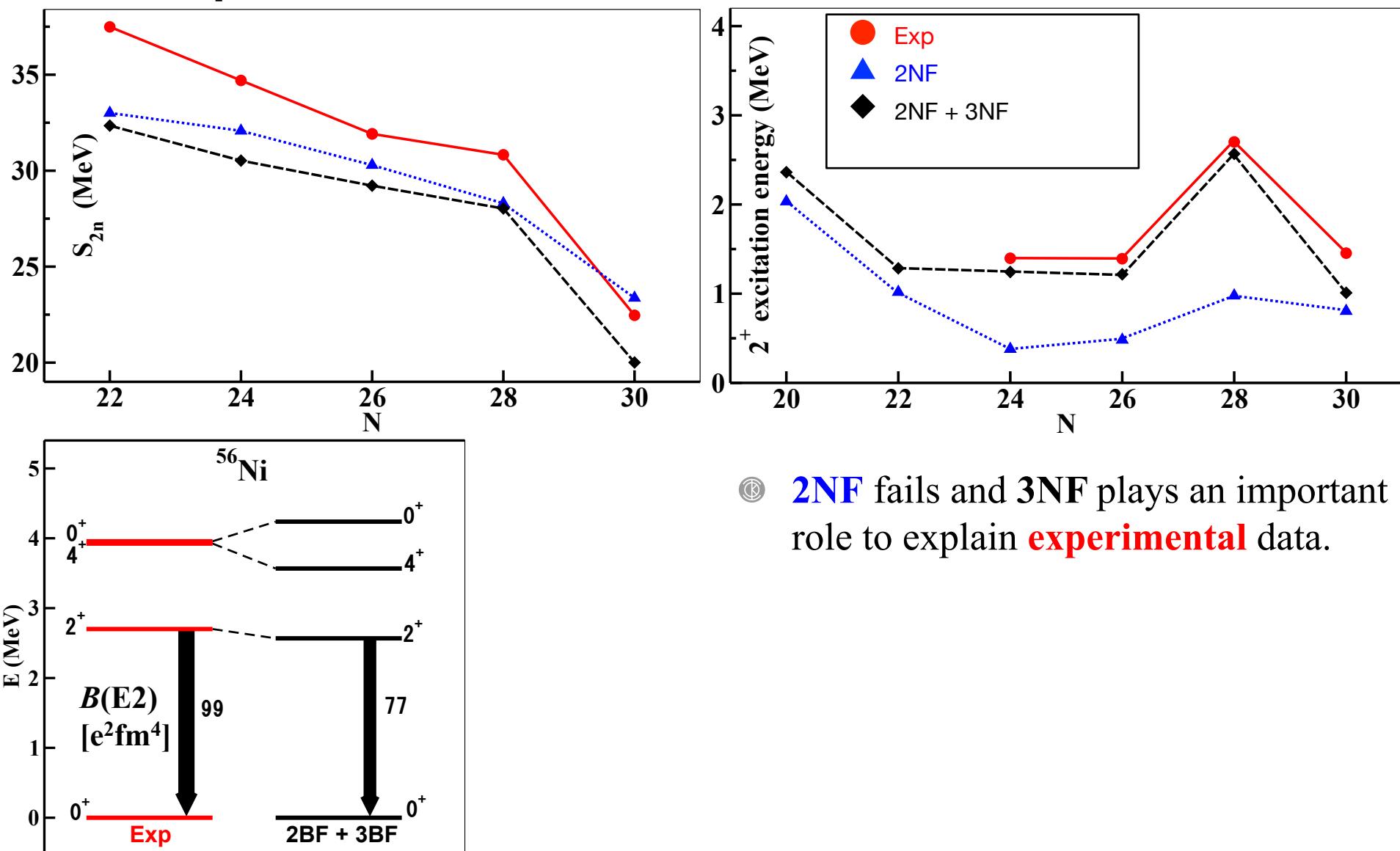
- Even **2NF** reasonably accounts for **experimental** behavior of $2n$ -separation energies S_{2n} and closure properties.

Ca isotopes

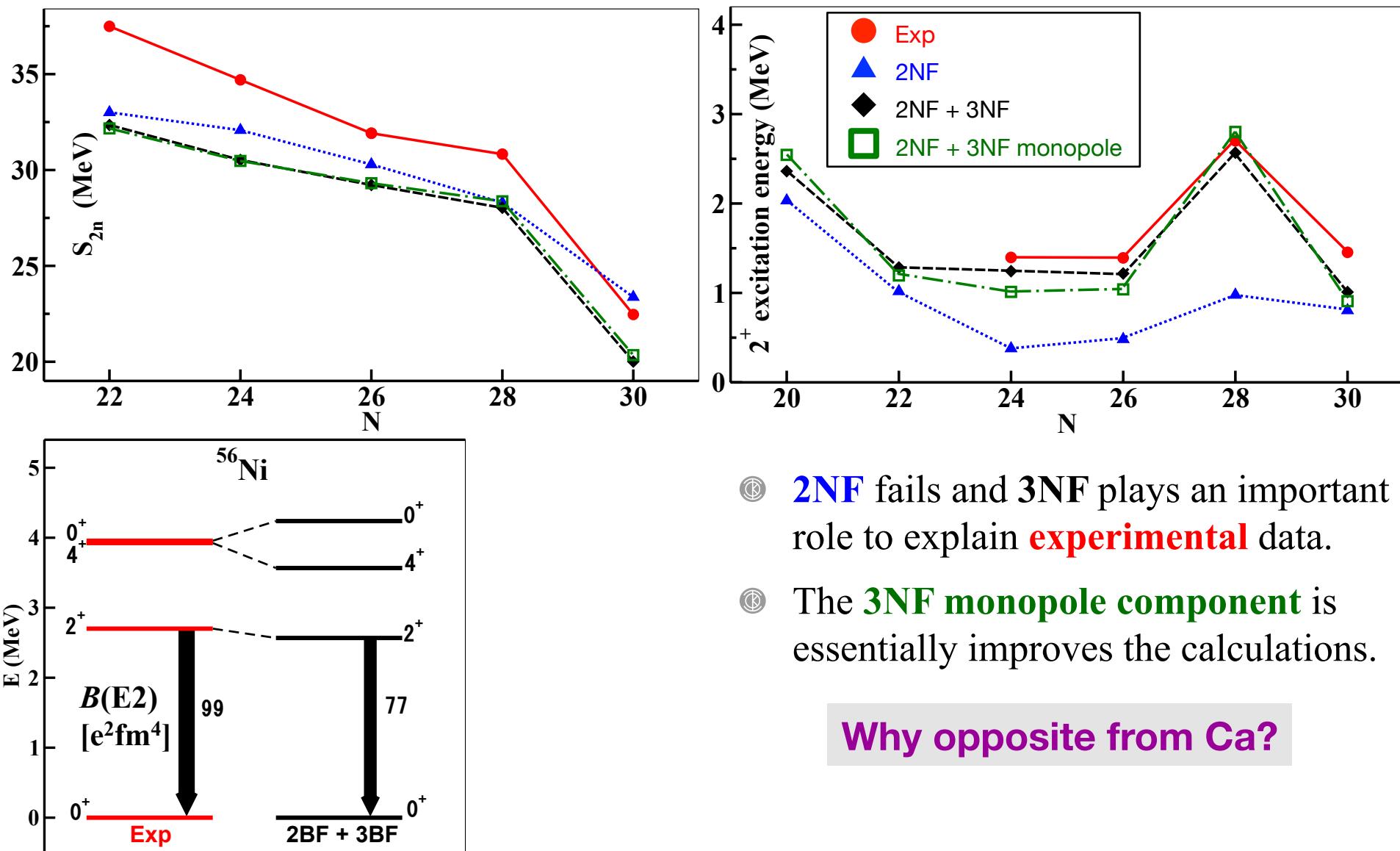


- ⌚ Even **2NF** reasonably accounts for **experimental** behavior of $2n$ -separation energies S_{2n} and closure properties.
- ⌚ The **3NF monopole component** is small except for ^{48}Ca .

Ni isotopes



Ni isotopes



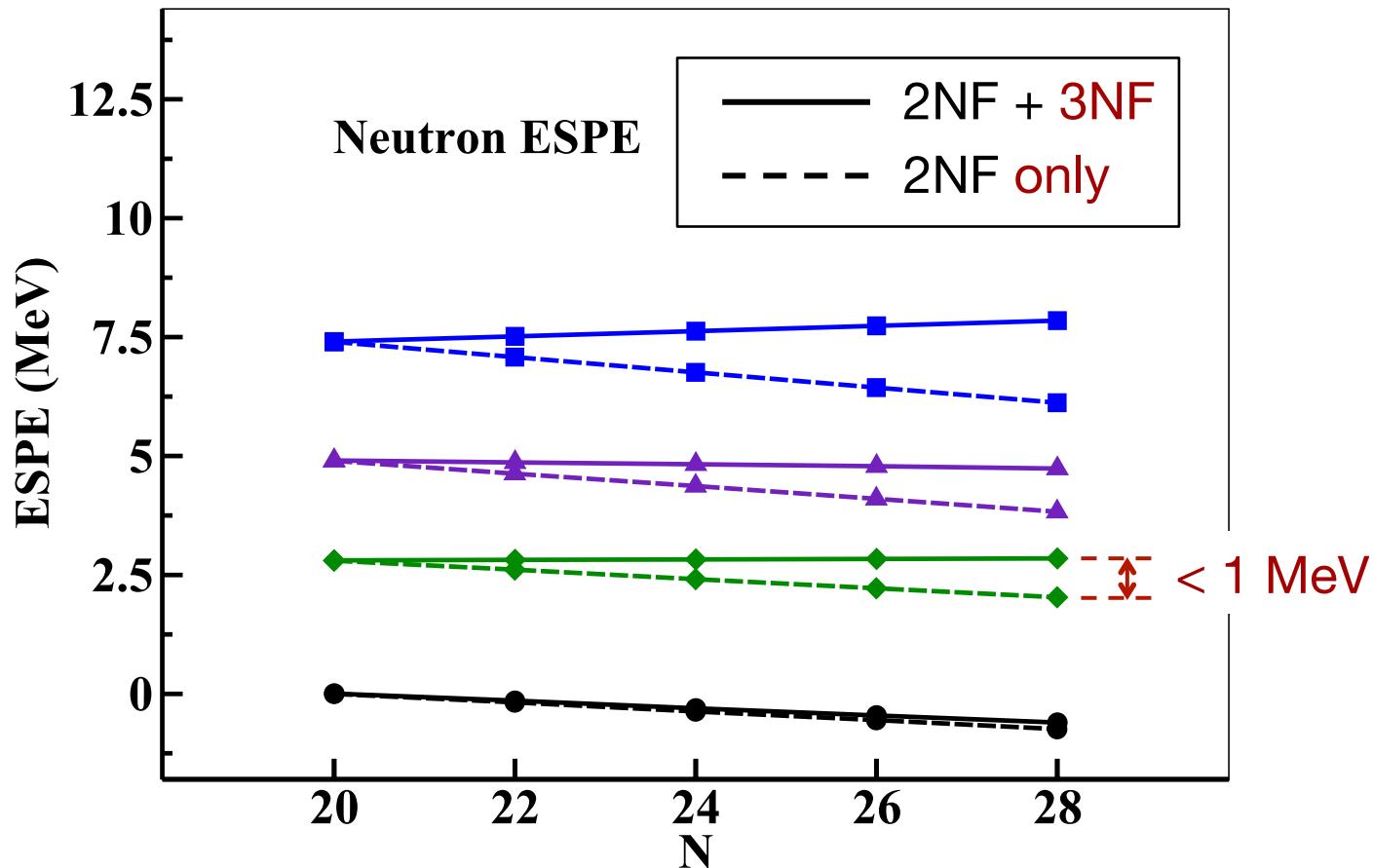
- ⌚ **2NF** fails and **3NF** plays an important role to explain **experimental** data.

- ⌚ The **3NF monopole component** is essentially improves the calculations.

Why opposite from Ca?

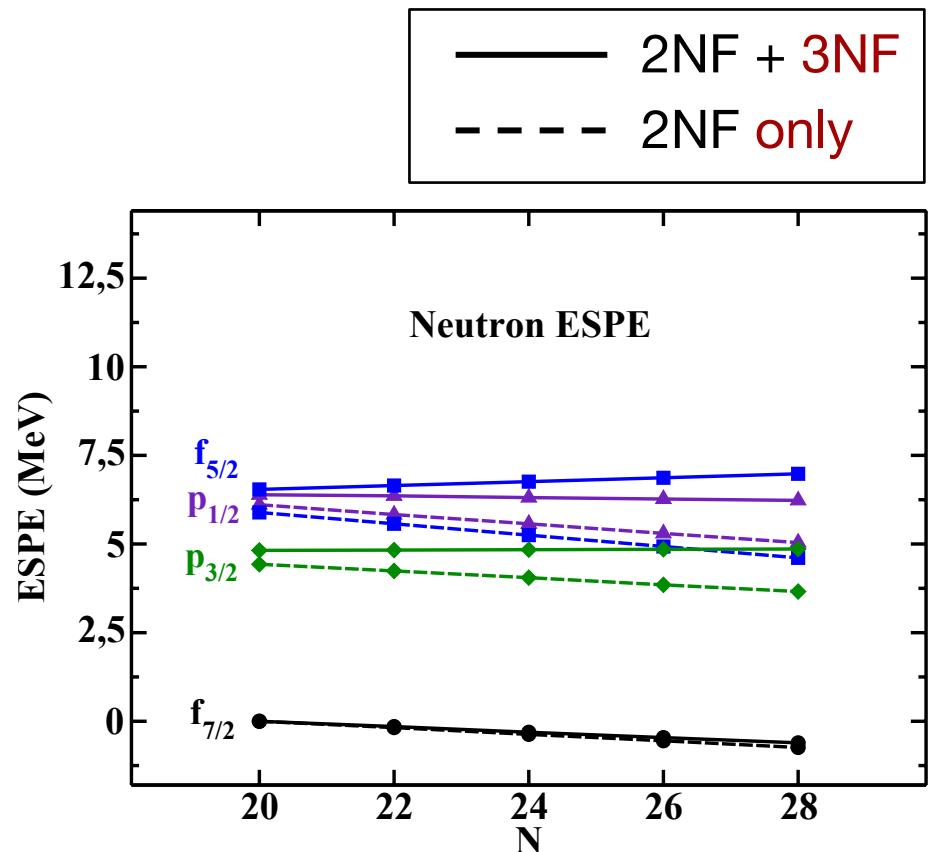
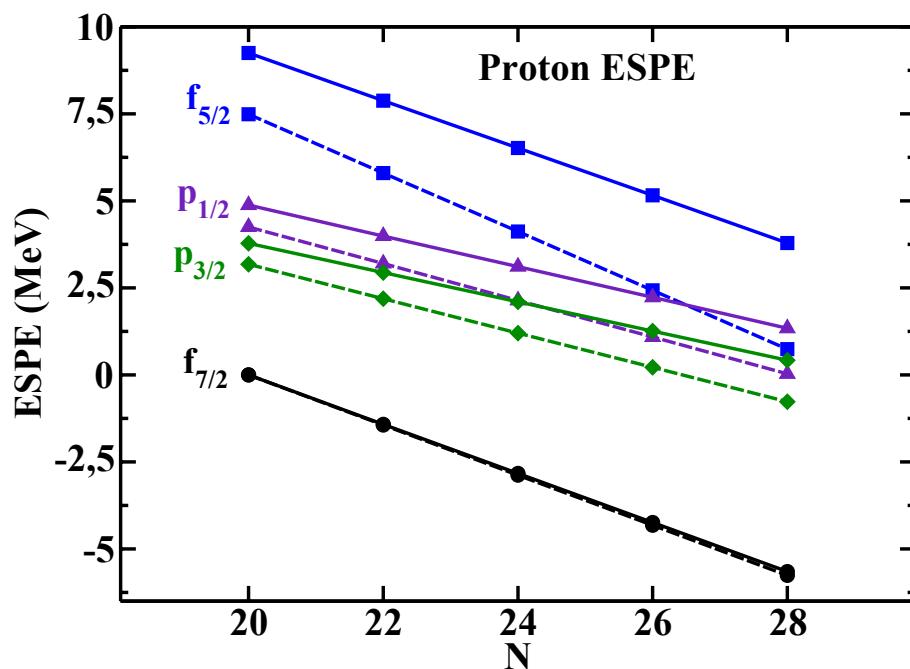
ESPEs relevant for Ca isotopes

- Very small difference from 2NF to 2NF + 3NF.



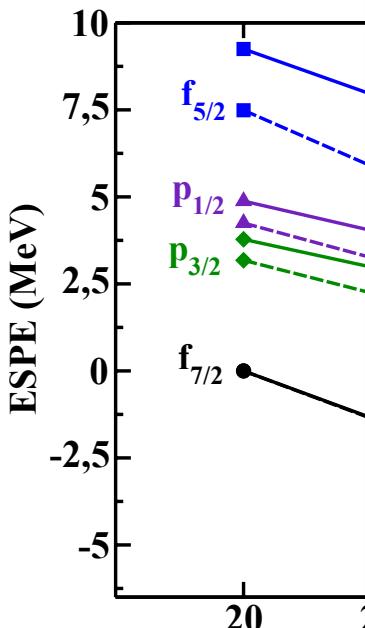
ESPEs relevant for Ni isotopes

- ⌚ Drastic evolution of ESPEs due to 3NF.



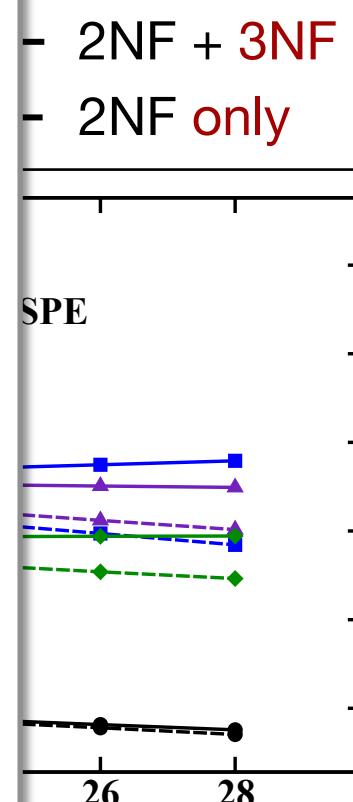
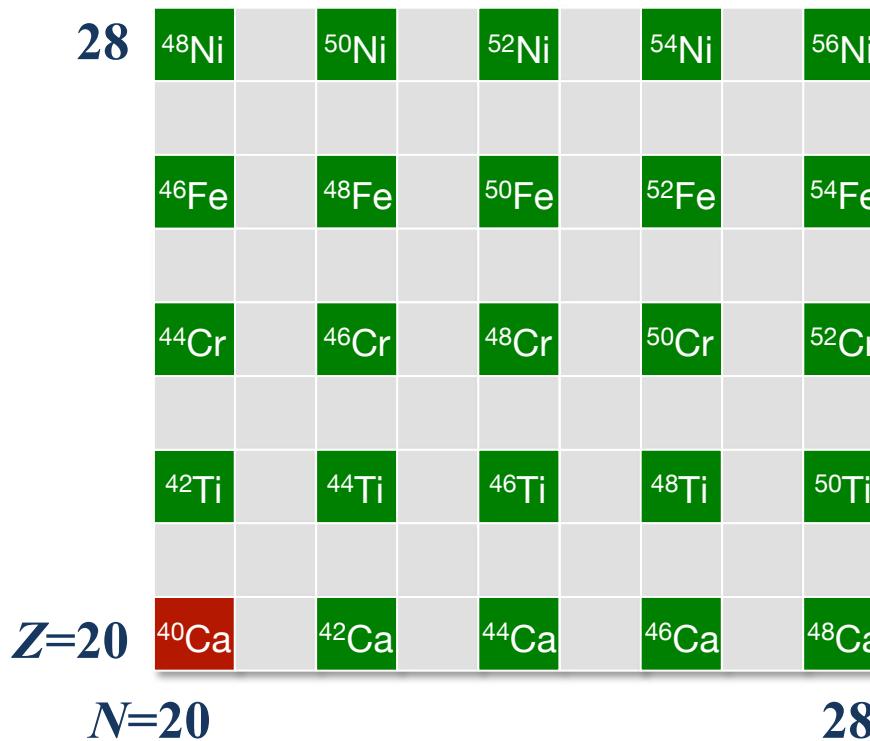
ESPEs relevant

- Drastic evolution



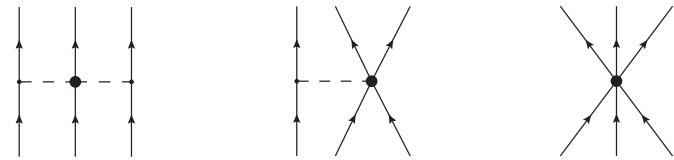
Similar results for other *fp*-shell nuclei.

→ Well described
 { monopole component
 { ESPE
 { shell closure



Summary

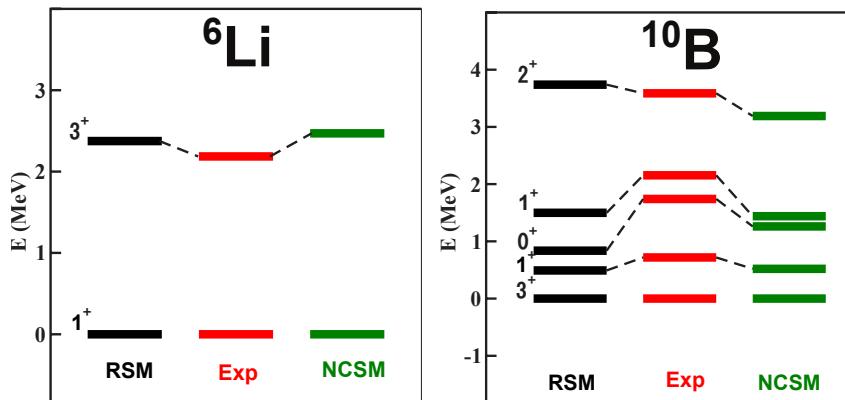
Chiral N²LO 3NF for shell model



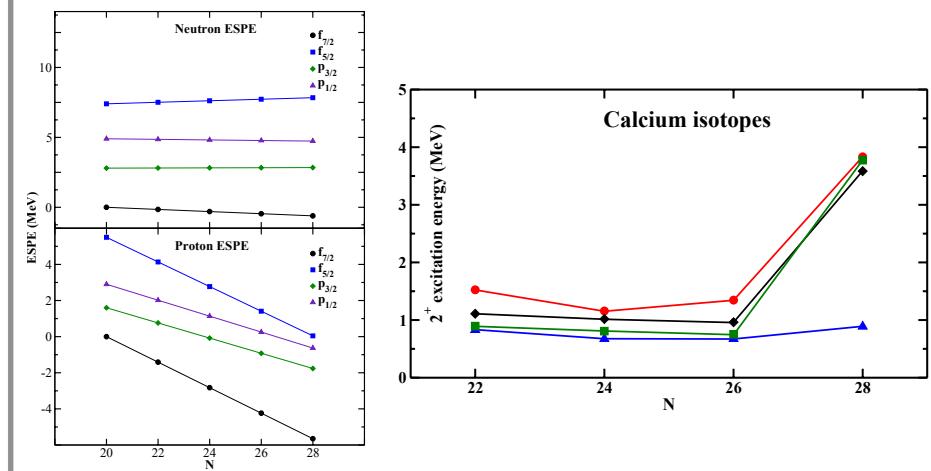
3-body MEs with nonlocal regulator

$$A \left\langle \left[\left[\bullet \bullet \right] \bullet \right]_{JT} \middle| V_{3N} \left| \left[\left[\bullet \bullet \right] \bullet \right]_{JT} \right\rangle_A$$

Benchmark test for *p*-shell



Monopole properties of *fp*-shell



→ Our RSM calculations with 3NF are satisfactorily comparable to the *ab initio* results.

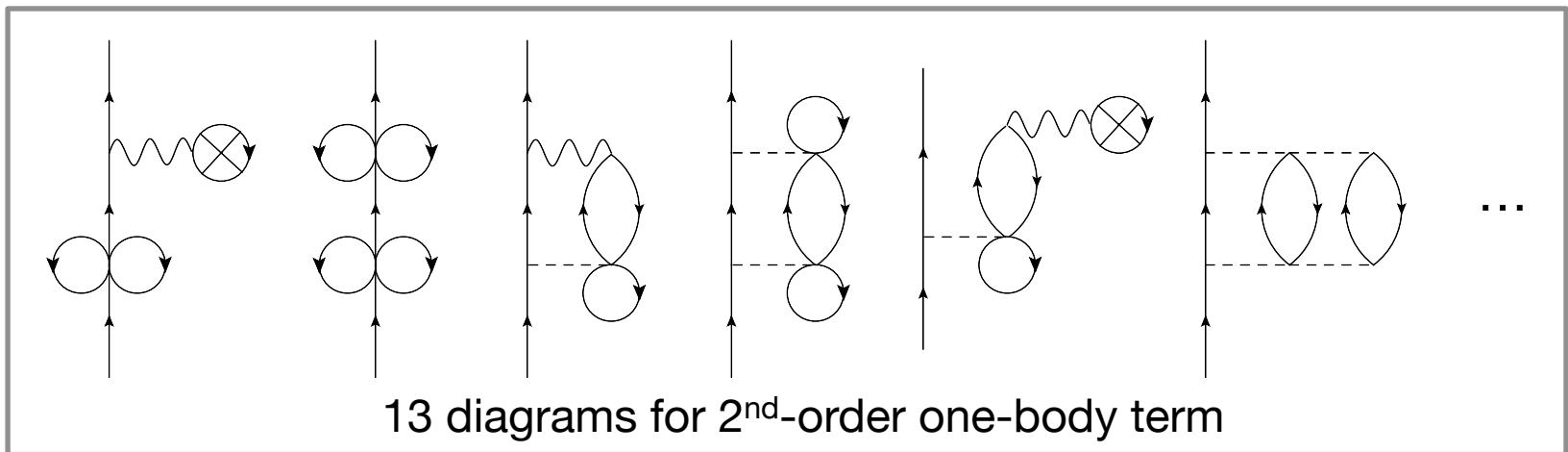
T. Fukui *et al.*, Phys. Rev. C 98, 04430 (2018).

→ The 3NF-induced monopole Hamiltonian is essential to explain the measured shell evolution.

Y. Z. Ma *et al.*, arXiv:1812.03284.

Extension of theoretical framework

- ◎ Contribution of the $g_{9/2}$ orbit
- ◎ Beyond 1st-order contribution



Study of neutrinoless double β decay

RSM calculations with chiral 3NF
for ^{76}Ge , ^{82}Se , ^{130}Te , ^{136}Xe , etc.

3NF in open quantum system

Gamow shell model calculations with chiral 3NF.